# INTRODUCTION

If the contents of this work were to be judged by its title, there is little doubt that it would be classified among those theoretical studies which are useful for clarifying one's thoughts and setting them in order but have little prospect of practical application.

Those who are unfamiliar with demographic methods would first of all ask what is meant by a "stable population"; and when told that this is the name given to a limit population to which actual populations tend when their mortality and fertility remain constant, they would no doubt feel that this was a thoroughly abstract concept. For, to approach a limit is to come closer and closer to it without ever reaching it, like a hyperbola which approaches its asymptotes without ever touching them. If they were told that fertility and mortality would have to remain unchanged for about 100 years before an actual population came close enough to a stable population, they would be quite sure that this was a mathematical tool having only the most tenuous contacts with reality.

Demographers, and particularly those interested in theoretical demography, would take a somewhat more flexible attitude which would not differ fundamentally from that held by non-specialists.

They would point out that the limit stable state reached by a population is a function of the initial conditions. They would see it less as an abstract concept than as a consequence of those initial conditions, and hence as something providing information on the latter. Indeed, the concept of a stable population has for long been accepted as one of the tools of the demographer. Work of the kind carried out by Alfred J. Lotka has demonstrated the genuine value and usefulness of computing a stable population, given a set of initial conditions. The characteristics of a stable population computed in this way would reveal the properties of the corresponding set of demographic conditions. The net reproduction rate, the intrinsic rate of natural increase and its two components of mortality and fertility are now generally accepted as indices which are universally utilized.

Most demographers would agree, however, that the stable populations which they compute are very seldom found in reality. Any comparison of a stable population with the actual population in an attempt to estimate the demographic conditions of the population is therefore doomed to failure. If the actual demographic conditions are known, the corresponding stable population can be computed; and by comparing the stable state with the real population what may be regarded as transitional in the real population can be determined. The reverse is, however, fundamentally impossible. A stable state can be reached by starting from many actual populations, but it would be completely fortuitous if an actual population identical with a stable population were to be found. Nothing could be learnt of demographic conditions in the actual population from a mere knowledge of demographic conditions in the stable population thus identified. Nevertheless, we shall try to show, in the following pages, that the concept of a stable population has many practical applications.

In the first place, there is another theoretical concept, which relates to populations with unchanging age structures. Such populations possess some of the properties of a stable population, and there are many actual examples of them in the world, at least as a first approximation. It can be demonstrated that there are, at all times, in such populations the same relations among fertility, mortality and age structure as in a stable population. Consequently, so far as any phenomena involving age structure and its relations to mortality and fertility are concerned, populations with constant age structures can be assimilated to stable populations.[1] In order to express the fact that populations with unchanging age structures possess only some of the properties of stable populations, they have been given the name of *semi-stable populations*.

Many actual populations have age structures which, while not absolutely unchanging, change only slightly, and thus the concept of stable populations, which appeared at first to be a mathematical concept scarcely encountered in reality, becomes, through the theory of semi-stable populations, a familiar and common tool in population analysis. Most populations of developing countries, or almost three-quarters of the world population, may be considered semi-stable.

The constancy of the age structure of most populations of developing countries can be interpreted in terms of fertility and mortality, and this interpretation leads us to a further notion, that of the *quasi-stable population*. One of the fundamental characteristics of the present demographic evolution of the world is the decline in mortality. Starting from a level of mortality corresponding to an expectation of life at birth of about 25 years, the populations of the various areas of the world have been moving with varying degrees of rapidity towards a level corresponding to an expectation of life at birth of about 75 years. Observations show that this decline in mortality almost always follows the same pattern. It is, therefore, possible to construct a series of model life tables reflecting the various phases of mortality of the human species as experienced in the past.

As long as mortality remains within the "universe" defined above, variations in mortality have little effect on the age structure of a population, the latter being determined primarily by variations in fertility. The result is that, in populations where fertility remains unchanged while mortality varies within the universe of the life tables defined above, the age structure changes only slightly. Such populations have been given the name of *quasi-stable populations*. Roughly speaking, they may be

[1] This cannot be done where a phenomenon involves other factors, such as the past history of an individual. The average number of surviving children per family, the proportion of widowers and widows in the population, the proportion of orphans, and the absolute numbers of population, births, deaths and so forth, are examples of such phenomena.

considered to represent a particular series of semi-stable populations.

It can now be appreciated why little change occurs in the age structure of populations of developing countries. After a very long period during which the fertility and mortality of such populations remained, on the whole, relatively unchanged, mortality began to decline while fertility remained constant and the populations became quasi-stable.

It is worth noting that the concepts of a *stable population* and a *semi-stable population* are theoretical, and the study of their properties is a problem of pure mathematics. The concept of a *quasi-stable population*, on the other hand, is based on experience. The properties of quasi-stable populations are based on empirical data, which may be revised as a result of further experiments. There now follows a summary of the contents of the various chapters of this work.

