Chapter II

ESTIMATION OF FERTILITY BASED ON INFORMATION ABOUT CHILDREN EVER BORN

A. BACKGROUND OF METHODS

1. *Nature of information on children ever born*

This chapter describes methods of fertility estimation based on data about children ever born. The number of children ever born to a particular woman is an aggregate measure of her lifetime fertility experience up to the moment at which the data are collected. This number conveys no information about timing, whether on a personal scale, such as age or duration of marriage, or on an external scale, such as calendar years. When women are grouped according to some other variable, such as age or duration of marriage, the average number of children ever borne by the group, also known as their average parity, can be computed by dividing the total number of children borne by the women in the group by the total number of women in the group. The result is a measure of the average lifetime fertility experience of the survivors of a birth or marriage cohort, but as before it conveys no information about the timing of the births. When average-parity data are used for analytical purposes, additional information about timing, either personal or external, is introduced from other sources or inferred from additional assumptions. Strictly speaking, the average parities refer to the survivors of particular cohorts, though it is generally assumed that the distortion introduced by female mortality may be disregarded.

The foregoing general account gives some indication of the strengths and weaknesses of data on children ever born. Their main strength is that no dating is involved, so that the data cannot be distorted by dating errors. This strength is also, of course, a weakness, as by themselves the data refer to no clearly defined time period, providing no information about age patterns or time trends of fertility. Another weakness is that the information must generally be collected in the form of numbers, which may be subject to relatively greater errors than is information derived from questions with a simple "yes" or "no" response.

The simplest way of collecting information on children ever born is by the question "How many children have you ever borne alive?", the expected answer being a number. Note that this question concerns only children born alive, excluding stillbirths and other foetal deaths. It is important to adhere to this definition because all the methods of analysis currently available are based strictly on live births, and because it is likely that the completeness of reporting of stillbirths may vary widely from one society to another. The simple question quoted above has been extensively used, but examination of the results it yields suggests that children are sometimes omitted, particularly by women aged 35 and over. Children who have left home or who have died are particularly likely to be omitted, so that questions focusing on these two groups may be expected to produce better results (practice seems to support this expectation, although it is difficult to disentangle the effects of improved education and improved phrasings of the basic question). It is therefore recommended that when the time allotted to each interview and the questionnaire space permit, the basic question be broken down into three parts, becoming "Of the children you have ever borne alive, how many: 1. Are still living at home with you?; 2. Are still living, but elsewhere, in some other household?; 3. Have died?". This form of the basic question has the advantage of providing the additional information necessary for the estimation of child mortality (see chapter III). A further refinement is to collect information on children ever born classified by sex, asking the three questions presented above for children of each sex. Data classified by sex provide a basis for the estimation of child mortality by sex and prove useful in the assessment of data quality.

It is also necessary to decide from which women the information should be collected. In general, the best procedure is to put the necessary questions to all women over the age of 15, regardless of their marital status. In some societies, however, it is culturally impossible to ask women who have never been married whether they have had any children; and in these cases, the questions can only be put to women who have been married at least once (ever-married women). It is important to recognize, however, that in this case the value of the information collected will vary inversely with the incidence of premarital fertility.

Data on children ever born are generally tabulated by five-year age groups of women, or by five-year duration of marriage groups if the necessary data are available. A simple tabulation of number of women, number of children ever born, and average parity for each group should be supplemented by a tabulation of women classified by both age or duration group and by the number of children ever born (0, 1, 2 ... 10, 11+, not stated). When data by sex are available, all these tabulations should be repeated by sex of children.

A special form of data on children ever born is collected by inquiries about fertility histories or pregnancy histories which obtain information about the date of occurrence of each birth or about the termination of each pregnancy, and often inquire as well about the subsequent fate of the child, recording the date of death when appropriate. The collection of a fertility history is time-consuming and hence fairly expensive, so that it has been limited in practice to relatively small-scale sample surveys. It is clear that this form of collection of data on children ever born also provides information about the timing of fertility, but at the cost of recording specific dates which are more likely to be subject to error.

A thorough analysis of the data gathered by fertility histories often requires fairly detailed tabulations of the data and access to them in machine-readable form. Thanks to the vast effort undertaken by the World Fertility Survey, the availability of both fertility-history data and methods designed specifically to analyse them has been greatly enhanced. Because the detailed description of these methods is beyond the scope of this *Manual*, the reader is referred to the series of publications issued as part of the World Fertility Survey. Of special interest is the collection of scientific reports which present detailed examples of the analysis and evaluation of data on fertility histories. Some of these analyses are cited in this chapter and others are listed at the end of the chapter.

2. *Typical errors in data on children ever born*

Average parities for groups of women calculated from data on children ever born can be distorted either by errors in the number of children reported or by errors in the classification of women in particular groups.

The most important error in the number of children reported is due to omission. Women tend to omit some of their live-born children, particularly those living in other households and those who have died, with the result that the proportion omitted tends to increase with age of mother. The symptoms of such omission are average parities that fail to increase rapidly enough as age increases; and, in some cases, average parities for women aged 40-44 and 45-49 may actually fall below that for women aged 35-39 even when there is no reason to suppose that fertility has been rising. Similar biases may be observed for duration-of-marriage groups 15-19 and higher. The effects of omission, if limited to women over 35, are not very serious because most methods of analysis make little use of the data referring to these women. However, in order to minimize omission at younger ages (under 35), it is advisable to stress at the data-collection stage the importance of obtaining accurate information from all women, regardless of age.

Another error in the reported children ever born arises from the inclusion of stillbirths or late foetal deaths among live-born children. The possible upward effect of this error on average parity is small, but stress should be laid during the data-collection process on including only live-born children. (A live-born child is generally defined as one who cries after birth.)

A third error affecting the recorded number of children ever born is introduced when the parity of a sizeable proportion of women is not recorded, that is, when there is a non-trivial proportion of women whose parity is not stated. If these women have in fact borne some children, their inclusion in the denominator of average parity, but the exclusion of their children from the numerator, will bias average parity downward. If it can be assumed that the women who provided information are representative, in parity terms, of those who did not, an unbiased estimate of average parity can be obtained by dividing the reported number of children by the number of women reporting. However, in many surveys there seems to be a disproportionate tendency for childless women to be tabulated as "not stated". A plausible explanation for this tendency has been suggested by El-Badry,[1] who argues that interviewers tend to leave blank the space for recording children ever born in cases where the true number is zero; at the coding stage, such blanks are interpreted as non-response, thus causing a net transfer of women from the zero category to the non-stated category. In such circumstances, average parity would be inflated if women tabulated as "not stated" were subtracted from the denominator. El-Badry also proposed a method by which the true incidence of non-response may be estimated by considering the relationship between the proportion of women with no children and the proportion of women whose parity was not stated; his method of adjustment is fully described in annex II. Its use is recommended when the relationship mentioned is linear in nature; otherwise, it is recommended that women whose parity is not stated should be included in the denominator when calculating average parities.

Misclassification errors arise from misreporting the age or duration of marriage of the women considered. When grouped data are used, errors arise only in so far as a woman is transferred from one group to another. The effects of such errors are complex, but certain principles can be outlined. Random errors in reporting age or duration are likely to have a slight equalizing effect on average parities, since a transfer upward that probably reduces the average parity of the higher group is likely to be matched by a transfer downward that probably increases the average parity of the lower group. However, the overall effect is expected to be small, since the misclassified women are likely to be close to the boundary of adjacent groups, with those transferred upward having parities above the average and those transferred downward having parities below the average for women of their true ages.

In contrast, a systematic transfer upward, in age or duration terms, is likely to reduce average parities for all groups, with the effect declining as age or duration increases, until it disappears in the age or duration group into which only women of completed fertility are shifted. Similarly, a systematic transfer downward will increase average parities, the effect again declining as age or duration increases, until it disappears for that age or duration group containing the point at which all childbearing has ceased. A systematic tendency to shift

[1] M. A. El-Badry, "Failure of enumerators to make entries of zero: errors in recording childless cases in population censuses", *Journal of the American Statistical Association*, vol. 56, No. 296 (December 1961), pp. 909-924.

age or duration upward until reaching some boundary, followed thereafter by a tendency to shift downward, concentrates respondents in a central group at the expense of the extremes and may result in relatively low average parities for groups below the boundary, an approximately correct average parity for the group that contains the boundary and relatively high average parities for the groups that lie above the boundary. In this case, groups lying above the upper limit of childbearing would exhibit correct average parities if age misstatement were the only reporting problem.

Because marriage is a more recent and more personally memorable event than one's own birth, it would seem that data classified by duration of marriage might be less distorted by dating errors than data classified by age, if duration is measured from a reported date of marriage. However, such data suffer from possible ambiguity about the date of marriage. The analyst is interested in the length of time since sexual relations began; but in some societies the onset of intercourse may predate formal marriage, and in others it may not occur immediately upon formal marriage. Further confusion exists in the case of remarriages, since the date reported may be that of the second or most recent marriage. This problem may be minimized by asking a question that refers specifically to the first marriage. Hence, it is important that interviewers and survey planners be aware of the conceptual problems related to this topic in order to devise the best data-gathering mechanisms to capture the information required.

It is generally assumed, when analysing data on children ever born, that the effects of mortality among women are negligible. The issue is important if one is comparing the average parity of a cohort at two different times, since the normal assumption is that the change in average parity between the two points is accounted for entirely by fertility in the intervening period. Ignoring mortality effects, one thus assumes that those members of the cohort who did not survive the period experienced similar fertility levels up to the time of their deaths as did the survivors, an assumption that is not likely to hold strictly in practice. It is not clear, however, in what direction the effects of mortality on average parity would be: if high-parity women experience above-average mortality risks, then the average parity reported by older women will underestimate the true level of cohort fertility; on the other hand, in developed countries, unmarried women experience higher mortality than married women, suggesting that low-parity women may be subject to higher mortality risks, in which case reported average parity will overestimate cohort fertility. However, the effects of mortality on average parity are likely to be very small, since in most countries today the mortality risks experienced by women in their childbearing years are fairly low.

Migration poses a problem similar to that introduced by mortality, though potentially more serious, particularly at the subnational level. Average parities for particular areas may be distorted by the migration of women not typical of the area, and changes between two points in time may also be distorted by migration. Thus, for example, the average parity for a city experiencing an influx of migrants may be inflated by the arrival of high-parity rural women. The problem arises, of course, from the timeless nature of the data on average parity, compounded, in this case, by a lack of information on place of previous residence. There is no way to resolve this problem other than by using areas that are not much affected by net migration or by resorting to other types of data. It would seem attractive to tabulate women by birthplace and children ever born by birthplace of mother, and to limit analysis to women born in the area being considered; but the estimates obtained would still not necessarily represent fertility in the area considered because immigrants may bring with them not only high historical fertility—and hence above-average parity—but above-average current fertility. Furthermore, if fertility behaviour is affected by place of residence, fertility estimates derived from data classified by place of birth would not represent adequately the current regional fertility differentials that are of greatest interest.

3. *Organization of this chapter*

All the estimation methods presented in this chapter have one characteristic in common: they use data on children ever born. However, the methods can be separated into categories according to the exact type of data they require (whether classified by age or by duration of marriage, for example). Sections B-D present the available methods divided into these categories. To aid the user in selecting the method best suited for a particular application, brief descriptions of each section are given below:

Section B. Methods of the Brass type based on comparison of cumulated age-specific fertility rates with reported average parities. This section presents several methods based on the idea, first proposed by Brass,[2] of comparing reported average parities with those estimated from period age-specific fertility rates. Their main characteristic, therefore, is that they require the availability of at least two types of information on fertility: children ever born for at least one point in time; and age-specific fertility rates referring to some period of interest. Variations of the basic method arise because of variations in the assumptions underlying it or because of the greater or lesser availability of data. In general, the methods presented in section B are ordered on the basis of their data requirements, that is, those presented earlier usually require less information than those presented later. Table 5 lists the data requirements of each method. It should be noted that all methods described in this section use data classified by age;

Section C. Estimation of age-specific fertility from the increments of cohort parities between two surveys. The method presented in section C is based exclusively on data on children ever born. Independently calculated

[2] William Brass, "Uses of census or survey data for the estimation of vital rates" (E/CN.14/CAS.4/V57), paper prepared for the African Seminar on Vital Statistics, Addis Ababa, 14-19 December 1964.

TABLE 5. SCHEMATIC GUIDE TO CONTENTS OF CHAPTER II

Section	Subsection and method	Type of input data	Estimated parameters
B. Methods of the Brass type based on comparison of period fertility rates with reported average parities	B.2. P/F ratio method based on data about all children	Children ever born classified by five-year age group of mother Births in a year classified by five-year age group of mother Women by five-year age group Total population	Adjusted age-specific fertility rates Adjusted total fertility Adjusted birth rate
	B.3. P_1/F_1 ratio method: first births	Women with at least one child classified by five-year age group First births in a year by five-year age group of mother Women classified by five-year age group	Adjusted first-birth age-specific fertility rates Adjusted overall proportion of mothers
	B.4. P/F ratio method for a hypothetical intersurvey cohort	Children ever born classified by five-year age group of mother from two surveys or censuses five or 10 years apart Births in the year preceding each survey or census classified by five-year age group of mother or, failing that, an estimate of intersurvey age-specific fertility rates (from vital registration data, for example) The number of women enumerated by each survey or census, classified by five-year age group	Adjusted intersurvey fertility schedule Adjusted intersurvey total fertility
	B.5. P/F ratio method for true cohorts	Children ever born classified by five-year age group of mother from a census Births registered during each of the 15 or 20 years preceding the census, classified by five-year age group of mother Women classified by five-year age group enumerated by the census gathering information on children ever born and by censuses taken during the 15 or 20 years preceding it	Estimates of completeness of birth registration Adjusted age-specific fertility rates for some period preceding the census Adjusted total fertility for the same period Adjusted birth rates for the same period
	B.6. P/F ratio method for hypothetical intercensal cohorts using registered births	Children ever born classified by five-year age group of mother from two censuses five or 10 years apart Births registered during each of the years of the intercensal period, classified by five-year age group of mother Women classified by five-year age group from the two censuses The total population according to each census	An estimate of the completeness of birth registration Adjusted intercensal age-specific fertility rates Adjusted intercensal total fertility Adjusted intercensal birth rate
C. Estimation of age-specific fertility from the increment of cohort parities between two surveys	C.2. Use of parity increments	Children ever born classified by five-year age group of mother, from two surveys or censuses five or 10 years apart Women classified by five-year age group from the two surveys or censuses	Intersurvey age-specific fertility rates Intersurvey total fertility
D. Estimation of fertility from information on children ever born classified by duration of marriage	D.2. Estimation of a natural fertility level	Children ever born classified by five-year duration of marriage group of mother Ever-married women classified by five-year duration of marriage group Total female population classified by five-year age group and by marital status (single, married, widowed and divorced) First age at which a significant number of marriages occurs The total population	Adjusted marital age-specific fertility rates Adjusted age-specific fertility rates Adjusted total fertility Adjusted birth rate

TABLE 5 *(continued)*

Section	*Subsection and method*	*Type of input data*	*Estimated parameters*
D. Estimation of fertility from information on children ever born classified by duration of marriage *(continued)*	D.3. *P/F* ratio method for data by duration of marriage	Children ever born classified by five-year duration of marriage group of mother Births in a year classified by five-year duration of marriage group Ever-married women classified by five-year duration of marriage group The total population	Adjusted duration-specific fertility rates Adjusted marital total fertility Adjusted birth rate

age-specific fertility rates are not necessary. All input data need to be classified by age;

Section D. Estimation of fertility from information on children ever born classified by duration of marriage. When data on children ever born classified by the duration of the mother's marriage are available, two methods of fertility estimation can be used. The simplest method requires only the data on children ever born; a more elaborate method permits a comparison of the Brass type between parity information and parity equivalent measures derived from a duration-specific fertility schedule. The latter method requires, therefore, information both on children ever born and on the number of births in a given year classified by mother's duration of marriage (see table 5).

B. METHODS OF THE BRASS TYPE BASED ON COMPARISON OF PERIOD FERTILITY RATES WITH REPORTED AVERAGE PARITIES

1. *General description of methods of the Brass type*

The total number of children ever borne by a group of women of a given age is a record of their total childbearing experience from the beginning of their reproductive life to their current age. The average number of children ever born, obtained by dividing the number of reported children by the number of women, is therefore a measure of the fertility experience of the cohort of women, though it is a measure of the level of fertility only, containing no information about its timing. If it is assumed that the fertility experience of those women who die is the same up to the age at death as that of those who survive, the average number of children born provides a mortality-free measure of cohort fertility.

A similar period measure may be obtained from age-specific fertility rates. If such rates are cumulated upward from the age at which childbearing begins, taking due account of the width of the age interval for which the rates are specific, the results obtained can be interpreted as the average number of children that would have been borne by women experiencing those fertility rates from the beginning of childbearing to the upper age boundary of the highest age group included in the cumulation.

The availability of information about both lifetime fertility, from a survey question about number of children ever born, and current fertility, from a survey question about births in the past year or date of the most recent birth, or from vital registration data, makes possible a powerful consistency check, whereby current fertility rates can be cumulated and compared with average parity. Such a comparison clearly uses both cohort rates and period rates, but it is valuable even if the two are not expected to be consistent because of changing fertility.

This comparison of lifetime with current fertility data can also provide a method of adjustment for cases where the data are distorted by typical errors. Information on children ever born is frequently distorted by omission, but this omission, perhaps of long-dead children or of those who have left the parental home, is most marked for older women; the reports of younger women, up to age 30 or 35, may be fairly reliable. Information on current fertility from a question on births occurring during the 12 months preceding a survey may be distorted by a misperception of the length of the reference period, so that the reported births correspond to an ill-defined period whose average length may be either shorter or longer than a year. If information on current fertility comes from a vital registration system, the level of the reported fertility rates may be distorted by general omission. If these errors in the information on current fertility may be assumed to be roughly constant with respect to age (an assumption that is particularly appropriate in the case of reference-period error), the age pattern of observed current fertility can be accepted as correct although its level may be distorted.

Cumulated current fertility may be compared with the reported lifetime fertility of women younger than 30 or 35 in order to obtain an adjustment factor for the level of the current fertility rates, which, once adjusted for level, provide a better estimate of actual current fertility. In order for this adjustment to be valid, it must be assumed that the fertility of younger women has not changed appreciably, for if it had changed, their lifetime fertility could not be expected to be consistent with cumulated current fertility rates. Furthermore, when dealing with data classified by five-year age group, cumulated current fertility rates provide an estimate of the average number of children ever borne by women who have reached the end of each age group, whereas parity data provide an estimate of the average number borne by women whose ages vary over the range of the age group. Therefore, a process of interpolation is required to ensure that the figures cover a comparable age range.

The essence of the Brass[3] fertility estimation procedure is the adjustment of the age pattern of fertility derived from information on recent births by the level of fertility implied by the average parity of women in age

[3] *Ibid.*

groups 20-24, 25-29, and perhaps 30-34. Several extensions of the original procedure have been proposed. First, if the reasoning outlined above may be applied to all births, it may also be applied to births of any particular birth order; and cumulated birth-order-specific fertility rates should be comparable with the proportions of women reporting at least that many children ever born. Such a comparison may be particularly revealing in the case of first births. Secondly, if data on children ever born are available from two surveys five or 10 years apart, and average fertility rates may be calculated for the same period, the assumption of constant fertility in the recent past can be relaxed, because average parity for a hypothetical intersurvey cohort can be constructed and compared with parity equivalents derived from intersurvey fertility rates. Lastly, the assumption of constant fertility may be relaxed if information on fertility rates during the past 15 or 20 years is available from a vital registration system or some other data source. The observed fertility rates for a series of true cohorts can be cumulated through the recent past for comparison with the average parities reported by women in successive age groups at the end-point. These useful extensions of the Brass method are described and illustrated below, following the description of the original technique. First, however, some general points that apply to all the versions discussed are considered.

