# II. THE COHORT COMPONENT METHOD FOR MAKING POPULATION PROJECTIONS

## A. Introduction

Preparing population projections is a critical first step in incorporating population concerns into comprehensive planning. Many projections relevant to this type of planning, such as projections of employment, incomes and consumption directly or indirectly make use of population projections. For example, projections of employment are normally made in conjunction with projections of labour force, which are in turn often derived using population projections.

A variety of methods can be used to project a nation's population. Some methods directly project the total population given the initial size of the population and assumptions on future rates of population growth.1/ The cohort component method, however, can project population by age and sex employing the initial age and sex structure of the population together with assumptions on the future components of population change, such as fertility and mortality.

This chapter describes a variant of the cohort component method which can be used to make a projection either of the national population or of urban and rural populations. The method is capable of projecting the structure of the population by age and sex along with various indicators of population size, structure and change. The chapter will not describe methods which can only project the total population, since the projected population structures are a critical prerequisite for many planning exercises using population projections.

The population structures projected by the method can be used as inputs for projecting the number of households, school enrolment, and the size and composition of the labour force. Projections of the population size can be employed to project household incomes as well as household consumption and savings. Likewise, the results can be used as an input for projecting government consumption and investment in the health sector.

The major strength of this technique is its ability to project a population in a straightforward and unambiguous manner. The technique does not embody restrictive or arbitrary assumptions and generates results which faithfully reflect the initial population structure and the fertility, mortality and migration conditions specified by the user. It yields projection results which are indispensable to any planning exercise seeking to take the future population change into account. These features make this technique fundamental for integrating population factors into development planning.

It is important, however, not to confuse these strengths with the capacity to provide sound forecasts of future demographic change. A projection prepared by this technique may not necessarily be an accurate prediction of future population change. Given the initial population, the technique can only provide an indication of the future population, if the components of population change turn out to be as specified by the initial assumptions. The degree to which a specific projection will correspond to future demographic events will depend on the accuracy with which the initial population was specified and trends in the components of population change were forecast.

In general, the decision on whether to apply the technique at all, or whether to use it in order to project only the national population depends on whether it is possible to prepare sound inputs. Under most conditions, the preparation of those inputs would require a substantial amount of information, especially if projections of urban and rural populations are sought. National statistical or census offices and national research institutions, as well as offices within the United Nations system, often have prepared estimates of the major demographic parameters. These estimates can frequently serve as a guide or basis for peparation of demographic inputs into the projection process.

A population projection requires many computations in order to arrive at a projection, particularly at a projection of urban and rural populations. This apparent problem is compounded by the fact that, for many planning purposes, a single projection will not suffice. However, these tedious calculations are readily performed with the aid of an electronic computer and, fortunately, several computer programs have been developed for preparing population projections. Some of those programs are for use on mainframe computers and others for use on microcomputers (box 15). A computer program that will correspond to the description of the cohort component method contained herein will be developed in the future for microcomputers.

The remainder of this chapter initially sets forth the procedures which make up the cohort component method. Then, it describes the inputs required and discusses how these inputs can be prepared. Finally, the chapter presents examples showing how the technique is used to prepare projections.

## B. The technique

### 1. Overview

As an introduction to the cohort component method, this overview indicates the types of inputs it requires along with the types of outputs it can generate. In addition, the overview outlines the computational steps involved.

#### (a) Inputs

In order to project the national population, the following types of inputs will be required:

Box 15

Computer programs for population projections

One of the first computer programs for making population projections was developed by the United Nations Population Division in 1973. a/ The program, which was designed for preparing national projections on a mainframe computer, was revised in 1982 and again in 1987. b/ The 1987 program is available in two versions, which can be respectively used on a mainframe computer and a microcomputer.

Another early mainframe-based program was developed by Shorter and Pasta, for preparing national projections and certain subnational projections, such as projections of the urban population. c/ This program was recently adapted for a microcomputer. d/

A few other microcomputer-based programs for making national population projections have been recently published. One of those programs, based on the United Nations projection methodology was prepared by The Futures Group. e/ Another program was recently published by the United Nations Economic and Social Commission for Asia and the Pacific. f/

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a/ "A computer program for population projections using the component method", Population Division working paper (ESA/P/WP.50, June 1973).

b/ United Nations, "A user's manual to the population projection computer program of the Population Division of the United Nations" (ESA/P/WP.77, 26 January 1982); "United Nations population projection computer program" (forthcoming).

c/ Frederic Shorter and David Pasta, Computational Methods for Population Projections: with Particular Reference to Development Planning (New York, The Population Council, 1974).

d/ Frederick C. Shorter, David Pasta and Robert Sendek, Computational Methods for Population Projections: With Particular Reference to Development Planning, 1st ed. with supplement added (New York, The Population Council, 1987).

e/ The Futures Group, A Demographic Projection Model for Development Planning for the IBM PC Micro-computer (Glastonbury, Connecticut, November 1986).

f/ Economic and Social Commission for Asia and the Pacific, "ESCAP/POP: a computer program for projecting populations by age and sex", Population Research Leads, No. 22 (Bangkok, 1986).

(i) Initial age and sex structure;
(ii) Assumptions on mortality;
(iii) Assumptions on fertility;
(iv) For an open population, assumptions on international migration.

If a projection of urban and rural populations is sought, the requisite inputs will include those listed under (i) through (iv) for each population -- urban and rural. In addition the inputs will include assumptions on internal (urban-rural) migration.

Box 16 lists the types of inputs required to make a population projection (national or urban-rural). Since the cohort component method is described in this chapter as a tool for preparing quinquennial population projections, assumptions on the components of population change would be for dates five years apart or for the intervening five-year time interval. As indicated in box 16, these assumptions can be expressed in terms of a variety of mortality, fertility and migration measures.

(b) Outputs

Where the cohort component method is used to project the national population, the types of results can include projections of the following:

(i) Age and sex structure of the population;
(ii) Various population aggregates, such as the population size, young- and old-age population, and the number of women in the childbearing ages;
(iii) Indicators of the population structure, such as the proportions of population in broad age groups (0-14, 15-64 and 65+) and the sex ratio of the population;
(iv) Rates of population change due to births, deaths and, where appropriate, international migration.

Where the technique is employed to project urban and rural populations, the results can include projections of the variables listed under (i) to (iv), both for the national population and for urban and rural populations. In addition, the results can include indicators of the population distribution, such as proportions of the national population, urban and rural, plus rates of population change due to internal (urban-rural) migration.

The types of results that can be obtained by the method, which are listed in box 17, would be for the dates five years apart or the intervening projection intervals.

(c) Computational steps

Projecting the population with the cohort component method involves a sequence of computational steps that are repeated for successive projection intervals, which in this description of the method are five-year time intervals. 2/ The steps use assumptions on future demographic conditions to modify the age and sex structure of the population as well as to derive

Box 16

Inputs for applying the cohort component method

1. Initial age and sex structure of the population (national or urban and rural)

2. Assumptions on mortality (national or urban and rural):

   Survival ratios by age and sex; or

   Expectations of life at birth by sex; or

   Infant mortality rates by sex and
   Expectations of life at age 5 by sex

3. Assumptions on fertility (national or urban and rural):

   Fertility rates by age; or

   Total fertility rates and
   Proportionate fertility rates by age

4. Assumptions on international migration (national or urban and rural; if population is open to international migration):

   Net international migration rates by age and sex; or

   Total net international migration rates by sex and
   Proportionate net inernational migration rates by age, by sex;
   or

   Net change to the population due to international migration by
   age and sex

5. Assumptions on internal migration (for urban or rural population;
   if urban and rural populations are being projected):

   Net internal (urban-rural) migration rates by age and sex; or

   Total net internal (urban-rural) migration rates by sex and
   Proportionate net internal (urban-rural) migration rates by age,
   by sex; or

   Net change to the population due to internal (urban-rural)
   migration by age and sex

Box 17

Outputs of the cohort component method

1. Age and sex structure of the population (national or urban, rural and national)

2. Population aggregates (national or urban, rural and national):

   Population size
   Population in selected broad age groups
   Mid-interval population size
   Number of person-years-lived
   Population growth
   Births
   Deaths
   Net change due to migration (international, internal and/or combined international and internal)

3. Indicators of the population structure (national or urban, rural and national):

   Proportions by broad age groups
   Dependency ratios
   Median age of the population
   Proportion of women in childbearing ages
   Sex ratio of the population

4. Indicators of the population distribution (national; if urban and rural populations are being projected):

   Proportion urban
   Proportion rural

5. Rates of population change (national or urban, rural and national):

   Crude birth rate
   Crude death rate
   Rate of natural increase
   Crude net migration rates (international, internal and/or combined international and internal)
   Rate of population growth

various indicators of the population size, structure and changes. They result in the projected age and sex structures for the end of the projection intervals along with different types of indicators pertaining to the same dates or the intervals themselves.

Typically, a projection of population prepared in connection with a development plan will be made for a 15- or 20-year time period. A projection over a period of this length will be made even when the planning exercise is limited to a medium term, such as a five-year period, and does not include any perspective planning. This is so since for a variety of purposes, a longer-term population perspective will be needed. In view of this, making a projection by the cohort component method will require repeating the sequence of steps for several five-year projection intervals.

## 2. National population

This section will initially elaborate the steps involved in deriving the age and sex structure along with other results that can be obtained in the course of projecting the national population closed to international migration. This section will then discuss additional steps needed to project a national population open to international migration. The steps used to derive rural and urban populations will be described in a later section.

### (a) Closed population

The procedure involved in making a projection of the national population closed to international migration over a five-year projection interval (t to t+5) includes, among others, steps needed to derive the age and sex structure of the population at the end of that interval. These steps can be divided into two groups: (i) those which yield the segment of the population structure at age five and above; and (ii) steps to derive the segment of that structure below age five. The procedure also includes steps to calculate various population aggregates and indicators of population structure and change. The steps that make up this procedure are summarized in box 18, and a subset of them, used to calculate the age and sex structure of the population, are also depicted diagramatically in figure II.

#### (i) Segment of the population structure at age 5 and over

##### a. Survival ratios

The nature of the first step in deriving the age and sex structure will depend on whether the mortality assumptions are formulated in terms of survival ratios (box 19) or summary mortality measures, such as expectations of life at birth. 3/ If they are specified in terms of survival ratios, these can be used directly as inputs into the projection process as shown in equations (2), (3) and (8).

Alternatively, if mortality assumptions are formulated using, say, expectations of life at birth by sex, then survival ratios first need to be

Box 18

Computational steps to project a national population closed to international migration

The steps used to project a national population closed to international migration over a five-year projection interval are:

(1) Use mortality assumptions to derive survival ratios by age and sex for the interval.

(2) Apply the survival ratios to the age and sex structure of the population at the beginning of the interval to obtain the segment of the age and sex structure of the population at age 5 and above at the end of the interval.

(3) Use fertility assumptions to derive fertility rates by age for the interval.

(4) Use the fertility rates, the numbers of women in the childbearing ages at the beginning and the end of the interval, and the sex ratio at birth to calculate the numbers of births by sex occurring during the interval.

(5) Apply appropriate survival ratios to the numbers of births by sex to obtain the segment of the age and sex structure below age 5 at the end of the interval.

(6) Derive various population aggregates, such as the population size, population of broad age groups and the numbers of births and deaths.

(7) Calculate various indicators of the population structure, such as proportions of population in various broad age groups, dependency ratios and the sex ratio of the population.

(8) Compute various rates of population change such as the crude birth and death rates as well as rates of natural increase and population growth.

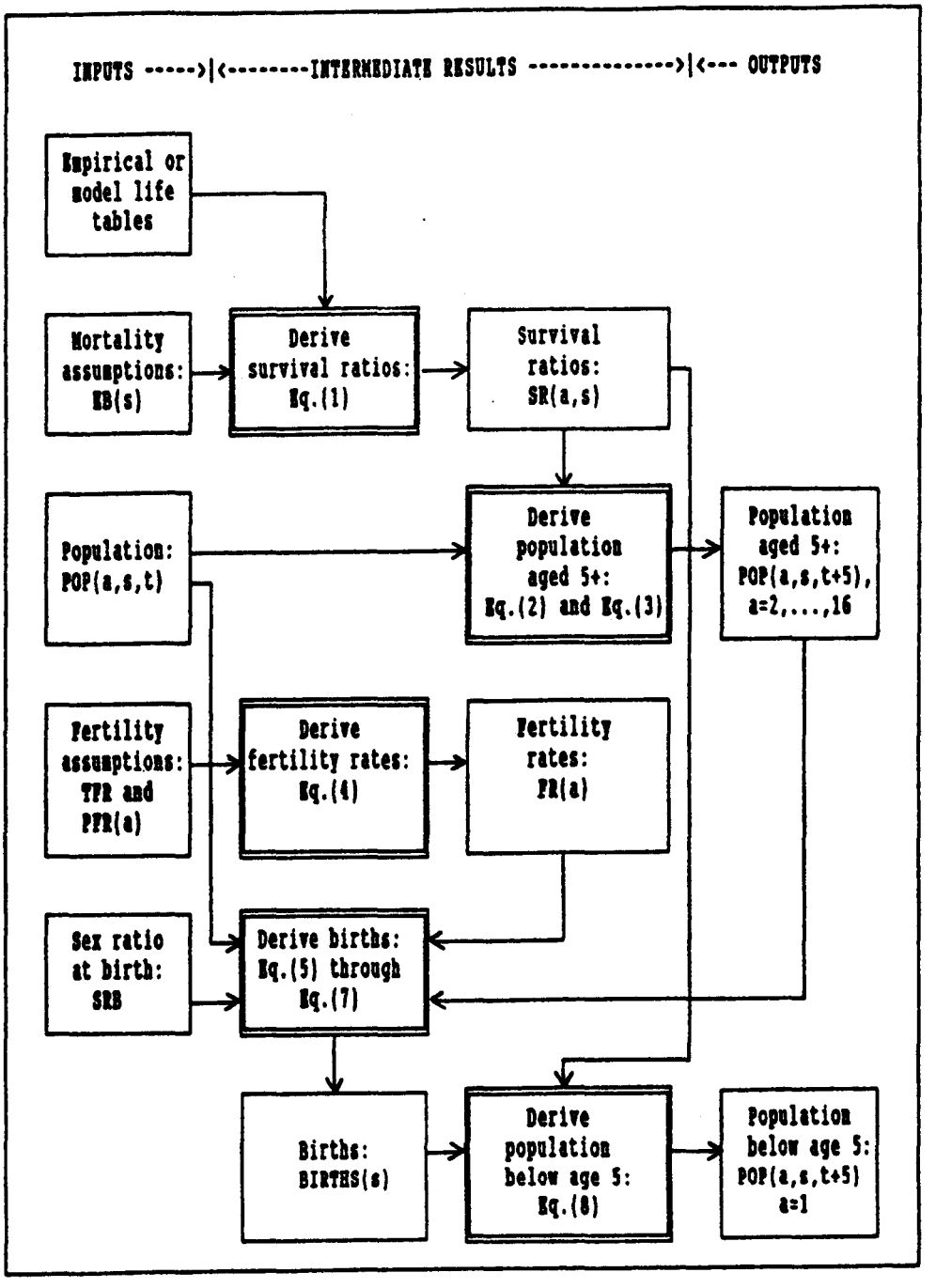
derived from those expectations. Written in general terms, the survival ratios would be calculated as follows:

$$SR(a,s) = T\,[EB(s)]; \qquad (1)$$

$$a = 1,\ldots,16;$$

$$s = 1,2,$$

Figure II. Steps to derive age and sex structure of the national population closed to international migration at the end of projection interval t to t+5



Box 19

Glossary

Expectation of life at birth
: The average number of years a member of a cohort of births would be expected to live if the cohort were subject to the mortality conditions specified by a particular set of age-specific mortality rates. It is denoted by the symbol $e_0$ in the life table notation.

Life table
: A listing of the number of survivors at different ages (up to the highest age attained) in a hypothetical cohort subject from birth to a particular set of age-specific mortality rates. The rates are usually those observed in a given population during a particular period of time. The tabulations commonly accompanying a life table include other features of the cohort's experience.

Model life table
: An expression of typical mortality experience derived from a group of observed life tables.

Survival ratio
: The probability of survival between one age or age group and another; when computed for age groups the ratios correspond to those of the person-years-lived function, ${}_nL_x$, of a life table.

where:

| | |
|---|---|
| a = 1,...,16 | are five-year age groups 0-4,...,75+, 4/ |
| s = 1,2 | are male and female sexes, |
| SR(a,s) | is the survival ratio representing the probability of survival over the interval among persons who belong to age group a and sex s at the end of the interval, 5/ |
| EB(s) | is the expectation of life at birth of sex s specified for the interval, and |
| T | is the transformation of expectations of life at birth by sex into survival ratios using selected life tables. (See annex I to this chapter, Description of a life table.) |

For either sex, the first survival ratio, obtained by means of a transformation indicated in equation (1) i.e., for a = 1, represents the probability of survival between birth occurring during the interval and age 0-4 at its end. The remaining ratios indicate the probabilities of survival between years of age 0-4, 5-9,..., 70+ at the beginning of the interval and the respective ages 5-9, 10-14, ..., 75+ at its end.

The transformation shown by equation (1), will typically amount to deriving survival ratios within the selected family of model life tables that correspond to expectations of life at birth given by assumptions. 6/ This derivation may involve an interpolation among survival ratios at appropriate levels of mortality in the model life tables employed. The computations involved in this transformation that use a linear interpolation are described in section C. Also, described in annex II to this chapter are calculations involved in a related transformation of infant mortality rates (box 20) and expectations of life at age 5 into survival ratios.

b. Population aged 5 and over

The numbers of survivors who belong to the various five-year age groups at age 5 and over at the end of the five-year projection interval (except those in the open age group) are calculated by applying the survival ratios to the numbers of persons belonging to corresponding five-year age groups at the beginning of the interval:

$$POP(a,s,t+5) = POP(a-1,s,t) \cdot SR(a,s); \qquad (2)$$

$$a = 2,\ldots,15;$$

$$s = 1,2,$$

where:

t is the year of the projection period,

POP(a,s,t+5) is the population (survivors) of age group a and sex s at the end of the interval, and

POP(a-1,s,t) is the population of age group (a-1) and sex s at the beginning of the interval.

The number of survivors in the open (eldest) age group at the end of the five-year projection interval is obtained by applying survival ratios to the numbers of persons who at the beginning of the interval belong either to the open age group or to the age group preceding it:

$$POP(16,s,t+5) = \left[ \sum_{a=15}^{16} POP(a,s,t) \right] \cdot SR(16,s); \qquad (3)$$

$$s = 1,2.$$

## Box 20

## Glossary

**Age-specific fertility rate**
The number of births occurring during a specified period to women of a given age or age group, divided by the number of person-years-lived during that period by women of that age or age group. When an age-specific fertility rate is calculated for a calendar year, the number of births to women of the given age is usually divided by the mid-year population of women of that age.

**Childbearing span**
The age span within which women are capable of bearing children, generally taken to be from age 15 to age 49 or, sometimes, to age 44.

**Expectation of life at exact age x**
The average number of years a person of exact age x would be expected to live if subjected to the mortality conditions specified by a particular set of age-specific mortality rates at age x and above. It is denoted by the symbol $e_x$ in the life table notation.

**Infant mortality rate**
The ratio of the number of deaths of children under one year of age occurring in a given year to the number of births in the same year. Also used in a more rigorous sense to mean the number of deaths that would occur to children under one year of age in a life table with a radix of 1,000. In this sense, it is denoted by the symbol $_1q_0$.

**Proportionate age-specific fertility rate**
The rate calculated by dividing a particular age-specific fertility rate by the sum of age-specific fertility rates across the childbearing ages. The sum of all proportionate age-specific fertility rates equals one.

**Sex ratio at birth**
The number of male births for each female birth, conventionally multiplied by 100.

**Total fertility rate**
The average number of children that would be born per women if all women lived to the end of their childbearing years and bore children according to a given set of age-specific fertility rates. This rate can be computed as the sum of fertility rates by single year of age or the sum of fertility rates by five-year age group, multiplied by 5.

Taken together, the steps described by equations (2) and (3) yield the structure of the population at age 5 and over at the end of the interval.

(ii) Segment of the population structure below age 5

The numbers of persons below age 5 at the end of a five-year projection interval consist of survivors of children born during the interval. Therefore, in order to obtain those numbers, it is first necessary to compute the numbers of births by sex occurring during the interval and then multiply those numbers by suitable survival ratios. The numbers of births are calculated from fertility rates based on fertility assumptions, the numbers of women in the childbearing ages and the sex ratio at birth.

a. Fertility rates

Fertility assumptions can be specified using age-specific fertility rates or summary fertility measures, such as the total fertility rate along with proportionate age-specific fertility rates. If the assumptions are expressed in terms of age-specific fertility rates, those rates can be directly used in the projection, as shown in equation (6).

Where fertility assumptions are formulated using summary measures, the age specific fertility rates ought to be first computed from those measures. Thus, if the assumptions are given in terms of the total fertility rate and proportionate fertility rates by age, the derivation of age-specific fertility rates for a given five-year projection interval amounts to scaling the proportionate rates with a factor that equals the total fertility rate divided by the number of years in the projection interval, 5:

$$FR(a) = (TFR/5) \cdot PFR(a); \qquad (4)$$

$$a = 4,\ldots,10,$$

where:

FR(a) is the average annual fertility rate of age group a for the interval,

TFR is the total fertility rate specified for the interval, and

PFR(a) is the proportionate fertility rate of age group a for the interval.

b. Births

To calculate the number of births occurring during the interval using the age-specific fertility rates, it is first necessary to derive the numbers of women in the various five-year age groups of the childbearing span at the mid-point of the interval. For each age group, this is done by calculating

the geometric mean of the number of women at the beginning and at the end of the time interval: 7/

$$MIPOP(a,2) = [\ (POP(a,2,t) \cdot (POP(a,2,t+5)\ ]^{1/2}; \qquad (5)$$

$$a = 4,\ldots,10,$$

where:

MIPOP(a,2) is the mid-interval number of women of age group a.

Given the age-specific fertility rates and the mid-interval numbers of women of the childbearing span, the number of births taking place during the interval is:

$$BIRTHS = 5 \cdot [\ \sum_{a=4}^{10} FR(a) \cdot MIPOP(a,2)\ ], \qquad (6)$$

where:

BIRTHS is the number of births occurring during the interval.

As suggested by equation (6), the number of births for a given five-year projection interval equals the sum of the products of the age-specific fertility rates for the interval and the mid-interval numbers of women by age, multiplied by the length of the interval, five.

To calculate the numbers of children who survive to the end of this interval, it is necessary that the total number of births be disaggregated by sex. This can be done by using proportions of births by sex, derived from an assumed sex ratio at birth:

$$BIRTHS(s) = BIRTHS \cdot PBS(s); \qquad (7)$$

$$s = 1,2,$$

where:

$$PBS(s) = \begin{cases} SRB/(100 + SRB), \text{ when } s = 1 \\ 100/(100 + SRB), \text{ when } s = 2, \end{cases}$$

and where:

BIRTHS(s) is the number of births of sex s occurring during the interval,

PBS(s) is the proportion of births of sex s, and

SRB is the sex ratio at birth.

c. Population below age 5

For each sex, the population aged 0-4 at the end of the interval is obtained by applying survival ratios to the numbers of births:

$$POP(1,s,t+5) = BIRTHS(s) \cdot SR(1,s); \qquad (8)$$

$$s = 1,2.$$

This step completes the derivation of the age and sex structure of the closed population at the end of the five-year projection interval.

(iii) Other results

Once the age and sex structure of the population is derived for the end of a given five-year projection interval, it is possible to calculate many indicators that are useful for planning and policy making. Some of these indicators refer to population aggregates, some refer to the population structure and some refer to rates of population change.

a. Population aggregates

The age and sex structures can be used to calculate a number of different population aggregates, among which are the population size and the number of persons in special age intervals--the young-age population, the working-age population and the old-age population along with the school-age population and the number of women in the childbearing ages. All these aggregates refer to the end of the five-year projection interval. Among the other aggregates which can be calculated are the mid-interval population size and the total person-years-lived by the population during the interval. Yet another group of aggregates includes total births, deaths and the growth of the population.

i. Population size

The population size can be obtained by aggregating the numbers of persons projected for the end of the interval across the age groups and sexes:
where:

$$POP(t+5) = \sum_{a=1}^{16} \sum_{s=1}^{2} POP(a,s,t+5), \qquad (9)$$

where:

$POP(t+5)$ is the size of the population at the end of the interval.

ii. Young-age population

The young-age population can be calculated as the sum of all persons below age 15:

$$YAP(t+5) = \sum_{a=1}^{3} \sum_{s=1}^{2} POP(a,s,t+5), \quad (10)$$

where:

YAP(t+5) is the young-age population at the end of the interval.

iii. Working-age population

The working-age population is normally defined as the population within the age interval 15-64 and can be calculated as:

$$WAP(t+5) = \sum_{a=4}^{13} \sum_{s=1}^{2} POP(a,s,t+5), \quad (11)$$

where:

WAP(t+5) is the working-age population at the end of the interval.

iv. Old-age population

The old-age population conventionally includes persons aged 65 and over and can be obtained as:

$$OAP(t+5) = \sum_{a=14}^{16} \sum_{s=1}^{2} POP(a,s,t+5), \quad (12)$$

where:

OAP(t+5) is the old-age population at the end of the interval.

v. School-age population

The school-age population is conventionally defined as population within the age range 5-24 and can, therefore, be obtained by adding up all persons within this age interval:

$$SAP(t+5) = \sum_{a=2}^{5} \sum_{s=1}^{2} POP(a,s,t+5), \quad (13)$$

where:

SAP(t+5) is the school-age population at the end of the interval.

vi. Women of the childbearing ages

The number of women of the childbearing ages, which is typically defined as the number of women between age 15 and age 49 is calculated as:

$$WCA(t+5) = \sum_{a=4}^{10} POP(a,2,t+5), \quad (14)$$

where:

WCA(t+5) is the number of women in the childbearing ages at the end of the interval.

vii. Mid-interval population size

The mid-interval population size is calculated as the geometric mean of the population sizes at the beginning and the end of the interval respectively:

$$MIPOP = [\, POP(t) \cdot POP(t+5) \,]^{1/2}, \quad (15)$$

where:

MIPOP is the mid-interval population size.

viii. Total number of person-years-lived

The total number of person-years-lived by the population, which is the number of years lived by all members of the population during the interval, can be obtained as the product of the mid-interval population size and the length of the interval, 5:

$$NPYL = MIPOP \cdot 5, \quad (16)$$

where:

NPYL is the total number of person-years-lived by the population during the interval.

ix. Population growth

The growth of the population over a specified interval equals the difference between the population sizes at the end and the beginning of the interval respectively:

$$POPGR = POP(t+5) - POP(t), \quad (17)$$

where:

POPGR is the population growth over the interval.

x. Births

The number of births occurring during the interval (BIRTHS) is calculated as shown earlier (equation (7)).

xi. Deaths

The number of deaths can be obtained as the difference between the number of births and the population growth:

$$DEATHS = BIRTHS - POPGR, \quad (18)$$

where:

DEATHS is the number of deaths occurring during the interval.

b. Indicators of the population structure

Indicators of the age and sex structure of the population can play an important role in planning and policy formulation. Among those indicators are the proportions of the population in broad age groups, such as 0-14, 15-64 and 65+ (which include young-age, working-age and old-age populations, respectively). Other important indicators of age structure include dependency ratios such as the young-age dependency ratio, the old-age dependency ratio and the total dependency ratio. Yet another indicator of the age structure is the median age of the population. These various age structure indicators can be supplemented by two more indicators. These are the proportion of women of the childbearing ages in the total population, an indicator of the age and sex structure, and the sex ratio of the population, an indicator of the sex structure.

i. Proportions by broad age groups

The proportions by broad age groups (0-14, 15-64 and 65+) are obtained by dividing the numbers of persons in these broad age groups by the population size:

the proportion at young age:

$$PYA(t+5) = YAP(t+5) / POP(t+5), \quad (19)$$

the proportion at working age:

$$PWA(t+5) = WAP(t+5) / POP(t+5), \quad (20)$$

and the proportion at old age:

$$POA(t+5) = OAP(t+5) / POP(t+5), \quad (21)$$

where:

PYA(t+5) is the proportion of the population at young age (age group 0-14) at the end of the interval,

PWA(t+5) is the proportion of the population at working age (age group 15-64) at the end of the interval, and

POA(t+5) is the proportion of the population at old age (age group 65+) at the end of the interval.

ii. Dependency ratios

Dependency ratios include the young-age dependency ratio, the old-age dependency ratio and the total dependency ratio. They are respectively calculated by dividing the young-age population, the old-age population and the sum of these two populations by the working-age population:

the young-age dependency ratio:

$$YADR(t+5) = YAP(t+5) / WAP(t+5), \quad (22)$$

the old-age dependency ratio:

$$OADR(t+5) = OAP(t+5) / WAP(t+5), \quad (23)$$

and the total dependency ratio:

$$TDR(t+5) = [\ YAP(t+5) + OAP(t+5)\ ] / WAP(t+5), \quad (24)$$

where:

YADR(t+5) is the young-age dependency ratio at the end of the interval,

OADR(t+5) is the old-age dependency ratio at the end of the interval, and

TDR(t+5) is the total dependency ratio at the end of the interval.

iii. Median age of the population

The median age of the population is computed using the standard formula for computing the median age from grouped data. 8/ If applied to the population age structure, this formula is:

$$MAPOP(t+5) = (a'-1) \cdot 5 + [ ( POP(t+5)/2 - \sum_{a=1}^{a'-1} \sum_{s=1}^{2} POP(a,s,t+5) ) / \sum_{s=1}^{2} POP(a',s,t+5) ] \cdot 5, \quad (25)$$

where:

MAPOP(t+5) is the median age of the population at the end of the interval, and

a' is the five-year age group containing the member of the population who is older than one half of the population and younger than the other half.

In equation (25), the first term on the right-hand size, $(a'-1) \cdot 5$, represents the lower limit of the five-year age group containing the middle member of the population. The term, $\sum_{a=1,a'-1}\sum_{s=1,2}POP(a,s,t+5)$, stands for the number of persons in all five-year age groups preceding the age group containing the middle member, and the term, $\sum_{s=1,2}POP(a',s,t+5)$ is the number of persons in that latter age group.

iv. Proportion of women in the childbearing ages

The proportion of women who are in the childbearing period is obtained as a ratio of the number of women in childbearing ages to the population size:

$$PWCA(t+5) = WCA(t+5) / POP(t+5), \quad (26)$$

where:

PWCA(t+5) is the proportion of women in the childbearing ages at the end of the interval.

v. Sex ratio of the population

The sex ratio of the population (box 21) is calculated as the ratio of the number of males in the population to the number of females:

$$SRP(t+5) = [ ( \sum_{a=1}^{16} POP(a,1,t+5) ) / ( \sum_{a=1}^{16} POP(a,2,t+5) ) ] \cdot 100, \quad (27)$$

where:

SRP(t+5) is the sex ratio of the population at the end of the interval.

Box 21

Glossary

Crude birth rate
The number of births in a population during a specified period divided by the number of person-years-lived by the population during the same period. It is frequently expressed as births per 1,000 population.

Crude death rate
The number of deaths in a population during a specified period divided by the number of person-years-lived by the population during the same period. It is frequently expressed as deaths per 1,000 population.

Rate of natural increase
The difference between the births and deaths occurring during a given period divided by the number of person-years-lived by the population during the same period. This rate, which specifically excludes changes resulting from migration, is equal to the difference between the crude birth rate and the crude death rate.

Rate of population growth
The increase or decrease of a population in a specified period divided by the number of person-years-lived by the population during the same period. The increase in a population is the result of a surplus (or deficit) of births over deaths and a surplus (or deficit) of immigrants over emigrants.

