

TABLE 13. LUXEMBOURG MALES, 1946-1949. COMPUTATION OF SURVIVAL RATIOS BY SHORT-CUT AND BY LONG PROCEDURE

Age (x)	1,000 $m_x$	Conversion factor	Estimates by short-cut procedure			Results of computation by long procedure	
			1,000 $a_x$	$p_x$	$P_x$ ( $P_b=0.9351$ ) <sup>a</sup>	$p_x^d$	$P_x^d$ ( $P_b=0.9354$ )
0.....	71.1	0.89	63.3	0.9367	0.9861 <sup>b</sup>	0.9370	0.9862
1- 4.....	3.1	3.9	12.1	0.9879		0.9880	
5- 9.....	1.4	5	7.0	0.9930	0.9938	0.9930	0.9937
10-14.....	1.1	5	5.5	0.9945	0.9932	0.9945	0.9933
15-19.....	1.6	5	8.0	0.9920	0.9900	0.9920	0.9901
20-24.....	2.4	5	12.0	0.9880	0.9878	0.9881	0.9879
25-29.....	2.5	5	12.5	0.9875	0.9858	0.9876	0.9859
30-34.....	3.2	5	16.0	0.9840	0.9820	0.9841	0.9822
35-39.....	4.0	5	20.0	0.9800	0.9758	0.9802	0.9761
40-44.....	5.7	5	28.5	0.9715	0.9622	0.9719	0.9626
45-49.....	9.6	4.9	47.0	0.9530	0.9488	0.9530	0.9490
50-54.....	11.3	4.9	55.4	0.9446	0.9276	0.9449	0.9284
55-59.....	18.6	4.8	89.3	0.9107	0.8908	0.9109	0.8918
60-64.....	27.5	4.7	129.2	0.8708	0.8398	0.8709	0.8412
65-69.....	42.5	4.5	191.2	0.8088	0.7642	0.8071	0.7676
70-74.....	65.2	4.3	280.4	0.7196	0.6530	0.7187	0.6649
75-79.....	103.4	4	413.6	0.5864	0.4984	0.5900	0.5224
80-84.....	173.4	3.4	589.6	0.4104	0.2987 <sup>c</sup>	0.4078	0.3075
85+.....	260.9	...	1,000	...		...	

<sup>a</sup> Computed by formula 1 in the accompanying text.

<sup>b</sup>  $P_{0-4}$ , computed by formula 2.

<sup>c</sup>  $P_{80+}$ , computed by formula 3.

<sup>d</sup> Computed from the  $q_x$  values in table 11, column 3.

<sup>e</sup> From table 11, column 6.

#### IV. ESTIMATING CURRENT LEVELS AND FUTURE TRENDS OF SURVIVAL RATIOS WITH THE USE OF MODEL LIFE TABLES

181. The main part of the procedure of a population projection by sex-age groups consists in multiplying the numbers of various cohorts living at a given time by appropriate survival ratios. The needed ratios can often be worked out individually for a particular population projection, but the procedure is greatly simplified when reference is made to a system of model life tables.

182. Tabulated values for a system of model life tables are found in the appendix, with a note explaining how they were constructed. To facilitate their use in formulating assumptions relating to the future trend of mortality, the tables have been so arranged that they can be regarded as representing successive stages in a process of declining mortality.

183. The idea of a coherent system of model life tables and of some of its uses is developed in part A of this chapter. Part B describes how, with this system, under various conditions and with varying amounts of statistical information, suitable survival ratios can be quickly obtained. The application of survival ratios to an actual population project is illustrated in part C, consideration also being given to possible variations in the systematic assumption of future changes in mortality.

##### A. THE MODEL LIFE TABLES CONCEIVED AS ONE SYSTEM

184. Each model life table is designed to represent a typical combination of age-sex specific functions of mor-

tality, or survival, corresponding to a given general level of mortality. For present purposes, the general mortality level has been determined in terms of  ${}^0e_0$ , the expectation of life at birth, for both sexes combined. Actually, the combination of mortality rates, age group by age group, in any given instance, will always differ more or less from any pattern taken as typical for the same general mortality level. It may therefore be necessary to refer to more than one model life table, and perhaps also to make interpolations between two successive tables in estimating the appropriate combination of rates for a given case.

185. A generalization is here made as to the manner in which mortality may decline, during successive five-year time periods, from the conditions of one model life table to those of the next table in the sequence. This generalization requires a rather liberal interpretation. It is not asserted that mortality will always decline in this particular way. It may decline more slowly, more rapidly, or with different rapidity for different age groups. The model assumption is merely one which is plausible under some of the more typical conditions to be found in the world today, and it can be modified as required. Apart from its uses in the estimation of future mortality trends, this model assumption also serves as the link by which the several model life tables are tied together into a coherent sequence.

## 1. Gains in life expectation: the past record

186. A substantial decline in rates of mortality is one of the outstanding phenomena of the modern era. This record of past achievement is relevant to considerations of the future, but is not susceptible of very accurate assessment. Adequate mortality statistics covering a sufficiently long period have not become available for a representative majority of the world's populations. In order to obtain a very rough indication of the manner in which mortality has changed in those areas for which fairly reliable statistics exist, the official life tables published in the United Nations *Demographic Yearbook* for various countries have been used. Areas were selected for which both an earlier and a more recent life table had been reproduced in the Yearbook, and these areas were grouped geographically as follows:

I. *English-speaking countries of European settlement overseas.* Europeans of the Union of South Africa, Canada, United States, Australia and New Zealand.

II. *Northern and Northwestern Europe.* Denmark, Finland, Iceland, Norway, Sweden, England and Wales, Northern Ireland and Scotland.

III. *Western and Central Europe.* Austria, Belgium, France, Germany, Netherlands.

IV. *Southern and Eastern Europe.* Bulgaria, Greece, Italy, Portugal, Spain and USSR.

V. *Latin America and Antilles.* Federal District of Brazil, Mexico, Jamaica, Trinidad and Tobago, Argentina, Chile and British Guiana.

VI. *Asia.* India, Japan, Ceylon and Cyprus.

187. For each area, two life tables of the present century were selected, namely, the earliest and the most recent, except where the area had been affected by a substantial boundary change. The central dates and the expectations of life at birth were averaged, within each group, both for the earlier and the more recent set of life tables, with results as shown below:

Group of countries	Earlier life tables		More recent life tables		Average gain in $\text{‰}$ per annum
	Average date	Average $\text{‰}$	Average date	Average $\text{‰}$	
I.....	1912	57	1949	68	0.30
II.....	1909	53	1946	66	0.35
III.....	1907	50	1946	65	0.39
IV.....	1910	41	1934	51	0.42
V.....	1917	38	1946	50	0.41
VI.....	1914	40	1950	55	0.42

188. It will be noted that, with the exception of the first two groups, where expectation of life was fairly high at the outset, the average annual gain in  $\text{‰}$  has been more or less consistently 0.4 years. The data are too few and too unrepresentative to permit a conclusion of general validity, but they lend some support to the view that expectation of life tends to increase at a fairly uniform rate so long as a high figure has not been attained. More painstaking studies of past trends in mortality have led to a similar conclusion. Moreover, it has been shown that gains in the expectation of life have been increasingly rapid in the course of the first half of this century.<sup>38</sup> Available evidence, however, is not sufficiently comprehensive to permit an exact inference as to

<sup>38</sup> United Nations, *Age and Sex Patterns of Mortality* (ST/ SOA/ Series A, Population Studies, No. 22).

the manner in which mortality declines have accelerated in recent times, nor whether they tend to be more rapid in areas of high mortality than in those of low mortality. Estimates for the future must rely on the scanty evidence of the past, taking into consideration the special circumstances affecting the prospects of future mortality decline in specific circumstances.

## 2. The model assumption of future mortality decline

189. In the past, mortality generally declined only at a slow rate where it was at a high level. As witnessed by recent experience in many parts of the world, this need no longer be so. It is now possible to cut down mortality quickly from a high level, even in poverty-stricken areas. It can, therefore, be reasonably assumed that the future mortality declines in areas where mortality is now high or moderately high will be more rapid than they were in areas where mortality conditions were similar during the first half of this century.

