Chapter VI

DERIVATION OF A SMOOTH LIFE TABLE FROM A SET OF SURVIVORSHIP PROBABILITIES

A. BACKGROUND OF METHODS

1. *Necessity of smoothing and completing sets of survivorship probabilities*

There are several situations in which one can obtain estimates of life-table probabilities of survivorship, $l(x)$, but in which one would still want to smooth[1] these estimates with reference to a model life table. For example, one could calculate a life table directly by converting observed ${}_nm_x$ values into ${}_nq_x$ and hence into $l(x)$ values. In other instances, the $l(x)$ values available may not define a complete life table because some may apply to childhood and some to advanced adult ages (as those obtained from maternal orphanhood data). In the most common situation, one would estimate childhood survivorship probabilities from information on children ever born and children surviving (see chapter III) and estimate adult survivorship probabilities from information on spouse survival or on parental survival (see chapter IV), and the age ranges covered by these sets of estimates will not exhaust the 0-85 life-table range. In such cases, both an element of smoothing and some interpolation or extrapolation to obtain missing $l(x)$ values may be necessary.

In yet other situations, one may wish to derive complete sets of life-table probabilities of survivorship, $l(x)$, from childhood mortality estimates and conditional survivorship probabilities for adults (such as the probability of surviving from age A to age B, $l(B)/l(A)$). Conditional probabilities are usually obtained from information on widowhood or orphanhood (see chapter IV) or by the use of some death-distribution method (see chapter V). The problem one faces in this circumstance is to link the two independent estimates. Again, this linkage is usually accomplished by reference to a model life table.

2. *Organization of this chapter*

This chapter focuses on methods to derive complete life tables from partial information on survivorship probabilities. Two types of methods are distinguished: those which allow the smoothing and interpolation of $l(x)$ values (called the "SI methods"); and those which, in addition to incorporating some element of smoothing and allowing interpolation, permit the linkage of direct or unconditional probabilities of survivorship, $l(x)$, with conditional probabilities, i.e., the ratios of $l(x)$ values (called the "L methods"). The sections presenting each type of method are described below:

TABLE 134. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER VI

Section	*Subsection and method*	*Type of input data*	*Estimated parameters*
B. Smoothing and interpolation of an incomplete set of survivorship probabilities		A set of $l(x)$ values. It may be the complete $l(x)$ function for $x = 1, 5, 10, ..., 80$, or some subset of those values A reliable, complete set of $l(x)$ values to be used as standard	Parameters α and β defining the fitted $l(x)$ function in the logit system generated by the selected standard A complete life table
C. Linkage of child survivorship probabilities with conditional adult survivorship probabilities	C.2. Using the logit system	A set of $l(z)$ values, usually for childhood ages A set of conditional survivorship probabilities, of the form $l(x)/l(y)$ A reliable, complete set of $l(x)$ values to be used as standard	Parameters α and β defining the fitted $l(x)$ function in the logit system generated by the selected standard A complete life table
	C.3. Using the Coale-Demeny models	A set of $l(z)$ values, usually for childhood ages A set of conditional survivorship probabilities, of the form $l(x)/l(y)$	A complete life table

Section B. Smoothing and interpolation of an incomplete set of survivorship probabilities. This section presents a smoothing and interpolation method based on the use of the logit life-table system. It requires as input a set of survivorship probability estimates, $l(x)$.

Section C. Linkage of child survivorship probabilities with conditional adult survivorship probabilities. This section presents two linkage methods based, respectively,

[1] The term "to smooth" is used in this *Manual* in its most general sense to mean the elimination or minimization of irregularities often present in reported data or in preliminary estimates obtained from them. In this sense, the set of possible "smoothing techniques" encompasses a wide variety of procedures, ranging from the fitting of models to simple averaging. The traditional smoothing techniques applied to age distributions and to observed age-specific mortality rates are part of this set, but they do not exhaust it. The somewhat rougher procedures described in this *Manual* are necessary because the basic data available are both deficient and incomplete.

on the use of the logit life-table system and the Coale-Demeny life tables. Both require as input a set of child survivorship probabilities, $l(z)$, and a set of conditional survivorship probabilities of the form $l(x)/l(y)$. See table 134 for more details.

B. SMOOTHING AND INTERPOLATION OF AN INCOMPLETE SET OF SURVIVORSHIP PROBABILITIES

1. *Basis of method and its rationale*

Perhaps the simplest technique for smoothing and interpolating between a sequence of $l(x)$ values is provided by the logit system described in chapter I, subsection B.4. Because all life-table $l(x)$ functions belonging to the same logit system are related linearly on the logit scale, one way of smoothing the observed $l(x)$ values would be to plot their logit transformations against those of some standard life table. If the estimated life table does conform to the logit system generated by this standard, then the plotted points should form a fairly straight line with slope β and intercept α; any method of line-fitting can then be used to estimate the actual values of these parameters. However, when the plotted points depart from a straight line, the problem of selecting the best fit is less tractable. If the observed deviations from linearity are systematic, as, for example, when the deviations become larger as age increases or as age decreases, or when the plot is decidedly curvilinear, the use of a different standard should be considered. If, on the other hand, the deviations from a linear trend seem to be random in nature, the exclusion of some of the points may be necessary before a line is fitted to the remaining points. In such a case, the use of fitting techniques (such as regression) that assume some homogeneity in the errors affecting the data is not warranted, because the errors involved are likely to have different variances at different points and these variances cannot be estimated from the data usually available. Hence, in cases of rough overall linearity, where distortions are mostly due to errors, rather crude fitting techniques (such as those described in chapter V, subsection C.3), coupled with the judicious selection of the most reliable points, is probably the most acceptable procedure to follow.