Sex ratio of the population
The number of males in the population for each female, conventionally multiplied by 100.

### c. Rates of population change

The projections made thus far make it possible to derive the following average annual rates of population change: the crude birth rate, the crude death rate, the rate of natural increase and the rate of population growth.

i. Crude birth rate

The crude birth rate for a five-year projection interval (t to t+5) is obtained by dividing the average annual number of births by the mid-interval population and multiplying by 1,000:

$$CBR = [\ (BIRTHS/5)\ /\ MIPOP\ ] \cdot 1{,}000, \qquad (28)$$

where:

CBR is the crude birth rate for the interval.

ii. Crude death rate

The crude death rate for an interval is obtained by dividing the average annual number of deaths by the mid-interval population and multiplying by 1,000:

$$CDR = [\ (DEATHS/5)\ /\ MIPOP\ ] \cdot 1{,}000, \qquad (29)$$

where:

CDR is the crude death rate for the interval.

iii. Rate of natural increase

The average annual rate of natural increase, which expresses the change in the population size resulting from births and deaths, is calculated as the difference between the crude birth rate and the crude death rate:

$$RNI = CBR - CDR, \qquad (30)$$

where:

RNI is the rate of natural increase for the interval.

iv. Rate of population growth

The average annual rate of population growth, is calculated for the five-year projection interval as follows:

$$GRP = [\ \ln(\ POP(t+5)/POP(t)\ )\ /\ 5\ ] \cdot 1{,}000, \qquad (31)$$

where:

GRP is the average annual growth rate of the population for the interval, and

ln is the natural logarithm.

In a closed population, the growth rate of the population equals the rate of natural increase.

(b) Open population

The procedure used to project an open national population over a five-year projection interval (t to t+5) utilizes all the steps described above along with additional steps relating to international migration (box 22). The steps used in this type of projection are summarized in box 23, while a subset of steps employed to derive the age and sex structure are also depicted diagramatically in figure III. The discussion that follows will focus on those steps related to international migration.

International migration can be measured in a variety of ways from the perspective of a country which is being affected by it. Thus, the measurement may seek to quantify both gains and losses to the country's population due to international migration or assess net changes to the population (gains minus losses) caused by it. The measurement, which should be typically for a fixed time interval, may concern the numbers of persons gained and lost or the net change due to migration during the interval. Alternatively, it may focus on the gains and losses or net changes to the population at the end of the time interval caused by migration occurring during the interval.

There are many different measures of international migration, which is partly a consequence of a variety of ways of measuring this type of population mobility. Those measures include, for example, gross and net international migration rates which, respectively, measure gains and losses and net changes due to migration. Depending on the types of data at hand, the rates that are derived from them may measure gains and losses or net changes to the population caused by migration over a given time interval. Alternatively, they may measure gains and losses or net changes to the population at the end of the time interval in question resulting from migration occurring during the interval.

This discussion of the cohort component method will make use of international migration measures which quantify net changes to the surviving population at the end of a given time interval due to international migration occurring over the interval. Such measures, which will be used here in connection with international migration assumptions, include age-specific net international migration rates. They may also include selected summary international migration measures, such as the total net international migration rate and proportionate age-specific net international migration rates, both of which are specific by sex. Furthermore, they may include the numbers of survivors by age and sex at the end of the time interval that the population lost and/or gained on balance due to international migration that took place during the interval.

## Box 22

## Glossary

**Age-specific net international migration rate**
The net gain or loss to the survivors of a given age or age group at the end of a specified period due to international migration occurring during that period divided by the number of survivors of that age or age group.

**Crude net international migration rate**
The net change (loss or gain) to the population due to international migration during a specified period, divided by the number of person-years-lived by the population during the same period. It is frequently expressed as net change due to international migration per 1,000 population.

**International migration**
Movements of population across national boundaries. It is designated as emigration from the standpoint of the nation from which the movement occurs and as immigration from that of the receiving nation.

**Proportionate age-specific net international migration rate**
The rate calculated by dividing a particular age-specific net international migration rate by the sum of age-specific net international migration rates across all ages or age groups. The sum of proportionate age-specific net international migration rates across all ages equals one.

**Reverse survival**
A procedure to estimate an earlier population from an observed population, allowing for those members of the population who would have died according to observed or assumed mortality conditions. It can be used to estimate the number of births occurring over a specified time interval from the observed number of survivors of those births at the end of the interval. Similarly, the procedure can be employed to calculate net changes to the population due to migration occurring during a given time interval from the observed changes in the numbers of survivors in the population at the end of the interval resulting from migration.

**Total net international migration rate**
The sum of age-specific net international migration rates across all ages or age groups.

Box 23

## Computational steps to project a national population open to international migration

The steps used to project a national population open to international migration over a five-year projection interval are:

(1) Use mortality assumptions to derive survival ratios by age and sex for the interval.

(2) Apply the survival ratios to the age and sex structure of the population at the beginning of the interval to obtain the numbers of survivors by age and sex at age 5 and over at the end of the interval.

(3) Use international migration assumptions to derive net international migration rates by age and sex for the end of the interval.

(4) Use net international migration rates to modify the numbers of survivors at age 5 and over and thus derive the segment of the age and sex structure of the population over that age range at the end of the interval.

(5) Use fertility assumptions to derive fertility rates by age for the projection interval.

(6) Use the fertility rates, the average numbers of women in the childbearing ages at the beginning and the end of the interval and the sex ratio at birth to obtain the numbers of births by sex occurring during the interval.

(7) Apply appropriate survival ratios to the numbers of births by sex to obtain the numbers of survivors by sex below age 5 at the end of the interval.

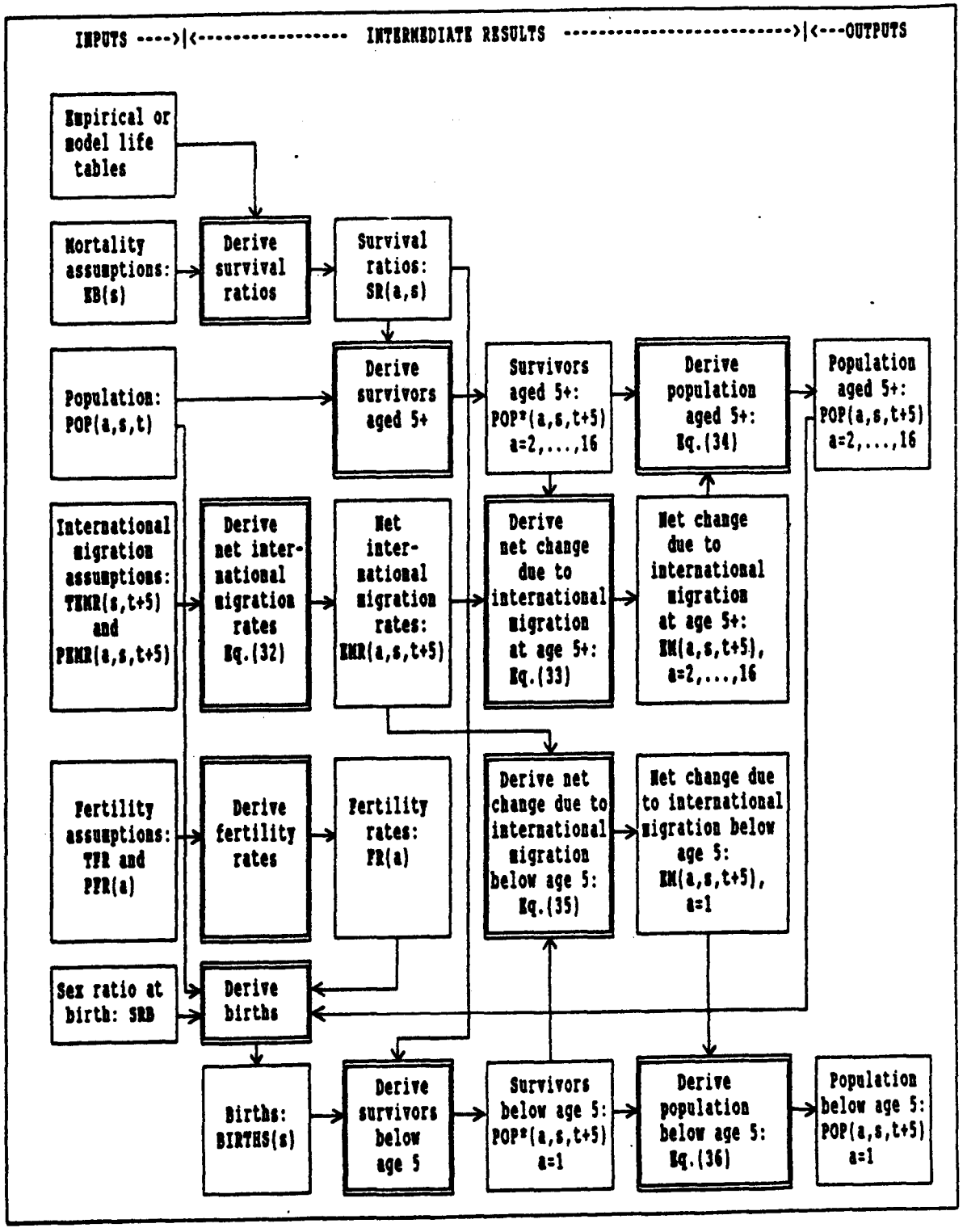
(8) Use appropriate net international migration rates to modify those survivors and in the process obtain the segment of the age and sex structure of the population below age 5 at the end of the interval.

(9) Derive various population aggregates, such as the population size, population of broad age groups and the numbers of births and deaths along with the net change due to international migration.

(10) Calculate various indicators of the population structure, such as proportions of population in various broad age groups, dependency ratios and the sex ratio of the population.

(11) Compute various rates of population change, including the crude birth, death and international migration rates as well as rates of natural increase and population growth.

**Figure III.** Steps to derive age and sex structure of the national population open to international migration at the end of projection interval t to t+5



(i) Segment of the population structure at age 5 and over

a. Survival ratios and survivors aged 5 and over

The first steps in computing the age and sex structure of an open population at the end of a five-year projection interval involve calculating survival ratios for the projection interval and the number of survivors aged 5 and over at the end of the interval. These steps are identical to those described by equation (1) and equations (2) and (3), respectively. The numbers of survivors aged 5 and over should, however, be further modified using net international migration rates.

b. Net international migration rates

If international migration assumptions are formulated using age-specific net international migration rates, the rates can be used as a direct input into the projection, as indicated in equations (33) and (35). Where those assumptions are specified in terms of the numbers of survivors lost and/or gained at the end of the projection interval, the numbers can be used directly as shown in equations (34) and (36). Alternatively, if the assumptions are specified in terms of the summary measures, such as those mentioned above, age specific net international migration rates ought to be first derived from those measures.

Among the summary measures used here, the total net international migration rate for a given sex is a sum of age-specific net international migration rates of that sex. This rate is a measure of the extent to which survivors of a given sex at the end of the five-year time interval are affected by international migration. (The total net international migration rate is not affected by the age structure of the population.) Proportionate age-specific net international migration rates for a specific sex represent the age patterns of net international migration rates for that sex.

Using these migration measures, age-specific net international migration rates are obtained by multiplying the proportionate age-specific rates by the total net internal migration rates:

$$EMR(a,s,t+5) = TEMR(s,t+5) \cdot PEMR(a,s,t+5); \qquad (32)$$

$$a = 1,\ldots,16;$$

$$s = 1,2,$$

where:

EMR(a,s,t+5) is the net international (external) migration rate applying to survivors of age group a and sex s at the end of the interval,

TEMR(s,t+5) is the total net international (external) migration rate applying to survivors of sex s at the end of the interval, and

PEMR(a,s,t+5) is the proportionate net international (external) migration rate applying to survivors of age group a and sex s at the end of the interval.

c. Population aged 5 and over

To derive the segment of the age and sex structure at age 5 and over, it is first necessary to calculate the net changes among the survivors due to international migration. These changes are obtained by multiplying the numbers of survivors by the net international migration rates:

$$EM(a,s,t+5) = POP^{*}(a,s,t+5) \cdot EMR(a,s,t+5); \quad (33)$$

$$a = 2,\ldots,16;$$

$$s = 1,2,$$

where:

EM(a,s,t+5) is the net change due to international (external) migration among survivors of age group a and sex s at the end of the interval, and

POP*(a,s,t+5) is the number of survivors of age group a and sex s at the end of the interval. 9/

Following this, the numbers of survivors are modified by these net changes:

$$POP(a,s,t+5) = POP^{*}(a,s,t+5) + EM(a,s,t+5); \quad (34)$$

$$a = 2,\ldots,16;$$

$$s = 1,2,$$

This yields the age and sex structure of the open population at age 5 and above at the end of the interval.

(ii) Segment of population structure below age 5

a. Fertility rates, births and survivors below age 5

In order to obtain the segment of the population structure below age 5, it is first necessary to calculate fertility rates and the numbers of births along with the numbers of survivors below that age. The calculations can be performed using the steps indicated by equations (4) through (8).

b. Population below age 5

To complete the derivation of the segment of the population structure below age 5, the net international migration rates are initially used to compute net changes due to migration among survivors below age 5:

$$EM(1,s,t+5) = POP^{*}(1,s,t+5) \cdot EMR(1,s,t+5); \qquad (35)$$

$$s = 1,2.$$

Then, population below age 5 is obtained by modifying the numbers of survivors by those net changes:

$$POP(1,s,t+5) = POP^{*}(1,s,t+5) + EM(1,s,t+5); \qquad (36)$$

$$s = 1,2.$$

This completes the derivation of the age-sex structure of the open national population at the end of the interval.

(iii) Other results

After the age and sex structure of the population is derived for the end of a given five year projection interval, indicators of the population size, structure and change can be calculated. The indicators include all those that are obtained in the course of projecting a closed population, plus a few additional ones.

a. Population aggregates

With but two exceptions, the population aggregates obtained in this type of projection can be calculated as in the closed population, using steps indicated by equations (9) through (17). The total number of deaths cannot be obtained using equation (18), and is calculated in a different way. Furthermore, one additional aggregate is obtained in the course of this projection, which is the net change in the population due to international migration. This net change needs to be calculated before calculating the number of deaths.

i. Change due to international migration

The net change to the population resulting from international migration during a projection interval can be obtained by a procedure which involves a reverse survival of the net gains or losses due to migration among the survivors at the end of the interval. The reverse survival is carried out over two and a half years, until the mid-point of the projection interval. The results, which represent net gains or losses to the population classified by age and sex due to international migration, are then aggregated across ages and sexes:

$$NCDEM = \sum_{a=1}^{16} \sum_{s=1}^{2} EM(a,s,t+5) \;/\; SRF(a,s) \qquad (37)$$

where for each s:

$$SRF(a,s) = \begin{cases} 0.67 + 0.33 \cdot SR(1,s), \text{ when } a = 1 \\ ( 1 + SR(a,s) ) \;/\; 2, \text{ when } 1 < a \leq 16, \underline{10}/ \end{cases}$$

and where:

NCDEM is the net change in the population due to international (external) migration during the interval, and

SRF(a,s) is the survival ratio factor used to reverse survive the net change due to international migration among the survivors of age group a and sex s.

ii. Deaths

The number of deaths occurring in an open population during a five-year projection interval is obtained in a way to ensure that this number will reflect the fact that the population gains or loses numbers owing to international migration. The number of deaths can be obtained as the difference between the number of births and the population growth plus the net change due to international migration:

$$DEATHS = BIRTHS - POPGR + NCDEM \qquad (38)$$

b. Indicators of the population structure

The indicators of the age and sex structure of the population calculated as part of a projection of an open population are identical to those that can be obtained in the course of projecting a closed population. The steps used to calculate the indicators are described by equations (19) through (27).

c. Rates of population change

The rates computed in the course of projecting an open population include those obtained as part of a projection of a closed population as described by equations (28) through (31). In addition, this type of projection makes it possible to calculate the crude net international migration rate.

### i. Crude net international migration rate

The average annual crude net international migration rate for a given projection interval is computed as the average annual net change due to international migration divided by the mid-interval population and multiplied by 1,000:

$$CEMR = [\ (NCDEM/5)\ /\ MIPOP\ ] \cdot 1{,}000, \qquad (39)$$

where:

CEMR is the crude net international (external) migration rate referring to the interval.

For an open population, the crude net international migration rate must equal the difference between the rate of growth of population and the rate of natural increase.

## 3. Urban and rural populations

If the projections are needed for two or more subnational populations, one may utilize a projection of the national population along with projected ratios of subnational populations to the national population. 11/ Alternatively, one can use a variant of the cohort component method. 12/

For many planning exercises, projections of population totals at the national or subnational level without projecting age and sex composition are not sufficient. Hence, this section describes a variant of the cohort component method which can be used to project the age and sex structures of the national population or of urban and rural populations separately.

The procedure described here is similar to that employed to project the national population, except that additional steps are needed in order to introduce internal migration (box 24) into the projection process. In this description of the cohort component method, internal migration denotes migration between urban and rural areas. As in the case of the national population, the steps involved in making a projection of urban and rural populations depend on whether international migration is assumed to be among the components of population change.

### (a) Closed populations

The procedure to project urban and rural populations closed to international migration consists of steps that yield segments of age and sex structures of the populations at age 5 and over and below age 5. Those steps are shown schematically in figure IV. The procedure also includes steps to compute population aggregates and indicators of population structure and change. The discussion of the procedure will emphasize the steps related to internal migration.

Box 24

Glossary

**Age-specific net internal migration rate**

The net gain or loss to the survivors of a given age or age group within a given location, such as urban or rural areas at the end of a specified period due to internal migration occurring during that period, divided by the number of survivors of that age or age group within that location.

**Crude net internal migration rate**

The net change (loss or gain) to the population residing within a given geographical or residential location due to internal migration during a specified period, divided by the number of person-years-lived by th population of that location during the same period. It is frequently expr sed as net change due to internal migration per 1,000 population.

**Internal migration**

Movements of population within the national boundaries involving relatively permanent changes in residence. It is designated as out-migration from the standpoint of the location from which the movement occurs and as in-migration from that of the receiving nation.

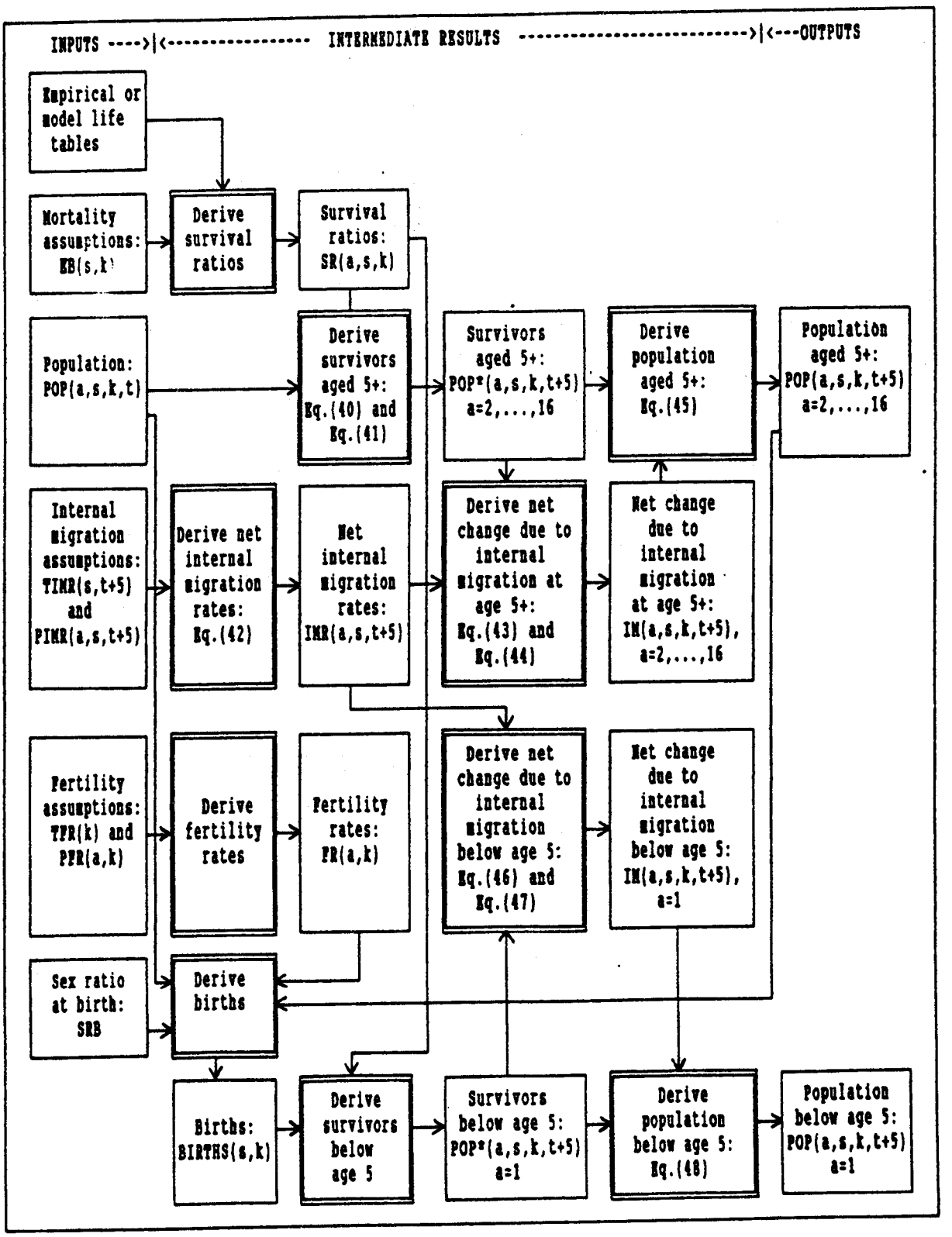
**Proportionate age-specific net internal migration rate**

The rate calculated by dividing a particular age-specific net internal migration rate by the sum of age-specific net internal migration rates across all ages or age groups. The sum of proportionate age-specific net internal migration rates across all ages equals one.

**Total net internal migration rate**

The sum of age-specific net internal migration rates across all ages or age groups.

Figure IV. Steps to derive age and sex structures of urban and rural populations closed to international migration at the end of projection interval t to t+5



Internal migration can be measured in a variety of ways from the perspective of geographical or residential locations within a given country. For any specific location, the measurement may seek to quantify both gains and losses to the population of the location concerned due to internal migration. Alternatively, it may seek to assess net changes to the population (gains minus losses) caused by internal migration. The measurement, which should normally be for a given time interval, may concern the numbers of persons gained and lost or the net change due to migration during the interval. Alternatively, it may concentrate on the gains and losses or net changes to the survivors in the given population at the end of the time interval caused by migration occurring during the interval.

Among the various measures of internal migration are gross and net internal migration rates. Depending on the types of data available, these rates may measure, say losses to the population in each given location (vis-à-vis those in other locations) or net changes to the population of each location caused by migration over a given time interval. Alternatively, these rates may measure losses or net changes to the populations of various locations at the end of a given time interval caused by migration occurring during the interval.

This discussion will employ internal migration measures that quantify changes to the survivors in the population of a given residential location at the end of a time interval which are caused by internal migration occurring over that interval. Such measures, which will be used in relation to the internal migration assumptions, include age-specific net internal migration rates by sex which apply to one of the two locational populations, for example the rural population. They can include selected summary internal migration measures, such as the total net internal migration rate and proportionate age-specific net internal migration rates, both of which are specific by sex. These measures can be also defined with respect to the rural population. Also, the measures may include the numbers of survivors by age and sex at the end of the interval that the rural population loses and/or gains on balance through internal migration.

(i) Segments of population structures at age 5 and over

a. Survival ratios

If mortality assumptions are expressed in selected summary measures of mortality such as expectations of life at birth, a projection of urban and rural populations for a given projection interval begins with the derivation of survival ratios for that interval. The survival ratios are derived separately for urban and rural areas by means of an urban-rural counterpart of the transformation described in equation (1).

b. Survivors aged 5 and over

In each population the numbers of survivors at age 5 and over at the end of the interval are calculated by means of urban-rural counterparts of steps described by equations (2) and (3). In particular, the numbers of survivors over this age span, except those at the open age group, are obtained as:

$$POP^{*}(a,s,k,t+5) = POP(a-1,s,k,t) \cdot SR(a,s,k); \quad (40)$$

$$a = 2,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2,$$

where:

$k = 1,2$ are urban and rural locations,

$POP^{*}(a,s,k,t+5)$ is the population (survivors) of age group a and sex s in location k at the end of interval,

$POP(a-1,s,k,t)$ is the population of age group (a-1) and sex s in location k at the beginning of the interval, and

$SR(a,s,k)$ is the survival ratio representing the probability of survival over the interval among persons who belong to age group a and sex s in location k at the end of the interval.

The numbers of survivors in the open age group is obtained as:

$$POP^{*}(16,s,k,t+5) = \left[\sum_{a=15}^{16} POP(a,s,k,t)\right] \cdot SR(16,s,k); \quad (41)$$

$$s = 1,2;$$

$$k = 1,2.$$

After deriving the numbers of survivors at age 5 and over in each location, these numbers must be modified using net internal migration rates.

c. Net internal migration rates

Where internal migration assumptions are specified in terms of age-specific net internal migration rates, they can be used directly in the projection, as shown in equations (43) and (46). Where the assumptions are formulated using the summary internal migration rates, examples of which are introduced above and used in this description, those measures need to be transformed into age-specific net internal migration rates.

The summary internal migration measures used here refer to the rural population. The total net internal migration rate for a given sex is the sum of the age specific net internal migration rates of that sex. It measures the extent to which rural survivors of a given sex are affected by internal

migration. Proportionate age specific net internal migration rates for a specific sex represent the age pattern of net internal migration rates for that sex.

Using these measures, age specific net internal migration rates are obtained through multiplication of the proportionate rates by age by the total net internal migration rate:

$$IMR(a,s,t+5) = TIMR(s,t+s) \cdot PIMR(a,s,t+5); \quad (42)$$

$$a = 1,\ldots,16;$$

$$s = 1,2,$$

where:

IMR(a,s,t+5) is the net internal migration rate applying to rural survivors of age group a and sex s at the end of the interval,

TIMR(s,t+5) is the total net internal migration rate applying to rural survivors of sex s at the end of the interval, and

PIMR(a,s,t+5) is the proportionate net internal migration rate applying to rural survivors of age group a and sex s at the end of the interval.

d. Population aged 5 and over

To derive the numbers of persons aged 5 and over at the end of the projection interval, it is first necessary to calculate net changes to urban and rural survivors at age 5 and above due to internal migration. In particular, net changes among the rural survivors are obtained by multiplying the numbers of rural (k=2) survivors by the net internal migration rates as follows:

$$IM(a,s,2,t+5) = POP^{*}(a,s,2,t+5) \cdot IMR(a,s,t+5); \quad (43)$$

$$a = 2,\ldots,16;$$

$$s = 1,2,$$

where:

IM(a,s,k,t+5) is the net change due to internal migration among survivors of age group a, sex s and location k at the end of the interval.

Net changes among urban survivors due to internal migration equal those among rural survivors, but with an opposite sign. This is so since any gain (loss) to urban survivors arising from internal migration equals the loss (gain) to rural survivors. Therefore:

$$IM(a,s,1,t+5) = - IM(a,s,2,t+5); \quad (44)$$

$$a = 2,\ldots,16;$$

$$s = 1,2.$$

Following this, the numbers of urban and rural survivors are modified by these net changes:

$$POP(a,s,k,t+5) = POP^{*}(a,s,k,t+5) + IM(a,s,k,t+5); \quad (45)$$

$$a = 2,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2.$$

This yields the age and sex structures of urban and rural populations aged 5 and over at the end of the interval.

(ii) Segments of population structures below age 5

a. Fertility rates, births and survivors below age 5

Deriving the segments of age and sex structures below age 5 for the populations of the two locations initially involves the same steps as those described in the projection of the national population closed to international migration. For each location, these steps include deriving age-specific fertility rates, the numbers of births by sex, and the numbers of survivors below age 5. The steps involved are the urban-rural counterparts of the steps described by equations (4) through (8).

b. Population below age 5

To allow for the effect of internal migration the numbers of survivors below age 5, net changes resulting from internal migration among these survivors must be calculated. Net changes among the rural survivors are obtained as follows:

$$IM(1,s,2,t+5) = POP^{*}(1,s,2,t+5) \cdot IMR(1,s,t+5); \quad (46)$$

$$s = 1,2,$$

and those for urban survivors are:

$$IM(1,s,1,t+5) = - IM(1,s,2,t+5); \quad (47)$$

$$s = 1,2.$$

Lastly, for each sex within each location the numbers of survivors below age 5 are modified by these net changes:

$$POP(1,s,k,t+5) = POP^{*}(1,s,k,t+5) + IM(1,s,k,t+5); \quad (48)$$

$$s = 1,2;$$

$$k = 1,2.$$

This completes the derivation of the age and sex structures of urban and rural populations closed to international migration at the end of a five-year projection interval.

### (iii) Other results

The urban-rural projection can yield other results for those two populations as well as for the national population. Those other results include:

a. Population aggregates;
b. Indicators of population structures;
c. Indicators of population distribution;
d. Rates of population change.

Most of the results obtained under these four categories are similar to those discussed earlier in connection with the projection of the national population. Most of them are, however, calculated for the urban and rural, as well as for the national population.

#### a. Population aggregates

The population aggregates obtained in the course of a projection of urban and rural populations include all those listed earlier, which in the case of this projection are for the urban, rural and the national population. Those aggregates, except the numbers of deaths, are calculated using steps similar to those indicated by equations (9) through (17). The numbers of deaths in urban and rural populations are calculated by means of a step which is similar to that described by equation (35), while the number of deaths in the national population equals the sum of urban and rural deaths. In addition, the population aggregates include net changes in urban and rural populations due to internal migration.

i. Change due to internal migration

For a given interval, the net change to the rural population due to internal migration is derived in a way similar to that of calculating net change to the national population due to international migration. In particular, the change due to internal migration can be obtained by a reverse survival of net changes due to internal migration among rural (k=2) survivors at the end of the interval, followed by aggregation:

$$NCDIM(2) = \sum_{a=1}^{16} \sum_{s=1}^{2} IM(a,s,2,t+5) \;/\; SRF(a,2,s), \tag{49}$$

where for each s:

$$SRF(a,2,s) = \begin{cases} 0.67 + 0.33 \cdot SR(1,2,s), & \text{when } a = 1 \\ ( 1 + SR(a,2,s) ) \;/\; 2, & \text{when } 1 < a \leq 16, \end{cases}$$

and where:

NCDIM(k) is the net change to the population of location k due to internal migration occurring during the interval, and

SRF(a,2,s) is the survival ratio factor used to reverse survive the net change due to internal migration among the rural survivors of age group a and sex s.

As the gain (loss) to the rural population due to internal migration equals the loss (gain) to the urban population, net change to the urban population due to internal migration equals the net change to the rural population with an opposite sign:

$$NCDIM(1) = - NCDIM(2). \tag{50}$$

ii. Deaths

The number of deaths in each population are calculated in a way similar to that used to obtain the number of deaths in an open national population. The number of deaths in either population equals the difference between the number of births and the population growth plus the net change in the population due to internal migration:

$$DEATHS(k) = BIRTHS(k) - POPGR(k) + NCDIM(k); \tag{51}$$

$$k = 1,2,$$

where:

DEATHS(k) is the number of deaths occurring in location k during the interval,

BIRTHS(k) is the number of births occurring in location k during the interval,

POPGR(k) is the population growth in location k over the interval.

b. Indicators of the population structure

The indicators of population structure calculated for the urban and rural populations are the same as those obtained for the national population. However, those indicators are calculated separately for the urban, rural and national populations, using steps similar to those indicated by equations (19) through (27).

c. Indicators of the population distribution

In addition to other indicators, a projection of urban and rural populations makes it possible to compute the proportions of the national population urban and rural, indicators of the population distribution.

i. Proportions urban and rural

The proportion urban (k=1) is obtained as a ratio of the urban population size to the national population size:

$$PURB(t+5) = POP(1,t+5) \;/\; POP(t+5), \qquad (52)$$

where:

PURB(t+5) is the proportion of the national population urban at the end of the interval, and

POP(k,t+5) is the size of the population of location k at the end of the interval.