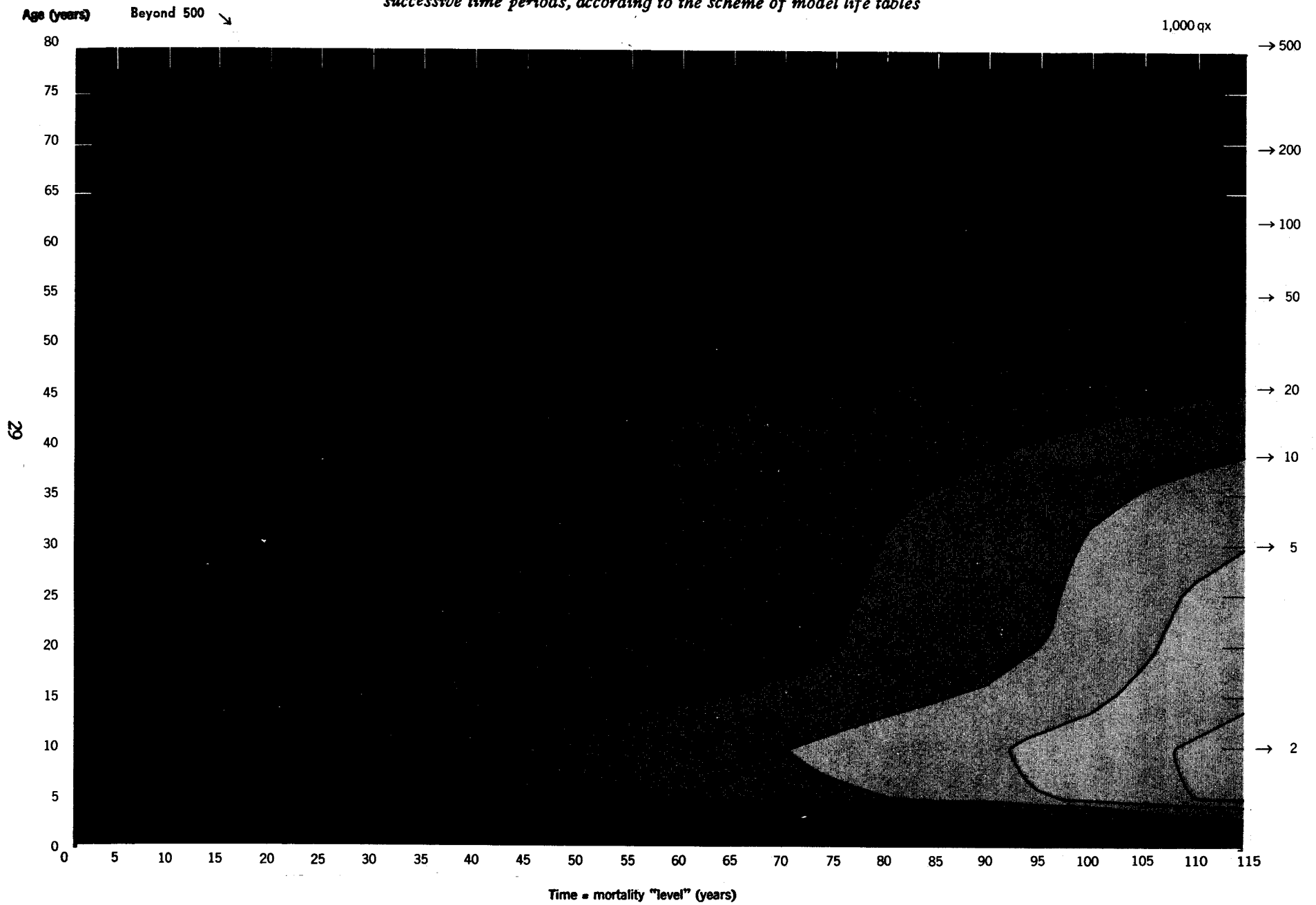
190. Important progress is still to be expected in areas where mortality is already low. Present knowledge and means for the prevention and cure of disease are still not exploited to the utmost in low-mortality countries, and new discoveries are still being made. In terms of expectation of life, however, the effects of further reductions in mortality will be less striking in the future than they were in the past. Where mortality is already very low, deaths below the age of 50 have become so infrequent that even their complete elimination—if that were possible—would result in comparatively little increase in the expectation of life. There are also some prospects for the reduction of mortality at older ages, for which death rates continue to be comparatively high, but progress in this field will probably not be rapid in the near future.

191. If a simple assumption of possible future changes in mortality under the most typical current conditions is to be made, one can envisage a uniform rate of increase in expectation of life at birth up to a certain level, beyond which any further increase will be at a slower pace. It is not suggested that such an assumption would be valid for every population, irrespective of the circumstances. Special factors affecting the prospects in each case must be taken into account.

192. The assumption proposed as a model is an annual gain of 0.5 years in expectation of life, wherever the expectation is less than 55 years. When an expectation of 55 years is attained, the model assumption implies a slight acceleration in gains<sup>39</sup> until the expectation approaches 65 years. Thereupon, the rate of gain slows down and becomes slight when the expectation has risen substantially higher than 70 years. The model life tables presented in the appendix have been arranged in conformity with this assumption. They are spaced at such intervals that, in conformity with the assumption, mortality would pass from the level indicated by one table to that of the next within a five-year period.

<sup>39</sup> The reason, as explained in the appendix, is that at this level of  $\text{‰}$  past observations have indicated an acceleration in the decrease of infant mortality, requiring some departure from the otherwise rather rigid scheme by which the model life tables were constructed. While, for other ages, mortality declines at a rate consistent with an annual gain of 0.5 year in  $\text{‰}$ , the more rapid decline of infant mortality entails a somewhat more rapid rise in the expectation of life at birth.

Diagram 1. Contour chart of life-table death rates ( $1,000q_x$ ) for males in five-years age groups at successive time periods, according to the scheme of model life tables



193. As there are 24 model life tables, their spacing by five-year time intervals implies a total time span of 115 years, during which mortality would decline from the highest to the lowest level indicated. It is not assumed that in any case mortality would indeed decline with such uniformity over so long a period. For shorter periods, however, the trends shown by the sequence are characteristic of typical situations now existing in the world.

### 3. *The model mortality surface*

194. A three-dimensional model is required for the graphic representation of changes in mortality, specific by age, in the course of time, the three dimensions being age, time, and the value of the specific rate. If such a model were carved out of a block of wood, mortality being expressed by height, time by length and age by width, the upper surface would be very uneven, as the height of the mortality rate varies both with time and with age. The shape of this "mortality surface" can be described after the fashion of a topographical map, by means of contour lines on a two-dimensional chart of time and age, the lines joining points at which the mortality rates are equal. The time-variation of life-table death rates ( $1,000q_x$ ) for five-year age groups of males, according to the model system of life tables, is so represented in diagram 1. The horizontal axis measures time and the vertical axis age, while the curving lines connect points where the mortality rate is at the level indicated by the figure marked against each line. Following these curves across the chart from left to right, one sees how the range of ages subjected to relatively high mortality narrows progressively as time passes, while the range of ages having lower mortality rates is widened.

195. To return to the three-dimensional wooden model, of which the surface contours are represented by this chart, a length-wise cut through it, parallel to the time-axis, would show the trend of a certain age-specific death rate in the course of time. A cross-cut, on the other hand, would show the curve of age-specific mortality rates prevailing at a given moment of time.

196. One cut through the model which is of special interest is a cut in a diagonal direction, like the one indicated by the dotted line. Such a cut, at one-half of a right angle, forms a line along which age advances at the same rate as time. It therefore represents the mortality experience, in the course of a life span, of a cohort of persons born at the same time. The cut along the dotted line, for example, would show the experience of persons born in the year 0, 85 years old in the year 85. Since general mortality conditions are assumed to improve while the cohort grows older, its mortality experience at any age will differ from the rate which prevailed among persons who were at that age when this cohort was born. In a population projection, where each age group currently living is traced into the future, it is this type of mortality experience, relating to the same generation of individuals, which is of interest.

### 4. *Relation of specific life-table functions to the general mortality level*

197. In the model life tables, general mortality levels have been conveniently defined by  ${}^{\circ}e_0$ , the expectation of life at birth for both sexes. Moreover, a scale has been attached to the progression of  ${}^{\circ}e_0$ , in accordance with an

assumption of typical current declines in mortality. The general mortality level can therefore also be designated by the number of the year in this time-scale, beginning with 0 where  ${}^{\circ}e_0$  is 20, and ending with 115 where  ${}^{\circ}e_0$  attains 73.9.