In the application of this smoothing procedure, the selection of an appropriate standard is of great importance, since only when the mortality pattern of the standard resembles that of the observed $l(x)$ function will the linear relationship on the logit scale be evident. The most common sources of standards are sets of model life tables. The general standard proposed by Brass[2] is always one possibility to be considered (see chapter I, table 2), as are the four families of the Coale-Demeny[3] life tables (see annex XI) and the United Nations model patterns for developing countries[4] (see chapter I, subsection B.5). In using these models, it is helpful to remember that the general standard has a pattern very similar to that of the West model in the Coale-Demeny set, and that the logit transformations of life tables at different levels within the same family of Coale-Demeny models have an approximately linear relationship to one another, but that linearity is not maintained when the logit transformations of life tables from different families are compared.

2. *Data required*

The data listed below are required for this method:

(*a*) A set of $l(x)$ values, estimated either directly from observed data or by using any of the procedures described in this *Manual*. Values at five-year intervals (5, 10, 15, 20 and so on) are sufficient, but the complete range from 0 to 80 or 85 need not be covered;

(*b*) A standard life table, which may be selected from the Coale-Demeny models, the United Nations models for developing countries, the general standard or any reliable life table thought to approximate the pattern of mortality in the population being studied.

3. *Computational procedure*

The steps of the computational procedure are described below.

Step 1: calculation of logit transformation of the estimated survivorship probabilities. If one denotes by $\lambda(x)$ the logit transformation of the survivorship probabilities, $l(x)$, then

$$\lambda(x)=\text{logit}(1.0-l(x))=0.5\ \ln((1.0-l(x))/l(x)). \tag{B.1}$$

Step 2: plot of logit transformation of the estimated life table against logit transformation of the standard. The logit transformations of the $l(x)$ values for the life table of the population under study should be plotted against those of the standard. If the relationship is linear (or approximately so, with no systematic deviations), then the parameter values α and β can be estimated. If systematic distortions are evident, a different standard should be employed, though departures from linearity may arise from errors in the observed $l(x)$ values rather than from the use of an unsuitable standard.

Step 3: estimation of parameter values. If the plot in step 2 reveals a nearly linear relationship, then estimates of the parameter values, α and β, the constant term and the slope of the line representing this relationship, may be obtained by almost any line-fitting procedure: either by least squares; or by the procedures described in chapter V, subsection C.3. If certain groups of $l(x)$ values are thought to be more reliable than others, then only those values should be used in fitting a line, as is illustrated in the detailed example given below.

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[2] William Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data* (Chapel Hill, North Carolina, Carolina Population Center, Laboratories for Population Studies, 1975).

[3] Ansley J. Coale and Paul Demeny, *Regional Model Life Tables and Stable Populations* (Princeton, New Jersey, Princeton University Press, 1966).

[4] *Model Life Tables for Developing Countries* (United Nations publication, Sales No. E.81.XIII.7).

Step 4: computation of smoothed life-table values. Once estimates of the parameters α and β are available, the smoothed life-table values, $l^*(x)$, are obtained as

$$l^*(x) = (1.0 + \exp(2\alpha + 2\beta\lambda_s(x)))^{-1} \quad \text{(B.2)}$$

where $\lambda_s(x)$ denotes the logit transformation of the standard life table at age x. Note that $l(0)$ and $l(\omega)$, where ω is the highest age that may be attained, cannot be calculated by using equation (B.2); instead, the radix of the life table, $l(0)$, is set equal to one, and $l(\omega)$ is set equal to zero.

TABLE 135. CHILDHOOD SURVIVORSHIP PROBABILITIES FOR FEMALES, ESTIMATED USING TRUSSELL COEFFICIENTS WITH DATA CLASSIFIED BY AGE AND BY MARRIAGE DURATION OF MOTHER, PANAMA, 1976

	Estimates based on data classified by:					
	Duration			*Age*		
Age *z* *(1)*	*Childhood survivorship probability* *l(z)* *(2)*	*West level* *(3)*	*Reference date* *(4)*	*Childhood survivorship probability* *l(z)* *(5)*	*West level* *(6)*	*Reference date* *(7)*
2	0.9688	21.3	1975.3	0.9560	20.2	1974.3
3	0.9505	20.0	1973.0	0.9405	19.2	1972.5
5	0.9428	19.7	1970.6	0.9324	19.0	1970.3
10	0.9238	18.9	1968.3	0.9091	18.1	1967.9
15	0.9026	18.1	1965.6	0.9035	18.1	1965.3
20	...	...	...	0.8691	17.0	1962.4

4. *A detailed example*

The case of female mortality in Panama is considered in this example. In chapters III and IV, the data gathered by the Demographic Survey of Panama in 1976 were used to estimate female child and adult survivorship. Table 135 shows estimates of female $l(z)$ values for childhood ages obtained both from data classified by age of mother (see chapter III, subsection B.3) and from those classified according to the duration of mother's first marriage (see chapter III, subsection C.3). Each $l(z)$ value presented in table 135 is accompanied by the mortality level it implies in the West family of Coale-Demeny life tables and by its reference period, both estimated as described in chapter III.