The proportion rural can be calculated as a complement of the proportion urban:

$$PRUR(t+5) = 1 - PURB(t+5), \qquad (53)$$

where:

PRUR(t+5) is the proportion of the national population rural at the end of the interval.

d. Rates of population change

In the course of a projection of urban and rural populations closed to international migration, it is possible to derive for the urban, rural and the national populations, all those rates of population change that can be computed as part of projecting the national population closed to international migration. In addition, the crude net internal migration rate can be computed for urban and rural populations.

i. Crude net internal migration rate

For each population, the average annual crude net internal migration rate over a given projection interval is computed by dividing the average annual net change due to internal migration by the mid-interval population and multiplying by 1,000:

$$CIMR(k) = [\ (NCDIM(k) / 5) / MIPOP(k)\ ] \cdot 1{,}000; \qquad (54)$$

$$k = 1,2,$$

where:

CIMR(k) is the crude net internal migration rate of the population of location k for the interval, and

MIPOP(k) is the mid-interval population size of the population of location k.

(b) Open populations

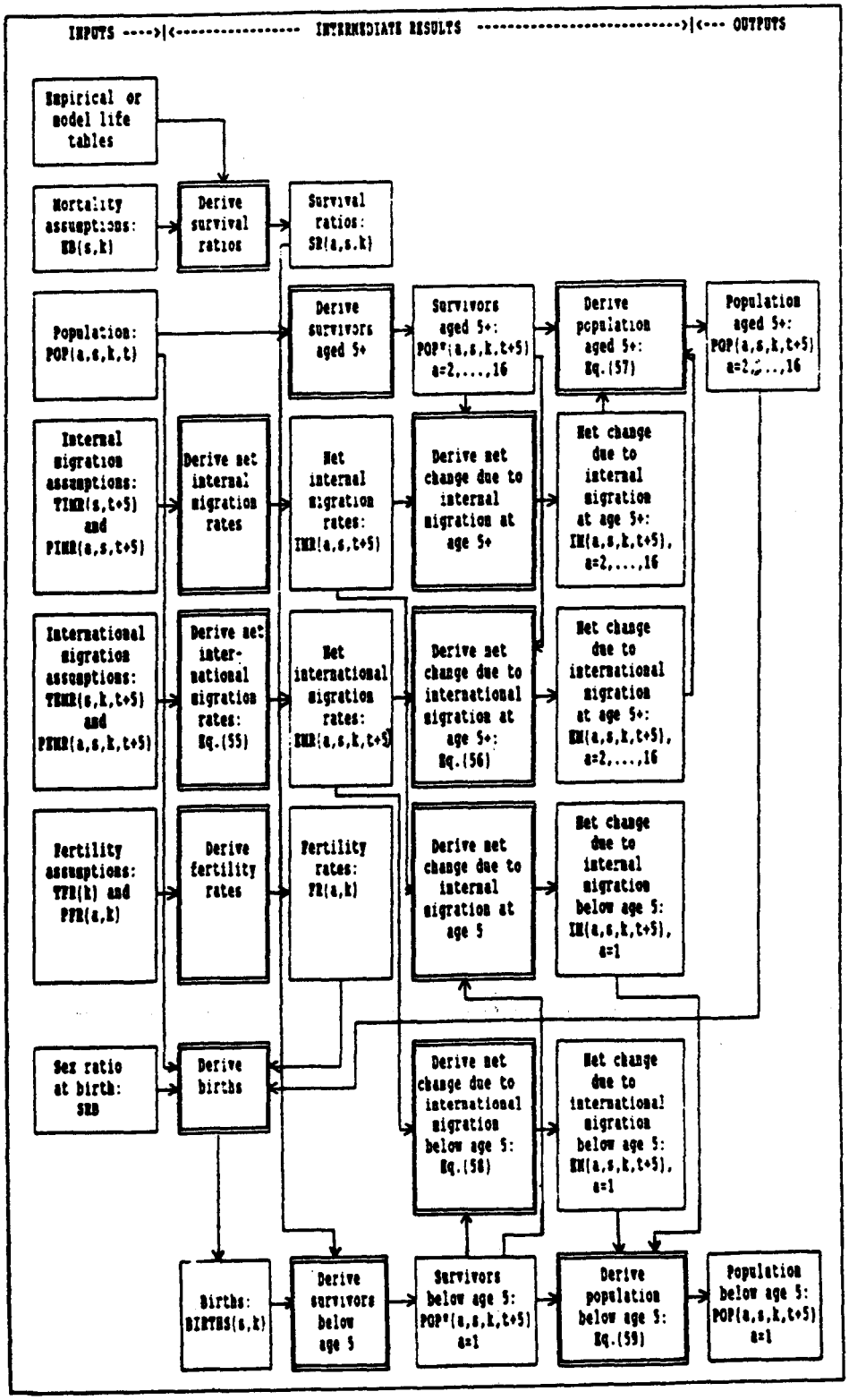
The procedure employed to project urban and rural populations open to international migration over a five-year projection interval makes use of all the steps described above and additional steps relating to international migration. Those steps, which are used to derive age and sex structures of urban and rural populations at the end of the interval, are depicted diagramatically in figure V.

(i) Segments of population structures at age 5 and over

a. Survival ratios, survivors aged 5 and over, and net internal migration rates

Steps involved in computing age and sex structures of urban and rural populations open to international migration initially include those required to compute survival ratios for both locations--urban and rural. In addition, they include steps to compute the numbers of survivors aged 5 and over (equations (40) and (41)) and net internal migration rates (equation (42)). If the international migration assumptions are formulated using relevant summary measures, further steps include the calculation of net international migration rates.

**Figure V.** Steps to derive age and sex structures of urban and rural populations open to international migration at the end of projection interval t to t+5



b. Net international migration rates

If the international migration assumptions for urban and rural populations are specified in terms of total net international migration rates and age specific proportionate net international migration rates, the international migration rates are obtained using an urban-rural counterpart of the step indicated by equation (32). In particular, the rates are computed as:

$$EMR(a,s,k,t+5) = TEMR(s,k,t+5) \cdot PEMR(a,s,k,t+5); \quad (55)$$

$$a = 1,\ldots,16;$$

$$s = 1,2,$$

$$k = 1,2,$$

where:

EMR(a,s,k,t+5) is the net international (external) migration rate applying to survivors of age group a and sex s in location k at the end of the interval,

TEMR(s,k,t+5) is the total net international (external) migration rate applying to survivors of sex s in location k at the end of the interval, and

PEMR(a,s,k,t+5) is the proportionate net international (external) migration rate applying to survivors of age group a and sex s in location k at the end of the interval.

c. Population aged 5 and over

To derive the numbers of persons aged 5 and over at the end of the projection interval, it is necessary to calculate net changes due to internal migration among urban and rural survivors at age 5 and over. Those net changes are obtained as indicated by equations (43) and (44). It is also necessary to derive net changes due to international migration among the survivors at age 5 and over using urban-rural counterparts of steps described by equation (33). In particular:

$$EM(a,s,k,t+5) = POP^{*}(a,s,k,t+5) \cdot EMR(a,s,k,t+5); \quad (56)$$

$$a = 2,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2,$$

where:

EM(a,s,k,t+5) is the net change due to international (external) migration among survivors of age group a and sex s in location k at the end of the interval.

Then, the numbers of urban and rural survivors at age 5 and above are modified using net changes due to both internal and international migration:

$$POP(a,s,k,t+5) = POP^{*}(a,s,k,t+5) + IM(a,s,k,t+5) + EM(a,s,k,t+5); \quad (57)$$

$$a = 2,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2,$$

where:

IM(a,s,k,t+5) is the net change due to internal migration among survivors of age group a and sex s in location k at the end of the interval.

This step completes the derivation of segments of the population structures at age 5 and above.

(ii) Segments of population structures below age 5

a. Fertility rates, births and survivors below age 5

The derivation of the segments of population structures below age 5 begins with calculations of fertility rates, numbers of births and the numbers of survivors below age 5 in urban and rural populations, respectively. The steps involved are identical to those used to project urban and rural populations closed to international migration.

b. Population below age 5

The numbers of survivors below age 5 are next used along with net internal migration rates to compute net changes due to internal migration among those survivors. The computations are described by equations (46) and (47). Also, the numbers of survivors are used along with net international migration rates to compute net changes among the survivors due to this type of migration:

$$EM(1,s,k,t+5) = POP^{*}(1,s,k,t+5) \cdot EMR(1,s,k,t+5); \quad (58)$$

$$s = 1,2.$$

$$k = 1,2.$$

The numbers of urban and rural survivors below age 5 are then modified using net changes due to both internal and international migration:

$$POP(1,s,k,t+5) = POP^*(1,s,k,t+5) + IM(1,s,k,t+5) + EM(1,s,k,t+5); \quad (59)$$

$$s = 1,2;$$

$$k = 1,2.$$

This completes the derivation of age and sex structures of urban and rural populations open to international migration at the end of a five-year projection interval. The step is followed by aggregation of those structures across locations to obtain the age and sex structure of the national population.

(iii) Other results

This type of projection yields population aggregates, indicators of the population structure and distribution as well as rates of population change. As in the case of an urban-rural projection closed to international migration, most of those other results obtained in the current type of projection are for the urban, rural and the national populations.

a. Population aggregates

Population aggregates include all those obtained in an urban-rural projection closed to international migration. In addition, they include two additional aggregates, which are net changes to population due to international migration and net changes to population resulting from combined internal and international migration. The two indicators are respectively calculated for the urban, rural and the national population and for the urban and rural populations.

Net changes to urban and rural populations due to international migration are obtained using urban-rural counterparts of the steps described by equation (37). The net change to the national population is the sum of the net changes to the urban and rural populations. On the other hand, net changes to those two populations resulting from internal migration are obtained using steps indicated by equations (49) and (50). Lastly, net changes due to combined migration for the two populations are obtained as sums of net changes resulting from international and internal migration.

Only after net changes due to migration are calculated, is it possible to derive the numbers of births in the urban, rural and the national populations. In the two former populations, those numbers are derived by taking into account net changes due to both internal and international migration. The number of deaths in the national population equals the sum of the numbers of deaths in urban and rural populations.

i. Deaths

The number of deaths in urban or rural population equals the difference between the number of births and the population growth, plus net changes due to internal and international migration:

$$DEATHS(k) = BIRTHS(k) - POPGR(k) + NCDIM(k) + NCDEM(k); \quad (60)$$

$$k = 1,2,$$

where:

NCDEM(k) is the net change in the population of location k due to international (external) migration during the interval.

b. Indicators of the population structure

These indicators include all those that can be calculated in the course of a projection of urban and rural populations closed to international migration.

c. Rates of population change

Two rates, in addition to those computed in the course of a projection of urban and rural populations closed to international migration can be calculated. They are crude net international migration rates and crude net combined migration rates. The former rates are derived for the urban, rural and the national population. The latter rates are computed for the urban and rural population.

Crude net international migration rates are obtained using an urban-rural equivalent of the step indicated by equation (40). To obtain crude net combined migration rates, it is first necessary to compute crude net internal migration rates, using the steps described by equation (51).

i. Crude net combined migration rate

For the population of either location, this rate is derived as a sum of crude net international and internal migration rates:

$$CCMR(k) = CEMR(k) + CIMR(k); \quad (61)$$

$$k = 1,2,$$

where:

CCMR(k) is the crude net combined migration rate of the population of location k for the interval, and

CEMR(k) is the crude net international (external) migration rate of the population of location k for the interval.

## C. The inputs

This section will discuss issues relating to the inputs used with the cohort component method. In particular, it will list the types of inputs required to make a projection with the method and describe how those inputs can be prepared.

### 1. Types of inputs required

The inputs required to apply the cohort component technique depend on the type of projection that one wishes to make. As indicated in the preceding section, the projection can be that of the national population, which can be either closed or open to international migration. Alternatively, the projection can be that of urban and rural populations, which can also be either affected or unaffected by international migration.

#### (a) National projection

To project the national population closed to international migration, the inputs must include:

(i) The initial age and sex structure;
(ii) Assumptions on mortality;
(iii) Assumptions on fertility.

To project an open national population, the inputs should also include:

(iv) Assumptions on international migration.

The initial population structure should pertain to the mid-point of the initial year of the plan, denoted below as year 0. The population shall be disaggregated into the standard five-year age groups, 0-4, 5-9, ..., 75+.

Assumptions on mortality and fertility should refer to the consecutive five-year projection intervals, 0-5, 5-10,... Where the measures used to formulate international migration assumptions are of the type used in section B (see equation (32)), those assumptions ought to refer to the end of the consecutive five-year projection intervals although they represent migration conditions during the intervals.

#### (b) Urban-rural projections

In order to project urban and rural populations which are closed to international migration, the following inputs would be needed for each population:

(i) The initial age and sex structure;
(ii) Assumptions on mortality;
(iii) Assumptions on fertility.

Also, the inputs should include:

(iv) Assumptions on internal migration, for, say, rural population.

If urban and rural populations open to international migration are to be projected, the inputs would need also to include for each population:

(v) International migration assumptions.

The urban and rural age and sex structures should refer to the initial year of the plan. Similarly, mortality and fertility assumptions ought to refer to the five-year time intervals of the plan horizon. Where the internal and international migration assumptions are specified in terms of the measures used in section B (equations (42) and (55)), the assumptions ought to refer to the end of the five-year intervals.

(c) Measures used in assumptions

One may formulate assumptions on the future trends in the components of population change using different demographic measures as long as those measures can be used to derive the basic inputs for the projection--survival ratios and fertility and migration rates. The measures chosen in this presentation enable one to specify both the level as well as the pattern of the relevant rates.

Thus, the total fertility rate and proportionate age-specific fertility rates make it possible to specify the level of fertility as well as its pattern by age. The use of expectations of life at birth by sex enables one to specify both the level and sex pattern of mortality. Alternatively, the use of infant mortality rates and expectations of life at exact age 5 by sex makes it possible to choose both the level and the age-sex pattern of mortality. Where infant mortality rates and expectations of life at age 5 are used, they are employed, respectively, as indicators of mortality below 5 and mortality at age 5 and above. 13/ The use of total net migration rates along with proportionate age-specific net migration rates by sex allow the user to specify both the level and age-sex pattern of migration over time. This is true for both international and internal migration.

The choice of mortality measures would normally be based on an analysis involving the observed life tables for the population concerned and the model life tables of different families. The appropriate mortality measure to use in any given application of the method will depend on whether or not the family of model life tables selected for the projection closely approximates the observed mortality experience by age in the population concerned. Where the observed age pattern of mortality is well approximated by the selected family of life tables over the entire age range, it will be appropriate to use expectations of life at birth. However, where the observed pattern is closely approximated only within age groups 0-4 and 5+, which may often be the case in developing countries, the use of infant mortality rates and expectations of life at age 5 would be preferable. In this instance, these two mortality measures can be used as representations of mortality conditions below age 5 and at age 5 and above.

## 2. Preparation of the inputs

The preparation of inputs for a projection would include the preparation of the age and sex structure of the population for the initial year of the plan. Also, it would involve formulating assumptions on the components of population change during the plan horizon.

### (a) Initial age and sex structure(s)

If a population projection is to be used in a development plan, it will usually be convenient to start that projection in the initial year of the plan. Since at the time of plan preparation this initial year will lie in the future, it will be necessary to make a preliminary projection of the age and sex structure(s) for the initial year of the plan by using the age and sex structure(s) of the most recent population census or survey.

In rare circumstances, where the census or survey precedes the initial year of the plan by a multiple of five (say, five or ten) years, the initial age and sex structure(s) could be derived directly from the preliminary projection, by carrying this projection forward until the initial year. Where the time difference between the initial year of the plan and the year of the most recent census or survey is not a multiple of five, the preliminary projection can be carried out beyond the initial year of the plan and the age and sex structure(s) for the initial year can be derived by interpolation.

In many developing countries population censuses are defective due to under-enumeration, over-enumeration, and/or age misreporting. Where these defects are believed to be quantitatively important, it is essential to adjust the census population structure(s) before using it (them) to prepare the preliminary projection. Various procedures have been expressly designed to address these problems (See, for example, Ewbank, 1981; and United Nations, 1983, pp. 241-249).

Where the preliminary projection is made for urban and rural populations, the original census or survey structures of population for the two areas should be based on a de jure (or place of permanent residence) enumeration. Furthermore, where appropriate, they may be based on definitions of urban and rural populations utilizing the community population size as the classification criterion.

### (b) Estimates and assumptions for the preliminary projection

Available observations on the relevant demographic measures will never refer to a time period that comes all the way up to the initial year of the plan. Therefore, it is necessary first to make a preliminary projection covering the period from the most recent census or survey to the initial year of the plan period. The following comments describe data needs related to the formulation of assumptions to be used with such a preliminary projection.

In order to formulate reliable assumptions for making the preliminary population projection, it is necessary to have the most accurate available information on recent levels and trends of fertility and mortality. In some countries, reliable information on recent fertility and mortality will be available from vital statistics collected through a vital registration system or a sample survey registration scheme. Where reliable information is lacking and no estimates of recent fertility and mortality are available, an indirect estimation of their levels and trends may prove necessary. Fortunately, a number of techniques are available for indirect estimation of the fertility and mortality measures used in this chapter (United Nations, 1967 and 1983).

Where the population to be projected is open to international migration, it will be necessary to have information or estimates on the relevant measures of international migration for recent years. The international migration measures used in this chapter could be derived from census or survey information on immigration and emigration for the country concerned. The data should refer to survivors among immigrants and emigrants, make it possible to classify them by age and sex at the time of census or survey, and include information on the date of international migration. In addition, if one needs international migration measures for urban and rural areas, the data should also contain information on the previous location of residence of emigrants and the current location of residence of immigrants.

International migration data are often the weakest among population statistics in developing countries, especially where international migration is quantitatively unimportant or where the Government lacks the means to or interest in collecting those data. In view of this, it may often prove necessary to estimate international migration measures by indirect means. The measures used in this chapter can be obtained using one of the standard methods of indirect migration estimation--the survival ratio method of migration estimation (United Nations, 1970). However, the method can only yield estimates for the national population, but not for urban and rural populations. To apply the method, age and sex structures of the population must be available from two censuses, preferably taken a multiple of five years apart. Intercensal mortality estimates must also be available.

If projections of urban and rural populations are sought, inputs for the preliminary projections would also need to make use of observations or estimates of recent internal (urban-rural) migration. Where retrospective internal migration data are available from population censuses or surveys, observations on internal migration measures used in the chapter can be readily obtained. Along with standard information on age, sex and residence at the time of census or survey, the data should include the following information relating to persons who have moved prior to the census or survey: the date of the move (or whether the change of residence occurred in the five years before the census or survey) along with residence (rural or urban) before migration.

Where this type of data is unavailable, it will be necessary to estimate internal migration indirectly. The measures used in the chapter could be obtained by the survival-ratio method of migration estimation. This method requires age and sex structures of the rural population based on adjacent

censuses, along with an estimate of intercensal mortality for the rural areas. These measures can only be estimated if the country remained closed to any significant international migration during the intercensal period.

The data required to prepare a preliminary projection might be difficult to obtain, especially if an urban-rural projection is sought. This may be particularly true where the population to be projected is open to international migration. In such situations, the planner may need to adopt a pragmatic approach to filling data gaps even if there is not a very solid basis for doing so. For example, national estimates of mortality may be used in place of missing urban and rural estimates in order to make an urban-rural projection. Using national estimates would be justified if there is some evidence that urban-rural mortality differentials are relatively small.

## (c) Assumptions for the plan projection

### (i) National projection

To formulate assumptions for the national projection over the plan period, it is normally necessary to consider expected socio-economic trends as well as social, economic and population policies to be implemented over the plan horizon. Where feasible, the likely impact of these trends and policies on the components of population change should be assessed on the basis of research findings concerning the determinants of fertility, mortality and international migration. In doing so, it will be necessary to allow for the fact that the effects of various factors on the components of change will be felt after a delay, the length of which may vary from one factor to another.

#### a. Mortality

In order to formulate mortality assumptions it will often be necessary to assess the effect on mortality of likely changes in such factors as female education, availability of and access to health services, household incomes and environmental conditions. Thus, if the proportion of females receiving formal education is expected to rise over the plan horizon, the mortality assumptions should normally allow for rising survival ratios or increases in expectations of life at birth, but only after a time lag. This is so since higher maternal education is generally associated with improvements in household sanitary conditions, better child nutrition and a greater demand for health services catering to children.

#### b. Fertility

Fertility assumptions could be formulated by identifying likely developments in educational attainment, health status, and labour force participation among women, as well as by considering likely changes in child education, infant and childhood mortality, urbanization and family planning. If, for example, educational attainment and labour force participation among women, as well as school enrolment among children, are expected to rise during

the plan period, the fertility assumptions could allow for a falling total fertility rate. Similarly, if rapid urbanization is envisaged, assumptions would generally allow for falling fertility. An assumption of falling fertility could also be made if the Government and/or non-governmental organizations were expected to pursue an effective family planning programme. The effects of these various factors may be subject to lags of different duration.

Some of the factors considered here as direct determinants of fertility may also influence fertility indirectly, through their effect on the age of marriage. In particular, increases in female labour force participation and urbanization may both influence age of marriage among females, leading to lower proportions married at early childbearing ages and to lower fertility rates at those ages. The same effect could occur as a result of increases in school attendance among females and the resultant increases in their educational attainment. Some of these effects can be felt however, after considerable delays.

Improvements in infant and child survivorship could also directly contribute to a fertility reduction. Where fertility is uncontrolled and breast-feeding is widespread and lengthy, fewer deaths in infancy and early childhood, as a rule, may lead to fewer births, for biological reasons. Even if fertility is subject to control, lower infant and early childhood mortality may bring about lower fertility. Under such conditions, the improved survivorship would enable parents to achieve the desired number of surviving children through fewer births. However, child survivorship often improves hand-in-hand with maternal health, in which case, these health improvements would tend to increase fertility through increased fecundity (the biological capacity to conceive and bear children) (United Nations, 1987).

#### c. International migration

International migration assumptions, especially for small- to medium-size countries experiencing considerable international migration, may have to be rather speculative. For countries that are net importers of labour, the assumptions might be formulated by taking into account expected trends in the domestic labour requirements as well as domestic labour force trends. For labour exporting countries, domestic unemployment and labour earnings as well as likely economic trends in the countries that are net importers of workers ought to be considered. Where international migration is subject to considerable governmental control, these assumptions should reflect expected government policies towards population movements across its national boundaries.

### (ii) Urban-rural projections

#### a. Fertility, mortality and international migration

Where the inputs are prepared for the projection of urban and rural populations, mortality, fertility and international migration assumptions can

be formulated as for the national population. However, in this case, they should reflect likely changes in the relevant factors and policies and their impact in rural and urban areas separately.

b. Internal migration

To formulate internal (urban-rural) migration assumptions, it would be necessary to identify likely changes in the factors that bear upon decisions to migrate between urban and rural areas. In most circumstances, it will prove necessary to take into account expected trends in employment, underemployment and open unemployment in both urban and rural areas. Also, it would be desirable to consider household incomes and the cost of living as well as the availability of educational, health, and housing services in those areas. For example, where urban-rural disparities in employment and earning opportunities as well as differentials in the availability of services are large and expected to remain so, internal migration assumptions would have to allow for a sizeable net outmigration from rural areas.

Also, it would often be important to consider the relative size of the urban population and the concomitant absorptive capacity of the urban centres. Where the urban population is relatively small, the assumptions might need to provide for a relatively small net outflow from the rural areas in view of the limited absorptive capacity of the urban areas. Normally, these assumptions should not imply rates of growth of the urban population that surpass, say, the highest rates on record in the country concerned or those in similar countries.

In addition, where they are present, the effects of active government policies that directly impact on urban-rural migration would need to be taken into account. Such policies might include the repatriation of the residents of urban squatter settlements to the rural areas from which they came or the resettling of populations among geographical regions that may involve the crossing of urban-rural boundaries.

(d) Revising assumptions for the plan projection

In most planning exercises population projections will be prepared prior to making projections of the socio-economic variables. As a result, before formulating assumptions for the population projection it will normally be necessary to informally predict the future changes in socio-economic variables which are likely to have a demographic impact, including those to be subsequently projected. Furthermore, it would be necessary to infer the likely effects of those changes on the components of population change. In view of this, at least initially, the demographic assumptions may lead to population projections which appear implausible in the context of subsequent socio-economic projections.

Once the formal projections of the socio-economic variables are made, it may prove useful to re-examine the assumptions underlying the population projection, and if necessary, to revise them and to project population again. This iterative approach would allow each new set of demographic assumptions to reflect the changes in the socio-economic projections. Repeating population and socio-economic projections in an iterative fashion is also likely to lead to a greater consistency among the various projection outcomes as well as assumptions.

## D. Illustrative examples of projections

Three examples are presented in this section to illustrate the preparation of different types of population projections using the cohort component method. The first example will show how to project a closed national population. The second will project an open national population. The third example will illustrate the projection of urban and rural populations which are closed to international migration.

Each example will describe the required calculations for the first five-year projection interval, 0-5. Projection results for a 20-year projection period will also be presented since planners will often find it useful to project population over a 20-year period even in planning exercises that focus exclusively on the medium term.

### 1. National population

This section will first illustrate a projection of the national population which is closed to international migration. Then it will describe a projection of such a population open to international migration.

#### (a) Closed population

This example will first show how to derive the segment of the age and sex structure at age 5 and over at the end of the projection interval 0-5. Next, it will illustrate how to obtain the segment of the population structure below age 5. Lastly, other results obtained for the interval 0-5 will be calculated. The example will be based on the inputs contained in table 1, which include the initial age and sex structure of the population and assumptions on mortality and fertility.

The initial population structure and the assumptions selected for this example portray an age and sex structure and trends in mortality and fertility representative of a country that has already experienced substantial mortality decline along with a limited fertility reduction. As indicated in figure VI, the lower end of the age pyramid suggests some recent fertility reduction. Figure VII shows a continuation of the mortality decline associated with some deceleration in that decline. And figure VIII indicates a further decline in fertility, which accelerates over the projection period.

Table 1. Inputs for making a projection of the national population closed to international migration

(a) Initial population by age and sex (thousands):

| Age group | Male | Female |
|---|---|---|
| 0-4 | 784.5 | 750 7 |
| 5-9 | 740.0 | 694.8 |
| 10-14 | 624.0 | 580.5 |
| 15-19 | 486.8 | 448 3 |
| 20-24 | 402.2 | 359.8 |
| 25-29 | 381.5 | 348.3 |
| 30-34 | 358.7 | 352 2 |
| 35-39 | 312.4 | 300.9 |
| 40-44 | 223.8 | 206.7 |
| 45-49 | 152.1 | 158.5 |
| 50-54 | 192.5 | 174.6 |
| 55-59 | 157.0 | 164.0 |
| 60-64 | 128.1 | 126.1 |
| 65-69 | 90.1 | 105.6 |
| 70-74 | 44.2 | 63.5 |
| 75+ | 38.6 | 69.4 |

(b) Mortality assumptions:

| Time interval | Expectations of life at birth (years) | |
|---|---|---|
| | Male | Female |
| 0-5 | 51.63 | 53 01 |
| 5-10 | 55.65 | 57 06 |
| 10-15 | 59.19 | 60 47 |
| 15-20 | 62.16 | 63 13 |

(c) Fertility assumptions:

| Time interval | Total fertility rates | Proportionate age-specific fertility rates by age | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 |
| 0-5 | 5.57 | 0.081 | 0.251 | 0.245 | 0.195 | 0.143 | 0 059 | 0.026 |
| 5-10 | 5.16 | 0.087 | 0.259 | 0.246 | 0.190 | 0.139 | 0 058 | 0 022 |
| 10-15 | 4.54 | 0.092 | 0.268 | 0.249 | 0.183 | 0.133 | 0.056 | 0.019 |
| 15-20 | 3.79 | 0.097 | 0.277 | 0.254 | 0.177 | 0.124 | 0.054 | 0.016 |

