A growth model for a two-sector economy with endogenous productivity

Codrina Rada

Abstract

A growth model is developed for an open dual economy. The economy expands due to a higher growth rate of labour productivity in the modern sector through the Kaldor-Verdoorn channel and higher effective demand through a Keynesian channel. The model incorporates a retardation mechanism affecting the slopes of productivity and output growth schedules as labour surplus and economies of scale diminish. A wage or profit-led regime and initial conditions may give rise to: de-industrialization in terms of both output and employment; a growth trap sustaining a situation of structural heterogeneity; or sustainable employment and adequate output and productivity growth.

JEL Classification: O11; O41; O47

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Codrina Rada

The present paper discusses dynamic structural change, employment and growth in a dual economy with abundant labour surplus, thereby addressing the reality that exists in many of today’s developing countries. The premise is both old and new. It is old in the sense that economic theory—as it appears in the works of classical economists (Lewis, 1963; 1979), neo-classical economists (Jorgenson, 1961; 1967) or Keynesians (Kaldor, 1957; 1978)—has long recognized the importance of intersectoral dynamics for growth as well as the interaction between sectors and the overall economy. The premise is new (or rather “renewed”) as a result of the structural changes that have recently been taking place in developing economies, and more specifically in some of the fast-growing South and East Asian economies.

We will elaborate more on the ideas behind this paper below, but first the reader should have an understanding of what is meant by structural change. Following Pieper (2001), “economic structure is defined as the sectoral composition of output, employment and labor productivity and its evolution over time”. Several empirical studies (Cripps and Tarling, 1973; Pieper, 2001) have gathered evidence that shows a significant and positive association between changes in the employment and productivity in the industrial (or modern) sector and overall macroeconomic performance. These stylized facts, undisputed at the theoretical level, prompt us to explore the dynamics associated with sectoral composition and sectoral changes in output and employment in order to understand different patterns of growth and development.

Theoretical considerations

One of the central questions raised repeatedly in development economics concerns the mechanisms through which an economy can grow and at the same time lead to a more productive use of underutilized resources, in particular an underutilized labour force. This is another way of saying that development economics is about identifying structural changes that lead to higher growth rates while simultaneously contributing to a decline in the numbers of underemployed and unemployed. An overall improvement in economic and social sustainability will take place only if both of these outcomes are attained. Concerns of this nature are directly relevant to the case of developing countries, where a large portion of the labour force is stuck in a slow-growing or stagnant informal or subsistence sector.

The interest of development economics in the analysis of structural changes has emerged from a generally accepted theoretical insight as well as from stylized facts (known as Kaldor’s “first” and “third” laws (Thirlwall, 1983)), which contend that the more rapidly the high-productivity sectors expand—i.e., the manufacturing or the capitalist sector—and the more rapidly labour transfers from low- to high-productivity sectors, the faster the economy will grow. The preoccupation with the capitalist sector is due to its higher productivity growth, which results from increasing returns to scale (a fact observed long ago by Adam Smith, and later by Young (1928)) and gains from innovations and learning-by-doing. It follows, therefore, that the capitalist sector contributes dynamically to overall output and productivity growth (Pieper, 2001).

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1 I am grateful to Lance Taylor and Duncan Foley whose comments and guidance are invaluable, as well as to Willi Semmler for his useful suggestions. I would also like to thank José Antonio Ocampo who suggested this line of research.

2 Throughout the paper the terms “subsistence”, “non-tradable”, “informal” and the label “N” are used interchangeably to refer to the low-productivity sector; the terms “modern”, “tradable”, “capitalist” and the label “T” refer to the high-productivity sector.
In the realm of theory, classic economists argue that in an economy “employment expands in a capitalist sector as capital formation occurs” (Lewis, 1963). They further claim that capital formation will take place at the rate allowed by available savings. Finally, classic economics concludes that the solution to an economy’s slow growth and underemployment lies in a successful increase of savings and, therefore, investment rates.

Kaldor (1957) and other Keynesians (Thirlwall, 1986; Dutt, 1992) agree that the modern sector indeed leads growth and job creation but conclude that the “combustible” for growth is represented by two “fundamental sources of autonomous demand”: exports and investment. Keynesians see the problem not in the inability of an economy to increase the savings share or the scarcity of capital but rather in the available demand which they consider to be the driving force in the growth process. This paper builds mostly on the latter premise.

One stylized fact evident in developing countries is the duality of the economic system. An established modern or capitalist sector exists alongside a subsistence sector and offers, to use Lewis’s classification, “good” and “bad” jobs, respectively. In line with much of the literature on labour markets, we assume that workers always choose the better-paid jobs available in the capitalist sector. This means that whatever drives labour demand in the capitalist sector determines employment in the subsistence sector as well.

From a growth-accounting perspective, a change in employment is equal to the difference between the rates of growth of output and labour productivity. If employment is not assumed to be predetermined, the analysis can be reduced to finding those forces that act upon output and productivity. Along Keynesian lines of thought, it has been postulated that investment and exports are main sources of output growth. In addition, the economic literature backed by empirical evidence is confident that a productivity increase causes higher output. The reverse, known as the Kaldor-Verdoorn (KV) relationship, has been less explored and allows instead for a positive feedback loop from output to productivity growth. Productivity is therefore endogenous, as in the new growth theory; in this case, though, it is determined from a demand-side perspective. What is important to observe, however, is the reciprocal causality between output and productivity growth.

One scenario might be as follows: higher investment leads to an increase in the growth rate of output and, subsequently, productivity (this based on the KV relationship). Employment may or may not follow the same upward trend, depending on whether output grows faster than productivity. For the sake of argument, let us assume that output expansion is greater than that of productivity, a condition which will draw in labour. A transfer of labour from the low-wage, low-productivity subsistence sector to the high(er)-wage, high-productivity modern sector has an impact on growth through two main channels: first, by means of a more productive use of labour; and, second, via the effective demand.

Another scenario might start with a rise in the level of exports, for example, which are set to shift the aggregate demand schedule outwards and thereby produce an increase in productivity growth rate via the KV relationship. The rest of the scenario unfolds as described above. Alternatively, the model we develop here may describe the situation of labour-shedding in the capitalist sector. Two main causes could lead to such an outcome: an increase in productivity at a rate faster than that of output, or a slowdown in output growth.

3 Where “good” and “bad” jobs are differentiated based on the money-wage earned.
The creative component of the capitalist sector does not always fit comfortably within an economy or a society. Technological advancement meant to provide smarter and cheaper ways to produce goods at times causes an initial displacement of employment from the capitalist to the subsistence sector. Such a displacement of workers from good to bad jobs has obvious negative implications in the short run; however, economic theory and history shows how Schumpeterian “creative destruction” prevails in the long run inasmuch as enough employment is created through the expansion of markets and products. In other words, the economy may face a loss-win situation for employment productivity in the short run but a win-win situation in longer run.

However, if the loss of employment is the result of an exogenous shock, such as a sudden collapse in external demand, the negative long-term impact on economic performance of developing economies can be significant: the displaced workers end up in the low-productivity subsistence sector, being either underemployed or unemployed; the overall productivity follows a downward trend; the loss of good jobs also acts along Keynesian lines by reducing the effective demand and causing a further decline in output;\(^4\) and, finally, lower output growth spreads into falling rates of productivity growth.

Such a vicious cycle that harms long-run growth seems to have been a characteristic of many Latin American economies since the 1980s and has been a reality for sub-Saharan Africa for most of the period since the slowdown in growth in the late 1970s. Empirical studies have also shown that developing countries are much more prone to volatility and instability (Pritchett, 2000; Ocampo and Parra, 2005), with exogenous shocks having a deeper and longer impact on economic performance (Easterly and others, 1993; Morley and Vos, 2004). It is therefore important to understand how an initial negative displacement can trigger a chain of back-to-back effects on demand, productivity and employment growth, which in turn can determine either a slow recovery or a prolonged stagnation of economic growth.

**Stylized facts**

This section attempts to answer questions regarding the novelty of the thinking behind the present paper. The answer is revealed by exploring a few simple stylized facts about employment and productivity trends in developing countries. First, the reader’s interest and attention is drawn to excerpts from the 2004/5 report of the International Labour Organization on *Global Employment Trends* (ILO, 2005) and the 2005 Asian Development Bank report on *Key Indicators* for Asian economies (ADB, 2005). These sources show that “out of a total labour force of 1.7 billion in the DMCs,\(^5\) around 500 million are underutilized in terms of being either unemployed or underemployed…” (ADB, 2005);\(^6\) “During the 1990s, own-account and family workers” represented nearly two-thirds of the total non-agricultural labour force in Africa, half in South Asia, a third in the Middle East…” (ILO, 2005); and “In Latin America the urban informal economy was the primary job generator during the 1990s....urban informal employment in Africa was estimated to absorb about 60 per cent of the urban labour force and generate more than 93 per cent of all new jobs in the region in the 1990s” (ILO, 2005).