Table 136 shows a set of $l(x)$ values for females estimated from data on the widowhood status of male respondents classified by duration of marriage. Again, each of the estimates presented is accompanied by the mortality level it implies in the West models and by its time reference period (see chapter IV, subsection C.3 (c)(iii)). As these tables indicate, the two distinct sets of estimates, those referring to childhood and those referring to adult ages, cover only the age range from 2 to 40. Furthermore, since different estimates refer to different time periods, it is clear that for a given period, say from 1970 to 1975, no complete set of $l(x)$ estimates is available covering that age range. Hence, it is necessary to employ the procedure described in this section to derive from this partial information on survivorship probabilities a complete life table referring to the recent past.

TABLE 136. FEMALE SURVIVORSHIP PROBABILITIES ESTIMATED FROM DATA ON THE WIDOWHOOD STATUS OF MALE RESPONDENTS CLASSIFIED BY DURATION OF MARRIAGE, PANAMA, 1976

Age *x* *(1)*	*Survivorship probability* *l(x)* *(2)*	*West mortality level* *(3)*	*Reference date* *(4)*
20	0.9365	20.2	1975.6
25	0.9160	19.6	1973.3
30	0.8955	19.2	1971.1
35	0.8744	18.9	1968.9
40	0.8491	18.6	1967.0

The steps of the computational procedure are described below.

Step 1: calculation of logit transformation of the estimated survivorship probabilities. Equation (B.1) is used to calculate the logit transformation of each survivorship probability, $l(x)$, given in tables 135 and 136. The resulting $\lambda(x)$ values are shown in table 137. As an example, the logit transformation of $l(30)$ is shown below:

$$\lambda(30) = \text{logit}(1.0 - 0.8955) = 0.5 \ln((1.0 - 0.8955)/0.8955)$$
$$= 0.5 \ln(0.1167) = -1.0741.$$

Column (2) of table 137 gives the logit transformations of the $l_s(x)$ probabilities corresponding to the standard used. In this case, the general standard has been selected as an adequate representation of the pattern of mortality prevalent in the population under study; therefore, the $\lambda_s(x)$ values listed in table 137 have been copied from table 2 (see chapter I).

TABLE 137. LOGIT TRANSFORMATION OF THE ESTIMATED AND STANDARD SURVIVORSHIP PROBABILITIES, PANAMA, 1976

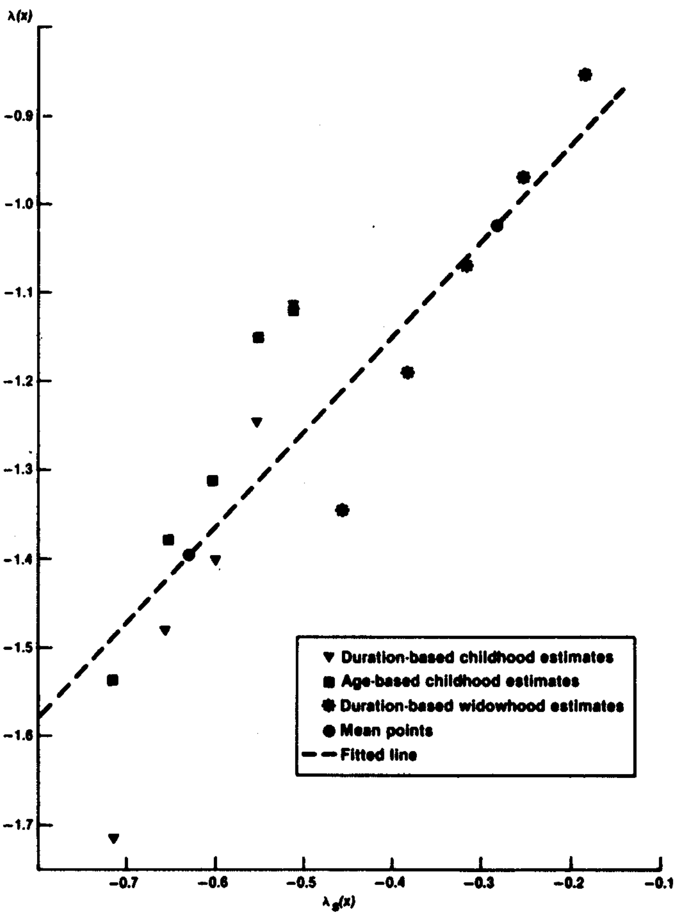
Age *x* *(1)*	*General standard* $\lambda_s(x)$ *(2)*	*Duration-based estimate* $\lambda(x)$ *(3)*	*Age-based estimate* $\lambda(x)$ *(4)*
2	−0.7152	−1.7178	−1.5393[a]
3	−0.6552	−1.4775[a]	−1.3802[a]
5	−0.6015	−1.4012[a]	−1.3121[a]
10	−0.5498	−1.2476[a]	−1.1513
15	−0.5131	−1.1132	−1.1184
20	−0.4551	−1.3456	−0.9465
25	−0.3829	−1.1946[a]	...
30	−0.3150	−1.0741[a]	...
35	−0.2496	−0.9702[a]	...
40	−0.1816	−0.8638[a]	...