**Figure VI. Initial age and sex structure of the population**

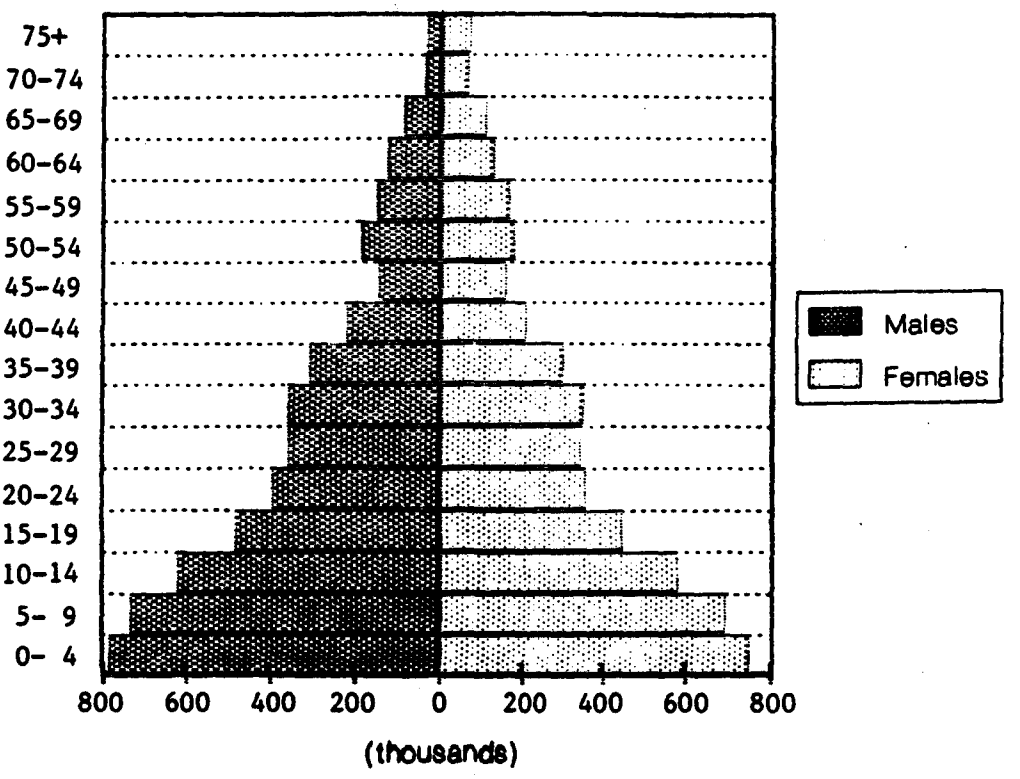


**Figure VII. Expectations of life at birth**

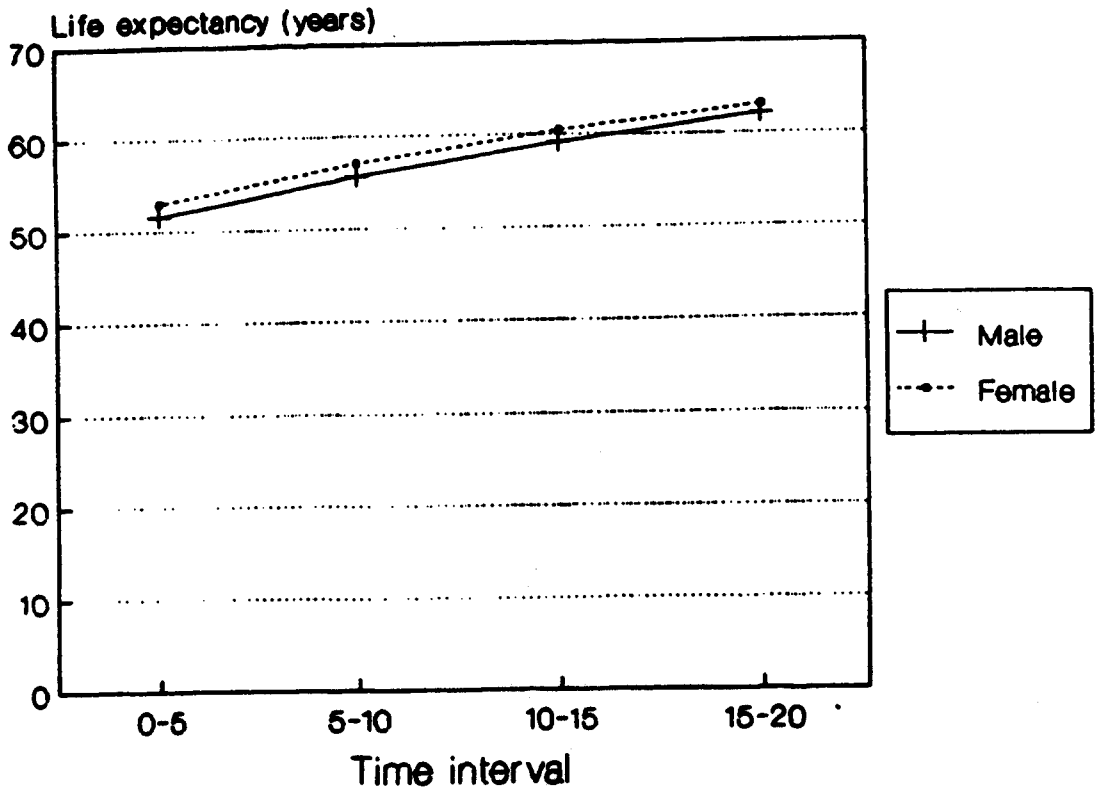
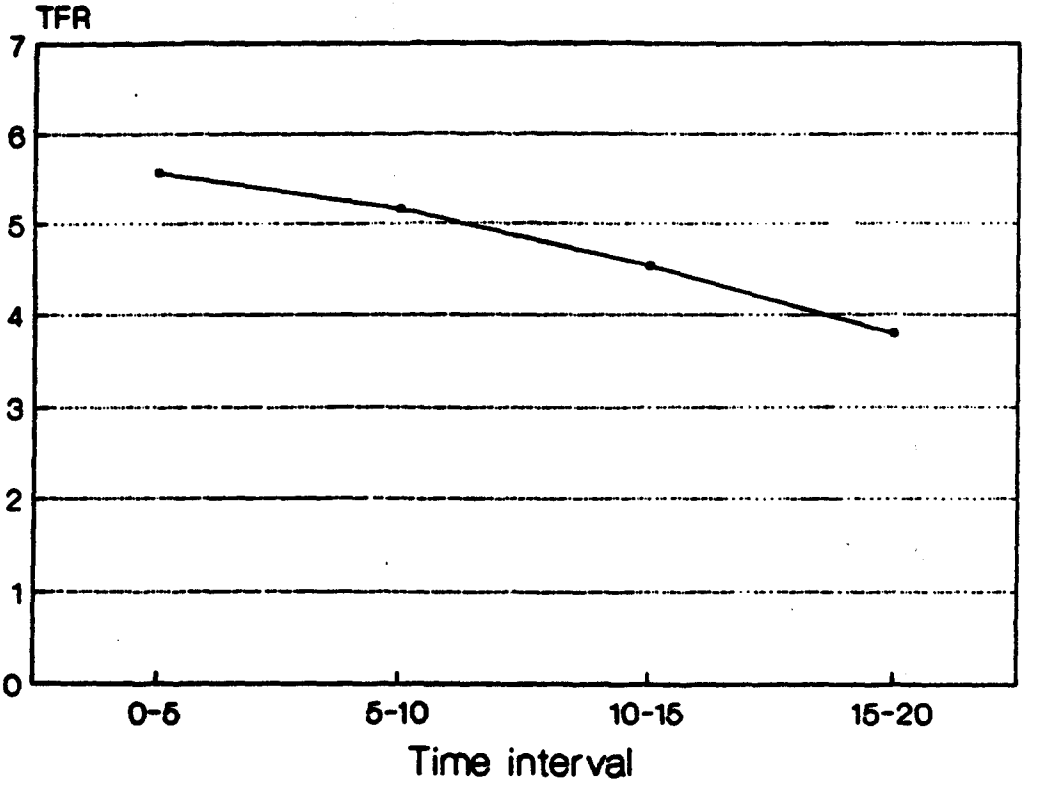


**Figure VIII. Total fertility rate**



(i) Segment of the population structure at age 5 and over

To derive the segment of the population structure at age 5 and above, it is initially necessary to obtain survival ratios from mortality assumptions. These survival ratios are then multiplied by the appropriate populations to determine the number of survivors.

a. Survival ratios

The procedure to calculate survival ratios from expectations of life at birth using a selected family of model life tables is the same for each sex. Its application will be illustrated using the expectation for males, 51.63 years (table 1), and the United Nations family of South Asian model life tables (United nations, 1981). 14/ Since the use of a suitable family of model life tables in a population projection may be fairly important, box 25 describes a procedure to select the most suitable family.

Table 2 illustrates the way male survival ratios are calculated using an expectation of life at birth and a family of model life tables. Columns 2 and 3 show relevant male survival ratios (panel A) and selected numbers of person-years-lived (panel B) in the South Asian family that correspond to male expectations of life at birth 51.0 and 52.0. It is noted that all survival ratios shown in the two columns, except those in the last row (70+ to 75+), along with the numbers of person-years-lived come directly from the model life tables in question.

The survival ratios shown in the last row are not available in the model life tables and, therefore, must be calculated using the numbers of person-years-lived at ages 70 and 75 (panel B). At each level of expectation of life at birth, this ratio is obtained as the ratio of the number of person-years-lived at age 75 and over to the number of person-years-lived at age 70 and above. Thus, at the expectation of life at birth 51.0 (column 2), the ratio in question, 0.5485, is obtained as 176,292 divided by 321,294, where 321,394 is the number of person-years-lived at age 70 and over and 176,292 is the number of person-years-lived at age 75 and above.

Survival ratios that correspond to male expectation of life at birth 51.63 (column 4) can be obtained by linear interpolation of survival ratios at expectations of life 51.0 and 52.0. The interpolation factor is 0.37, which is obtained as (52.0 - 51.63)/(52.0 - 51.0). Using this factor, the first survival ratio 0.83874, shown in column 4, is calculated as:

$$0.83874 = (0.37)\,(0.8342) + (1 - 0.37)\,(0.8414), \qquad (1)$$

where 0.8342 and 0.8414 are the survival ratios at expectations of life 51.0 and 52.0, respectively. The remaining survival ratios for the expectation of life 51.63 are obtained in the same way.

Box 25

## Selecting a family of model life tables

There are a number of alternative ways of selecting a family of model life tables that best represents the age pattern of mortality in a given population. These different ways are variants of the same procedure which utilizes an empirical life table, such as the national life table for a given sex. They also make use of model life tables for the same sex that belong to different families of model life tables, such as the United Nations or the Coale-Demeny model life tables. a/

One way to select a family of model life tables was recently described by the United Nations. b/ The procedure utilizes age-specific central death rates ($_nm_x$ values) or age-specific probabilities of dying ($_nq_x$ values) to compute indices of similarity between empirical and model age patterns of death rates or probabilities of dying.

An index of similarity, which is computed for each of the families considered can be obtained for the entire age span or selected portions of it. An index for the entire span can be computed as follows:

$$I = \sum_{x=0}^{w} | e_0 (x,x+n) - M | / N$$

where:

| | |
|---|---|
| I | is the index of similarity for a given family of model life tables, |
| x = 0,1,5,...,w | are the lower limits of age groups used in a life table 0, 1-4, 5-9, ..., w+ , where w is the lower limit of the open age group, |
| $e_0$ (x,x+n) | is the expectation of life at birth in the family of model life tables used which corresponds to the empirical death rate or probability of dying in age group x to x+n, |
| M | is the median of all different $e_0(x,x+n)$ values, and |
| N | is the number of age groups within the entire age span. |

(continued)

Box 25 (continued)

Once the values of the index of similarity are computed using different families of model life tables, one has a basis for selecting the most representative of the families. The family that yields the lowest value of the index is the one that best fits the empirical age pattern of mortality.

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a/ See Model Life Tables for Developing Countries (United Nations publication, Sales No. E.81.XIII.7). Also, Ansley J. Coale, Paul Demeny and Barbara Vaughan, Regional Model Life Tables and Stable Populations, 2nd ed. (New York, Academy Press, 1983).

b/ MORTPAK-LITE: The United Nations Software Package for Mortality Measurement; Interactive Software for the IBM-PC and Compatibles (United Nations publication, Sales No. E.88.XIII.2).

b. Population aged 5 and over

When the survival ratios are applied to the population at the beginning of a given projection interval, the result is population aged 5 and over at the end of the interval. In particular, when the relevant survival ratios (table 3, column 4) are applied to the population in year 0 (column 3), one obtains the age and sex structure of the population at age 5 and over in year 5 (column 5). 15/

For example, the number of males aged 5-9 in year 5, 737.0 (column 5), is calculated as the product of the number of males aged 0-4 in year 0, 784.5 (column 3) and the survival ratio among males aged 5-9, 0.93949 (column 4):

$$737.0 = (784.5)\ (0.93949). \qquad (2)$$

The number of males aged 75+ in year 5, 45.6, is the product of the number of males aged 70+ in year 0, 82.8 (obtained as 44.2 + 38.6) and the appropriate survival ratio, 0.55073:

$$45.6 = (82.8)\ (0.55073). \qquad (3)$$

Table 2. Calculating survival ratios; results for males; projection interval 0-5

| | Age | Expectations of life at birth | | |
|---|---|---|---|---|
| | | 51.0 a/ | 52.0 a/ | 51.63 b/ |
| | (1) | (2) | (3) | (4) |
| PANEL A: Survival ratios | Birth to 0-4 | 0.8342 | 0.8414 | 0.83874 |
| | 0-4 to 5-9 | 0.9371 | 0.9409 | 0.93949 |
| | 5-9 to 10-14 | 0.9858 | 0.9867 | 0.98637 |
| | 10-14 to 15-19 | 0.9909 | 0.9914 | 0.99122 |
| | 15-19 to 20-24 | 0.9891 | 0.9897 | 0.98948 |
| | 20-24 to 25-29 | 0.9870 | 0.9876 | 0.98738 |
| | 25-29 to 30-34 | 0.9840 | 0.9848 | 0.98450 |
| | 30-34 to 35-39 | 0.9793 | 0.9803 | 0.97993 |
| | 35-39 to 40-44 | 0.9710 | 0.9723 | 0.97182 |
| | 40-44 to 45-49 | 0.9582 | 0.9599 | 0.95927 |
| | 45-49 to 50-54 | 0.9378 | 0.9399 | 0.93912 |
| | 50-54 to 55-59 | 0.9089 | 0.9115 | 0.91054 |
| | 55-59 to 60-64 | 0.8665 | 0.8696 | 0.86845 |
| | 60-64 to 65-69 | 0.8064 | 0.8102 | 0.80879 |
| | 65-69 to 70-74 | 0.7290 | 0.7334 | 0.73177 |
| | 70+ to 75+ | 0.5485 | 0.5520 | 0.55073 |
| PANEL B: Person-years lived at indicated age and above | 70 | 321394 | 336971 | |
| | 75 | 176292 | 185997 | |

a/ From South Asian model life tables.
b/ Calculated by interpolation.

Table 3. Deriving the segment of age and sex structure of the national population closed to international migration at age 5 and over: end of projection interval 0-5

| Sex | Age group | Population in year 0 a/ (thousands) | Survival ratios b/ | Population in year 5 c/ (thousands) |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| Male | 0-4 | 784.5 | | |
| | 5-9 | 740.0 | 0.93949 | 737.0 |
| | 10-14 | 624.0 | 0.98637 | 729.9 |
| | 15-19 | 486.6 | 0.99121 | 618.5 |
| | 20-24 | 402.2 | 0.98948 | 481.5 |
| | 25-29 | 361.5 | 0.98738 | 397.1 |
| | 30-34 | 358.7 | 0.98450 | 355.9 |
| | 35-39 | 312.4 | 0.97993 | 351.5 |
| | 40-44 | 223.6 | 0.97182 | 303.6 |
| | 45-49 | 152.1 | 0.95927 | 214.5 |
| | 50-54 | 192.5 | 0.93912 | 142.8 |
| | 55-59 | 157.0 | 0.91054 | 175.3 |
| | 60-64 | 128.1 | 0.86845 | 136.3 |
| | 65-69 | 90.1 | 0.80879 | 103.6 |
| | 70-74 | 44.2 | 0.73177 | 65.9 |
| | 75+ | 38.6 | 0.55073 | 45.6 |
| Female | 0-4 | 750.7 | | |
| | 5-9 | 694.8 | 0.93874 | 704.7 |
| | 10-14 | 580.5 | 0.98621 | 685.2 |
| | 15-19 | 448.3 | 0.99031 | 574.9 |
| | 20-24 | 359.8 | 0.98701 | 442.5 |
| | 25-29 | 348.3 | 0.98521 | 354.5 |
| | 30-34 | 352.2 | 0.98311 | 342.4 |
| | 35-39 | 300.9 | 0.97981 | 345.1 |
| | 40-44 | 206.7 | 0.97541 | 293.5 |
| | 45-49 | 158.5 | 0.96831 | 200.2 |
| | 50-54 | 174.6 | 0.95422 | 151.2 |
| | 55-59 | 164.0 | 0.92892 | 162.2 |
| | 60-64 | 126.1 | 0.88943 | 145.9 |
| | 65-69 | 105.6 | 0.83374 | 105.1 |
| | 70-74 | 63.5 | 0.75775 | 80.0 |
| | 75+ | 69.4 | 0.54846 | 72.9 |

a/ From table 1.
b/ From table 2, col. 4.
c/ For age groups:
5-9 through 70-74:
(Entry in a preceding row of col. 3) . (Entry in the given row of col. 4).

75+:
(The sum of entries corresponding to age groups 70-74 and 75+ in col. 3) . (Entry corresponding to the latter of age groups in col. 4).

(ii) Segment of the population structure below age 5

To obtain the segment of the population structure below age five at the end of a given five year projection interval, it is necessary first to calculate the total number of births for the interval using, among other things, fertility rates for the interval. It will then be necessary to calculate the number of survivors among these births.

a. Fertility rates

In this example, the fertility assumptions are specified in terms of the total fertility rate and proportionate age specific fertility rates. Using these measures, age specific fertility rates can be calculated as shown in table 4. In particular, any age specific fertility rates (column 3) can be obtained as a product of the given total fertility rate divided by 5 and the corresponding proportionate age specific fertility rate (column 2).

For example, the fertility rate for the age group 20-24, 0.2796, is obtained as:

$$0.2796 = (5.57/5)\ (0.251), \qquad (4)$$

where 5.57 is the total fertility rate, shown in table 1, and 0.251 is the proportionate fertility rate for the age group 20-24.

b. Births

Before the number of births can be calculated, it is necessary to derive the mid-interval numbers of women of the childbearing period. These numbers can be obtained as geometric means of the numbers of women in various five-year age groups at the beginning and the end of the interval (table 5).

For example, the number of women for age group 15-19, 507.7, shown in column 4 is calculated as:

$$507.7 = [(448.3)\ (574.9)]^{1/2}, \qquad (5)$$

where 448.3 is the number of women aged 15-19 at the beginning of the interval (column 2) and 574.9 is the corresponding number at the end of the interval (column 3).

The number of births is calculated from the fertility rates and the mid-interval numbers of women as illustrated in table 6. The number is obtained as five times the sum of the products of the age-specific fertility rates (column 2) and the mid-interval numbers of women by age (column 3). The sum of the products (column 4) stands for the average annual number of births which is multiplied by 5 to calculate the total number of births for the interval:

Table 4. Calculating age-specific fertility rates; projection interval 0-5

| Age group | Proportionate age-specific fertility rates a/ | Age-specific fertility rates b/ |
|---|---|---|
| (1) | (2) | (3) |
| 15-19 | 0.081 | 0.0902 |
| 20-24 | 0.251 | 0.2796 |
| 25-29 | 0.245 | 0.2729 |
| 30-34 | 0.195 | 0.2172 |
| 35-39 | 0.143 | 0 1593 |
| 40-44 | 0.059 | 0 0657 |
| 45-49 | 0.026 | 0.0290 |

a/ From table 1.
b/ (Total fertility rate/5) . (Col. 2).

Table 5. Calculating mid-interval numbers of women of childbearing ages; projection interval 0-5

(Thousands)

| Age group | Women of childbearing age | | |
|---|---|---|---|
| | In year 0 a/ | In year 5 b/ | At mid-point of time interval 0-5 c/ |
| (1) | (2) | (3) | (4) |
| 15-19 | 448.3 | 574.9 | 507.7 |
| 20-24 | 359.8 | 442.5 | 399.0 |
| 25-29 | 348.3 | 354.5 | 351.4 |
| 30-34 | 352.2 | 342.4 | 347.3 |
| 35-39 | 300.9 | 345.1 | 322.2 |
| 40-44 | 206.7 | 293.5 | 246.3 |
| 45-49 | 158.5 | 200.2 | 178.1 |

a/ From table 1.
b/ From table 3, col. 5.
c/ $\{ (\text{Col. 2}) \cdot (\text{Col. 3}) \}^{1/2}$.

Table 6. Calculating the average annual number of births: projection interval 0-5

| Age group | Age-specific fertility rates a/ | Mid-interval number of women b/ (thousands) | Numbers of births c/ (thousands) |
|---|---|---|---|
| (1) | (2) | (3) | (4) |
| 15-19 | 0.0902 | 507.7 | 45.8 |
| 20-24 | 0.2796 | 399.0 | 111.6 |
| 25-29 | 0.2729 | 351.4 | 95.9 |
| 30-34 | 0.2172 | 347.3 | 75.4 |
| 35-39 | 0.1593 | 322.2 | 51.3 |
| 40-44 | 0.0657 | 246.3 | 16.2 |
| 45-49 | 0.0290 | 178.1 | 5.2 |
| Total | | | 401.4 |

a/ From table 4, col. 3.
b/ From table 5, col. 4.
c/ (Col. 2) . (Col. 3).

$$2{,}007.0 = (5)\ (401.4), \qquad (6)$$

where 401.4 is the average annual number of births (total in column 4).

In order to calculate the numbers of survivors aged 0-4 by sex, the total number of births by sex should be subdivided by sex. The total number of births is disaggregated using proportions of births by sex, derived from the assumed value of the sex ratio at birth (105). The proportion male based on this sex ratio at birth is:

$$0.5122 = 105 / (\ 100 + 105\ ),$$

and the proportion female is:

$$0.4878 = 100 / (\ 100 + 105\ ).$$

The number of males born during the interval is therefore:

$$1{,}028.0 = (0.5122)\ (2{,}007.0), \qquad (7)$$

and the number of females born is:

$$979.0 = (0.4878)\ (2{,}007.0). \qquad (7)$$

c. Population below age 5

To derive the population below age 5 by sex at the end of a five-year time interval, it is necessary to multiply the numbers of births by sex by the appropriate survival ratios. This is so since in a closed population the number of persons aged 0-4 at the end of a five-year time interval consists of the survivors of children born during the interval.

Thus, the number of male children aged 0-4 at the end of the interval, 862.2 (column 4 of table 7), is obtained as:

$$862.2 = (1{,}028.0)\ (0.83874), \qquad (8)$$

where 1,028.0 (column 2) is the number of male births and 0.83874 (column 3) is survival ratio for males.

The number of female children aged 0-4 at the end of the interval is calculated in the same manner:

$$828.9 = (979.0)\ (0.84666). \qquad (8)$$

This completes the projection of the age and sex structure of the closed population for the end of the projection interval 0-5.

When repeated over subsequent time intervals, the calculations illustrated above for the interval 0-5 produce projected population structures for the end of intervals 5-10, 10-15, and so on. Thus, a projection over a 20-year time

Table 7. Deriving the segment of age and sex structure of the national population closed to international migration below age 5: end of projection interval 0-5

| Sex | Births for interval 0-5 a/ (thousands) | Survival ratios b/ | Population aged 0-4 in year 5 c/ (thousands) |
|---|---|---|---|
| (1) | (2) | (3) | (4) |
| Male | 1028.0 | 0.83874 | 862.2 |
| Female | 979.0 | 0.84666 | 828.9 |

a/ Derivation illustrated in text.
b/ From table 2, col. 4.
c/ (Col. 3) . (Col. 4).

period produces the age and sex structures presented in table 8. The age and sex structure at the end of the 20-year interval is depicted in figure IX.

(iii) Other results

A variety of indicators may be calculated along with population structures, which include:

a. Population aggregates;
b. Indicators of population structure;
c. Rates of population change.

The calculation of those indicators will be illustrated below for the projection interval 0-5 and the results will be presented in table 9 along with indicators for the subsequent projection intervals.

a. Population aggregates

A number of population aggregates can be obtained by adding up the numbers of persons in designated age and sex groups.

i. Population size

The population size at the end of the interval 0-5--that is, in year 5, 11,210.4 (table 9)--is the result of adding up the numbers of persons of both sexes and the various age groups at that date. The growth of the population over the projection period is illustrated in figure X.

ii. Young-age population

The young-age population for the end of the interval, 4,547.9, is obtained by summing up the numbers of persons of both sexes in age groups 0-4 through 10-14.

iii. Working-age population

The working-age population, 6,189.4, is found by adding up the numbers of persons of both sexes in age groups 15-19 through 60-64.

iv. Old-age population

The old-age population, 473.1, is obtained by summing up the numbers of both sexes in age groups 65-69 through 75+. The growth of the young age, working age and old age populations over the projection period is illustrated in figure XI.

Table 8. Projected national population closed to international migration, by age and sex

(Thousands)

| Sex | Age group | Year | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 5 | 10 | 15 | 20 |
| Male | 0-4 | 784.5 | 862.2 | 957.3 | 1022.5 | 1012.7 |
| | 5-9 | 740.0 | 737.0 | 822.2 | 923.3 | 994.4 |
| | 10-14 | 624.0 | 729.9 | 729.3 | 815.6 | 917.5 |
| | 15-19 | 486.6 | 618.5 | 724.9 | 725.4 | 812.0 |
| | 20-24 | 402.2 | 481.5 | 613.4 | 720.1 | 721.5 |
| | 25-29 | 361.5 | 397.1 | 476.7 | 608.5 | 715.5 |
| | 30-34 | 358.7 | 355.9 | 392.2 | 472.0 | 603.6 |
| | 35-39 | 312.4 | 351.5 | 350.2 | 387.1 | 466.9 |
| | 40-44 | 223.6 | 303.6 | 343.4 | 343.5 | 381.0 |
| | 45-49 | 152.1 | 214.5 | 293.2 | 333.5 | 335.1 |
| | 50-54 | 192.5 | 142.8 | 203.2 | 279.9 | 320.3 |
| | 55-59 | 157.0 | 175.3 | 131.5 | 189.0 | 262.4 |
| | 60-64 | 128.1 | 136.3 | 154.4 | 117.4 | 170.5 |
| | 65-69 | 90.1 | 103.6 | 112.4 | 129.5 | 99.9 |
| | 70-74 | 44.2 | 65.9 | 77.7 | 86.2 | 101.4 |
| | 75+ | 38.6 | 45.6 | 63.0 | 81.4 | 99.1 |
| Female | 0-4 | 750.7 | 828.9 | 916.3 | 974.5 | 961.3 |
| | 5-9 | 694.8 | 704.7 | 789.6 | 882.6 | 945.8 |
| | 10-14 | 580.5 | 685.2 | 697.4 | 783.3 | 877.0 |
| | 15-19 | 448.3 | 574.9 | 680.2 | 693.5 | 779.9 |
| | 20-24 | 359.8 | 442.5 | 569.3 | 675.3 | 689.6 |
| | 25-29 | 348.3 | 354.5 | 437.5 | 564.5 | 670.7 |
| | 30-34 | 352.2 | 342.4 | 349.9 | 433.2 | 560.0 |
| | 35-39 | 300.9 | 345.1 | 337.0 | 345.5 | 428.7 |
| | 40-44 | 206.7 | 293.5 | 338.2 | 331.6 | 340.8 |
| | 45-49 | 158.5 | 200.2 | 285.7 | 330.7 | 325.2 |
| | 50-54 | 174.6 | 151.2 | 192.3 | 276.1 | 320.9 |
| | 55-59 | 164.0 | 162.2 | 141.9 | 181.9 | 262.8 |
| | 60-64 | 126.1 | 145.9 | 146.3 | 129.5 | 167.6 |
| | 65-69 | 105.6 | 105.1 | 124.0 | 126.4 | 113.4 |
| | 70-74 | 63.5 | 80.0 | 81.7 | 98.5 | 102.3 |
| | 75+ | 69.4 | 72.9 | 86.4 | 97.5 | 116.1 |

**Figure IX. Terminal age and sex structure of the population**

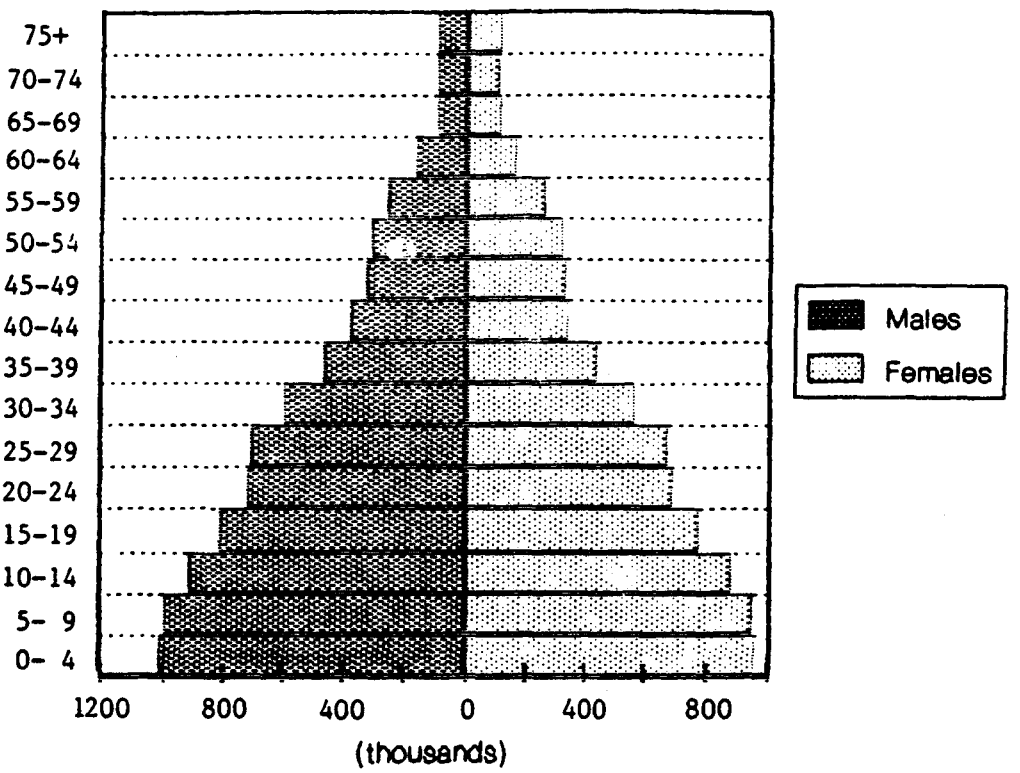


Table 9. Population aggregates, indicators of the population structure and rates of population change of the national population closed to international migration

| Indicators | Year | | | | |
|---|---|---|---|---|---|
| | 0 | 5 | 10 | 15 | 20 |
| Population aggregates (thousands) | | | | | |
| Population size | 10000.0 | 11210.4 | 12619.0 | 14159.4 | 15675.6 |
| Young-age | 4174.5 | 4547.9 | 4912.1 | 5401.8 | 5708.7 |
| Working-age | 5414.1 | 6189.4 | 7161.4 | 8138.2 | 9335.0 |
| Old-age | 411.4 | 473.1 | 545.2 | 619.5 | 632.2 |
| School-age | 4336.2 | 4974.2 | 5626.3 | 6219.1 | 6737.7 |
| Women of childbearing age | 2174.7 | 2553.1 | 2997.8 | 3374.3 | 3794.9 |
| Mid-interval population size | | 10587.9 | 11893.9 | 13367.0 | 14898.2 |
| Number of person-years lived | | 52939.6 | 59469.3 | 66835.1 | 74491.1 |
| Population growth | | 1210.4 | 1408.6 | 1540.4 | 1516.2 |
| Births | | 2007.0 | 2157.0 | 2244.0 | 2179.0 |
| Deaths | | 796.6 | 748.4 | 703.6 | 662.8 |
| Indicators of the population structure | | | | | |
| Proportions by broad age groups | | | | | |
| At young age (0-14) | 0.42 | 0.41 | 0.39 | 0.38 | 0.36 |
| At working age (15-64) | 0.54 | 0.55 | 0.57 | 0.57 | 0.60 |
| At old age (65+) | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| Dependency ratios | | | | | |
| Young-age | 0.77 | 0.73 | 0.69 | 0.66 | 0.61 |
| Old-age | 0.08 | 0.08 | 0.08 | 0.08 | 0.07 |
| Total | 0.85 | 0.81 | 0.76 | 0.74 | 0.68 |
| Median age of population | 19.4 | 19.4 | 20.0 | 20.9 | 21.9 |
| Proportion of women of childbearing age | 0.22 | 0.23 | 0.24 | 0.24 | 0.24 |
| Sex ratio of the population | 104 | 104 | 104 | 104 | 105 |
| Rates of population change (per thousand) | | | | | |
| Birth rate | | 37.9 | 36.3 | 33.6 | 29.3 |
| Death rate | | 15.0 | 12.6 | 10.5 | 8.9 |
| Natural increase | | 22.9 | 23.7 | 23.0 | 20.4 |
| Population growth | | 22.9 | 23.7 | 23.0 | 20.3 |

Figure X. Population size, school-age population and number of women of childbearing ages