198. If the relationships among the mortality rates for different age groups in a given case follow a typical pattern, that is, the pattern of any one of the model life tables, then the general mortality level can be readily found if one of the age-specific rates is given. For instance, if the age-specific death rate ( $1,000 m_x$ ) for males aged 20-24 years in a given country is known to be 10.21, it is found on the corresponding line in appendix table I that this value relates to mortality level 35, where  ${}^{\circ}e_0$  equals 37.5 years. It can then be said that this particular age-specific rate is typical of an expectation of life at birth of 37.5 years.

199. In practice, when all age-specific rates for a given country have been traced in this manner, it may be found, for instance, that the rates for young persons are generally characteristic of one mortality level, while those for older people agree more nearly with typical rates for a different mortality level. The age-specific functions of an entire life table for the given country can then be estimated by reference to the model tables for both mortality levels.

200. The values for two levels of general mortality can also be interpolated. Let it be supposed that the age-specific death rate for males aged 20-24 years is approximately 10 per 1,000. A value of exactly 10.00 is  $21/89$  of the way between 10.21, the value for mortality level 35, and 9.32, found at level 40, and so corresponds to a level of 36.18 for which—using the same interpolation— ${}^{\circ}e_0$  would be 35.59 years. Such fine interpolation, however, is not warranted; the nearest integral figure, 36 in the present instance, is sufficient.

201. The procedure is illustrated in table 14, with the age-specific mortality rates ( $1,000 m_x$ ) computed for the male population of Luxembourg in table 10. In this example, minor fluctuations in the reference to general mortality levels might be largely explained by the small numbers of deaths from which some of the specific rates were computed, but it is noticeable, in contrast with average conditions, that male mortality in Luxembourg during that period was comparatively high at older ages in view of its comparatively low level at young ages.

202. The procedure described above is useful for three purposes. In the first place, it serves as a check on the reliability of the statistics: if the general mortality "level" to which the specific observed rates are referred fluctuates sharply from one age group to the next, there is good reason to doubt the accuracy of the data. It is quite unlikely, for example, that, in any actual population, men aged 20-24 years die as if general mortality were very high while men aged 25-29 die at a rate typically observed where general mortality is very low.

203. In the second place, the peculiarities of mortality conditions appear in the general trend, age group by age group, which runs through the resulting series. Thus, in the example of table 14, it appears that the mortality at older ages corresponds to  ${}^{\circ}e_0$  of 55-60 years only, while the mortality at younger ages is more characteristic of  ${}^{\circ}e_0$  near 66. The situation studied here deviates, in fact, from the average pattern.

TABLE 14. REFERRAL OF AGE-SPECIFIC DEATH RATES OF LUXEMBOURG MALES, 1946-1949, TO CHARACTERISTIC GENERAL MORTALITY LEVELS

Age (years)	1,000 $m_x$	Values found in appendix table I				Interpolated level <sup>b</sup>	Corresponding $e_{60}$ <sup>b</sup>
		Preceding value		Following value			
		Level	1,000 $m_x$	Level	1,000 $m_x$		
0.....	71.1	80	90.18	85	70.10	85	63
1- 4.....	3.1	95	3.67	100	2.45	97	69
5- 9.....	1.4	90	1.45	95	1.15	91	66
10-14.....	1.1	85	1.30	90	1.09	90	66
15-19.....	1.6	90	1.81	95	1.49	93	67
20-24.....	2.4	90	2.52	95	2.02	91	66
25-29.....	2.5	90	2.59	95	2.08	91	66
30-34.....	3.2	85	3.33	90	2.79	86	64
35-39.....	4.0	80	4.50	85	3.88	84	63
40-44.....	5.7	80	5.79	85	5.07	81	61
45-49.....	9.6	70	9.97	75	9.07	72	56
50-54.....	11.3	80	11.78	85	10.81	82	62
55-59.....	18.6	70	20.00	75	18.53	75	58
60-64.....	27.5	75	27.69	80	26.05	76	58
65-69.....	42.5	70	44.32	75	42.02	74	57
70-74.....	65.2	70	68.14	75	65.01	75	58
75-79.....	103.4	65	107.64	70	103.28	70	55
80-84.....	173.4	55	175.67	60	169.30	57	49
85+.....	260.9	65	264.90	70	259.83	69	54

<sup>a</sup> Computed in table 10.

<sup>b</sup> Approximate, to the nearest unit.

204. Finally, as will presently be shown, this procedure makes it possible to infer the corresponding survival ratios, needed for the population projection, without a separate computation.

205. One important point, however, should be noted. This procedure is valuable in dealing with the ratio-functions of the life table, such as  $m_x$ ,  $q_x$  and  $P_x$ , the value of which for any age is mathematically independent of its value for other ages. The same interpretation is not impossible if the cumulated life-table functions, such as  $l_x$  and  $L_x$ , are referred to the levels noted in the model life tables: these latter functions depend not only on mortality for the particular age, but also on mortality at other ages.

#### 5. Inference of specific $P_x$ values from general mortality levels

206. One major advantage in using the model life tables is that most of the sequence of computations—described in chapter III—by which  $P_x$ -values are obtained can be dispensed with. Once the general mortality levels corresponding to specific values of either  $m_x$  or  $q_x$  have been obtained (as in the preceding example), the corresponding values of  $P_x$  can be read off, or interpolated, in accordance with these “levels” from appendix table V.

207. One further fact, however, must still be considered: the survival ratios ( $P_x$ ) relate to mortality conditions in two successive age groups, while death rates (whether  $m_x$  or  $q_x$ ) relate to mortality in one age group only. Theoretically, the  $m_x$ -values or  $q_x$ -values of the

same population should indicate identical mortality levels.<sup>35</sup> The levels corresponding to age-specific survival ratios ( $P_x$ ), on the other hand, will be intermediate between the mortality levels for the given and the next higher age group, according to  $m_x$  (or  $q_x$ ).

208. This fact is demonstrated in table 15, with the  $P_x$ -values which were computed by the methods of the preceding chapter (see table 11). The levels of these values were located, by interpolations like those of table 14, between the nearest values in appendix table V. These levels of the computed  $P_x$ -values are indeed intermediate between the levels of the  $m_x$ -values in the two successive age groups. Moreover, proceeding from age group to age group, the sequence of levels indicated by  $P_x$  is smoother than the sequence according to  $m_x$ . Roughly the same effect is obtained directly by averaging levels of two successive age groups according to  $m_x$ . If the averaging is properly done, separate computation of  $P_x$ -values is no longer necessary.

209. To start with, then, only the sequence of either  $m_x$  (or  $q_x$ ) is required. Their levels are found by reference to appendix tables I, or II. These levels are then averaged for every successive pair of age groups. Using the averaged levels, the values of  $P_x$  are then found in appendix table V.

210. A subtle theoretical problem relates to the exact method of averaging. If the calculation were to be carried out with great refinement and accuracy, simple averaging would introduce undesired distortions. On the

<sup>35</sup> Small differences in the indicated “level” result if the mode of transformation of  $m_x$  into  $q_x$  differs from that used in the construction of the model life tables.

TABLE 15. COMPARISON OF GENERAL MORTALITY LEVELS IN RESPECT OF SURVIVAL RATIOS FOR LUXEMBOURG MALES, 1946-1949, AS DETERMINED BY TWO DIFFERENT PROCEDURES

Age (years)	Determination from $P_x$ values already computed		Determination from $m_x$ values	
	$P_x^a$	Corresponding level <sup>b</sup>	Level in respect of $m_x^c$	Same, averaged for successive age groups <sup>d</sup>
	( $P_b = 0.9354$ )	(88)		(91)
0.....	0.9862	94	85	91
1-4.....				
5-9.....			90	
10-14.....			92	
15-19.....	0.9901	92	93	92
20-24.....	0.9879	91	91	91
25-29.....	0.9859	89	91	88.5
30-34.....	0.9822	85	86	85
35-39.....	0.9761	82	84	82.5
40-44.....	0.9626	76	81	76.5
45-49.....	0.9490	77	72	77
50-54.....	0.9284	78	82	78.5
55-59.....	0.8918	75	75	75.5
60-64.....	0.8412	74	76	75
65-69.....	0.7676	74	74	74.5
70-74.....	0.6649	72	75	72.5
75-79.....	0.5224	66	70	65
80-84.....	0.3075	67	57	65
85 and over			69	

<sup>a</sup> As computed in table 11.  
<sup>b</sup> Interpolated according to values in appendix table V.  
<sup>c</sup> Taken from table 14, where interpolation was made from values in appendix table I.  
<sup>d</sup> Method of averaging explained in accompanying text.

other hand, if simplicity is more important than mathematical precision, the use of simple arithmetic averages of the specific levels, according to  $m_x$  (or  $q_x$ ), of two adjacent age groups can be regarded as yielding adequate results for the levels of  $P_x$ , from  $P_{5-9}$  to  $P_{70-74}$ . Three of the four extreme survival ratios,  $P_b$  (i.e., survival from a five-year period of births),  $P_{0-4}$ ,  $P_{75-79}$ , and  $P_{80+}$ , relate to age groups which do not coincide with the age groups of  $m_x$  (or  $q_x$ ). For these, the averaging procedure must be varied somewhat. The procedure suggested is as follows: (a) assume the average of levels of  $m_0$ ,  $m_{1-4}$  and  $m_{5-9}$  both for  $P_b$  and  $P_{0-4}$ , and (b) assume the average of levels of  $m_{75-79}$ ,  $m_{80-84}$  and  $m_{85+}$  both for  $P_{75-79}$  and  $P_{80+}$ .