Reported fertility rates are used to estimate the average cumulated fertility or parity equivalent, F, that women in each age interval would have if they had been subject throughout their lives to the reported rates. Two problems arise, however, in obtaining a value of F that is comparable to the average parity, P, reported by women of each age group. First, because fertility data are ordinarily tabulated by five-year age group, cumulating the reported age-specific fertility rates and multiplying by five yields estimates of the parity or cumulated fertility that women experiencing those rates would achieve by the end of each five-year age group (that is, by exact ages 20, 25, 30 etc. when conventional five-year age groups are used). These estimates are not comparable with the average parities calculated from data on children ever born, because the latter values represent the mixed experience of women of different exact ages. Hence, some procedure is required for estimating the average cumulated fertility or parity within each age group from knowledge of the values that the cumulated fertility schedule takes at the end-points of the age groups considered. Secondly, when the current fertility schedule is obtained from a survey question on births during the 12 months preceding the survey or on date of the most recent birth, the births are generally tabulated by the mother's age at the time of the survey, not at the time of the birth. If one assumes that births in a given year are uniformly distributed in time, the women who had a birth in the 12 months preceding the survey were, on average, six months younger at the time of the birth than at the time of the interview. Therefore, the age-specific fertility rates that can be calculated from data on children born during the year before the survey classified by age of mother at the time of the survey correspond to unorthodox age intervals whose limits are (14.5, 19.5), (19.5, 24.5), ..., (44.5, 49.5), rather than to the usual intervals with end-points (15, 20), (20, 25), ..., (45, 50). When the source of information on current fertility is a vital registration system, this second problem should not arise, since births are supposed to be recorded near the time of occurrence and therefore the reported age of mother is likely to be the age she had at the time of the birth. When registered births are used, however, late registrations should be excluded; otherwise, they might seriously distort the age pattern of fertility.

An interpolation procedure based on model fertility schedules has been devised to allow the estimation of parity equivalents (F) for the usual five-year age groups of women from the cumulated fertility schedule. A similar procedure that takes into account the problem of age groups displaced by six months and produces the desired estimates of F (parity equivalents) has also been developed. Hence, two variants of the procedure are available: one suited for use with fertility rates calculated from vital registration data, that is, with births tabulated by age of mother at delivery; and the other suited for use with fertility rates calculated from reported births for a 12-month period tabulated by age of mother at the end of the period.

The Brass fertility estimation method and the variants presented in this chapter are best suited for estimating fertility in countries where massive systematic age-misreporting is not apparent. Its application to populations where age is poorly reported is likely to yield biased results. Fertility estimates obtained by applying the original method to populations where either marital fertility or age at marriage has been changing rapidly in the recent past may also be subject to bias, since it would no longer be valid to assume that the "historical" pattern of fertility implied by the reported average parities is equal to that embodied by the current fertility schedule. However, when a fertility decline is due mainly to effective use of contraception at relatively older ages, the method described here may still yield valid results if the adjustment factor is selected on the basis of information pertaining to the youngest age groups (20-24 is recommended).

2. *The P/F ratio method based on data about all children*

(a) *Basis of method and its rationale*

The original P/F ratio method or Brass method seeks to adjust the level of observed age-specific fertility rates, which are assumed to represent the true age pattern of fertility, to agree with the level of fertility indicated by the average parities of women in age groups lower than ages 30 or 35, which are assumed to be accurate. Measures of average parity equivalents, F, comparable to reported average parities, P, are obtained from period fertility rates by cumulation and interpolation (these measures are effectively averages of the cumulated fertility schedule over age groups). Ratios of average parities (P) to the estimated parity equivalents (F) are calculated age group by age group, and an average of the ratios obtained for younger women is used as an adjustment factor by which all the observed period fertility rates are multiplied. Note that P/F ratios are generally calculated for the entire age range from 15 to 49, even though not all the ratios are used for adjustment purposes. This practice is recommended because the pattern of the ratios with age may reveal data errors or fertility trends. During successful application of this method, the age pattern of the period fertility rates is combined with the level implied by the average parities of younger women to derive a set of fertility rates that is generally more reliable than either of its constituent parts.

(b) *Data required*

The following data are required for this method:

(*a*) The number of children ever born classified by five-year age group of mother;

(*b*) The number of children born during the year preceding the survey or census classified by five-year age group of mother, or the number of registered births in the year of the census, also classified by five-year age group of mother;

(*c*) The total number of women in each five-year age group (irrespective of marital status);

(*d*) The total population if the birth rate is to be estimated.

(c) *Computational procedure*

Every function in this section is indexed by a variable (i or j) the values of which represent the age groups being considered. Table 6 summarizes the relationship between the index number and the age group.

TABLE 6. CORRESPONDENCE BETWEEN INDICES AND AGE GROUPS

Index value i or j (1)	*Age group (2)*
0	10-14
1	15-19
2	20-24
3	25-29
4	30-34
5	35-39
6	40-44
7	45-49

The steps in the procedure are described below.

Step 1: calculation of reported average parities. The reported average parity of women in age group i is denoted by $P(i)$. Its value is obtained by dividing the total number of children ever born to women in age group i by the total number of women in that age group (whether married or single, fertile or not). See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the use of the adjustment method proposed by El-Badry (see annex II).

Step 2: calculation of a preliminary fertility schedule from information on births in the past year or from registered births. The fertility rate of women in age group i is denoted by $f(i)$. This value is computed for each i by dividing the number of births occurring to women in age group i during the year preceding the interview by the total number of women (whether childless or not, ever married or not) in that age group. In the case of registered births, the births by age group recorded for a calendar year should be divided by an estimate of the mid-year female population of the age group (usually obtained from a census).

Step 3: calculation of cumulated fertility schedule for a period. To calculate this schedule, denoted by $\phi(i)$, the fertility rates computed in step 2 are added, beginning with $f(1)$ (or with $f(0)$ if its value is not zero) and ending with $f(i)$. The value of this sum mutiplied by five is an estimate of cumulated fertility up to the upper limit of age group i. The formal definition of $\phi(i)$ is

$$\phi(i)=5\left[\sum_{j=0}^{i} f(j)\right]. \qquad \text{(B.1)}$$

Step 4: estimation of average parity equivalents for a period. Average parity equivalents, $F(i)$, are estimated by interpolation using the period fertility rates $f(i)$ and the cumulated fertility values $\phi(i)$ calculated in previous steps. Several procedures have been proposed for this interpolation. Brass[4] uses a simple polynomial model of fertility to calculate the relationship between average parity and cumulated fertility for successive age groups for a range of age locations of the fertility model. Coale and Trussell[5] propose fitting a second-degree polynomial to three consecutive values of $\phi(i)$ and estimating the average parity of women of an age group within the range by evaluating the integral of the polynomial; in an actual application, $F(i)$ is obtained as

$$F(i)=\phi(i-1)+af(i)+bf(i+1) \qquad \text{(B.2)}$$

where a and b are constants whose values are shown in table 7 for i = 1, 2, ..., 6. $F(7)$ is obtained as

$$F(7)=\phi(6)+a^{*}f(6)+b^{*}f(7) \qquad \text{(B.3)}$$

and the values of a^* and b^* are also displayed in table 7. A somewhat more accurate procedure is based on the general principle underlying equation (B.2), but it allows the constants a and b to vary with i. The interpolation equation used is

$$F(i)=\phi(i-1)+a(i)f(i)+b(i)f(i+1)+c(i)\phi(7). \qquad \text{(B.4)}$$

Values of the parameters a, b and c were estimated by using least-squares regression to fit equation (B.4) to a large number of model cases constructed using the

[4] *Ibid.*

[5] Ansley J. Coale and T. James Trussell, "Model fertility schedules: variations in the age structure of childbearing in human population", *Population Index*, vol. 40, No. 2 (April 1974), pp. 185-258.

TABLE 7. COEFFICIENTS FOR INTERPOLATION BETWEEN CUMULATED FERTILITY RATES TO ESTIMATE PARITY EQUIVALENTS

Age group (1)	Index i (2)	Equation No. (3)	Coefficients: $a(i)$ (4)	$b(i)$ (5)	$c(i)$ (6)
(a) *Fertility rates calculated from births in a 12-month period by age of mother at end of period*					
15-19-40-44	1-6	B.2	3.392	−0.392	-
45-49	7	B.3	0.392	2.608	-
15-19	1	B.4	2.531	−0.188	0.0024
20-24	2	B.4	3.321	−0.754	0.0161
25-29	3	B.4	3.265	−0.627	0.0145
30-34	4	B.4	3.442	−0.563	0.0029
35-39	5	B.4	3.518	−0.763	0.0006
40-44	6	B.4	3.862	−2.481	−0.0001
45-49	7	B.4	3.828	0.016[a]	−0.0002
(b) *Fertility rates calculated from births by age of mother at delivery*					
15-19-40-44	1-6	B.2	2.917	−0.417	-
45-49	7	B.3	0.417	2.083	-
15-19	1	B.4	2.147	−0.244	0.0034
20-24	2	B.4	2.838	−0.758	0.0162
25-29	3	B.4	2.760	−0.594	0.0133
30-34	4	B.4	2.949	−0.566	0.0025
35-39	5	B.4	3.029	−0.823	0.0006
40-44	6	B.4	3.419	−2.966	−0.0001
45-49	7	B.4	3.535	−0.007[a]	−0.0002

[a] This coefficient should be applied to $f(i-1)$, not $f(i+1)$, that is, to $f(6)$ instead of $f(8)$.

Coale-Trussell[6] fertility model. Note that an additional constant term, $c(i)\phi(7)$, is introduced in equation (B.4). This term is effectively an estimated coefficient, $c(i)$, weighted by the observed total fertility rate, $\phi(7)$. In theory, the inclusion of a constant term in equation (B.4) is unsatisfactory because, if $f(i)$ and $f(i+1)$ were zero, $F(i)$ should be identical to $\phi(i-1)$. In practice, however, such degenerate fertility schedules are not encountered; and the restrictions imposed by such theoretical considerations do not warrant the loss of flexibility they would imply in obtaining the best possible fit to the model data.

Table 7 shows the values of the coefficients required for the use of equation (B.4). The table is divided into two parts: the first part presents coefficients for use with fertility rates derived from births in the 12 months before a survey tabulated by age of mother at the time of the survey; and the second presents coefficients for use with fertility rates calculated from births classified by age of mother at the time of delivery.

Step 5: calculation of a fertility schedule for conventional five-year age groups. When age-specific fertility rates have been calculated from births in a 12-month period classified by age of mother at the end of the period, they are specific for unorthodox age groups that are shifted by six months. A fertility schedule for conventional five-year age groups, $f^+(i)$, can be estimated by weighting the rates referring to unorthodox age groups according to equations (B.5) and (B.6), and using the coefficients displayed in table 8. Note that when fertility rates have been calculated from births classified by age of mother at the time of delivery, this step is not required:

TABLE 8. COEFFICIENTS FOR CALCULATION OF WEIGHTING FACTORS TO ESTIMATE AGE-SPECIFIC FERTILITY RATES FOR CONVENTIONAL AGE GROUPS FROM AGE GROUPS SHIFTED BY SIX MONTHS

Age group (1)	Index i (2)	Coefficients: $x(i)$ (3)	$y(i)$ (4)	$z(i)$ (5)
15-19	1	0.031	2.287	0.114
20-24	2	0.068	0.999	−0.233
25-29	3	0.094	1.219	−0.977
30-34	4	0.120	1.139	−1.531
35-39	5	0.162	1.739	−3.592
40-44	6	0.270	3.454	−21.497

$$f^+(i)=(1-w(i-1))f(i)+w(i)f(i+1) \quad (B.5)$$

where $f(i)$ and $f^+(i)$ are, respectively, the unadjusted and adjusted age-specific fertility rates; and the weighting factor, $w(i)$, is calculated as

$$w(i)=x(i)+y(i)f(i)/\phi(7)+z(i)f(i+1)/\phi(7). \quad (B.6)$$

The values of $x(i)$, $y(i)$ and $z(i)$ were obtained by fitting equation (B.6) by least-squares regression to the same model cases used in deriving the coefficients presented in table 7. No weighting factor is needed for $i=7$, as childbearing is assumed to cease after age 50; and $f^+(7)$ is therefore taken to be $(1-w(6))f(7)$. Births reported to women under age 15 can be included among those reported by women aged 15-19.

Step 6: adjustment of period fertility schedule. With the quantities computed in steps 1-4, the ratios $P(i)/F(i)$ are calculated. Ideally, these ratios should be fairly simi-

[6] *Ibid.*

lar for different values of i, although if children ever born are increasingly omitted by older women, the ratios will tend to decrease as age increases (especially over ages 30 or 35). In practice, however, they are often far from being constant, even below age 35; and one can be satisfied if $P(2)/F(2)$ and $P(3)/F(3)$ are reasonably consistent. If this is the case, either one of them can be used as an adjustment factor for the period fertility rates. If they are not very similar, a weighted average of the two can be used (using as weights the number of women in age groups 20-24 and 25-29 as a proportion of all women aged 20-29), as can a simple, unweighted average of them. However, if there is evidence suggesting that the population is experiencing a fertility decline affecting mainly women in the older age groups, the value of $P(2)/F(2)$ is recommended as an adjustment factor because it is less likely to be affected by the decline. In general, $P(1)/F(1)$ should be disregarded because of the intrinsic difficulty in estimating $F(1)$, and the P/F ratios for age groups over 30 cannot be regarded as reliable due to the possible omission of children ever born. Naturally, the more consistent the set of ratios obtained, the more confidence one can have in the adjustment factor selected. Certain patterns of variation of the ratios with age may also reveal the types of problems present. For example, a recent decline in fertility tends to produce a sequence of P/F ratios that increases with age.

Once an adjustment factor has been chosen (one may denote it by K), an adjusted fertility schedule is computed by multiplying the fertility rates for conventional age groups, $f^{+}(i)$, if the rates were originally for age groups shifted by six months, but $f(i)$ if the rates were originally for conventional age groups, by K, to yield adjusted $f^{*}(i)$ values:

$$f^{*}(i) = Kf^{+}(i), \text{ or } f^{*}(i) = Kf(i). \quad \text{(B.7)}$$

Once all the $f^{*}(i)$ values are available, one may calculate total fertility, TF, which is defined as

$$TF = 5\left[\sum_{i=1}^{7} f^{*}(i)\right]. \quad \text{(B.8)}$$

An estimate of the adjusted birth rate can be obtained by multiplying each of the adjusted fertility rates by the number of women in the relevant age group to estimate numbers of births, adding these results for all ages and then dividing their sum by the total population.

The calculation of these and other parameters is described in the following example.

(d) *A detailed example*

Table 9 shows data on the number of children ever born and children born in the year preceding the survey for women who were interviewed during a demographic survey conducted in Bangladesh in 1974.

The steps of the calculation are given below.

Step 1: calculation of reported average parities. Values of the reported average parities, $P(i)$, are obtained by dividing the numbers listed in column (3) (children ever born) of table 9 by those appearing in column (2) (number of women). Results are given in table 10; shown below is the way in which $P(3)$ was obtained (it will be recalled that index 3 refers to age group 25-29):

TABLE 9. CHILDREN EVER BORN AND BIRTHS IN THE PAST YEAR, BY AGE GROUP OF MOTHER, BANGLADESH, 1974

Age group (1)	Number of women (2)	Children ever born (3)	Births in past year (4)
15-19	3 014 706	1 160 919	320 406
20-24	2 653 155	4 901 382	609 269
25-29	2 607 009	9 085 852	561 494
30-34	2 015 663	9 910 256	367 833
35-39	1 771 680	10 384 001	237 297
40-44	1 479 575	9 164 329	95 357
45-49	1 135 129	6 905 673	38 125

$$P(3) = (9{,}085{,}852/2{,}607{,}009) = 3.485.$$

In the case of Bangladesh, data were available to apply the El-Badry correction for non-response. The estimated level of non-response was so low, however, as to be insignificant. The figures on number of women shown in column (2) of table 9 therefore include all women, even those for whom parity was not stated.

TABLE 10. AVERAGE PARITIES, PERIOD FERTILITY RATES AND CUMULATED FERTILITY, BY AGE GROUP OF MOTHER, BANGLADESH, 1974

Age group (1)	Index i (2)	Average parity per woman $P(i)$ (3)	Period fertility rate $f(i)$ (4)	Cumulated fertility $\phi(i)$ (5)
15-19	1	0.385	0.1063	0.5315
20-24	2	1.847	0.2296	1.6795
25-29	3	3.485	0.2154	2.7565
30-34	4	4.917	0.1825	3.6690
35-39	5	5.861	0.1339	4.3385
40-44	6	6.194	0.0644	4.6605
45-49	7	6.084	0.0336	4.8285

Step 2: calculation of preliminary fertility schedule. The values of this schedule, denoted by $f(i)$, are computed by dividing the entries in column (4) (births in the past year) of table 9 by those in column (2) (number of women). The value of $f(3)$, for example, is calculated as

$$f(3) = (561{,}494/2{,}607{,}009) = 0.2154.$$

Other values of $f(i)$ are given in table 10.

Step 3: calculation of cumulated fertility schedule. The values of $\phi(i)$, the cumulated fertility schedule, are obtained by adding the values of $f(j)$, beginning with $j=1$ and ending with $j=i$, and then multiplying this sum by five (this number is used because five-year age groups are being considered). Final results are shown in column (5) of table 10. As an example, $\phi(4)$ is computed as

$$\phi(4) = 5(0.1063 + 0.2296 + 0.2154 + 0.1825$$
$$= 5(0.7338) = 3.6690.$$

Step 4: estimation of average parity equivalents for a period. Period fertility rates were calculated from births in the 12 months preceding the survey, tabulated by age of mother at the time of the survey; therefore, coefficients from part (a) of table 7 should be used to estimate the current average parity equivalents, $F(i)$. The computation of $F(1)$, $F(4)$ and $F(7)$ is illustrated, using the values of $\phi(i)$ and $f(i)$ listed in table 10 (full results are shown in column (4) of table 11):

$$F(1)=\phi(0)+a(1)f(1)+b(1)f(2)+c(1)\phi(7)$$

$$=0.0+(2.531)(0.1063)+(-0.188)(0.2296)+$$

$$(0.0024)(4.8285)=0.237;$$

$$F(4)=\phi(3)+a(4)f(4)+b(4)f(5)+c(4)\phi(7)$$

$$=2.7565+(3.442)(0.1825)+(-0.563)(0.1339)+$$

$$(0.0029)(4.8285)=3.323;$$

$$F(7)=\phi(6)+a(7)f(7)+b(7)f(6)+c(7)\phi(7)$$

$$=4.6605+(3.828)(0.0336)+(0.016)(0.0644)+$$

$$(-0.0002)(4.8285)=4.789.$$

TABLE 11. AVERAGE PARITIES, ESTIMATED PARITY EQUIVALENTS AND P/F RATIOS, BANGLADESH, 1974

Age group (1)	Index i (2)	Average parity per woman $P(i)$ (3)	Estimated parity equivalent $F(i)$ (4)	P/F ratio $P(i)/F(i)$ (5)
15-19	1	0.385	0.237	1.624
20-24	2	1.847	1.209	1.528
25-29	3	3.485	2.338	1.491
30-34	4	4.917	3.323	1.480
35-39	5	5.861	4.094	1.432
40-44	6	6.194	4.503	1.376
45-49	7	6.084	4.789	1.270

Step 5: calculation of a fertility schedule for conventional five-year age groups. Because what is being considered now is a case in which births in the past year were tabulated by age of mother at the time of the survey, the reported period rates, $f(i)$, need to be converted into a fertility schedule, $f^+(i)$, for conventional age groups. Conversion is carried out by using equations (B.5) and (B.6). A detailed example of the calculation of $f^+(1)$, $f^+(4)$ and $f^+(7)$ is given below; other values of f^+ are shown in table 12. The totals of columns (3) and (4) of table 12, labelled $f(i)$ and $f^+(i)$, respectively, do not quite agree because of rounding:

$$(i=1)\quad w(1)=x(1)+y(1)f(1)/\phi(7)+z(1)f(2)/\phi(7)$$

$$=0.031+(2.287)(0.1063)/(4.8285)+$$

$$(0.114)(0.2296)/(4.8285)$$

$$=0.087$$

$$f^+(1)=(1.0)(0.1063)+(0.087)(0.2296)=0.1263$$

$$(i=4)\quad w(4)=x(4)+y(4)f(4)/\phi(7)+z(4)f(5)/\phi(7)$$

$$=0.120+(1.139)(0.1825)/(4.8285)+$$

$$(-1.531)(0.1339)/(4.8285)$$

$$=0.121$$

$$f^+(4)=(1-w(3))f(4)+w(4)f(5)$$

$$=(0.889)(0.1825)+(0.121)(0.1339)$$

$$=0.1784$$

$$(i=7)\quad f^+(7)=(1-w(6))f(7)=(0.834)(0.0336)$$

$$=0.0280.$$

Step 6: adjustment of period fertility schedule. The first step in selecting an adjustment factor K for the converted fertility rates obtained in the previous step is to calculate the P/F ratios. They are shown in column (5) of table 11.