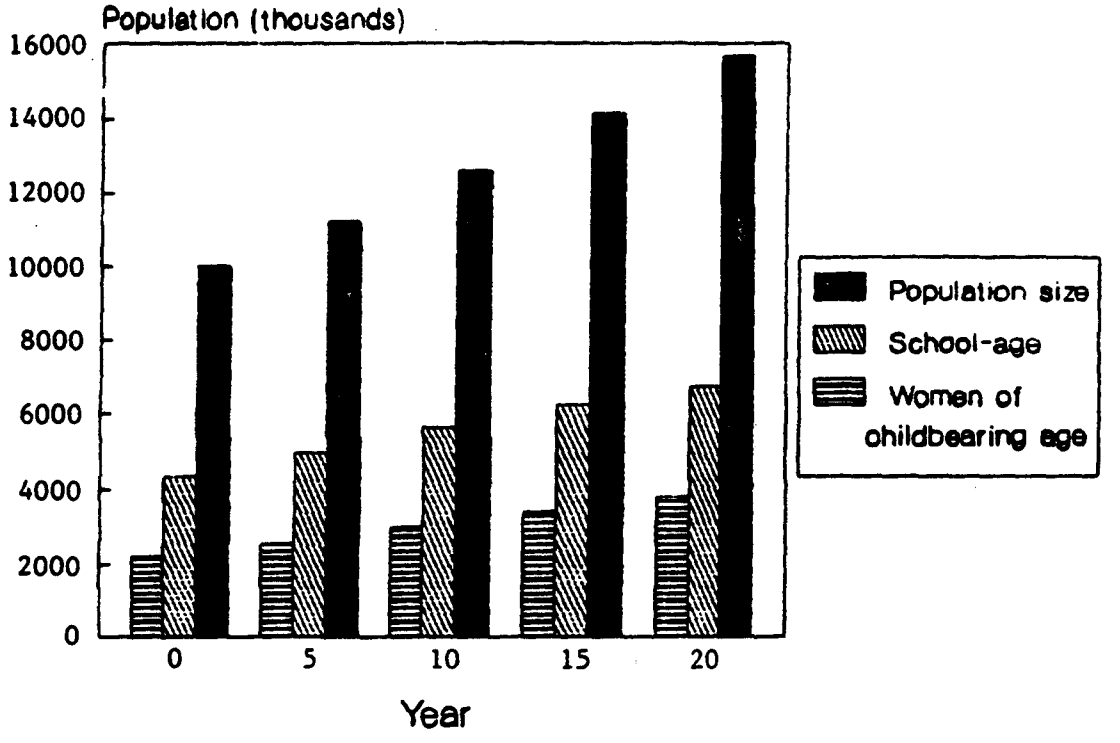
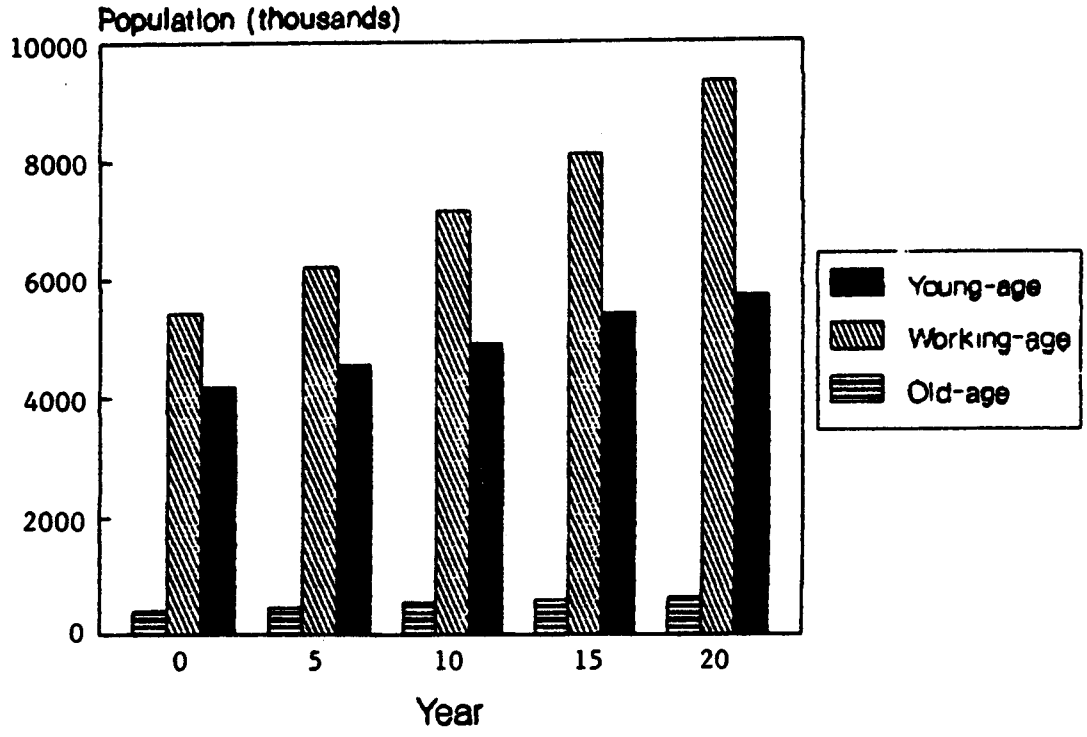


Figure XI. Young-age population, working-age population and old-age population



v. School-age population

The school-age population at the end of the interval, 4,974.2, is derived by adding up the numbers of persons of both sexes in age groups 5-9 through 20-24.

vi. Women of the childbearing ages

The number of women of the childbearing ages, 2,553.1, is the sum of the numbers of women in age groups 15-19 through 45-49. The growth of the school-age population and women of the childbearing ages is illustrated in figure X.

vii. Mid-interval population size

The mid-interval population size for the interval 0-5, 10,587.9, is calculated as the geometric mean of the population sizes at the beginning and the end of the interval--10,000.0 and 11,210.4, respectively:

$$10{,}587.9 = [(10{,}000.0)\ (11{,}210.4)]^{1/2}. \qquad (15)$$

viii. Total number of person-years-lived

The number of person-years-lived by the population during the interval, 52.939.6, is calculated as 5 times the mid-interval population size:

$$52{,}939.6 = (10{,}587.9)\ (5). \qquad (16)$$

ix. Population growth

Among the aggregates indicating changes in the population size, the population growth, 1,210.4, is obtained as the difference between the population size at the end of the interval--11,210.4--and the population size at the beginning--10,000.0:

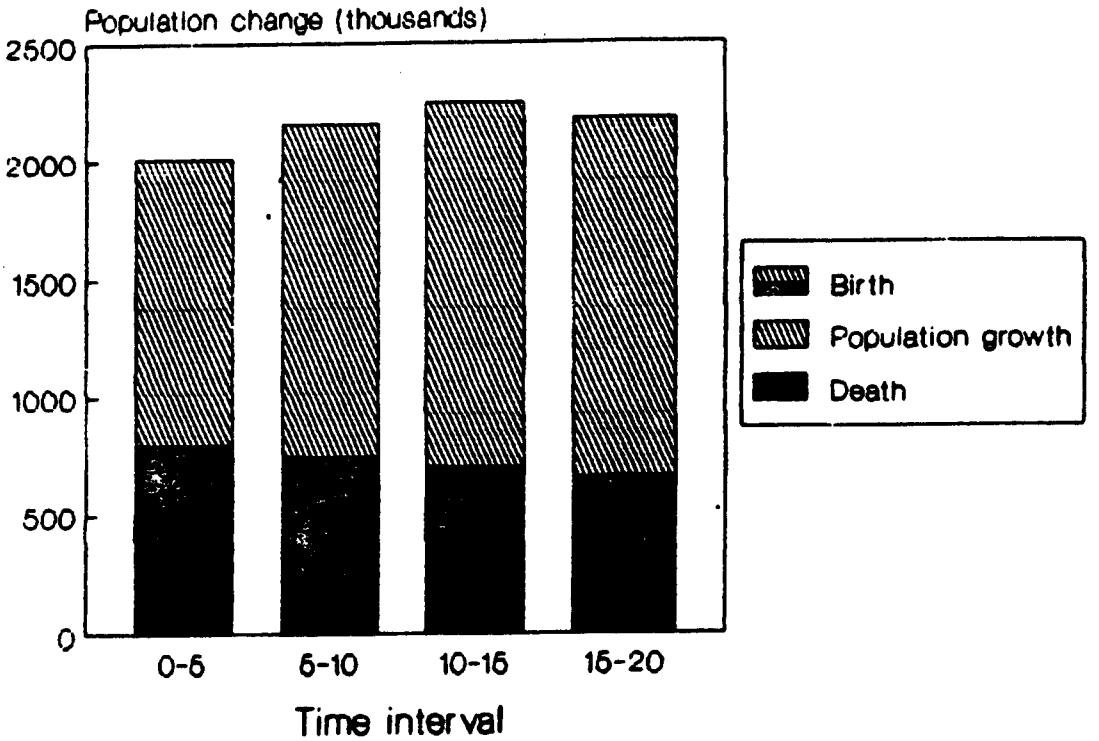
$$1{,}210.4 = 11{,}210.4 - 10{,}000.0. \qquad (17)$$

Population growth during each five-year interval of the projection period is illustrated in figure XII as the difference between births and deaths during the interval.

x. Births

The number of births, 2,007.0, is calculated in the course of deriving the age and sex structure of the population.

**Figure XII. Population growth, number of births and number of deaths**



xi. Deaths

The number of deaths, 796.6, is obtained as the difference between the number of births, 2,007.0, and the population growth, 1,210.4:

$$796.6 = 2{,}007.0 - 1{,}210.4. \quad (18)$$

Births and deaths are illustrated in figure XII.

b. Indicators of the population structure

i. Proportions by broad age groups

The first group of indicators of the population structure--proportions by broad age groups (0-14, 15-64 and 65+)--is obtained by dividing the young-age population, the working-age population and the old-age population, respectively, by the population size. In the course of the illustrative projection, those proportions are calculated for the end of the interval 0-5 as follows:

The proportion at young age:

$$0.41 = 4{,}547.9 / 11{,}210.4, \quad (19)$$

where 4,547.9 is the size of the young-age population;

The proportion at working age:

$$0.55 = 6{,}189.4 / 11{,}210.4, \quad (20)$$

where 6,189.4 is the size of the working-age population; and

The proportion at old age:

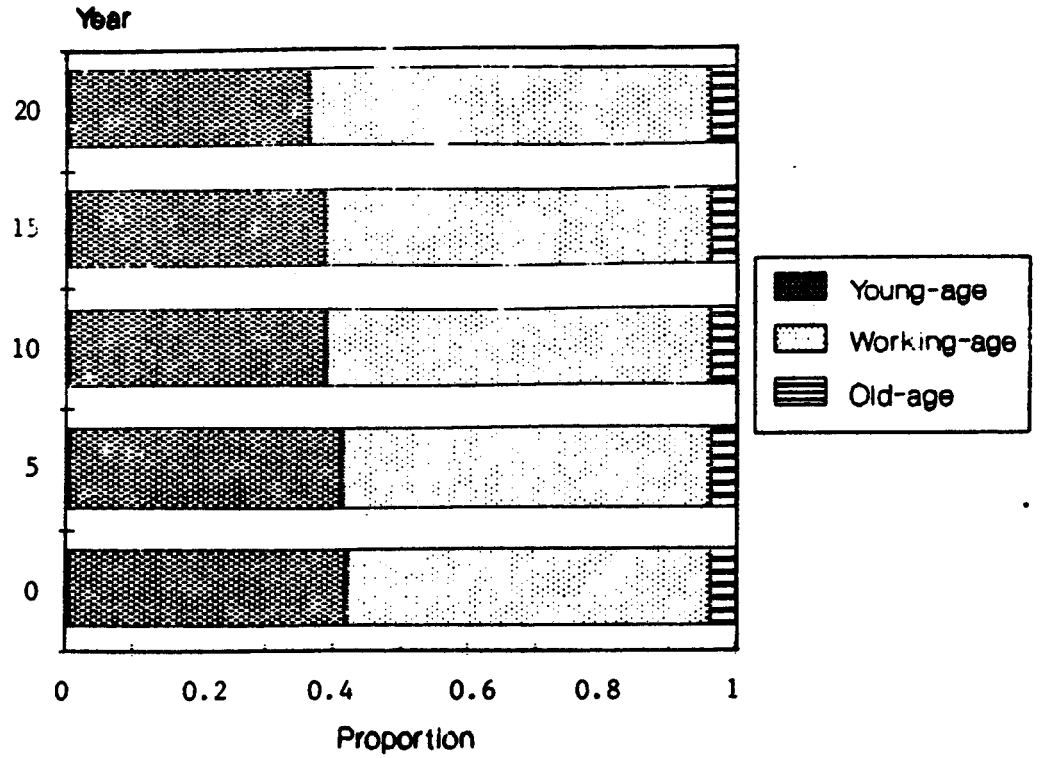
$$0.04 = 473.1 / 11{,}210.4, \quad (21)$$

where 473.1 is the size of the old-age population. The total population size is 11,210.4. These proportions at five-year intervals are shown in table 9 and illustrated in figure XIII.

ii. Dependency ratios

Dependency ratios are obtained in a similar manner. Thus, the three dependency ratios--the young-age dependency ratio, the old-age dependency ratio and the total dependency ratio--for the end of the interval 0-5 are calculated as:

Figure XIII. Proportions of the population in broad age groups (0-14, 15-64 and 65+)



The young-age dependency ratio:

$$0.73 = 4,547.9 / 6.189.4, \quad (22)$$

where 4,547.9 is the young age population;

The old-age dependency ratio:

$$0.08 = 473.1 / 6,189.4, \quad (23)$$

where 473.1 is the old-age population; and

The total dependency ratio:

$$0.81 = (4,547.9 + 473.1) / 6,189.4, \quad (24)$$

and where in each expression 6,189.4 is the working-age population.

The three dependency ratios are shown for dates five years apart in table 9 and figure XIV.

### iii. Median age of the population

The median age of the population is computed using the formula for calculating the median age from the grouped data. For the end of the 0-5 interval, the median age of the population, 19.4, is computed as follows:

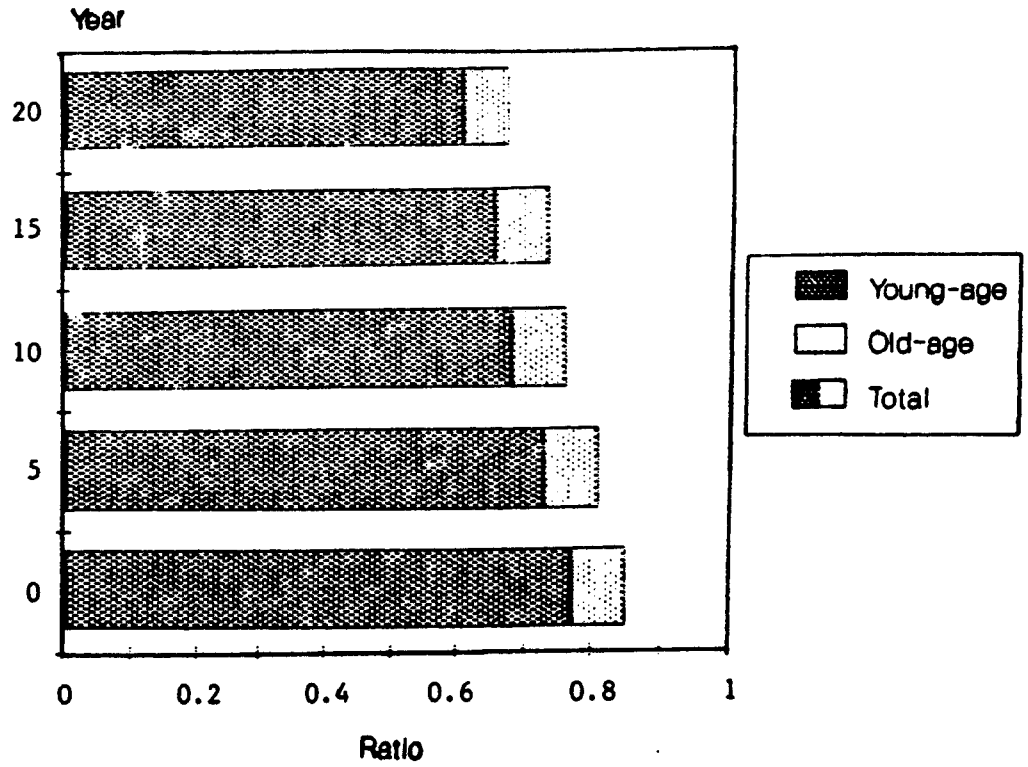
$$19.4 = (4 - 1) (5) + [ (11,210.4 / 2 - 4,547.9) / 1,193.4 ] (5). \quad (25)$$

Among the numbers used in the expression, the 4 in the first parenthesis stands for the fourth five-year age group, 15-19, which contains the middle member of the population; this term is multiplied by 5, the length of the five-year age group. The result, 15, is the lower limit of the five-year age group containing the middle number. The population size at the end of the interval 0-5 is 11,210.4. The number of persons in the population below age 15 (prior to the fourth age group) is 4,547.9, and 1,193.4 is the number of persons in the 15-19 age group. These two numbers are obtained by adding up the numbers of persons of both sexes in age groups 0-4 through 10-14 and within the age group 15-19, respectively. Again, the term is multiplied by 5, the length of the five-year age group.

### iv. Proportion of women in the childbearing ages

The proportion of women in the childbearing ages is obtained as the number of women of that age divided by the population size. For the end of the 0-5 interval, this proportion is:

**Figure XIV. Dependency ratios**



$$0.23 = 2{,}553.1 \;/\; 11{,}210.4, \qquad (26)$$

where 2.553.1 is the number of women in childbearing years and 11,210.4 is the population size.

### v. Sex ratio of the population

The sex ratio is a ratio of the number of males in the population to the number of females. The sex ratio at the end of the 0-5 interval is:

$$104 = (5{,}711.3 \;/\; 5{,}489.1) \cdot 100, \qquad (27)$$

where 5,721.3 and 5,489.1 are, respectively, the numbers of males and females in the population at that date.

## c. Rates of population change

In the case of a projection of the national population closed to international migration, these rates include the crude birth rate and the crude death rate along with the rate of natural increase and the rate of population growth.

### i. Crude birth rate

The average annual crude birth rate for the interval 0-5 is calculated as follows :

$$37.9 = [\ (\ 2{,}007.0 \;/\; 5\ ) \;/\; \ 10{,}587.9\ ] \cdot 1{,}000, \qquad (28)$$

where 2,007.0 is the number of births for the interval, 5 is the length of the interval, and 10,587.9 is the mid-interval population size.

### ii. Crude death rate

The average annual crude death rate for the same interval is calculated as:

$$15.0 = [\ (\ 796.6 \;/\; 5\ ) \;/\; \ 10{,}587.9\ ] \cdot 1{,}000, \qquad (29)$$

where 796.6 is the total number of deaths for the interval, 5 is the length of the interval, and 10,587.9 is the mid-interval population size.

### iii. Rate of natural increase

The rate of natural increase, which equals the difference between the crude birth rate and the crude death rate, is:

$$22.9 = 37.9 - 15.0, \quad (30)$$

where 37.9 and 15.0 are crude birth and death rates.

iv. Rate of population growth

Lastly, the rate of population growth is obtained from the population sizes at the beginning and the end of the interval, using the exponential growth-rate formula. For the period 0-5, this rate is obtained as:

$$22.9 = [\ \ln (\ 11{,}210.4 \ / \ 10{,}000.0\ )\ /\ 5\ ] \cdot 1{,}000, \quad (31)$$

where 11,210.4 and 10,000.0 are the population sizes in year 5 and 0, respectively, and 5 is the length of the interval. This rate, along with crude birth and death rates and the rate of natural increase is shown in table 9. As can be observed, in a closed population, the rate of natural increase is equal to the rate of population growth.

The crude birth rate, the crude death rate, the rate of natural increase and the rate of population growth are shown in table 9 and are illustrated in figure XV.

(b) Open population

A number of steps used to project an open national population are the same as those used to project a closed population. Therefore, this example will not illustrate those steps, but emphasize the steps relating to international migration. The example will make use of the inputs of table 1 along with those of table 10, in which assumptions on international migration are given for a 20-year time interval.

(i) Segment of the population structure at age 5 and over

a. Survival ratios and survivors aged 5 and over

The survival ratios are computed exactly as described for the closed population. The steps used to derive survivors aged 5 and over at the interval's end are identical to those employed to obtain population aged 5 and over in the closed population. As this example uses the same initial population structure and mortality assumptions as the previous one, the numbers of survivors aged 5 and over (table 11, column 3) are identical to the numbers obtained earlier (table 3, column 5). In a projection of an open population those numbers of survivors have to be modified using net international migration rates.

**Figure XV. Rates of population change**

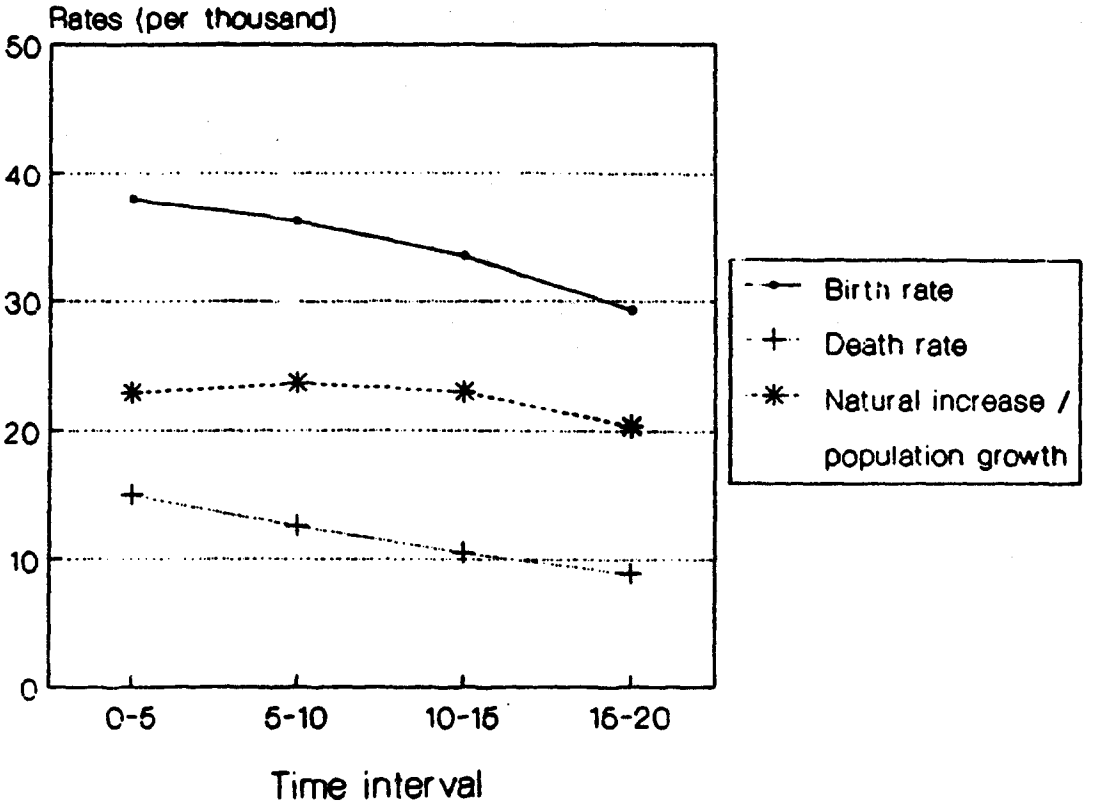


Table 10. International migration assumptions for making a projection of the national population

| Year | Total net international migration rates | Proportionate age-specific net international migration rates | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0-4 | 5-9 | 10-14 | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 | 50-54 | 55-59 | 60-64 | 65-69 | 70-74 | 75+ |
| | | | | | | | | Male | | | | | | | | | |
| 5 | -0.1949 | 0.023 | 0.024 | 0.027 | 0.009 | 0.040 | 0.229 | 0.296 | 0.190 | 0.125 | 0.074 | -0.005 | -0.008 | -0.008 | -0.006 | -0.005 | -0.003 |
| 10 | -0.2252 | 0.022 | 0.023 | 0.027 | 0.009 | 0.039 | 0.231 | 0.290 | 0.195 | 0.127 | 0.074 | -0.004 | -0.008 | -0.008 | -0.006 | -0.005 | -0.002 |
| 15 | -0.2061 | 0.022 | 0.024 | 0.028 | 0.009 | 0.038 | 0.228 | 0.288 | 0.196 | 0.128 | 0.074 | -0.004 | -0.008 | -0.008 | -0.006 | -0.005 | -0.002 |
| 20 | -0.1842 | 0.022 | 0.023 | 0.027 | 0.009 | 0.038 | 0.221 | 0.294 | 0.201 | 0.129 | 0.074 | -0.003 | -0.008 | -0.008 | -0.006 | -0.006 | -0.005 |
| | | | | | | | | Female | | | | | | | | | |
| 5 | -0.0712 | 0.050 | 0.050 | 0.041 | 0.028 | 0.177 | 0.220 | 0.208 | 0.158 | 0.114 | 0.057 | -0.015 | -0.024 | -0.019 | -0.016 | -0.014 | -0.013 |
| 10 | -0.1049 | 0.050 | 0.050 | 0.042 | 0.028 | 0.177 | 0.219 | 0.210 | 0.160 | 0.110 | 0.056 | -0.015 | -0.024 | -0.019 | -0.016 | -0.015 | -0.015 |
| 15 | -0.1149 | 0.049 | 0.049 | 0.041 | 0.027 | 0.168 | 0.215 | 0.214 | 0.166 | 0.112 | 0.056 | -0.014 | -0.023 | -0.018 | -0.016 | -0.014 | -0.014 |
| 20 | -0.1155 | 0.046 | 0.046 | 0.038 | 0.024 | 0.152 | 0.208 | 0.222 | 0.176 | 0.118 | 0.059 | -0.013 | -0.021 | -0.017 | -0.014 | -0.013 | -0.013 |

Table 11. Deriving the segment of age and sex structure of the national population open to international migration at age 5 and over; end of projection interval 0-5

| Sex | Age group | Survivors in year 5 a/ (thousands) | Net international migration rates b/ | Net change due to international migration among survivors c/ (thousands) | Population in year 5 d/ (thousands) |
|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) |
| Male | 5-9 | 737.0 | -0.0047 | -3.44 | 733.6 |
| | 10-14 | 729.9 | -0.0053 | -3.83 | 726.1 |
| | 15-19 | 618.5 | -0.0018 | -1.08 | 617.4 |
| | 20-24 | 481.5 | -0.0078 | -3.75 | 477.7 |
| | 25-29 | 397.1 | -0.0445 | -17.68 | 379.4 |
| | 30-34 | 355.9 | -0.0576 | -20.49 | 335.4 |
| | 35-39 | 351.5 | -0.0369 | -12.99 | 338.5 |
| | 40-44 | 303.6 | -0.0243 | -7.38 | 296.2 |
| | 45-49 | 214.5 | -0.0144 | -3.09 | 211.4 |
| | 50-54 | 142.8 | 0.0010 | 0.14 | 143.0 |
| | 55-59 | 175.3 | 0.0016 | 0.27 | 175.6 |
| | 60-64 | 136.3 | 0.0016 | 0.21 | 136.6 |
| | 65-69 | 103.6 | 0.0012 | 0.12 | 103.7 |
| | 70-74 | 65.9 | 0.0010 | 0.06 | 66.0 |
| | 75+ | 45.6 | 0.0006 | 0.03 | 45.6 |
| Female | 5-9 | 704.7 | -0.0036 | -2.51 | 702.2 |
| | 10-14 | 685.2 | -0.0029 | -2.00 | 683.2 |
| | 15-19 | 574.9 | -0.0020 | -1.15 | 573.7 |
| | 20-24 | 442.5 | -0.0126 | -5.57 | 436.9 |
| | 25-29 | 354.5 | -0.0157 | -5.55 | 348.9 |
| | 30-34 | 342.4 | -0.0147 | -5.02 | 337.4 |
| | 35-39 | 345.1 | -0.0112 | -3.88 | 341.2 |
| | 40-44 | 293.5 | -0.0081 | -2.38 | 291.1 |
| | 45-49 | 200.2 | -0.0041 | -0.81 | 199.3 |
| | 50-54 | 151.2 | 0.0011 | 0.16 | 151.4 |
| | 55-59 | 162.2 | 0.0017 | 0.28 | 162.5 |
| | 60-64 | 145.9 | 0.0014 | 0.20 | 146.1 |
| | 65-69 | 105.1 | 0.0011 | 0.12 | 105.3 |
| | 70-74 | 80.0 | 0.0010 | 0.08 | 80.1 |
| | 75+ | 72.9 | 0.0009 | 0.07 | 73.0 |

a/ Calculations not illustrated.
b/ From table 12, col. 3.
c/ (Col. 3) . (Col. 4).
d/ (Col. 3) + (Col. 5).

b. Net international migration rates

In this example, the international migration assumptions are formulated using total net international migration rates and proportionate age specific net international migration rates. These measures need first to be used to derive age specific net international migration rates, the calculation of which is illustrated for males in table 12. Each rate (column 3) is obtained by multiplying a corresponding proportionate age specific rate (column 2) by the given total net international migration rate.

For example, the net international migration rate for males in the age group 0-4 at the end of the 0-5 projection interval, -0.0045, is obtained as:

$$-0.0045 = (-0.1949)\ (0.023), \qquad (32)$$

where -0.1949 is the total net international migration rate for males for year 5, shown in table 10, while 0.023 is the proportionate rate for males aged 0-4 for that date.

c. Population aged 5 and over

To modify the numbers of survivors aged 5 and over for international migration, it is necessary to apply these rates to the numbers of survivors and derive net changes due to international migration among those survivors. The numbers of survivors are then modified by the net changes. In this example, in table 11, net international migration rates (column 4) are applied to the numbers of survivors (column 3) to obtain net changes due to international migration among the survivors (column 5).

Hence, for age group 5-9, the number of male survivors, 737.0, is multiplied by the net international migration rate, -0.0047, to get the net change due to international migration, -3.44:

$$-3.44 = (737.0)\ (-0.0047). \qquad (33)$$

These net changes are further used to modify the numbers of survivors and the result is population aged 5 and over at the end of the interval (column 6). Thus, the number of males aged 5-9, 733.6, would equal the sum of the male survivors, 737.0, and the net change due to international migration among males, -3.44:

$$733.6 = 737.0 + (-3.44). \qquad (34)$$

This completes the projection of the structure of the population age 5 and over for the end of the interval 0-5.

(ii) Segment of the population structure below age 5

a. Fertility rates, births and survivors below age 5

In order to derive the population below age 5, age-specific fertility rates are first calculated from the fertility assumptions as in the case of

Table 12. Calculating age-specific net international migration rates; results for males; end of projection interval 0-5

| Age group | Proportionate net international migration rates a/ | Net international migration rates b/ |
|---|---|---|
| (1) | (2) | (3) |
| 0-4 | 0.023 | -0.0045 |
| 5-9 | 0.024 | -0.0047 |
| 10-14 | 0.027 | -0.0053 |
| 15-19 | 0.009 | -0.0018 |
| 20-24 | 0.040 | -0.0078 |
| 25-29 | 0.229 | -0.0445 |
| 30-34 | 0.296 | -0.0576 |
| 35-39 | 0.190 | -0.0369 |
| 40-44 | 0.125 | -0.0243 |
| 45-49 | 0.074 | -0.0144 |
| 50-54 | -0.005 | 0.0010 |
| 55-59 | -0.008 | 0.0016 |
| 60-64 | -0.008 | 0.0016 |
| 65-69 | -0.006 | 0.0012 |
| 70-74 | -0.005 | 0.0010 |
| 75+ | -0.003 | 0.0006 |

a/ From table 10.
b/ (Total net international migration rate) . (Col. 2).

the closed population. Then, the number of births is derived as in the preceding example, except that in this example the mid-interval numbers of women reflect the impact of international migration occurring during the interval. This number is further disaggregated by sex, using the assumed sex ratio at birth. The resultant numbers of births by sex differ somewhat from those obtained for the closed population owing to the difference in the numbers of women in the childbearing period.

To obtain the numbers of survivors age 0-4 by sex at the end of the interval (table 13, column 2), the survival ratios are applied to the numbers of births classified by sex. The numbers of survivors obtained in the current example are somewhat smaller than in the preceding example (see table 7, column 4) owing to the net emigration of women in the childbearing ages.

b. Population below age 5

To obtain the numbers of persons aged 0-4 by sex at the end of the interval in an open population, the numbers of survivors must be modified for international migration. This is done in the same way as for the numbers of survivors aged 5 and over, as illustrated in table 11. In particular, net international migration rates for the age group 0-4 by sex (table 13, column 3) are first applied to the numbers of survivors age 0-4 by sex (column 2) to calculate the net changes among those survivors due to international migration (column 4).