In addition, the two reports gather evidence which, according to expectations, shows that there is an inverse relationship between an increase in formal-sector employment and poverty. A transfer of labour to better pay is instrumental for a decline in poverty rates around the world, which remain high despite global economic growth. As Fields (2004) points out, the “poor are poor because they earn little…\(^4\)

---

4 Unless the decline in employment leads to a lower wage share, which would stimulate investment to compensate sufficiently for the initial decline in effective demand of wage earners.

5 Developing member countries of the Asian Development Bank.

6 Where Asia’s labour force of 1.7 billion accounts for about 57.3 per cent of the world’s total (ADB, 2005).

7 Two categories which account for a broad definition of underemployment (ILO, 2005)
from the work they do”. The ramifications of poverty on the quality of human capital are well known and history has taught us that an educated labour force is a necessary, albeit insufficient, ingredient for growth. It follows that development and growth are processes that are rooted in a circular causality that builds upon its own outcomes.

The empirical evidence also provides a disturbing aspect of the recent experience of developing countries: contrary to expectations, higher economic growth rates and a diminishing informal sector employment have generally not been observed.

Evidence in support of a positive relationship between output and formal employment growth can be found in some rapidly growing Asian countries such as the Republic of Korea (South Korea) or Taiwan Province of China, while other countries, although experiencing substantial growth, have failed to narrow the share of their informal sector in employment. Among these are India and Viet Nam (see ADB, 2005; and Amin, 2002). As a result of these rather discouraging developments of the last decade and a half, the policy agenda in developing countries has become mindful of the reality of jobless growth. It is of even greater concern since in the long run growth without expansion of employment in the fast-growing sectors is unsustainable.

The stylized facts just discussed underline not only the abundance of labour surplus but also its upward trend in most of the developing countries during the last decade, even at times when growth has accelerated. In addition, this situation points to the failure of these economies to reach the potential growth otherwise attainable in the presence of job creation. In this vein, ILO (2005) recommends that “in order to harness the development potential of structural changes, however, developing countries, in particular, must focus on a two-pronged strategy of improving the productivity of workers in dynamic niche industries and, at the same time, focusing on those sectors of the economy where the majority of labour is concentrated. This focus would give them the tools to move from low- to high-productivity activities”.

Rada and Taylor (2006a) present some figures on employment growth and structural change as well as on the contribution of sectoral productivity to overall growth for several regions in Asia, based on the decomposition techniques laid out in Berg and Taylor (2001) and Taylor (2004). The analysis indicates that, in line with Kaldor’s stylized facts, the productivity growth in the manufacturing sector drives the overall productivity growth. The economic tigers’ region has been doing relatively well for the last three decades in terms of productivity growth as well as job creation in the context of substantial structural change, whereas the results for the rest of the regions are ambiguous. South Asia, and in particular India, has experienced strong tendencies towards jobless growth which has driven much of the increase in informal-sector employment despite a decreasing agricultural share in total output.

If the experience of some of the Asian countries in terms of negative employment growth is rather recent, countries in other continents have been experiencing the burden of employment and productivity slowdown for much longer: sub-Saharan African has been in a “lose-lose” situation with falling employment and productivity (Pieper, 2001); following the macro shocks of the mid-1980s, Latin America has not yet managed to recover its growth rates fully with regard to both productivity and employment, with periods of stagnation alternating with downfalls in output.

8 The quotation by Fields is taken from ADB (2005).
9 Here, the paper includes South Korea, Malaysia, Singapore and Taiwan Province.
When comparing the few successful cases with most of the developing countries, the immediate question is why some countries do better than others? The present paper attempts to answer this question from the point of view of dynamic structural changes that take place in terms of employment, output and productivity. We start with a simple model in level terms based on a two-sector open economy Social Accounting Matrix (SAM) and solve it for short-term equilibrium. We then proceed by differentiating the model in order to analyse the short-run adjustment process. Finally, we develop the complete model by incorporating the KV relationship and the effective demand factor within the context of a retardation mechanism. The latter is introduced as a feature of the long-term development process of an economy as observed by Gerschenkron (1962). The complete model allows us to understand how the forces described above may or may not lead to higher and sustainable economic growth.

The accounting of the model

The present paper studies a two-sector, two-commodity open economy that functions according to the SAM presented in Table 1:

<table>
<thead>
<tr>
<th>Costs</th>
<th>Use of income</th>
<th>Accumulation</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>N</td>
<td>Y_{WT}</td>
<td>Y_II</td>
</tr>
<tr>
<td>NT</td>
<td>Subsistence</td>
<td>PTCT_{WT}</td>
<td>PTCT_{II}</td>
</tr>
<tr>
<td>NT</td>
<td>Subsistence</td>
<td>PTCT_{WT}</td>
<td>PTCT_{II}</td>
</tr>
<tr>
<td>Income Labor (T)</td>
<td>w_T b_T X_T</td>
<td>P_T X_T</td>
<td>Y_{WT}</td>
</tr>
<tr>
<td>Profit (T)</td>
<td>\pi v_T X_T</td>
<td>P_T X_T</td>
<td>Y_II</td>
</tr>
<tr>
<td>Labour (NT)</td>
<td>w_N (L - b_T X_T)</td>
<td>P_N C_N \times T</td>
<td>Y_{WN}</td>
</tr>
<tr>
<td>Foreign</td>
<td>\beta v_T s_A X_T</td>
<td>s_A T</td>
<td>Y_F</td>
</tr>
<tr>
<td>TOTAL</td>
<td>P_T X_T</td>
<td>P_N X_N</td>
<td>Y_{WT}</td>
</tr>
</tbody>
</table>

Each sector produces its own distinct good. Workers, regardless of the sector, consume both products, which are not perfect substitutes. The tradable or modern sector employs labour and capital and it imports part of the inputs. Within the modern sector, there are two classes: the capitalists, who own the capital, conduct investment, consume the tradable good only and save; and the workers, who earn a wage which they spend entirely on consumption of both sectors’ products. This assumption is in line with the classical view that workers do not save or, if they do, the saving is at a level that can be ignored. Finally, the tradable good can be consumed, invested or exported.

In the subsistence sector, there is only one factor of production: labour, which earns a wage. Subsistence sector performance is determined by a productivity level equivalent to the product wage, which can be increased either by consuming more of the tradable sector’s good—using the efficiency
wage argument—or as a result of lower employment in the sector (see below for details). Finally, there is also the foreign sector, which supplies intermediate inputs used in the production of the tradable good.

**The modern sector**

In the case of the modern sector, we follow the standard practice of setting up the price in the tradable sector as a function of the wage, \( w_T \), labour-output ratio, \( b_T = \frac{L_T}{X_T} \), a constant markup, \( \tau \), and the cost of imported intermediate inputs, \( eP^*a \), where \( e \) is the exchange rate, \( P^* \) is the price in terms of foreign currency and \( a \) is the share of imported inputs in relation to total inputs:

\[
P_T = (1 + \tau)(w_Tb_T + eP^*a)
\]

which can also be written in terms of the profit share \( \pi \) as:

\[
P_T = \frac{(w_Tb_T + eP^*a)}{1 - \pi}
\]

In level terms, labour productivity is simply equal to the inverse of the labour-to-output share, which can further be written as:

\[
\varepsilon_{LT} = \frac{1}{b_T}
\]

Given the rule (2.1.1), price levels incorporate any fluctuations of costs, while responding negatively to labour productivity. With productivity pro-cyclical during the upswing, the price level declines.

Workers in the modern sector earn an institutionally set wage, \( w_T \), and consume according to:

\[
Y_w = (P_Tc^{T\_w} + P_Nc^{N\_w})L_T = w(tb_T X_T)
\]

where \( Y_w \) is the wage bill, \( P_T \) and \( P_N \) are prices of tradable and non-tradable goods and \( c^{T\_w}, c^{N\_w} \) are consumption levels of the two products by a worker in the modern sector.