[a] Values used in fitting procedure.

Step 2: plot of logit transformation of the estimated life table against logit transformation of the standard. The points defined by each pair of values $[\lambda_s(x), \lambda(x)]$ are plotted in figure 16. Note that because of the nature of the data used in deriving the different $l(x)$ estimates, at least three different sets of points have been identified in the plot. The points corresponding to the two sets of child mortality estimates cluster, in general, to the bottom left of the graph. They are similar, but the desirable degree of coincidence between one set and the other is not achieved. Notably deviant points are those associated with the $l(2)$ estimate obtained from data classified by duration and with the $l(x)$ estimates for ages 15 and 20 (even the point associated with $l(10)$ of the age-based estimates is somewhat out of line with the rest). The estimates derived from widowhood data, on the other hand, cluster near the upper right-hand side of the graph. They display a fairly linear trend, but the point associated with $l(20)$, and even perhaps that associated with $l(25)$, would have to be disregarded in order for the widowhood-based estimates to be considered a continuation of a straight line defined by the points derived from child survivorship probabilities.

A rapid appraisal of the West mortality levels associated with each estimate and of their respective reference dates suggests that most of the deviant points among the childhood estimates are associated with periods preceding 1970 or following 1975, and that they usually correspond to mortality levels (in the Coale-Demeny set) substantially lower or higher than the rest. Deviant points in the case of the widowhood-based estimates are associated with the most recent dates (after 1975) and correspond to fairly high mortality levels. On the basis of these observations, the points indicated by footnote a in table 137 are chosen as the most reliable indicators of child and adult mortality during, roughly, the period 1969-1975.

Figure 16. Plot of the logit transfomation of the estimated female survivorship probabilities, $l(x)$, against those of the general standard, Panama, 1976



Step 3: estimation of parameter values. Having selected the most reliable set of points representing survivorship probabilities for childhood and adult ages, a line is fitted to them by using group means (see chapter V, subsection C.3). Means must be obtained both for the logit transformations of the estimated survivorship probabilities and for those of the standard. If one denotes the means of the former values by $\Theta(2,3,3,5,5,10)$ and $\Theta(25,30,35,40)$, and those of the latter by $\Theta_s(2,3,3,5,5,10)$ and $\Theta_s(25,30,35,40)$, then the line passing through the points defined by these mean values satisfies the following two equations:

$$\Theta(2,3,3,5,5,10) = \alpha + \beta\Theta_s(2,3,3,5,5,10), \quad \text{(B.3)}$$

and

$$\Theta(25,30,35,40) = \alpha + \beta\Theta_s(25,30,35,40). \quad \text{(B.4)}$$

Hence, an estimate of the parameter β is given by

$$\beta^* = \frac{\Theta(2,3,3,5,5,10) - \Theta(25,30,35,40)}{\Theta_s(2,3,3,5,5,10) - \Theta_s(25,30,35,40)}$$

$$= \frac{-1.3930 + 1.0257}{-0.6297 + 0.2823} = \frac{-0.3673}{-0.3474} = 1.0573, \quad \text{(B.5)}$$

and parameter α is then found by substituting the estimated value of β into either equation (B.3) or equation (B.4) as follows:

$$\alpha^* = \Theta(2,3,3,5,5,10) - \beta^*\Theta_s(2,3,3,5,5,10)$$

$$= -1.3930 - [(1.0573)(-0.6297)] = -0.7272,$$

or

$$\alpha^* = \Theta(25,30,35,40) - \beta^*\Theta_s(25,30,35,40)$$

$$= -1.0257 - [(1.0573)(-0.2823)] = -0.7272.$$

Step 4: computation of smoothed life-table values. Once best estimates of α and β have been calculated, the full set of estimated smoothed life-table survivorship probabilities, $l^*(x)$, shown in column (3) of table 138, can be obtained by using equation (B.2). For example, $l^*(20)$ is calculated as

$$l^*(20) = \frac{1.0}{1.0 + \exp(2(-0.7272) + 2(1.0573)(-0.4551))}$$

$$= 0.918.$$

It should be stated that this procedure may not be robust to different choices of groups in fitting a line to the logit points. For example, if instead of choosing as the point representative of childhood mortality the mean of those associated with the age-based $l(2)$, $l(3)$ and $l(5)$, and with the duration-based $l(3)$, $l(5)$ and $l(10)$, just the mean of the three duration-based estimates had been used, β^* would have been 1.0934 instead of its previous value of 1.0573. Similarly, α^* would have been slightly higher, with an estimated value of −0.7170. Hence, unless one is fairly certain that a particular group as a whole is correct, other combinations should be tried in order to assess the sensitivity of the final life table to alternative choices. In this case, however, most of the reasonable choices produce fairly similar life tables.

