Chapter VIII

ESTIMATION OF FERTILITY BY REVERSE-SURVIVAL METHODS

A. BACKGROUND OF METHODS

1. *Meaning of reverse survival*

In a closed population, children currently aged x are just the survivors of the births that occurred x years ago. From this fact it is easily inferred that the number of births occurring x years ago can be estimated by using life-table survivorship probabilities to "resurrect" numerically those no longer present among the population aged x. This method of estimation is known as "reverse survival" or "reverse projection" because the population now aged x is "survived" or "reverse-projected" to age $x-t$ by moving it, with a suitable life table, t years into the past.

It is immediately evident that if the single-year age distribution of a population enumerated at t_0 is available, it is potentially possible to estimate the number of births occurring during each of the 15 or 20 years preceding t_0. Methods that exploit this possibility are described in this chapter. However, before proceeding with their description, it is worth noting that they are all heavily dependent upon the accuracy of the reported age distribution of the population being studied. Errors in age-reporting or differential completeness of enumeration affecting certain age groups, especially the younger ones, are certain to bias the estimates obtained. Because these types of deficiencies are all too frequently characteristic of the data sets available, reverse-survival methods are often ineffective in producing reliable fertility estimates. Their usefulness depends mainly upon the fact that they often provide independent fertility estimates which can be used to assess the plausibility of those obtained by other means.

2. *Organization of this chapter*

Only two estimation methods based on reverse survival are presented in this chapter. The first method illustrates the basic principle underlying other, more sophisticated procedures. It allows the estimation of the average birth rate during the five or 10 years preceding enumeration from the population classified by five-year age group. Variations of this basic method arise mainly because of the increasing amount of data available or because of the increasing detail with which it is classified. The second method presented here depends upon the latter aspect, because it requires detailed tabulations of enumerated children classified both according to their own age and to that of their mother. To aid the user in selecting the method appropriate for a particular case, table 153 shows their data requirements and the

TABLE 153. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER VIII

Section	*Type of input data*	*Estimated parameters*
B. Estimation of birth rate by reverse survival of the population under age 10	Enumerated population under age 10 classified by five-year age group Total population enumerated at two points in time, or that enumerated at one point and its growth rate Probabilities of child survivorship ($l(2)$, $l(3)$ or $l(5)$, for example)	The average birth rate during the two five-year periods immediately preceding the time of enumeration
C. The own-children method of fertility estimation	Enumerated population under age 10 or 15, classified by single year of age and by single year of age of mother Children under age 10 or 15 whose mother's age is unknown, classified by single year of own age Women aged 15-59 or 15-64, classified by single year of age Probabilities of child survivorship Probabilities of adult female survivorship (a life table for females)	Age-specific fertility rates for each of the 10 or 15 years preceding enumeration Total fertility for each of the 10 or 15 years preceding enumeration

types of estimates they yield; and the sections describing these methods are listed below:

Section B. Estimation of birth rates by reverse survival of the population under age 10. This section presents the basic reverse-survival method. It requires as input the population classified by five-year age group and survivorship probabilities referring to childhood;

Section C. The own-children method of fertility estimation. This method allows the estimation of annual age-specific fertility rates for the 10 or 15 years preceding the time of enumeration. It requires that the enumerated children be classified by age of mother at the time of enumeration.

B. ESTIMATION OF BIRTH RATES BY REVERSE SURVIVAL OF THE POPULATION UNDER AGE 10

1. *Basis of method and its rationale*

The estimation of the number of births occurring x years before enumeration from the enumerated population aged x is a well-known possibility that has been widely exploited. However, practice has shown that the estimates obtained in this manner are often not very useful due to the severe age-reporting errors generally present in the basic data. Problems are especially acute when the reverse-projected data are classified by single year of age, since age-heaping is likely to produce spurious peaks and troughs in the estimates obtained. Furthermore, because censuses often fail to enumerate children completely, especially those aged 0 or 1, it is frequent for the estimated number of births referring to the year or two immediately preceding the time of enumeration to be too low.

In order to avoid some of these problems or to minimize their effect on the final estimates, grouped data are often used. Five-year age groups are most commonly selected; they yield estimates of the average annual number of births occurring during the two five-year periods immediately preceding the time of enumeration. According to the observations made above, the average annual number of births estimated by reverse-projecting the population aged from 0 to 4 is likely to underestimate the true number of births occurring during the five-year period immediately preceding enumeration. A better estimate may be expected from the reverse projection of the population in age group 5-9. However, estimates of numbers of births obtained from the population aged 5-9 years are likely to be more affected by errors in the estimation of mortality and may also be exaggerated by age-reporting errors (heaping at age 5, for example).

Besides being very sensitive to the presence of age-misreporting, the estimation of fertility by reverse survival is also dependent upon the type of mortality estimates used. By its very nature, reverse survival cannot be performed without a life table, covering at least the ages of childhood; and adequate information allowing the direct construction of resonable life tables is lacking in most countries where reverse-survival estimates of fertility are needed. Therefore, reverse survival is often applied in conjunction with other methods, particularly those which permit the indirect estimation of child mortality (see chapter III). All of these methods assume that the pattern of mortality in the population studied conforms to a certain model. As one would expect, the choice of model affects the fertility estimates yielded by reverse survival, the differential effects being greater the further one reverse-projects the observed population into the past.

In spite of all these caveats, the estimation of the birth rate by reverse survival is described here because of its traditional importance and because it provides a valuable tool for the detection of inconsistencies. For example, suppose that b_1, an independently obtained estimate of the birth rate for the five-year period preceding a census, is smaller than b_2, the estimate obtained by reverse-survival of the population aged 0-4. Since b_2 is usually a lower bound for the birth rate, any estimate for the same period lower than b_2 would be suspect. Of course, one should also explore the possibility of having overestimated the mortality level used in reverse surviving the population aged 0-4 years.

The value of reverse-survival procedures may also be increased if they can be applied to a series of censuses to obtain estimates for overlapping periods. In such circumstances, it may be worth while to reverse-project the population in age groups older than the traditional age groups 0-4 and 5-9 in order to obtain a range of birth-rate estimates for particular periods. Thus, if three census enumerations spaced by 10-year intervals are available, three estimates can be made of the birth rates during the periods from zero to four years and from five to nine years before the first census. For the five years before the first census, estimates can be based on the population aged 0-4 at the first census, 10-14 at the second census and 20-24 at the third census, whereas for the period from five to nine years before the first census, birth-rate estimates can be based on the population aged 5-9 at the first census, 15-19 at the second census and 25-29 at the third census. Such a procedure has been applied by Shorter and Macura[1] to the quinquennial censuses of Turkey taken between 1935 and 1975.

This extension of the basic procedure requires substantially more information than does the simpler procedure. The older the population being considered, the greater the influence of the survivorship estimates used to calculate the number of births the survivors represent, and of the growth rate estimates used to calculate the population denominators. For most developing countries, the level of child mortality in the 1940s or 1950s is known only to a very rough approximation; hence, the reverse projection of a recent census for some 30 years, or the reverse projection of an earlier census for a shorter period, is fraught with uncertainty. Consistency between estimates for earlier periods does not provide

[1] Frederic C. Shorter and Miroslav Macura, *Trends in Fertility and Mortality in Turkey, 1935-1975*, Committee on Population and Demography Report No. 8 (Washington, D.C., National Academy Press, 1982).

any evidence in their support, because any error in specifying past mortality risks will affect all the estimates in the same direction. Migration also represents a problem; and any evidence suggesting a significant level of migration during the period being considered will make it necessary to attempt some correction, not only at the aggregate level but at the level of each age group. On the positive side, the effects of age-reporting errors should be reduced by considering data from several censuses, since different age groups determine the different estimates for the same period; however, it is possible that a sequence of 10-year intercensal intervals, combined with heavy heaping on ages ending in zero, may give rise to consistently high estimates of the birth rate for periods determined by age groups that include zero as an age ending (except for the estimate based on the population aged 0-4), while producing consistently low birth-rate estimates for the intervening periods, associated with age groups including five as an age ending (except possibly for age group 5-9). Given a range of birth-rate estimates for each period, the problem arises how to arrive at a "best" estimate; the most obvious possibility is to take the mean of the available estimates, though given the likely nature of the errors involved, the median may be a better indicator. It may be mentioned that changes in enumeration completeness from one census to the next have relatively little impact on the procedure, so long as the age distributions do not change markedly from one census to the next and intercensal survival is not used as a basis for the estimation of survivorship probabilities.

2. *Data required*

The following data are required for this method:

(*a*) The population under age 10, classified by age (five-year age groups are sufficient, though single-year data are preferable) and by sex;

(*b*) The total population at the time of enumeration, t_0;

(*c*) An estimate of the growth rate;

(*d*) Estimates of mortality parameters that would permit the construction of a life table up to age 10. A value of $l(2)$ obtained from information on children ever born and surviving (see chapter III) is adequate.

3. *Computational procedure*

Step 1: calculation of life-table estimates of person-years lived. In order to reverse-project to birth the population in age groups 0-4 and 5-9, one only needs values of ${}_5L_0$ and ${}_5L_5$, the person-years lived by the stationary population constituting the life table between birth and exact age 5, and between age 5 and exact age 10, respectively. Usually, but not necessarily, these values are obtained by assuming that the mortality level associated with $l(2)$ remained constant during the 10 years preceding enumeration. Under this assumption, the actual calculation of ${}_5L_0$ and ${}_5L_5$ is carried out by interpolating between the printed values of the Coale-Demeny model life tables.[2] Naturally, the family of models selected should be that used in estimating $l(2)$. In some cases, the mortality level selected may be the mean of those associated with the $l(2)$, $l(3)$ and $l(5)$ values yielded by the indirect estimation of child mortality (see chapter III). This mean level may represent more closely the true mortality level prevalent during the decade preceding enumeration, especially when there is evidence suggesting a recent decline in child mortality.

Step 2: estimation of mid-period populations. Since this method is directed towards the estimation of an average annual birth rate for the periods from t_0-5 to t_0 and from t_0-10 to t_0-5, t_0 being the date of enumeration, an estimate of the total population at the mid-points of these periods is required. Perhaps the simplest way of estimating these mid-period populations is by using the equation

$$N_M = N_0 \exp[r\ (t_M - t_0)] \qquad \text{(B.1)}$$

where t_M is the mid-point of the period being considered; N_0 is the total count yielded by enumeration; and r is an estimate of the growth rate. As usual, the growth rate is estimated from knowledge of the total population at two points in time, t_0 and t_1. In such a case,

$$r = \ln [N_1/N_0]/(t_1 - t_0) \qquad \text{(B.2)}$$

where N_1 is the total count at time t_1 and N_0 is that at time t_0.

Of course, there are several other ways of estimating a mid-period population; but, in general, they require far more effort than the intrinsic roughness of the method at hand would warrant. For this reason, they are not described here.

Step 3: estimation of average annual birth rates for the two five-year periods preceding the census. The average annual number of births for the first period, from t_0-5 to t_0, is

$$B_1 = {}_5N_0/{}_5L_0 \qquad \text{(B.3)}$$

where ${}_5N_0$ is the population in age group 0-4; and ${}_5L_0$ is the life-table estimate obtained in step 1.