Capital owners save part of their income and consume the rest:

\[
Y_\pi = sY_s + (1 - s)Y_s = \pi P_T X_T
\]

where \( Y_\pi \) is the capital owner’s income, \( s = \frac{S}{Y_\pi} \) is the saving propensity, \( S_\pi \) is the saving from profits and \( (1 - s) = c = \frac{C^{T\_x}}{Y_\pi} \) is the propensity to consume. The output of the modern sector can be calculated either from the demand or production side as:

\[
P_T X_T = P_Tc^{T\_w} + P_Nc^{N\_w} + P_tX_T + P_tI_T + P_tE_T = w_Tb_T X_T + \pi P_T X_T + eP^*aX_T
\]

where upper cases for consumption variables now stand for the sum of all individual consumption levels. Finally, the excess demand in the modern sector is derived from the equality between consumption and income, or uses of income and sources of income after dividing (2.1.5) by \( P_T \):

\[
C^{T\_w} + C^{T\_x} + I_T + E_T - X_T = ED_T
\]

**The subsistence sector**

The subsistence sector produces a non-tradable good, with labour as the only factor of production. We start from a simple accounting identity, \( X_N = X_N \), and multiply the right-hand side by \( L_N/L_N \) to obtain:

\[
X_N = \varepsilon_N L_N
\]
where $\epsilon_N = X_N / L_N$ is labour productivity, $X_N$ is the quantity of output and $L_N$ is the amount of employment in the sector. In other words, subsistence-sector output is determined by available labour and its productivity. Given the price level, $P_N$, the value of total output is:

$$P_N X_N = P_N \epsilon_N L_N$$  \hspace{1cm} (2.2.2)

We will see that the wage level is simply equal to labour productivity, or the quantity produced by an individual worker, multiplied by the price,\textsuperscript{10} or $w_N = P_N \epsilon_N$. Wage in terms of the subsistence good increases only if productivity increases.\textsuperscript{11} In turn, productivity responds positively to the share of employment in modern sector, $\hat{\lambda} = L_T / L$, for reasons which are explained below. A central feature of the model is the assumption that $L_T + L_N = L$, which allows us to account for the entire labour force, employed and underemployed.\textsuperscript{12} A transfer of labour from the subsistence to the modern sector or vice versa is accounted for through changes in productivity, wages and, therefore, demand.

The logic can be explained as follows: since the marginal product of labour in the subsistence sector where capital is limited is close to zero (Sen, 1966), a transfer of labour to the modern sector eventually determines a rise in the average product in the subsistence sector since a smaller number of workers are able to obtain the same amount of output as before. If the transfer of labour to the modern sector continues, productivity and consequently real wage continue to grow. The increase of real wage in the subsistence sector may hurt the modern sector’s ability to attract workers from the subsistence sector if the productivity increase is not sufficiently offset by a decrease in prices (Lewis, 1963) to avoid a move of terms of trade vis-à-vis the modern sector.

In the longer run, labour becomes more productive by increasing the consumption of the modern sector good; this can happen as a result of higher wages. We will assume that the more of the tradable good that is consumed, the more productive the labour becomes. This concept is related to the efficiency-wage concept which states that higher wages bring about an improvement in workers’ health, education or training, which in turn positively feed into his/her capacity to work more efficiently. It also compensates for the “lack” of capital in our model. Growth in productivity in the subsistence sector can be understood as a positive function of growth in wage level and a negative function of growth of labour force:

$$\hat{\xi}_{LN} = f(\hat{L}_N, \hat{w}_N)$$  \hspace{1cm} (2.2.3)

where $\hat{\xi}_{LN}$ is labour productivity growth in the non-tradable sector and $\hat{L}_N$ and $\hat{w}_N$ are employment growth and wage growth, respectively. Given (2.2.3), the expression for output growth acquires an intrinsic dynamic. Later in the paper, we will use a linear function to approximate (2.2.3).

The functioning of the subsistence sector’s market presented thus far and the implicit adjustment mechanism is a medium- or long-run process. In the short-run, market clearing and the overall equilibrium takes place through adjustments in the price level as presented in detail in the next section.

Similar to the modern sector, output can be expressed either in terms of cost of production or as the sum of demand by workers in both the subsistence and the modern sector according to:

$$P_N X_N = P_N C^N w_T + P_N C^N w_N = w_N (L - b_T X_T)$$  \hspace{1cm} (2.2.4)

\textsuperscript{10} Price in the modern sector is determined via the standard markup relation and is a function of movements in wage and productivity (assuming a constant markup). In the subsistence sector, on the other hand, markup is assumed to be zero and it is the wage level that is derived given price level and productivity.

\textsuperscript{11} We also note that the real wage is none other than productivity.

\textsuperscript{12} The model does not distinguish between unemployed and underemployed.
where $C_w^N, C_{wN}^N$ are consumption levels of the subsistence sector good by the workers in the modern sector and subsistence sector, respectively.

**The foreign sector**

The foreign sector trades with our economy, consuming $P_T E_T$ and supplying its own good:

$$eP^* aX_T = Y_F$$

where $a$ is the share of imported inputs in total output, $e$ is the exchange rate and $Y_F$ is the foreign-sector income. The exported amount, $E_T$, could depend on $eP_T^*/P_T$, with productivity growth helping exports. We can express the difference between the income and the consumption to obtain the foreign sector’s savings as:

$$eP^* aX_T - P_T E_T = S_F$$

**Investment**

The macro balance requires that investment equals saving:

$$S_T + S_F = P_T I_T$$

(2.4.1)

By inserting (2.3.2) into (2.4.1), we obtain:

$$s\pi P_T X_T + eP^* aX_T - P_T E_T - P_T I_T = 0$$

(2.4.2)

Investment is carried out by the capitalists in the tradable sector, where the equality between investment and saving holds according to (2.4.2). One option is to make investment a function of available saving, as in a Solow type of closure, and thereby a function of prices and productivity, given that more saving is available when the economy increases its exports. If the closure is Keynesian (the option chosen for this paper), then investment is an independent function that drives output, $X_T$, along the lines of effective demand.

By dividing (2.4.2) by $P_T$, we obtain:

$$s\pi X_T + eP^* / P_T aX_T - E_T - I_T = 0$$

(2.4.3)

This signifies that real exports and investment must be equal to real domestic saving plus imports.

**Excess demand and short-term equilibrium**

In this section we discuss the short-run market clearing process in the two sectors based on the excess demand functions for each market.

**Workers’ demand functions**

Workers in both sectors consume a minimum required amount of the subsistence sector’s good, $c_0$. The remaining income is divided between the consumption of both sectors’ goods based on constant shares. In order to describe this behaviour analytically, we use a simple linear expenditure system (LES) as first.

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13 The LES model has a number of deficiencies when applied to actual data, as described in Parks (1969); however, it best serves our purpose of parsimoniously modelling consumer behaviour in the subsistence sector.
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laid out in Stone (1954) and formally derived from the utility maximization function below. The rationale behind using an LES is based on the stylized fact that the subsistence sector mainly produces food, a good considered necessary for workers in both sectors. This is not a far stretch from reality: individuals consume the amount they need to survive and if they have anything left over, they can divide it between consumption of goods from both sectors. Assuming that the entire wage is spent, as the wage increases the individual “consumes more of both goods but proportionally more of one good” (Varian, 1992), in our case the modern sector’s good. The derivation of the workers’ demand functions that are used to obtain the excess demand functions as laid out in the next section are presented in the appendix.

Excess demand functions

As noted above, the macro equilibrium condition when excess demands in the two sectors are zero, $ED_N = ED_T = 0$, is given by the saving-investment balance (2.4.3) while the SAM in table 1 balances.

Similar to other fixed-flex price models as first put forth by Hicks (1965) and later incorporated into different models such as those developed by Taylor (1983) and Thirlwall (1986), we assume that in the subsistence sector the price level adjusts in the short term to bring excess demand to zero, whereas in the modern sector, given excess capacity, output, $X_T$, is the variable that clears the market.\(^{14}\)

In the subsistence sector, the excess demand comes from workers in both sectors and is equal to the difference between the aggregate demand and aggregate supply:

$$c^N_{wT}L_T + c^N_{wN}L_N - X_N = ED_N \quad (3.2.1)$$

Using the demand expressions obtained above and introducing them into (3.2.1) after aggregating individual consumption, we obtain:

$$\frac{1}{P_N}(1 - \alpha)w_T b_T X_T + \alpha c_0 b_T X_T - \beta (\epsilon_N - c_0)(L - b_T X_T) - X_N = ED_N \quad (3.2.2)$$

Considering that $w_N = P_N\epsilon_N$, the excess demand for the subsistence good can also be written as follows:

$$\frac{1}{P_N}(1 - \alpha)w_T b_T X_T - \beta (\epsilon_N - c_0)(L - b_T X_T) = ED_N \quad (3.2.3)$$

In solving for the adjustable variable, the price level, $P_N$, when $ED_N = 0$, we obtain:

$$P_N = \frac{(1 - \alpha)w_T b_T X_T}{\beta (\epsilon_N - c_0)(L - b_T X_T) - \alpha c_0 b_T X_T} \quad (3.2.4)$$

In the tradable sector, the excess demand is:

$$\frac{\alpha (w_T - P_N c_0) b_T}{P_T} X_T + \frac{\beta (w_N - P_N c_0)}{P_T} (L - b_T X_T) + \frac{(1 - s)\alpha X_T}{P_T} + E_T + I_T - X_T = ED_T$$

\(^{14}\) Another possibility in the subsistence sector would be for the real wage or productivity to be the market-clearing variable after fixing either the nominal wage or the price level, so that $\epsilon_N = w_N / P_N$. However, we feel that productivity in the subsistence sector is a slow variable owing to the lack of technological input and is therefore unable to adjust fast enough in the short run to clear markets.
In reduced form and solving for output level, $X_T$, we obtain:

$$X_T = \frac{E + I + \beta L(e_N) c_0 P_N / P_T}{1 - (s) \pi - \alpha b_T (w_T - P_N c_0) / P_T + \beta b_T (e_N) c_0 P_N / P_T}$$

### Comparative statics and short-run macroeconomics

At this stage, it is of interest to analyse how the two adjusting variables respond to a disequilibrium following an exogenous shock. This type of exercise may shed light on the economic mechanisms and linkages between different factors, and it becomes more than a theoretical venture in cases where the underlying assumptions are true to the reality of the economies it is claiming to examine. As it is set up, the model attempts to describe a developing economy where excess capacity in the tradable sector would imply an adjustment in quantity, i.e., in output, whereas a competitive market for the non-tradable product would indicate a nominal adjustment, i.e., in price.