Thus, for males aged 0-4, the net change due to international migration, -3.83, equals the number of survivors, 857.0, times the net international migration rate, -0.0045:

$$-3.83 = (857.0)\ (-0.0045). \qquad (35)$$

These changes are then used to modify the numbers of survivors. Thus, males aged 0-4, 853.1, equals the number of survivors, 857.0, plus net change due to international migration, -3.83:

$$853.1 = 857.0 + (-3.83). \qquad (36)$$

This completes the derivation of the age and sex structure of an open national population at the end of the 0-5 projection interval. When the steps described above are applied to the following five-year time intervals, 5-10, 10-15,..., the result is age and sex structures at the end of those intervals. The age and sex structures obtained as part of this illustrative projection are displayed in table 14.

(iii) Other results

It is possible to compute a variety of other results in the course of a projection of an open national population. Those other results, which are shown in table 15, include all those types of results obtained as part of a projection of the closed national population (table 9), plus some additional

Table 13. Deriving the segment of age and sex structure of the national population open to international migration below age 5: end of projection interval 0-5

| Sex | Survivors aged 0-4 in year 5 a/ (thousands) | Net international migration rates b/ | Net changes due to international migration among survivors c/ (thousands) | Population aged 0-4 in year 5 d/ (thousands) |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| Male | 857.0 | -0.0045 | -3.83 | 853.1 |
| Female | 823.9 | -0.0036 | -2.93 | 821.0 |

a/ Calculations not illustrated.
b/ From table 12, col. 3.
c/ (Col. 2) . (Col. 3).
d/ (Col. 2) + (Col. 4).

Table 14. Projected national population open to international migration, by age and sex

(Thousands)

| Sex | Age group | Year | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 5 | 10 | 15 | 20 |
| Male | 0-4 | 784.5 | 853.1 | 934.6 | 986.0 | 967.2 |
| | 5-9 | 740.0 | 733.6 | 809.4 | 897.0 | 954.9 |
| | 10-14 | 624.0 | 726.1 | 721.5 | 798.2 | 886.9 |
| | 15-19 | 486.6 | 617.4 | 719.6 | 716.3 | 793.4 |
| | 20-24 | 402.2 | 477.7 | 606.9 | 709.3 | 707.5 |
| | 25-29 | 361.5 | 379.4 | 448.4 | 573.9 | 676.0 |
| | 30-34 | 358.7 | 335.4 | 350.4 | 417.8 | 538.5 |
| | 35-39 | 312.4 | 338.5 | 315.6 | 331.8 | 398.0 |
| | 40-44 | 223.6 | 296.2 | 321.3 | 301.4 | 318.7 |
| | 45-49 | 152.1 | 211.4 | 281.3 | 307.3 | 290.1 |
| | 50-54 | 192.5 | 143.0 | 200.5 | 268.8 | 295.3 |
| | 55-59 | 157.0 | 175.6 | 131.9 | 186.7 | 252.4 |
| | 60-64 | 128.1 | 136.6 | 154.9 | 117.9 | 168.7 |
| | 65-69 | 90.1 | 103.7 | 112.7 | 130.1 | 100.4 |
| | 70-74 | 44.2 | 66.0 | 77.9 | 86.6 | 101.9 |
| | 75+ | 38.6 | 45.6 | 63.1 | 81.6 | 99.5 |
| Female | 0-4 | 750.7 | 821.0 | 894.3 | 938.7 | 917.0 |
| | 5-9 | 694.8 | 702.2 | 777.9 | 856.5 | 906.2 |
| | 10-14 | 580.5 | 683.2 | 691.9 | 768.1 | 847.3 |
| | 15-19 | 448.3 | 573.7 | 676.3 | 685.9 | 762.6 |
| | 20-24 | 359.8 | 436.9 | 557.6 | 658.3 | 669.9 |
| | 25-29 | 348.3 | 348.9 | 422.1 | 539.2 | 638.1 |
| | 30-34 | 352.2 | 337.4 | 336.8 | 407.6 | 521.1 |
| | 35-39 | 300.9 | 341.2 | 326.5 | 326.2 | 395.2 |
| | 40-44 | 206.7 | 291.1 | 330.6 | 317.1 | 317.4 |
| | 45-49 | 158.5 | 199.3 | 281.8 | 321.1 | 308.9 |
| | 50-54 | 174.6 | 151.4 | 191.8 | 272.7 | 312.1 |
| | 55-59 | 164.0 | 162.5 | 142.4 | 181.9 | 260.2 |
| | 60-64 | 126.1 | 146.1 | 146.9 | 130.3 | 167.9 |
| | 65-69 | 105.6 | 105.3 | 124.4 | 127.1 | 114.2 |
| | 70-74 | 63.5 | 80.1 | 81.9 | 99.0 | 103.0 |
| | 75+ | 69.4 | 73.0 | 86.6 | 97.9 | 116.8 |

Table 15. Population aggregates, indicators of the population structure and rates of population change of the national population open to international migration

| Indicators | Year | | | | |
|---|---|---|---|---|---|
| | 0 | 5 | 10 | 15 | 20 |
| **Population aggregates (thousands)** | | | | | |
| Population size | 10000.0 | 11092.6 | 12319.7 | 13638.2 | 14907.2 |
| Young-age | 4174.5 | 4519.2 | 4829.6 | 5244.5 | 5479.5 |
| Working-age | 5414.1 | 6099.7 | 6943.6 | 7771.5 | 8792.0 |
| Old-age | 411.4 | 473.7 | 546.6 | 622.3 | 635.8 |
| School-age | 4336.2 | 4950.8 | 5561.1 | 6089.6 | 6528.7 |
| Women of childbearing age | 2174.7 | 2528.5 | 2931.7 | 3255.4 | 3613.2 |
| Mid-interval population size | | 10532.1 | 11690.1 | 12962.2 | 14258.6 |
| Number of person-years lived | | 52660.7 | 58450.3 | 64811.0 | 71292.9 |
| Population growth | | 1092.6 | 1227.1 | 1318.5 | 1269.0 |
| Births | | 1994.8 | 2116.0 | 2174.1 | 2089.6 |
| Deaths | | 793.3 | 739.2 | 689.8 | 645.5 |
| Net change due to international migrtion | | -108.9 | -149.7 | -165.8 | -175.1 |
| **Indicators of the population structure** | | | | | |
| Proportions by broad age groups | | | | | |
| At young age (0-14) | 0.42 | 0.41 | 0.39 | 0.38 | 0.37 |
| At working age (15-64) | 0.54 | 0.55 | 0.56 | 0.57 | 0.59 |
| At old age (65+) | 0.04 | 0.04 | 0.04 | 0.05 | 0.04 |
| Dependency ratios | | | | | |
| Young-age | 0.77 | 0.74 | 0.70 | 0.67 | 0.62 |
| Old-age | 0.08 | 0.08 | 0.08 | 0.08 | 0.07 |
| Total | 0.85 | 0.82 | 0.77 | 0.75 | 0.70 |
| Median age of population | 19.4 | 19.3 | 19.8 | 20.6 | 21.5 |
| Proportion of women of childbearing age | 0.22 | 0.23 | 0.24 | 0.24 | 0.24 |
| Sex ratio of the population | 104 | 103 | 103 | 103 | 103 |
| **Rates of population change (per thousand)** | | | | | |
| Birth rate | | 37.9 | 36.2 | 33.5 | 29.3 |
| Death rate | | 15.1 | 12.6 | 10.6 | 9.1 |
| Natural increase | | 22.8 | 23.6 | 22.9 | 20.3 |
| International migration | | -2.1 | -2.6 | -2.6 | -2.5 |
| Population growtn | | 20.7 | 21.0 | 20.3 | 17.8 |

ones. The discussion below will explain how the number of deaths and the net change due to international migration are calculated. The calculation of the crude net rate of international migration will also be illustrated.

a. Population aggregates

i. Change due to international migration

The change due to international migration is derived by reverse surviving net changes due to international migration by age and sex, followed by adding up the results. The reverse survival is carried out over a two-and-a-half-year period, up to the mid-point of the five year interval, using the survival ratios by age and sex.

Table 16 illustrates the way the change to the national population due to international migration is computed. Net changes due to international migration among the survivors by age and sex are shown in column 3. The survival ratios for the interval are shown in column 4. These are used to produce by reverse survival net changes due to migration among various age and sex groups (column 5). The total net change, -108.9, is calculated as the sum of the net changes for each age group (total in column 5).

The way individual net changes are calculated by reverse survival can be illustrated in relation to males as follows. For males aged 0-4, the net change due to international migration, -4.05, is:

$$-4.05 = (-3.83) / [\ 0.67 + (0.33)\ (0.83874)\ ], \qquad (37)$$

where -3.83 is the net change due to international migration among male survivors age 0-4 and 0.83874 is the survival ratio relating to males of that age at the end of the interval.

For males aged 5-9, the net change due to international migration, -3.55, is:

$$-3.55 = (-3.44) / [\ (1 + 0.93949) / 2\ ], \qquad (37)$$

where -3.44 is the relevant net change due to international migration among survivors and 0.93949 is the survival ratio, respectively. The results for age groups 10-14 and beyond are obtained in a way similar to that for the age group 5-9.

ii. Deaths

The number of deaths in an open population is calculated by taking into account the fact that the population loses or gains numbers through migration. The number of deaths for the period 0-5, 793.3, can be obtained as:

$$793.3 = 1{,}994.8 - 1{,}092.6 + (-108.9), \qquad (38)$$

Table 16. Computing net change due to international migration; projection interval 0-5

| Sex | Age group | Net change to the survivors due to international migration a/ (thousands) | Survival ratios b/ | Net change to the population due to international migration c/ (thousands) |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| Male | 0-4 | -3.83 | 0.83874 | -4.05 |
| | 5-9 | -3.44 | 0.93949 | -3.55 |
| | 10-14 | -3.83 | 0.98637 | -3.86 |
| | 15-19 | -1.08 | 0.99121 | -1.09 |
| | 20-24 | -3.75 | 0.98948 | -3.76 |
| | 25-29 | -17.68 | 0.98738 | -17.80 |
| | 30-34 | -20.49 | 0.98450 | -20.65 |
| | 35-39 | -12.99 | 0.97993 | -13.12 |
| | 40-44 | -7.38 | 0.97182 | -7.49 |
| | 45-49 | -3.09 | 0.95927 | -3.15 |
| | 50-54 | 0.14 | 0.93912 | 0.14 |
| | 55-59 | 0.27 | 0.91054 | 0.29 |
| | 60-64 | 0.21 | 0.86845 | 0.23 |
| | 65-69 | 0.12 | 0.80879 | 0.13 |
| | 70-74 | 0.06 | 0.73177 | 0.07 |
| | 75+ | 0.03 | 0.55073 | 0.03 |
| Female | 0-4 | -2.93 | 0.84666 | -3.09 |
| | 5-9 | -2.51 | 0.93874 | -2.59 |
| | 10-14 | -2.00 | 0.98621 | -2.01 |
| | 15-19 | -1.15 | 0.99031 | -1.15 |
| | 20-24 | -5.57 | 0.98701 | -5.61 |
| | 25-29 | -5.55 | 0.98521 | -5.59 |
| | 30-34 | -5.02 | 0.98311 | -5.06 |
| | 35-39 | -3.88 | 0.97981 | -3.92 |
| | 40-44 | -2.38 | 0.97541 | -2.41 |
| | 45-49 | -0.81 | 0.96831 | -0.82 |
| | 50-54 | 0.16 | 0.95422 | 0.17 |
| | 55-59 | 0.28 | 0.92892 | 0.29 |
| | 60-64 | 0.20 | 0.88943 | 0.21 |
| | 65-69 | 0.12 | 0.83374 | 0.13 |
| | 70-74 | 0.08 | 0.75775 | 0.09 |
| | 75+ | 0.07 | 0.54846 | 0.09 |
| | Total | | | -108.90 |

a/ From table 11, col. 5 and table 13, col. 4.
b/ Calculations not illustrated.
c/ (Col. 3)/(0.67 + (0.33) . (Col. 4)), when age group is 0-4, and (Col. 3)/((1 + (Col. 4))/2), when age group is 5-9 or over.

where 1,994.8 is the number of births, 1,092.6 is the population growth and and -108.9 is the net change due to international migration during the period.

b. Rates of population change

i. Crude net international migration rate

This rate is calculated from the net change due to international migration and the mid-interval population size. For the interval 0-5, the crude net international migration rate, -2.1, is obtained as:

$$-2.1 = [\ (\ -108.9/5\ )\ /\ \ 10{,}532.1\ ]\ \cdot\ 1{,}000, \qquad (39)$$

where -108.9 is the net change to the national population due to international migration, 5 is the length of the projection interval, and 10,532.1 is the mid-interval population size.

## 2. Urban and rural populations

This section will describe a projection of urban and rural populations closed to international migration. A companion projection of urban and rural populations open to international migration will not be illustrated since it would introduce few new principles. Furthermore, such a projection would only rarely be prepared, partly due to excessive data requirements relating to migration inputs.

### (a) Closed populations

The example given to illustrate this type of projection will use the illustrative inputs shown in tables 17 through 19. It will draw on the two preceding illustrative projections as appropriate, but will not deal in detail with all the steps which are urban-rural counterparts of calculations described as part of those projections. The focus of this example will be on internal migration and how this component of population change is introduced into the projection process. For the sake of brevity, the example will present only calculations that refer to females. As in the previous examples, the illustrative calculations will be presented for the interval 0-5. Complete projection results will be, however, given for the entire 20-year period.

#### (i) Segments of population structures at age 5 and over

a. Survival ratios and survivors aged 5 and over

The first steps performed in the course of making a projection of urban and rural populations closed to international migration are similar to those made in order to project a closed national population. They include the derivation of survival ratios from mortality assumptions and the use of these

Table 17. Inputs for making a projection of urban and rural populations closed to international migration: inputs for urban areas

(a) Initial population by age and sex (thousands):

| Age group | Male | Female |
|---|---|---|
| 0-4 | 184.8 | 175.6 |
| 5-9 | 181.7 | 173.1 |
| 10-14 | 170.0 | 154.8 |
| 15-19 | 181.7 | 137.4 |
| 20-24 | 201.3 | 110.9 |
| 25-29 | 134.5 | 103.2 |
| 30-34 | 126.2 | 102.4 |
| 35-39 | 109.9 | 96.7 |
| 40-44 | 77.7 | 63.0 |
| 45-49 | 51.6 | 50.0 |
| 50-54 | 59.6 | 49.2 |
| 55-59 | 47.1 | 52.4 |
| 60-64 | 31.8 | 33.3 |
| 65-69 | 26.1 | 32.6 |
| 70-74 | 12.8 | 19.6 |
| 75+ | 11.2 | 21.4 |

(b) Mortality assumptions:

| Time interval | Expectations of life at birth (years) | |
|---|---|---|
| | Male | Female |
| 0-5 | 56.19 | 57.54 |
| 5-10 | 58.76 | 60.12 |
| 10-15 | 60.93 | 62.28 |
| 15-20 | 63.31 | 64.26 |

(c) Fertility assumptions:

| Time interval | Total fertility rates | Proportionate age-specific fertility rates by age | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 |
| 0-5 | 4.00 | 0.096 | 0.251 | 0.245 | 0.186 | 0.149 | 0.063 | 0.010 |
| 5-10 | 3.80 | 0.100 | 0.255 | 0.248 | 0.184 | 0.143 | 0.061 | 0.009 |
| 10-15 | 3.50 | 0.104 | 0.258 | 0.252 | 0.182 | 0.137 | 0.059 | 0.008 |
| 15-20 | 3.10 | 0.107 | 0.262 | 0.257 | 0.179 | 0.130 | 0.057 | 0.008 |

Table 18 Inputs for making a projection of urban and rural populations closed to international migration: inputs for rural areas

(a) Initial population by age and sex (thousands):

| Age group | Male | Female |
|---|---|---|
| 0-4 | 599.7 | 575.1 |
| 5-9 | 558.3 | 521.7 |
| 10-14 | 454.0 | 425.7 |
| 15-19 | 304.9 | 310.9 |
| 20-24 | 200.9 | 248.9 |
| 25-29 | 227.0 | 245.1 |
| 30-34 | 232.5 | 249.8 |
| 35-39 | 202.5 | 204.2 |
| 40-44 | 145.9 | 143.7 |
| 45-49 | 100.5 | 108.5 |
| 50-54 | 132.9 | 125.4 |
| 55-59 | 109.9 | 111.6 |
| 60-64 | 96.5 | 92.8 |
| 65-69 | 64.0 | 73.0 |
| 70-74 | 31.4 | 43.9 |
| 75+ | 27.4 | 48.0 |

(b) Mortality assumptions:

| Time interval | Expectations of life at birth (years) | |
|---|---|---|
| | Male | Female |
| 0-5 | 49.78 | 51.32 |
| 5-10 | 53.87 | 55.48 |
| 10-15 | 57.84 | 59.25 |
| 15-20 | 61.08 | 62.20 |

(c) Fertility assumptions:

| Time interval | Total fertility rates | Proportionate age-specific fertility rates by age | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 |
| 0-5 | 6.40 | 0.076 | 0.253 | 0.245 | 0.197 | 0.141 | 0.057 | 0.031 |
| 5-10 | 6.20 | 0.079 | 0.263 | 0.249 | 0.192 | 0.135 | 0.055 | 0.027 |
| 10-15 | 5.60 | 0.082 | 0.273 | 0.253 | 0.187 | 0.129 | 0.053 | 0.023 |
| 15-20 | 4.60 | 0.085 | 0.282 | 0.257 | 0.182 | 0.122 | 0.052 | 0.020 |

Table 19. Inputs for asking a projection of urban and rural populations closed to international migration: internal migration assumptions

| Year | Total net internal migration rates | Proportionate age-specific net internal migration rates by age | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0-4 | 5-9 | 10-14 | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 | 50-54 | 55-59 | 60-64 | 65-69 | 70-74 | 75+ |
| **Male** | | | | | | | | | | | | | | | | | |
| 5 | -1 5604 | 0 020 | 0.028 | C. 108 | C. 158 | C 140 | 0.094 | 0.085 | 0 072 | 0.056 | 0 046 | 0 039 | 0.035 | 0 038 | 0 031 | 0.029 | 0.021 |
| 10 | -1 5013 | 0 020 | 0.028 | 0 108 | 0 158 | C 140 | C 094 | 0.085 | 0 072 | 0 056 | 0.046 | 0.039 | 0.035 | 0.038 | C 031 | 0 029 | C.021 |
| 15 | -1.4492 | 0 020 | 0.028 | 0.108 | 0.158 | C 140 | C 094 | 0.085 | 0 072 | 0 056 | 0 046 | 0.039 | 0.035 | 0.038 | 0.031 | 0 029 | 0.021 |
| 20 | -1.4412 | 0.020 | 0.028 | 0 108 | 0.158 | C 140 | 0.094 | 0.085 | 0.072 | 0 056 | 0 046 | 0.039 | 0.035 | 0.038 | 0.031 | 0.029 | 0.021 |
| **Female** | | | | | | | | | | | | | | | | | |
| 5 | -1.2431 | C 015 | 0.018 | 0 089 | C 103 | C 129 | 0 117 | 0.097 | 0 079 | 0 067 | 0 059 | 0.049 | 0.045 | 0 049 | 0.040 | 0 037 | 0 027 |
| 10 | -1.3704 | 0 015 | 0.018 | C 089 | C 103 | C 129 | 0.117 | 0 097 | 0.079 | 0.067 | 0 059 | C 049 | 0.045 | 0 049 | 0 040 | 0 037 | 0 027 |
| 15 | -1 2255 | 0 015 | 0.018 | C 089 | C 103 | C 129 | 0.117 | 0.097 | 0.079 | 0 067 | 0.059 | 0.049 | 0.045 | 0 049 | C 040 | 0.037 | 0 027 |
| 20 | -1.2115 | 0.015 | 0.018 | C 089 | 0 103 | 0 129 | 0.117 | 0.097 | 0.079 | 0.067 | 0.059 | 0.049 | 0 045 | 0 049 | 0.040 | 0.037 | 0.027 |

ratios to calculate the numbers of survivors aged 5 and over at the end of the interval. Each step is carried out for both urban and rural populations. When these steps are performed using the inputs of tables 17 through 19, the results obtained for females are those shown in table 20.

In particular, the table shows for both rural and urban areas the survival ratios applying to women (column 4) and the numbers of female survivors aged 5 and over (column 5). These results along with those obtained for males, but not shown in table 20, can be derived using the steps similar to those illustrated earlier with respect to the closed national population. The numbers of survivors need to be further modified using net internal migration rates.

b. Net internal migration rates

The derivation of net internal migration rates is in principle identical with that used to obtain net international migration rates. Net internal migration rates are calculated for each sex by multiplying the age specific proportionate net internal migration rates by age of each sex by the total net internal migration rate for that sex. Table 21 illustrates how age specific net internal migration rates for females are calculated for the projection interval 0-5.

In particular, the proportionate rates (column 2) are all multiplied by the given total net internal migration rate to obtain age-specific net internal migration rates (column 3). Thus, the rate for the age group 0-4, -0.019, is obtained as:

$$-0.019 = (-1.2431)\ (0.015), \qquad (42)$$

where -1.2431 is the total net internal migration rate, shown in table 19, and 0.015 is the proportionate migration rate for age group 0-4.

c. Population aged 5 and over

To derive population aged 5 and over in urban and rural areas, net changes among rural survivors aged 5 and over due to internal migration are first calculated. They are obtained by applying net internal migration rates to the numbers of rural survivors aged 5 and above. In particular, for rural female survivors, net changes at age 5 and over (table 22, column 5) are obtained as the products of the numbers of survivors (column 3) and appropriate net internal migration rates (column 4).

For example, the net change among rural female survivors aged 5-9, -11.81, is obtained as:

$$-11.81 = (536.2)\ (-0.022), \qquad (43)$$

Table 20. Deriving the numbers of survivors in urban and rural populations closed to international migration at age 5 and over; results for females; end of projection interval 0-5

| Location | Age group | Population in year 0 a/ (thousands) | Survival ratios b/ | Survivors in year 5 c/ (thousands) |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| Urban | 0-4 | 175.6 | | |
| | 5-9 | 173.1 | 0.95413 | 167.5 |
| | 10-14 | 154.8 | 0.99003 | 171.4 |
| | 15-19 | 137.4 | 0.99297 | 153.7 |
| | 20-24 | 110.9 | 0.99068 | 136.1 |
| | 25-29 | 103.2 | 0.98923 | 109.7 |
| | 30-34 | 102.4 | 0.98749 | 101.9 |
| | 35-39 | 96.7 | 0.98474 | 100.8 |
| | 40-44 | 63.0 | 0.98069 | 94.8 |
| | 45-49 | 50.0 | 0.97415 | 61.4 |
| | 50-54 | 49.2 | 0.96166 | 48.1 |
| | 55-59 | 52.4 | 0.93929 | 46.2 |
| | 60-64 | 33.3 | 0.90357 | 47.3 |
| | 65-69 | 32.6 | 0.85191 | 28.4 |
| | 70-74 | 19.6 | 0.77970 | 25.4 |
| | 75+ | 21.4 | 0.56701 | 23.2 |
| Rural | 0-4 | 575.1 | | |
| | 5-9 | 521.7 | 0.93235 | 536.2 |
| | 10-14 | 425.7 | 0.98462 | 513.7 |
| | 15-19 | 310.9 | 0.98912 | 421.1 |
| | 20-24 | 248.9 | 0.98542 | 306.4 |
| | 25-29 | 245.1 | 0.98345 | 244.8 |
| | 30-34 | 249.8 | 0.98125 | 240.5 |
| | 35-39 | 204.2 | 0.97778 | 244.3 |
| | 40-44 | 143.7 | 0.97328 | 198.7 |
| | 45-49 | 108.5 | 0.96595 | 138.8 |
| | 50-54 | 125.4 | 0.95134 | 103.2 |
| | 55-59 | 111.6 | 0.92504 | 116.0 |
| | 60-64 | 92.8 | 0.88409 | 98.7 |
| | 65-69 | 73.0 | 0.82705 | 76.8 |
| | 70-74 | 43.9 | 0.74980 | 54.7 |
| | 75+ | 48.0 | 0.54193 | 49.8 |

a/ From tables 17 and 18.
b/ Derivation not illustrated.
c/ For age groups:
5-9 through 70-74:
(Entry in a preceding row of col. 3) . (Entry in the given row of col. 4).

75+:
(The sum of entries corresponding to age groups 70-74 and 75+ in col. 3) . (Entry corresponding to the latter of age groups in col. 4).

Table 21. Computing net internal migration rates; results for females; end of projection interval 0-5

| Age group | Proportionate net internal migration rates a/ | Net internal migration rates b/ |
|---|---|---|
| (1) | (2) | (3) |
| 0-4 | 0.015 | -0.019 |
| 5-9 | 0.018 | -0.022 |
| 10-14 | 0.069 | -0.086 |
| 15-19 | 0.103 | -0.128 |
| 20-24 | 0.129 | -0.160 |
| 25-29 | 0.117 | -0.145 |
| 30-34 | 0.097 | -0.120 |
| 35-39 | 0.079 | -0.098 |
| 40-44 | 0.067 | -0.083 |
| 45-49 | 0.059 | -0.073 |
| 50-54 | 0.049 | -0.061 |
| 55-59 | 0.045 | -0.056 |
| 60-64 | 0.049 | -0.061 |
| 65-69 | 0.040 | -0.050 |
| 70-74 | 0.037 | -0.046 |
| 75+ | 0.027 | -0.034 |

a/ From table 19.
b/ (Total net internal migration rate) . (Col. 2).

Table 22. Deriving age and sex structures of urban and rural populations closed to international migration at age 5 and over; results for females; end of projection interval 0-5

| Location | Age group | Survivors in year 5 a/ (thousands) | Net internal migration rates b/ | Net change due to internal migration among survivors c/ (thousands) | Population in year 5 d/ (thousands) |
|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) |
| Urban | 5-9 | 167.5 | | 11.81 | 179 4 |
| | 10-14 | 171 4 | | 44.21 | 215 6 |
| | 15-19 | 153 7 | | 53.94 | 207 7 |
| | 20-24 | 136 1 | | 49.06 | 185 2 |
| | 25-29 | 109.7 | | 35 52 | 145 2 |
| | 30-34 | 101 9 | | 28 88 | 130 8 |
| | 35-39 | 100 8 | | 23.96 | 124 8 |
| | 40-44 | 94.8 | | 16.51 | 111.3 |
| | 45-49 | 61 4 | | 10.14 | 71 5 |
| | 50-54 | 48 1 | | 6.30 | 54 4 |
| | 55-59 | 46.2 | | 6.50 | 52 7 |
| | 60-64 | 47 3 | | 6.02 | 53 4 |
| | 65-69 | 28 4 | | 3.84 | 32 2 |
| | 70-74 | 25.4 | | 2.52 | 27 9 |
| | 75+ | 23.2 | | 1.69 | 24 9 |
| Rural | 5-9 | 536.2 | -0.022 | -11.81 | 524 4 |
| | 10-14 | 513 7 | -0 086 | -44 21 | 469 5 |
| | 15-19 | 421 1 | -0 128 | -53.94 | 367 1 |
| | 20-24 | 306 4 | -0.160 | -49.06 | 257 3 |
| | 25-29 | 244.8 | -0.145 | -35.52 | 209 3 |
| | 30 34 | 240.5 | -0.120 | -28.88 | 211 6 |
| | 35-39 | 244.3 | -0 098 | -23 96 | 220 3 |
| | 40-44 | 198.7 | -0 083 | -16.51 | 182 2 |
| | 45-49 | 138.8 | -0.073 | -10.14 | 128 7 |
| | 50-54 | 103.2 | -0.061 | -6.30 | 96 9 |
| | 55-59 | 116.0 | -0 056 | -6.50 | 109 5 |
| | 60-54 | 98.7 | -0.061 | -6 02 | 92 6 |
| | 65-69 | 76.8 | -0.050 | -3.84 | 72 9 |
| | 70-74 | 54.7 | -0.046 | -2 52 | 52.2 |
| | 75+ | 49.8 | -0.034 | -1.69 | 48 1 |

a/ Table 20, col. 5.
b/ Table 21, col. 3.
c/ For rural: (Col. 3) . (Col. 4); for urban: (-1) . (Col. 5 under 'rural').
d/ (Col. 3) + (Col. 5).

where 536.2 is the number of female survivors aged 5-9 and -0.022 is the net internal migration rate applying to these survivors.

Further, net changes due to internal migration among urban survivors are obtained as those among rural survivors. However, they will have the opposite sign since any loss (gain) to rural survivors resulting from internal migration equals the gain (loss) to urban survivors. Therefore, net changes among urban female survivors (table 22, column 5) equal those among rural female survivors, with an opposite sign.

For example, net change among urban female survivors aged 5-9, 11.81, equals that among rural female survivors of the same age, -11.81, with an opposite sign:

$$11.81 = - (11.81). \qquad (44)$$

The net changes due to internal migration are next added to the survivors aged 5 and over in each population. Thus, the rural female population aged 5-9, 524.4, is obtained as the sum of the number of survivors, 536.2, and the net change due to internal migration, -11.81:

$$524.4 = 536.2 + (-11.81). \qquad (45)$$

The result is the age and sex structures of the two populations at age 5 and over at the end of the five-year projection interval. The age structures of females in urban and rural populations obtained through this process for the end of the interval 0-5 are shown in column 6 of table 22.

(ii) Segments of population structures below age 5

a. Fertility rates, births and survivors below age 5

To obtain the segments of the population structures below age 5 in urban and rural areas it is necessary to calculate fertility rates, the numbers of births by sex and the numbers of survivors aged 0-4 by sex. The calculations for each location are performed as in the projection of the national population. The numbers of female survivors aged 0-4 for urban and rural areas obtained by those steps are shown in table 23, column 2.

b. Population below age 5

These numbers of survivors are further used to compute net changes due to internal migration. The net changes are in turn employed to modify the numbers of survivors and thus take into account the impact of this type of migration. In table 23, the results of these calculations are illustrated for urban and rural females.

Table 23. Deriving age and sex structures of urban and rural populations closed to international migration below age 5; results for females: end of projection interval 0-5

| Location | Survivors aged 0-4 in year 5 a/ (thousands) | Net internal migration rates b/ | Net change due to internal migration c/ (thousands) | Population aged 0-4 in year 5 d/ (thousands) |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| Urban | 214.7 | | 11.58 | 226.3 |
| Rural | 609.0 | -0.019 | -11.58 | 597.4 |

a/ Calculations not illustrated.
b/ From table 21, col. 3.
c/ For rural: (Col. 2) . (Col. 3); for urban: (-1) . (Col. 4 under 'rural').
d/ (Col.2 ) + (Col. 4).

The net change in the number of rural female survivors aged 0-4 due to internal migration, -11.58 (column 4), is equal to the number of rural female survivors of that age, 609.0 (column 2), multiplied by the net internal migration rate, -0.019 (column 3):

$$-11.58 = (609.0)\ (-0.019). \qquad (46)$$

The net change in the number of urban female survivors aged 0-4 due to internal migration would, therefore, equal 11.58 (column 4):

$$11.58 = -\ (-11.58). \qquad (47)$$

The numbers of females aged 0-4 in urban and rural areas are then obtained by adding the net changes to the numbers of survivors. Thus, the number of rural females aged 0-4, 597.4 (column 5), is found as follows:

$$597.4 = 609.0 + (-11.58), \qquad (48)$$

where 609.0 is the number of survivors and -11.58 is the net change due to internal migration.

The calculations illustrated above yield the age and sex structures of the urban and rural populations at the end of the interval 0-5. When repeated over subsequent time intervals, they project the population structures for each area in years 10, 15 and beyond. A projection of age and sex structures of urban and rural populations for a 20-year time interval is presented in tables 24 and 25. Table 26 shows age and sex structures of the national population, which are obtained by aggregating urban and rural structures.