Though the P/F ratios show a fairly marked decline from age 35 onward, probably in this case because of

TABLE 12. REPORTED PERIOD FERTILITY RATES, FERTILITY RATES FOR CONVENTIONAL AGE GROUPS, ADJUSTED FERTILITY RATES AND ESTIMATED NUMBER OF BIRTHS, BANGLADESH, 1974

Age group (1)	Index i (2)	Reported fertility rate $f(i)$ (3)	Fertility rate for conventional age groups $f^+(i)$ (4)	Adjusted fertility rate $f^*(i)=Kf^+(i)$ ($K=1.500$) (5)	Estimated number of births $b(i)=f(i)FPOP(i)$ (6)
15-19	1	0.1063	0.1263	0.1895	571 287
20-24	2	0.2296	0.2323	0.3485	924 625
25-29	3	0.2154	0.2131	0.3197	833 461
30-34	4	0.1825	0.1784	0.2676	539 391
35-39	5	0.1339	0.1282	0.1923	340 694
40-44	6	0.0644	0.0595	0.0893	132 126
45-49	7	0.0336	0.0280	0.0420	47 675
TOTAL		0.9657	0.9658	1.4489	3 389 259
Total fertility		4.83	4.83	7.24	

omission of children ever borne by women over age 35, the ratios for the crucial age range 20-34 are fairly consistent. The adjustment factor has therefore been calculated as the average of the three ratios for women in this age range:

$$K = (1.528 + 1.491 + 1.480)/3 = 1.500.$$

Had the ratios not been so consistent, K would have been calculated as a weighted average of $P(2)/F(2)$ and $P(3)/F(3)$, the weights being the number of women in each age group as a proportion of the women in both age groups. As an illustration, in the example being considered,

$$K^* = (1.528)(2{,}653{,}155)/(2{,}653{,}155 + 2{,}607{,}009)$$
$$+ (1.491)(2{,}607{,}009)/(2{,}653{,}155 + 2{,}607{,}009)$$
$$= 1.510.$$

Adjusted age-specific fertility rates for conventional age groups, $f^*(i)$, are obtained by multiplying the $f^+(i)$ values by the adjustment factor K. Final values for $f^*(i)$ are shown in column (5) of table 12.

Total fertility, TF, can be estimated by multiplying the sum of the adjusted age-specific fertility rates $f^*(i)$ by five:

$$TF = 5(1.4489) = 7.24.$$

An adjusted birth rate can be obtained by calculating the number of births that would occur to the population being considered if it were subject to the adjusted fertility rates and by dividing the total number of births by the total population. Numbers of births by age are shown in column (6) of table 12. The total number of births is the sum of all these entries; and because the total population considered is 71,315,944, an adjusted value of b is given by

$$b = 3{,}389{,}259/71{,}315{,}944$$
$$= 0.0475.$$

Lastly, the general fertility rate can be calculated by dividing the total number of births by the number of women considered (women aged 15-49):

$$GFR = 3{,}389{,}259/14{,}676{,}917 = 0.2309.$$

3. *Comparison of current first-birth fertility with the reported proportion of mothers*

(a) *Basis of method and its rationale*

The consistency of information on current and retrospective fertility can also be checked by using data referring only to first births. In the same way that cumulated fertility rates based on all births are a measure closely associated with parity, cumulated first-birth fertility rates can be regarded as a measure of the probability of having had at least one child by the upper age-limit used in the cumulation. The consistency of recent and lifetime fertility information can then be checked by comparing cumulated and interpolated first-birth rates for a recent period with the reported proportions of women in each age group who have had at least one child. Assuming that any discrepancy between these two proportions is due to errors in the reporting of events, rather than to changing fertility, an adjustment factor may be obtained and an adjusted first-birth fertility schedule may be computed. This method of adjustment is, in fact, just a special case of the all-children method described in subsection B.2.

Computationally speaking, these two methods are very similar and there is also some similarity between the assumptions on which they are based. For example, both assume that the type of fertility measured (first child or all children) has remained constant in the recent past and that errors in current fertility are a result of a misperception of the reference period. However, the first-child method is less likely to be affected by changing marital fertility than is the method based on data for all children. Indeed, if a fertility decline is brought about by the use of contraception either to limit family size or to increase the spacing between children, the performance of the first-child method will not be affected; even if all childbearing after the first birth stopped, the first-birth comparison would remain valid. Any change affecting the timing of first births, on the other hand, would bias the results yielded by this method. Therefore, its use is not recommended when there is evidence of rapidly changing age at marriage or of a change in the interval between marriage and the onset of childbearing.

Another advantage of the first-birth method is that it is based on data that are more likely to be accurate. For example, the retrospective information it uses is the proportion of women who have had at least one child. This proportion is only distorted by women of parity one or higher who report themselves as childless, or by women of zero parity reporting themselves to be of parity one or higher. It is not affected by the actual number of children reported by these women and should therefore be more reliable than the equivalent information used by the all-children method (average reported parity). In particular, it should not suffer from the typical tendency of older women to omit some of the children they have had.

Unfortunately, the first-birth method does not solve the main problem: it does not produce an adjustment factor for all births. Logically, an adjustment factor computed from first-birth data applies only to first births. To estimate an adjustment factor applicable to all births from first-birth data one needs to make additional assumptions about the relationship between the errors affecting the reporting of first births and those affecting the reporting of all subsequent births. Although it may be reasonable to suppose that reference-period error is not much affected by birth order, other types of error may vary with birth order; and first-birth fertility rates may be relatively inflated if too many women report themselves as having only one child. In this respect, it is important to point out that, in the case of the first-birth method, the two pieces of information being used (proportions of mothers and first births in a year) are not strictly independent from the point of view of the information gathered to produce them. Thus, a birth occurring during the year of interest is identified as "first" only if the woman reporting it also reports that her parity is one. Hence, both the answer to the question about children ever born and that to the one about births in the past year are necessary to classify the data used. Therefore, unlike the method based on all births, errors in reporting children ever born may affect the estimated first-birth fertility rates, making the comparison of the observed proportions of mothers with the proportions implied by those rates somewhat less satisfactory.

In spite of these shortcomings, whenever the data required to apply this method have been tabulated, its application is recommended because it can provide useful indications about the overall quality of the data at hand. For example, since in most populations over 90 per cent of all women eventually become mothers, recent first-birth fertility rates that imply a lower proportion of eventual mothers are suspect. Furthermore, since no more than 100 per cent of all women may become mothers, adjustment factors for first-birth fertility rates that imply an eventual overall proportion of mothers higher than one cannot be accepted.

Lastly, the P/F ratios calculated by using data on all births are very often lower than those estimated on the basis of first-birth information. This outcome may be explained by a tendency to report more accurately recent first births than those of higher orders, or by the tendency of women whose first child has died to report a subsequent and recent birth as being the first. Yet, whatever the mechanism, whenever the completeness level of first-birth fertility rates is greater than that of all births, it seems safe to assert that it indicates an upper bound for the completeness of the latter. In other words, the adjustment factor derived from data on first births can, whenever the P_1/F_1 ratios are, on average, lower than those for all births, be regarded as a lower bound for the adjustment factor required by all births.

(b) *Data required*

The data required for this method are described below:

(*a*) The number of first births occurring in a given year, obtained either from a survey or from a registration system, classified by five-year age group of mother;

(*b*) Total number of women of reproductive ages (normally between ages 10 or 15 and 50) classified by five-year age group;

(*c*) The number of women in each age group who have borne at least one child during their lifetime;

(*d*) The reported birth rate or enough information to calculate it (i.e., the total number of births in a given year and the total population in that year).

(c) *Computational procedure*

As mentioned before, the computational procedure is exactly the same as that followed when estimating an adjustment factor for all births (see subsection B.2(c), except that the data used in this case refer only to first births. For the sake of completeness, the most important steps are summarized below.

Step 1: calculation of reported proportion of mothers in each age group. As in the all-children method, index $i = 1$ refers to age group 15-19, $i = 2$ to age group 20-24 and so on (see table 6). The proportion of mothers in each group, $P_1(i)$, is calculated by dividing the number of women who reported having borne at least one child by the total number of women in each age group. It should be noted that in this case $P_1(i)$ represents the proportion of women in each age group who have had at least one child and is equivalent to average parity in the all-children method. For the treatment of those whose parity is not stated in the calculation of the proportion of mothers in each age group, see subsection A.2 and annex II concerning the El-Badry correction.

Step 2: calculation of period first-birth fertility schedule. One computes this schedule, $f_1(i)$, by dividing the number of first births occurring in a given year to women in age group i by the total number of women in that age group.

Step 3: calculation of cumulated first-birth fertility schedule for a period. This schedule, denoted by $\phi_1(i)$, is five times the sum of the values of $f(j)$ from the youngest age group up to and including age group i, that is,

$$\phi_1(i) = 5\sum_{j=0}^{i} f_1(j). \tag{B.9}$$

Step 4: estimation of equivalent proportions of women with at least one child from information for a period. These proportions, denoted by $F_1(i)$, are estimated by interpolation within the cumulated fertility schedule $\phi_1(i)$. The interpolation procedure is the same as that described in subsection B.2(c) for the method based on all births, except that first-birth fertility rates are substituted for the usual all-birth fertility rates. The general form of the interpolation equation is

$$F_1(i) = \phi_1(i-1) + a(i)f_1(i) + b(i)f_1(i+1) + c(i)\phi_1(7). \tag{B.10}$$

The constants $a(i)$, $b(i)$ and $c(i)$ are presented in table 7; if births in a 12-month period have been classified by age of mother at the end of the period, as is normally the case with census or retrospective survey data, constants from part (a) of the table should be used; if births in a 12-month period have been classified by age of mother at the time of the birth, as is usually the case with registration data, constants from part (b) of the table should be used.

Step 5: calculation of a first-birth fertility schedule for conventional five-year age groups. When births in a year have been tabulated by mother's age at the end of the year, the reported first-birth fertility rates calculated in step 2 will refer to unconventional age groups roughly six months younger than the usual groups. A first-birth schedule for conventional age groups, $f_1^+(i)$, can be obtained by applying equations (B.5) and (B.6), and the constants given in table 8, to the reported schedule. Normally, however, the interest in first-birth fertility rates is limited to the consistency check described below, so that it is often not necessary to convert the reported schedule into a schedule for conventional age groups.

Step 6: selection of an adjustment factor for period fertility. Possible adjustment factors are obtained by calculating the $P_1(i)/F_1(i)$ ratios. $P_1(1)/F_1(1)$ is usually disregarded because the number of events in age group 15-19 is small, and the interpolation procedure is insufficiently flexible to replicate rapid increases with age in the rates for young women. When the assumptions of constant first-birth fertility in the past and relatively good reporting are correct, all values of P_1/F_1 after the first should be much the same. Furthermore, since the proportion of women with at least one child is not expected to decline with age, there is no reason for preferring early values of P_1/F_1 to later values. It is therefore recommended that the adjustment factor K_1 be calculated as the average of any group of consistent ratios, if such a group exists. This adjustment factor can then be multiplied by the observed $\phi_1(7)$ to obtain an adjusted proportion of women who, according to current rates, will become mothers.

Step 7: adjustment of fertility parameters referring to all births. Using the adjustment factor obtained in the previous step, the birth and general fertility rates derived from reported births, and total fertility, can be multiplied by K_1 to obtain what may be interpreted as lower bounds for their true values if first births appear to be more completely reported than all births (that is, if the P_1/F_1 ratios are lower than the P/F ratios). Strictly speaking, the birth rate and the general fertility rate should be computed by adjusting first the age-specific fertility rates based on all births, then calculating the implied number of births by multiplying by the number of women in each age group, cumulating the results, and then dividing by the relevant denominator (the total mid-year population if a birth rate is being calculated or the female population aged 15-49 if the general fertility rate is of interest), though in practice the accuracy gained by this procedure would be small. An adjusted first-birth fertility schedule, $f_1^*(i)$, can also be obtained if required, by multiplying $f_1^+(i)$ (or $f_1(i)$, if data from a vital registration system are being used) by K_1.

(d) *A detailed example*

Table 13 shows data obtained during a demographic survey conducted in Bangladesh in 1974. Data from the same survey were used to illustrate an application of the all-children method in subsection B.2 (d).

The calculations for this example are described below.

Step 1: calculation of reported proportion of mothers. Column (3) of table 14 shows the values for the proportion of mothers, $P_1(i)$, obtained by dividing the numbers appearing in column (3) (women with at least one child) of table 13 by those listed in column (2) (total number of women). Thus, for example, $P_1(2)$ is obtained as

$$P_1(2) = 2{,}092{,}845/2{,}653{,}155 = 0.789.$$

TABLE 13. NUMBER OF WOMEN, NUMBER OF WOMEN WITH AT LEAST ONE CHILD, AND NUMBER OF FIRST BIRTHS DURING THE 12 MONTHS PRECEDING THE SURVEY, BY AGE GROUP, BANGLADESH, 1974

Age group (1)	Number of women (2)	Number of women who reported at least one child (3)	First births during 12 months before the survey (4)
15-19	3 014 706	854 760	224 716
20-24	2 653 155	2 092 845	153 140
25-29	2 607 009	2 416 367	37 050
30-34	2 015 663	1 926 214	6 306
35-39	1 771 680	1 716 374	2 180
40-44	1 479 575	1 425 415	0
45-49	1 135 129	1 090 567	192

TABLE 14. PROPORTION OF WOMEN WITH AT LEAST ONE CHILD, PERIOD FIRST-BIRTH FERTILITY RATES AND CUMULATED FIRST-BIRTH FERTILITY SCHEDULE, BY AGE GROUP, BANGLADESH, 1974

Age group (1)	Index i (2)	Proportion of women with at least one child $P_1(i)$ (3)	Observed first-birth fertility rate $f_1(i)$ (4)	Cumulated first-birth fertility schedule $\phi_1(i) = 5\sum_{j=1}^{i} f_1(j)$ (5)
15-19	1	0.284	0.0745	0.3725
20-24	2	0.789	0.0577	0.6610
25-29	3	0.927	0.0142	0.7320
30-34	4	0.956	0.0031	0.7475
35-39	5	0.969	0.0012	0.7535
40-44	6	0.963	0.0000	0.7535
45-49	7	0.961	0.0002	0.7545

Step 2: calculation of period first-birth fertility schedule. The period fertility schedule, $f_1(i)$, for first births is computed by dividing the number of first births in a year (listed in column (4) of table 13) by the total number of women in each age group (listed in column (2) of table 13). As an example, $f_1(3)$ is computed as

$$f_1(3) = 37{,}050/2{,}607{,}009 = 0.0142.$$

The complete set of $f_1(i)$ values is presented in column (4) of table 14.

Step 3: calculation of cumulated first-birth fertility schedule for a period. This schedule, denoted by $\phi_1(i)$, is calculated by adding the first i values of $f_1(j)$ (listed in column (4) of table 14) and multiplying the result by five (since each fertility rate applies to a five-year age group). The calculation of $\phi_1(2)$ and $\phi_1(4)$ is illustrated below:

$$\phi_1(2) = 5(0.0745 + 0.0577) = 0.6610$$

$$\phi_1(4) = 5(0.0745 + 0.0577 + 0.0142 + 0.0031) = 0.7475.$$

Of course, $\phi_1(4)$, for example, could also be calculated as

$$\phi_1(4) = \phi(3) + 5 f_1(4) = 0.7320 + 5(0.0031) = 0.7475.$$

All values of $\phi_1(i)$ are listed in column (5) of table 14.

Step 4: estimation of equivalent proportions of women with at least one child from current information. Because the data used here were obtained from a sample survey and the recorded age of mother was, on average, six months older than her age when her first child was born, constants from part (a) of table 7 are substituted into equation (B.10) to calculate the proportions of women with at least one child, $F_1(i)$. The calculations of $F_1(1)$, $F_1(3)$ and $F_1(7)$ are shown below:

$$F_1(1) = \phi_1(0) + a(1)f_1(1) + b(1)f_1(2) + c(1)\phi_1(7)$$

$$= 0.0 + (2.531)(0.0745) + (-0.188)(0.0577) + (0.0024)(0.7545)$$

$$= 0.1795$$

$$F_1(4) = \phi_1(3) + a(4)f_1(4) + b(4)f_1(5) + c(4)\phi_1(7)$$

$$= 0.7320 + (3.442)(0.0031) + (-0.563)(0.0012) + (0.0029)(0.7545)$$

$$= 0.7442$$

$$F_1(7) = \phi_1(6) + a(7)f_1(7) + b(7)f_1(6) + c(7)\phi_1(7)$$

$$= 0.7535 + (3.828)(0.0002) + (0.016)(0.0) + (-0.0002)(0.7545)$$

$$= 0.7541.$$

Note that $\phi_1(0)$ is assumed to be zero because no data are available for women younger than 15. If data for these women were available, the value of $\phi_1(0)$ would have to be calculated. The values of $F_1(i)$ are shown in column (4) of table 15.

TABLE 15. REPORTED AND EQUIVALENT PROPORTIONS OF MOTHERS, BY AGE GROUP, AND P_1/F_1 RATIOS, BANGLADESH, 1974

Age group (1)	*Index* i (2)	*Reported proportion of mothers* P_1 (3)	*Equivalent proportion of mothers* F_1 (4)	P_1/F_1 *ratio* (5)
15-19	1	0.284	0.180	1.578
20-24	2	0.789	0.566	1.394
25-29	3	0.927	0.716	1.295
30-34	4	0.956	0.744	1.285
35-39	5	0.969	0.752	1.289
40-44	6	0.963	0.753	1.279
45-49	7	0.961	0.754	1.275

Step 5: calculation of first-birth fertility schedule for conventional age groups. The procedure used in step 5 of the example on all births (subsection B.2 (d)) could be repeated here in order to convert the reported first-birth fertility schedule for age groups that are shifted by six months to a schedule for conventional age groups, $f_1^+(i)$. However, since the first-birth fertility schedule has no intrinsic value, the process of conversion is not generally justified. This step is therefore omitted.