To begin with, one may observe in (3.2.3) that excess demand occurs when output in the modern sector expands. The adjusting variable, $P_N$, increases to clear the market following a clockwise shift of the output schedule. The curve for the subsistence sector is therefore positively sloped as shown in figure 1 below.

An exogenous increase in the subsistence sector’s labour productivity causes excess supply, driving down prices and moving the terms of trade in favour of the modern sector as the N-sector schedule shifts downwards.

The analysis of the adjustment taking place in the modern sector is more cumbersome. An increase in the price of the subsistence sector good may or may not determine a rise in $X_T$ in (3.2.6), depending on how strong the Engel effects are. A higher $P_N$ raises the wage in the subsistence sector in accordance with (2.2.3) and therefore contributes to a higher demand for the tradable good coming from the $N$ sector (0K?). As laid out by Taylor (1983), an increase in $P_N$ weakens the demand for the $T$ good that comes from the modern sector itself. If the Engel effects are strong, lower demand from the $T$ sector will be larger than the contribution to demand by the $N$ sector and, as a result, $X_T$ will have to diminish. Conversely, when the Engel effects are weak, a higher $P_N$ will have an expansionary effect on $X_T$.

In figure 1, we illustrate the case where a higher $P_N$ leads to excess demand for the $T$-sector good and therefore to higher output. Graphically, this is shown by a counterclockwise rotation of the $N$-sector schedule which increases $X_T$ and moves the equilibrium from point B to point C.

Now let us see how the markets adjust to re-establish equilibrium following shocks from changes in exogenous variables. A positive shock to demand in the $T$ sector as a result of higher exports shifts the schedule for that sector to the right, as supply is trying to match the increase in demand. At the same time, a higher output level in the modern sector implies higher demand for the subsistence good, which creates an imbalance in the $N$ sector. Excess demand becomes zero only through an increase in $P_N$. Visually, this represents a move of equilibrium from point B to point A in figure 1.
A higher wage in the modern sector, $w_T$, on the other hand triggers excess demand in both sectors. Equilibrium in the subsistence sector is re-established through a higher price level, $P_N$. In the modern sector, the imbalance can be redressed based on a combination of adjustments in price and quantity. Price level, $P_T$, goes up owing to the markup rule. If the increase is not sufficient enough to bring down the excess demand, supply will have to adjust in order to clear the market. It follows that an exogenous shock stemming from an increase in wage level in the modern sector will move the macro equilibrium upwards to a higher combination of prices and output—the optimal case following an increase in the wage level. Depending on the institutional setting (the bargaining power of workers in the modern sector, for example), structural inflation may occur together with a sharp decrease in demand from the $N$ sector, where purchasing power spirals downwards with the increase in the price of the modern sector’s good. In turn, the upswing in the price level cuts into the excess demand in the modern sector by shifting the schedule downwards.

Finally, we have established that higher productivity in the $N$ sector creates excess supply which can be corrected by a decrease in the price level, $P_N$. Since $\epsilon_N$ is present in both functions with a positive influence on $X_T$, the macro adjustment is more complicated. First, a clockwise rotation for the subsistence sector schedule leads to a decrease in $P_N$ and, consequently, in $X_T$, which is depicted in figure 1 as a movement from point C to point B. Lower prices for the subsistence good depress demand for the modern good coming from the $N$ sector but stimulate demand coming from the $T$ sector. In conclusion, there may in fact be excess demand for the modern good resulting from the direct positive impact of $\epsilon_N$ and the indirect impact on the real wage in the modern sector. Hence, the modern sector schedule rotates clockwise, re-establishing the equilibrium at a higher $P_N$ and $X_T$. 

---

**Figure 1: Short-term equilibrium determination in the modern and subsistence sectors**

- $P_N$: Price level in the modern sector
- $X_T$: Output of the modern sector
- $T$ Sector: Graph of the modern sector
- $N$ Sector: Graph of the subsistence sector
- $A$, $B$, $C$: Points on the schedule
A model of growth and employment in a two-sector economy

The core of the model of growth and employment in a two-sector economy lies in the dynamic feedback mechanism that exists between labour productivity and output as put forth by the KV technical progress function. The KV relationship, as mentioned in the introduction, captures the idea of cumulative causation which follows from the interaction between the “structural dynamics and macroeconomic performance [which] can be formalized in terms of a dual link between economic growth and productivity” (Ocampo, 2005).

The channels through which productivity or technical change determine higher output growth are well known to the reader from the standard economic literature and refer mainly to the effects of technical change on return to investment and factor supplies. The reverse positive linkages from output to productivity have been investigated to a lesser extent. We summarize them here as they appear in Ocampo (2005): “… dynamic economies of scale of a microeconomic character, associated with learning and induced innovations; those associated with the exploitation of intra- and intersectoral external economies […]; and the positive links generated by variations in underemployment.” Kaldor (1978) states that “learning is the product of experience— which means, as Arrow [1962] has shown, that productivity tends to grow faster, the faster output expands; it also means that the level of productivity is a function of cumulative output (from the beginning) rather than of the rate of production per unit of time.” The effects of increasing returns to scale, dating back to Adam Smith’s famous pin-factory example and later to Young (1928), emphasize the benefits some sectors may acquire due to the expansion of the industry sector as a whole. The presence of the “right” dynamics between output and productivity in the modern sector makes the transfer of labour surplus to high-productivity sectors possible, which as explained earlier causes a higher overall growth rate and increases intersectoral linkages. Alternatively, the failure of the economy to transfer labour from low- to high-productivity sectors hinders the long-run growth process by establishing “structural heterogeneity”, a term used by the Latin American structuralists to describe the existence of high-productivity enclaves within a sluggish overall economy.

Along these lines, the model allows for an important subsistence sector which provides more than just an elastic labour supply. An understanding of the complementarities between the modern and subsistence sectors goes beyond the question of labour supply or balanced growth, the latter being an analytical tool developed to accommodate parsimonious growth models and debated on both theoretical and factual grounds. At the theoretical level, Canning (1988) argues that “with increasing returns in some sectors, and diminishing returns (due to a fixed factor [land]) in others, balanced equilibrium growth is not possible”. Stylized facts, as discussed above, show that growth is not at all balanced between the two sectors, and a rapidly growing modern sector may exist without many spillovers into the labour-abundant subsistence sector. Finally, the growth experiences of developing countries seem to follow trajectories that start with spurts in the growth rates which are difficult to explain by assuming a smooth, balanced growth (Pritchett, 2000).

Determining productivity, output and employment in the modern sector

One important feature of our model is that it accounts for the entire labour force and does not assume unemployment in the standard sense. Similar to the assumption of full employment, as in the Solow model, the growth rate of overall employment is exogenous and equals the growth rate of population, or $\dot{L} = n$. The concern however is not with the exogenous expansion of the labour force but rather with its distribution among sectors with different productivity levels. In this context, we allow for underemployment, which includes those who cannot find work in the modern sector and end up in the subsistence sector.
From the differentiation of the expression of labour productivity, the rate of employment growth in the tradable sector is derived as \( \dot{L}_T = \dot{X}_T - \dot{\xi}_LT \), where \( \dot{\xi}_LT \) is the growth rate of labour productivity.

According to the KV technical progress function, the growth rate of labour productivity in the modern sector is linear in output growth:

\[
\dot{\xi}_LT = \xi LT + \gamma_0 \dot{X}_T
\]  

(4.1.1)

where the productivity trend term, \( \xi LT \), may respond to human capital growth, industrial policy, technological advancement or international openness. Using the KV function from (4.1.1) in the expression for the growth rate of employment, we find that employment in the modern sector depends on the slope of the KV schedule and the growth rate of output in the sector as well as on the initial or incoming growth rate of labour productivity according to:

\[
\dot{L}_T = (1 - \gamma_0) \dot{X}_T - \xi LT
\]

(4.1.2)

There is labour shedding in the sector if \( (1 - \gamma_0) \dot{X}_T < \xi LT \), meaning that either the economy has a high incoming growth rate of labour productivity and/or a high KV coefficient in the context of insufficient effective demand. A simple numerical exercise shows that if \( \xi LT = .02 \) and \( \gamma_0 = .4 \), given a labour force growth rate of 2 per cent, the output in the modern sector must be somewhere in range of 7 per cent in order to have labour transfer from the low-productivity subsistence sector to the high-productivity modern sector. Is such a rate of growth viable for a developing country? Although the growth performances of countries such as those in South-East Asia, or China and India at times, show that achieving such a growth rate is possible, for the large majority of developing countries a sustainable growth of 7 per cent per year remains unattainable. Furthermore, empirical evidence shows that for some of the rapidly growing countries it is not always true that growth rates in the range of 7 per cent (consider India, for example) are enough to generate employment if dynamic linkages among sectors are missing.