6. The assumption of future trends in survival ratios

211. The model assumption of future mortality declines is directly implied in the sequence of model life tables, according to which survival ratios pass, at the end of five years, from the level of one model life table to that of the next. They can be referred to accordingly, adding 5, in respect of every age group, to the number of the mortality level for every five-year interval.

212. In table 15, the general levels of  $P_x$  have been established, age group by age group, for Luxembourg males in 1946-1949. If mortality is assumed to decline, from that period onward, in accordance with the model assumption, the future development is shown in table 16. The mortality level has been raised by 5 years in every successive period and the corresponding values of  $P_x$  were inferred, by interpolation where necessary, from appendix table V. Actually, the work can be considerably shortened by rounding the mortality levels to the nearest multiple of 5. No interpolations are then necessary, while the loss of accuracy is only slight.

TABLE 16. SURVIVAL RATIOS OF LUXEMBOURG MALES IN SUCCESSIVE FUTURE PERIODS IN ACCORDANCE WITH THE MODEL ASSUMPTION OF MORTALITY DECLINES

Age (years)	1946-49 Level	1951-54		1956-59		1961-64	
		Level	$P_x$	Level	$P_x$	Level	$P_x$
( $P_b$ ).....	(91)	(96)	(0.9600)	(101)	(0.9691)	(106)	(0.9753)
0-4.....	91	96	0.9875	101	0.9914	106	0.9940
5-9.....	90.5	95.5	0.9950	100.5	0.9964	105.5	0.9975
10-14.....	91.5	96.5	0.9945	101.5	0.9959	106.5	0.9971
15-19.....	92	97	0.9923	102	0.9942	107	0.9958
20-24.....	91	96	0.9903	101	0.9928	106	0.9947
25-29.....	88.5	93.5	0.9884	98.5	0.9908	103.5	0.9929
30-34.....	85	90	0.9849	95	0.9874	100	0.9897
35-39.....	82.5	87.5	0.9794	92.5	0.9822	97.5	0.9848
40-44.....	76.5	81.5	0.9669	86.5	0.9706	91.5	0.9737
45-49.....	77	82	0.9533	87	0.9573	92	0.9600
50-54.....	78.5	83.5	0.9341	88.5	0.9387	93.5	0.9429
55-59.....	75.5	80.5	0.8991	85.5	0.9049	90.5	0.9103
60-64.....	75	80	0.8502	85	0.8573	90	0.8639
65-69.....	74.5	79.5	0.7784	84.5	0.7872	89.5	0.7955
70-74.....	72.5	77.5	0.6762	82.5	0.6866	87.5	0.6965
75-79.....	65	70	0.5326	75	0.5449	80	0.5566
80+.....	65	70	0.3186	75	0.3272	80	0.3352

213. Where there is reason to expect that the future trends will follow a different course, this model assumption need not be adhered to, but can be varied in several ways, according to the circumstances of the case. Among possible modifications of the model assumption, the following might be considered:

(a) *Assumption of slower, or faster, mortality decline.* If there is reason to believe that mortality will decline more, or less, rapidly than would be expected under typical conditions, the mortality levels from which the survival ratios are inferred may be raised by more or less than 5 during each successive period.

214. One possible assumption, in the example of Luxembourg, might be that mortality in 1946-1949 was still affected by somewhat abnormal post-war conditions. Unusually rapid improvement might then be expected in the first five-year period following 1946-1949, after which declines in mortality might revert to the more typical pattern of the model assumption.

(b) *Assumption of normalization of mortality pattern.* Observed differences in the levels attributed to  $P_x$  for various age groups may be ascribed to abnormal conditions adversely affecting certain age groups. In this event, it may be reasonable to assume that the mortality of the affected age groups will decrease more rapidly than that of other age groups, so that the future pattern of specific mortality conditions will conform more closely to that of a model life table.

215. In the example of Luxembourg, 1946-1949, it might be assumed that abnormal conditions existed in that period which were prejudicial to the health of older persons. Mortality rates at older ages might then be assumed to decrease more rapidly than at younger ages, until they conform to the same level.

(c) *Assumption of differences in the mortality experience of each cohort.* Under certain conditions, the mor-

tality of an age group may be more strongly affected by its past experience than by the current conditions affecting the entire population. For example, a cohort exposed to certain health hazards in childhood or in a time of war may have suffered permanent damage to health as a result of which it will be subject to heavier mortality risks at any subsequent age than other cohorts. Some knowledge of the prevailing causes of death and of their incidence among different age groups is necessary to evaluate this possibility in a particular case. If it appears that a certain cohort has been affected in this way, the assumption as to the future progression from one mortality level to the next should be applied to the figures for that same cohort as it grows older, instead of the figures for a given age group at successive points of time.

216. This procedure is illustrated in table 17 with reference to the mortality estimates for Luxembourg males. It is assumed for the purpose of this illustration that the comparatively heavy mortality among older persons in 1946-1949 was attributable to an impairment of their state of health resulting from special debilitating influences at some time earlier in their lives. When the same ages are attained by more recent generations whose health was not similarly damaged, it is presumed that the mortality rates for those ages will decline more rapidly than the model assumption would allow. With respect to the youngest cohorts, not yet born at the initial date, mortality is assumed to conform to the model assumption.

#### B. ESTIMATION OF SURVIVAL RATIOS UNDER DIVERSE PRACTICAL CONDITIONS

217. The method described in the preceding pages, namely the estimation of survival ratios by referral to characteristic mortality "levels" of the model life tables, can be applied wherever adequate statistics for the com-

TABLE 17. SURVIVAL RATIOS OF LUXEMBOURG MALES IN SUCCESSIVE FUTURE PERIODS IN ACCORDANCE WITH THE ASSUMPTION OF MORTALITY DECLINE BY COHORTS

Age (years)	1946-49 level	1951-54		1956-59		1961-64	
		Level	$P_x$	Level	$P_x$	Level	$P_x$
( $P_b$ ).....	(91)	(96)	(0.9600)	(101)	(0.9691)	(106)	(0.9753)
0-4.....	91	96	0.9875	101	0.9914	106	0.9940
5-9.....	90.5	96	0.9952	101	0.9965	106	0.9976
10-14.....	91.5	95.5	0.9942	101	0.9958	106	0.9970
15-19.....	92	96.5	0.9920	100.5	0.9937	106	0.9955
20-24.....	91	97	0.9908	101.5	0.9930	105.5	0.9945
25-29.....	88.5	96	0.9896	102	0.9924	106.5	0.9939
30-34.....	85	93.5	0.9866	101	0.9901	107	0.9922
35-39.....	82.5	90	0.9808	98.5	0.9852	106	0.9884
40-44.....	76.5	87.5	0.9712	95	0.9759	103.5	0.9804
45-49.....	77	81.5	0.9529	92.5	0.9614	100	0.9664
50-54.....	78.5	82	0.9326	86.5	0.9369	97.5	0.9461
55-59.....	75.5	83.5	0.9026	87	0.9066	91.5	0.9113
60-64.....	75	80.5	0.8509	88.5	0.8618	92	0.8664
65-69.....	74.5	80	0.7794	85.5	0.7889	93.5	0.8018
70-74.....	72.5	79.5	0.6805	85	0.6917	90.5	0.7021
75-79.....	65	77.5	0.5508	84.5	0.5664	90	0.5779
(80+).....	65	70	(0.3186)	82.5	(0.3388)	89.5	(0.3486)

putation of age-specific death rates, or some elements of a life table, are available. Estimating problems, however, arise where not all of this statistical detail is available or where the statistics are inaccurate or incomplete. The results, in some instances, cannot attain the same degree of accuracy as where the statistics are complete, detailed and accurate. Nevertheless, if based on good judgement, they will usually be adequate for estimating future population.