For the period from t_0-10 to t_0-5, the equivalent average annual number of births is

$$B_2 = {}_5N_5/{}_5L_5 \qquad \text{(B.4)}$$

where ${}_5N_5$ is the population in age group 5-9. If the radix, $l(0)$, of the life table being used were not one, equations (B.3) and (B.4) would have to be modified as follows:

$$B_1 = {}_5N_0\, l(0)/{}_5L_0$$

and

$$B_2 = {}_5N_5\, l(0)/{}_5L_5. \qquad \text{(B.5)}$$

[2] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University, 1966).

Once B_1 and B_2 are calculated, the birth rate for each period is obtained by dividing these values by the corresponding mid-period populations calculated in step 2.

4. *A detailed example*

Data gathered by the 1960 census of Brazil are used to illustrate the application of this method. The steps of the procedure are described below.

Step 1: calculation of life-table estimates of person-years lived. The data on children ever born and surviving collected by the 1960 census of Brazil were used in chapter III, subsection E.4(b) to estimate probabilities of survivorship in childhood with respect to the West model. In chapter VII, subsection C.5, however, it was shown that the South model provided a better representation of child mortality in this country. Therefore, to obtain estimates of ${}_5L_0$ and ${}_5L_5$, $l(2)$ was re-estimated using the South model. Its value, 0.8491, is consistent with mortality level 15.21 in the South family of model life tables. This level is used now to estimate ${}_5L_0$ and ${}_5L_5$ by interpolation. Table 154 shows the ${}_nL_x$ values appearing in the Coale and Demeny model life tables at levels 15 and 16.

Since the Coale-Demeny model life tables only contain ${}_1L_0$ and ${}_4L_1$, one needs to calculate ${}_5L_0$ by adding these values. Therefore, for males,

$$ {}_5L_0^{15} = {}_1L_0^{15} + {}_4L_1^{15} = 0.9164 + 3.3204 = 4.2368 $$

and

$$ {}_5L_0^{16} = 0.9229 + 3.3851 = 4.3080. $$

Using these values, interpolation may now be carried out as shown below:

$$ {}_5L_0^m = 0.79(4.2368) + 0.21(4.3080) = 4.2518. $$

Other values of ${}_5L_x$ are obtained in a similar fashion. They are shown in columns (2) and (5) of table 155.

TABLE 154. VALUES OF PERSON-YEARS LIVED FROM EXACT AGE x TO $x+5$ BY A STATIONARY POPULATION, SOUTH MODEL LIFE TABLES

Person-years lived (1)	Males: Level 15 (2)	Males: Level 16 (3)	Females: Level 15 (4)	Females: Level 16 (5)
${}_1L_0$..........	0.9164	0.9229	0.9271	0.9329
${}_4L_1$..........	3.3204	3.3851	3.3726	3.4360
${}_5L_5$..........	4.0161	4.1163	4.0832	4.1835

Step 2: estimation of mid-period populations. Table 156 shows the population counts produced by the censuses of Brazil since 1950. Intercensal growth rates for each period and sex are also given. Each growth rate has been calculated according to the equation:

$$ r = \ln [N_1/N_0]/(t_1 - t_0). $$

Thus, for example, for males during the period 1950-1970, the growth rate was

$$ r = \ln [45{,}754{,}659/25{,}885{,}001]/20.167 = 0.0282. $$

Examination of the set of growth rates given in table 156 shows that population growth in Brazil slowed somewhat during the period 1960-1970. The lower growth rate for males observed during that period, however, seems suspect. Sex differences in growth rates diminish when the 20-year period is considered. Because there are reasons to believe that the 1960 census might not be of comparable quality with the others, the growth rate selected in this case is based on the estimates for 1950-1970. The average of the male and female growth rates, amounting to 0.0285, is considered representative of the growth rate experienced by the Brazilian population around 1960. Therefore, the mid-year populations desired are estimated by

$$ N_1 = N_0 \exp[-0.0285(2.5)] $$

and

$$ N_2 = N_0 \exp[-0.0285(7.5)] $$

TABLE 155. ESTIMATION OF NUMBER OF BIRTHS BY REVERSE SURVIVAL, BRAZIL, 1960
(Thousands)

Age x (1)	Males: Person-years lived from x to $x+5$ ${}_5L_x^m$ (2)	Males: Reported population (thousands) ${}_5N_x$ (3)	Males: Estimated number of births (thousands) B_i (4)	Females: Person-years lived from x to $x+5$ ${}_5L_x^f$ (5)	Females: Reported population (thousands) ${}_5N_x$ (6)	Females: Estimated number of births (thousands) B_i (7)
0...............	4.2518	5 688	1 337.8	4.3142	5 506	1 276.3
5...............	4.0371	5 171	1 280.9	4.1043	4 988	1 215.3

TABLE 156. TOTAL POPULATION AT CENSUS DATES AND INTERCENSAL GROWTH RATES BY SEX, BRAZIL

Census date (1)	Total population: Males (2)	Total population: Females (3)	Intercensal period (4)	Growth rate: Males (5)	Growth rate: Females (6)
1 July 1950.........................	25 885 001	26 059 396	1950-1960	0.0298	0.0294
1 Sept. 1960.........................	35 059 546	35 131 824	1960-1970	0.0266	0.0282
1 Sept. 1970.........................	45 754 659	46 586 897	1950-1970	0.0282	0.0288

TABLE 157. BIRTH RATES BY SEX, ESTIMATED BY REVERSE SURVIVAL, BRAZIL, 1960

Period i (1)	Males: Estimated population (thousands) N_i (2)	Males: Estimated number of births (thousands) B_i (3)	Males: Estimated birth rate b_i (4)	Females: Estimated population (thousands) N_i (5)	Females: Estimated number of births (thousands) B_i (6)	Females: Estimated birth rate b_i (7)
1	32 647.9	1 337.8	0.0410	32 714.9	1 276.3	0.0390
2	28 314.5	1 280.9	0.0452	28 372.6	1 215.3	0.0428

where N_0 is the reported population at the time of the census. Values of N_0 (the population in 1960) for each sex are 35,059,546 males and 35,131,824 females. Hence, N_1 and N_2 for males (in thousands) are

$$N_1 = (35{,}060)(0.9312) = 32{,}647.9;$$

$$N_2 = (35{,}060)(0.8076) = 28{,}314.5.$$

The corresponding values for females are shown in column (5) of table 157.

Step 3: estimation of average annual birth rates for the two five-year periods preceding the census. Table 155 shows the number of males and females enumerated in age groups 0-4 and 5-9, denoted by ${}_5N_x$. Using equations (B.3) and (B.4), average annual births for each period are obtained. The case of male births is illustrated below:

$$B_1 = 5{,}688/4.2518 = 1{,}337.8;$$

$$B_2 = 5{,}171/4.0371 = 1{,}280.9.$$

Once the average annual number of births for each period is estimated, calculation of the birth rates is straightforward. For example, in the case of males:

$$b_1 = B_1/N_1 = 1{,}337.8/32{,}647.9 = 0.0410;$$

$$b_2 = B_2/N_2 = 1{,}280.9/28{,}314.5 = 0.0452.$$

The corresponding estimates for females are shown in column (7) of table 157.

Estimates for both sexes can now be obtained by adding male and female births for each period, and the male and female mid-period populations separately. Then, as usual, the birth rates are found as the ratios of total births in a period to total mid-period population. Table 158 summarizes results for both sexes.

TABLE 158. BIRTH RATES FOR BOTH SEXES COMBINED, ESTIMATED BY REVERSE SURVIVAL, BRAZIL, 1960

Period i (1)	Estimated population (thousands) N_i (2)	Estimated number of births (thousands) B_i (3)	Estimated birth rate b_i (4)
1	65 362.8	2 614.1	0.0400
2	56 687.1	2 496.2	0.0440

When the birth rates estimated by reverse survival are compared with those obtained by stable-population analysis in chapter VII, subsection C.5, $b_m = 0.0438$, $b_f = 0.0416$ and $b_t = 0.0427$, stable estimates prove in every instance to be higher than the reverse-survival estimates for the five-year period immediately preceding the census (since the census took place on 1 September 1960, this period extends from 2 September 1955 to 1 September 1960 and is denoted by 1956-1960). However, the stable estimates are much lower than the reverse-survival estimates for the period 1951-1955. If one accepts the stable estimates as true, it would appear that both the male and female populations aged 0-4 in 1960 were underenumerated by approximately 6 per cent, while those in age group 5-9 were overenumerated by about 3 per cent. Although part of the apparent excess in age group 5-9 may be caused by upward transfers of younger children, it is clear that if the stable estimates are correct, transfers alone cannot explain the deficit observed in age group 0-4. Hence, one must accept either that fertility fell during the five-year period immediately preceding the 1960 census or that the birth-rate estimates obtained by reverse-surviving the population aged from 0 to 4 constitute a lower bound for the true birth rate of the Brazilian population during the period 1956-1960. As is often the case, the latter possibility is more likely.

C. THE OWN-CHILDREN METHOD OF FERTILITY ESTIMATION

1. *Basis of method and its rationale*

The own-children method permits the estimation of age-specific fertility rates for the 10 or 15 years preceding a census or survey from information on the enumerated number of children classified by single year of age and single year of age of mother. In order to obtain the desired age-specific fertility estimates, the own-children method calls for the reverse-projection of these children to the time of their birth. It can be viewed, therefore, as a specific application of the method presented in the previous section. The point where the two methods differ most markedly is on the type of data they require, and probably the greatest innovation introduced by the proponents of the own-children method is the exploitation of seldom-used census information for fertility estimation purposes.[3] Indeed, the "own-children

[3] Lee-Jay Cho, "The own-children approach to fertility estimation: an elaboration", *International Population Conference, Liège, 1973* (Liège, International Union for the Scientific Study of Population, 1973), vol. 2, pp. 263-280.

tabulation" (children by single year of age and single year of age of mother) which constitutes the basis of this method can only be made if enumerated children are linked in some way to their mothers. When the method was first proposed, censuses did not usually include a direct question making this linkage. It was therefore necessary to infer the mother-child link from information on relationship to the head of the household and from the compatibility between the age of the presumed mother and those of her children. Hence, the traditional own-children method involves more than an estimation procedure; it also includes a set of criteria to perform the mother-child linkage in any given case. Because the linkage process falls outside the scope of this *Manual* and because in recent censuses and surveys a direct question identifying the mother of each enumerated child has often been included (making the linkage unnecessary), the matching criteria are not presented here.

As mentioned above, age-specific fertility rates are estimated essentially by the reverse projection of enumerated children to the time of their birth. Therefore, these estimates are usually not as smooth as one would like because, just as in the case where reverse projection is used to estimate birth rates by single years, they are derived from enumerated children classified by single year of age, so that differential completeness of enumeration, age-misreporting and age-heaping will affect them substantially. Hence, it is not unusual to find that the fertility rates estimated for the year immediately preceding the time of enumeration are too low or that those obtained from children aged 5 or 10 are too high. Averaging the results that refer to contiguous age groups is a way of reducing the effects of age-heaping.