Chapter I begins with a clarification of a point of terminology, since Lotka's definition of a stable population is not precisely what is stated above. Lotka defines a stable population as a particular case of a broader category of populations, called *Malthusian populations*. A Malthusian population is a population where the survivorship curve and the age structure are invariable.[2] There are an infinite number of such populations, which can be classified, initially, in two ways:

(1) Given a survivorship function, there corresponds a sub-set (H) of all the Malthusian populations based upon that survivorship function. By varying the survivorship function, an infinite number of sub-sets (H) embracing all Malthusian populations will then be obtained;

(2) Given an age structure, there corresponds a sub-set (F) of all the Malthusian populations based upon that age structure. By varying the age structure an infinite number of sub-sets (F) covering all Malthusian populations will then be obtained.

In a given sub-set ($H_0$) or ($F_0$) an additional condition is needed in order to determine a particular population. The stable population of a sub-set ($H_0$) is a particular population for which the fertility function is known. It can be demonstrated that for any given fertility function there exists one, and only one, stable population.

By varying the fertility function, on the other hand, all the populations of the sub-set (H) can be obtained. Thus, any Malthusian population can be considered a stable population, and indeed there are an infinite number of ways of doing this. That is why the two concepts are often confused, and such confusion has, in fact, become traditional. In this introduction, following the tradition, we have made references to a stable population, whereas in some cases the expression "Malthusian population" would have been more accurate. For the same reason, the expression "stable population" in the title of this work is not entirely in keeping with the contents. The expression "Malthusian population" can lead to confusion, since it may suggest a population practising birth control, whereas the subject of this manual is quite different. In order, therefore, to avoid using a title which might be misunderstood, we have preferred to be somewhat imprecise. In the body of this text, however, it is obviously necessary to distinguish clearly between the concept of a Malthusian population and that of a stable population, and this is the subject of the first chapter.

Lotka's definition of a stable population shows that such a population merely represents a particular case of a broader category, i.e., *the populations of a sub-set (H) or (F) which satisfy a given condition.* This general problem is posed in chapter I, but the study of the possible solutions to it is left to other chapters.

Lotka does not, however, simply define the concept of a stable population as a particular case of a Malthusian population. After asking the question of what happens to a population whose survivorship and fertility functions remain invariable, he demonstrates that such a population approaches the stable population defined by the sub-set of Malthusian populations based on the known survivorship function and the fertility function. A stable population thus appears as a limit state, and it was in this form that is was referred to at the beginning of this introduction.

This second definition of stable populations suggests a method of computing their characteristics. By simply computing far enough into the future a projection based on conditions of constant mortality and fertility, one gradually sees emerging from the projection the stable population corresponding to the given mortality and fertility. Chapter I ends with a description of such a method and establishes the properties of stable populations from actual examples of projections. These properties are established without the use of mathematical notation. It is somewhat like a person who, in attempting to demonstrate that the altitudes of a triangle meet at a point, begins by drawing triangles of different shapes and graphically verifies his thesis in each of the triangles he has drawn.

While this procedure is convenient for an understanding of the mechanism followed by a population in becoming stable, it is not suitable for practical computations,[3] which are based on the results of a mathematical analysis of "renewable resources". Chapter II describes and applies these methods. In this chapter we start from Lotka's definition of a stable population; that is to say, we take sub-sets (H) of Malthusian populations which have a given survivorship function. First, the formulae for the computation of the demographic characteristics of a particular population of the set satisfying a given condition are indicated. The chapter then goes on to actual applications of such formulae, beginning with the case where the given condition is knowledge of the fertility function, as in the case of a stable population. Various other conditions, such as knowledge of the crude death rate, crude birth rate and so forth are examined.

In chapter III, the same examples are taken up, but they are now considered as limit states of the process of demographic evolution. This chapter therefore deals with the theoretical aspect of the projections studied empirically in chapter I. By removing the condition of invariability

[2] It can be demonstrated that such a population has a constant rate of natural variation, and it is for this reason that such populations are known as Malthusian populations, Malthus having dealt in his works with populations increasing at a constant rate.

[3] The use of electronic computers may invalidate this statement, since such machines enable projections to be obtained in a few seconds, and the empirical method referred to in chapter I then becomes very easy to apply. See in this connexion the article by Nathan Keyfitz, "*L'utilisation des machines électroniques pour les calculs démographiques*", *Population* (Paris), No. 4, 1964, pp. 673-682.

of the age structure, a process of demographic evolution can be made to correspond to each of the examples given in chapter II. First, processes with constant survivorship and fertility functions leading at their limit to stable populations are studied. The chapter then proceeds to study other examples, such as processes with constant survivorship functions and crude birth rates, processes with constant survivorship functions and crude death rates, and so forth.

Chapter IV returns to the problem of determining a Malthusian population satisfying a given condition, this time by taking the sub-sets (F) defined by a given age structure rather than the sub-sets (H) defined by a given survivorship function.