### (iii) Other results

A variety of results in addition to the population structures can be derived as part of a projection of urban and rural populations. Most of these results are of the same type as those obtained in the course of projecting a national population and can be obtained as illustrated in the two preceding examples. Those results are shown in tables 27 through 29, for the urban, rural and the national population.

#### a. Population aggregates

Most population aggregates can be found using the same steps as in the case of the national population closed to international migration. The calculation of an additional aggregate obtained in this type of projection will be shown here - net changes to urban and rural populations due to internal migration. The way the numbers of deaths are calculated in these populations will also be illustrated.

Table 24. Projected urban population closed to international migration, by age and sex

(Thousands)

| Sex | Age group | Year | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 5 | 10 | 15 | 20 |
| Male | 0-4 | 184.8 | 244.1 | 332.7 | 413.3 | 458.2 |
| | 5-9 | 181.7 | 201.1 | 259.5 | 346.9 | 426.7 |
| | 10-14 | 170.0 | 272.6 | 285.2 | 343.5 | 432.8 |
| | 15-19 | 181.7 | 279.7 | 378.8 | 384.6 | 447.3 |
| | 20-24 | 201.3 | 246.1 | 348.4 | 446.3 | 450.7 |
| | 25-29 | 134.5 | 228.4 | 277.0 | 381.7 | 480.5 |
| | 30-34 | 126.2 | 162.5 | 247.3 | 298.9 | 406.5 |
| | 35-39 | 109.9 | 149.9 | 180.9 | 259.4 | 313.8 |
| | 40-44 | 77.7 | 124.7 | 163.5 | 191.3 | 265.8 |
| | 45-49 | 51.6 | 85.1 | 132.9 | 170.9 | 196.7 |
| | 50-54 | 59.6 | 54.7 | 88.3 | 136.1 | 173.3 |
| | 55-59 | 47.1 | 61.6 | 55.0 | 87.9 | 134.8 |
| | 60-64 | 31.6 | 47.2 | 60.5 | 53.1 | 84.7 |
| | 65-69 | 26.1 | 29.8 | 42.8 | 54.7 | 47.9 |
| | 70-74 | 12.8 | 21.7 | 25.2 | 35.5 | 45.6 |
| | 75+ | 11.2 | 14.7 | 22.3 | 29.4 | 40 5 |
| Female | 0-4 | 175.6 | 226.3 | 311.4 | 387.0 | 428.6 |
| | 5-9 | 173.1 | 179.4 | 232.0 | 314.2 | 389.2 |
| | 10-14 | 154.8 | 215.6 | 226.8 | 276.4 | 359.1 |
| | 15-19 | 137.4 | 207.7 | 280.3 | 284.6 | 337.4 |
| | 20-24 | 110.9 | 185.2 | 270.3 | 341.3 | 346.4 |
| | 25-29 | 103.2 | 145.2 | 224.4 | 310.7 | 386.0 |
| | 30-34 | 102.4 | 130.8 | 171.1 | 247.6 | 338.0 |
| | 35-39 | 96.7 | 124.8 | 151.7 | 186.3 | 262.8 |
| | 40-44 | 63.0 | 111.3 | 142.4 | 164.4 | 196.7 |
| | 45-49 | 50.0 | 71.5 | 123.2 | 153.3 | 173.2 |
| | 50-54 | 49.2 | 54.4 | 77.4 | 128.8 | 159.3 |
| | 55-59 | 52.4 | 52.7 | 56.9 | 79.5 | 130.6 |
| | 60-64 | 33.3 | 53.4 | 54.7 | 56.9 | 79.1 |
| | 65-69 | 32.6 | 32.2 | 50.3 | 51.5 | 53.2 |
| | 70-74 | 19.6 | 27.9 | 28.4 | 43.1 | 44.7 |
| | 75+ | 21.4 | 24.9 | 32.7 | 37.9 | 50.7 |

Table 25. Projected rural population closed to international migration, by age and sex

(Thousands)

| Sex | Age group | Year | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 5 | 10 | 15 | 20 |
| Male | 0-4 | 599.7 | 611.6 | 617.8 | 608.3 | 553.0 |
| | 5-9 | 558.3 | 534.6 | 555.2 | 569.0 | 566.4 |
| | 10-14 | 454.0 | 457.1 | 442.4 | 464.4 | 477.1 |
| | 15-19 | 304.9 | 338.8 | 345.8 | 339.1 | 357.0 |
| | 20-24 | 200.9 | 235.5 | 265.0 | 273.6 | 269.1 |
| | 25-29 | 227.0 | 169.1 | 200.0 | 226.9 | 234.8 |
| | 30-34 | 232.5 | 193.5 | 145.5 | 173.5 | 197.3 |
| | 35-39 | 202.5 | 201.7 | 169.5 | 128.5 | 153.7 |
| | 40-44 | 145.9 | 179.1 | 180.1 | 152.6 | 116.1 |
| | 45-49 | 100.5 | 129.5 | 160.5 | 162.8 | 138.8 |
| | 50-54 | 132.9 | 88.3 | 115.1 | 144.1 | 147.4 |
| | 55-59 | 109.9 | 113.8 | 76.7 | 101.2 | 127.8 |
| | 60-64 | 96.5 | 89.2 | 93.9 | 64.3 | 85.9 |
| | 65-69 | 64.0 | 73.7 | 69.6 | 74.7 | 52.0 |
| | 70-74 | 31.4 | 44.2 | 52.3 | 50.7 | 55.6 |
| | 75+ | 27.4 | 31.0 | 40.7 | 51.7 | 58.3 |
| Female | 0-4 | 575.1 | 597.4 | 599.5 | 587.6 | 531.8 |
| | 5-9 | 521.7 | 524.4 | 551.8 | 562.5 | 556.4 |
| | 10-14 | 425.7 | 469.5 | 469.4 | 500.9 | 512.0 |
| | 15-19 | 310.9 | 367.1 | 399.7 | 407.6 | 436.5 |
| | 20-24 | 248.9 | 257.3 | 299.0 | 333.7 | 341.7 |
| | 25-29 | 245.1 | 209.3 | 213.3 | 253.7 | 284.4 |
| | 30-34 | 249.8 | 211.6 | 178.9 | 185.8 | 222.0 |
| | 35-39 | 204.2 | 220.3 | 185.4 | 159.3 | 166.2 |
| | 40-44 | 143.7 | 182.2 | 195.8 | 167.2 | 144.3 |
| | 45-49 | 108.5 | 128.7 | 162.6 | 177.4 | 152.0 |
| | 50-54 | 125.4 | 96.9 | 115.0 | 147.4 | 161.6 |
| | 55-59 | 111.6 | 109.5 | 85.0 | 102.5 | 132.3 |
| | 60-64 | 92.8 | 92.6 | 91.6 | 72.6 | 88.5 |
| | 65-69 | 73.0 | 72.9 | 73.8 | 74.8 | 60.2 |
| | 70-74 | 43.9 | 52.2 | 53.3 | 55.6 | 57.5 |
| | 75+ | 48.0 | 48.1 | 53.9 | 59.6 | 65.5 |

Table 26. Projected national population closed to international migration, by age and sex, obtained in the course of projecting urban and rural populations

(Thousands)

| Sex | Age group | Year | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 5 | 10 | 15 | 20 |
| Male | 0-4 | 784.5 | 855.7 | 950.5 | 1021.6 | 1011.2 |
| | 5-9 | 740.0 | 735.7 | 814.7 | 915.9 | 993.1 |
| | 10-14 | 624.0 | 729.6 | 727.7 | 807.9 | 910.0 |
| | 15-19 | 486.6 | 618.5 | 724.6 | 723.6 | 804.3 |
| | 20-24 | 402.2 | 481.6 | 613.4 | 719.9 | 719.8 |
| | 25-29 | 361.5 | 397.5 | 477.0 | 608.6 | 715.3 |
| | 30-34 | 358.7 | 356.0 | 392.8 | 472.4 | 603.8 |
| | 35-39 | 312.4 | 351.6 | 350.4 | 387.9 | 467.5 |
| | 40-44 | 223.6 | 303.7 | 343.6 | 343.8 | 381.9 |
| | 45-49 | 152.1 | 214.6 | 293.4 | 333.8 | 335.5 |
| | 50-54 | 192.5 | 142.9 | 203.4 | 280.2 | 320.6 |
| | 55-59 | 157.0 | 175.3 | 131.7 | 189.1 | 262.7 |
| | 60-64 | 128.1 | 136.4 | 154.4 | 117.5 | 170.6 |
| | 65-69 | 90.1 | 103.5 | 112.4 | 129.4 | 99.9 |
| | 70-74 | 44.2 | 66.0 | 77.5 | 86.2 | 101.2 |
| | 75+ | 38.6 | 45.6 | 63.0 | 81.1 | 98.7 |
| Female | 0-4 | 750.7 | 823.7 | 910.9 | 974.6 | 960.4 |
| | 5-9 | 694.8 | 703.7 | 783.7 | 876.7 | 945.6 |
| | 10-14 | 580.5 | 685.1 | 696.2 | 777.3 | 871.1 |
| | 15-19 | 448.3 | 574.8 | 680.0 | 692.2 | 773.8 |
| | 20-24 | 359.8 | 442.5 | 569.2 | 675.0 | 688.2 |
| | 25-29 | 348.3 | 354.5 | 437.7 | 564.4 | 670.4 |
| | 30-34 | 352.2 | 342.4 | 350.0 | 433.4 | 559.9 |
| | 35-39 | 300.9 | 345.1 | 337.1 | 345.7 | 429.0 |
| | 40-44 | 206.7 | 293.6 | 338.2 | 331.6 | 341.0 |
| | 45-49 | 158.5 | 200.2 | 285.9 | 330.7 | 325.3 |
| | 50-54 | 174.6 | 151.3 | 192.4 | 276.3 | 321.0 |
| | 55-59 | 164.0 | 162.2 | 142.0 | 181.9 | 262.9 |
| | 60-64 | 126.1 | 146.0 | 146.3 | 129.6 | 167.6 |
| | 65-69 | 105.6 | 105.1 | 124.2 | 126.3 | 113.4 |
| | 70-74 | 63.5 | 80.2 | 81.6 | 98.7 | 102.1 |
| | 75+ | 69.4 | 73.1 | 86.6 | 97.5 | 116.1 |

Table 27. Population aggregates, indicators of the population structure and rates of population change for the urban population closed to international migration

| Indicators | Year | | | | |
|---|---|---|---|---|---|
| | 0 | 5 | 10 | 15 | 20 |
| **Population aggregates (thousands)** | | | | | |
| Population size | 2983.4 | 4067.0 | 5334.3 | 6697.3 | 8140.9 |
| Young-age | 1040.0 | 1339.1 | 1647.6 | 2081.3 | 2494.6 |
| Working-age | 1819.7 | 2576.9 | 3485.0 | 4363.6 | 5363.6 |
| Old-age | 123.7 | 151.2 | 201.7 | 252.1 | 282.6 |
| School-age | 1310.9 | 1787.4 | 2281.3 | 2737.8 | 3189.6 |
| Women of childbearing age | 663.6 | 976.5 | 1363.4 | 1688.2 | 2040.5 |
| Mid-interval population size | | 3483.3 | 4657.7 | 5977.1 | 7383.9 |
| Number of person-years lived | | 17416.6 | 23288.7 | 29885.4 | 36919.5 |
| Population growth | | 1083.6 | 1267.3 | 1363.0 | 1443.6 |
| Births | | 503.8 | 690.3 | 857.1 | 943.5 |
| Deaths | | 194.6 | 228.7 | 259.9 | 281.9 |
| Net change due to internal migration | | 774.4 | 805.7 | 765.8 | 782.0 |
| **Indicators of the population structure** | | | | | |
| Proportions by broad age groups | | | | | |
| At young age (0-14) | 0.35 | 0.33 | 0.31 | 0.31 | 0.31 |
| At working age (15-64) | 0.61 | 0.63 | 0.65 | 0.65 | 0.66 |
| At old age (65+) | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 |
| Dependency ratios | | | | | |
| Young-age | 0.57 | 0.52 | 0.47 | 0.48 | 0.47 |
| Old-age | 0.07 | 0.06 | 0.06 | 0.06 | 0.05 |
| Total | 0.64 | 0.58 | 0.53 | 0.53 | 0.52 |
| Median age of population | 22.1 | 22.4 | 22.9 | 23.8 | 25.0 |
| Proportion of women of childbearing age | 0.22 | 0.24 | 0.26 | 0.25 | 0.25 |
| Sex ratio of the population | 117 | 121 | 119 | 119 | 118 |
| **Rates of population change (per thousand)** | | | | | |
| Birth rate | | 28.9 | 29.6 | 28.7 | 25.6 |
| Death rate | | 11.2 | 9.8 | 8.7 | 7.6 |
| Natural increase | | 17.8 | 19.8 | 20.0 | 17.9 |
| Internal migration | | 44.5 | 34.6 | 25.6 | 21.2 |
| Population growth | | 62.0 | 54.3 | 45.5 | 39.0 |

Table 28. Population aggregates, indicators of the population structure and rates of population change for the rural population closed to international migration

| Indicators | Year 0 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| **Population aggregates (thousands)** | | | | | |
| Population size | 7016.6 | 7130.6 | 7258.0 | 7433.5 | 7503.1 |
| Young-age | 3134.5 | 3194.6 | 3236.1 | 3292.7 | 3196.7 |
| Working-age | 3594.4 | 3614.0 | 3678.4 | 3773.8 | 3957.4 |
| Old-age | 287.7 | 322.1 | 343.6 | 367.1 | 349.1 |
| School-age | 3025.3 | 3184.3 | 3328.3 | 3450.8 | 3516.2 |
| Women of childbearing age | 1511.1 | 1576.5 | 1634.7 | 1684.7 | 1747.1 |
| Mid-interval population size | | 7073.4 | 7194.0 | 7345.2 | 7468.2 |
| Number of person-years lived | | 35366.9 | 35970.1 | 36726.1 | 37341.1 |
| Population growth | | 114.0 | 127.4 | 175.5 | 69.6 |
| Births | | 1493.0 | 1455.4 | 1388.4 | 1234.1 |
| Deaths | | 604.6 | 522.3 | 447.1 | 382.5 |
| Net change due to internal migration | | -774.4 | -805.7 | -765.8 | -782.0 |
| **Indicators of the population structure** | | | | | |
| Proportions by broad age groups | | | | | |
| At young age (0-14) | 0.45 | 0.45 | 0.45 | 0.44 | 0.43 |
| At working age (15-64) | 0.51 | 0.51 | 0.51 | 0.51 | 0.53 |
| At old age (65+) | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |
| Dependency ratios | | | | | |
| Young-age | 0.87 | 0.88 | 0.88 | 0.87 | 0.81 |
| Old-age | 0.08 | 0.09 | 0.09 | 0.10 | 0.09 |
| Total | 0.95 | 0.97 | 0.97 | 0.97 | 0.90 |
| Median age of population | 18.0 | 17.6 | 17.6 | 17.8 | 18.5 |
| Proportion of women of childbearing age | 0.22 | 0.22 | 0.23 | 0.23 | 0.23 |
| Sex ratio of the population | 99 | 96 | 95 | 93 | 92 |
| **Rates of population change (per thousand)** | | | | | |
| Birth rate | | 42.2 | 40.5 | 37.8 | 33.0 |
| Death rate | | 17.1 | 14.5 | 12.2 | 10.2 |
| Natural increase | | 25.1 | 25.9 | 25.6 | 22.8 |
| Internal migration | | -21.9 | -22.4 | -20.9 | -20.9 |
| Population growth | | 3.2 | 3.5 | 4.8 | 1.9 |

Table 29. Population aggregates, indicators of the population structure and distribution, and rates of population change for the national population closed to international migration, obtained in the course of projecting urban and rural populations

| Indicators | Year | | | | |
|---|---|---|---|---|---|
| | 0 | 5 | 10 | 15 | 20 |
| **Population aggregates (thousands)** | | | | | |
| Population size | 10000.0 | 11197.5 | 12592.3 | 14130.8 | 15644.0 |
| Young-age | 4174.5 | 4533.5 | 4883.7 | 5374.0 | 5691.4 |
| Working-age | 5414.1 | 6190.7 | 7163.5 | 8137.6 | 9321.1 |
| Old-age | 411.4 | 473.5 | 545.3 | 619.2 | 631.4 |
| School-age | 4336.2 | 4971.5 | 5609.5 | 6188.5 | 6705.9 |
| Women of childbearing age | 2174.7 | 2553.1 | 2998.1 | 3373.0 | 3787.6 |
| Mid-interval population size | | 10581.8 | 11874.4 | 13339.4 | 14868.2 |
| Number of person-years lived | | 52909.1 | 59372.2 | 66696.9 | 74340.8 |
| Population growth | | 1197.5 | 1394.8 | 1538.5 | 1513.2 |
| Births | | 1996.7 | 2145.7 | 2245.5 | 2177.6 |
| Deaths | | 799.2 | 750.9 | 707.0 | 664.4 |
| **Indicators of the population structure** | | | | | |
| Proportions by broad age groups | | | | | |
| At young age (0-14) | 0.42 | 0.40 | 0.39 | 0.38 | 0.36 |
| At working age (15-64) | 0.54 | 0.55 | 0.57 | 0.58 | 0.60 |
| At old age (65+) | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| Dependency ratios | | | | | |
| Young-age | 0.77 | 0.73 | 0.68 | 0.66 | 0.61 |
| Old-age | 0.08 | 0.08 | 0.08 | 0.08 | 0.07 |
| Total | 0.85 | 0.81 | 0.76 | 0.74 | 0.68 |
| Median age of population | 19.4 | 19.5 | 20.0 | 21.0 | 22.0 |
| Proportion of women of childbearing age | 0.22 | 0.23 | 0.24 | 0.24 | 0.24 |
| Sex ratio of the population | 104 | 104 | 104 | 104 | 105 |
| **Indicators of the population distribution** | | | | | |
| Proportions of the national population | | | | | |
| Urban | 0.30 | 0.36 | 0.42 | 0.47 | 0.52 |
| Rural | 0.70 | 0.64 | 0.58 | 0.53 | 0.48 |
| **Rates of population change (per thousand)** | | | | | |
| Birth rate | | 37.7 | 36.1 | 33.7 | 29.3 |
| Death rate | | 15.1 | 12.6 | 10.6 | 8.9 |
| Natural increase | | 22.6 | 23.5 | 23.1 | 20.4 |
| Population growth | | 22.6 | 23.5 | 23.1 | 20.3 |

i. Change due to internal migration

The net change to the rural population is computed by reverse surviving net changes to rural survivors over a two-and-a-half-year interval and by aggregating the results. Table 30 illustrates how net changes to female rural survivors are calculated by reverse survival.

Thus, the net change to the rural female population aged 0-4 due to internal migration, -12.24 (column 4), is obtained as:

$$-12.24 = (-11.58) / [\ 0.67 + (0.33)\ (0.83618)\ ], \qquad (49)$$

where -11.58 (column 2) is the net change due to internal migration among survivors and 0.83618 (column 3) is the survival ratio.

The net change to the rural female population aged 5-9 due to internal migration, -12.22, is obtained as:

$$-12.22 = (-11.81) / [\ (1 + 0.93235) / 2\ ], \qquad (49)$$

where -11.81 and 0.93235 are, respectively, net change among the survivors and the survival ratio. Net changes due to internal migration relating to other age groups are computed in a way identical to that for the age group 5-9.

The net change to female rural population due to internal migration is -317.8 (total in column 4). A similarly calculated net change to male rural population amounts to -456.6. As a result, the net change to the rural population equals -774.4. The net change to the urban population is, therefore, 774.4:

$$774.4 = -(-774.4). \qquad (50)$$

ii. Deaths

The number of deaths in the urban population for the interval 0-5, 194.6, is obtained as:

$$194.6 = 503.8 - 1{,}083.6 + 774.4, \qquad (51)$$

where 503.8 is the number of births, 1,083.6 is the population growth, and 774.4 is the net change due to internal migration in the urban population during that interval.

The number of deaths in the rural population, 604.5, is obtained in the same way:

$$604.5 = 1{,}493.0 - 114.0 - (-774.4), \qquad (51)$$

where 1,493.0, 114.0 and -774.4 are the number of births, the population growth and the net change due to internal migration in the rural population.

Table 30. Computing net change due to internal migration: results for rural females; projection interval 0-5

| Age group | Net changes to the survivors due to internal migration a/ (thousands) | Survival ratios b/ | Net change to the population due to internal migration c/ (thousands) |
|---|---|---|---|
| (1) | (2) | (3) | (4) |
| 0-4 | -11.58 | 0.83618 | -12.24 |
| 5-9 | -11.81 | 0.93235 | -12.22 |
| 10-14 | -44.21 | 0.98462 | -44.56 |
| 15-19 | -53.94 | 0.98912 | -54.24 |
| 20-24 | -49.06 | 0.98542 | -49.42 |
| 25-29 | -35.52 | 0.98345 | -35.82 |
| 30-34 | -28.88 | 0.98125 | -29.16 |
| 35-39 | -23.96 | 0.97778 | -24.23 |
| 40-44 | -16.51 | 0.97328 | -16.73 |
| 45-49 | -10.14 | 0.96595 | -10.32 |
| 50-54 | -6.30 | 0.95134 | -6.46 |
| 55-59 | -6.50 | 0.92504 | -6.75 |
| 60-64 | -6.02 | 0.88409 | -6.39 |
| 65-69 | -3.84 | 0.82705 | -4.20 |
| 70-74 | -2.52 | 0.74980 | -2.88 |
| 75+ | -1.69 | 0.54193 | -2.20 |
| Total | | | -317.82 |

a/ From table 22, col. 5 and table 23, col. 4.
b/ From table 20, col. 4, for all age groups except the first.
c/ (Col. 2)/(0.67 + (0.33) . (Col.3)), when age group is 0-4, and (Col.2)/((1 + (Col. 3))/2), when age group is 5-9 or over.

b. Indicator of the population distribution

i. Proportions urban and rural

The proportion urban of the national population is calculated by dividing the urban population size by the total population size. This proportion is obtained as:

$$0.36 = 4{,}067.0 \;/\; 11{,}197.5, \qquad (52)$$

where 4,067.0 and 11,197.5 are, respectively, the urban and total population sizes at the end of the interval 0-5.

The proportion rural equals the complement of the proportion urban:

$$0.64 = 1 - 0.36. \qquad (53)$$

c. Rates of population change

i. Crude net internal migration rates

The crude net internal migration rate for the urban population is calculated as follows:

$$44.5 = [\; (774.4 \;/\; 5) \;/\; 3{,}483.3 \;] \cdot 1{,}000, \qquad (54)$$

and the crude net internal migration rate for the rural population is calculated as:

$$-21.9 = [\; (-774.4 \;/\; 5) \;/\; 7{,}073.4 \;] \cdot 1{,}000. \qquad (54)$$

In these calculations, 774.4 and -774.4 are net changes to urban and rural populations due to internal migration during the interval 0-5, and 5 is the length of the interval; 3,483.3 and 7,073.4 are mid-interval urban and rural population sizes, respectively.

## E. Summary

This chapter has described the cohort component method for making population projections. Such projections are a basic pre-requisite for integrating population considerations into comprehensive planning. The method can be used to make a projection of a national population or of urban and rural populations. These populations could be either closed or open to international migration. The method can be employed to make a projection over one or several consecutive five-year projection intervals and thus generate projection results for dates five years apart and the intervening projection intervals. A list of projection results that can be obtained by the method as described in this chapter is presented in box 26.

Box 26

Outputs of the cohort component method

1. Age and sex structure of the population (national or urban, rural and national)

2. Population aggregates (national or urban, rural and national):

Population size:

- Young-age population
- Working-age population
- Old-age population

- School-age population
- Women of childbearing ages

Mid-interval population size
Number of person-years-lived

Population growth
Births
Deaths
Net change due to international migration (if population open to international migration)
Net change due to internal migration (if urban and rural populations are being projected; urban and rural only)
Net change due to combined international and internal migration (if urban and rural populations open to international migration are being projected; urban and rural only)

3. Indicators of the population structure (national or urban, rural and national)

Proportions by broad age groups:

- Proportion at young age (0-14)
- Proportion at working age (15-64)
- Proportion at old age (65 +)

(continued)

Box 26 (continued)

Dependency ratios:

- Young-age dependency ratio
- Old-age dependency ratio
- Total dependency ratio

- Median age of the population
- Proportion of women in childbearing ages
- Sex ratio of the population

4. Indicators of the population distribution (national; if urban and rural populations are being projected)

   - Proportion urban
   - Proportion rural

5. Rates of population change (national or urban, rural and national, except where stated otherwise)

   - Crude birth rate
   - Crude death rate
   - Rate of natural increase
   - Crude net international migration rate (if population open to international migration)
   - Crude net internal migration rate (urban and rural only; if urban and rural populations are being projected)
   - Crude net combined international and internal migration rate (urban and rural only; if urban and rural populations open to international migrations are being projected)
   - Rate of population growth

Procedures used to make national or urban-rural projections, closed or open to international migration have been presented. Also, the types of inputs used with the method have been discussed along with issues relating to the preparation of the inputs. Lastly, these projection examples have been introduced and described with a view to illustrating the use of the method to project population. The examples were those of making projections of the national population closed and open to international migration, and of a projection of urban and rural populations closed to international migration.

## F. Notation and equations

### 1. Indices, variables and special symbols

(a) List of indices

| | |
|---|---|
| a = 1,...,16 | are five-year age groups 0-4,...,75+ |
| k = 1,2 | are urban and rural locations |
| s = 1,2 | are male and female sexes |
| t | is the year of the projection period |

(b) List of variables

| | |
|---|---|
| BIRTHS | is the number of births occurring during the interval |
| BIRTHS(s) | is the number of births of sex s occurring during the interval |
| BIRTHS(k) | is the number of births occurring in location k during the interval |
| CBR | is the crude birth rate for the interval |
| CCMR(k) | is the crude net combined migration rate of the population of location k for the interval |
| CDR | is the crude death rate for the interval |
| CEMR | is the crude net international (external) migration rate for the interval |
| CEMR(k) | is the crude net international (external) migration rate of the population of location k for the interval |
| CIMR(k) | is the crude net internal migration rate of the population of location k for the interval |
| DEATHS | is the number of deaths occurring during the interval |
| DEATHS(k) | is the number of deaths occurring in location k during the interval |
| EB(s) | is the expectation of life at birth of sex s specified for the interval |

| | |
|---|---|
| EM(a,s,k,t+5) | is the net change due to international (external) migration among survivors of age group a and sex s in location k at the end of the interval |
| EM(a,s,t+5) | is the net change due to international (external) migration among survivors of age group a and sex s at the end of the interval |
| EMR(a,s,k,t+5) | is the net international (external) migration rate applying to survivors of age group a and sex s in location k at the end of the interval |
| EMR(a,s,t+5) | is the net international (external) migration rate applying to survivors of age group a and sex s at the end of the interval |
| FR(a) | is the average annual fertility rate of age group a for the interval |
| GRP | is the average annual growth rate of the population for the interval |
| IM(a,s,k,t+5) | is the net change due to internal migration among survivors of age group a and sex s in location k at the end of the interval |
| IMR(a,s,t+5) | is the net internal migration rate applying to rural survivors of age group a and sex s at the end of the interval |
| MAPOP(t+5) | is the median age of the population at the end of the interval |
| MIPOP | is the mid-interval population size |
| MIPOP(a,2) | is the mid-interval number of women of age group a |
| MIPOP(k) | is the mid-interval size of the population of location k |
| NCDEM | is the net change in the population due to international (external) migration during the interval |
| NCDEM(k) | is the net change in the population of location k due to international (external) migration during the interval |
| NCDIM(k) | is the net change to the population of location k due to internal migration occurring during the interval |

| | |
|---|---|
| NPYL | is the total number of person-years-lived by the population during the interval |
| OADR(t+5) | is the old-age dependency ratio at the end of the interval |
| OAP(t+5) | is the old-age population at the end of the interval |
| PBS(s) | is the proportion of births of sex s |
| PEMR(a,s,k,t+5) | is the proportionate net international (external) migration rate applying to survivors of age group a and sex s in location k at the end of the interval |
| PEMR(a,s,t+5) | is the proportionate net international (external) migration rate applying to survivors of age group a and sex s at the end of the interval |
| PFR(a) | is the proportionate fertility rate of age group a pertaining to the interval |
| PIMR(a,s,t+5) | is the proportionate net internal migration rate applying to rural survivors of age group a and sex s at the end of the interval |
| POA(t+5) | is the proportion of the population at old age (age group 65+) at the end of the interval |
| POP(a,s,k,t+5) | is the population (survivors) of age group a and sex s in location k at the end of interval |
| POP(a,s,t+5) | is the population (survivors) of age group a and sex s at the end of the interval |
| POP(a-1,s,k,t) | is the population of age group (a-1) and sex s in location k at the beginning of the interval |
| POP(a-1,s,t) | is the population of age group (a-1) and sex s at the beginning of the interval |
| POP(k,t+5) | is the size of the population of location k at the end of the interval |
| POP(t+5) | is the size of the population at the end of the interval |
| POPGR | is the population growth over the interval |
| POPGR(k) | is the population growth in location k over the interval |

| | |
|---|---|
| PRUR(t+5) | is the proportion of the national population rural at the end of the interval |
| PURB(t+5) | is the proportion of the national population urban at the end of the interval |
| PWA(t+5) | is the proportion of the population at working age (age group 15-64) at the end of the interval |
| PWCA(t+5) | is the proportion of women in the childbearing ages at the end of the interval. |
| PYA(t+5) | is the proportion of the population at young age (age group 0-14) at the end of the interval |
| RNT | is the rate of natural increase for the interval |
| SAP(t+5) | is the school-age population at the end of the interval |
| SR(a,s) | is the survival ratio representing the probability of survival over the interval among persons who belong to age group a and sex s at the end of the interval |
| SR(a,s,k) | is the survival ratio representing the probability of survival over the interval among persons who belong to age group a and sex s in location k at the end of the interval |
| SRB | is the sex ratio at birth |
| SRF(a,s) | is the survival ratio factor used to reverse survive the net change due to international migration among survivors of age group a and sex s at the end of the interval |
| SRF(a,2,s) | is the survival ratio factor used to reverse survive the net change due to internal migration among the rural survivors of age group a and sex s at the end of the interval |
| SRP(t+5) | is the sex ratio of the population at the end of the interval |
| TDR(t+5) | is the total dependency ratio at the end of the interval |
| TEMR(s,k,t+5) | is the total net international (external) migration rate applying to survivors of sex s in location k at the end of the interval |

| | |
|---|---|
| TEMR(s,t+5) | is the total net international (external) migration rate applying to survivors of sex s at the end of the interval |
| TFR | is the total fertility rate specified for the interval |
| TIMR(s,t+5) | is the total net internal migration rate applying to rural survivors of sex s at the end of the interval |
| WAP(t+5) | is the working-age population at the end of the interval |
| WCA(t+5) | is the number of women in the childbearing ages at the end of the interval |
| YADR(t+5) | is the young-age dependency ratio at the end of the interval |
| YAP(t+5) | is the young-age population at the end of the interval |

(c) List of special symbols

| | |
|---|---|
| ln | is the natural logarithm |
| T | is the transformation of expectations of life at birth by sex into survival ratios using selected life tables |
| a' | is the five-year age group containing the member of the population who is older than one half of the population and younger than the other half |