The own-children method is attractive because it permits the detailed estimation of fertility from data that are almost invariably collected by censuses and because it does not depend upon any assumptions about fertility trends and is not very sensitive to assumptions about recent changes in the level of mortality. However, it clearly requires detailed estimates of mortality for both children and females, referring at least to the decade preceding the time of enumeration. If only one set of mortality rates is used in reverse projection of the population, mortality is implicitly assumed to have remained constant during the period considered. Yet, the method itself does not require such an assumption and one is free to use different mortality schedules for different periods when evidence of changes in mortality exists.

The own-children method is also appealing because, in principle, it is capable of estimating fertility at different points in time and, therefore, of estimating trends as well as levels. The existence of age-heaping and differential completeness of enumeration, however, often frustrates these hopes. For instance, given the known deficiencies of census enumerations, it would be naïve to interpret the drop in fertility rates during the two or three years immediately preceding a census as an indication of the occurrence of a fertility decline. Such a spurious trend is more likely to be caused by the underenumeration of young children.

In practical applications, the own-children method provides a reasonable estimate of overall fertility level (especially when only the estimates derived from enumerated children above age 3 are considered) and a rough idea of trends. In populations where age-reporting is accurate (those of Chinese origin, for example), it performs remarkably well. In traditional applications of the own-children method (where the child-mother linkage has to be performed indirectly), the estimates it yields tend to be affected by what is known as the "grandmother effect" caused by the erroneous allocation of children to their grandmothers. This type of error is reduced when a direct question linking children to their mothers is asked.

The own-children method was first developed to study fertility differentials between different subgroups of a population;[4] and it remains an important tool for studies of this kind. However, in using it for this purpose one must bear in mind two points: first, if there is reason to believe that the subgroups considered are subject to different mortality levels or patterns, it is important to incorporate different mortality estimates for each subgroup in the application of the method; secondly, if the subpopulations under study are not closed, careful interpretation of the results obtained is required. For example, consider the estimation of fertility by the own-children method for the rural and urban populations of a country at a given point in time, t. When one reverse-projects the urban population of time t to time $t-a$, say, the result does not necessarily represent the urban population at time $t-a$. Both would be the same only if the urban population had been closed between time $t-a$ and time t, or if there were no fertility or mortality differentials between the urban and rural populations of the country in question.

Lastly, it must be pointed out that the own-children method of fertility estimation provides valuable independent estimates of total fertility that may be used to assess the quality or plausibility of those obtained by other means.

2. *Data required*

The data required for this method are described below:

(*a*) The enumerated children (persons under 15) whose mother was identified, classified according to single year of own age and single year of age of mother;

(*b*) Children (persons under 15) whose mother could not be identified, classified by single year of age;

(*c*) All women (irrespective of whether they are mothers), classified by single year of age. In general, these data are needed only for the age range from η to $\nu+\gamma$, where η and ν are the lower and upper limits of the reproductive age span, respectively, and γ is the number of age groups used in classifying the children. If one assumes that η and γ equal 15, and ν equals 49, the

[4] Lee-Jay Cho, Wilson H. Grabill and Donald J. Bogue. *Differential Current Fertility in the United States* (Chicago, University of Chicago, Community and Family Study Center, 1971).

number of women aged from 15 to 64, classified by single year of age, is sufficient;

(*d*) Estimates of child survivorship. In fact, the probabilities of surviving from birth to the age group from x to $x+1$, ${}_1L_x$, for $x = 1, 2, ..., 15$, and for the 15 or so years preceding the point of enumeration are required. In practice, however, a complete set of ${}_1L_x$ is rarely available. Therefore, the computational procedure described below includes some steps for calculating these probabilities from other indirect information;

(*e*) Estimates of female adult mortality. Again, the probabilities of surviving from age η to exact age x, for $x = \eta + 1, \eta + 2, ..., \nu + \gamma$, and for the 15 years or so preceding the time of enumeration are required. Methods for calculating them from other indirect information are described below.

3. *Computational procedure*

The steps of the computational procedure are given below:

Step 1: Redistribution of children with unidentified mother. This step can only be performed if the children whose mother could not be identified at the matching stage (probably because the mother had died or because she did not live in the same household as her child or children) are tabulated by single year of age. In general, it is important to incorporate these children in the application of the own-children method, since their elimination will certainly lead to underestimates of fertility levels. The purpose of this step is to estimate the probable distribution of the unmatched children according to age of mother on the basis of information on children whose mother could be identified.

If one denotes by U_x the number of unmatched children aged x, and by C_x^a the children aged x whose mother's age at the time of enumeration was a, then an estimate of U_x^a, the number of unmatched children whose mother's age was a can be obtained by

$$U_x^a = C_x^a U_x / \sum_{a=\eta}^{\nu+\gamma} C_x^a \tag{C.1}$$

where η and ν are the lower and upper limits of the childbearing ages, respectively, and γ is the number of age groups used in classifying the children.

If one extends the notation used so that

$$C_x = \sum_{a=\eta}^{\nu+\gamma} C_x^a, \tag{C.2}$$

equation (C.1) can be rewritten as

$$U_x^a / U_x = C_x^a / C_x, \tag{C.3}$$

that is, it states algebraically that the distribution of unmatched children of age x according to age of mother is identical to that observed among children of age x whose mothers were identified. Although this identity may not hold exactly in reality, it seems the most reasonable approximation given the data available.

Step 2: estimation of survivorship probabilities for children. The probabilities of surviving from birth to the age group from x to $x+1$ for $x = 0, 1, 2, ..., (\gamma-1)$, denoted by ${}_1L_x$, are needed to reverse-project the enumerated children to the time of their birth. According to the data available, estimates of these probabilities may be obtained by a variety of procedures, only one of which is described here. It assumes that a good estimate of the mean level of child mortality prevalent in the population under study during the 15 years or so preceding enumeration is available (see chapter III). This estimate may be expressed either as a level within any of the Coale-Demeny families of model life tables, or as a pair of parameters α and β defining a life table in one of the four logit systems generated from level 16 female life tables of the Coale-Demeny models (see chapter I, subsection B.4).

When a Coale-Demeny level is provided as an estimate of child mortality, it is possible to transform it into equivalent ${}_1L_x$ values by identifying the α and β parameters that define that level within the logit system generated by female level 16 of the life-table family selected. Table 159 presents the α and β values that identify the printed male and female levels of the life tables for the 0-15 age range. Linear interpolation between these values makes the identification of α and β straightforward.

Using these α and β values in conjunction with the suitably interpolated standard ($l(x)$ values by single year of age for female level 16 of each of the Coale-Demeny families of model life tables are listed in annex XI) and the inverse logit transformation, an estimated life table $l^*(x)$ by single year can be calculated and the desired ${}_1L_x$ values obtained.

When estimated child mortality is already expressed as a pair of α and β values, only the inversion of the logit transformation is necessary to obtain $l^*(x)$.

Although this procedure yields ${}_1L_x$ values that are slightly different from those which would be obtained by direct interpolation between the different levels of the model life tables themselves, it does not require an unmanageably large set of constants that would make its computer implementation awkward. Furthermore, since the standard used has already been interpolated by single year of age, derivation of $l^*(x)$ estimates by single year is relatively simple. These reasons recommend it for inclusion here.

Step 3: estimation of survivorship probabilities for adult females. In order to reverse-project the female population to each of the γ years preceding the time of enumeration, survivorship probabilities of the type ${}_1L_a / {}_1L_{a-x}$ for $0 \leqslant x \leqslant \gamma$ and $\eta \leqslant a \leqslant \nu+\gamma$ are required. Just as in the case of children, several procedures may be used to estimate the set of female ${}_1L_x$ values by single year of age. The one presented here is analogous to that described above for children, and it also assumes that the mean level of adult mortality prevalent in the female population under consideration during the 10 or 15 years preceding enumeration is known (the estimation methods described in chapters IV and V may be used to

TABLE 159. VALUES OF PARAMETERS α AND β DETERMINING THE COALE-DEMENY MODEL LIFE TABLES FOR CHILDHOOD IN THE LOGIT SYSTEM GENERATED BY FEMALE LEVEL 16 IN THE CORRESPONDING FAMILY

	North model				South model			
	Females		Males		Females		Males	
Level (1)	Parameter α (2)	Parameter β (3)	Parameter α (4)	Parameter β (5)	Parameter α (6)	Parameter β (7)	Parameter α (8)	Parameter β (9)
1	1.5228	1.5134	1.5159	1.4177	1.9152	2.1520	1.7322	1.9218
2	1.4007	1.4670	1.3896	1.3715	1.7623	2.0593	1.5933	1.8436
3	1.2892	1.4264	1.2749	1.3312	1.6225	1.9762	1.4660	1.7734
4	1.1857	1.3904	1.1690	1.2957	1.4924	1.9002	1.3475	1.7091
5	1.0882	1.3578	1.0698	1.2636	1.3693	1.8292	1.2354	1.6491
6	0.9952	1.3277	0.9755	1.2343	1.2512	1.7617	1.1281	1.5921
7	0.9054	1.2998	0.8849	1.2071	1.1364	1.6962	1.0240	1.5371
8	0.8177	1.2734	0.7969	1.1815	1.0235	1.6319	0.9221	1.4832
9	0.7312	1.2479	0.7104	1.1571	0.9111	1.5677	0.8210	1.4296
10	0.6450	1.2232	0.6246	1.1333	0.7921	1.4963	0.7257	1.3797
11	0.5580	1.1985	0.5385	1.1100	0.6645	1.4145	0.6084	1.3076
12	0.4695	1.1737	0.4513	1.0865	0.5376	1.3345	0.4921	1.2374
13	0.3596	1.1330	0.3461	1.0503	0.4098	1.2548	0.3757	1.1684
14	0.2411	1.0862	0.2390	1.0112	0.2790	1.1738	0.2576	1.0990
15	0.1221	1.0425	0.1296	0.9733	0.1433	1.0896	0.1361	1.0279
16	0.0000	0.9999	0.0194	0.9399	0.0000	1.0000	0.0096	0.9534
17	−0.1287	0.9564	−0.0974	0.9053	−0.1542	0.9023	−0.1246	0.8736
18	−0.2683	0.9093	−0.2242	0.8676	−0.3245	0.7920	−0.2708	0.7850
19	−0.4251	0.8552	−0.3666	0.8235	−0.4805	0.7055	−0.4555	0.6749
20	−0.6115	0.7862	−0.5332	0.7679	−0.6413	0.6200	−0.6073	0.5980
21	−0.8490	0.6882	−0.7402	0.6906	−0.8107	0.5345	−0.7683	0.5201
22	−1.1956	0.5211	−1.0234	0.5669	−0.9917	0.4484	−0.9406	0.4419
23	−1.4662	0.4285	−1.2580	0.4977	−1.1863	0.3636	−1.1277	0.3632
24	−1.7829	0.3389	−1.5493	0.4207	−1.3997	0.2805	−1.3339	0.2856