Chapter V defines the concept of a "semi-Malthusian population" (or "semi-stable population", to use the not quite accurate terminology employed above). This, it should be recalled, is a population with an unchanging age structure. Whereas the earlier chapters described the well-known demographic works with a minimum of explanations because the full demonstration could always be found in the original works, this was not the case as regards chapter V. As the concept of a semi-Malthusian population is new in demography, it is necessary each time to justify the formulae used. Naturally, therefore, chapter V is highly theoretical.

The concept of a semi-Malthusian population has many practical applications. In the first place, it leads to a broadening of Lotka's definition of a Malthusian population. This is the subject of chapter VI, which takes up the problem of determining a Malthusian population satisfying a given condition within a new series of sub-sets (G) where the age structure of deaths is constant. However, the importance of the concept of a semi-Malthusian population lies in the fact that it opens the way to a solution of the problem from which this manual takes its title, namely, how to use the concept of a stable population for the purpose of estimating the fertility and mortality of actual populations.

In order to solve this problem, two lines of reasoning could legitimately be adopted according to whether the actual populations are assimilated to semi-Malthusian or to quasi-stable populations. The two methods are applied to populations whose age structure has changed little over time.

When a population is assimilated to a semi-Malthusian population, the little variation in its age structure is interpreted as the imperfect materialization of an invariable age distribution, while when a population is assimilated to a quasi-stable population, the little variation in its age structure is interpreted as the materialization of a population with constant fertility but with a mortality varying over the possible range of human mortality.

In the first case, when the actual population is assimilated to a semi-Malthusian population, it is convenient to treat the actual population as a Malthusian population for which certain demographic characteristics other than mortality and fertility are known, and the question is what can be said about the unknown mortality and fertility. This is the general problem studied in chapters II, IV and VI, and everything stated in those chapters is applicable to actual populations.

In the second case, when the actual population is assimilated to a quasi-stable population, the levels of mortality and fertility are sought along totally different lines. As the concept of a quasi-stable population is based on experience, it is by comparing actual populations with empirically constructed quasi-stable populations that the appropriate quasi-stable population to be assimilated to given actual population is determined. The mortality and fertility of this assimilated quasi-stable population are thus the estimated values sought. However, it is necessary first to construct schemes and framework of quasi-stable populations.

There are an infinite number of ways of calculating such schemes. In order to reduce the number, it may be noted that quasi-stable populations are always close to the stable populations of the moment. It was therefore assumed that stable populations, computed as having mortality levels within the range of human mortality, conveniently represented the possible range of quasi-stable populations. There remained the task of defining the possible range of human mortality. It was stated earlier that a series of model life tables representing the various phases of changes in human mortality could be conceived and constructed. In reality, and more exactly, however, human mortality vacillates around these successive phases, and it is therefore necessary to construct, in addition to the series of model life tables referred to, two other series reflecting tendencies deviating upwards and downwards. These two series mark the limits of the possible range of variation of actual human mortality. Details of the methods used to construct these three series of mortality tables will be found in annex II. It should be noted immediately that the intermediate model life table is practically indentical with the series of model tables computed by the United Nations in *Manual III*.[4]

With the aid of the three series of model life tables, three sets of stable populations, which were designated as the intermediate, the upward-deviating and the downward-deviating frameworks, were constructed. Methods of estimating mortality and fertility were developed by comparing actual populations with the populations of these three frameworks. This is the subject of chapter VII. These methods presuppose that the mortality sought conforms to the series of model life tables employed in the construction of the framework used in finding the quasi-stable population to be assimilated to the actual population. This is an assumption which cannot be verified, since the mortality is not known. It is therefore necessary to verify the fact that the assumption does not have any significant effect on the estimated values sought, and this is the subject of chapter VIII.

Finally, the manual has four annexes. Annex I deals with certain aspects, hitherto neglected by theoretical demographers, of the theory of a stable population considered as the limit of the process of evolution with constant levels of mortality and fertility. These aspects are not directly connected with the subject of this manual, and the results of annex I cannot be used to estimate the mortality and fertility of actual populations, but they are nevertheless the natural complement of the work referred to in chapter III. This annex involves the use of highly theoretical and mathematical notation and methods, and leads to the very concrete notion of the "growth poten-

[4] *Methods of Estimating Population, Manual III: Methods for Population Projections by Sex and Age* (United Nations publication, Sales No.: 56.XIII.3).

tial" of a population — a most convenient concept, whose use is considered in chapter I and the computation of which presents no difficulty.

It has already been stated that annex II deals with the methods used in constructing the series of model life tables applicable for the range of possible variations of human mortality. This annex merely gives a summary of more extensive work on the subject published elsewhere.

Annex III gives the characteristics of the framework of intermediate model stable populations, while annex IV gives the characteristics of the upward-deviating and downward-deviating frameworks.