Step 6: selection of an adjustment factor for period fertility. Ratios of retrospective, P_1, to period, F_1, proportions of mothers are calculated for each age group by dividing the values of P_1 in column (3) of table 15 by those of F_1 in column (4). Except for those for the first two age groups, the ratios are very consistent; excluding those two, the average of the remaining five ratios is 1.285. Note that two general observations made earlier about the results from the first-births method have been fulfilled: first, the ratios remain rather constant as age increases; and secondly, the ratios are, in general, lower than those obtained using all births. Taking the average ratio computed above as an adjustment factor K_1 for the first-birth fertility rates, and adjusting $\phi_1(7)$ by it, one obtains an estimate of the proportion of women who would ultimately become mothers equal to $(1.285)(0.7545) = 0.970$. Since this proportion cannot theoretically exceed 1.0, and in practice is always somewhat lower than 1.0 because of sterility, the adjusted proportion is as high as can be expected; the estimated adjustment factor of 1.285 for first births could not thus be any larger.

Step 7: adjustment of fertility parameters referring to all births. As mentioned earlier, if first births appear to be better reported than all births, K_1 may also be used as a conservative adjustment factor for parameters obtained from data on all births. The following values can therefore be regarded as lower bounds for the true values:

$$b = (0.0313)(1.285) = 0.0402;$$

$$TF = (4.8285)(1.285) = 6.20;$$

$$GFR = (0.1519)(1.285) = 0.1952.$$

In the case of Bangladesh, these values are all smaller than those obtained in subsection B.2 (d), because the adjustment factor used is considerably lower. The very marked difference between the first-birth and all-birth adjustment factors is to be noted. Since the evidence does not support the existence of a fertility decline in Bangladesh that would artificially inflate the P/F ratios for all births, the fact that the adjustment factor derived from all births is larger than that derived from first births suggests that it should be preferred as an adjustment for overall fertility. It is not possible to establish with certainty what mechanism or set of mechanisms leads to the large difference between the two sets of ratios, but it is likely to be related to the fact that, as pointed out earlier, first-birth fertility rates are not entirely independent of the data on children ever born. The interplay between errors in reported children ever born and births in the past year may well lead to first-birth fertility rates that, as in this case, appear to be more complete than those referring to all births.

Hence, in the case of Bangladesh, the P/F ratios based on all births should be preferred as a basis for the adjustment of overall fertility rates, with the proviso that the adjustment factor derived from them should not be applied to fertility rates specific by birth order. The first-birth method has been useful in establishing that the data are not as internally consistent as one would wish and that the adjustment factor obtained from all births is acceptable, in that it is not obviously inconsistent with other evidence.

4. *Comparison of period fertility rates with average parities for a hypothetical cohort*

(a) *Basis of method and its rationale*

It has been stressed above that the estimation of an adjustment factor for period fertility on the basis of the comparison of cumulated period fertility rates with lifetime average parities is only valid if fertility has been approximately constant during the 15 years or so preceding the time at which the data were collected. If fertility has been changing, cumulated period fertility rates cannot be expected to equal lifetime fertility; and an adjustment factor calculated on the basis of the comparison of the two will reflect not only possible data errors but the effects of changes through time. Hence, its use for correction purposes will tend to obscure the effects of those changes.

One way of avoiding this problem is to compute average parities that refer to the fertility experienced during a particular period and to compare those parities with cumulated average fertility rates measured during the same period. Suitable parities, referring to a particular period rather than to lifetime experience, can be computed if data on children ever born classified by age of mother are available from two surveys; in such a case, the average parities that a hypothetical cohort subject to intersurvey fertility would exhibit can be constructed on the basis of the intersurvey parity increments for true cohorts.

With an interval of five or 10 years between the surveys, the survivors of a cohort of women at the first survey can be identified at the second, and the change in the average parity of the cohort can be calculated. The resulting sequence of parity increments for different cohorts during the period between the surveys can then be cumulated to calculate average parities for a hypothetical cohort experiencing the level of intersurvey fertility implicit in the observed parity increments. Other uses of average parities for hypothetical cohorts are discussed in section C. Note that in deriving this measure of intersurvey fertility it is assumed that mortality and migration have no effect on actual parity distributions; that is, it is assumed that the average parity of those women who die or migrate between the surveys is not significantly different from the average parity at comparable ages of those women who are alive and present at the end of the period.

The "period" fertility rates from which parity equivalents are to be derived for comparison with the intersurvey parities should ideally refer to the entire intersurvey period. Suitable rates can be obtained if registered births classified by age of mother are available for each calendar year of the period. In this case, all births recorded during the period for each age group can be calculated by addition over calendar years; and average intersurvey fertility rates can be obtained by dividing the births by the number of woman-years lived in each age group, estimated from the female population enumerated at the end-points of the period. A simpler, and generally adequate, procedure is to calculate age-specific fertility rates only for the first and last years of the period, and to approximate average intersurvey rates by the arithmetic mean of these two sets. If registered births are not available, but the two surveys gathered data on births in the past year, age-specific fertility rates for the period may by approximated in the same way by averaging the rates observed at the beginning and at the end of the period. When the births during the 12 months preceding each survey are tabulated by age of mother at the time of the survey, the observed fertility rates will correspond to age groups displaced by six months, and the analysis performed will have to take this fact into account. It is, of course, important that the sets of fertility rates being averaged be consistent with respect to age classification before they are averaged; if they are not consistent at first, because one refers to age groups displaced by six months and the other does not, the former set should be adjusted before proceeding. If age-specific fertility rates for the end-points of the period are not available, a set of rates referring approximately to the mid-point of the period could be used. It should be remembered that only the pattern of the intersurvey age-specific fertility rates is important in applying the Brass method, so that if this pattern was more or less constant over the period, the exact reference date of the rates used does not matter.

Once the intersurvey parities and intersurvey fertility rates have been calculated, the cumulation and interpolation of the latter, and their comparison with the average parities, are carried out exactly as described above in subsection B.2(*c*).

(b) *Data required*

The data required are described below:

(*a*) The number of children ever born classified by five-year age group of mother, taken from two surveys or censuses five or 10 years apart;

(*b*) The number of births during the year preceding each survey classified by five-year age group of mother, or registered births by five-year age group of mother for each intersurvey year (if registered births are used, the female population enumerated by censuses at the end-points of the period considered is also necessary);

(*c*) The number of women in each five-year age group from both surveys or censuses;

(*d*) If the birth rate is to be calculated, the total population recorded by each survey or census.

(c) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities from each survey. The average parities obtained from the first survey are denoted by $P(i, 1)$, and those from the second survey by $P(i, 2)$. In both cases, they are computed by dividing the reported number of children ever born to women in age group i by the total number of women in age group i. See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the possible application of the El-Badry correction.

Step 2: calculation of average parities for a hypothetical intersurvey cohort. The way in which these parities are calculated depends upon the length of the intersurvey interval. If this interval is five years, all the survivors of age group i at the first survey belong to age group $i+1$ at the second survey, and the parity increment between the surveys for the corresponding cohort is equal to $P(i+1, 2)-P(i, 1)$. Such increments can be calculated for each age group, and the hypothetical-cohort parities are then obtained by successively cumulating them. Thus, if the parity increment for the cohort of age group i at the first survey is denoted by $\Delta P(i+1)$, and the parity of age group i for the hypothetical cohort is denoted by $P(i, s)$ (where the s stands for "synthetic"), one has

$$\Delta P(i+1)=P(i+1,2)-P(i,1) \text{ for } i=1 \dots 6 \qquad (B.11)$$

and

$$P(i,s)=\sum_{j=1}^{i} \Delta P(j). \qquad (B.12)$$

The parity increment $\Delta P(i+1)$ for the youngest age group ($i=0$) is taken as being directly equal to $P(1, 2)$. If fertility is changing rapidly, this value of $\Delta P(1)$ will reflect period rates somewhat closer to the second survey than to the mid-point of the interval, slightly over-allowing therefore for the change in fertility.

If the intersurvey interval is 10 years, then the survivors of the initial cohort of age group i in the first survey will be the women in age group $(i+2)$ in the second; and the hypothetical cohort parities are obtained by cumulating two parallel sequences of parity increments. Once more, for the youngest age groups, $\Delta P(1)$ is taken as being equal to $P(1, 2)$ and $\Delta P(2)$ to $P(2, 2)$. Other parity increments are calculated as

$$\Delta P(i+2)=P(i+2,2)-P(i,1) \text{ for } i=1 \dots 5. \qquad (B.13)$$

Hypothetical-cohort parities for even-numbered age groups are obtained by summing the parity increments for even-numbered age groups, whereas those for odd-numbered age groups are obtained by summing parity increments for odd-numbered age groups. Thus,

$$P(2,s)=\Delta P(2)=P(2,2)$$

$$P(4,s)=\Delta P(2)+\Delta P(4)$$

and

$$P(6,s)=\Delta P(2)+\Delta P(4)+\Delta P(6)$$

whereas

$$P(1,s)=\Delta P(1)=P(1,2)$$

$$P(3,s)=\Delta P(1)+\Delta P(3)$$

$$P(5,s)=\Delta P(1)+\Delta P(3)+\Delta P(5)$$

and

$$P(7,s)=\Delta P(1)+\Delta P(3)+\Delta P(5)+\Delta P(7).$$

Step 3: calculation of intersurvey fertility schedule. The method of calculating this schedule, denoted by $f(i)$, depends upon the data available. One possible procedure is to calculate age-specific fertility rates referring roughly to the first and last years of the intersurvey period by using data on the reported number of births during the year preceding each survey. In such a case, for each survey one would divide the reported births for each five-year age group of mother by the reported number of women in the same age group and then obtain age-specific fertility rates for the intersurvey period by calculating the arithmetic mean of each pair of end-point rates. Because of age-group incompatibilities, it is important to avoid combining a schedule derived from a question on births in the past year with another based on registered births; either consistent sources of current fertility rates must be used or the schedule based on births in the past year must be adjusted for the fact that its age classification is likely to be displaced by six months (see step 5 of subsection B.2 (c)). Only when compatibility has been ensured by such an adjustment may averaging be performed. If data on births classified by age of mother are not available for the end-points of the intersurvey period, the use of an age-specific fertility schedule referring approximately to the middle of the period would be acceptable.

Step 4: calculation of cumulated fertility for the hypothetical intersurvey cohort. The calculation of cumulated fertility, denoted by $\phi(i)$, is exactly the same as step 3 in subsection B.2 (c); and its description is not repeated here.

Step 5: estimation of average parity equivalents for the hypothetical intersurvey cohort. The estimation of these equivalents, denoted by $F(i)$, is performed exactly as described in step 4 of subsection B.2 (c). It is not described again here.

Step 6: calculation of a fertility schedule for the usual five-year age groups. This step, which is omitted when the source of current fertility data is a vital registration system, is exactly the same as step 5 described in subsection B.2 (c). Its description is not repeated at this time.

Step 7: adjustment of the intersurvey fertility schedule. This step is exactly the same as step 6 of subsection B.2 (c), and its description is omitted.

(d) *A detailed example*

Data from Thailand for 1960 and 1970 permit the

TABLE 16. FEMALE POPULATION, CHILDREN EVER BORN AND REGISTERED BIRTHS, BY AGE GROUP OF WOMEN, THAILAND, 1960 AND 1970

Age group (1)	Female population 1960 (2)	Female population 1970 (3)	Number of children ever born 1960 (4)	Number of children ever born 1970 (5)	Number of registered births 1960 (6)	Number of registered births 1970 (7)
15-19	1 236 294	1 885 371	88 610	246 839	49 799	95 532
20-24	1 204 153	1 361 717	1 038 074	1 353 569	238 243	299 335
25-29	1 046 464	1 143 377	2 402 581	2 754 376	250 736	267 369
30-34	869 876	1 077 088	3 290 345	4 112 920	183 152	213 814
35-39	679 940	957 607	3 388 799	4 866 424	119 887	154 439
40-44	563 812	766 332	3 232 209	4 537 467	49 348	63 259
45-49	482 966	597 454	2 855 997	3 656 488	10 356	12 745

application of the hypothetical-cohort technique. The censuses in both those years published data on children ever born classified by age of mother, and registered births classified in the same way are available for 1960 and 1970 (years that closely approximate the exact end-points of the intercensal period). Although the intercensal period is not exactly 10 years in length—the census dates were 15 April 1960 and 1 April 1970—it is close enough to being exactly 10 years for a cohort in 1960 to be identified as a cohort 10 years older in 1970. The basic data required are shown in table 16 and the steps of the calculations are given below.

Step 1: calculation of reported average parities from each survey. The two sets of average parities at the end-points of the intercensal period are calculated by dividing the number of children ever born, given in columns (4) and (5), by the total number of women, shown in columns (2) and (3), for each age group. Results are shown in table 17. As an example, the average parity for age group 2 from the 1960 census, denoted by $P(2, 1)$, is obtained as

$$P(2, 1) = 1{,}038{,}074/1{,}204{,}153 = 0.8621,$$

and the average parity for age group 4 from the 1970 census, denoted by $P(4, 2)$, is obtained as

$$P(4, 2) = 4{,}112{,}920/1{,}077{,}088 = 3.8186.$$

TABLE 17. REPORTED AVERAGE PARITIES, 1960 AND 1970, AND PARITIES FOR THE HYPOTHETICAL INTERCENSAL COHORT, BY AGE GROUP, THAILAND

Age group (1)	Average parity 1960 P(i, 1) (2)	Average parity 1970 P(i, 2) (3)	Cohort parity increment ΔP(i) (4)	Parity for hypothetical cohort P(i, s) (5)
15-19	0.0717	0.1309	(0.1309)	0.1309
20-24	0.8621	0.9940	(0.9940)	0.9940
25-29	2.2959	2.4090	2.3373	2.4682
30-34	3.7825	3.8186	2.9565	3.9505
35-39	4.9840	5.0819	2.7860	5.2542
40-44	5.7328	5.9210	2.1385	6.0890
45-49	5.9135	6.1201	1.1361	6.3903

Step 2: calculation of average parities for a hypothetical intersurvey cohort. The intercensal interval in the case under consideration is 10 years, so the second way of deriving hypothetical-cohort parities, employing two different sums of parity increments, is used. The hypothetical-cohort parity for age group i is denoted by $P(i, s)$. The first two values are obtained directly from the parities reported at the second census; thus,

$$P(1, s) = \Delta P(1) = P(1, 2) = 0.1309$$

and

$$P(2, s) = \Delta P(2) = P(2, 2) = 0.9940.$$

Subsequent values of $P(i, s)$ are obtained by summing certain cohort parity increments, one sum using only increments derived from odd-numbered age groups and another only those from even-numbered age groups. The parity increments, denoted by $\Delta P(i)$ and shown in table 17, are calculated as

$$\Delta P(i+2) = P(i+2, 2) - P(i, 1).$$

Thus, for example, the parity increment for $i = 5$ is calculated as

$$\Delta P(5) = P(5, 2) - P(3, 1) = 5.0819 - 2.2959 = 2.7860.$$

Average parities for a hypothetical intercensal cohort are then obtained by cumulating the cohort parity increments. Since the intercensal period is 10 years, two sums are required: parities for odd-numbered age groups are obtained by adding odd-numbered parity increments; while those for even-numbered age groups are obtained by summing even-numbered parity increments. The resulting average parities are shown in column 5 of table 17; the following two examples illustrate the computational procedure:

$$P(3, s) = \Delta P(1) + \Delta P(3) = 0.1309 + 2.3373 = 2.4682;$$

$$P(6, s) = \Delta P(2) + \Delta P(4) + \Delta P(6)$$

$$= 0.9940 + 2.9565 + 2.1385$$

$$= 6.0890.$$

Step 3: calculation of the intersurvey fertility schedule. Table 16 shows the number of births registered in 1960 and 1970 classified by age of the mother and also the female population enumerated by the 1960 and 1970 censuses classified according to the same age groups. Age-specific fertility rates for 1960 and 1970 are obtained by dividing the registered births for each age group, given in columns (6) and (7), by the enumerated female population of the same age group, shown in columns (2) and (3). Thus, if $f(i, 1)$ denotes the age-specific fertility rate of age group i in 1960 and $f(i, 2)$ denotes the corresponding rate in 1970,

$$f(2, 1) = 238{,}243/1{,}204{,}153 = 0.1979$$

and

$$f(4, 2) = 213{,}814/1{,}077{,}088 = 0.1985.$$

Values of $f(i, j)$ for all age groups are shown in columns (2) and (3) of table 18. Average fertility rates for the intercensal period 1960-1970, $f(i)$, are then obtained by summing the rates for 1960 and 1970 for each age group and dividing by two. The final $f(i)$ values are shown in column (4) of table 18. The next examples illustrate the computational procedure:

$$f(2) = (f(2, 1) + f(2, 2))/2$$

$$= (0.1979 + 0.2198)/2 = 0.2088$$

and

$$f(5) = (f(5, 1) + f(5, 2))/2$$

$$= (0.1763 + 0.1613)/2 = 0.1688.$$

TABLE 18. AGE SPECIFIC FERTILITY RATES, 1960 AND 1970, AND AVERAGE FERTILITY RATES FOR THE INTERCENSAL PERIOD, THAILAND

	Age-specific fertility rates		
Age group (1)	1960 $f(i, 1)$ (2)	1970 $f(i, 2)$ (3)	1960-1970 $f(i)$ (4)
15-19........	0.0403	0.0507	0.0455
20-24........	0.1979	0.2198	0.2088
25-29........	0.2396	0.2338	0.2367
30-34........	0.2105	0.1985	0.2045
35-39........	0.1763	0.1613	0.1688
40-44........	0.0875	0.0825	0.0850
45-49........	0.0214	0.0213	0.0214

It should be mentioned that this estimate of the average intercensal fertility schedule, being just an arithmetic mean of the schedules observed at the two end-points, is only approximate in nature. If the age patterns of the fertility schedules at the end-points are very different, it is likely that the age pattern of fertility may have changed sharply during the intercensal period; and a better estimate of an average intercensal fertility schedule would be obtained by averaging the schedules observed during each year of the period or by calculating directly an intercensal schedule for the period by dividing all the births recorded during the period for each age group of mother by an estimate of person-years lived by women in each age group during the same period. In the case of Thailand, the age-specific fertility rates for 1960 and 1970 shown in table 18 are fairly similar and do not suggest the existence of a sharp change in the age pattern of fertility over the intercensal period, so that the $f(i)$ estimates listed in table 18 are adequate.

Fertility rates for 1960 and 1970 have been calculated by using the births registered during each year and the female population enumerated by each census. Strictly speaking, these rates should have been calculated by using estimates of the female population at the middle of each year, rather than the populations enumerated by censuses taking place in April. Since the mid-year populations would be slightly larger than those enumerated, the fertility rates calculated using them as denominators would be slightly lower. However, if the mid-year populations are estimated by applying the same growth rate to the enumerated number of women in each age group, the resulting fertility rates will just be constant multiples of those obtained in table 18 and their age pattern will not be affected. Since only this pattern is relevant in applying the P/F ratio method, the results yielded by it will not be essentially affected by the lack of adjustment of the female population, although, of course, the estimates of the completeness of birth registration obtained are valid only with respect to the population actually used as denominator. In this case, the use of the enumerated population instead of that corresponding to the mid-year would increase the completeness estimates only by about three quarters of 1 per cent.

Step 4: calculation of cumulated fertility for the hypothetical intersurvey cohort. The intercensal fertility rates, $f(i)$, shown in column (4) of table 18 are multiplied by the width of the age group to which they refer and

TABLE 19. CUMULATED FERTILITY SCHEDULE, PARITY EQUIVALENTS, HYPOTHETICAL-COHORT PARITIES, P/F RATIOS AND ADJUSTED INTERCENSAL FERTILITY RATES, THAILAND, 1960-1970

Age group (1)	Cumulative fertility schedule $\phi(i)$ (2)	Parity equivalent $F(i)$ (3)	Hypothetical-cohort parity $P(i, s)$ (4)	P/F ratio $P(i, s)/F(i)$ (5)	Adjusted intercensal fertility rate $f^{*}(i)$ (6)
15-19........	0.2275	0.0632	0.1309	2.071	0.0606
20-24........	1.2715	0.7193	0.9940	1.382	0.2781
25-29........	2.4550	1.8679	2.4682	1.321	0.3153
30-34........	3.4775	2.9747	3.9505	1.328	0.2724
35-39........	4.3215	3.9218	5.2542	1.340	0.2248
40-44........	4.7465	4.5482	6.0890	1.339	0.1132
45-49........	4.8535	4.8206	6.3903	1.326	0.0285

summed successively to produce the values of the cumulated fertility schedule, $\phi(i)$. Because all age groups in this instance are five years in length, the process just described is equivalent to that illustrated below for $i = 2$ and 5, where cumulation of the values of $f(i)$ is carried out first and multiplication by the length of the age groups later:

$$\phi(2) = 5(0.0455 + 0.2088) = 1.2715$$

$$\phi(5) = 5(0.0455 + 0.2088 + 0.2367 + 0.2045 + 0.1688)$$

$$= 5(0.8643) = 4.3215.$$

The complete set of $\phi(i)$ values is shown in column (2) of table 19.