	East model				West model			
	Females		Males		Females		Males	
Level	Parameter α (10)	Parameter β (11)	Parameter α (12)	Parameter β (13)	Parameter α (14)	Parameter β (15)	Parameter α (16)	Parameter β (17)
1	1.6288	1.6374	1.5657	1.4345	1.5841	1.5351	1.4869	1.3604
2	1.5087	1.5919	1.4406	1.3937	1.4627	1.4915	1.3656	1.3210
3	1.3973	1.5506	1.3260	1.3566	1.3510	1.4528	1.2546	1.2860
4	1.2926	1.5124	1.2192	1.3225	1.2466	1.4178	1.1512	1.2543
5	1.1927	1.4766	1.1182	1.2906	1.1477	1.3856	1.0536	1.2252
6	1.0962	1.4425	1.0214	1.2604	1.0527	1.3556	0.9604	1.1981
7	1.0022	1.4097	0.9276	1.2313	0.9604	1.3270	0.8701	1.1723
8	0.9092	1.3773	0.8356	1.2029	0.8696	1.2994	0.7817	1.1475
9	0.8164	1.3450	0.7445	1.1746	0.7794	1.2724	0.6941	1.1231
10	0.7227	1.3121	0.6530	1.1461	0.6886	1.2453	0.6064	1.0987
11	0.6111	1.2634	0.5442	1.1017	0.5962	1.2177	0.5178	1.0741
12	0.4938	1.2095	0.4378	1.0587	0.5010	1.1891	0.4270	1.0486
13	0.3762	1.1577	0.3307	1.0173	0.3950	1.1528	0.3042	1.0006
14	0.2563	1.1066	0.2213	0.9763	0.2576	1.0939	0.1898	0.9608
15	0.1320	1.0548	0.1075	0.9344	0.1315	1.0469	0.0812	0.9243
16	−0.0002	0.9998	−0.0132	0.8901	−0.0002	0.9998	−0.0335	0.8871
17	−0.1436	0.9403	−0.1443	0.8411	−0.1430	0.9487	−0.1552	0.8498
18	−0.3066	0.8692	−0.2902	0.7851	−0.3024	0.8904	−0.2913	0.8064
19	−0.5002	0.7782	−0.4591	0.7164	−0.4904	0.8174	−0.4495	0.7522
20	−0.7449	0.6516	−0.6673	0.6219	−0.7269	0.7161	−0.6420	0.6794
21	−0.9617	0.5743	−0.8465	0.5734	−0.9943	0.6105	−0.8531	0.6106
22	−1.1980	0.4951	−1.0589	0.5131	−1.2530	0.5302	−1.0701	0.5550
23	−1.4724	0.4107	−1.3061	0.4497	−1.5638	0.4421	−1.3340	0.4910
24	−1.7961	0.3254	−1.6041	0.3816	−1.9451	0.3475	−1.6616	0.4211

ascertain this level). Estimates of female adult mortality can be expressed as a level within a given family of the Coale-Demeny life tables, or as a pair of α and β parameters identifying a life table within one of the four logit systems generated by the four female level 16 life tables of the Coale-Demeny families. The transformation of such mortality estimates into single-year ${}_1L_x$ values is carried out as described in step 2, but the identification of the α and β parameters on the basis of a given Coale-Demeny level must be performed by using table 160, specifically designed to cover age range 10-64 for females. Further details about this calculation procedure are presented in the example given below.

Step 4: reverse survival of children. Just as in the case of the simple reverse-survival method, the number of children born in year $t-x$ (t being the time of enumeration) to women aged $a-x$ is estimated by

$$M_{t-x}^{a-x} = (C_x^a + U_x^a)/{}_1L_x. \qquad \text{(C.4)}$$

TABLE 160. VALUES OF PARAMETERS α AND β DETERMINING THE COALE-DEMENY MODEL LIFE TABLES FOR ADULT FEMALES IN THE LOGIT SYSTEM GENERATED BY FEMALE LEVEL 16 IN THE CORRESPONDING FAMILY

	North model		South model		East model		West model	
Level (1)	Parameter α (2)	Parameter β (3)	Parameter α (4)	Parameter β (5)	Parameter α (6)	Parameter β (7)	Parameter α (8)	Parameter β (9)
1	1.3129	1.2924	1.3433	1.3948	1.2698	1.2146	1.3431	1.2829
2	1.1827	1.2331	1.2148	1.3328	1.1495	1.1673	1.2112	1.2248
3	1.0654	1.1829	1.0984	1.2796	1.0400	1.1268	1.0922	1.1757
4	0.9582	1.1401	0.9916	1.2335	0.9389	1.0919	0.9833	1.1337
5	0.8590	1.1033	0.8924	1.1932	0.8445	1.0618	0.8825	1.0978
6	0.7662	1.0718	0.7991	1.1578	0.7556	1.0359	0.7881	1.0671
7	0.6785	1.0447	0.7107	1.1268	0.6711	1.0139	0.6988	1.0408
8	0.5949	1.0216	0.6260	1.0995	0.5900	0.9952	0.6137	1.0186
9	0.5146	1.0022	0.5443	1.0755	0.5117	0.9799	0.5318	1.0001
10	0.4367	0.9862	0.4663	1.0572	0.4355	0.9677	0.4523	0.9852
11	0.3608	0.9735	0.3896	1.0434	0.3637	0.9644	0.3747	0.9737
12	0.2862	0.9641	0.3133	1.0312	0.2928	0.9664	0.2983	0.9658
13	0.2165	0.9680	0.2369	1.0208	0.2215	0.9703	0.2232	0.9631
14	0.1463	0.9767	0.1599	1.0122	0.1493	0.9768	0.1532	0.9759
15	0.0745	0.9873	0.0816	1.0059	0.0757	0.9864	0.0783	0.9867
16	0.0006	1.0007	0.0015	1.0021	0.0003	1.0003	0.0012	1.0015
17	−0.0757	1.0180	−0.0815	1.0015	−0.0777	1.0202	−0.0787	1.0218
18	−0.1552	1.0412	−0.1677	1.0067	−0.1586	1.0486	−0.1622	1.0502
19	−0.2383	1.0733	−0.2602	1.0107	−0.2432	1.0901	−0.2500	1.0911
20	−0.3258	1.1199	−0.3606	1.0105	−0.3320	1.1527	−0.3429	1.1531
21	−0.4180	1.1915	−0.4700	1.0107	−0.4317	1.2259	−0.4461	1.2397
22	−0.5137	1.3166	−0.5914	1.0109	−0.5479	1.2997	−0.5697	1.3245
23	−0.6440	1.4082	−0.7290	1.0105	−0.6815	1.3966	−0.7133	1.4410
24	−0.7919	1.5299	−0.8889	1.0084	−0.8394	1.5284	−0.8866	1.6056

Note that the number of unmatched children whose mother was estimated to be aged a at the time of enumeration must be added to the children whose mother was identified and aged a before the whole is reverse-survived. Note also that equation (C.4) must be applied for x ranging from 0 to $(\gamma-1)$ and for all age groups of mother $(\eta \leq a \leq \nu+\gamma)$.

The quantity M_{t-x}^{a-x} represents the number of births occurring during a year $(t-x-1, t-x)$ to women who are between exact ages a and $a+1$ at time t. Therefore, M_{t-x}^{a-x} are births to women whose ages ranged between $a-x-1$ and $a-x+1$ (exclusive) during the year $(t-x-1, t-x)$, so that some adjustment is necessary to obtain the births occurring during that year to women whose ages ranged only between $a-x$ and $a-x+1$. The simplest type of adjustment consists of taking the average of M_{t-x}^{a+1-x} and M_{t-x}^{a-x} to represent the desired number of births. This average is denoted by B_{t-x}^{a-x}.

Step 5: reverse survival of adult females. Women aged a at time t, W_t^a, must be reverse-survived to times $t-x$, for $0 \leq x \leq \gamma$. The equation used in this case is

$$W_{t-x}^{a-x} = W_t^a({}_1L_{a-x}/{}_1L_a) \qquad (C.5)$$

or, equivalently,

$$W_{t-x}^{a-x} = {}_1L_{a-x}(W_t^a/{}_1L_a). \qquad (C.6)$$

Thus, when one is performing the calculations by hand, it is easier to calculate first the set of $W_t^a/{}_1L_a$ values and then to multiply them by ${}_1L_{a-x}$ for x ranging from 0 to γ.

Note that the value of W_{t-x}^{a-x} obtained in this way represents the number of women whose ages ranged between $a-x$ and $a-x+1$ (exclusive) at exact time $t-x$. Yet, the number of women that should be used as denominator for B_{t-x}^{a-x} in calculating age-specific fertility rates is the mid-period female population during the year $(t-x-1,\ t-x)$. This population, denoted by N_{t-x}^{a-x}, can be estimated, as usual, by

$$N_{t-x}^{a-x} = (W_{t-x-1}^{a-x} + W_{t-x}^{a-x})/2.0. \qquad (C.7)$$

Step 6: calculation of age-specific fertility rates. The calculation of fertility rates specific by single year of age and time period is now straightforward, since

$$f_{t-x}(a) = \frac{B_{t-x}^a}{N_{t-x}^a} \qquad (C.8)$$

where $f_{t-x}(a)$ denotes the fertility rate corresponding to age a during the year $(t-x-1, t-x)$. However, due to age-reporting errors, rates by single year are likely to be erratic. Furthermore, since other methods of estimation usually yield fertility rates specific by five-year age group, it may be necessary, for comparison purposes, to calculate conventional five-year age group estimates. This calculation is carried out by cumulating the single-year fertility schedules just estimated to obtain $F_{t-x}(a)$, that is,

$$F_{t-x}(a) = \sum_{y=\eta}^{a-1} f_{t-x}(y) \qquad (C.9)$$

and then calculating the usual five-year estimates, denoted by $f_{t-x}(i)$, by differencing

$$f_{t-x}(i) = (F(10+5(i+1)) - F(10+5i))/5.0$$
$$\text{for } i = 1, 2, \ldots, 8. \qquad (C.10)$$

TABLE 161. OWN-CHILDREN DATA, WITH CHILDREN CLASSIFIED BY SINGLE YEAR OF AGE AND SINGLE YEAR AGE OF MOTHER, COLOMBIA, 1978