Let us take on the challenge of decomposing the macro picture along KV and Keynesian lines of thought in order to understand what it takes to spark an interaction between output and productivity growth that also provides a transfer of labour to better-paying, high-productivity sectors.

To extract the details, we solve for the output level in the tradable sector as a positive function of demand stemming from investment, exports and as a negative function of savings from profits based on the macro equilibrium condition (2.4.3) and obtain:

\[
X_T = (E_T + I_T) / (s\pi + eP^* / P_T a)
\]

(4.1.3)

The total differentiation of (4.1.3) in respect of exports, investment, the savings and the exchange rates renders the following expression for output growth:

\[
\dot{X}_T = (1 - \mu_1) \dot{E}_T + \mu_1 \dot{I}_T + \mu_2 \sigma [\dot{\omega}_T - \dot{\xi}_LT] - (1 - \mu_2) \dot{\theta}
\]

(4.1.4)

where \( \mu_1 = l / (l + E) \) and \( \mu_2 = s\pi / (s\pi + eP^* / Pa) \). Expression (4.1.4) is derived by assuming that the growth rate of savings from profits is negatively related to the wage share, \( \dot{s} = -\sigma \dot{\psi} \), where \( \dot{\psi} = \dot{\omega}_T - \dot{\xi}_LT \) is the growth rate of wage share and \( c \) is the marginal effect of an increase in wage share on saving rate.

Along Keynesian lines, output is determined by the aggregate demand, similar to the specification used in Rada and Taylor (2006b), from whom we borrow the investment and export behavioural functions.
Let us assume that the effective demand stemming from investment responds positively to faster output growth and negatively to wage share:

\[ \hat{I} = \hat{I}_0 + \phi_X \hat{X} - \phi_V \hat{\psi}_{LT} \quad (4.1.5) \]

where \( \hat{I}_0 \) is a trend rate of growth of investment demand and \( \psi_X, \phi_V \) are the effects of output growth and wage share growth, respectively, on the rate of investment.

Export sales on the other hand are crowded out by higher domestic demand and respond negatively to a higher wage share, but are stimulated by an increase in competitiveness due to a depreciation of the exchange rate:

\[ \hat{E} = -\theta_X \hat{X}_T - \theta_V \hat{\psi} + \theta_e \hat{e} \quad (4.1.6) \]

Solving for \( \hat{X}_T \) using the behavioural functions for the growth rates of exports and investment, we obtain the following expression:

\[
\hat{X}_T = \frac{\mu_1}{1 - \mu_0 \theta_X + (1 - \mu_1) \theta_X} + \frac{(1 - \mu_1) \theta_V + \mu_1 \phi_V - \mu_2 \sigma \hat{e}}{1 - \mu_0 \theta_X + (1 - \mu_1) \theta_X} \left[ \hat{X}_{LT} - \hat{\omega}_T \right] + \frac{(1 - \mu_1) \theta_e - (1 - \mu_2) \hat{e}}{1 - \mu_0 \theta_X + (1 - \mu_1) \theta_X}
\]

or

\[
\hat{X} = \chi_1 \hat{I}_0 + \chi_2 [\hat{X}_{LT} - \hat{\omega}_T] + \chi_3 \hat{e}
\]

where \( \chi_1 = \frac{\mu_1}{1 - \mu_0 \theta_X + (1 - \mu_1) \theta_X}, \quad \chi_2 = \frac{(1 - \mu_1) \theta_V + \mu_1 \phi_V - \mu_2 \sigma}{1 - \mu_0 \theta_X + (1 - \mu_1) \theta_X}, \quad \chi_3 = \frac{(1 - \mu_1) \theta_e - (1 - \mu_2)}{1 - \mu_0 \theta_X + (1 - \mu_1) \theta_X} \)

The sign of \( \chi_1 \) is positive, in line with the expectations that an increasing rate of investment leads to higher output growth. Investment provides a strong stimulus to output when the denominator is small, which further implies that the accelerator, \( \psi_X \), is large and that the exports are not excessively crowded out (low \( \theta_X \)).

The significance of the wage share (or productivity and wage) on output growth is indicated by \( \chi_2 \) and depends (apart from the above-mentioned impacts on the denominator) on how strongly investment and exports respond to a higher wage share. If the growth rate of the wage share has a weak impact on investment (low \( \phi_V \)) and exports (low \( \theta_V \)), coupled with a high savings propensity from profits (\( \sigma \)), \( \chi_2 \) could be either negative or positive, but in any case will be very small. The foregoing is the case with respect to wage-led demand growth. Conversely, in the case of a profit-led economy, when the wage share has a substantial negative impact on both investment and export growth and the denominator is small, \( \chi_2 \) will be positive and greater than 1. The implications of wage-led or profit-led cases for our model are presented in detail in the next section. A discussion and comprehensive model on the wage-led/profit-led distinction for an open economy can also be found in Foley and Michl (1999) and Foley and Taylor (2004).

Finally, we analyse the effects of exchange-rate depreciation on output. As explored in the economic literature (Krugman and Taylor, 1978) an exchange-rate devaluation does not always have an expansionary effect. In (4.1.7), \( \chi_3 \) can be either positive or negative depending on how strongly the devaluation of the exchange rate stimulates exports relative to the increase in the costs of imported inputs and income redistribution effects. If domestic production relies strongly on imported inputs (as is the case in many developing countries) a higher exchange rate raises the costs of imported inputs, thereby cutting into the profitability and, consequently, the output growth. An exchange-rate depreciation also acts on the
income distribution through a decrease in the level of real income. If the demand elasticity of imports is low, a devaluation does not lead to lower levels of imported inputs and therefore a contraction in output may take place. Lower profitability and pressure from costs as a result of an increase in the exchange rate translate into labour shedding from the tradable to the non-tradable sector.

Using (4.1.1), (4.1.2) and (4.1.7), we solve the system for the growth rate of output, labour productivity and employment simultaneously:

\[
\hat{X}_T = \frac{1}{1-\gamma_0\chi_2} [\chi_2\hat{z}_{LT} + \chi_1\hat{r}_0 + \chi_3\hat{e} - \chi_2\omega_T] \\
\hat{\xi}_{LT} = \frac{1}{1-\gamma_0\chi_2} [\tilde{z}_{LT} + \gamma_0(\chi_1\hat{r}_0 + \chi_3\hat{e} - \chi_2\omega_T)] \\
\hat{L}_T = \frac{1}{1-\gamma_0\chi_2} [(1-\gamma_0)(\chi_1\hat{r}_0 - \chi_2\omega_T + \chi_3\hat{e}) - (1-\chi_2)\tilde{z}_{LT}] 
\]  

(4.1.8)  
(4.1.9)  
(4.1.10)

Similar to Rada and Taylor (2006b) the joint determination of output, employment and productivity based on (4.1.8)-(4.1.10) is presented in figures 2a and 2b. Employment is the endogenous variable here and is determined at the intersection of the KV and output schedules. The employment growth contours depict the trajectory along which \( \hat{L}_T \) is constant. As the equilibrium point shifts to the right or downwards, employment expands at a faster rate.

If the slope of \( \hat{X}_T \) is smaller than \( 45^0 \), as in figure 2a, an upward shift of the KV schedule raises employment growth\(^{15}\) as the new equilibrium point is situated on a lower employment contour line, or to the right. When \( \hat{X}_T \) intersects the \( 45^0 \) line from below, as in figure 2b, an increase in productivity growth takes place at the expense of employment as the new equilibrium is now placed on a higher employment growth contour. A final observation concerns the stability of the system: it is rendered unstable when the KV schedule intersects the output growth schedule from below.

Figure 2a: Determination of productivity, output and employment in the modern sector (when the slope is smaller than \( 45^0 \))  
Figure 2b: Determination of productivity, output and employment in the modern sector (when the slope is larger than \( 45^0 \))

\(^{15}\) Or it slows down the decrease of employment, depending on whether the equilibrium point is above or below the \( 45^0 \) line through the origin.
Let us see now how shifts in the exogenous variables affect the growth rates of employment, output and productivity. That the trend in investment acts positively on all three variables is straightforward to establish.