Step 5: estimation of average parity equivalents for the hypothetical intersurvey cohort. Since, in this case, information on births by age of mother was obtained from a registration system, the reported age of mother is likely to be the one she had at the time of the birth. Therefore, the values of average parity equivalents, $F(i)$, are calculated by substituting the coefficients from part (b) of table 7 in equation (B.4), which has the form:

$$F(i) = \phi(i-1) + a(i)f(i) + b(i)f(i+1) + c(i)\phi(7).$$

As examples,

$$F(3) = \phi(2) + (2.760)f(3) + (-0.594)f(4) +$$

$$(0.0133)\phi(7)$$

$$= 1.2715 + (2.760)(0.2367) + (-0.594)(0.2045) +$$

$$(0.0133)(4.8535)$$

$$= 1.8679$$

and

$$F(7) = \phi(6) + (3.535)f(7) + (-0.007)f(6) +$$

$$(-0.0002)(4.8535)$$

$$= 4.7465 + (3.535)(0.0214) + (-0.007)(0.0850) +$$

$$(-0.0002)(4.8535)$$

$$= 4.8206.$$

All values of $F(i)$ are shown in column (3) of table 19.

Step 6: calculation of a fertility schedule for the usual five-year age groups. This step is omitted because in this case the information on births by age of mother was obtained from a registration system where women are likely to report the age they had at the time of delivery.

Step 7: adjustment of the intercensal fertility schedule. Comparable values of $P(i)$ and $F(i)$ are now available for the period 1960-1970, so that P/F ratios can be calculated and an adjustment factor K can be selected from them. Column (3) of table 19 shows the $F(i)$ values, column (4) shows the $P(i,s)$ values copied from table 17 and column (5) shows the $P(i,s)/F(i)$ ratios. These ratios are reasonably consistent, except for the first, the value of which suggests the existence of less complete registration of births by very young mothers; part of this above-average omission affects the P/F ratio for the second age group through its dependence upon $\phi(1)$. Since most of the P/F ratios are consistent, the way in which an adjustment factor is selected is not of great importance; the average of the ratios for age groups 3-6 is likely to be as satisfactory as any, so

$$K = (1.321 + 1.328 + 1.340 + 1.339)/4.0 = 1.332.$$

Column (6) of table 19 shows the adjusted fertility schedule, $f^*(i)$, for the period 1960-1970. Total fertility may be estimated either by summing the $f^*(i)$ values and multiplying by five or by multiplying $\phi(7)$ by K. In either case, the estimate of TF obtained is 6.46, compared with the unadjusted value of 4.85. K is an adjustment factor for registered births, so its reciprocal, $1/K$, is an estimate of the completeness of birth registration, found to be 75.1 per cent (this completeness is measured with respect to the female population enumerated by the censuses). The intercensal birth rate may be estimated by summing total births registered during the years 1960-1969, multiplying the total by K, and dividing by the person-years lived by the entire population from 1960 to 1970.

5. *Comparison of mean number of births registered by a cohort of women with the reported average parity of the same cohort*

(a) *Basis of method and its rationale*

Subsection B.4 presented a method for comparing average parities with average parity equivalents derived from period fertility rates without the necessity of assuming constant fertility. The most important aspect of this method is that average parities are calculated for a period rather than for a series of cohorts. It requires, however, that data on children ever born be available for two points in time, five or 10 years apart. If only one source of data on children ever born exists, or if the intersurvey period is not five or 10 years in length, an alternative procedure that does not require the assumption of constant fertility may be used. However, this procedure requires the availability of a fairly long series of annual data on registered births classified by age of mother. This method makes use of the cohort nature of reported average parities and compares them with parity equivalents obtained from the recorded fertility rates pertaining to the relevant cohorts.

If one considers women aged 30-34 at some census, a year before the census they were aged 29-33, then 10 years before the census they were aged 20-24, and 20 years before the census they were aged 10-14. Therefore, assuming that childbearing effectively begins at age 15, the children ever born reported by these women at the time of the census reflect the cumulated fertility experience of the women over the preceding 20 years. If mortality and migration are assumed to be unrelated to the fertility experience of women, and fertility rates can be calculated for those 20 years, average parity equivalents for each cohort can be constructed and compared with the reported average parity of women at the time of the census. This method is mainly of use with data on births from a vital registration system, which is normally the only source of information about births over a 20-year period; but if fertility schedules are available from other sources for regular five-year intervals, there is no reason to prevent the use of such schedules. The description here, however, is given in terms of data from a vital registration system.

The difficulty with applying this general idea is that a cohort represented by a conventional five-year age group at the time of the census would not have been a conventional five-year age group in earlier years. Thus, the population in age group 30-34 at the time of a census would have been aged 29-33 a year earlier, 28-32 two years earlier and so on. If births are tabulated by single year of age of mother, this problem is not serious, because single-year fertility rates can be calculated for each year and then summed by cohort with relative ease. The calculations would be lengthy, however, and age-heaping might have a non-trivial effect on the fertility rates, so it is convenient to have an approach that can be applied to rates for conventional five-year age groups. Such a procedure is described here.

(b) *Data required*

The data required for this method are described below:

(*a*) The number of children ever born by five-year age group of mother, taken from a census;

(*b*) Registered births by five-year age group of mother for each of 15 or 20 years preceding the census;

(*c*) The number of women in each age group from the census, and from one or more earlier censuses, to allow the estimation of the female population by five-year age group for each of the 15 or 20 years preceding the final census.

(c) *Computational procedure*

The following steps are required for the computational procedure.

Step 1: calculation of reported average parities. Average parities for each age group from the final census, denoted by $P(i)$, are obtained by dividing the number of children ever born reported by women in each age group by the total number of women in each age group. See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the use of the El-Badry correction (described in annex II).

Step 2: estimation of mid-year female population by age group for each year preceding the census. The exact procedure to be followed in estimating the series of mid-year female populations by age group depends upon the dates of the census enumerations available, so an attempt is made here to describe the procedure in general terms. It is assumed that census enumerations cover, or almost cover, the 20 years or so for which registered fertility rates are to be cumulated. The reference date of each census should then be calculated in terms of years, the decimal part being obtained by dividing the number of days from 1 January to the date of the census by 365, the number of days in a year. The exponential growth rate, $r(i)$, of each age group i is then obtained by dividing the difference between the natural logarithms of the female population of age group i at the second and first censuses by the length of the intercensal period in years, as shown in equation (B.14):

$$r(i)=(\ln N(i,2)-\ln N(i,1))/(t_2-t_1) \quad \text{(B.14)}$$

where $N(i,j)$ is the female population of age group i at census j; t_1 is the date of the first census expressed in decimals; and t_2 is the date of the second census expressed in the same fashion. The required denominators for each year can then be calculated for each mid-year between t_1 and t_2 by expanding exponentially the initial population for the period using equation (B.15):

$$N(i,\tau)=N(i,1)\exp(r(i)(\tau+0.5-t_1)) \quad \text{(B.15)}$$

where $N(i,\tau)$ is the female population of age group i required as denominator for calendar year τ.

The objective of this method is to measure the completeness of birth registration, with a view to adjusting births registered during a recent period for omission, and thus to estimate the recent levels of fertility. The effects of other errors, such as changes in the completeness of census enumeration through time, should therefore be allowed for before cumulating age-specific fertility rates for comparison with average parities. Hence, when there is evidence suggesting that changes in the completeness of enumeration have taken place, it is desirable to adjust the censuses before calculating the population denominators. However, it is not necessary to adjust each census for absolute underenumeration; it is only necessary to ensure that the completeness of enumeration of the different censuses shall be the same.

Step 3: calculation of age-specific fertility rates from births registered during the years preceding the census. Age-specific fertility rates are to be calculated for calendar years, so it is convenient to cumulate the rates to the end of each year. The census providing average parities is unlikely to have as reference date exactly the end of a year, but fortunately average parities for a specified age group change slowly even when fertility is changing rapidly. The parities from the census can therefore be regarded as referring to the year-end nearest to the census date, and registered rates can be cumulated up to the relevant year-end. Thus, if the census date is on or before 30 June, registered fertility rates would be cumulated to the end of the preceding calendar year, whereas if the census date is after 30 June, registered fertility rates would be cumulated to the end of the calendar year during which the census took place.

Age-specific fertility rates are required for a total of 20 calendar years. The rate for age group i and calendar year τ, $f(i, \tau)$, is calculated as

$$f(i, \tau) = B(i, \tau)/N(i, \tau) \qquad \text{(B.16)}$$

where $B(i, \tau)$ is the number of births registered in calendar year τ as having occurred to women of age group i.

If registered births by age of mother are not available for a few of the 20 calendar years required, the application of the method will be only slightly affected if rates for the odd blank year are estimated from neighbouring rates. For example, if registered births are not available for one year in the series, the fertility rates for that year can be estimated as the average of the rates in the preceding and following years. Or if fertility rates are only available for the last 16 of the 20 years required, the rates for the earliest available year can be adopted for the four preceding years without much danger of introducing sizeable errors. However, such extrapolation is more dangerous if more recent years are involved because imputation in this case affects more age groups, is likely to cover more of the years of peak childbearing and is less likely to reflect adequately any changes in fertility that might have actually taken place.

Step 4: cumulation of registered fertility for different female birth cohorts to estimate parity equivalents. As mentioned in step 3, reported average parities are assumed to refer exactly to the end of a calendar year, whereas age-specific fertility rates calculated from registered births refer to whole calendar years. Hence, each age-specific fertility rate encompasses the child-bearing experience of two female birth cohorts if the latter are defined with respect to age at the end of a calendar year. It is therefore necessary to split the observed age-specific fertility rates into two parts, each contributed by a cohort for which reported average parity is available, in order to estimate parity equivalents, $F(i)$. For example, one may assume that average parities by five-year age group are available for the end of year t. Women aged 25-29 at the end of year t were aged 24-28 at the beginning of year t, so during year t their cumulated fertility would have been increased by most of the age-specific fertility rate for age group 25-29 in year t (but not by all of it, because some of those aged 29 years who give birth in year t would be age 30 by the end of the year) and by a small amount of the rate for age group 20-24 (to allow for births to those aged 24 years who were 25 by the end of the year). Assuming that, within each age group, the distribution of women by age is rectangular and their fertility by age is constant, 90 per cent of $f(3, t)$, the rate for women 25-29 in year t, and 10 per cent of $f(2, t)$ contribute to the cumulated fertility of women aged 25-29 years at the end of year t. The same women would have begun year $t-1$ aged 23-27 and ended it aged 24-28, so their cumulated fertility would have a rather greater contribution from age group 20-24 and a rather smaller contribution from age group 25-29 than in the previous case; making the same assumptions as above, their cumulated cohort fertility would increase by 70 per cent of $f(3, t-1)$ and by 30 per cent of $f(2, t-1)$ during year $t-1$. It should be noted in passing that the proportions of these age-specific fertility rates that are not contributed to the cohort in question are contributed to another cohort; for example, 30 per cent of $f(3, t-1)$ is added to the cumulated fertility of the cohort aged 30-34 at the end of year t, and 70 per cent of $f(2, t-1)$ is added to that of the cohort aged 20-24. This example shows that the essence of the calculation of cumulated cohort fertility lies in splitting by cohort the fertility rates for each age group and calendar year into two parts and then summing the portions relevant to each cohort.

Unfortunately, the simple assumption made above concerning the constancy of fertility within each age group is not satisfactory for splitting the age-specific fertility rates for younger women; and since the recent fertility experience of these women is that of greatest interest, an alternative estimation procedure is necessary. Hill[7] devised such a procedure. It is based on a set of separation factors derived from the Brass[8] fertility polynomial. The set of factors used in any particular instance depends upon the general shape of the age-specific fertility schedule proposed by Brass and on its specific age-location in the case at hand. The general equation defining cumulated fertility in terms of separation factors and observed annual age-specific fertility rates is

$$F(i, t) = \sum_{j=1}^{i} \left[\sum_{m=1}^{5} (s(m, j) f(j, t-k) + (1 - s(m, j-1)) f(j-1, t-k) \right] \qquad \text{(B.17)}$$

where $F(i, t)$ = cumulated fertility of women of age group i at the end of year t;

$s(m, j)$ = separation factor for location m of age group j;

$f(j, t-k)$ = age-specific fertility rate for age group j in year $t-k$, where $k = 5(j-1-i)+m$.

Values of $s(m, j)$ are listed in table 20, and they should be selected for each calendar year t according to the value of $f(1, t)/f(2, t)$.

Equation (B.17) states that cumulated cohort fertility at the end of year t is the sum of all the portions of the registered fertility rates contributed by women of the cohort. The number of years the sum will cover depends upon the age of the cohort at the end of year t. The oldest members of age group 15-19 at time t would have

[7] Kenneth H. Hill, "Methods for estimating fertility trends using WFS and other data", *World Fertility Survey Conference; Record of Proceedings*, London, 7-11 July 1980 (Voorburg, The Hague, International Statistical Institute, 1981), vol. 3, pp. 455-508.

[8] William Brass, *Methods for Estimating Fertility and Mortality from Limited and Defective Data* (Chapel Hill, N.C., Carolina Population Center, Laboratories for Population Studies, 1975).

TABLE 20. SEPARATION FACTORS FOR SPLITTING ANNUAL AGE-SPECIFIC FERTILITY RATES BY COHORT

Age group of fertility rate $f(j)$ (1)	Age of cohort on 31 December (2)	Index m (3)	Separation factors $s(m, j)$ for $f(1)/f(2)$ = 0.7 (4)	0.6 (5)	0.5 (6)	0.4 (7)	0.3 (8)	0.2 (9)	0.1 (10)
15-19	11-15	1	0.062	0.046	0.022	0.007	-	-	-
	12-16	2	0.196	0.180	0.127	0.065	0.013	-	-
	13-17	3	0.406	0.365	0.304	0.222	0.124	0.027	-
	14-18	4	0.626	0.593	0.544	0.474	0.378	0.239	0.040
	15-19	5	0.870	0.857	0.836	0.807	0.764	0.691	0.557
20-24	16-20	1	0.094	0.090	0.086	0.082	0.077	0.069	0.059
	17-21	2	0.288	0.279	0.270	0.261	0.249	0.233	0.209
	18-22	3	0.488	0.478	0.468	0.458	0.444	0.426	0.398
	19-23	4	0.693	0.684	0.677	0.668	0.657	0.642	0.619
	20-24	5	0.898	0.894	0.891	0.888	0.883	0.877	0.868
25-29	21-25	1	0.108	0.105	0.103	0.101	0.099	0.096	0.093
	22-26	2	0.320	0.314	0.309	0.304	0.299	0.293	0.285
	23-27	3	0.525	0.519	0.513	0.507	0.501	0.494	0.486
	24-28	4	0.723	0.717	0.713	0.708	0.703	0.698	0.690
	25-29	5	0.910	0.908	0.906	0.904	0.902	0.900	0.897
30-34	26-30	1	0.121	0.118	0.116	0.114	0.112	0.109	0.107
	27-31	2	0.351	0.344	0.338	0.334	0.329	0.324	0.318
	28-32	3	0.562	0.553	0.547	0.542	0.536	0.530	0.523
	29-33	4	0.752	0.746	0.741	0.736	0.731	0.727	0.721
	30-34	5	0.923	0.920	0.918	0.916	0.914	0.912	0.910
35-39	31-35	1	0.145	0.139	0.134	0.130	0.127	0.124	0.120
	32-36	2	0.404	0.390	0.380	0.371	0.364	0.356	0.349
	33-37	3	0.622	0.606	0.595	0.585	0.576	0.568	0.559
	34-38	4	0.801	0.788	0.779	0.771	0.764	0.758	0.750
	35-39	5	0.942	0.937	0.933	0.930	0.928	0.925	0.922
40-44	36-40	1	0.227	0.198	0.181	0.169	0.159	0.151	0.143
	37-41	2	0.577	0.517	0.482	0.456	0.435	0.417	0.399
	38-42	3	0.805	0.745	0.707	0.680	0.657	0.636	0.617
	39-43	4	0.933	0.893	0.866	0.845	0.828	0.812	0.797
	40-44	5	0.989	0.977	0.967	0.960	0.953	0.947	0.941

reached 15 at the beginning of year $t-4$, so that if no childbearing is assumed to occur before age 15, the sum need only be calculated for the years from $t-4$ to t. For age group 30-34, on the other hand, childbearing occurs during the years from $t-19$ to t, so the sum begins in year $t-19$ with a small portion of the age-specific fertility rate for women aged 15-19 and continues through 20 years, to finish with a large portion of the rate in year t for women aged 30-34, plus a small portion of the rate in year t for women aged 25-29.

In conclusion, it should be mentioned that although the form of equation (B.17) may give the impression that its application is very complicated, the basic idea underlying it is fairly simple, and if working-sheets are laid out following the procedure described in the detailed example the necessary calculations are tedious but straightforward.

Step 5: estimation of completeness of birth registration. The cumulated cohort fertility from registered births, $F(i)$, calculated in the previous step has been constructed so as to be comparable to reported cohort parity, $P(i)$, at the final census. Therefore, the ratio $F(i)/P(i)$ provides a measure of the average completeness of registration of the births that occurred to cohort i. If the completeness of registration had remained approximately constant over a period of 15 years or so, the F/P ratios should have more or less the same values for all cohorts, and an average of the ratios for age groups 20-24, 25-29 and 30-34 can be adopted as an estimate of the completeness of birth registration over the period. Hence, its reciprocal can be used as an adjustment factor for any or all of the age-specific fertility schedules calculated in step 3. If the completeness of birth registration has improved over time, the F/P ratios for the younger cohorts will be higher than for older cohorts. In such a case, the most recent fertility schedule (referring to the year of the final census) may be adjusted by $P(2)/F(2)$, the ratio reflecting the most recent level of completeness; $P(1)/F(1)$ should not be used in general as an adjustment factor because of the intrinsic difficulty in approximating $F(1)$ accurately. When the F/P ratios indicate that completeness has been improving through time, no obvious basis exists for adjusting the fertility schedules referring to earlier years.

(d) *A detailed example*

The data required for the application of the method described in this section are available for Thailand prior to 1970. Table 21 shows registered births classified by five-year age group of mother for the period 1950-1969 (only the births for women under 35 are shown, since the cumulation of cohort fertility is only carried out up to the cohort of women aged 30-34 at the end of the period). Although not shown, some of the births registered could not be classified according to age of mother because the latter information was missing. Therefore, there is a question as to what to do with these births. If their proportion over all registered births is not large and it does not change much over time, the difference introduced by distributing them according to births of known age of mother will be minor. Hence, the simplest procedure is to exclude them, in which case the estimates of registration completeness obtained refer only to births of known age of mother. If the births with unspecified age of mother are a substantial proportion of all registered births, or if their proportion changes over time, their redistribution and addition to the births with mothers of known age is likely to affect the results; because there is usually no sound basis to determine how their redistribution with respect to age of mother should be performed, the safest procedure is to exclude them from the analysis. In table 21, they have been excluded. Table 22 shows the female population enumerated by the censuses of 1947, 1960 and 1970, and the number of children ever born classified by age of mother from the 1970 census.