Age of mother	Number of children, by age of child																Number of women
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
15	13	7	0	2	2	2	1	4	3	4	2	1	3	1	3	3	755
16	12	3	0	2	0	2	1	1	1	0	0	0	2	0	0	1	696
17	23	16	6	1	0	0	0	0	0	0	1	0	0	0	0	0	686
18	58	36	17	3	0	3	1	2	0	0	0	0	0	0	0	0	706
19	66	46	24	13	11	1	3	1	0	0	0	0	1	0	0	1	538
20	77	55	45	33	19	12	2	1	0	2	2	1	1	1	0	0	602
21	78	71	56	47	48	17	7	5	3	0	1	2	1	1	0	0	488
22	84	80	76	73	46	26	18	15	3	0	0	0	0	0	1	0	534
23	84	85	80	84	61	53	29	24	7	9	1	2	0	2	1	0	488
24	93	63	78	72	56	48	45	34	17	9	8	3	0	1	1	1	411
25	91	84	87	83	69	71	55	52	31	21	5	5	2	1	0	0	464
26	73	67	65	70	66	70	61	55	41	24	17	11	1	1	2	0	393
27	58	61	70	58	63	79	64	64	47	28	27	16	11	5	2	1	339
28	83	71	77	81	94	80	87	91	80	60	42	34	16	8	3	2	442
29	48	58	52	59	68	64	77	75	61	66	48	50	23	23	6	4	330
30	46	60	70	62	82	86	86	86	82	74	69	50	45	31	20	8	403
31	42	39	42	36	44	44	55	66	63	56	57	46	43	24	12	8	243
32	45	50	67	54	66	65	73	82	79	91	78	64	63	66	38	30	343
33	51	33	37	51	46	63	53	58	65	66	65	67	55	50	58	24	272
34	34	34	42	50	46	48	50	72	62	74	68	67	65	46	57	31	257
35	33	29	33	30	58	55	60	51	80	63	73	61	77	63	66	40	317
36	31	28	40	36	39	43	40	61	71	54	70	61	83	73	69	61	272
37	28	23	37	44	43	47	49	60	61	66	74	79	80	79	74	68	291
38	25	35	28	38	49	49	56	66	65	69	69	70	85	65	87	85	345
39	17	21	25	22	38	25	38	49	48	40	52	50	62	55	68	59	216
40	31	19	33	35	44	39	49	70	65	57	66	75	79	77	80	74	347
41	8	7	22	22	22	22	27	32	30	39	38	32	46	40	46	45	173
42	5	6	11	23	18	26	26	43	38	44	50	33	62	68	47	75	242
43	13	9	15	22	21	31	37	42	46	36	60	59	50	47	66	73	221
44	2	5	1	14	17	15	28	26	24	34	43	38	43	46	42	53	182
45	6	2	9	10	16	19	31	24	42	33	52	45	77	60	55	70	287
46	4	4	6	5	11	14	15	13	31	18	29	23	31	33	36	36	158
47	2	3	1	9	13	19	24	32	38	32	28	36	52	40	40	50	215
48	3	4	4	7	7	18	22	28	44	37	40	41	58	59	47	59	263
49	0	0	2	3	2	7	8	16	15	19	21	23	40	41	39	32	156
50	3	0	2	0	4	6	7	12	11	20	29	29	43	41	47	41	302
51	1	1	1	0	2	2	2	3	10	11	11	14	25	25	21	22	127
52	0	1	1	2	2	3	7	6	8	9	12	19	22	12	29	26	169
53	0	0	0	0	0	4	1	2	2	6	5	12	12	20	19	22	150
54	0	0	0	1	2	2	4	4	3	4	9	10	14	15	19	19	146
55	1	0	1	1	0	2	1	2	1	5	4	8	10	13	17	24	183
56	0	0	0	0	0	1	1	2	1	1	3	4	3	9	8	11	133
57	0	0	0	0	1	0	0	0	2	1	2	2	3	4	9	15	138
58	0	0	0	0	0	1	1	3	1	7	4	4	2	6	10	14	126
59	0	0	0	0	0	1	0	0	1	0	1	1	2	1	2	4	82
60	0	0	0	0	0	1	1	1	1	1	2	1	3	4	6	5	207
61	0	0	0	0	0	0	0	0	0	1	0	0	1	2	1	2	86
62	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	3	107
63	0	0	0	0	0	0	2	0	0	0	2	0	0	1	1	2	90
64	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	77
Not known[a]	33	50	64	94	101	110	145	155	145	133	171	161	244	212	226	250	-
TOTAL	1 405	1 266	1 327	1 352	1 397	1 396	1 450	1 593	1 529	1 424	1 511	1 410	1 641	1 472	1 482	1 455	-

[a] Age of mother could not be determined.

In some cases, errors in the reported ages of young children may produce sequences of five-year age group fertility rates that are still fairly erratic. To obtain a smoother sequence of estimates, averages of the rates for adjacent years are often calculated. Their calculation is illustrated in the detailed example given below.

4. *A detailed example*

The data presented in table 161 serve to illustrate the application of the own-children method of fertility estimation. They were obtained from a survey conducted in Colombia in 1978. That survey included a question on identification of mother at the household level. (The data shown in table 161 have not been weighted.)

Step 1: redistribution of children with unidentified mother. The penultimate row of table 161 gives the number of children whose mother could not be found in the household in which they were enumerated. They are to be redistributed over the columns above them (representing the distribution of all other children with respect to mother's age). The last row of table 161 gives the totals of each column. In the notation used above, these totals equal $C_x + U_x$ for $0 < x < 15$. According to equation (C.3),

$$U_x^a = C_x^a(U_x / C_x), \qquad (C.11)$$

and since, as noted in step 4 of the computational procedure, the number of children who are to be reverse-projected to birth is $C_x^a + U_x^a$, this quantity can be expressed as

$$C_x^a + U_x^a = C_x^a(1.0 + U_x / C_x), \qquad (C.12)$$

that is, for a given age of children (a column in table 161), the desired $C_x^a + U_x^a$ values can be obtained merely by multiplying every entry C_x^a by the factor $K_x = (1.0 + U_x / C_x)$.

For example, consider children aged 8 years. According to table 161, a total of 1,529 children aged 8 were enumerated. Of these, 145 had no mother present. Therefore, those with identified mother amount to 1,384 $= C_x$. Hence,

$$K_x = 1.0 + U_x / C_x = 1.0 + 145/1{,}384 = 1.1048.$$

Values of K_x for $0 < x < 15$ are given in table 162. When making the calculations by hand, it is not necessary to calculate the U_x^a values proper. It suffices to use the factors K_x to adjust at a later stage (step 4) the estimated number of births resulting from the reverse survival of children whose mothers were identified.

Step 2: estimation of survivorship probabilities for children. Assume that by other methods based on independent data sources (such as children ever born and surviving) it is known that the level of child mortality prevalent among the Colombian population during the period 1963-1978 is approximately equal to 17.2 in the North family of Coale-Demeny models for both sexes combined. According to table 159, the values of parameters α and β associated with levels 17 and 18, North model, sex female are

$$\alpha_{17}^f = -0.1287 \quad \text{and} \quad \beta_{17}^f = 0.9564;$$

$$\alpha_{18}^f = -0.2683 \quad \text{and} \quad \beta_{18}^f = 0.9093.$$

Thus, the α and β values associated with level 17.2 for females can be obtained by interpolation as follows:

$$\alpha_{17.2}^f = (0.2)(-0.2683) + (0.8)(-0.1287) = -0.1566$$

and

$$\beta_{17.2}^f = (0.2)(0.9093) + (0.8)(0.9564) = 0.9470.$$

Following the same procedure in the case of males, values of α and β for males for level 17.2 are estimated as

$$\alpha_{17.2}^m = -0.1228 \quad \text{and} \quad \beta_{17.2}^m = 0.8978.$$

Using these values of the α and β parameters and the standard selected, level 16 for females in the North family of model life tables (see annex XI), $l(x)$ values for ages from 1 to 16 can be calculated using the inverse logit transformation (see chapter I, subsection B.4). The equations needed are

$$\lambda(x) = \alpha + \beta\,\lambda_s(x) \qquad (C.13)$$

and

$$l(x) = [1.0 + \exp(2.0\,\lambda(x))]^{-1} \qquad (C.14)$$

where $\lambda(x)$ denotes the logit transformation of $l(x)$; and equation (C.14) defines the inverse of the logit transformation. The subindex s is used to denote the standard.

For example, to estimate $l(2)$ for females, one notes first that $\lambda_s(2) = -1.1332$ (copied from annex XI). Therefore,

$$\lambda_f(2) = -0.1566 + 0.9470(-1.1332) = -1.2297,$$

so that

$$l_f(2) = \frac{1.0}{1.0 + \exp(-2.4594)} = 0.9213.$$

In an analogous way, $l(2)$ for males may be calculated as follows:

$$\lambda_m(2) = -0.1228 + 0.8978(-1.1332) = -1.1402,$$

so

$$l_m(2) = \frac{1.0}{1.0 + \exp(-2.2804)} = 0.9072.$$

TABLE 162. VALUES OF THE EXPANSION FACTORS, K_x, USED IN ADJUSTMENT OF OWN-CHILDREN DATA FOR THE EXISTENCE OF CHILDREN WITHOUT MOTHER, COLOMBIA, 1978

Age of child x (1)	Expansion factor K_x (2)
0	1.0241
1	1.0411
2	1.0507
3	1.0747
4	1.0779
5	1.0855
6	1.1111
7	1.1078
8	1.1048
9	1.1030
10	1.1276
11	1.1289
12	1.1747
13	1.1683
14	1.1799
15	1.2075

Hence $l_b(2)$, the probability of surviving from birth to exact age 2 for both sexes combined may be calculated as

$$l_b(2) = \frac{(1.05)(0.9072)+(0.9213)}{2.05} = 0.9141,$$

assuming that the sex ratio at birth is 1.05 males per female.

Table 163 gives other $l(x)$ values for males, females and both sexes combined. Once these values have been calculated, the desired survivorship probabilities, ${}_1L_x$, are obtained by averaging contiguous $l(x)$ values. For example,

$${}_1L_7 = \frac{l(7)+l(8)}{2.0} = \frac{0.8819+0.8786}{2.0} = 0.8803.$$

TABLE 163. ESTIMATES OF PROBABILITIES OF SURVIVING, $l(x)$ AND ${}_1L_x$, FOR CHILDREN, NORTH MODEL, COLOMBIA, 1978

	Probability of surviving to exact age x			*Probability of surviving to age group from x to x + 1, both sexes*
Age x	*Female l(x) (2)*	*Male l(x) (3)*	*Both sexes l(x) (4)*	${}_1L_x$ *(5)*
0	1.0000	1.0000	1.0000	0.9478
1	0.9365	0.9242	0.9302	0.9222
2	0.9213	0.9072	0.9141	0.9090
3	0.9114	0.8965	0.9038	0.8999
4	0.9038	0.8882	0.8959	0.8928
5	0.8979	0.8818	0.8897	0.8877
6	0.8939	0.8776	0.8856	0.8838
7	0.8904	0.8738	0.8819	0.8803
8	0.8872	0.8704	0.8786	0.8772
9	0.8845	0.8674	0.8757	0.8745
10	0.8820	0.8648	0.8732	0.8721
11	0.8799	0.8625	0.8710	0.8700
12	0.8779	0.8605	0.8690	0.8681
13	0.8762	0.8586	0.8672	0.8663
14	0.8745	0.8568	0.8654	0.8646
15	0.8728	0.8551	0.8637	0.8627
16	0.8709	0.8530	0.8617	-

The only exception is ${}_1L_0$, the calculation of which requires an estimate of the separation factor for deaths under age one. This factor can be estimated using the equations proposed by Coale and Demeny[5]; the form and coefficients for these equations are shown in table 164. In the case at hand, $q_f(1) = 1.0 - l_f(1) = 0.0635$ and $q_m(1) = 0.0758$, so both are less than 0.100. Hence, the coefficients for case A are used to estimate S for each sex separately as follows:

$$S_f = 0.05 + 3.0(0.0635) = 0.2405$$

$$S_m = 0.0425 + 2.875(0.0758) = 0.2604,$$

so that

$$\begin{aligned} {}_1L_0^f &= S_f + (1.0 - S_f)(l_f(1)) \\ &= 0.2405 + (1.0 - 0.2405)(0.9365) \\ &= 0.9518 \end{aligned}$$

$$\begin{aligned} {}_1L_0^m &= S_m + (1.0 - S_m)(l_m(1)) \\ &= 0.2604 + (1.0 - 0.2604)(0.9242) \\ &= 0.9439, \end{aligned}$$

and

$$\begin{aligned} {}_1L_0^b &= [(1.05)(0.9439) + 0.9518]/2.05 \\ &= 0.9478. \end{aligned}$$

This and other values of ${}_1L_x$ for both sexes combined are given in column (5) of table 163.