TABLE 138. ESTIMATED SMOOTHED LIFE-TABLE SURVIVORSHIP PROBABILITIES FOR FEMALES OBTAINED BY USE OF DIFFERENT SMOOTHING AND LINKAGE METHODS, PANAMA, 1976

Age x (1)	General standard $\lambda_s(x)$ (2)	Estimated smoothed survivorship probabilities, $l^*(x)$: Smoothing and interpolation method (3)	Linkage methods: Logit system (4)	Coale-Demeny (5)
0	-	1.000	1.000	1.000
2	−0.7152	0.951	0.948	0.948
5	−0.6015	0.939	0.940	0.940
10	−0.5498	0.932	0.935	0.934
20	−0.4551	0.918	0.927	0.923
30	−0.3150	0.893	0.913	0.911
40	−0.1816	0.863	0.897	0.893
50	−0.0212	0.817	0.875	0.859
60	0.2100	0.733	0.835	0.789
70	0.5818	0.556	0.752	0.636
80	1.2375	0.238	0.551	0.342

C. LINKAGE OF CHILD SURVIVORSHIP PROBABILITIES WITH CONDITIONAL ADULT SURVIVORSHIP PROBABILITIES

1. *Basis of method and its rationale*

As stated in chapter IV, orphanhood or widowhood data properly reflect only adult mortality experience. Hence, one should be able to obtain better estimates of conditional survivorship probabilities (for example, $l(35)/l(20)$) than for unconditional $l(x)$ values. Nevertheless, one is most often interested in the unconditional probabilities, since they define a conventional life table. It is useful, therefore, to have some procedure for linking conditional probabilities of adult survivorship, of the form $l(x)/l(y)$, with other information about survivorship, particularly that in childhood. Not only would such a procedure make possible the derivation of a complete life table, but it would also incorporate some element of smoothing over the range of survivorship estimates available.

If a number of survivorship probabilities of the type $l(x)/l(y)$ are available for different values of x, but for a fixed value of y, all that is needed in order to obtain estimates of $l(x)$ is an estimate of $l(y)$. Information about child survivorship—say, an estimate of the probability of surviving from birth to age 2, $l(2)$—can then be coupled with a system of model life tables to estimate a value of $l(y)$, which can in turn be multiplied by the adult survivorship probabilities to obtain $l(x)$. The logit system provides a convenient basis for such a procedure, and an iterative process that also introduces a powerful smoothing component is described below. An alternative procedure using the Coale-Demeny model life tables is also presented below (in subsection C.3).

2. *Linkage method using a logit life-table system*

(a) *Data required*

The data required for use of the first linkage method, that utilizing a logit life-table system, are listed below:

(*a*) Conditional probabilities of adult survivorship, usually in the form of $l(x)/l(y)$ ratios estimated by using the widowhood or orphanhood methods described in chapter IV;

(*b*) An estimate of child survivorship. Survivorship probabilities $l(z)$ for childhood ages can be estimated by using the methods described in chapter III;

(*c*) A standard model life table. The general standard, one of the Coale-Demeny models or of the United Nations new models, or any other reliable life table thought to approximate the mortality experience of the population in question may be used as standard.

(b) *Computational procedure*

In the two-parameter logit life-table system, any pair of survivorship probabilities, one from birth and another conditional on attaining a certain age, uniquely determine values of the parameters α and β defining a life table in the system. However, because one of the probabilities is conditional, the values of these parameters have to be estimated iteratively, as there is no way of solving for them algebraically. The procedure described here provides a way of finding satisfactory values of α and β when a number of conditional survivorship probabilities, all referring to the same population, are available, without having to find parameter values for each one, a process that would be tedious without a computer. The steps of the computational procedure are described below.

Step 1: initial estimate of parameter α. Given an estimate of $l(z)$ for children, such as $l(2)$ or the average of a group of estimates, the initial estimate of parameter α is obtained as

$$\alpha_1 = \lambda(z) - \lambda_s(z)$$

under the assumption that $\beta_1 = 1.0$.

Step 2: initial estimate of survivorship probability appearing as denominator. Given the value of α_1 estimated in step 1 and continuing to assume that $\beta_1 = 1.0$, a first estimate of this survivorship probability, $l(y)$, denoted by $l_1(y)$, is obtained from equation (B.2):

$$l_1(y) = (1.0 + \exp(2\alpha_1 + 2\lambda_s(y)))^{-1}.$$

Step 3: initial estimates of survivorship probabilities from birth. The initial approximation to $l(y)$, $l_1(y)$, is now used to calculate initial estimates of the survivorship probabilities from birth, $l(x)$, for each value of x from the observed ratios $l(x)/l(y)$:

$$l_1(x) = \frac{l(x)}{l(y)} l_1(y). \qquad \text{(C.1)}$$

Step 4: modified estimate of parameter β. If one denotes by $\lambda_1(x)$ the logit transformation of $l_1(x)$, each pair of points $[\lambda_s(z), \lambda(z)]$ and $[\lambda_s(x), \lambda_1(x)]$ determines parameters α and β with respect to the standard being used. The main interest at present is in parameter β, which can be found as

$$\beta_2(x) = \frac{(\lambda(x) - \lambda(z))}{(\lambda_s(x) - \lambda_s(z))}. \qquad \text{(C.2)}$$

A single estimate of β_2 can then be obtained by averaging the $\beta_2(x)$ values. It is often the case that the values of $\beta_2(x)$ vary sharply with x, and it may be decided that a best estimate of β_2 can be obtained by averaging the $\beta_2(x)$ values after excluding obvious outliers, such as the highest and lowest values.