1. *Estimates using data on the population and on deaths by sex and age*

218. As an example of how specific death rates affected by inaccurate age declarations can be utilized, a calculation is carried out in table 18 with the statistics of population by sex and age from the Egyptian census of 1947 and Egyptian statistics on deaths by sex and age. These figures fluctuate considerably from one age group to another, in a manner which must be attributed to inaccurate age statements. As might be expected, the resulting references of the specific death rates to general "levels" of mortality also fluctuate widely. The sequence is partly smoothed as a result of the averaging process by which the "levels" of the  $P_x$ -values (survival ratios) are found, but is still rather irregular. It is not to be expected that mortality in Egypt should conform precisely to the pattern of any one model. But it is reasonable

to expect, if the data were accurate, that deviations from the model patterns would not be abrupt, or irregular, from one age group to another.

219. In order to correct the series of "levels" to which the  $P_x$  values for the different age groups should relate, it is desirable to have some information about the manner in which ages of the population and of decedents were mis-stated, and about the completeness of enumeration and death registration at certain particular ages. Without such knowledge, one might suspect both an under-enumeration of infants and an under-registration of infant deaths; the mortality rate for infants, as well as for small children, might be either too high or too low depending on which of these deficiencies is greater. Likewise, one might presume a tendency to exaggerate old age, both in the census and in death reports. If the overstatement of old age is greater in the death returns than in the census, the recorded death rates for old persons will tend to be exaggerated. In fact, for ages 85 and over, an implausibly high death rate is noted. On the other hand, if the age at death is often much overstated, too few deaths are likely to be reported at younger ages, say, between 50 and 75 years. Similar reasoning may be applicable when contrasting the mortality "levels" found for males and females of the same ages. Death reporting for females may perhaps be less complete than for males.

TABLE 18. ESTIMATION LEVELS FOR SURVIVAL RATIOS FROM AGE-SPECIFIC DEATH RATES ACCORDING TO STATISTICS OF EGYPT, 1947

Age in years (x)	Males				Females			
	1,000 $m_x$	Mortality level for			1,000 $m_x$	Mortality level for		
		$m_x$	P			$m_x$	$P_x$	
			Computed	Adjusted			Computed	Adjusted
0.....	219.2	43	34 <sup>b</sup>	35 <sup>c</sup>	{ 197.3 47.2 }	41	38 <sup>b</sup>	40 <sup>c</sup>
1-4.....	52.3	14	34 <sup>d</sup>	35		19	38 <sup>d</sup>	45
5-9.....	5.9	45	40	40	4.7	54	54	50
10-14.....	5.0	35	43	40	3.5	54	61	55
15-19.....	5.2	51	52.5	45	3.5	68	70.5	55
20-24.....	7.1	54	50.5	45	4.1	73	69.5	55
25-29.....	8.6	47	43.5	45	5.5	66	59.5	55
30-34.....	10.8	40	40	45	8.2	53	54.5	55
35-39.....	12.4	40	41.5	45	8.1	56	54	55
40-44.....	14.2	43	47	45	9.9	52	59	60
45-49.....	15.2	51	50.5	50	8.7	66	64	60
50-54.....	20.3	50	57	50	12.7	62	74.5	60
55-59.....	22.1	64	66	50	11.0	87	86	60
60-64.....	30.3	68	63.5	50	18.0	85	82	60
65-69.....	50.2	59	62.5	50	31.5	79	85.5	60
70-74.....	70.7	66	53.5	50	45.7	92	77	60
75-79.....	134.2	41	45 <sup>e</sup>	50	100.2	62	66 <sup>e</sup>	60
80-84.....	183.6	49	45 <sup>f</sup>	50	{ 141.7 723.0 }	70	66 <sup>f</sup>	60
85+.....	664.8	t						

<sup>a</sup> Taken from United Nations *Demographic Yearbook, 1952*.

<sup>b</sup> Level for  $P_b$ ; taken as average of levels for  $m_0$ ,  $m_{1-4}$  and  $m_{5-9}$ .

<sup>c</sup> Adjusted level for  $P_b$ .

<sup>d</sup> Level for  $P_{0-4}$ ; taken as average of levels for  $m_0$ ,  $m_{1-4}$  and  $m_{5-9}$ .

<sup>e</sup> Level for  $P_{75-79}$ ; taken as average of levels for  $m_{75-79}$  and  $m_{80-84}$ .

<sup>f</sup> Specific death rate evidently too high, being outside the range of the model life tables.

<sup>t</sup> Level for  $P_{80+}$ ; taken as average of levels for  $m_{75-79}$  and  $m_{80-84}$ .



220. Though this type of reasoning does not permit any very definite conclusions, it does offer some guidance for adjusting the calculated mortality levels in order to make them more plausible. There seems to be some indication that the level advances from childhood to mature adult ages. In other words, an impression is gained that child mortality is rather high in Egypt in relation to the somewhat lower relative mortality levels in later years of life. The "adjusted" series of levels, for estimating the values of  $P_x$ , is clearly arbitrary and it cannot be claimed that it is accurate, but it probably represents an improvement over the clearly inaccurate original data. The improvement may be sufficient for the purpose of a population projection. The irregularities in the series of "levels" calculated from moderately inaccurate statistics are often not as great as in the present example. The series may then be smoothed by a moving average or some simple formula.

221. A somewhat different problem arises where the series of mortality levels obtained directly from the data is fairly smooth, with the exception of a few age groups. The statistics of Thailand are a case in point. In that country, an infant mortality rate of 79.8 per 1,000 was registered in 1947. This rate appears very low indeed in relation to the death rates of adults, though the latter rates show no marked inconsistencies among

themselves. As appears from the calculations shown in table 19, the mortality levels for ages 10 to 55 correspond fairly well to a typical pattern. There appears to be little need to revise them, except perhaps by some slight smoothing with a moving average. Yet, the rates for early childhood and advanced ages appear markedly too low. They can hardly be regarded as consistent with the generally observed pattern. One possible explanation is that, while death registration is fairly accurate at most ages, it is incomplete for infants, small children, and old persons. This hypothesis has been used in substituting apparently more consistent mortality levels for the latter age groups, while accepting those which were calculated for the age range from 10 to 55 years. This hypothesis, which has been adopted in the absence of any specific knowledge of the functioning of death registration in the country, gives results that appear to be at least consistent and plausible.

222. The two examples given do not exhaust the types of difficulties which arise when an effort is made to estimate the mortality levels for various age groups from faulty data. Familiarity with the statistical procedures of the country, and with the kinds of errors to which the data are subject, is evidently important for a realistic adjustment. But even where such knowledge is quite limited, reasonable adjustments can be expected

TABLE 19. ESTIMATION OF LEVELS FOR SURVIVAL RATIOS FROM AGE-SPECIFIC DEATH RATES ACCORDING TO STATISTICS OF THAILAND, 1947

Age in years (x)	Males				Females			
	Mortality level for				Mortality level for			
	1,000 $m_x$	$m_x$	$P_x$		1,000 $m_x$	$m_x$	$P_x$	
			Computed	Substituted			Computed	Substituted
0.....	57.1	88	60 <sup>b</sup>	45 <sup>a</sup>	{ 50.4 16.2 7.1	87	62 <sup>b</sup>	45 <sup>a</sup>
1-4.....	18.1	57	60 <sup>d</sup>	45			62 <sup>d</sup>	45
5-9.....	7.7	34	35	45			43	45
10-14.....	4.8	36	42.5	45	4.2	47	52.5	50
15-19.....	5.4	49	48.5	45	4.7	58	56.5	55
20-24.....	7.3	48	45.5	45	6.8	55	51	50
25-29.....	9.3	43	43	45	9.0	47	47	45
30-34.....	10.1	43	40.5	40	9.7	47	45.5	45
35-39.....	13.1	38	39	40	11.2	44	43.5	45
40-44.....	15.0	40	41	40	12.6	43	43	45
45-49.....	18.8	42	42.5	40	13.3	43	46.5	45
50-54.....	23.4	43	47.5	45	16.2	50	51	50
55-59.....	26.9	52	53.5	45	21.0	52	55	50
60-64.....	36.1	55	58.5	45	27.7	58	61.5	50
65-69.....	48.6	62	61.5	45	38.3	65	66	50
70-74.....	74.4	61	...	45	59.8	67	...	50
75-79.....	123.1 <sup>e</sup>	..	...	{ 45	106.3 <sup>f</sup>	..	...	{ 50
80+.....				{ 45				

<sup>a</sup> Data from United Nations *Demographic Yearbook, 1952*.