TABLE 164. COEFFICIENTS FOR ESTIMATION OF SEPARATION FACTORS FOR AGE GROUP 0-1, COALE-DEMENY MODEL LIFE TABLES

	Females		*Males*	
	Coefficients		*Coefficients*	
Model (1)	a_1 *(2)*	b_1 *(3)*	a_1 *(4)*	b_1 *(5)*
Case A: q(1) < 0.100				
North, South and West	0.0500	3.000	0.0425	2.875
East	0.0100	3.000	0.0025	2.875
Case B: q(1) ≥ 0.100				
North, South and West	0.3500	0.000	0.3300	0.000
East	0.3500	0.000	0.2900	0.000

Equation: $S = a_1 + b_1 q(1)$

Step 3: estimation of survivorship probabilities for adult females. The procedure followed in estimating these probabilities is very similar to that illustrated in the previous step. The main difference is that only females need to be considered and the translation of a given mortality level into values of α and β is made by using table 160. In the case at hand, assume that independent information (orphanhood data, for example) has shown that female adult mortality in Colombia during the period of interest (1963-1978) is well represented by level 16.6 of the West model life tables. Then, according to table 160, the α and β values associated with levels 16 and 17 are, respectively,

$$\alpha_{16} = 0.0012 \quad \text{and} \quad \beta_{16} = 1.0015;$$

$$\alpha_{17} = -0.0787 \quad \text{and} \quad \beta_{17} = 1.0218.$$

The values corresponding to level 16.6 are obtained by interpolation as follows:

$$\alpha_{16.6} = 0.4(0.0012) + 0.6(-0.0787) = -0.0467,$$

$$\beta_{16.6} = 0.4(1.0015) + 0.6(1.0218) = 1.0137.$$

[5] *Op. cit.*

Once these α and β values have been calculated, equations (C.13) and (C.14) can be used to estimate $l(x)$ by single year of age. For example, for $x = 35$, annex XI shows that $\lambda_s(35) = -0.6661$ for the West model. Hence,

$$\lambda(35) = -0.0467 + 1.0137(-0.6661) = -0.7219$$

and

$$l(35) = \frac{1.0}{1.0 + \exp(2.0(-0.7219))} = 0.8091.$$

Other values of $l(x)$ are shown in columns (2) and (5) of table 165. From these values, ${}_1L_x$ values are calculated by taking successive averages. For example,

$${}_1L_{35} = (0.8091 + 0.8048)/2.0 = 0.8070.$$

The complete sequence of L_x values is given in columns (3) and (6) of table 165.

Step 4: reverse survival of children. Using the C_x^a values shown in table 161 and the ${}_1L_x$ estimates presented in table 163, the reverse-projection of enumerated children is straightforward. For example, to estimate the births occurring in 1970 to women aged 25, one proceeds as follows:

$$M_{1970}^{25} = K_8(C_8^{33}/{}_1L_8) = 1.1048(65/0.8772) = 81.87,$$

$$M_{1970}^{26} = K_8(C_8^{34}/{}_1L_8) = 1.1048(62/0.8772) = 78.09,$$

so that

$$B_{1970}^{25} = (M_{1970}^{25} + M_{1970}^{26})/2.0$$

$$= (81.87 + 78.09)/2.0 = 79.98.$$

The full set of B_{t-x}^a values is given in table 166. Note, however, that the headings used in table 166 do not correspond exactly to the notation used so far. The B_{1970}^a estimates, for example, appear in the column labeled "1969/70". This heading is used to suggest explicitly that the births being estimated occurred during parts of both the 1969 and the 1970 calendar years. The exact portions involved depend upon the date on which enumeration (the survey in this case) took place in 1978. Only if enumeration had taken place 31 December 1978 would the B_{1970}^a estimates refer exclusively to 1970. Although the table headings used may be slightly confusing for the beginner, they have been adopted because they match those appearing in the output of computer programs that implement the own-children method and because, in fact, they are more accurate than the algebraic notation being used. The latter does not refer explicitly to two contiguous calendar years in each case only because such practice would render it even more awkward and imposing.

Lastly, note should be taken that in table 166 the entries for women aged 14 are not the result of averaging births belonging to two different age groups. They are just the values of M_{t-x}^{a-x} when $a - x = 15$. Since they represent boundary values, there is always some uncer-

TABLE 165. ESTIMATES OF PROBABILITIES OF SURVIVING, $l(x)$ AND ${}_1L_x$, FOR ADULT FEMALES, WEST MODEL, COLOMBIA, 1978

Age x (1)	Probability of surviving to exact age x $l(x)$ (2)	Probability of surviving to age group from x to $x+1$ ${}_1L_x$ (3)	Age x (4)	Probability of surviving to exact age x $l(x)$ (5)	Probability of surviving to age group from x to $x+1$ ${}_1L_x$ (6)
15	0.8735	0.8725	41	0.7824	0.7800
16	0.8714	0.8703	42	0.7776	0.7751
17	0.8691	0.8679	43	0.7726	0.7701
18	0.8667	0.8655	44	0.7675	0.7649
19	0.8642	0.8629	45	0.7622	0.7595
20	0.8616	0.8602	46	0.7567	0.7539
21	0.8587	0.8573	47	0.7510	0.7480
22	0.8558	0.8542	48	0.7449	0.7418
23	0.8526	0.8510	49	0.7386	0.7353
24	0.8494	0.8478	50	0.7320	0.7285
25	0.8461	0.8445	51	0.7250	0.7214
26	0.8428	0.8411	52	0.7177	0.7138
27	0.8394	0.8380	53	0.7099	0.7059
28	0.8366	0.8345	54	0.7018	0.6975
29	0.8323	0.8305	55	0.6931	0.6885
30	0.8286	0.8268	56	0.6839	0.6791
31	0.8249	0.8230	57	0.6742	0.6690
32	0.8211	0.8192	58	0.6638	0.6583
33	0.8172	0.8152	59	0.6527	0.6469
34	0.8132	0.8112	60	0.6410	0.6348
35	0.8091	0.8070	61	0.6285	0.6218
36	0.8048	0.8027	62	0.6151	0.6081
37	0.8005	0.7983	63	0.6010	0.5935
38	0.7961	0.7939	64	0.5859	0.5779
39	0.7916	0.7893	65	0.5698	-
40	0.7870	0.7847			

TABLE 166. ESTIMATED NUMBER OF BIRTHS IN SINGLE YEAR PERIODS, BY SINGLE YEAR OF AGE OF MOTHER, COLOMBIA, 1963/64-1977/78

Age of mother at birth of child	Year of birth of child and estimated number of births, B^a_{t-x}														
	1963/1964	1964/1965	1965/1966	1966/1967	1967/1968	1968/1969	1969/1970	1970/1971	1971/1972	1972/1973	1973/1974	1974/1975	1975/1976	1976/1977	1977/1978
14	8.19	10.79	14.89	14.28	6.47	11.35	8.82	18.88	8.80	14.68	13.29	3.58	6.94	3.39	14.05
15	17.75	20.91	18.27	17.52	14.23	18.92	15.12	24.55	15.72	17.74	18.12	9.56	13.29	10.73	13.51
16	21.84	36.42	26.40	32.45	28.45	28.39	30.23	36.51	29.56	26.30	40.46	27.48	23.70	29.36	18.91
17	34.13	37.10	46.02	54.52	44.62	32.80	45.35	54.13	46.53	48.32	56.76	47.79	39.88	46.30	43.76
18	65.53	60.70	59.56	64.90	58.20	55.51	55.43	67.35	62.88	61.77	64.61	71.68	58.38	57.03	66.99
19	78.49	78.24	71.74	62.30	75.66	79.48	79.99	74.90	72.95	72.78	70.65	93.78	76.30	71.14	77.26
20	83.96	64.75	79.86	71.39	81.48	88.31	88.81	97.56	78.61	86.24	75.48	93.19	90.17	85.26	83.74
21	92.15	73.52	81.22	85.02	87.30	82.01	90.07	104.49	94.95	91.13	81.52	92.59	91.33	93.16	87.52
22	97.61	91.73	96.11	86.96	92.47	92.73	91.33	101.34	103.13	97.25	77.90	91.39	95.37	83.56	90.77
23	109.89	102.52	108.29	83.07	86.01	99.04	89.44	95.67	102.50	88.07	94.81	76.46	87.86	83.00	95.63
24	105.80	97.12	110.32	79.18	91.18	88.31	90.70	93.16	88.67	91.74	97.83	83.03	78.03	85.26	99.41
25	101.02	80.94	111.67	90.86	92.47	86.42	79.99	88.12	80.49	79.51	90.58	83.63	84.97	72.27	88.60
26	86.00	89.03	99.49	96.70	93.12	73.81	89.44	81.83	79.23	66.67	76.09	72.28	74.57	74.53	70.78
27	63.48	78.91	95.43	77.88	92.47	75.70	95.11	77.42	64.77	78.29	66.43	58.54	70.52	72.83	76.18
28	77.13	72.84	84.60	81.12	78.25	85.16	83.14	70.50	69.17	67.89	67.63	53.76	64.74	66.62	70.78
29	73.72	77.56	73.09	69.44	76.31	68.76	79.36	76.16	62.88	63.00	55.56	62.72	63.01	55.90	50.79
30	66.21	62.73	75.80	42.18	67.25	61.19	71.18	79.31	55.97	59.94	62.80	60.33	60.12	50.25	47.54
31	62.11	71.49	62.94	59.71	56.91	60.56	71.18	72.39	66.03	55.05	58.58	47.79	45.66	46.86	47.00
32	51.87	62.73	81.22	62.95	71.13	52.36	59.84	74.90	59.11	58.72	49.52	39.42	43.35	37.83	51.87
33	59.38	49.24	73.09	53.87	66.61	50.47	42.83	64.20	54.71	45.26	55.56	47.79	42.20	35.57	45.92
34	58.70	66.77	56.17	44.13	61.43	44.16	52.91	47.21	47.79	39.14	52.54	48.98	44.51	32.18	36.20
35	58.70	67.45	74.45	38.29	52.38	42.26	44.09	53.50	33.33	37.31	49.52	35.84	37.57	28.80	34.58
36	46.41	55.31	66.33	49.97	36.86	32.17	41.57	42.80	39.62	29.36	39.86	34.05	30.64	32.75	31.88
37	34.13	44.51	56.17	41.54	43.97	31.54	45.98	31.47	40.87	34.86	24.15	34.05	33.53	31.62	28.63
38	32.76	24.96	46.02	33.75	39.45	43.53	43.46	23.29	37.10	28.13	23.55	26.88	31.79	22.58	22.69
39	25.94	21.58	31.81	27.91	32.33	35.33	51.65	28.32	28.93	20.80	22.95	26.88	19.07	14.68	25.93
40	24.57	23.61	23.01	21.42	25.87	24.60	37.16	37.77	24.52	20.18	19.93	21.50	15.03	7.34	21.07
41	17.06	18.88	17.60	20.12	14.87	19.56	16.38	27.70	28.93	20.18	16.30	14.34	9.25	8.47	7.02
42	11.60	14.84	16.24	14.28	10.99	12.62	13.23	17.62	18.87	22.63	14.49	8.96	5.78	7.90	9.72
43	12.97	8.77	8.80	11.68	9.05	9.46	11.34	9.44	9.43	15.29	12.08	8.36	8.67	3.95	8.10
44	8.19	6.74	4.06	7.79	8.41	6.31	6.30	5.66	5.66	7.95	5.43	9.56	4.05	3.39	4.32
45	5.46	4.72	3.38	3.89	4.53	5.68	3.15	5.04	5.66	4.89	3.62	5.97	2.89	3.95	5.40
46	4.78	3.37	2.71	3.89	3.23	3.78	2.52	3.78	5.03	3.06	3.62	1.79	3.47	3.95	3.24
47	1.37	4.05	3.38	3.24	3.88	1.26	1.26	3.78	3.14	4.28	2.42	0.00	2.31	2.26	2.70
48	1.37	1.35	2.71	1.30	3.23	5.05	1.89	2.52	3.14	3.67	1.21	1.19	1.73	0.00	1.62
49	0.68	0.67	0.68	0.65	1.94	4.42	1.89	1.26	1.26	2.45	1.21	1.19	1.16	0.56	1.62

Note: The figures in this table were generated by computer. They may not coincide with those given in the text due to rounding and truncation in intermediate calculations.