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities. The average parities by age group of mother, $P(i)$, are calculated for 1970 from the data displayed in table 22 by dividing the reported number of children ever born in each age group (column (5)) by the total number of women in each age group in 1970 (column (4)). Results for each age group are shown in column (6) of table 22; as an example,

$$P(4)=(4{,}112{,}920/1{,}077{,}088)=3.819.$$

Step 2: estimation of mid-year female population by age group for each year preceding the census. Table 22 shows the reference dates of the population censuses of 1947, 1960 and 1970. These dates can be converted into units of years by calculating the number of days from the beginning of the year to the census date and dividing it by 365 (or 366 in a leap year). The reference date of the 1947 census was 23 May 1947. There are 31 days in January, 28 in February (29 in a leap year), 31 in March, 30 in April; and in this case, 23 of the 31 days in May are needed. The decimal portion of the census date is thus (31 + 28 + 31 + 30 + 23)/365, or 143/365, = 0.392. Hence, the full decimal date is 1947.392. Because 1960 was a leap year, the decimal portion of the date is (31 + 29 + 31 + 25)/366 = 0.317, and the full decimal date is 1960.317. The 1970 date is 1970.249.

At this point, any adjustments necessary to improve the consistency of the completeness of enumeration of the censuses should be applied. Application of a method described in chapter IX suggests that the 1970 census was about 5 per cent less complete than the 1960 census, so the raw 1970 census figures appearing in column (4) of table 22 have been adjusted by a factor of 1.05 and are shown in column (7) of the same table. The 1947 and 1960 censuses appear to have achieved approximately the same level of completeness. Thus, columns (2), (3) and (7) of table 22 are regarded as showing an approximately consistent set of population figures.

Age-specific population growth rates are then calculated for both intercensal periods using equation (B.14) and the exact census dates calculated above. Thus, for the period 1947-1960 and for the female population aged 15-19 ($i=1$):

$$r(1)=[\ln(1{,}236{,}294)-\ln(979{,}613)]/[1960.317-1947.392]$$

$$=[14.02763-13.79491]/12.925$$

$$=0.01801.$$

The growth rates for all age groups and for both intercensal periods are shown in table 23; it will be noticed

TABLE 21. REGISTERED BIRTHS, BY FIVE-YEAR AGE GROUP OF MOTHER, THAILAND, 1950-1969

Year (1)	Age group 15-19 (2)	20-24 (3)	25-29 (4)	30-34 (5)
1950	32 352	139 307	131 729	101 258
1951	31 422	140 562	137 998	107 652
1952	31 083	150 681	141 402	104 569
1953	35 097	172 015	160 075	110 508
1954	38 528	189 795	182 055	125 260
1955	40 323	196 868	185 435	127 204
1956	44 611	216 365	207 804	143 083
1957	44 160	210 923	208 855	147 567
1958	43 607	209 405	212 248	153 627
1959	45 445	224 553	235 221	170 281
1960	49 799	238 243	250 736	183 152
1961	51 048	239 183	253 566	181 700
1962	53 686	246 039	265 988	196 153
1963	57 335	253 232	277 504	205 771
1964	64 153	270 198	299 972	231 271
1965	75 314	265 482	289 760	227 407
1966	81 238	260 053	274 279	215 850
1967	84 833	268 539	277 377	222 601
1968	92 338	295 524	288 379	234 962
1969	91 788	285 713	267 990	215 613

TABLE 22. TOTAL POPULATION AND FEMALE POPULATION AGED 15-34, BY AGE GROUP, FOR THE CENSUS YEARS 1947, 1960 AND 1970; CHILDREN EVER BORN IN 1970 AND ADJUSTED POPULATION IN 1970, THAILAND

Age group (1)	Enumerated female population 1947 23 May (2)	1960 25 April (3)	1970 1 April (4)	Children ever born 1970 (5)	Average parity, 1970 (6)	Adjusted female population, 1970 (7)
15-19	979 613	1 236 294	1 885 371	246 839	0.131	1 979 640
20-24	792 366	1 204 153	1 361 717	1 353 569	0.994	1 429 803
25-29	638 334	1 046 464	1 143 377	2 754 376	2.409	1 200 546
30-34	570 842	869 876	1 077 088	4 112 920	3.819	1 130 942
Total population	8 681 257	13 103 767	17 273 512			18 137 188

that they are rather variable, suggesting the existence of differential age-misreporting or fluctuations by age in enumeration completeness. However, beyond noting the possibility of errors, not much else can be done.

Mid-year population denominators can now be calculated for each year, from 1950 to 1969, by applying equation (B.15). A few examples will make the procedure clear. The female population aged 15-19 in mid-1950 (that is, at 1950.5 in decimal terms) is needed.

TABLE 23. AGE-SPECIFIC GROWTH RATES FOR THE FEMALE POPULATION, 1947-1960 AND 1960-1970, AFTER ADJUSTMENT, THAILAND

Age group	Index i	Growth rate of female population	
		1947-1960	1960-1970
(1)	(2)	(3)	(4)
15-19	1	0.01801	0.04740
20-24	2	0.03238	0.01729
25-29	3	0.03824	0.01383
30-34	4	0.03259	0.02643

The growth rate for age group 15-19 between 1947 and 1960 is 0.01801, and the period from the 1947 census to mid-1950 is 1950.5 − 1947.392 years. Hence, the estimated female population in 1950.5, $N(1, 1950)$, is obtained from the 1947 census population, $N(1, 1947)$, as

$$N(1, 1950) = N(1, 1947) \exp((0.01801)(1950.5 - 1947.392))$$
$$= (979{,}613) \exp((0.01801)(3.108))$$
$$= (979{,}613)(1.05757)$$
$$= 1{,}036{,}009.$$

For 1959 and age group 25-29,

$$N(3, 1959) = N(3, 1947) \exp((0.03824)(1959.5 - 1947.392))$$
$$= (638{,}334)(1.588849)$$
$$= 1{,}014{,}216.$$

For 1960, however, the 1960-1970 growth rate would be used:

$$N(3, 1960) = N(3, 1960) \exp((0.01383)(1960.5 - 1960.317))$$
$$= (1{,}046{,}464)(1.002534)$$
$$= 1{,}049{,}116.$$

The estimated mid-year populations are shown in table 24. Note that, for the sake of clarity, the calculations have been performed with a higher number of significant digits than is really required. For efficiency of calculation, it would be worth working with only four significant digits, rounding the female populations in each case to thousands.

Step 3: calculation of age-specific fertility rates from registered births. Age-specific fertility rates for each year are calculated by dividing the number of births registered for each age group of women (table 21) by the estimated mid-year female population of the age group (table 24). Thus, the age-specific fertility rate for age group 15-19 in 1950 is calculated as

$$f(1, 1950) = B(1, 1950/N(1, 1950)$$
$$= 32{,}352/1{,}036{,}009$$
$$= 0.0312.$$

TABLE 24. ESTIMATED MID-YEAR FEMALE POPULATION, BY AGE GROUP, THAILAND, 1950-1969

Year	Age group			
	15-19	20-24	25-29	30-34
(1)	(2)	(3)	(4)	(5)
1950	1 036 009	876 258	718 892	631 692
1951	1 054 838	905 095	746 915	652 618
1952	1 074 008	934 882	776 030	674 237
1953	1 093 526	965 649	806 280	696 573
1954	1 113 399	997 428	837 710	719 648
1955	1 133 633	1 030 254	870 364	743 488
1956	1 154 235	1 064 159	904 291	768 117
1957	1 175 211	1 099 181	939 541	793 562
1958	1 196 568	1 135 355	976 165	819 851
1959	1 218 313	1 172 719	1 014 216	847 010
1960	1 247 065	1 207 969	1 049 116	874 094
1961	1 307 599	1 229 036	1 063 726	897 504
1962	1 371 071	1 250 471	1 078 539	921 541
1963	1 437 625	1 272 280	1 093 559	946 222
1964	1 507 409	1 294 469	1 108 788	971 564
1965	1 580 581	1 317 045	1 124 229	997 585
1966	1 657 304	1 340 015	1 139 885	1 024 303
1967	1 737 752	1 363 385	1 155 760	1 051 736
1968	1 822 105	1 387 163	1 171 855	1 079 904
1969	1 910 552	1 411 355	1 188 174	1 108 826

Similarly, the rate for those aged 25-29 years in 1960 is calculated as

$$f(3, 1960) = B(3, 1960)/N(3, 1960)$$

$$= 250{,}736/1{,}049{,}116$$

$$= 0.2390.$$

All values of $f(i,j)$ are shown in table 25. For the period 1960-1969, age-specific fertility rates for women aged 35-39, 40-44 and 45-49 are also shown. These rates are not needed to apply the method described here, but they are needed in applying that described in subsection B.6.

Step 4: cumulation of registered fertility for different female birth cohorts to estimate parity equivalents. Some of the age-specific fertility rates shown in table 25 are not required in cumulating the fertility for the cohorts of interest since they reflect entirely the childbearing of cohorts older than 30-34 in 1970; to be specific, the rates above the dotted lines are not needed and have been calculated in part for the sake of completeness and in part to be used in subsection B.6. All the rates below the lines, however, need to be split between two cohorts in order to estimate parity equivalents, $F(i)$; and the portions corresponding to each cohort need to be cumulated separately. A convenient way of carrying out this process is to work with five-year periods. Table 26 shows the recommended way of laying out the calculations. The 20-year period considered is divided into four subperiods, each five years in length; within each period, each year is identified by an index value m ranging from 1 to 5, indicating the location of the cohorts with respect to the age groupings of the age-specific fertility rates. Thus, for 1969, m is equal to 5; and the cohorts are, on average, six months younger than the ages indicated by the age groups of the fertility rates; the cohort aged 30-34 at the end of 1969 began the year aged 29-33, for instance. For 1965, m is equal to 1; and the cohorts are, on average, 4.5 years younger than the ages indicated by the fertility rate groupings. The index m is used to select the separation factors necessary to split the observed fertility rates (see table 20). The other indices employed in table 26 are: k, a measure of years before the census; i, the cohort index; and j, which indicates the age range of the age-specific fertility rates being split.

For each group of five calendar years, the procedure is the same, though the number of rates that are split declines by one each time one moves five years into the past. The first step is to calculate, for each year, the value of $f(1)/f(2)$, the ratio of the age-specific fertility rate for women aged 15-19 to that for women aged 20-24. This ratio is an indicator of the age pattern of early childbearing, and its value is necessary for interpolating between the columns of table 20. Then, the $f(1, \tau)$ fertility rates for each of the five years preceding the census are split and the portion of each rate belonging to the census cohort aged 15-19, and its complement, belonging to the census cohort aged 20-24, are identified. The separation factors for these rates are obtained from table 20 and depend upon the index m, the age group j and the value of $f(1)/f(2)$. For 1969, m is 5 and $f(1)/f(2)$ is 0.2372. The required separation factor is therefore a value between 0.764 for an $f(1)/f(2)$ of 0.3, and 0.691

TABLE 25. AGE-SPECIFIC FERTILITY RATES CALCULATED FROM REGISTERED BIRTHS AND INTERPOLATED MID-YEAR FEMALE POPULATION, THAILAND, 1950-1969

	Age group						
Year (1)	*15-19 (2)*	*20-24 (3)*	*25-29 (4)*	*30-34 (5)*	*35-39 (6)*	*40-44 (7)*	*45-49 (8)*
1950..........	0.0312	0.1590	0.1832	0.1603	-	-	-
1951..........	0.0298	0.1553	0.1848	0.1650	-	-	-
1952..........	0.0289	0.1612	0.1822	0.1551	-	-	-
1953..........	0.0321	0.1781	0.1985	0.1586	-	-	-
1954..........	0.0346	0.1903	0.2173	0.1741	-	-	-
1955..........	0.0356	0.1911	0.2131	0.1711	-	-	-
1956..........	0.0386	0.2033	0.2298	0.1863	-	-	-
1957..........	0.0376	0.1919	0.2223	0.1860	-	-	-
1958..........	0.0364	0.1844	0.2174	0.1874	-	-	-
1959..........	0.0373	0.1915	0.2319	0.2010	-	-	-
1960..........	0.0399	0.1972	0.2390	0.2095	0.1751	0.0870	0.0213
1961..........	0.0390	0.1946	0.2384	0.2025	0.1626	0.0796	0.0197
1962..........	0.0392	0.1968	0.2466	0.2129	0.1753	0.0855	0.0216
1963..........	0.0399	0.1990	0.2538	0.2175	0.1803	0.0876	0.0219
1964..........	0.0426	0.2087	0.2705	0.2380	0.1965	0.0935	0.0228
1965..........	0.0476	0.2016	0.2577	0.2280	0.1884	0.0926	0.0235
1966..........	0.0490	0.1941	0.2406	0.2107	0.1736	0.0874	0.0238
1967..........	0.0488	0.1970	0.2400	0.2117	0.1717	0.0869	0.0247
1968..........	0.0507	0.2130	0.2461	0.2176	0.1795	0.0906	0.0234
1969..........	0.0480	0.2024	0.2255	0.1945	0.1591	0.0824	0.0225

TABLE 26. CALCULATION OF PARITY EQUIVALENTS FOR DIFFERENT FEMALE BIRTH COHORTS, THAILAND, 1950-1969

Age groups i (1)	j (2)	−k= / m= / f(1)/f(2)= (3)	1965 / −4 / 1 / 0.2361 (4)	1966 / −3 / 2 / 0.2524 (5)	1967 / −2 / 3 / 0.2477 (6)	1968 / −1 / 4 / 0.2380 (7)	1969 / 0 / 5 / 0.2372 (8)	Sum / - (9)
				(a) *1965-1969*				
1	1	$f(1)$	0.0476	0.0490	0.0488	0.0507	0.0480	-
		$s(m,1)$	0.0000	0.0070	0.0730	0.2920	0.7180	-
		Cohort 1	0.0000	0.0003	0.0036	0.0148	0.0345	0.0532
		Cohort 2	0.0476	0.0487	0.0452	0.0359	0.0135	0.1909
2	2	$f(2)$	0.2016	0.1941	0.1970	0.2130	0.2024	-
		$s(m,2)$	0.0720	0.2410	0.4350	0.6480	0.8790	-
		Cohort 2	0.0145	0.0468	0.0857	0.1380	0.1779	0.4629
		Cohort 3	0.1871	0.1473	0.1113	0.0750	0.0245	0.5452
3	3	$f(3)$	0.2577	0.2406	0.2400	0.2461	0.2255	-
		$s(m,3)$	0.0970	0.2960	0.4970	0.7000	0.9010	-
		Cohort 3	0.0250	0.0712	0.1193	0.1723	0.2032	0.5910
		Cohort 4	0.2327	0.1694	0.1207	0.0738	0.0223	0.6189
4	4	$f(4)$	0.2280	0.2107	0.2117	0.2176	0.1945	-
		$s(m,4)$	0.1100	0.3270	0.5330	0.7290	0.9130	-
		Cohort 4	0.0251	0.0689	0.1128	0.1586	0.1776	0.5430
		Cohort 5	0.2029	0.1418	0.0989	0.0590	0.0169	0.5195
5	5	$f(5)$	0.1884	0.1736	0.1717	0.1795	0.1591	-
		$s(m,5)$	0.1100	0.3270	0.5330	0.7290	0.9130	-
		Cohort 5	0.0207	0.0568	0.0915	0.1309	0.1453	0.4452
		Cohort 6	0.1677	0.1168	0.0802	0.0486	0.0138	0.4271
6	6	$f(6)$	0.0926	0.0874	0.0869	0.0906	0.0824	-
		$s(m,6)$	0.1250	0.3600	0.5720	0.7600	0.9260	-
		Cohort 6	0.0116	0.0315	0.0497	0.0689	0.0763	0.2380
		Cohort 7	0.0810	0.0559	0.0372	0.0217	0.0061	0.2020
7	7	$f(7)$	0.0235	0.0238	0.0247	0.0234	0.0225	-
		$s(m,7)$	0.1540	0.4260	0.6460	0.8180	0.9490	-
		Cohort 7	0.0036	0.0101	0.0160	0.0191	0.0214	0.0702
		Cohort 8	(0.0199)	(0.0137)	(0.0087)	(0.0043)	(0.0011)	(0.0477)

Age groups i (1)	j (2)	−k= / m= / f(1)/f(2)= (3)	1960 / −9 / 1 / 0.2023 (4)	1961 / −8 / 2 / 0.2004 (5)	1962 / −7 / 3 / 0.1992 (6)	1963 / −6 / 4 / 0.2005 (7)	1964 / −5 / 5 / 0.2041 (8)	Sum / - (9)
				(b) *1960-1964*				
2	1	$f(1)$	0.0399	0.0390	0.0392	0.0399	0.0426	-
		$s(m,1)$	0.0000	0.0000	0.0270	0.2400	0.6940	-
		Cohort 2	0.0000	0.0000	0.0011	0.0096	0.0296	0.0403
		Cohort 3	0.0399	0.0390	0.0381	0.0303	0.0130	0.1603
3	2	$f(2)$	0.1972	0.1946	0.1968	0.1990	0.2087	-
		$s(m,2)$	0.0690	0.2330	0.4260	0.6420	0.8770	-
		Cohort 3	0.0136	0.0453	0.0838	0.1278	0.1830	0.4535
		Cohort 4	0.1836	0.1493	0.1130	0.0712	0.0257	0.5428
4	3	$f(3)$	0.2390	0.2384	0.2466	0.2538	0.2705	-
		$s(m,3)$	0.0960	0.2930	0.4940	0.6980	0.9000	-
		Cohort 4	0.0229	0.0699	0.1218	0.1772	0.2435	0.6353
		Cohort 5	0.2161	0.1685	0.1248	0.0766	0.0271	0.6131
5	4	$f(4)$	0.2095	0.2025	0.2129	0.2175	0.2380	-
		$s(m,4)$	0.1090	0.3240	0.5300	0.7270	0.9120	-
		Cohort 5	0.0228	0.0656	0.1128	0.1581	0.2171	0.5764
		Cohort 6	0.1867	0.1369	0.1001	0.0594	0.0209	0.5040
6	5	$f(5)$	0.1751	0.1626	0.1753	0.1803	0.1965	-
		$s(m,5)$	0.1240	0.3560	0.5680	0.7580	0.9250	-
		Cohort 6	0.0217	0.0579	0.0996	0.1367	0.1818	0.4977
		Cohort 7	0.1534	0.1047	0.0757	0.0436	0.0147	0.3921
7	6	$f(6)$	0.0870	0.0796	0.0855	0.0876	0.0935	-
		$s(m,6)$	0.1510	0.4170	0.6360	0.8120	0.9470	-
		Cohort 7	0.0131	0.0332	0.0544	0.0711	0.0885	0.2603
		(Cohort 8)	(0.0739)	(0.0464)	(0.0311)	(0.0165)	(0.0050)	(0.1729)

TABLE 26 *(continued)*

Age groups *i* (1)	*j* (2)	(3)	Year 1955 (4)	1956 (5)	1957 (6)	1958 (7)	1959 (8)	Sum (9)
		$-k=$	-14	-13	-12	-11	-10	
		$m=$	1	2	3	4	5	
		$f(1)/f(2)=$	0.1863	0.1899	0.1959	0.1974	0.1948	.
				(c) *1955-1959*				
3	1	$f(1)$	0.0356	0.0386	0.0376	0.0364	0.0373	.
		$s(m, 1)$	0.0000	0.0000	0.0260	0.2340	0.6840	.
		Cohort 3	0.0000	0.0000	0.0010	0.0085	0.0255	0.0350
		Cohort 4	0.0356	0.0386	0.0366	0.0279	0.0118	0.1505
4	2	$f(2)$	0.1911	0.2033	0.1919	0.1844	0.1915	.
		$s(m, 2)$	0.0680	0.2310	0.4250	0.6410	0.8770	.
		Cohort 4	0.0130	0.0470	0.0816	0.1182	0.1679	0.4277
		(Cohort 5)	0.1781	0.1563	0.1103	0.0662	0.0236	0.5345

Age groups *i* (1)	*j* (2)	(3)	Year 1950 (4)	1951 (5)	1952 (6)	1953 (7)	1954 (8)	Sum (9)
		$-k=$	-19	-18	-17	-16	-15	
		$m=$	1	2	3	4	5	
		$f(1)/f(2)=$	0.1962	0.1919	0.1793	0.1802	0.1818	.
				(d) *1950-1954*				
4	1	$f(1)$	0.0312	0.0298	0.0289	0.0321	0.0346	.
		$s(m, 1)$	0.0000	0.0000	0.0210	0.2000	0.6670	.
		Cohort 4	0.0000	0.0000	0.0006	0.0064	0.0231	0.0301
		(Cohort 5)	0.0312	0.0298	0.0283	0.0257	0.0115	0.1265

Note: i = cohort index; j = age range of the age-specific fertility rates being split; k = measure of years before the census; m = index value indicating the location of cohorts with respect to the age groupings of the age-specific fertility rates; $f(1)/f(2)$ = ratio of the age-specific fertility rate for women aged 15-19 to that for women aged 20-24.

for an $f(1)/f(2)$ of 0.2. The required value can be found by linear interpolation, as shown below:

$$s(5, 1) = (0.764)(0.372) + (0.691)(1.0 - 0.372) = 0.718.$$

Note that the interpolation factor, 0.372, is simply the value of $f(1)/f(2)$, 0.2372, less the lower bound of the interval, 0.2, and divided by the width of the interval, 0.1. It is thus simple to calculate by moving the decimal point of $f(1)/f(2)$ one place to the right and taking only the decimal part of the resulting number. The portion of the fertility rate belonging to the youngest female census cohort considered is then found by multiplying this rate by the separation factor; for 1969, the resulting portion of $f(1, 1969)$ to be cumulated into $F(1)$ is (0.718)(0.0480), or 0.0345. The remainder, 0.0135, is the portion of $f(1,1969)$ contributed by the second census cohort (aged 20-24), and it is recorded for later cumulation. In the case of the first cohort, all its childbearing has occurred during the five years immediately preceding the census, so its cumulated fertility at the end of 1969, $F(1)$, is just the sum of all the portions belonging to the cohort for the years from 1965 through 1969:

$$F(1) = 0.0 + 0.0003 + 0.0036 + 0.0148 + 0.0345 = 0.0532.$$

This sum appears in column (9) of table 26, part (a). The complementary portions of each rate are also cumulated; and their sum, 0.1909, is recorded in column (9) for later use.