<sup>b</sup> Level for  $P_b$ ; taken as average of levels for  $m_0$ ,  $m_{1-4}$  and  $m_{5-9}$ .

<sup>c</sup> Substituted level for  $P_b$ .

<sup>d</sup> Level for  $P_{0-4}$ ; taken as average of levels for  $m_0$ ,  $m_{1-4}$  and  $m_{5-9}$ .

<sup>e</sup> Death rate for ages "75 and over"; rate is evidently too low: if this were the value of

$m_{75-79}$ , the corresponding level would be 50; the value of  $m_{75+}$ , for the same level, would be 157.7 per 1,000.

<sup>f</sup> Death rate for ages "75 and over"; rate is evidently too low: if this were the value of  $m_{75-79}$ , the corresponding level would be 56; the value of  $m_{75+}$ , for the same level, would be considerably greater.

to give a closer approximation to reality than unadjusted, faulty series.

223. Another problem arises where the registration of deaths is generally deficient for all age groups. In such cases, the age distribution of registered deaths may very well be approximately the same as the age distribution of all deaths, and the levels of mortality shown by the data may be consistent between age groups, but these levels are nevertheless too low.

224. Where there is good reason to believe that a substantial number of deaths escape registration, a coefficient has to be found by which to multiply the reported number of deaths in each age group in order to estimate the true total at all ages. In some countries, studies have been made of the degree of completeness of death registration. Where there is no country-wide estimate of the degree of completeness, it may be possible to arrive at a rough estimate by comparing the data for certain parts of the country, where the registration is known to be more complete, with those for the remaining areas.

225. Where death registration is confined to a particular portion of the country, such as a "registration area", it can sometimes be assumed that mortality conditions in the remainder of the country are roughly similar. Rates computed for the registration area can then be applied to the country as a whole. Sometimes, however, the registration area, or the area of presumably most accurate registration, represents comparatively advanced communities, where death rates may very well be lower than elsewhere in the country. It may then be appropriate to suppose that the same mortality conditions will be attained in the country as a whole after a certain number of years. Survival ratios computed for the area where registration is most accurate can then be used as the estimated ratios for the entire population as of a certain future date.<sup>86</sup>

226. Where only a relatively small portion of the deaths are registered, it is often not possible to relate these deaths to any precise segment of the population. It may still be possible to utilize the registration figures to represent the distribution by ages of decedents, but no significant death rates can be computed directly. In such a situation, it is still possible to assess the approximate mortality level under which the observed distribution of deaths by age would be expected. The estimating methods required in this connexion depend on the use of theoretical population models and are somewhat complex. They will be discussed in another publication.

## 2. Estimates using detailed data on population and summary data on deaths

227. The case considered here is one where statistics are available on the sex-age composition of the popula-

<sup>86</sup> This type of assumption was made in the United Nations population projection for Guatemala. A life table was found only for one province of the country, which contains the capital city. Application of this table to the population of the whole country indicated that mortality in this province was comparatively low and that, in conformity with general mortality assumption, the same level of mortality might be attained by the country's total population about 5 years later. The life table was then used in the projection by shifting its time-reference 5 years into the future.

tion but not of deaths. This is the case if only the total numbers of deaths are recorded, and also if the crude death rate or some other mortality measure, such as the expectation of life at birth, has been estimated. If the expectation of life is known or has been estimated, the problem is simple: mortality may then be assumed to conform to the model life table for which  $^0e_0$  is the same. In the absence of more specific information, this is perhaps the best estimate that can be made.

228. Given the numbers of the population in each sex-age group and the total number of deaths, a model life table can be found, by trial and error, which would result in the same number of deaths. The age-specific death rates ( $m_x$ ) for one of the models in appendix table I are multiplied with numbers of the population in the same sex-age groups and the results added; this process is repeated with other model life tables until the one is found which yields most nearly the given number of deaths.

229. The same procedure is also applicable where statistics of deaths are available by very broad age groups only. For example, the statistics of deaths for British Guiana in 1946 were tabulated in these age groups only: 0, 1-4, 5-14, 15-44, 45-79, and 80 and over, though a more detailed age distribution of the population is available from the census of that year. According to tabulated data, 618 deaths occurred among males aged 15-44. With the census figures and the specific death rates of several model life tables, the following results are obtained:

Age (years)	Male population	Mortality level 50		Mortality level 55	
		1,000 $m_x$	Deaths	1,000 $m_x$	Deaths
15-19.....	17,770	5.32	95	4.79	85
20-24.....	15,658	7.67	120	6.93	109
25-29.....	12,864	8.04	103	7.22	93
30-34.....	13,165	8.64	114	7.70	101
35-39.....	11,080	9.77	108	8.67	96
40-44.....	10,861	11.96	130	10.66	116
TOTAL 15-44	—	—	670	—	600

It appears, then, that for males aged 15-44 the mortality level was somewhat less than 55.<sup>87</sup> Similar computations can be carried out for other sex-age groups.

## 3. Estimates using detailed population statistics only

230. The level of mortality can also be estimated by means of detailed population statistics only, in the absence of any statistics relating to deaths. Two cases may be distinguished: (a) where population statistics by sex and age are available for one date only, and (b) where such statistics have been obtained for at least two dates.

231. The first of these cases again requires the use of methods that depend on hypothetical model populations. This subject is to be dealt with in another publication. Since the effect of mortality on the age structure of population is comparatively slight, this method is rarely sufficient for a reliable estimate of the mortality level. Under certain conditions, however, it may be the only method available. In the second case, where census statistics on the population by sex and age at two different dates are available, mortality levels can often be esti-

<sup>87</sup> This conclusion evidently depends on the supposition that both the statistics of population and of deaths are accurate.

mated with more confidence. An example is presented in table 20 with the census statistics for Iceland of December 1940 and December 1950. Actually, statistics of deaths in Iceland are available, but they are disregarded here in order to illustrate the method. It is assumed for the purpose of the illustration that the effects of migration to and from Iceland during the period in question were negligible and that the census statistics are entirely accurate. In countries lacking death statistics, where this method would be appropriate, the quality of census statistics is likely, in general, to be inferior to that of the Icelandic data, but moderately accurate census data are sufficient to give useful results. It will be seen that the results for Iceland are affected in some instances by the chance fluctuations of small numbers; this complication would not occur in the case of a country with a sizable population.

232. The first step is to obtain ten-year survival ratios for the various five-year age groups, by dividing the numbers of males and females of each age group in 1950 by the corresponding figures for the same cohorts, ten years younger, in 1940. When this has been done, it is necessary to obtain the values corresponding to  $P_b$ , that is, the ratios of survivors to ages 5-9 and 0-4 in 1950, among births during 1941-1945 and 1946-1950, respectively. In a country where adequate statistics of deaths are lacking, there will ordinarily be no adequate statistics of births which could be related to the census figures for the purpose of computing these ratios. It may be possible, however, to estimate the numbers of births by such methods as are to be discussed in chapter V. For the present example, the numbers of births registered in 1941-1945 and 1946-1950 (15,459 and 18,935, respectively) are introduced into the computations. The registered numbers are available for each sex separately, but here it is assumed that the sex ratio of births is 105 males per 100 females, in order to illustrate the application of an assumption which can validly be used where birth registration statistics are lacking but estimates of total numbers of births are available. The estimated  $P_b$  values will be only slightly affected by any error in the assumed sex ratio of births.

233. The ten-year survival ratios so computed from the data for Iceland are shown in columns 6 and 7 of table 20. The ratio obtained for females aged 5-9 in 1940 and 15-19 in 1950 is obviously too high, even exceeding unity, which is manifestly impossible. On the other hand, the ratios for males aged 15-29 in 1940 and females aged 15-24 in 1940 appear to be too low. These peculiarities may be due to emigration and, especially in the case of females, to mis-statement of ages. The latter presumption is confirmed by the impossible survival ratio of females from ages 5-9; too many females may have been reported in the 15-19 year age group at both censuses, with the result that computed survival to these ages results in an excessive figure, while computed survival from these ages yields a figure that is too low. For the most advanced ages, the computations could not be carried out in the absence of data on numbers of persons aged 85-89, 90-94, and 95 and over, in 1950.