TABLE 167. ESTIMATED NUMBER OF WOMEN IN SINGLE-YEAR PERIODS, BY SINGLE-YEAR AGE OF WOMAN, COLOMBIA, 1963/64-1977/78

Age of woman	Estimated number of women, N^a_{t-x}														
	1963/1964	1964/1965	1965/1966	1966/1967	1967/1968	1968/1969	1969/1970	1970/1971	1971/1972	1972/1973	1973/1974	1974/1975	1975/1976	1976/1977	1977/1978
14	405.24	408.38	381.10	444.44	452.11	462.60	523.96	522.15	554.84	578.52	629.14	702.12	695.17	727.96	756.64
15	385.94	404.36	407.49	380.27	443.48	451.13	461.60	522.82	521.02	553.63	577.27	627.78	700.59	693.66	726.38
16	340.54	384.96	403.34	406.47	379.31	442.36	449.99	460.43	521.51	519.70	552.23	575.81	626.19	698.83	691.91
17	309.85	339.65	383.95	402.28	405.39	378.31	441.19	448.80	459.22	520.13	518.33	550.78	574.29	624.54	696.98
18	325.61	308.99	338.71	382.89	401.16	404.27	377.26	439.97	447.56	457.95	518.69	516.89	549.25	572.70	622.81
19	280.66	324.63	308.06	337.69	381.73	399.96	403.06	376.13	438.65	446.21	456.57	517.13	515.34	547.60	570.98
20	305.21	279.75	323.57	307.06	336.59	380.50	398.66	401.75	374.91	437.22	444.77	455.09	515.45	513.67	545.83
21	313.61	304.18	278.81	322.48	306.03	335.46	379.21	397.32	400.39	373.64	435.75	443.27	453.56	513.72	511.94
22	300.41	312.52	303.12	277.83	321.36	304.95	334.28	377.89	395.93	398.99	372.34	434.22	441.72	451.97	511.92
23	340.01	299.29	311.35	301.98	276.79	320.15	303.81	333.03	376.47	394.44	397.50	370.94	432.60	440.06	450.28
24	300.22	338.73	298.16	310.17	300.85	275.75	318.95	302.67	331.78	375.05	392.96	396.00	369.55	430.97	438.41
25	302.27	299.05	337.41	297.00	308.97	299.68	274.68	317.71	301.49	330.49	373.59	391.43	394.46	368.11	429.29
26	279.25	301.06	297.84	336.06	295.81	307.72	298.47	273.57	316.43	300.28	329.16	372.09	389.85	392.87	366.63
27	223.75	278.22	299.95	296.75	334.82	294.72	306.59	297.37	272.57	315.26	299.17	327.94	370.72	388.42	391.43
28	250.01	222.82	277.05	298.70	295.51	333.42	293.49	305.31	296.13	271.43	313.94	297.92	326.57	369.17	386.79
29	217.97	248.81	221.75	275.73	297.26	294.09	331.82	292.08	303.85	294.71	270.13	312.44	296.49	325.01	367.40
30	254.58	217.00	247.71	220.76	274.50	295.94	292.78	330.34	290.78	302.49	293.40	268.92	311.05	295.17	323.56
31	241.75	253.41	216.00	246.57	219.75	273.24	294.58	291.43	328.82	289.44	301.10	292.05	267.69	309.62	293.82
32	203.58	240.60	252.21	214.98	245.40	218.70	271.94	293.18	290.05	327.27	288.07	299.67	290.66	266.42	308.15
33	261.69	202.61	239.46	251.01	213.96	244.23	217.66	270.65	291.79	288.67	325.71	286.70	298.25	289.28	265.15
34	229.83	260.37	201.59	238.25	249.75	212.88	243.00	216.57	269.29	290.32	287.22	324.07	285.26	296.75	287.82
35	252.85	228.64	259.02	200.55	237.02	248.45	211.78	241.74	215.45	267.89	288.82	285.73	322.39	283.78	295.21
36	237.05	251.53	227.45	257.67	199.50	235.79	247.16	210.67	240.49	214.33	266.50	287.31	284.25	320.71	282.30
37	164.78	235.75	250.15	226.20	256.26	198.41	234.49	245.80	209.52	239.17	213.15	265.04	285.74	282.69	318.96
38	178.34	163.87	234.45	248.77	224.95	254.85	197.31	233.20	244.45	208.37	237.85	211.98	263.58	284.16	281.13
39	166.49	177.31	162.92	233.09	247.33	223.65	253.37	196.17	231.85	243.03	207.16	236.47	210.75	262.05	282.52
40	186.42	165.52	176.28	161.97	231.73	245.89	222.35	251.89	195.03	230.50	241.62	205.95	235.09	209.52	260.52
41	180.05	185.31	164.53	175.22	161.00	230.34	244.42	221.01	250.39	193.86	229.12	240.17	204.72	233.69	208.26
42	155.85	178.92	184.14	163.50	174.12	159.99	228.90	242.88	219.62	248.81	192.64	227.68	238.66	203.43	232.22
43	153.14	154.85	177.77	182.95	162.44	173.00	158.96	227.42	241.31	218.21	247.21	191.40	226.21	237.12	202.12
44	121.70	152.10	153.80	176.57	181.72	161.35	171.83	157.89	225.88	239.68	216.73	245.54	190.11	224.68	235.52
45	171.99	120.84	151.03	152.72	175.32	180.44	160.21	170.62	156.77	224.29	237.99	215.20	243.80	188.76	223.10
46	175.05	170.70	119.93	149.90	151.57	174.00	179.08	159.00	169.34	155.60	222.61	236.21	213.59	241.98	187.35
47	117.53	173.68	169.37	118.99	148.72	150.39	172.64	177.68	157.76	168.01	154.38	220.86	234.36	211.92	240.08
48	121.53	116.57	172.26	167.99	118.02	147.51	149.16	171.23	176.23	156.47	166.64	153.12	219.06	232.45	210.19
49	104.76	120.46	115.55	170.75	166.51	116.99	146.22	147.85	169.73	174.69	155.10	165.18	151.78	217.14	230.41

Note: The figures in this table were generated by computer. They may not coincide with those given in the text due to rounding and truncation in intermediate calculations.

tainty about their accuracy, and it is perhaps better to ignore them when computing total fertility.

Step 5: reverse survival of adult females. Using the ${}_1L_x$ values shown in table 165, the reverse projection of females given in table 161 is carried out by using equation (C.5), or (C.6) when the calculations are being made systematically in two steps by hand. As an example, the female population aged 25 at the middle of a year (1969/70) is calculated. As intermediate steps one needs to compute W^{25}_{1969} and W^{25}_{1970} by

$$W^{25}_{1970} = W^{33}_{1978}({}_1L_{25}/{}_1L_{33}) = 272(0.8445/0.8152) = 281.78$$

$$W^{25}_{1969} = W^{34}_{1978}({}_1L_{25}/{}_1L_{34}) = 257(0.8445/0.8112) = 267.55$$

and then to compute their average as

$$N^{25}_{1970} = (281.78 + 267.55)/2.0 = 274.66.$$

All values of N^{a}_{t-x} are given in table 167.

Step 6: calculation of age-specific fertility rates. The calculation of age-specific fertility rates by single years, denoted by $f_{t-x}(a)$, using equation (C.8), is simple. For example,

$$f_{t-x}(25) = B^{25}_{1970}/N^{25}_{1970} = 79.98/274.66 = 0.2912.$$

Other values of $f_{t-x}(a)$ are shown in table 168. From these values, the calculation of five-year age-specific fertility rates, denoted by $f_{t-x}(i)$, is carried out using a modification of equation (C.10) as illustrated below:

$$f_{1970}(1) = (f_{1970}(15) + f_{1970}(16) + f_{1970}(17) +$$

$$+ f_{1970}(18) + f_{1970}(19))/5.0$$

$$= (0.0327 + 0.0672 + 0.1028 + 0.1469 +$$

$$+ 0.1985)/5.0 = 0.1096.$$

Other estimates of $f_{t-x}(i)$ are given in table 169. They are obtained by averaging the relevant $f_{t-x}(a)$ values, as specified by the next general equation:

$$f_{t-x}(i) = \sum_{a=10+5i}^{14+5i} f_{t-x}(a) \qquad \text{(C.15)}$$

Tables 168 and 169 also show values of total fertility. In both cases, they are obtained by adding the $f_{t-x}(a)$ values or the $f_{t-x}(i)$ values multiplied by five to take into account the fact that the latter values represent five-year averages. The difference between the total fertility values appearing in table 168 and those shown in table 169 is due to the fact that the values given in table 168 include the fertility rates for age 14, whereas those shown in table 169 do not.

Note that every set of fertility rates and total fertility estimates is labelled with a pair of years, indicating the period to which the rates refer. According to the notation used in this section, the second or last year of each period is that denoted by $(t-x)$.

Consider the sequence of total fertility estimates obtained by the application of the own-children method (given in table 169). In general, these estimates tend to increase as one moves into the past. There are, however, some exceptions to a monotonical increase. The estimates for the single-year periods 1965/66 and 1967/68 are higher than those of the neighbouring years. They are estimates derived from children whose reported ages in 1978 were 12 and 10 years, respectively. Obviously, age-heaping at these preferred ages is the cause of the relatively high total fertility estimates associated with 1965/66 and 1967/68. The relatively low total fertility estimated for 1966/67 is probably also due to age-reporting errors: avoidance of age 11 in 1978. To smooth out some of the peaks and troughs observed in the raw total fertility estimates, averages of the estimates for contiguous years may be calculated. The estimates shown in table 170 are obtained in this way; for example,

$$f_{1967-1969}(3) = (f_{1966/67}(3) + f_{1967/68}(3) + f_{1968/69}(3))/3.0$$

$$= (0.2759 + 0.2823 + 0.2549)/3.0 = 0.2710$$

and

$$TF_{1967-1969} = (TF_{1966/67} + TF_{1967/68} + TF_{1968/69})/3.0$$

$$= (5.8168 + 6.1170 + 5.5807)/3.0 = 5.8382.$$

The smoothed total fertility estimates given in table 170 increase steadily as one moves further into the past; that is, they are consistent with the existence of a long-term decline in fertility. The levels they imply are, on the whole, satisfactory but should not be interpreted too strictly. In particular, it appears likely that the strong attraction of age 12 may bias the 1964-1966 estimate upward, and that selective omission of young children may bias downward the estimate for 1976-1978. Comparison of these estimates with those derived from other sources and by other methods is necessary in order to validate them.