A rise in the incoming growth rate of labour productivity $\xi_{LT}$ is expansionary in terms of both output and productivity growth but affects employment growth differently subject to the value of $\chi$. Employment growth slows down in the case of a wage-led economy when $1 - \chi > 0$, whereas in a profit-led economy, which is equivalent to $\chi > 1$, job creation takes place, as output expands at a faster rate than productivity.

Similarly, a steep slope given by $\gamma_0$ in the KV relationship leads to a lower rate of employment growth since it is true that $\frac{\partial L_T}{\partial \gamma_0 < 0}$. The KV slope is flatter when, as according to Ocampo (2005) “(i) both micro- and mesoeconomic economies of scale are not too strong; (ii) labour underemployment is moderate; and (iii) fixed factors are not very important in the long run”. Finally, exchange-rate depreciation affects all three growth rates in the same manner, by stimulating them when $\chi > 0$ or by causing them to decrease when $\chi < 0$.

**Determining productivity, output and employment in the subsistence sector**

Wages and output in the subsistence sector are endogenously determined by changes in prices and productivity. To remind our reader, wages equal productivity, or the quantity of goods produced by the worker, multiplied by the price determined on the market, in keeping with $w_N = P_N \varepsilon_N$. The value of output is obtained simply as the wage each worker earns multiplied by total number of workers, or $P_N \lambda = a N L_N$.

By differentiating these expressions, we obtain the growth rates for the subsistence sector’s wage and output:

$$\dot{w}_N = \dot{P}_N + \xi_{LN}$$

and

$$\dot{X}_N = \xi_{LN} + \dot{L}_N = \xi_{LN} + \frac{n - \dot{L}_T}{1 - \lambda}$$

The subsistence sector’s output expands at a faster rate when productivity and employment grow at a more rapid pace, as seen in (4.2.2). The growth of labour productivity on the other hand is restricted by the fact that land (or any other factor of production used in the informal sector) is considered to be fixed, while technological improvement takes place only over the long term. As a result, the marginal product of labour is insignificant when there is abundant labour surplus. Sen (1966) defines “surplus labor as that part of the labor force in this peasant economy that can be removed without reducing the total amount of output produced, even when the amount of other factors is not changed”. This is analogous with the concept of diminishing returns to scale. A transfer of labour from the subsistence sector does not have any effect on total output when a labour surplus exists; however, it does cause a rise in the average product of labour (or labour productivity) and therefore in the real wage. Overall, the volume of output\(^\text{16}\)

\(^{16}\)Higher productivity, independent of the transfer of labour, negatively affects prices in the non-tradable sector in the short term. Depending on the trade-off between a price decrease and a productivity increase, the value of $\dot{X}_N$ can be positive or negative.
remains constant if productivity simply responds to lower labour inputs. Output changes only when the growth rate of productivity increases following, for example, a higher intake of the modern sector’s good, which as already mentioned can be described as a type of “investment”, while the wage can be described as an efficiency wage.\(^{17}\) Combining these ideas, labour productivity growth in the subsistence sector responds negatively to employment and positively to consumption or wage growth:

\[ \xi_{LN} = \bar{\xi}_{LN} - \gamma_1 \hat{L}_N + \gamma_2 \dot{\theta}_N \]  

(4.2.3)

where \( \bar{\xi}_{LN} \) is incoming labour productivity growth. The use of the equation (4.2.3) transforms (4.2.1) and (4.2.2) from mere identities into behavioural functions.

To summarize, the model for the subsistence sector contains three endogenous variables, \( \xi_{LN}, \dot{\theta}_N \) and \( \hat{X}_N \); and three relationships (4.2.1), (4.2.3) and (4.2.4), based on which we solve for the endogenous variables as follows:

\[
\dot{\theta}_N = \frac{\bar{\xi}_{LN} + \bar{\xi}_{LN} + \gamma_1 \hat{L}_N}{1 - \gamma_2} - \frac{\gamma_1 (n - \lambda T)}{(1 - \gamma_2)(1 - \lambda)} \\
\hat{X}_N = \frac{\bar{\xi}_{LN} + \gamma_2 \dot{\theta}_N + (1 - \gamma_1 - \gamma_2) \left( \frac{n - \lambda T}{1 - \lambda} \right)}{1 - \gamma_2} \\
\xi_{LN} = \frac{\bar{\xi}_{LN} + \gamma_2 \dot{\theta}_N - \gamma_1 \hat{L}_N}{1 - \gamma_2} 
\]  

(4.2.4)  

(4.2.5)  

(4.2.6)

In (4.2.4), the growth rate of the wage is inversely related to growth in employment in the subsistence sector, in accordance with a standard labour market supply-demand notion. With regard to our two-sector economy, it also means that higher employment in the modern sector drives up the level of wage in the subsistence sector.

The effect of employment on output expansion depends on the sign of \( 1 - \gamma_1 - \gamma_2 \). An increased \( \hat{L}_N \) may lead to lower output if \( 1 - \gamma_1 - \gamma_2 < 0 \). This occurs when the efficiency-wage mechanism is significant (high \( \gamma_2 \)) and when higher employment strongly affects productivity (high \( \gamma_1 \)). Such an outcome occurs in those economies where massive transfer of labour to the subsistence sector leads to a slowdown in average production to the point where it reaches a subsistence level of wage (see Mellor, 1963).

The situation with regard to the subsistence sector is presented in figure 3. A decrease in the growth rate of employment shifts the schedule downwards, thereby positively affecting the growth rate of labour productivity as well as wages (not shown in the diagram), the latter contributing to an improvement in productivity through the efficiency-wage argument. Finally, a lower growth of employment positively affects output growth (observed in the shift of \( \hat{X}_N \) to the right) if the increase in productivity is great enough to compensate for a decrease in the rate of growth of employment.

\(^{17}\) Naastepad (2005) has a similar specification; however, the interpretation is different from that contained in the present paper. In Naastepad’s model, productivity growth responds positively to wage growth, implying that there is “wage-led technological progress [which] measures the extent to which more expensive labour induces firms to intensify their search for and adoption of labour productivity-raising techniques”.

The reader should note that we are referring here to the value of the output and not to the volume, which is strictly a positive function of productivity.
Determining output growth in the two-sector economy

Let us postpone a thorough analysis of the dynamics and possibility of growth traps or growth spurts until the following section and instead analyse here how the exogenous variables perform in determining macro performance. In the light of the analysis of each of the two sectors, the macro picture of our dual economy growth rate can be decomposed as:

\[
\dot{X} = \mu_3 \dot{X}_T + (1 - \mu_3) \dot{X}_N = \mu_3 \dot{X}_T + (1 - \mu_3) [\xi_{LN} + \dot{L}_N]
\]

where \( \mu_3 = X_T / X \) is the share of modern sector output in relation to total output. Using the expressions we obtained for the growth rates of output in the two sectors, (4.3.1) becomes:

\[
\dot{X} = \mu_3 \left[ \frac{X_T (\xi_{LT} - \phi_T)}{1 - \gamma_0 X_2} + \frac{X_N \beta}{1 - \gamma_0 X_2} \right] + (1 - \mu_3) \left[ \frac{\xi_{LN} + \gamma_2 \hat{P}_N + (1 - \gamma_1 - \gamma_2) n - \dot{\lambda}}{\lambda} \right]
\]

In (4.3.2), total output responds positively to an increase in investment, incoming productivities in both sectors as well as the growth rate of employment in the modern sector (or negatively relative to the employment growth rate in the subsistence sector). Such a result is logical in the sense that the growth rate of an economy rises when the resources, i.e., labour, are utilized more productively. It follows that output growth in the modern sector contributes to economic growth through two channels: first, through its own growth rate; and, second, through a decline in underemployment which implies a better utilization of the labour force. This relationship between the sources of growth can be observed in figure 4.

A rising productivity schedule in the NE quadrant determines a higher growth rate of output, which subsequently drives up employment growth, as illustrated in the NW quadrant of figure 4. If the labour demand in the modern sector is strong enough to compensate for the labour force increase which implies a transfer of labour from the subsistence to the modern sector, productivity and, consequently, output will expand at a faster rate in the former, as shown by the downward shift of the productivity growth schedule in the SW quadrant. Finally, in this particular setting, the overall economy grows faster, as can be observed in the SE quadrant.
If on the other hand higher productivity growth in the modern sector does not lead to job creation but rather to job destruction, the overall economic growth rate could actually slow down. In figure 4, the output growth schedule would have to have a steeper slope than the employment growth contours, which would ensure that a shift upwards of the KV schedule, although leading to more growth, would come at the expense of a lower growth rate of employment (the employment growth schedule in the NW quadrant would have a negative slope in this case). This can be the curse of an increase in productivity or jobless growth, a much debated idea in both policy and academic circles (see ILO, 2005).