Step 5: a second estimate of parameter α. A second estimate of α, denoted by α_2, is obtained by repeating step 1, but now using the estimate of β_2 obtained in step 4 instead of the first assumed value of $\beta_1 = 1.0$. Thus,

$$\alpha_2 = \lambda(z) - \beta_2 \lambda_s(z). \qquad \text{(C.3)}$$

Step 6: second estimate of survivorship probability used as denominator. The new value of α, denoted by α_2, and the second approximation to β, denoted by β_2, are now used to obtain a revised value of $l(y)$, denoted by $l_2(y)$, as follows:

$$l_2(y) = (1.0 + \exp(2\alpha_2 + 2\beta_2 \lambda_s(y)))^{-1}. \qquad \text{(C.4)}$$

Step 7 and onward: further iteration. The iterative procedure continues, by repeating step 3 to obtain a second set of estimates of $l(x)$, then repeating step 4 to find a revised estimate of β, then re-estimating α in step 5 and $l(y)$ in step 6 with the new α and β values, and so on, until the first two or three decimal places of the estimate of β no longer change with continued iteration. This unchanging value of β, denoted by β^*, and the value of α^* it implies (calculated by using equation (C.3)) are then accepted as best estimates of the parameters defining a life table consistent with the available survivorship probabilities in the logit system being used.

With practice, the iterative process does not take long; four or five iterations are usually sufficient for the β estimate to converge. A slight short-cut can also be taken, since β^*, the unchanging estimate of β, is always further away, generally about half as far again, from the first assumed value of 1.0 than is the first estimate obtained. Thus, as an approximation,

$$\beta_3 = 1.5\beta_2 - 0.5. \qquad \text{(C.5)}$$

Then, instead of substituting the value of β_2 in step 5, a more rapid convergence can be achieved by substituting β_3 instead. Thereafter, however, no short-cuts can be taken.

(c) *A detailed example*

Table 139 shows conditional survivorship probabilities for the female population of Panama derived from data on the widowhood status of male respondents classified by age (see chapter IV, subsection B.3 (c) (iii)). Given the age-based estimates of childhood mortality shown in table 135, the way in which a full life table may be constructed is illustrated below.

TABLE 139. ESTIMATED CONDITIONAL FEMALE SURVIVORSHIP PROBABILITIES, $l(x)/l(20)$, AND CORRESPONDING MORTALITY LEVELS IN THE WEST FAMILY OF MODEL LIFE TABLES, PANAMA, 1976

Age *x* *(1)*	*Survivorship probabilities* $l(x)/l(20)$ *(2)*	*West mortality level* *(3)*
25	0.9962	22.2
30	0.9878	21.3
35	0.9775	21.0
40	0.9641	20.8
45	0.9417	20.4

Step 1: initial estimate of parameter α. The life table used as standard is, once more, the general standard presented in table 2 in chapter I. In the first iteration, an estimate of α_1 is obtained by assuming that $\beta_1 = 1.0$. If one assumes that the estimates of $l(2)$, $l(3)$ and $l(5)$ are reasonably accurate, then α_1 can be estimated as

$$\alpha_1 = \Theta(2, 3, 5) - \Theta_s(2, 3, 5)$$

$$= -1.4105 + 0.6573 = -0.7532$$

where $\Theta(2, 3, 5)$ denotes, as usual, the mean of the logit transformations of $l(2)$, $l(3)$ and $l(5)$, that is, the mean of $\lambda(2)$, $\lambda(3)$ and $\lambda(5)$ in this case.

Step 2: initial estimate of survivorship probability appearing as denominator. In this case, the survivorship ratios being used have as denominator the value of $l(20)$, so $y = 20$. A first estimate of $l(20)$ is obtained by setting $\beta_1 = 1.0$, and using the α_1 estimate from step 1:

$$l_1(20) = (1.0 + \exp(2(-0.7532) + 2(-0.4551)))^{-1}$$

$$= 0.9181.$$

Step 3: initial estimates of survivorship probabilities from birth. Each value of $l(x)/l(20)$ is multiplied by $l_1(20)$ in order to obtain first estimates of $l(x)$. The results are shown in table 140. The value for $x = 40$ is obtained as

$$l_1(40) = \frac{l(40)}{l(20)} l_1(20) = (0.9641)(0.9181) = 0.8851.$$

Step 4: modified estimate of parameter β. Each estimate of $l_1(x)$, in combination with the pooled estimates of $l(2)$, $l(3)$ and $l(5)$, implies a value of β which is equal to the ratio of the difference in the observed logit transfor-

TABLE 140. ITERATION PROCESS TO ESTIMATE THE α AND β PARAMETERS DEFINING A LIFE TABLE FOR FEMALES IN THE LOGIT SYSTEM GENERATED BY THE GENERAL STANDARD, PANAMA, 1976