234. The next step is to find the general levels of mortality which correspond to the ten-year survival ratios. Now, a ten-year survival ratio for a five-year group is equivalent to the product of five-year survival ratios for two successive five-year age groups. Hence,

the desired levels can be found by locating, in appendix table V, those pairs of successive five-year ratios ( $P_x$ -functions in the series of model life tables) which, when multiplied together, produce most nearly each of the computed ten-year ratios. The levels so found in this example are tabulated in columns 8 and 9 of table 20.

235. The simplest way of estimating the appropriate levels for five-year survival ratios from the levels obtained for the ten-year ratios is to average each successive pair of the latter levels. This procedure incidentally has the effect of "smoothing" the series. However, in this instance, critical examination of the results obtained in this manner showed that some of them were not acceptable. Those which were clearly inconsistent with other values in the series were replaced by figures which would make a consistent series. The results are tabulated in columns 10 and 11 of table 20.

236. The procedure which has been illustrated can also be applied where census age statistics are much less accurate than those of Iceland, where the time interval between censuses is not an exact multiple of five years, and where migration has been of some importance. Space does not permit a detailed discussion of all the techniques which might be applied to deal with such difficulties. Where age declarations are quite inaccurate, it is advisable to graduate the age statistics at each census with a rather refined formula before computing the ratios. Where the census interval is not a multiple of five years, the statistics of one of the censuses can be carried forward, by methods described in chapter II, to a date which is convenient for comparison with the other census; alternatively, single-year age data may be combined in five-year groups other than those terminating in 0-4 and 5-9 and further adjustments may be made in the computed survival ratios to obtain values corresponding to a multiple of five years. Where immigration has been of some importance but emigration has been negligible, survival ratios may be computed from census tabulations limited to the population born in the country, if such tabulations are available. Alternatively, ratios may be computed from statistics for the whole population and then adjusted to eliminate the estimated effects of migration.<sup>88</sup>

#### C. DERIVATION OF FUTURE SURVIVAL RATIOS FROM STATISTICS FOR COSTA RICA

237. The illustrative computations relating to the population of Costa Rica presented in chapter II resulted in a preliminary estimate of the sex-age composition as of mid-year 1955 (table 9). The estimate is subject to adjustment for the apparently incomplete enumeration of children in the 1950 census. The method of this adjustment will be explained in chapter V, after methods of estimating fertility have been considered. One of the requirements for such an adjustment, however, is an estimate of mortality levels. In the following pages, the method of estimating these levels and the corresponding survival ratios is explained.

<sup>88</sup> If it is desired to make projections on the assumption that migration will continue in the future to exert the same influence as it had during the period between the two census dates, the survival ratios affected by immigration or emigration as derived from the census data may be used without adjustment. See chapter VII.

TABLE 20. ESTIMATION OF MORTALITY LEVELS FROM CENSUS STATISTICS BY SEX AND AGE FOR ICELAND, 1940-1950

Age in 1940 (years)	Population, 1940 (and births, 1941-50)		Age in 1950 (years)	Population, 1950		10-year survival ratio		Approximate level of 10-year ratio		Estimated level of 5-year survival ratio	
	Males	Females		Males	Females	Males	Females	Males	Females	Males	Females
Unborn.....	(9,698)	(9,237)	0-4	9,466	8,813	(0.9761) <sup>a</sup>	(0.9541) <sup>a</sup>	(105) <sup>a</sup>	(90) <sup>a</sup>	(95) <sup>ab</sup>	(95) <sup>ab</sup>
	(7,918)	(7,541)	5-9	7,431	7,167	(0.9385) <sup>c</sup>	(0.9504) <sup>c</sup>	(95) <sup>c</sup>	(95) <sup>c</sup>	(95)	(95)
0-4.....	5,922	5,725	10-14	5,851	5,656	0.9880	0.9879	100	100	95	95 <sup>b</sup>
5-9.....	6,231	5,931	15-19	6,159	5,969	0.9884	1.0064	90	<sup>d</sup>	95	95 <sup>b</sup>
10-14.....	6,317	6,127	20-24	6,229	6,058	0.9861	0.9887	95	95	90	90 <sup>b</sup>
15-19.....	5,934	5,690	25-29	5,727	5,416	0.9651	0.9518	80	65	85 <sup>b</sup>	85 <sup>b</sup>
20-24.....	5,313	5,097	30-34	5,109	4,873	0.9616	0.9561	80	70	85 <sup>b</sup>	85 <sup>b</sup>
25-29.....	4,773	4,677	35-39	4,599	4,558	0.9635	0.9746	80	90	85	90
30-34.....	4,458	4,178	40-44	4,276	4,057	0.9592	0.9710	85	90	90	90
35-39.....	4,060	3,868	45-49	3,870	3,749	0.9532	0.9692	90	95	95	90
40-44.....	3,677	3,670	50-54	3,453	3,482	0.9391	0.9488	95	90	100	90
45-49.....	3,380	3,519	55-59	3,120	3,278	0.9231	0.9315	105	90	100	95
50-54.....	2,574	2,694	60-64	2,223	2,423	0.8636	0.8994	95	95	105	100
55-59.....	2,016	2,287	65-69	1,674	1,962	0.8304	0.8579	110	100	110	105
60-64.....	1,765	2,087	70-74	1,297	1,645	0.7348	0.7882	110	110	110 <sup>b</sup>	110
65-69.....	1,400	1,790	75-79	917	1,215	0.6550	0.6788	<sup>e</sup>	115	110 <sup>b</sup>	110
70-74.....	1,107	1,486	80-84	506	759	0.4571	0.5108	110	110	110 <sup>b</sup>	110 <sup>b</sup>
75-79 } 80-84 } 85+ }	1,378	2,307 <sup>f</sup>	{ 85-89 } { 90-94 } { 95+ }	300	593	(0.2177)	(0.2570)	{ <sup>g</sup> } { <sup>g</sup> } { <sup>g</sup> }	{ <sup>g</sup> } { <sup>g</sup> } { <sup>g</sup> }	110 <sup>b</sup>	110 <sup>b</sup>

<sup>a</sup> Five-year survival ratio ( $P_5$ ) from births in 1946-50 to ages 0-4 in 1950.

<sup>b</sup> Estimate arbitrarily inserted in order to obtain a consistent series.

<sup>c</sup> Ten-year survival ratio from births in 1941-45 to ages 5-9 in 1950.

<sup>d</sup> Computed survival ratio impossibly high.

<sup>e</sup> Computed survival ratio probably too high, and outside range of model tables.

<sup>f</sup> Level cannot be readily computed from the data.

238. Since the publication of the United Nations population projections for Central America,<sup>39</sup> an official life table for Costa Rica, relating to the 1949-1951 period, has become available and some of the relevant functions have been published in the United Nations *Demographic Yearbook, 1954*, though not separately for each sex. The  $l_x$ -function, as tabulated for both sexes combined, is used as a starting point for the present calculations.

239. Differences between successive values of  $l_x$  result in specific values of  $d_x$  and division of each  $d_x$  by the corresponding  $l_x$  results in  $q_x$ , the life-table mortality rate, as shown in table 21. The values of  $q_x$ , however, are for both sexes combined; they cannot be referred readily to the  $q_x$ -values for the model life tables (in appendix table II), which are given for each sex separately. As a rather rough working assumption, it can be supposed that each specific level of  $q_x$ , for the two sexes combined, approximates the average of the corresponding levels for the two sexes. The interpolations in the series of model life-table values, therefore, have to be made twice, once in respect of males and once in respect of females, and the average of the results is then taken as the required "level" in respect of each age group. It is not possible, on this basis, to find

whether the position of males or of females in respect of mortality is more favourable, and it will have to be assumed that, at each age, male and female mortality rates correspond to the same level. The corresponding levels in respect of  $P_x$  are then found by means of the averaging process described in paragraph 210 of this chapter.

240. The sequence of "levels" of  $P_x$ , found in this manner, shows no obvious signs of inconsistency. Death risks at the ages between 5 and 30 years in Costa Rica may well be low in comparison with the mortality level implied by the rates for earliest childhood and age groups over 40. The only abrupt change in the mortality levels ascribed to  $P_x$ -values is between ages 0-4 and 5-9, namely from 67.5 to 77.5. In part, this jump reflects the relatively high value of  $q_{1-4}$ , which corresponds to mortality level 61.5. It is unlikely that the registration of deaths of small children was excessive, but it is possible that some of the infants who died were reported as one year old. In the absence of any knowledge on this subject, the  $P_x$ -levels for these age groups are accepted as computed.