The same process is then repeated using the fertility rates for the next age group, 20-24. Separation factors are again obtained from table 20, on the basis of the values of m and of the observed $f(1)/f(2)$. Thus, for 1966, m is 2 and $f(1)/f(2)$ is 0.2524, so the interpolation factor is 0.524; and $s(m, 2)$ is obtained as

$$s(m, 2) = (0.249)(0.524) + (0.233)(1.0 - 0.524)$$
$$= 0.241.$$

The portion of $f(2, 1966)$ contributed by the cohort aged 20-24 at the time of the census (cohort 2) is thus $f(2, 1966)(0.241)$, or 0.0468; its complement, 0.1473, has been contributed by the third cohort (aged 25-29 in 1970). The total contribution of cohort 2 to the fertility rates for age group 20-24 during the period 1965-1969 is then found by adding the estimated contributions for each year of the period, giving 0.4629; the complement of this quantity, 0.5452, is the contribution of cohort 3. Both values are recorded in column (9).

It is now possible to calculate the fertility accumulated by the cohort aged 20-24 in 1970 during the five years preceding the census. It is the sum of the cohort's contribution to the age-specific fertility rates for those aged 15-19 (0.1909) and its contribution to the rates for those aged 20-24 (0.4629). This sum, 0.6538, is not yet an estimate of the total cumulated fertility for the cohort, since it does not include the cohort's childbearing experience during the period from 6 to 10 years before the census: the missing estimates are calculated in part (b) of table 26.

Similar calculations are carried out for the age-specific fertility rates for women aged 25-29 and 30-34 registered during the period 1965-1969, each rate being divided up by cohort contribution and the portions attributable to each cohort being cumulated over the five years. The portion of the rates for those aged 30-34

TABLE 27. CONTRIBUTIONS TO COHORT FERTILITY, BY AGE GROUP AND FIVE-YEAR PERIOD, THAILAND, 1950-1969

Age group (1)	Number of years ago (2)	Reference period (3)	Age group of cohort at end of 1969: 15-19 (4)	20-24 (5)	25-29 (6)	30-34 (7)
15-19	15-19	1950-1954	-	-	-	0.0301
	10-14	1955-1959	-	-	0.0350	0.1505
	5-9	1960-1964	-	0.0403	0.1603	-
	0-4	1965-1969	0.0532	0.1909	-	-
20-24	10-14	1955-1959	-	-	-	0.4277
	5-9	1960-1964	-	-	0.4535	0.5428
	0-4	1965-1969	-	0.4629	0.5452	-
25-29	5-9	1960-1964	-	-	-	0.6353
	0-4	1965-1969	-	-	0.5910	0.6189
30-34	0-4	1965-1969	-	-	-	0.5430
Parity equivalent, $F(i)$			0.0532	0.6941	1.7850	2.9483

that is contributed by women aged 35-39 at the time of the census is not needed in this application, but it has been calculated for the sake of completeness. The calculations in parts (a) and (b) of table 26 have in fact been extended to cover all the childbearing experience of cohorts aged 35-49 in 1970, mainly because these estimates are used later in subsection B.6.

Once the calculations for the five years immediately preceding the 1970 census have been completed, the same procedure is applied for each of the years of the period 1960-1964. The results are shown in part (b) of table 26. Note how the fertility rates for the oldest age group are excluded. Strictly speaking, only those for the first three age groups need to be included, since the oldest cohort considered, those aged 30-34 in 1970, was aged under 30 in 1965. Once the rates for the period from 6 to 10 years (1960-1964) before the census have been split and cumulated, the same procedure is applied to the rates for the period from 11 to 15 years before (it need only be applied to the first two age-specific fertility rates) as shown in part (c) of table 26 and to those for the period from 16 to 20 years before (for only the first age-specific rate) as shown in part (d) of the same table.

The final parity equivalents, $F(i)$, can now be obtained by summing the portions contributed by each cohort for each five-year period. To avoid errors, such portions can be copied from column (9) of table 26 (labelled "sum") and arranged in columns by cohort, age group of the fertility rates from which the contribution came and period to which the rates refer, as shown in table 27. Cumulation of the entries in each column leads to the desired $F(i)$ values.

It is worth taking note that table 27 may reveal other interesting features of the registration data. Consider, for example, the set of contributions to cohort fertility of the period fertility rates for age group 15-19; the diagonals show the change in such contributions through time. Reading from right to left, it is clear that these contributions have been increasing, indicating either rising fertility among this age group or improving registration of their births. Given the assumptions on which this estimation method is based, the former situation will not affect a final estimate of completeness of birth registration, but the latter will, and the estimates of completeness obtained in such circumstances will be averages of recent levels.

Step 5: estimation of completeness of birth registration. For each cohort, the completeness of birth registration is estimated as the ratio of the parity equivalent, $F(i)$, calculated from registered births to reported average parity, $P(i)$, as obtained from the census. Table 28 shows the results.

The estimates of completeness of registration of births for women aged 20-24, 25-29 and 30-34 are moderately consistent, suggesting an average level of completeness of around 74 per cent. The estimate for women aged 15-19 is very low, however, and the other estimates rise

TABLE 28. ESTIMATES OF COMPLETENESS OF BIRTH REGISTRATION, THAILAND, 1969

Age group in 1969 (1)	Reported average parity $P(i)$ (2)	Parity equivalent $F(i)$ (3)	Completeness of birth registration $F(i)/P(i)$ (4)
15-19	0.131	0.0532	0.406
20-24	0.994	0.6941	0.698
25-29	2.409	1.7850	0.741
30-34	3.819	2.9483	0.772

with age of woman. It seems likely that registration of births is substantially less complete among very young women and that this differential also reduces the completeness estimate derived from the reports of women aged 20-24. The estimates of completeness based on the reports of women aged 25-29 and 30-34 are affected relatively little by the excess omission at early ages, so that in this case a better estimate of average completeness of birth registration would be an average of 0.741 and 0.772, the final estimate therefore being 0.757. Hence, an estimate of fertility for 1969 could be obtained by inflating the registered age-specific fertility rates for that year by a factor of 1.0/0.757, or 1.321. It should be noted, however, that the adjusted fertility schedule might not be a good indicator of the age pattern of childbearing, because of the relatively higher omission of births by young women.

The results presented in table 28 do not suggest that birth registration completeness has been changing, so the adjustment factor of 1.321 can also be applied to the observed age-specific fertility rates for years preceding 1969. However, because the contributions of fertility rates registered before 1960 to cumulated cohort fertility are small, the estimated adjustment factor cannot be validly applied to the fertility rates registered before 1960. Although not observed in the case of Thailand, evidence of a trend towards more complete registration, such as a tendency for the estimates of completeness to decline with age, should warn against adjusting registered births for particular years.

6. *Comparison of the cohort fertility registered between two censuses with cohort parity increments*

(a) *Basis of method and its rationale*

When information on average parity from two censuses (or surveys) is available in such a way that cohort parity increments can be calculated (see subsection B.4) and age-specific fertility rates can be calculated from registered births for the intersurvey period, a more specific version of the method described in the previous section can be applied. The change in parity of a cohort as it ages from one census or survey to the next is a measure of the childbearing experience of the cohort during the interval. An equivalent measure can be calculated from registered births by splitting the fertility rates registered during the interval by cohort and cumulating the contributions of each cohort. The ratio of cumulated cohort fertility during the interval estimated from period rates, such as those calculated from registered births, to the cohort change in average parity from the beginning to the end of the interval provides a measure of completeness of birth registration specific both to a cohort and to a time period. The main assumptions made in the previous section in order to adjust period fertility on the basis of cumulated cohort fertility from the beginning of childbearing, namely, that registration completeness be constant both by age of mother and by period, are no longer required. On the other hand, parity increments are very sensitive to changes in the completeness of reporting of children ever born, as a result of which the estimates of registration completeness obtained by the method described in this section are also quite sensitive to such changes, which are generally most marked for older women.

(b) *Data required*

The data required for this method are described below:

(*a*) Children ever born classified by five-year age group of mother for two points in time, five or 10 years apart;

(*b*) Registered births classified by five-year age group of mother for each calendar year of the period between the two surveys;

(*c*) The total number of women in each age group at the beginning and end of the period, or enough information to estimate the mid-year female population by five-year age group for each year for which birth registration data are available.

(c) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities for the first and second surveys. Average parities obtained from the first survey are denoted by $P(i,1)$ and those obtained from the second by $P(i,2)$. In both cases, they are computed by dividing the reported number of children ever born to women in age group i by the total number of women in age group i. See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the possible application of the El-Badry correction (described in annex II).

Step 2: calculation of cohort parity increments. For each cohort of women of age group i at the second census or survey, the average parity of the same cohort at the first survey can be identified. Then, the cohort parity increment, $\Delta P(i)$, can be calculated as

$$\Delta P(i) = P(i,2) - P(i-n,1) \qquad \text{(B.18)}$$

where n is the number of five-year periods between the two surveys.

Step 3: estimation of mid-year female population by age group for each year of the intersurvey period. The procedure to be followed is exactly the same as that described in step 2 of subsection B.5(c). Its description is not repeated here.

Step 4: calculation of age-specific fertility rates from registered births for each year of the intersurvey period. Again, the procedure is exactly the same as that described in step 3 of subsection B.5 (c). Its description is omitted here.

Step 5: calculation of intersurvey increments in cohort fertility from registered births. In essence, the calculation of intersurvey increments in cohort fertility, denoted by $\Delta F(i)$, is very similar to the calculation of lifetime cohort parity equivalents from period rates. Each age-specific fertility rate for a given year of the intersurvey period (taken to be exactly five or 10 years in length) is split into two portions, according to the cohort structure of the age group to which the rate refers during the given calendar year; one portion is the period contribution to the fertility increment of one cohort and the other is the contribution to the fertility increment of the next cohort. The main differences between this procedure and that described in step 4 of subsection B.5 (c) is that here all calculations are limited to the intersurvey period, and all cohorts of reproductive age at the time of the second survey are considered. The splitting of age-specific fertility rates is carried out using the separation factors in table 20; the general definition of the cohort fertility increments is

$$\Delta F(i)=\sum_{j=1}^{n}\sum_{m=1}^{5}[s(m,k)f(k,h)$$

$$+(1-s(m,k-1))f(k-1,h)] \qquad (B.19)$$

where $\Delta F(i)$ = parity increment for cohort i at the second survey;

n = number of five-year periods in the intersurvey interval;

$s(m,k)$ = separation factor from table 20 required for age group k, where $k=(i-n+j)$;

$f(k,h)$ = age-specific fertility rate for age group k in year h, where $h=5(j-n-1)+m$ years before the second survey (assumed to have taken place at the end of a year).

The detailed example given below illustrates the way in which this equation is used in practice.

Step 6: estimation of completeness of birth registration. Estimates of the completeness of birth registration for the offspring of cohort i during the intersurvey period are obtained by dividing the fertility increment, $\Delta F(i)$, estimated for the cohort by the parity increment, $\Delta P(i)$, of the same cohort. If the estimates for different cohorts are consistent, the intersurvey births classified by age group can be inflated by an average of the $\Delta P(i)/\Delta F(i)$ ratios and adjusted age-specific fertility rates can be obtained by dividing the inflated number of births by an estimate of the female person-years lived in each age group over the intercensal period.

(d) *A detailed example*

The detailed example is again for Thailand, since data on children ever born are available both for 1960 and for 1970. Average parities for each census have already been presented in table 17 and annual age-specific fertility rates for the intercensal period can be found in table 25. However, some of the raw data necessary for the calculation of the latter rates are not presented. The steps followed in applying the method to this example are given below.

Step 1: calculation of reported average parities for the first and second surveys. The method of calculating average parities has been described in step 1 of subsection B.4 (c) and is not repeated here; the $P(i,t)$ values for 1960 and 1970 are shown in columns (2) and (3) of table 29. Note that the average parities are taken as referring to the year-end nearest to the census dates, in this case to the end of 1959 and the end of 1969, respectively (the exact census dates can be found in table 22).

Step 2: calculation of cohort parity increments. As the intersurvey interval in this case is 10 years, the n in equation (B.18) is 2. Thus, the parity increment for a cohort in age group i at the second survey is calculated by subtracting from the average parity of women of age group i at the second survey, $P(i,2)$, the average parity of the same cohort 10 years earlier, $P(i-2,1)$. For example, for the cohort aged 25-29 in 1970,

$$\Delta P(3)=P(3, 1970)-P(1, 1960)$$

$$=2.409-0.072=2.337.$$

The parity increments are shown in column (4) of table 29.

TABLE 29. REPORTED AVERAGE PARITIES, 1960 AND 1970; AND COHORT PARITY INCREMENTS DURING THE INTERCENSAL PERIOD, THAILAND

Age group (1)	Average parity 1960 (2)	Average parity 1970 (3)	Cohort parity increment 1960-1970 (4)
15-19	0.072	0.131	0.131
20-24	0.862	0.994	0.994
25-29	2.296	2.409	2.337
30-34	3.783	3.819	2.957
35-39	4.984	5.082	2.786
40-44	5.733	5.921	2.138
45-49	5.914	6.120	1.136

Step 3: estimation of mid-year female population by age group for each year of the intersurvey period. Most of the required estimates have already been made in step 2 of subsection B.5 (d), the results being shown in table 24. Since the procedure to estimate the remaining female populations is exactly the same as that used there, it is not illustrated again.

Step 4: calculation of age-specific fertility rates from registered births for each year of the intersurvey period. Most of the required fertility rates have already been calculated in step 3 of subsection B.5 (d) and all the required rates are shown in table 25. Although the raw data needed to calculate some of these rates have not been presented, the calculation procedure is identical to that illustrated in subsection B.5 (d) and is not repeated here.

Step 5: calculation of intersurvey increments in cohort fertility from registered births. In this example, the starting-point of the cumulation is the beginning of 1960 and the final point is the end of 1969. Fertility rates for the intervening decade have already been calculated and split by cohorts identified at the end of 1969 in table 26, parts (a) and (b). The cohort fertility increments, denoted by $\Delta F(i)$, can therefore be obtained simply by cumulating the portions of the observed age-specific fertility rates attributable to each cohort. Table 30 shows these portions arranged by cohort and five-year period (they were copied from column (9) of table 26, parts (a) and (b), labelled "Sum"). Table 30 also shows the sum, $\Delta F(i)$, of the portions corresponding to each cohort. The process of splitting fertility rates by cohort is exactly the same as that used in subsection B.5 (c) (step 4) to construct table 26, so it is not described again in detail. However, in order to enable the reader to grasp the nature of the cumulation carried out in this case, one

TABLE 30. CONTRIBUTIONS OF PERIOD FERTILITY TO INTERSURVEY COHORT FERTILITY BY FIVE-YEAR PERIOD AND ESTIMATED INTERSURVEY INCREMENTS IN COHORT FERTILITY, THAILAND, 1960-1969

Cohort's age group at end of 1969 (1)	Index i (2)	Contribution to cohort fertility from age group j: 1960-1964, j=i−2 (3)	1960-1964, j=i−1 (4)	1965-1969, j=i−1 (5)	1965-1969, j=i (6)	Estimated cohort fertility increment $\Delta F(i)$ (7)
15-19	1	-	-	-	0.0532	0.0532
20-24	2	-	0.0403	0.1909	0.4629	0.6941
25-29	3	0.1603	0.4535	0.5452	0.5910	1.7500
30-34	4	0.5428	0.6353	0.6189	0.5430	2.3400
35-39	5	0.6131	0.5764	0.5195	0.4452	2.1542
40-44	6	0.5040	0.4977	0.4272	0.2380	1.6667
45-49	7	0.3921	0.2603	0.2020	0.0702	0.9246

may consider the case of the cohort aged 30-34 at the end of 1969. At the beginning of 1960, this cohort was aged 20-24, and therefore all the annual fertility rates for age group 20-24 during the period 1960-1964 contributed a portion to the intercensal fertility of this cohort (the exact amount contributed by each rate is found in part (b) of table 26, in the panel corresponding to $f(2)$ and the line labelled "Cohort 4". When the annual contributions of the $f(2)$ rates are summed over the period 1960-1964, their total contribution to the intersurvey fertility of the cohort aged 30-34 at the end of 1969 is 0.5428. In a similar way, the total contribution of the $f(3)$ rates (those for age group 25-29) from 1960 to 1964 is 0.6353 of a child, and that of the same rates from 1965 to 1969 is 0.6189 of a child. Lastly, the total contribution of the annual $f(4)$ rates (for age group 30-34) during the period 1965-1969 is 0.5430 of a child. Hence, the total increment in intersurvey or intercensal fertility for the cohort aged 30-34 at the end of 1969 is 2.3400 births per woman.