Age x	Estimated conditional female survivorship probability $l(x)/l(20)$	First iteration $\alpha_1 = -0.7532$, $\beta_1 = 1.0000$, $l_1(20) = 0.9181$			Second iteration $\alpha_2 = -0.8695$, $\beta_2 = 0.8230$, $l_2(20) = 0.9233$			Third iteration $\alpha_3 = -0.9195$, $\beta_3 = 0.7470$, $l_3(20) = 0.9255$		
		$l_1(x)$	$\lambda_1(x)$	$\beta_2(x)$	$l_2(x)$	$\lambda_2(x)$	$\beta_3(x)$	$l_3(x)$	$\lambda_3(x)$	$\beta_4(x)$
25	0.9962	0.9146	−1.1856	0.820	0.9198	−1.2198	0.695	0.9220	−1.2349	0.640
30	0.9878	0.9069	−1.1382	0.796	0.9120	−1.1692	0.705	0.9142	−1.1830	0.665
35	0.9775	0.8974	−1.0843	0.800	0.9025	−1.1127	0.730	0.9047	−1.1253	0.700
40	0.9641	0.8851	−1.0208	0.819	0.8901	−1.0459	0.766	0.8923	−1.0572	0.743
45	0.9417	0.8646	−0.9270	0.879	0.8695	−0.9483	0.840	0.8715	−0.9571	0.824
AVERAGE				0.823			0.747			0.714

Age x	Estimated conditional female survivorship probability $l(x)/l(20)$	Fourth iteration $\alpha_4 = -0.9412$, $\beta_4 = 0.7140$, $l_4(20) = 0.9264$			Fifth iteration $\alpha_5 = -0.9497$, $\beta_5 = 0.7010$, $l_5(20) = 0.9267$			Sixth iteration $\alpha_6 = -0.9530$, $\beta_6 = 0.6960$, $l_6(20) = 0.9269$		
		$l_4(x)$	$\lambda_4(x)$	$\beta_5(x)$	$l_5(x)$	$\lambda_5(x)$	$\beta_6(x)$	$l_6(x)$	$\lambda_6(x)$	$\beta_7(x)$
25	0.9962	0.9229	−1.2412	0.617	0.9232	−1.2433	0.609	0.9234	−1.2447	0.604
30	0.9878	0.9151	−1.1888	0.648	0.9154	−1.1907	0.642	0.9156	−1.1920	0.638
35	0.9775	0.9056	−1.1305	0.687	0.9058	−1.1317	0.684	0.9060	−1.1329	0.681
40	0.9641	0.8931	−1.0614	0.734	0.8934	−1.0630	0.731	0.8936	−1.0640	0.728
45	0.9417	0.8724	−0.9612	0.817	0.8727	−0.9625	0.815	0.8729	−0.9634	0.813
AVERAGE				0.701			0.696			0.693

mations of the $l(x)$ estimates and the difference in those of the standard (see equation (C.2)). The logit transformations of the $l_1(x)$ values and the values of $\beta_2(x)$ implied by each are shown in table 140. As an example, for $x = 40$,

$$\lambda_1(40) = \text{logit}(1.0 - l_1(40)) = 0.5 \ln((1.0 - 0.8851)/0.8851)$$

$$= -1.0208;$$

and since $\lambda_s(40)$ is -0.1816, $\Theta(2, 3, 5)$ is -1.4105, and the equivalent value for the standard, $\Theta_s(2, 3, 5)$, is -0.6573, then

$$\beta_2(40) = \frac{-1.0208 - (-1.4105)}{-0.1816 - (-0.6573)} = 0.819.$$

The values of $\beta_2(x)$ are fairly consistent, except for $\beta_2(45)$. However, it is not sufficiently deviant to warrant its exclusion. Therefore, β_2 is estimated as the average of all $\beta_2(x)$ values, that is, 0.823.

Step 5: a second estimate of parameter α. Step 1 is now repeated, but using the estimate of $\beta_2 = 0.823$ instead of 1.0. Thus,

$$\alpha_2 = \Theta(2, 3, 5) - \beta_2 \Theta_s(2, 3, 5)$$

$$= -1.4105 - (0.823)(-0.6573)$$

$$= -0.8695.$$

Step 6: second estimate of survivorship probability to age 20. Step 2 is now repeated, but using the new values of $\alpha_2(-0.8695)$ and $\beta_2(0.823)$ obtained after the first complete iteration. Thus,

$$l_2(20) = (1.0 + \exp(2(-0.8695) + 2(0.823)(-0.4551)))^{-1}$$

$$= -0.9233.$$

Step 7 and onward: further iteration. Table 140 shows the intermediate results obtained during the first six iterations. The value of β is clearly converging towards 0.69, so this value can be adopted as an estimate of β^*. The value of α corresponding to β^* is then calculated using equation (C.3), which in this case yields a value of -0.957 for α^*.

Final step: calculation of fitted life table. Once final estimates of α and β have been arrived at, the estimated $l^*(x)$ function of the fitted life table can be calculated using equation (B.2); and, as usual, the other functions of the life table can then be derived from it. For example, for age 40,

$$l^*(40) = (1.0 + \exp(2(-0.957) + 2(0.69)(-0.1816)))^{-1}$$

$$= 0.8970.$$

The resulting $l^*(x)$ values are shown in column (4) of table 138.

(d) *Comment on the detailed example*

There are several points to note in this detailed example. First, the final α^* and β^* estimates are very far from the neutral values of 0.0 and 1.0, respectively, for the standard being used, suggesting that another standard might have been a better model of the estimates of survivorship being linked in this example. Secondly, the consistency of the $\beta_j(x)$ estimates with age becomes less and less satisfactory as the iteration proceeds, so that the initial range (from 0.796 to 0.879) widens greatly by iteration six (from 0.604 to 0.813). This is a further indication that the standard used may not be suitable. In this regard, it must be noted that no allowance has been made for the fact that different survivorship estimates refer to different periods. It is possible that the pattern of the standard used may not represent adequately the various mortality levels that have prevailed in the past. Thirdly, it will be seen that in this case six iterations were needed for β to converge; use of the proposed short-cut would have reduced the number somewhat. It may be pointed out in passing that the selection of $\beta_1 = 1.0$ as the beginning value of β has no impact on the final estimates to which the procedure converges.