However, by no means do we claim here that a technologically driven rise in productivity is unwanted. On the contrary, the core idea of the Kaldorian vision on growth is that the feedback between productivity in manufacturing and output growth represents the foundation of economic performance. Still, the jobless growth that has affected both developed and developing countries throughout the last decade is worrisome and calls for complementary macro policies. Dasgupta and Singh (2005) analyse the economy of India, a country where formal employment was roughly 8 per cent of total employment in 1999-2000. The authors look at how elasticities of employment with respect to output have changed over recent decades. They note that elasticities in both agriculture (primarily representing the informal sector) and manufacturing have decreased, while in some service sectors (mostly information technology (IT)) they have risen. A combination of slow structural change and the particular type of educated labour force demanded by the IT sector—the only robust sector that contributes to job creation—represents a challenge for the Indian policymakers over the next decade. The authors suggest that an increase in the Government’s contribution to demand growth and a strengthening in the dynamics between the IT and the manufacturing and agricultural sectors should qualify as policy priorities.
The retardation of the Kaldor-Verdoorn technical progress function and output growth

In this section, we will explore several aspects of growth dynamics characteristic of a developing economy. The economic literature on growth points to the presence of non-linearities which are increasingly making their way into both empirical and theoretical models. We attempt to do the same here but would first like to acknowledge that our exercise in this respect is a modest one. Second, any investigation of economic growth performance has to consider aspects related to the structural characteristics and institutional framework of the economy. Since it is impossible to account for all the factors important for growth, we limit the discussion to two distinct cases: the profit- and the wage-led economy. Since it is impossible to account for all the factors important for growth, we limit the discussion to two distinct cases: the profit- and the wage-led economy. In addition, we consider the implications that the initial or starting values of different parameters have on the long-term growth trajectories of an economy. Third, we introduce a retardation mechanism, an idea that goes back to Gerschenkron (1962), who argues that there may be a downward shift in the trend of productivity growth as a poor country catches up with a rich one. In our model, the retardation takes place commensurate with the country’s stage of development.

So far we have analysed (in a linear manner) how structural change and economic growth in the industrial sector interact to set off a feedback mechanism characteristic of a cumulative growth process. The model focuses on the relationship between labour productivity and output growth. As the economy approaches the level of development characteristic of a mature economy with a low labour surplus and high productivity levels, the cumulative effect of output growth on productivity ceases to be important. This development takes the form of a retardation process that appears as a result of technological “catch-up”, weakened economies of scale and intersectoral linkages as well as a diminishing labour surplus. In Kaldor’s view, the curve of the technical progress function “is likely to be convex upwards and flatten out altogether beyond a certain point” (Kaldor, 1957).

In this model, which tries to be parsimonious but at the same time realistic enough to offer a glimpse of what the retardation force could be, we choose the ratio of employment in the modern sector to the total labour force as the “variable to watch” in the medium and long run. In addition, introducing the retardation process brings non-linearities into the model as presented below.

Analytically, we allow the KV coefficient \( \gamma_0 \) to be written as a function of the modern sector’s employment share in total employment \( \lambda \):

\[
\gamma_0 = f(\lambda) \tag{5.1}
\]

where we limit \( \gamma_0 \) to being positive at all times. We do not derive an exact functional form for \( f(\lambda) \), but intuitively we choose a concave function (possibly quadratic). The technical progress coefficient increases when there is substantial underutilization of resources and economies of scale in the presence of strong feedbacks between the structural dynamics and macroeconomic performance. As the modern sector develops, the \( \gamma_0 \) coefficient reaches a certain maximum point, followed by a slow down. By combining (5.1) with \( \dot{\lambda} = \dot{L}_T - n \), we obtain a differential equation for the state variable \( \lambda \) in respect of its own level:

\[
\dot{\lambda} = \lambda \left[ \frac{(1-f(\lambda))A-B}{1-f(\lambda)\chi_2} - n \right] = \left( \frac{\lambda}{1-f(\lambda)\chi_2} \right) \left( A - B - n + f(\lambda)(n\chi_2 - A) \right) \tag{5.2}
\]

\[18\] Alternatively, we could use other variables such as the wage share for model closure. However, this could pose some problems in using the model empirically, given the data availability for the wage share in developing countries.
where \( A = \chi \dot{\lambda} - \chi_2 \dot{\theta} + \chi_3 \hat{e} \) and \( B = (1 - \chi_2) \delta LT \). (5.2) is a non-linear differential equation which may have three fixed points (considering that \( f(\lambda) \) is concave).

We then take into account only those solutions that are logical in economic terms, based on the restriction that \( 0 < \lambda < 1 \). Where there is no modern sector, the solution \( \lambda_1 = 0 \) requires that the economy remain in that state up to the point where “animal spirits” determine an exogenous shift in investment that subsequently draws in labour. \( f(\lambda_{2,3}) = \frac{A - B - n}{A - \chi_2 n} \) is another set of fixed points with \( \lambda_2 < \lambda_3 \). Further stability analysis explains the qualitative features of these solutions. 19

Two scenarios are possible when we consider the sign restrictions for \( \lambda \) and \( \gamma_0 \). If the origin \( \lambda_1 = 0 \) is a stable solution or a sink, \( \lambda_2 \) is unstable or a source, which ultimately implies a stable \( \lambda_3 \). Alternatively, the origin and \( \lambda_3 \) are unstable, which renders the middle equilibrium point stable. From mathematical point of view, it suffices to determine the stability of the system at the origin. To do so, we evaluate the sign of \( \frac{d \dot{\lambda}}{d \lambda} \) at zero. The partial derivative with respect to \( \lambda \) is:

\[
\frac{d \dot{\lambda}}{d \lambda} = a'(\lambda)\beta(\lambda) + \alpha(\lambda)\beta'(\lambda)
\]

where \( a(\lambda) = \frac{\lambda}{1 - f(\lambda)\chi_2} \) and \( \beta(\lambda) = A - B - n + f(\lambda)(n\chi_2 - A) \)

At the origin, we note that \( \frac{d \dot{\lambda}}{d \lambda} = a'(\lambda)\beta(\lambda) \), which takes the sign of \( \beta(\lambda) \) since \( a'(\lambda) \big|_{\lambda=0} \) is always larger than zero. 20 The exercise is then reduced to evaluating the sign of \( \beta(\lambda) \). If \( \beta(\lambda) > 0 \), the origin is a source, otherwise it is a sink. Figure 5 describes what we have just said in graphic form. \( \lambda_2 \), or point \( S \), is a sink, with arrows pointing towards it, while \( \lambda_3 \), or point \( U \), is a source, with the system diverging away from it.

In analytical terms, at point \( S \), the partial derivative, \( \frac{d \dot{\lambda}}{d \lambda} \), is smaller than zero. To the left of it, the effective demand in the modern sector expands fast enough (\( \dot{\lambda} > 0 \)) to absorb labour from the subsistence sector. Eventually, the cumulative effect of output on productivity growth causes the latter to catch up and the system subsequently settles at equilibrium, where employment in both sectors and population grow at the same rate, or \( \dot{\hat{L}}_T = \dot{\hat{L}}_N = n \). To the right of the fixed point, \( S \), output growth is not large enough to draw in labour, causing the share of modern-sector employment, \( \lambda \), to diminish. Pieper (2001) defines this situation as deindustrialization in terms of employment.

19 We should also note that if \( f(\lambda) = 1/\chi_2 \) the differential equation for \( \lambda \) is not defined.

20 To explain this, we express the derivative as \( a'(\lambda) \big|_{\lambda=0} = \frac{1 - f(\lambda)\chi_2 + f(\lambda)\chi_2}{(1 - f(\lambda)\chi_2)^2} \), which evaluated at zero and considering a quadratic concave function \( f(\lambda) = -a\lambda^2 + b\lambda + c \) becomes \( a'(\lambda) \big|_{\lambda=0} = \frac{1 - c\chi_2 + b\chi_2}{(1 - f(\lambda)\chi_2)^2} \). Several empirical studies approximate that \( \gamma_0 = f(\lambda) \) takes values in the range of 0.3 to 0.6, which further implies that \( c \) and \( b \) are smaller than one, and positive. This means that the numerator \( 1 - c\chi_2 + b\chi_2 \) is positive, unless \( \chi_2 \) is extremely high, which would be unlikely.
The second case, as illustrated in figure 6, where the origin is a sink. The lower equilibrium point, $U$, is an unstable solution for the system. If an economy is situated to the left of $U$, it falls into a growth trap and the modern sector disappears. In this case, deindustrialization takes place with regard to both employment and output, a situation that matches the sub-Saharan Africa experience of the last few decades.

It is of interest to see how the retardation scenario fits into the macro dynamics for the two-sector economy. Depending on the stability of fixed points and considering only the non-zero solutions, the retardation effect establishes above all that both the KV and output schedules follow a concave trajectory.$^{21}$ In the graphical representation, we depict only the situations where there are two common roots for output and productivity growth equations.

