VI. DESCRIPTION OF THE ANNEXES

The output of the model life table project appears in the five annexes to the report. Annex I presents the model life tables themselves. Annex II contains mortality indices by single years of age under age 5 suitable for application of Brass-type indirect estimation procedures. Five- and 10-year life tables survival ratios are presented in annexes III and IV, respectively. These two annexes should be useful for estimation of mortality from census age distributions. Annex V presents the life tables which constituted the refined data set for the project along with brief descriptions of the data sources, evaluations and adjustments.

This chapter briefly describes the formats of these tables and methods of computation when necessary.

ANNEX I. UNITED NATIONS MODEL LIFE TABLES

Male and female model life tables are presented for each of five patterns (the four regional patterns plus the general pattern) for life expectancies at birth between 35 years and 75 years at one-year increments. The life tables consist of nine columns defined as follows:

- AGE : The initial age of the age interval $(x, x + n)$ where x is the initial age and n is the length of the interval. The interval n equals five years with the exception of the first interval (one year), second interval (four years) and last interval (open-ended)
- M(X) : Central death rate for the age interval $(x, x + n)$; usual notation is ${}_nm_x$
- Q(X) : Probability of an individual at age x dying before the end of the age interval $(x, x + n)$; usual notation is ${}_nq_x$
- I(X) : Number of survivors at age x in a life table with radix (starting population) of 100,000 persons; usual notation is l_x
- D(X) : Number of deaths in age interval $(x, x + n)$; usual notation is ${}_nd_x$
- L(X) : Number of person-years lived in age interval $(x, x + n)$; usual notation is ${}_nL_x$
- T(X) : Number of person-years lived at ages x and older; usual notation is T_x
- E(X) : Expectation of life at age x; usual notation is e_x^0
- A(X) : Average number of years lived in the age interval $(x, x + n)$ by those dying during the age interval; usual notation is ${}_na_x$

Life tables were constructed by the 1-component model described in chapter II (see equation (2)). An iterative procedure was used to locate the value of a_{1j} which corresponds to the desired life expectancy at birth. The life table columns were calculated using a computer program written in the Population Division. Given ${}_nm_x$ as input,

$${}_nq_x = \frac{n \cdot {}_nm_x}{1 + (n - {}_na_x)\,{}_nm_x}\text{; and}$$

$$l_{x+n} = l_x(1 - {}_nq_x)$$

$${}_nd_x = l_x - l_{x+n}$$

$${}_nL_x = {}_na_x l_x + (n - {}_na_x)l_{x+n}$$

$$T_x = \sum_{a=x}^{w} L_x$$

$$e_x^0 = T_x / l_x$$

For ages 15 and over, the expression for ${}_na_x$ is derived from Greville[25] as ${}_na_x = 2.5 - (25/12)\,({}_nm_x) - k)$, where $k = 1/10\ ln\,({}_nm_{x+5}/{}_nm_{x-5})$. For ages 5 and 10, ${}_na_x = 2.5$ and for ages under 5, ${}_na_x$ values from the Coale and Demeny West region relationships are used.[26]

With ${}_nq_x$ as input, the procedure is identical, except that an iterative procedure is used to find the ${}_nm_x$ and ${}_nq_x$ values consistent with the given ${}_nq_x$ and with the Greville expression. To complete the life table, the last six ${}_nq_x$ values are used to fit the Makeham-type expression $({}_nq_x/(1 - {}_nq_x)) = A + Bc^x$. The fitting is done via non-linear least squares (Gauss-Newton iteration). Using this Makeham-type curve, ${}_nq_x$ values are extrapolated until there are no survivors remaining. These extrapolated ${}_nq_x$ values are converted to l_x and ${}_nL_x$ values with appropriate separation factors and the ${}_nL_x$ added up to obtain the final T_x.

ANNEX II. SINGLE-YEAR MORTALITY UNDER AGE 5

Probabilities of dying (${}_1q_x$) by single years of age for ages under 5 are presented along with the corresponding number of survivors at each age (lx). Single-year mortality rates were calculated according to the 3-parameter interpolation equation,[27]

$$ln\ {}_1q_x = -\,t_1(x + t_2)^{t_3}.$$

The interpolation equation has undergone considerable testing and performed very satisfactorily. Values of t_1, t_2 and t_3 were solved for which reproduce the ${}_1q_0$, ${}_4q_1$ and ${}_5q_5$ values of the given model life table. These interpolation parameters are also printed out.

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[25] T. N. E. Greville, "Short methods of constructing abridged life tables", *The Record of the American Institute of Actuaries*, vol. XXXII, part 1, No. 65 (June 1943), pp. 29-42.

[26] Coale and Demeny West region formulae are as follows. When ${}_1q_0 \geq 0.100$, then ${}_1a_0 = 0.33$ for males and 0.35 for females; ${}_4a_1 = 1.352$ for males and 1.361 for females. When ${}_1q_0 < 0.100$, ${}_1a_0 = 0.0425 + 2.875\,{}_1q_0$ for males and ${}_1a_0 = 0.050 + 3.00\,{}_1q_0$ for females; ${}_4a_1 = 1.653 - 3.013\,{}_1q_0$ for males and ${}_4a_1 = 1.524 - 1.627\,{}_1q_0$ for females. See A. J. Coale and P. Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, N.J., Princeton University Press, 1966), p. (20).

[27] This formula is the first term of a 3-term mathematical expression of the age curve of mortality, as presented in L. Heligman and J. H. Pollard, "The age pattern of mortality", *The Journal of the Institute of Actuaries*, vol. 107, part 1, No. 434 (June 1980), pp. 49-80.

Single-year values for both sexes combined were calculated assuming a sex ratio at birth of 1.05. Male and female values for the same life expectancy at birth were used for these computations. The resulting "both sexes" figures are probably of sufficient validity for use in Brass-type estimates although in individual cases readers may wish to improve their estimates through use of the regression equations provided in chapter V or of other external information on sex differentials.

ANNEXES III AND IV. FIVE- AND 10-YEAR LIFE TABLE SURVIVAL RATIOS

These tables present 5- and 10-year life table survival ratios. Five-year survival rates are calculated, in standard life table notation, as ${}_5L_{x+5}/{}_5L_x$ for ages $x=0, 5, \ldots, 75$. The first survival ratio presented is the probability of five annual cohorts of births surviving to age-group 0-4, i.e., ${}_5L_0/{}_5l_0$. The final survival ratio for the open-ended age group is T_{85}/T_{80}. Ten-year survival ratios are calculated similarly as ${}_5L_{x+10}/{}_5L_x$ for $x = 0, 5, \ldots, 70$; ${}_{10}L_0/{}_{10}l_0$ for the births, and T_{85}/T_{75} for the final open-ended group.

ANNEX V. DESCRIPTION OF LIFE TABLE CONSTRUCTION FOR THE INPUT LIFE TABLES

Annex V presents the life tables which constitute the input data set for the project along with brief descriptions of the data sources, evaluations and adjustments. The life tables are in the same format and were calculated in the same manner as the model tables contained in annex I.