The life table shown in column (4) of table 138 and implied by the values of α^* and β^* just estimated is moderately similar at earlier ages to the life table given in column (3) of the same table (generated by the smoothing and interpolation method presented in subsection B.3), but the two tables become increasingly different as age advances. The survivorship probabilities shown in column (4), generated by this linkage method, are always higher than those in column (3), generated by the smoothing and interpolation method, and become increasingly higher at older ages. Although it may not be immediately evident, the observed differences in the final life-table estimates are due in part to differences in the original survivorship estimates themselves and do not arise entirely from the smoothing and linking techniques used. It should also be pointed out that the $l(x)$ values given in column (4) of table 138 cannot be believed. It cannot be accepted that 55 per cent of women in Panama will actually reach age 80, and the mortality rates for advanced ages implied by the $l(x)$ function are unacceptably low. In this case, the errors in the data are such that a less flexible smoothing procedure than that based on the logit system is required. This example should serve as a warning that alternative estimation methods, even if they employ the same data (in this case the proportions of male respondents with surviving first wives), do not always yield the same results. Because general rules about the reliability of the results yielded by these methods cannot be given, it is recommended that as many techniques as possible be applied to a particular data set, so that decisions about the validity of the results can be based on a thorough appraisal.

3. *Use of Coale-Demeny model life tables*

In a one-parameter model life-table system such as that proposed by Coale and Demeny,[5] any survivorship probability, whether from birth or conditional on having reached a certain age, uniquely determines a life table within the system, once a family of models has been selected. Thus, each child survivorship probability from birth, $l(z)$, implies a life table, as does each conditional survivorship probability of the type $l(x)/l(y)$ (see chapters III and IV for descriptions of the procedure followed in determining these life tables). The information for the two distinct age ranges can be linked together by adopting the life table implied by the average level of the child mortality estimates up to age y (which can be regarded as the pivotal age) and then completing the life table over age y by applying the conditional probabilities of survivorship from age y consistent with the average level implied by the adult survivorship estimates.

The procedure can be illustrated by using again the data referring to Panamanian females. Table 135 shows female probabilities of surviving, $l(z)$, from birth to ages 2, 3 and 5, obtained from information on female children ever born and surviving classified by age of mother. Each $l(x)$ value implies a mortality level in a family of Coale-Demeny model life tables; column (6) shows the levels implied in the West family. The average female child mortality level is 19.5. The conditional female adult survivorship probabilities estimated from data on the widowhood status of male respondents classified by duration also imply mortality levels, as is shown in table 139; the average mortality level is 21.1. The data therefore suggest that, for females, and ignoring timing effects, child mortality in Panama is approximated by level 19.5 in the West family of model life tables for females, whereas adult mortality is approximated by level 21.1. A composite life table is then constructed by adopting a level 19.5 life table for females up to age 20, calculated by averaging the $l(x)$ functions of the life tables of levels 19 and 20 shown in table 236 (see annex VIII), from which age onward the life table is completed by applying survivorship probabilities from age 20 taken from a level 21.1 life table for females. The latter value can be calculated up to age 60 by weighting the values of $l(n)/l(20)$ for levels 21 and 22 in table 222 (see annex VII), those for level 22 being weighted by 0.1 and those for level 21 by 0.9 (that is, linear interpolation is used). The ${}_nq_x$ values of the resultant life table would show some irregularity around the pivotal age, but the quantitative importance of any discontinuity would be negligible. The final $l^*(x)$ values of the life table estimated in this manner are shown in column (5) of table 138.

In comparing these $l^*(x)$ estimates with those shown in column (4), it is remarkable that the latter values are quite different, especially at older ages. Since these two $l(x)$ functions have been fitted to exactly the same survivorship probabilities, the differences observed can only be ascribed to differences in the procedures used and in the models underlying them. Because the mortality pattern of the general standard is fairly similar to that of the West family of Coale-Demeny models, the choice of standard is unlikely to be the cause of the differences observed. It seems more likely that these divergences have arisen because the procedure based on the Coale-Demeny models introduces a more powerful smoothing component than does the one based on the logit system. Indeed, if one considers the West mortality levels associated with each of the conditional probabilities of survival presented in table 139, it is immediately evident that they cover a fairly wide range of levels. It is not surprising therefore that the shape of the general standard has to be twisted considerably in order to approximate them all at the same time (thus, the estimated β is substantially lower than one). In fitting Coale-Demeny life tables no such twisting is involved, since a single level (21.1 in this case) is used to represent

[5] *Op. cit.*

adult mortality in general. In this instance, the smoothed life table obtained by using the Coale-Demeny system is much more plausible than that obtained by using the logit system, which, being more flexible, incorporates data distortions as well as the actual underlying mortality pattern. This example, therefore, helps to underscore the fact that these methods of fitting cannot be used mechanically. The user must be well aware of the nature and significance of the data at hand in order to establish which elements are truly compatible and merit being used as input for the fitting procedures described here.