$^{21}$ The reader should note that we refer to a concave trajectory and not to a curve since the KV schedule is itself linear, but we have a change in the slope of the linear function which is the KV coefficient itself.
The subsequent discussion is meant to provide an indicative glimpse at the system’s dynamics and therefore does not take into account the caveats presented by choosing the $\gamma_0 = f(\lambda)$ function. We remind our reader that employment, and therefore the share $\lambda$, is the endogenous state variable which is determined by the movements in output and productivity growth rates. Figures 5 and 6 helped us observe the qualitative features of the system in a one-dimensional space; however, the stability and the dynamics of $\lambda$ are subject to the interaction between $\dot{X}_T$ and $\xi_{LT} + n$.

The curve for the output growth, $\dot{X}_T$, is depicted by a dotted line, while the productivity and labour force growth, $\xi_{LT} + n$, is represented by a solid line, as can be observed in figure 7. The reason we choose $\xi_{LT} + n$ instead of the KV equation is because macroeconomic equilibrium is reached when $\dot{X}_T = \xi_{LT} + n$, or equivalently when $\dot{L}_T = n$. The choice of $\xi_{LT} + n$ does not significantly change the analysis in terms of the KV relationship as the two functions are more or less equivalent given a constant labour force growth.

Figure 7 presents the two cases discussed above when the solution at origin in (5.2) is either a sink (left graph) or a source (right graph). The analysis of the trajectory which the modern sector follows is the same as that in figures 5 and 6.
Movements in modern sector’s variables drive the performance of the subsistence sector, as illustrated in the lower half of figure 7. We discuss here only one such example. We have established that, to the left of the unstable point, $U$, in the first graph, the industrial sector ceases to exist both in terms of output and employment. Further, the labour shed by the industrial sector finds its way to the subsistence sector, which causes a slowdown in $\xi_{Ln}$. Based on the analysis carried out above in the section on determining productivity, output and employment in the subsistence sector, a decrease in $X_N$ is likely to occur (as long as a productivity decline offsets the expansion in the labour force). Eventually, the entire labour force is absorbed in the only sector of the economy, the subsistence sector. Thereafter, the scenario becomes Malthusian in which, as stated by Mellor (1985), the labour force grows at the level permitted by the subsistence wage.

In the light of the above discussion, it is appropriate for policymakers to find the right mixture of policies that will shift the stable equilibrium point upwards and to the right so that higher economic performance is present together with a dominant modern sector. Hence, from this point of view, the case of a maquiladora economy is not perceived to be desirable in the long run. In terms of the diagrams in figure 7, the policy conundrum applies to point $S$ in both wage-led and profit-led economy.

Let us now see how the interplay between the KV technical progress function and output growth schedules, assisted by macroeconomic policies, may lead to such desirable results. A higher rate of investment or a favourable exchange-rate depreciation augments both $\bar{X}_F$ and $\xi_{LT}$; this can be visualized as an upward shift in output and productivity growth schedules. Since the KV coefficient is smaller than 1, it follows from (4.1.8) and (4.1.9) that the position of the output schedule will alter more than the KV schedule. The economy benefits from this in two ways. First, the stable equilibrium is now situated at a higher combination of output, productivity growth and employment share. Second, the space within which the economy may fall into a growth trap in the first graph (to the right of point $U$) is minimized, whereas the space that allows aggressive expansion of the industrial sector in the second graph is enlarged (to the right of point $U$).

A wave of innovations,23 captured by the incoming growth rate of productivity, $\xi_{LT}$, raises the intercept of the output and productivity growth equations and therefore shifts both schedules upwards. If innovation were to affect productivity growth only, an upward shift of the KV curve may have a negative impact in terms of the position of the stable equilibrium points. However, innovation and technology boost output growth as well. The final outcome depends on what type of economy we are dealing with. In a profit-led economy characterized by a $\chi_2$ that is close to or higher than 1, the effect on employment is weak or positive as the output schedule takes a larger leap upwards than does the KV schedule. In a wage-led economy, however, innovation might come with labour shedding and should therefore be supported by other expansionary policies designed to increase the effective demand.

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23 Ocampo (2005) discusses the effects of an inflow of innovations following trade liberalization and points to the possibility of a negative impact on the domestic dynamic linkages that may result in a situation that is less optimal than autarky.
Conclusions

In this paper, we use a parsimonious model to explore the mechanisms in place in the developing countries that may or may not deliver a sustainable growth process. In addition, we have tried to address an issue that has affected developing countries (and, more recently, developed ones) around the world over last two decades or so. The issue in question is the inability of the economic system to create productive employment. Its implications are numerous and important enough to go into in much more depth. However, we modestly attempt here to describe a macroeconomic mechanism that might lead to three possible situations: deindustrialization in terms of both output and employment in the presence of low or negative productivity growth; a growth trap sustaining a situation of structural heterogeneity where output and productivity expand but little employment is created; and, finally, the desired case of healthy and sustainable employment in the presence of reasonable output and productivity growth.

The main conclusions and insights are as follows: a) technological advancement is not only important but necessary in order to foster growth; b) full growth potential and its sustainability are in peril, however, unless faster growth contributes to productive employment; c) an active macroeconomic policy is necessary to strengthen the dynamic linkages between sectors; and, d) if it is to be effective, macroeconomic policy should focus on those sectors where the majority of the labour force is employed. This would require that policymakers have a long-run vision of the direction of structural change in the economy. Development policy in this regard should contribute to education and training of the underemployed to prepare them for the sectors that contribute dynamically to overall growth.
In formal terms, the utility function in the subsistence sector, as derived by Samuelson (1947) and Geary (1950-51) takes the following form:

$$U = \sum_{i=1}^{N} \beta_i \log(c_i - c_0)$$  \hspace{1cm} (3.1.1)

where $c_0$ is the necessary amount of the subsistence sector’s good only. One primary condition is that

$$\sum \beta_i = \beta_T + \beta_N = 1$$

which we will express from now on as $\beta_T = \beta$ and $\beta_N = 1 - \beta$. The budget constraint is shown as:

$$w_N = P_T c^T_{wn} + P_N c^N_{wn}$$  \hspace{1cm} (3.1.2)

Workers are maximizing their utility subject to the budget constraint, with the Lagrangean given by:

$$L = \sum_{i=1}^{N} \beta_i \log(c_i - c_0) - \ell (\sum_{T,N} P_i c_i - w_N)$$  \hspace{1cm} (3.1.3)

By solving the constrained maximization problem, we obtain the first-order conditions:

$$\frac{\partial L}{\partial c^T_{wn}} = \frac{\beta}{c^T_{wn}} - \ell P_T = 0$$

$$\frac{\partial L}{\partial c^N_{wn}} = \frac{1-\beta}{c^N_{wn} - c_0} - \ell P_N = 0$$  \hspace{1cm} (3.1.4)

$$\frac{\partial L}{\partial \lambda} = w_N - P_T c^T_{wn} - P_N c^N_{wn} = 0$$

From the condition that $\beta + (1-\beta) = 1$ and the first two partial derivatives, we obtain the following expression:

$$1 = \ell (w_N - P_N c_0)$$  \hspace{1cm} (3.1.5)

where the Lagrange multiplier is $\ell = 1/(w_N - P_N c_0)$, which we substitute in (3.1.4) to get the consumption of the two goods by the subsistence sector’s workers:

$$D(c^T_{wn}) = \frac{\beta (w_N - P_N c_0)}{P_T}$$  \hspace{1cm} (3.1.6)

$$D(c^N_{wn}) = \frac{(1-\beta)w_N}{P_N} + \beta c_0$$  \hspace{1cm} (3.1.7)

Consumption of both products is a positive function of the wage level. The lower the level of minimum consumption of the subsistence good, the higher will be the demand for the modern sector good and therefore the greater the trade between the two sectors. The derivation of the worker’s consumption in the $T$ sector is the same as for the $N$ sector and yields:

$$D(c^T_{wt}) = \frac{\alpha (w_T - P_N c_0)}{P_T}$$  \hspace{1cm} (3.1.8)

$$D(c^N_{wt}) = \frac{(1-\alpha)w_T}{P_N} + \alpha c_0$$
Bibliography:


Dasgupta Sukti and Singh Ajit (2005), ‘Manufacturing, Services, Jobless Growth and the Informal Economy: Will Services be the New Engine of Indian Economic growth?’, UN-WIDER.


Frenkel, Roberto and Taylor Lance (2005) Real Exchange Rate, Monetary Policy, and Employment, (mimeo) UN-DESA.


Morley Samuel and Vos Rob (2004), ‘Bad luck or wrong policies? External shocks, domestic adjustment, and the growth slowdown in Latin America and the Caribbean’, in Gauza et al. (ed.) Is Trade Liberalization good for Latin America’s Poor?, UNDP.


Ocampo José Antonio and Parra María Angela (2005), Explaining the Dual Divergence: Successes and Collapses in the Developing World since 1980, (mimeo), UN-DESA.


Pieper Ute (2001) Sectoral Regularities of Productivity Growth in Developing Countries-A Kaldorian Interpretation, MERIT.