To conclude, it must be pointed out that the own-children method, as described here, uses unchanging mortality schedules for the entire period under consideration. The modification of the method to the case in which mortality changes is straightforward, but it is not common to have the information required to estimate with some confidence a time-series of mortality schedules. Therefore, the version of the method described here is suited for many of the cases encountered in practice.

TABLE 168. ESTIMATED SINGLE-YEAR FERTILITY RATES, BY SINGLE-YEAR OF AGE OF WOMAN, COLOMBIA, 1963/64–1977/78

Age of woman	Estimated fertility rates, $f_{t-x}(a)$														
	1963/1964	1964/1965	1965/1966	1966/1967	1967/1968	1968/1969	1969/1970	1970/1971	1971/1972	1972/1973	1973/1974	1974/1975	1975/1976	1977/1978	1978/1979
14	0.0202	0.0264	0.0391	0.0321	0.0143	0.0245	0.0168	0.0362	0.0159	0.0254	0.0211	0.0051	0.0100	0.0047	0.0186
15	0.0460	0.0517	0.0448	0.0461	0.0321	0.0419	0.0327	0.0470	0.0302	0.0320	0.0314	0.0152	0.0190	0.0155	0.0186
16	0.0641	0.0946	0.0654	0.0798	0.0750	0.0642	0.0672	0.0793	0.0567	0.0506	0.0733	0.0477	0.0378	0.0420	0.0273
17	0.1101	0.1092	0.1199	0.1355	0.1101	0.0867	0.1028	0.1206	0.1013	0.0929	0.1095	0.0868	0.0694	0.0741	0.0628
18	0.2012	0.1965	0.1758	0.1695	0.1451	0.1373	0.1469	0.1531	0.1405	0.1349	0.1246	0.1387	0.1063	0.0996	0.1076
19	0.2797	0.2410	0.2329	0.1845	0.1982	0.1987	0.1985	0.1991	0.1663	0.1631	0.1547	0.1814	0.1481	0.1299	0.1353
20	0.2751	0.2314	0.2468	0.2325	0.2421	0.2321	0.2228	0.2428	0.2097	0.1972	0.1697	0.2048	0.1749	0.1660	0.1534
21	0.2938	0.2417	0.2913	0.2636	0.2853	0.2445	0.2375	0.2630	0.2372	0.2439	0.1871	0.2089	0.2014	0.1813	0.1710
22	0.3249	0.2935	0.3171	0.3130	0.2878	0.3041	0.2732	0.2682	0.2605	0.2437	0.2092	0.2105	0.2159	0.1849	0.1773
23	0.3232	0.3425	0.3478	0.2751	0.3107	0.3093	0.2944	0.2873	0.2723	0.2233	0.2385	0.2061	0.2031	0.1886	0.2124
24	0.3524	0.2867	0.3700	0.2553	0.3031	0.3203	0.2844	0.3078	0.2672	0.2446	0.2489	0.2097	0.2112	0.1978	0.2268
25	0.3342	0.2706	0.3310	0.3059	0.2993	0.2884	0.2912	0.2774	0.2670	0.2406	0.2425	0.2136	0.2154	0.1963	0.2064
26	0.3080	0.2957	0.3340	0.2877	0.3148	0.2398	0.2997	0.2991	0.2504	0.2220	0.2312	0.1943	0.1913	0.1897	0.1930
27	0.2837	0.2836	0.3182	0.2624	0.2762	0.2569	0.3102	0.2604	0.2376	0.2483	0.2220	0.1785	0.1902	0.1875	0.1946
28	0.3085	0.3269	0.3054	0.2716	0.2648	0.2554	0.2833	0.2309	0.2336	0.2501	0.2154	0.1805	0.1982	0.1805	0.1830
29	0.3382	0.3117	0.3296	0.2519	0.2567	0.2338	0.2392	0.2608	0.2070	0.2138	0.2057	0.2007	0.2125	0.1720	0.1382
30	0.2601	0.2891	0.3060	0.1911	0.2450	0.2068	0.2431	0.2401	0.1925	0.1982	0.2141	0.2243	0.1933	0.1702	0.1469
31	0.2569	0.2821	0.2914	0.2422	0.2590	0.2216	0.2416	0.2484	0.2008	0.1902	0.1945	0.1636	0.1706	0.1514	0.1600
32	0.2548	0.2607	0.3220	0.2928	0.2899	0.2394	0.2200	0.2555	0.2038	0.1794	0.1719	0.1316	0.1491	0.1420	0.1683
33	0.2269	0.2430	0.3052	0.2146	0.3113	0.2066	0.1968	0.2372	0.1875	0.1568	0.1706	0.1667	0.1415	0.1230	0.1732
34	0.2554	0.2565	0.2787	0.1852	0.2460	0.2074	0.2177	0.2180	0.1775	0.1348	0.1829	0.1511	0.1560	0.1085	0.1258
35	0.2322	0.2950	0.2874	0.1909	0.2210	0.1701	0.2082	0.2213	0.1547	0.1393	0.1714	0.1254	0.1165	0.1015	0.1171
36	0.1958	0.2199	0.2916	0.1939	0.1848	0.1364	0.1682	0.2032	0.1647	0.1370	0.1496	0.1185	0.1078	0.1021	0.1129
37	0.2071	0.1888	0.2246	0.1836	0.1716	0.1590	0.1961	0.1280	0.1951	0.1458	0.1133	0.1285	0.1173	0.1118	0.0898
38	0.1837	0.1523	0.1963	0.1357	0.1754	0.1708	0.2203	0.0999	0.1518	0.1350	0.0990	0.1268	0.1206	0.0795	0.0807
39	0.1558	0.1217	0.1952	0.1197	0.1307	0.1580	0.2038	0.1444	0.1248	0.0856	0.1108	0.1137	0.0905	0.0560	0.0918
40	0.1318	0.1426	0.1305	0.1322	0.1116	0.1001	0.1671	0.1499	0.1258	0.0876	0.0825	0.1044	0.0639	0.0350	0.0809
41	0.0948	0.1019	0.1070	0.1148	0.0924	0.0849	0.0670	0.1253	0.1155	0.1041	0.0712	0.0597	0.0452	0.0362	0.0337
42	0.0745	0.0829	0.0882	0.0873	0.0631	0.0789	0.0578	0.0726	0.0859	0.0910	0.0752	0.0394	0.0242	0.0389	0.0419
43	0.0847	0.0566	0.0495	0.0639	0.0557	0.0547	0.0713	0.0415	0.0319	0.0701	0.0489	0.0437	0.0383	0.0167	0.0401
44	0.0673	0.0443	0.0264	0.0441	0.0463	0.0391	0.0367	0.0359	0.0251	0.0332	0.0251	0.0389	0.0213	0.0151	0.0184
45	0.0317	0.0391	0.0224	0.0255	0.0258	0.0315	0.0197	0.0295	0.0361	0.0218	0.0152	0.0278	0.0119	0.0209	0.0242
46	0.0273	0.0198	0.0226	0.0260	0.0213	0.0218	0.0141	0.0238	0.0297	0.0197	0.0163	0.0076	0.0162	0.0163	0.0173
47	0.0116	0.0233	0.0200	0.0273	0.0261	0.0084	0.0073	0.0213	0.0199	0.0255	0.0156	0.0000	0.0099	0.0107	0.0113
48	0.0112	0.0116	0.0157	0.0077	0.0274	0.0342	0.0127	0.0147	0.0178	0.0235	0.0072	0.0078	0.0079	0.0000	0.0077
49	0.0065	0.0056	0.0059	0.0038	0.0117	0.0377	0.0129	0.0085	0.0074	0.0140	0.0078	0.0072	0.0076	0.0026	0.0070
TOTAL FERTILITY	6.6335	6.4407	7.0557	5.8489	6.1313	5.6052	5.8832	5.8516	5.2086	4.8188	4.6279	4.2691	4.0144	3.5487	3.7752

Note: The figures in this table were generated by computer. They may not coincide with those given in the text due to rounding and truncation in intermediate calculations.

TABLE 169. ESTIMATED SINGLE-YEAR FERTILITY RATES, BY FIVE-YEAR AGE GROUP, COLOMBIA, 1963/64-1977/78

Age group	Estimated fertility rate, $f_{t-x}(t)$														
	1963/1964	1964/1965	1965/1966	1966/1967	1967/1968	1968/1969	1969/1970	1970/1971	1971/1972	1972/1973	1973/1974	1974/1975	1975/1976	1976/1977	1977/1978
15-19	0.1402	0.1386	0.1278	0.1231	0.1121	0.1058	0.1096	0.1198	0.0990	0.0947	0.0987	0.0939	0.0761	0.0722	0.0703
20-24	0.3139	0.2792	0.3146	0.2679	0.2858	0.2821	0.2625	0.2738	0.2494	0.2306	0.2107	0.2080	0.2013	0.1837	0.1882
25-29	0.3145	0.2977	0.3236	0.2759	0.2823	0.2549	0.2847	0.2657	0.2391	0.2350	0.2233	0.1935	0.2015	0.1852	0.1831
30-34	0.2508	0.2663	0.3007	0.2252	0.2702	0.2164	0.2239	0.2398	0.1924	0.1719	0.1868	0.1675	0.1621	0.1390	0.1548
35-39	0.1949	0.1955	0.2390	0.1648	0.1767	0.1589	0.1993	0.1594	0.1582	0.1285	0.1288	0.1226	0.1106	0.0902	0.0985
40-44	0.0906	0.0857	0.0803	0.0885	0.0738	0.0715	0.0800	0.0850	0.0783	0.0772	0.0606	0.0572	0.0386	0.0284	0.0430
45-49	0.0177	0.0199	0.0173	0.0181	0.0225	0.0267	0.0133	0.0195	0.0222	0.0209	0.0124	0.0101	0.0107	0.0101	0.0135
TOTAL FERTILITY	6.6133	6.4143	7.0166	5.8168	6.1170	5.5807	5.8664	5.8155	5.1928	4.7934	4.6068	4.2640	4.0045	3.5441	3.7566

TABLE 170. ESTIMATED THREE-YEAR FERTILITY RATES, BY FIVE-YEAR AGE GROUP, COLOMBIA, 1964-1978

Age group	1964-1966	1967-1969	1970-1972	1973-1975	1976-1978
15-19	0.1355	0.1136	0.1095	0.0958	0.0729
20-24	0.3026	0.2786	0.2619	0.2164	0.1911
25-29	0.3120	0.2710	0.2632	0.2173	0.1899
30-34	0.2726	0.2373	0.2187	0.1754	0.1520
35-39	0.2098	0.1668	0.1723	0.1266	0.0997
40-44	0.0855	0.0779	0.0811	0.0650	0.0366
45-49	0.0183	0.0224	0.0184	0.0145	0.0114
TOTAL FERTILITY	6.6814	5.8382	5.6249	4.5547	3.7684