## 2. Equations

### National population

(a) Closed population

(i) Segment of the population structure at age 5 and over

a. Survival ratios

$$SR(a,s) = T\,[EB(s)]; \qquad (1)$$

$$a = 1,\ldots,16;$$

$$s = 1,2$$

b. Population aged 5 and over

$$POP(a,s,t+5) = POP(a-1,s,t) \cdot SR(a,s); \quad (2)$$

$$a = 2,\ldots,15;$$

$$s = 1,2$$

$$POP(16,s,t+5) = \left[ \sum_{a=15}^{16} POP(a,s,t) \right] \cdot SR(16,s); \quad (3)$$

$$s = 1,2$$

(ii) Segment of the population structure below age 5

a. Fertility rates

$$FR(a) = (TFR/5) \cdot PFR(a); \quad (4)$$

$$a = 4,\ldots,10$$

b. Births

$$MIPOP(a,2) = \left[ (POP(a,2,t) \cdot (POP(a,2,t+5) \right]^{1/2}; \quad (5)$$

$$a = 4,\ldots,10$$

$$BIRTHS = 5 \cdot \left[ \sum_{a=4}^{10} FR(a) \cdot MIPOP(a,2) \right] \quad (6)$$

$$BIRTHS(s) = BIRTHS \cdot PBS(s); \quad (7)$$

$$s = 1,2,$$

where:

$$PBS(s) = \begin{cases} SRB/(100 + SRB), & \text{when } s = 1 \\ 100/(100 + SRB), & \text{when } s = 2 \end{cases}$$

c. Population below age 5

$$POP(1,s,t+5) = BIRTHS(s) \cdot SR(1,s); \quad s = 1,2 \tag{8}$$

(iii) Other results

a. Population aggregates

i. Population size

$$POP(t+5) = \sum_{a=1}^{16} \sum_{s=1}^{2} POP(a,s,t+5) \tag{9}$$

ii. Young-age population

$$YAP(t+5) = \sum_{a=1}^{3} \sum_{s=1}^{2} POP(a,s,t+5) \tag{10}$$

iii. Working-age population

$$WAP(t+5) = \sum_{a=4}^{13} \sum_{s=1}^{2} POP(a,s,t+5) \tag{11}$$

iv. Old-age population

$$OAP(t+5) = \sum_{a=14}^{16} \sum_{s=1}^{2} POP(a,s,t+5) \tag{12}$$

v. School-age population

$$SAP(t+5) = \sum_{a=2}^{5} \sum_{s=1}^{2} POP(a,s,t+5) \tag{13}$$

vi. Women of the childbearing ages

$$WCA(t+5) = \sum_{a=4}^{10} POP(a,2,t+5) \tag{14}$$

vii. Mid-interval population size

$$MIPOP = [\ POP(t) \cdot POP(t+5)\ ]^{1/2} \quad (15)$$

viii. Total number of person-years-lived

$$NPYL = MIPOP \cdot 5 \quad (16)$$

ix. Population growth

$$POPGR = POP(t+5) - POP(t) \quad (17)$$

x. Births

xi. Deaths

$$DEATHS = BIRTHS - POPGR \quad (18)$$

b. Indicators of the population structure

i. Proportions by broad age groups

the proportion at young age:

$$PYA(t+5) = YAP(t+5) / POP(t+5) \quad (19)$$

the proportion at working age:

$$PWA(t+5) = WAP(t+5) / POP(t+5) \quad (20)$$

and the proportion at old age:

$$POA(t+5) = OAP(t+5) / POP(t+5) \quad (21)$$

ii. Dependency ratios

the young-age dependency ratio:

$$YADR(t+5) = YAP(t+5) / WAP(t+5) \quad (22)$$

the old-age dependency ratio:

$$OADR(t+5) = OAP(t+5) \;/\; WAP(t+5) \tag{23}$$

and the total dependency ratio:

$$TDR(t+5) = [\; YAP(t+5) + OAP(t+5) \;] \;/\; WAP(t+5) \tag{24}$$

iii. Median age of the population

$$MAPOP(t+5) = (a' - 1) \cdot 5 + [\; (POP(t+5)/2 - \sum_{a=1}^{a'-1} \sum_{s=1}^{2} POP(a,s,t+5) \;) \;/\; \sum_{s=1}^{2} POP(a',s,t+5) \;] \cdot 5 \tag{25}$$

iv. Proportion of women in the childbearing ages

$$PWCA(t+5) = WCA(t+5) \;/\; POP(t+5) \tag{26}$$

v. Sex ratio of the population

$$SRP(t+5) = [\; \sum_{a=1}^{16} POP(a,1,t+5) \;] \;/\; [\; \sum_{a=1}^{16} POP(a,2,t+5) \;] \tag{27}$$

c. Rates of population change

i. Crude birth rate

$$CBR = [\; (BIRTHS/5) \;/\; MIPOP \;] \cdot 1{,}000 \tag{28}$$

ii. Crude death rate

$$CDR = [\; (DEATHS/5) \;/\; MIPOP) \;] \cdot 1{,}000 \tag{29}$$

iii. Rate of natural increase

$$RNI = CBR - CDR \tag{30}$$

iv. Rate of population growth

$$GRP = [\ \ln( POP(t+5)/POP(t) ) / 5\ ] \cdot 1{,}000 \qquad (31)$$

(b) Open population

(i) Segment of the population structure at age 5 and over

a. Survival ratios and survivors aged 5 and over

b. Net international migration rates

$$EMR(a,s,t+5) = TEMR(s,t+5) \cdot PEMR(a,s,t+5); \qquad (32)$$

$$a = 1,\ldots,16;$$

$$s = 1,2$$

c. Population aged 5 and over

$$EM(a,s,t+5) = POP^{*}(a,s,t+5) \cdot EMR(a,s,t+5); \qquad (33)$$

$$a = 2,\ldots,16;$$

$$s = 1,2$$

$$POP(a,s,t+5) = POP^{*}(a,s,t+5) + EM(a,s,t+5); \qquad (34)$$

$$a = 2,\ldots,16;$$

$$s = 1,2$$

(ii) Segment of population structure below age 5

a. Fertility rates, births and survivors below age 5

b. Population below age 5

$$EM(1,s,t+5) = POP^{*}(1,s,t+5) \cdot EMR(1,s,t+5); \qquad (35)$$

$$s = 1,2$$

$$POP(1,s,t+5) = POP^{*}(1,s,t+5) + EM(1,s,t+5); \qquad (36)$$

$$s = 1,2$$

(iii) Other results

a. Population aggregates

i. Change due to international migration

$$NCDEM = \sum_{a=1}^{16} \sum_{s=1}^{2} EM(a,s,t+5) \,/\, SRF(a,s), \tag{37}$$

where for each s:

$$SRF(a,s) = \begin{cases} 0.67 + 0.33 \cdot SR(1,s), & \text{when } a = 1 \\ ( 1 + SR(a,s) ) \,/\, 2, & \text{when } 1 < a \leq 16 \end{cases}$$

ii. Deaths

$$DEATHS = BIRTHS - POPGR + NCDEM \tag{38}$$

b. Indicators of the population structure

c. Rates of population change

i. Crude net international migration rate

$$CEMR = [\ (NCDEM/5) \,/\, MIPOP\ ] \cdot 1{,}000 \tag{39}$$

## Urban and rural populations

(a) Closed populations

(i) Segments of population structures at age 5 and over

a. Survival ratios

b. Survivors aged 5 and over

$$POP^{*}(a,s,k,t+5) = POP(a-1,s,k,t) \cdot SR(a,s,k); \tag{40}$$

$$a = 2,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2$$

$$POP^{*}(16,s,k,t+5) = \left[ \sum_{a=15}^{16} POP(a,s,k,t) \right] \cdot SR(16,s,k); \quad (41)$$

$$s = 1,2;$$

$$k = 1,2$$

c. Net internal migration rates

$$IMR(a,s,t+5) = TIMR(s,t+s) \cdot PIMR(a,s,t+5); \quad (42)$$

$$a = 1,\ldots,16;$$

$$s = 1,2$$

d. Population aged 5 and over

$$IM(a,s,2,t+5) = POP^{*}(a,s,2,t+5) \cdot IMR(a,s,t+5); \quad (43)$$

$$a = 2,\ldots,16;$$

$$s = 1,2$$

$$IM(a,s,1,t+5) = - IM(a,s,2,t+5); \quad (44)$$

$$a = 2,\ldots,16;$$

$$s = 1,2$$

$$POP(a,s,k,t+5) = POP^{*}(a,s,k,t+5) + IM(a,s,k,t+5); \quad (45)$$

$$a = 2,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2$$

(ii) Segments of population structures below age 5

a. Fertility rates, births and survivors below age 5

b. Population below age 5

$$IM(1,s,2,t+5) = POP^{*}(1,s,2,t+5) \cdot IMR(1,s,t+5); \quad (46)$$

$$s = 1,2$$

$$IM(1,s,1,t+5) = - IM(1,s,2,t+5); \quad s = 1,2 \tag{47}$$

$$POP(1,s,k,t+5) = POP^{*}(1,s,k,t+5) + IM(1,s,k,t+5); \quad s = 1,2; \quad k = 1,2 \tag{48}$$

(iii) Other results

a. Population aggregates

i. Change due to internal migration

$$NCDIM(2) = \sum_{a=1}^{16} \sum_{s=1}^{2} IM(a,s,2,t+5) \,/\, SRF(a,2,s), \tag{49}$$

where for each s:

$$SRF(a,2,s) = \begin{cases} 0.67 + 0.33 \cdot SR(1,2,s), & \text{when } a = 1 \\ ( 1 + SR(a,2,s) ) \,/\, 2, & \text{when } 1 < a \leq 16 \end{cases}$$

$$NCDIM(1) = - NCDIM(2) \tag{50}$$

ii. Deaths

$$DEATHS(k) = BIRTHS(k) - POPGR(k) + NCDIM(k); \quad k = 1,2 \tag{51}$$

b. Indicators of the population structure

c. Indicators of the population distribution

i. Proportions urban and rural

$$PURB(t+5) = POP(1,t+5) \,/\, POP(t+5) \tag{52}$$

$$PRUR(t+5) = 1 - PURB(t+5) \tag{53}$$

d. Rates of population change

i. Crude net internal migration rate

$$CIMR(k) = [\ (NCDIM(k) / 5) / MIPOP(k)\ ] \cdot 1{,}000; \qquad (54)$$

$$k = 1,2$$

(b) Open populations

(i) Segments of population structures at age 5 and over

a. Survival ratios, survivors aged 5 and over, and net internal migration rates

b. Net international migration rates

$$EMR(a,s,k,t+5) = TEMR\ (s,k,t+5) \cdot PEMR(a,s,k,t+5); \qquad (55)$$

$$a = 1,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2$$

c. Population aged 5 and over

$$EM(a,s,k,t+5) = POP^{*}(a,s,k,t+5) \cdot EMR(a,s,k,t+5); \qquad (56)$$

$$a = 2,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2$$

$$POP(a,s,k,t+5) = POP^{*}(a,s,k,t+5) + IM(a,s,k,t+5) + EM(a,s,k,t+5); \qquad (57)$$

$$a = 2,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2$$

(ii) Segments of population structures below age 5

a. Fertility rates, births and survivors below age 5

b. Population below age 5

$$EM(1,s,k,t+5) = POP^{*}(1,s,k,t+5) \cdot EMR(1,s,k,t+5); \quad (58)$$

$$s = 1,2;$$

$$k = 1,2$$

$$POP(1,s,k,t+5) = POP^{*}(1,s,k,t+5) + IM(1,s,k,t+5) + EM(1,s,k,t+5); \quad (59)$$

$$s = 1,2;$$

$$k = 1,2$$

(iii) Other results

a. Population aggregates

i. Deaths

$$DEATHS(k) = BIRTHS(k) - POPGR(k) + NCDIM(k) + NCDEM(k); \quad (60)$$

$$k = 1,2$$

b. Indicators of the population structure

c. Rates of population change

i. Crude net combined migration rate

$$CCMR(k) = CEMR(k) + CIMR(k); \quad (61)$$

$$k = 1,2$$

## Notes

1/ For alternative methods of population projections see, for example, Henry S. Shryock, Siegel and Associates, 1973.

2/ An alternative description of the cohort component method would use a one-year time period as a projection interval.

3/ Several definitions found in the glossary boxes throughout this chapter were adapted from Shorter and Macura, 1982.

4/ In the present description, the age structure of the population is that involving five-year age groups ending with the open age group, 75+ . In the application of the cohort component method the open age group may actually be 65+, 70+ or 80+, instead of 75+.

5/ In equation (1), as well as in the remainder of the chapter, the notation used differs from the standard demographic notation. A uniform notation is being used throughout the manual, which is primarily designed to describe methods to project socio-economic variables rather than demographic variables.

6/ An alternative would be the use of an empirical life table, such as a national life table to derive survival ratios from given expectation of life at birth.

7/ The geometric mean is used in this and similar computations, since this type of mean rather than some others, such as the arithmetic mean, is compatible with the assumption of exponential growth of the population. In particular, if population grows at a constant exponential rate, the geometric mean of the population at the beginning and the end of the interval, represents the mid-interval population.

8/ For a discussion of the calculation of the median age from grouped data, see: Shryock, Siegel and Associates, 1973.

9/ The intermediate results obtained in calculating a particular variable in the course of a projection are marked with an asterisk, *. In this particular instance, the numbers of survivors obtained for the end of a five-year projection interval represent the intermediate result of projecting an open population and are, therefore, marked with an asterisk.

10/ These two expressions, applying to the five-year groups below age 5 and at age 5 and above, have been adopted from Shorter and others, 1987.

11/ An example of this category of methods is the so-called urban-rural growth differential method which derives proportions of the national population in urban and rural areas and then applies those proportions to the national population projection in order to calculate projected urban and rural populations. For the urban-rural growth differential method along with alternative techniques for making urban and rural population projections, see United Nations, 1974.

12/ An example of such a variant of the cohort component method is the multiregional population projection model elaborated in Rogers, 1975.

13/ For the use of infant mortality rates and expectations of life at five as measures of mortality below 5 and at age 5 and over, see Shorter and others, op. cit.

14/ In preparing a population projection using, among other things, model life tables, the planner can choose from among several families of model life tables. The United Nations model life tables include five such families, which are referred to as Latin American, Chilean, South Asian, Far Eastern and General. Another set of model life tables, developed by Coale and Demeny, consists of four families, called West, North, East and South. In connection with this set see, Ansley J. Coale, Paul Demeny and Barbara Vaughan, Regional Model Life Tables and Stable Populations, 2nd ed., (New York, Academic Press, 1983).

15/ The initial age and sex structure of the population, shown in table 1, is expressed in units of one thousand. In view of this, the numbers of persons in this illustrative example will be given in thousands.

## Annex I

### DESCRIPTION OF A LIFE TABLE

A life table is a table showing a listing of the number of survivors at different ages (up to the highest age attained) in a hypothetical cohort (box 27), typically of one sex. It represents an experience of a cohort subject from birth to a particular set of age-specific mortality rates, which are usually those observed in a given population during a particular time period. A life table also presents other aspects of the cohort's experience, such as probabilities of dying and the numbers of person-years-lived between specific years of age. The following is a typical life table:a/

| Age | $_nm_x$ | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.18522 | 0.16532 | 100000 | 16532 | 89254 | 4300001 | 43.000 |
| 1 | 0.04080 | 0.14733 | 83468 | 12297 | 301421 | 4210746 | 50.447 |
| 5 | 0.00710 | 0.03486 | 71171 | 2481 | 349653 | 3909326 | 54.929 |
| 10 | 0.00300 | 0.01488 | 68690 | 1022 | 340895 | 3559673 | 51.822 |
| 15 | 0.00437 | 0.02161 | 67668 | 1462 | 334845 | 3218778 | 47.567 |
| 20 | 0.00532 | 0.02626 | 66206 | 1739 | 326754 | 2883933 | 43.560 |
| 25 | 0.00564 | 0.02782 | 64467 | 1794 | 317904 | 2557179 | 39.667 |
| 30 | 0.00652 | 0.03207 | 62673 | 2010 | 308421 | 2239275 | 35.730 |
| 35 | 0.00731 | 0.03592 | 60663 | 2179 | 297956 | 1930854 | 31.829 |
| 40 | 0.00854 | 0.04183 | 58484 | 2446 | 286458 | 1632898 | 27.921 |
| 45 | 0.01080 | 0.05266 | 56037 | 2951 | 273132 | 1346440 | 24.028 |
| 50 | 0.01609 | 0.07751 | 53086 | 4115 | 255709 | 1073308 | 20.218 |
| 55 | 0.02449 | 0.11574 | 48972 | 5668 | 231407 | 817599 | 16.695 |
| 60 | 0.03773 | 0.17294 | 43304 | 7489 | 198476 | 586192 | 13.537 |
| 65 | 0.05517 | 0.24297 | 35815 | 8702 | 157737 | 387715 | 10.825 |
| 70 | 0.08240 | 0.34149 | 27113 | 9259 | 112366 | 229978 | 8.482 |
| 75 | 0.12232 | 0.46362 | 17854 | 8278 | 67672 | 117613 | 6.587 |
| 80 | 0.16561 | 0.57223 | 9577 | 5480 | 33089 | 49941 | 5.215 |
| 85 | 0.24310 | | 4097 | 4097 | 16851 | 16851 | 4.113 |

The various functions or columns in a life table are defined as follows:

Age — is the age interval (x to x + n) where x is the initial age of the interval and n is its length; the length equals five years with the exception of the first interval (one year), second interval (four years) and the last interval (open-ended),

$_nm_x$ — is the death rate (number of deaths per person-years-lived) between age x and x + n,

$_nq_x$ — is the probability of surviving between age x and x + n,

$l_x$ — is the number of survivors at age x out of an original cohort of 100,000 (or the radix of a different value),

Box 27

Glossary

**Age-specific mortality rate**
The number of deaths occurring during a specified period to persons (usually specified by sex) of a specified age or age group, divided by the number of person-years-lived during that period by the persons of that age or age group. When an age-specific mortality rate is calculated for a calendar year, the number of deaths to persons of the specified age is usually divided by the mid-year population of persons of that age.

**Cohort**
A group of individuals who experienced the same class of events in the same period. Thus, a birth or age cohort is a group of people born during a particular time period.

**Person-years-lived**
The number of years lived by a group of people, such as the national population or the urban population, during a specified period of time. When used in relation to a life table it represents the number of years lived by a hypothetical cohort between any two exact ages, and it is denoted by the symbol $_nL_x$.

**Radix**
The hypothetical birth cohort of a life table. Its common values are 1, 1,000 or 100,000.

$_nd_x$ is the number of deaths between age x and x + n,

$_nL_x$ is the number of person-years-lived between age x and x + n,

$T_x$ is the total number of person-years-lived from age x onward,

$e_x$ is the expectation of life at age x.

In addition to the columns shown in the above table, a typical life table would either include or be accompanied by a column showing survival ratios, defined as follows:

$_nP_x$ is the proportion of persons between ages x and x + n surviving until ages x + 5 to x + n + 5.

A column of survival ratios accompanying the above life table is:

| Age | | | $_nP_x$ |
|---|---|---|---|
| Birth | to | 0-4 | 0.7813 |
| 0-4 | to | 5-9 | 0.8950 |
| 5-9 | to | 10-14 | 0.9750 |
| 10-14 | to | 15-19 | 0.9823 |
| 15-19 | to | 20-24 | 0.9758 |
| 20-24 | to | 25-29 | 0.9729 |
| 25-29 | to | 30-34 | 0.9702 |
| 30-34 | to | 35-39 | 0.9661 |
| 35-39 | to | 40-44 | 0.9614 |
| 40-44 | to | 45-49 | 0.9535 |
| 45-49 | to | 50-54 | 0.9362 |
| 50-54 | to | 55-59 | 0.9050 |
| 55-59 | to | 60-64 | 0.8577 |
| 60-64 | to | 65-69 | 0.7947 |
| 65-69 | to | 70-74 | 0.7124 |
| 70-74 | to | 75-79 | 0.6022 |
| 75-79 | to | 80-84 | 0.4890 |
| 80+ | to | 85+ | 0.3374 |

The various life table functions are interrelated, which makes it possible to derive an entire life table from one particular function, such as $_nm_x$ or $_nq_x$. The relationships among the functions may often be useful where one wishes, on the basis of an existing life table, to derive entries that are not available in it. An example of such an entry, which is unavailable in the illustrative life table used above, is the death rate for the age interval 0-4, $_5m_0$. The relationships among various life table are as follows:

$$_nq_x = (n \cdot {}_nm_x) / [\, 1 + (n - {}_na_x \cdot {}_nm_x) \,],$$

$$l_{x+n} = l_x \cdot (1 - {}_nq_x),$$

$$_nd_x = l_x - l_{x+n},$$

$$_nL_x = {}_na_x \cdot l_x + (n - {}_na_x) \cdot l_{x+n},$$

$$T_x = \sum_{a=x}^{w} {}_nL_x, \quad \text{and}$$

$$e_x = T_x / l_x,$$

where $_na_x$'s are constants and w is the lower end of the open age interval.b/

## Notes

a/ This illustrative life table was published in *Model Life Tables for Developing Countries* (United Nations publication, Sales No. E.81.XIII.7).

b/ *Ibid.*, p. 31.

## Annex II

### DERIVING SURVIVAL RATIOS FROM INFANT MORTALITY RATES AND EXPECTATIONS OF LIFE AT AGE 5

This annex will show, using an illustrative example, how to derive survival ratios from infant mortality rates and expectations of life at age 5 employing a family of model life tables. The example, which will illustrate calculations for males, will use the male infant mortality rate, 156.7, and the male expectation of life at age 5, 63.6, along with the South Asian family of United Nations model life tables for males. a/

Where infant mortality rates and expectations of life at age 5 are used as measures to formulate mortality assumptions, the two measures can be used as representations of mortality below age 5 and at age 5 and above, respectively. Therefore, the use of these measures to derive survival ratios would involve locating segments of model life tables below age 5 and at age 5 and above that correspond to those measures. It would also involve making additional calculations using the two segments.

This annex will initially show how to derive survival ratios below age 5 and at age 5 and above. Then, it will illustrate how to obtain a survival ratio that bridges the two age segments.

### A. Survival ratio below age 5

The survival ratio below age 5, which indicates the probability of survival between a birth during a five-year interval and the population age 0-4 at the end of the interval, is obtained using the given infant mortality rate specified for the interval. Also derived along with this survival ratio are selected model life table entries needed to later calculate the survival ratio bridging the two age spans--below age 5 and at age 5 and above. Those entries include the number of survivors at age 5 and the numbers of person-years-lived at exact ages 0-1 and 1-5.

The derivation of the survival ratio below age 5 and the relevant life table entries that correspond to the male infant mortality rate 156.7 is illustrated in table 31. The derivation is based on a linear interpolation between entries in columns 2 and 3, which were taken from the male model life tables of the United Nations South Asian family at expectations of life at birth, 46 and 47. Note that the model life tables in question are those embodying infant mortality rates (161.43 and 155.91) which bracket the infant mortality rate selected for this illustration (156.7). The factor of interpolation used, 0.8569, is computed as:

$$0.8569 = (161.43 - 156.7) / (161.43 - 155.91),$$

Table 31 Deriving the segment of a model life table below age 5, using infant mortality rate, results for males

| | Age | Selected model life table entries at adjacent expectations of life at birth, 46 and 47 | | Various entries corresponding to infant mortality rate, 156 7 |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Expectation of life at birth | | 46 0 | 47 0 | |
| Infant mortality rate | | 161.43 | 155.91 | 156 7 |
| Survival ratios | Birth to 0-4 | 0 7960 | 0 8046 | 0 8035 |
| Survivors | 5 | 73974 | 75001 | 74854 |
| Person-years lived | 0-1 | 89184 | 89554 | 89501 |
| | 1-5 | 309256 | 312724 | 312228 |
| | 0-5 | | | 401729 |

where 161.43 and 155.91 are male model infant mortality rates embodied in the model life tables at expectations of life at birth, 46 and 47; 156.7 is the male infant mortality rate selected for this illustration.

With this factor of interpolation, the survival ratio sought, 0.8035, which is shown in column 4, is obtained as:

$$0.8035 = (0.8569)\ (0.8046) + (1 - 0.8569)\ (0.7969),$$

where 0.7969 and 0.8046 are, respectively, model survival ratios between birth and age 0-4 in male model life tables at expectations of life at birth, 46 and 47 years. Among the remaining entries sought--the number of survivors at age 5 (74,854) and the numbers of person-years-lived at exact years of age 0-1 and 1-5 (89,501 and 312,228), shown in column 4--are obtained in the same way.

The last result obtained, which is the number of person-years-lived at ages 0-5, 401,729, shown in column 4, is derived as follows:

$$401{,}729 = 89{,}501 + 312{,}228,$$

where 89,501 and 312,228 are the numbers of person-years-lived at ages 0-1 and 1-5.

## B. Survival ratios at ages 5 and above

The survival ratios at age 5 and over are obtained using the given expectation of life at age 5 specified for the five-year times interval in question. Also derived with those ratios are selected entries, some of which are required to calculate the survival ratio at the border of the two age spans.

The calculations of those survival ratios and life table entries are illustrated in table 32. Those calculations are made by means of a linear interpolation between entries in columns 2 and 3, which contain values taken from the male model life tables used in this example at expectations of life at birth, 59 and 60. Note that the expectations at age 5 embodied in these life tables (63.355 and 63.873) bracket the expectation of life at age 5 selected for this example (63.6). The factor of interpolation used is computed as follows:

$$0.5270 = (63.873 - 63.6) \,/\, (63.873 - 63.355),$$

where 63.355 and 63.873 are male model expectations of life at birth in the model life tables at expectations of life at birth, 59 and 60 (columns 2 and 3); 63.6 is the expectation of life at age 5 selected for this example.

This interpolation factor is used to obtain survival ratios ranging from the one for "5-9 to 10-14" through that for "65-69 to 70-74". Thus, for example, the first of these ratios, 0.9921, is obtained as:

$$0.9921 = (0.5270)\ (0.9918) + (1 - 0.5270)\ (0.9924),$$

Table 32. Deriving the segment of a model life table at age 5 and over using expectation of life at age 5; results for males

| | Age | Selected model life table entries at adjacent expectations of life at birth, 59 and 60 | | Various entries corresponding to expectation of life at age 5, 63.6 |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Expectation of life at birth | | 59.0 | 60.0 | |
| Expectation of life at age 5 | | 63.355 | 63.873 | 63.600 |
| Survival ratios | 5-9 to 10-14 | 0.9918 | 0.9924 | 0.9921 |
| | 10-14 to 15-19 | 0.9945 | 0.9949 | 0.9947 |
| | 15-19 to 20-24 | 0.9933 | 0.9938 | 0.9935 |
| | 20-24 to 25-29 | 0.9919 | 0.9924 | 0.9921 |
| | 25-29 to 30-34 | 0.9900 | 0.9907 | 0.9903 |
| | 30-34 to 35-39 | 0.9868 | 0.9877 | 0.9872 |
| | 35-39 to 40-44 | 0.9808 | 0.9819 | 0.9813 |
| | 40-44 to 45-49 | 0.9710 | 0.9724 | 0.9717 |
| | 45-49 to 50-54 | 0.9543 | 0.9563 | 0.9552 |
| | 50-54 to 55-59 | 0.9294 | 0.9320 | 0.9306 |
| | 55-59 to 60-64 | 0.8917 | 0.8950 | 0.8933 |
| | 60-64 to 65-69 | 0.8377 | 0.8418 | 0.8396 |
| | 65-69 to 70-74 | 0.7666 | 0.7716 | 0.7689 |
| | 70+ to 75+ | 0.5779 | 0.5819 | 0.5798 |
| Survivors | 5 | 86116 | 86934 | 86503 |
| Person-years lived | 5-10 | 428143 | 432401 | 430157 |
| | 70+ | 465675 | 487387 | |
| | 75+ | 269089 | 283592 | |

where 0.9918 and 0.9924 (columns 2 and 3) are the relevant model survival ratios.

To calculate the last of the survival ratios, which is one for "70+ to 75+", it is necessary first to calculate corresponding model survival ratios in the model life tables used. These survival ratios can be obtained from the numbers of person-years-lived at ages 70 and above and 75 and over. Thus, the survival ratio for "70+ to 75+" in the model life table at the expectation of life at birth 59, 0.5779, shown in column 2, is obtained as:

0.5779 = 269,689 / 465,675,

where 465,675 and 269,089 (column 2) are the numbers of person-years-lived at age 70 and above and 75 and over.

Given survival ratios for "70+ to 75+" in the two adjacent model life tables at expectations of life at birth, 59 and 60, the survival ratio that corresponds to the expectation of life at age 5, 63.6, is obtained in a similar way. This ratio, 0.5798, is obtained as:

0.5798 = (0.5270) (0.5779) + (1 - 0.5270) (0.5819).

This completes the calculation of survival ratios at age 5 and over.

In order to enable the calculation of the survival ratio spanning the age intervals below age 5 and at age 5 and above, it is also necessary to perform further calculations involving the model numbers of survivors at age 5 and the model numbers of person-years-lived at ages 5-10 (columns 2 and 3). These calculations yield the number of survivors at age 5, 86,503, and the number of person-years-lived at age 5-10, 430,157, shown in column 4, which correspond to the expectation of life at age 5, 63.6 (column 4). They are performed by means of a linear interpolation using the same interpolation factor, 0.5270.

### C. Survival ratio spanning age intervals below age 5 and age 5 and above

The survival ratio for "0-5 to 5-10" is obtained using the numbers of person-years-lived aged 0-5 and 5-10 which are respectively derived from the given infant mortality rate (156.7) and the expectation of life at age 5 (63.6). Also used in the calculation are the numbers of survivors at age 5 obtained using these two mortality measures.

The survival ratio sought, 0.9266, is calculated as follows:

0.9266 = [ (430,157) (74,854/86,503) ] / 401,729,

where 401,729 and 430,157 are the numbers of person-years-lived at ages 0-5 and 5-10, derived from the given infant mortality rate and the expectation of life at age 5. 74,854 and 86,503 are the numbers of survivors at age 5 derived from those two measures, respectively.

This completes the derivation of the survival ratios from infant mortality rates and the expectation of life at age 5.

## Note

a/ *Model Life Tables for Developing Countries* (United Nations publication, Sales No. E. 81. XII1.7).

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# III. THE HEADSHIP RATE METHOD FOR MAKING HOUSEHOLD PROJECTIONS

## A. Introduction

Household projections can be extremely useful for planning. Thus, the total number of households and the average household size (box 28), obtained as part of a projection, may be used as inputs for projections of household income or household consumption and savings. Those results may be employed as inputs into projections of government consumption and investment in the housing sector. They can also be used in a variety of other planning exercises.

Household projections can be prepared by the headship rate method, which is available in several versions. The simplest version derives a projection by applying assumed age- and sex-specific headship rates to the projected age and sex structures of the population (United Nations, 1973). A more complex version applies age-, sex- and marriage-specific headship rates to the projected population disaggregated by age, sex and marital status. 1/ Another version employs assumptions on age- and sex-specific headship rates along with assumptions on the proportionate size distributions of households classified by the age group of the household head (See, for example, Kono, 1981 and 1987). Yet another version of the method employs assumptions on headship rates by age, sex and household category. Among the categories are one-person households and households headed by males with the spouse present (Mason, 1986).

This chapter describes the simplest version of the headship rate method, which is least demanding in terms of both data and assumptions and, therefore, most readily applicable in planning. The method can be used to make a national projection or a projection for urban and rural areas, given requisite age and sex structures of the population along with assumptions on future trends in age- and sex-specific headship rates. The method yields the projected numbers of households by age and sex of household head, average household size and other relevant indicators.

The version of the headship-rate method described here requires limited data and simple calculations. The data include the numbers of household heads and their age, sex and location of residence along with information on the population classified by the same characteristics. Though the data requirements are not great, until recently the lack of requisite census or survey information relating to households made the method inapplicable in many developing countries. As these data are becoming more readily available, the method is likely to be increasingly used by planners in those countries.

A major limitation of the headship rate method (in all its versions) is that it generates the number of households in a mechanical way. This is so