241. The next point to consider is the assumption of mortality decline. There is no doubt that mortality has declined substantially in Costa Rica during recent years (see the death rates shown in part D of chapter II), and it is highly probable that the decline will continue.

<sup>39</sup> *The Population of Central America (including Mexico), 1950-1980 (ST/SOA/Ser.A, No. 16).*

TABLE 21. ESTIMATION OF MORTALITY LEVELS FOR COSTA RICA, 1949-1951, FROM LIFE-TABLE MORTALITY RATES ( $q_x$ ) FOR BOTH SEXES

Age in years (x)	$l_x$ (both sexes)	$d_x$ (both sexes)	$1,000 q_x$ (both sexes)	Mortality levels			$P_x$
				$q_x$ (males)	$q_x$ (females)	$q_x$ (both sexes)	
0.....	100,000	9,706	97.1	76	71	73.0	67.5 <sup>a</sup>
1-4.....	90,294	5,447	60.3	62	61	61.5	67.5
5-9.....	84,847	1,155	13.6	72	72	72	77.5
10-14.....	83,692	566	6.8	84	82	83	84.5
15-19.....	83,126	791	9.5	88	84	86	84
20-24.....	82,335	1,287	15.6	84	80	82	80
25-29.....	81,048	1,505	18.6	80	77	78.5	76.5
30-34.....	79,543	1,781	22.4	75	74	74.5	72.5
35-39.....	77,762	2,166	27.9	72	69	70.5	70
40-44.....	75,596	2,557	33.8	73	67	70	69
45-49.....	73,039	3,314	45.4	74	63	68.5	69
50-54.....	69,725	4,296	61.6	76	62	69	66
55-59.....	65,429	6,123	93.6	71	55	63	64
60-64.....	59,306	7,814	131.8	74	57	65.5	66
65-69.....	51,492	9,745	189.3	75	59	67	66.5
70-74.....	41,747	11,867	284.3	73	59	66	67
75-79.....	29,880	11,940	399.6	74	63	68.5	71
80-84.....	17,940	9,349	521.1	86	71	78.5	71
85+.....	8,591	8,591	1,000.0	..	..	...	..

<sup>a</sup> Estimated level for  $P_b$ .

Efforts are being made to improve public sanitation, to combat diseases, and to educate the population in matters of hygiene. The model assumption, implying gains of about 2½ years in life expectation every five years, is accepted here as appropriate to the conditions, although a study of health problems, medical and sanitary resources, and existing plans for the development of health programmes in the future might lead to a modification of this assumption.

242. In view of deviations of the age-specific mortality indices from average patterns, the further question arises whether the calculations relevant to future mortality should be carried out in terms of constant age groups, or in respect of cohorts which advance in age as time progresses. A study of conditions in the country might help to resolve this question also, by showing whether the relatively low mortality of the younger age groups appears to be the consequence of their having received better education or nutrition, for example, than the older generations. If so, the future changes in survival levels may better be estimated in relation to cohorts than to fixed age groups.

243. For the present purpose, the simpler procedure of estimates according to age groups is adopted. The alternative of mortality estimates by cohorts seems at least equally plausible, but it would involve complications.<sup>40</sup>

<sup>40</sup> First, for the period from 1949-1951 to 1955-1960, a 7½-year time interval must be allowed for, i.e., mortality levels have to

If mortality declines were estimated in relation to cohorts rather than age groups, the result would be more rapid mortality decline in the middle adult ages and probably also among children. The assumption of mortality decline in relation to fixed age groups, therefore, can be regarded as a rather conservative one.

244. The latter assumption is worked out in table 22. The mortality level for each age group is raised by 7.5 points from 1949-1951 to 1955-1960, and by 5 points for each succeeding period. The corresponding values of  $P_x$  are determined by reference to appendix table V, with interpolations wherever required. Actually, this degree of refinement is hardly necessary. Mortality levels rounded to the nearest multiple of 5 and  $P_x$ -values derived directly from the appropriate tabulated figures would give sufficiently accurate results.

be raised by 7.5 points, an assumption which does not fit well with the five-year age groups of cohorts. This problem could be resolved by shifting the mortality levels—diagonally for cohorts—first by five years, then by another five years, and taking the average of the two results. Second, lower levels, implying higher mortality, are indicated for the age groups under ten years in 1950 than for those from 10 to 25 years. Unless it is assumed that the cohorts under ten in 1950 do not share the same health advantages as the next older cohorts, the cohort method requires some assumption for a particularly rapid decline of their mortality during the next five years. Furthermore, some basis is required for estimating the initial mortality rates, at ages under ten years, of the cohorts yet to be born. It is difficult to make realistic assumptions in such matters without a full examination of the relevant conditions in the country.

TABLE 22. ESTIMATION OF FUTURE SURVIVAL RATIOS ( $P_x$ ) FOR COSTA RICA, IN ACCORDANCE WITH THE MODEL ASSUMPTION OF MORTALITY DECLINE BY FIXED AGE GROUPS

Age in years (x)	Mortality "levels" (either sex)					( Survival ratios )							
						Males				Females			
	1949/51	1955/60	1960/65	1965/70	1970/75	1955/60	1960/65	1965/70	1970/75	1955/60	1960/65	1965/70	1970/75
( $P_x$ ).....	(67.5)	(75.0)	(80.0)	(85.0)	(90.0)	(0.8877)	(0.9070)	(0.9262)	(0.9438)	(0.9036)	(0.9208)	(0.9380)	(0.9535)
0-4.....	67.5	75.0	80.0	85.0	90.0	0.9648	0.9708	0.9765	0.9818	0.9669	0.9731	0.9791	0.9844
5-9.....	77.5	85	90	95	100	0.9924	0.9937	0.9949	0.9963	0.9932	0.9948	0.9962	0.9972
10-14.....	84.5	92	97	102	107	0.9933	0.9947	0.9960	0.9972	0.9947	0.9960	0.9971	0.9978
15-19.....	84	91.5	96.5	101.5	106.5	0.9899	0.9920	0.9940	0.9957	0.9920	0.9940	0.9956	0.9968
20-24.....	80	87.5	92.5	97.5	102.5	0.9860	0.9886	0.9911	0.9934	0.9882	0.9908	0.9930	0.9948
25-29.....	76.5	84	89	94	99	0.9834	0.9861	0.9886	0.9911	0.9852	0.9879	0.9904	0.9925
30-34.....	72.5	80	85	90	95	0.9792	0.9822	0.9849	0.9874	0.9811	0.9842	0.9868	0.9893
35-39.....	70	77.5	82.5	87.5	92.5	0.9728	0.9762	0.9794	0.9822	0.9766	0.9798	0.9828	0.9854
40-44.....	69	76.5	81.5	86.5	91.5	0.9629	0.9669	0.9700	0.9737	0.9702	0.9737	0.9768	0.9796
45-49.....	69	76.5	81.5	86.5	91.5	0.9483	0.9529	0.9569	0.9607	0.9602	0.9642	0.9678	0.9709
50-54.....	66	73.5	78.5	83.5	88.5	0.9232	0.9289	0.9341	0.9887	0.9416	0.9467	0.9515	0.9558
55-59.....	64	71.5	76.5	81.5	86.5	0.8869	0.8939	0.9003	0.9060	0.9125	0.9194	0.9256	0.9314
60-64.....	66	73.5	78.5	83.5	88.5	0.8398	0.8478	0.8552	0.8619	0.8716	0.8800	0.8880	0.8951
65-69.....	66.5	74	79.0	84	89	0.7678	0.7775	0.7864	0.7947	0.8037	0.8141	0.8240	0.8328
70-74.....	67	74.5	79.5	84.5	89.5	0.6691	0.6805	0.6907	0.7003	0.7060	0.7185	0.7304	0.7411
75-79.....	71	78.5	83.5	88.5	93.5	0.5531	0.5642	0.5748	0.5848	0.5914	0.6048	0.6171	0.6288
80+.....	71	78.5	83.5	88.5	93.5	(0.3328)	(0.3404)	(0.3473)	(0.3537)	(0.3552)	(0.3629)	(0.3699)	(0.3762)