Step 6: estimation of the completeness of birth registration. Cohort-specific estimates of the completeness of birth registration between 1960 and 1969 can now be obtained by calculating the ratios of $\Delta F(i)$ to $\Delta P(i)$. Results are shown in table 31. Note that the estimates for the cohorts aged 15-19 and 20-24 are the same as those obtained in subsection B.5(d) (table 28) where they were

TABLE 31. ESTIMATES OF COMPLETENESS OF BIRTH REGISTRATION, THAILAND, 1960-1969

Cohort's age at end of 1969 (1)	Index i (2)	Intercensal cohort fertility increment $\Delta F(i)$ (3)	Intercensal cohort parity increment $\Delta P(i)$ (4)	Estimated completeness of birth registration $\Delta F(i)/\Delta P(i)$ (5)
15-19	1	0.053	0.131	0.405
20-24	2	0.694	0.994	0.698
25-29	3	1.750	2.337	0.749
30-34	4	2.340	2.957	0.791
35-39	5	2.154	2.786	0.773
40-44	6	1.667	2.138	0.780
45-49	7	0.925	1.136	0.814

based on the lifetime, rather than the intersurvey, estimates of cohort fertility. This outcome was to be expected because all the childbearing of these two cohorts is assumed to have taken place during the period 1960-1969, so that their lifetime and intersurvey fertility estimates are identical.

Two features of the results require some comment, namely, the low estimates of completeness for women under 25 and the relative consistency of the estimates for the central age range (25-44).

In interpreting the results of this method, it is important to bear in mind its assumptions. It has been assumed that the reporting of children ever born is complete at both censuses. However, if there is a tendency among older women to omit children ever born, cohort parity increments will be reduced and the estimates of registration completeness will be increased. A relatively minor deterioration of the completeness of parity reporting by all cohorts from one census to the next will produce estimates of completeness that increase with age. In a similar way, a relative amelioration of the completeness of parity reporting will lead to completeness estimates that decrease with age.

The second assumption made is that parity increments are unaffected by migration and mortality. If low-parity women are more likely to migrate than those of higher parities, areas of in-migration are likely to display reduced parity increments, and vice versa. If low-parity women are more likely to die than high-parity women, parity increments will be inflated. In the case of Thailand, neither of these two effects is likely to be important, since both international migration and mortality are low.

Another assumption of importance is that the denominators used are accurate. Age-reporting errors that are not the same when a birth is registered as when the population is enumerated will distort the pattern of period age-specific fertility rates. The rather wide variations exhibited by the age-specific growth rates given in table 23 suggest that there are problems with age-reporting, but their effects on the final estimates of completeness are very hard to predict. Denominators may also be distorted by changes in the completeness of enumeration from one census to the next; in the case of Thailand, the 1970 overall census count is about 5 per cent less complete than that yielded by the 1960 census; although the former count was adjusted for this change, differential completeness of enumeration by age group might still affect the results. Changes in enumeration completeness might also affect average parities; if women with children are more likely to be enumerated than women without, average parities will be inflated by omission.

However, none of these considerations explains adequately the low estimates of completeness obtained for the cohorts aged 15-19 and, to a lesser extent, those aged 20-24. Of course, the procedure used to split the period fertility rates is not perfect and it is most likely to be inaccurate at 15-19, but possible methodological inaccuracy cannot explain the large differential observed. Furthermore, since average parities were calculated without making any adjustment for non-response, they are more likely to be too small than too large. Hence, on the basis of this evidence alone, it would appear that birth registration is really less complete for young mothers than for the older group. Yet, those familiar with the data from Thailand and with the typical errors affecting age-reporting in East Asian countries may suggest another explanation for the outcome observed. It is customary in East Asian cultures to reckon age as of conception rather than as of birth. Therefore, when a question on age is posed (as during the 1960 census) the reports collected tend to reflect age as of the next birthday rather than age at the last birthday, as is normally expected. To avoid such misreporting problems, date of birth should be requested instead of age (as was done during the 1970 census). Because age is also the item recorded in the vital registration system, the direct intercensal comparison of average parity as reported in the 1960 and 1970 censuses with that reconstructed from vital registration is likely to yield biased results because of the change in the age-recording scheme. Furthermore, younger cohorts are those most likely to suffer from such biases, since age reported as of next birthday would make them lose systematically some of their high-fertility members. In this context, at least part of the low completeness estimates observed for the cohorts aged 15-19 and 20-24 at the time of the 1970 census may be due to non-comparable age-reporting schemes in the different data-collection systems producing the data used as input. It must be noted, however, that even when such deficiencies in the basic data are taken into account and some adjustment is performed to eliminate the biases they imply, the low completeness estimates associated with younger mothers persist, albeit at a somewhat lower level than that shown in this example.

C. Estimation of age-specific fertility from the increment of cohort parities between two surveys

1. *Basis of method and its rationale*

Data on children ever born tabulated by standard five-year age group of women for a single census or survey convey much information about the past fertility experience of the women. Unfortunately, however, if fertility has been changing, it is not possible to use the average parities of women in different age groups to obtain estimates of the age patterns of either cohort or period fertility.

Yet, if information on children ever born is available from two surveys approximately five or 10 years apart, the change in the average number of children ever borne by a particular cohort of women reflects their intercensal fertility; and it becomes possible to estimate an intercensal age-specific fertility schedule. Arretx[9] developed a method for using such information with a 10-year interval between the surveys; Coale and Trussell recently developed an elegant method based on the concept of the hypothetical cohort for using such information with an interval of five or 10 years between the surveys. The latter method is relatively simple and a variant of it is described here, illustrated by cases with such intervals.

The general warning given in subsection A.2 about the use of information on children ever born in estimating fertility should be kept in mind in this instance. There is a distinct tendency, even in countries with otherwise reasonably good data, for older women to omit some of their children, perhaps those who have died or those who have left home. As a result, average parities often fail to increase at a plausible rate, or may even decrease after age 35 or 40. The calculation of age-specific fertility rates from parities that suffer from such a degree of omission will result in underestimates of the fertility of older women; and if the error is relatively minor, its effects may not be obvious. Thus, fertility estimates based on average parities of older women must be interpreted with caution, particularly if they indicate low fertility in relation to that estimated from the reports of younger women. Average parities for a hypothetical cohort are in fact very sensitive to changes in parity reporting from one survey to the other, and the calculation of such parities provides a useful consistency check of the raw data.

The method based on the increment of cohort parities between two surveys estimates the average age-specific fertility rates in effect during the intersurvey period by constructing the average parities of a hypothetical, intersurvey cohort; a cumulated fertility schedule is then derived from these parities by interpolation, and age-specific fertility rates are obtained from cumulated fertility by successive subtraction.

The method is intended for situations in which it is possible to calculate average parities by age group of women for two points in time approximately five or 10 years apart. If the interval between the surveys is five years, the women in any five-year age group at the second survey represent the survivors of the women in the next younger five-year age group at the first survey. The difference in the average parity of the cohort between the first and the second surveys reflects its childbearing experience between the two surveys, if it is assumed that the women who died or migrated between the two surveys had, on average, lifetime fertility that

[9] Carmen Arretx, "Fertility estimates derived from information on children ever born using data from censuses", *International Population Conference, Liège, 1973* (Liège, International Union for the Scientific Study of Population, 1973), vol. 2, pp. 247-261.

was not systematically different from that of the native women who remained. By cumulating the intersurvey parity increments, it is possible to estimate average parities for a hypothetical cohort experiencing throughout its hypothetical lifetime the age-specific fertility rates in effect during the intersurvey period. If the length of this period is 10 years, a five-year age group at the second survey represents the survivors of the five-year age group who were two groups younger at the first survey; and it is still possible to calculate the cohort parity increment for each cohort in order to construct the average parities of a hypothetical intersurvey cohort. The method may be applied when the data come entirely or partially from nationally representative sample surveys, for although cohorts of particular individuals will not be identical on each occasion, their average parities will be representative of those of the sampled female population.

It is worth noting that although the strength of the intercensal parity evolution method is its robustness to changing fertility, the technique presented here can also be used to estimate age-specific fertility rates using parity data from only one census or survey when fertility has not been changing during the reproductive life spans of the women concerned. It may also be mentioned that the two data sets need not refer to two points exactly five or 10 years apart. Unless fertility is changing very rapidly, a four-year interval or an 11-year interval will provide reasonable estimates. In such a case, one is no longer following a cohort from survey to survey, but this factor is not very important because the average parity of an åge group will not change rapidly from one year to the next.

A final general observation is that if the required information is available, the El-Badry correction procedure to estimate the level of non-response, described in annex II, should be applied when calculating average parities by age group. For a further discussion of the treatment of women whose parity is not stated, see subsection A.2.

2. *Fertility estimation from the increment of cohort parities between two surveys*

(a) *Data required*

The data required for this method are described below:

(*a*) Children ever born classified by five-year age group of mother for two points in time approximately five or 10 years apart;

(*b*) Number of women aged 15-49, classified by five-year age group for the same two points in time.

(b) *Computational procedure*

The computational procedure is slightly different if the intersurvey period is five years than if it is 10 years in length. However, the only difference occurs in step 2, where the average parities of the hypothetical cohort are derived. Therefore, two versions of step 2 are described here: step 2A to be used with a five-year interval; and step 2B to be used with a 10-year interval. All other steps are described once.

Step 1: calculation of average parities for both surveys. The observed average parities from the first survey are denoted by $P(i, 1)$ and those from the second survey by $P(i, 2)$, where $i = 1$ indicates age group 15-19; $i = 2$ the 20-24 age group; and so on. In both cases, the average parities are computed by dividing the reported number of children ever born to women in age group i by the total number of women in age group i. See, however, the discussion in subsection A.2 concerning the treatment of women whose parity is not stated and the possible application of the El-Badry correction (annex II).

Step 2A: calculation of average parities for a hypothetical cohort: five-year intersurvey period. The survivors of cohort i at the first survey belong to age group $i+1$ at the second survey; and the parity increment for the cohort, denoted by $\Delta P(i+1)$, is equal to the average parity of the cohort at the second survey minus that at the first survey. That is,

$$\Delta P(i+1) = P(i+1, 2) - P(i, 1). \qquad \text{(C.1)}$$

Such parity increments are calculated for values of i from 1 to 6; the value of $\Delta P(1)$, corresponding to $i = 0$, may also be calculated from equation (C.1), with $P(0, 1)$, the average parity at the first survey of women aged 10-14, being taken as equal to zero; this is equivalent, of course, to taking $\Delta P(1)$ as being directly equal to $P(1, 2)$.

Once the cohort parity increments have been obtained from equation (C.1), the average parities for the hypothetical cohort are obtained by successive summation of the increments. Thus, on the basis of intersurvey fertility, the parity of women in age group 1, denoted by $P(1, s)$, is equal to $\Delta P(1)$, or, as it may also be written, $P(1, 2)$; the parity of women in age group 2 is equal to the intersurvey parity for age group 1 plus the intersurvey parity increment for age group 2 at the second survey; this increment is $\Delta P(2)$, so

$$P(2, s) = \Delta P(1) + \Delta P(2).$$

In general terms, one may write:

$$P(i, s) = \sum_{j=1}^{i} \Delta P(j). \qquad \text{(C.2)}$$

Step 2B: calculation of average parities for a hypothetical cohort: 10-year intersurvey period. In this case, the survivors of the initial cohort of age group i at the time of the first survey will be the women in age group $i+2$ at the time of the second survey, and the hypothetical-cohort parities are obtained by the cumulation of two parallel sequences of cohort parity increments. The parity increments are obtained by subtracting from the average parity of women of age group $i+2$ at the second survey the average parity of the women of age group i at the first survey. Thus,

$$\Delta P(i+2) = P(i+2, 2) - P(i, 1) \quad \text{for } i = 1, ..., 5. \qquad (C.3)$$

$\Delta P(1)$ and $\Delta P(2)$ are simply put equal to $P(1,2)$ and $P(2,2)$, respectively (this procedure will distort the results slightly when fertility is changing very rapidly). Hypothetical-cohort parities for even-numbered age groups are obtained by summing the parity increments for even-numbered age groups, whereas those for odd-numbered age groups are obtained by summing parity increments for odd-numbered age groups. Thus, for even numbers,

$$P(2,s) = \Delta P(2) \qquad (C.4)$$

$$P(4,s) = \Delta P(2) + \Delta P(4) \qquad (C.5)$$

and

$$P(6,s) = \Delta P(2) + \Delta P(4) + \Delta P(6) \qquad (C.6)$$

whereas

$$P(1,s) = \Delta P(1) \qquad (C.7)$$

$$P(3,s) = \Delta P(1) + \Delta P(3) \qquad (C.8)$$

$$P(5,s) = \Delta P(1) + \Delta P(3) + \Delta P(5) \qquad (C.9)$$

and

$$P(7,s) = \Delta P(1) + \Delta P(3) + \Delta P(5) + \Delta P(7) \qquad (C.10)$$

where $P(i,s)$ is the hypothetical average parity for age group i.

Step 3: interpolation between the hypothetical-cohort parities to estimate cumulated fertility. Average parities for the hypothetical cohort calculated in the previous step can be used to estimate cumulated fertility up to the exact ages of interest. The process of estimation followed is directly linked to that used in subsection B.2 (c) to estimate average parities from observed cumulated fertility. Indeed, it can be said that this process is just the inverse of that described in subsection B.2 (c), since the problem now is to go from average parities to cumulated fertility, while the P/F estimation method yields estimates of average parities from cumulated fertility.

One interpolation procedure described in subsection B.2 (c) fitted a second-order polynomial to sections of the cumulated fertility schedule and average parities were computed by integrating this polynomial over the desired age range. Hence, average parities can be approximated by integrals of polynomials whose coefficients define the shape of the cumulated fertility schedule, thus providing a means of estimating a value of this schedule at any exact age within the fitting range. Following this principle, an expression allowing the estimation of cumulated fertility from observed parities was obtained by fitting a third-order polynomial to successive average parities. It has the form:

$$\phi(i,s) = 0.9283P(i,s) + 0.4547P(i+1,s) - 0.0585P(i+2,s) - 0.3245\phi(i-1,s) \quad \text{for } i = 1, ..., 5. \qquad (C.11)$$

where $\phi(i,s)$ is fertility cumulated up to the upper limit of age group i.

When the performance of this estimating equation was tested by using the Coale-Trussell model fertility schedules, it was found that it did not perform very satisfactorily at the upper extreme of the fertile period. Therefore, an empirically derived correction was introduced to estimate $\phi(6,s)$, resulting in the following equation:

$$\phi(6,s) = 0.0209P(4,s) - 0.5574P(5,s) + 1.0478P(6,s) + 0.2869P(7,s) + 0.2018\phi(4,s). \qquad (C.12)$$

Unfortunately, no satisfactory correction was found for estimating $\phi(7,s)$ based on polynomial fits. Yet, since fertility rates are usually very low at ages 45 and older, it is recommended that $\phi(7,s)$ be estimated directly from the observed $P(7,s)$ by using the following equation:

$$\phi(7,s) = 1.007P(7,s). \qquad (C.13)$$

Step 4: calculation of intersurvey age-specific fertility rates. Intersurvey age-specific fertility rates, $f(i,s)$, are calculated by subtracting the cumulated fertility to the lower boundary of age group i from that to the upper boundary of the same group and dividing the difference by five. Thus, in general:

$$f(i,s) = (\phi(i,s) - \phi(i-1,s))/5. \qquad (C.14)$$

Note that cumulated fertility to age 15, which is denoted by $\phi(0,s)$, is generally assumed to be zero.

Step 5: calculation of total fertility and the birth rate. The estimated total fertility for the intersurvey period is equal to $\phi(7,s)$. To obtain an estimate of the birth rate, the mid-period female population classified by five-year age group and the mid-period total population are required. Both can be obtained by averaging, when censuses are available for the beginning and the end of the period. Then, the births occurring at the middle of the intersurvey period can be found by multiplying the estimated mid-period female population by the relevant intersurvey age-specific fertility rates and summing over all age groups. The birth rate is then obtained by dividing the total number of births by the mid-period population. However, when adequately spaced censuses are not available, the calculation of an acceptable mid-period population may not be possible. In such a case, birth-rate estimates may be obtained independently for the first and second surveys using the intersurvey age-

TABLE 32. FEMALE POPULATION AND CHILDREN EVER BORN, BY AGE GROUP, THAILAND, 1960, 1970 AND 1975

Age group (1)	Index i (2)	1960 Female population (3)	1960 Children ever born (4)	1970 Female population (5)	1970 Children ever born (6)	1975 Female population (7)	1975 Children ever born (8)
15-19	1	1 236 294	88 610	1 885 371	246 839	13 329	1 698
20-24	2	1 204 153	1 038 074	1 361 717	1 353 569	10 304	9 108
25-29	3	1 046 464	2 402 581	1 143 377	2 754 376	8 008	17 027
30-34	4	869 876	3 290 345	1 077 088	4 112 920	6 561	22 866
35-39	5	679 940	3 388 799	957 607	4 866 424	6 400	29 821
40-44	6	563 812	3 232 209	766 332	4 537 467	5 546	31 595
45-49	7	482 966	2 855 997	597 454	3 656 488	4 521	28 537

specific fertility rates obtained in step 4 and their average may be used as an estimate of the intersurvey birth rate.

(c) *Detailed examples*

Information on children ever born classified by age of mother is available from the 1960 and 1970 censuses of Thailand and from a large sample survey conducted in 1975. The 10-year cohort parity increment procedure may be applied between 1960 and 1970, and the five-year method between 1970 and 1975. The basic data are shown in table 32.

First to be considered are the calculations in the case of the 10-year interval in order to retain historical order.

Step 1: calculation of average parities for both surveys. Average parities are obtained by dividing the number of children ever born, classified by age group of women (shown in columns (4) and (6) of table 32 for 1960 and 1970, respectively), by the total number of women in each age group (shown in columns (3) and (5) of table 32 for 1960 and 1970, respectively). Results are shown in table 33; but, as an example, the average parity of women aged 25-29 in 1960 is obtained as

$$P(3, 1) = 2{,}402{,}581/1{,}046{,}464$$

$$= 2.2959.$$

Step 2B: calculation of average parities for a hypothetical cohort: 10-year intersurvey. Using equation (C.3), cohort parity increments are calculated by subtracting from each recorded parity in 1970 the parity of the corresponding cohort, 10 years younger, in 1960. Thus, in the case of age group 30-34 in 1970,

$$\Delta P(4) = P(4, 2) - P(2, 1)$$

$$= 3.8186 - 0.8621$$

$$= 2.9565.$$

Values of $\Delta P(i)$ for all age groups are shown in column (5) of table 33. Note that for age groups 1 and 2, the parity increment is taken as being equal to the observed parities for age groups 1 and 2 at the time of the second census.

Once the cohort parity increments have been calculated, the average parities for hypothetical cohorts are obtained by summation, as shown in equations (C.4)-(C.10). The average parities for even-numbered age groups are obtained by summing even-numbered cohort parity increments; thus, to calculate $P(4, s)$, using equation (C.5),

$$P(4, s) = \Delta P(2) + \Delta P(4)$$

$$= 0.9940 + 2.9565$$

$$= 3.9505.$$

For odd-numbered age groups, average parities for a hypothetical cohort are obtained by summing odd-numbered cohort parity increments; thus, to calculate $P(5, s)$, using equation (C.9),

$$P(5, s) = \Delta P(1) + \Delta P(3) + \Delta P(5)$$

$$= 0.1309 + 2.3373 + 2.7860$$

$$= 5.2542.$$

TABLE 33. AVERAGE PARITIES FOR 1960 AND 1970, COHORT PARITY INCREMENTS AND AVERAGE PARITIES FOR THE INTERSURVEY HYPOTHETICAL COHORT, THAILAND

Age group (1)	Index i (2)	Average parity 1960 (3)	Average parity 1970 (4)	Cohort parity increment (5)	Hypothetical-cohort parity (6)
15-19	1	0.0717	0.1309	0.1309	0.1309
20-24	2	0.8621	0.9940	0.9940	0.9940
25-29	3	2.2959	2.4090	2.3373	2.4682
30-34	4	3.7825	3.8186	2.9565	3.9505
35-39	5	4.9840	5.0819	2.7860	5.2542
40-44	6	5.7328	5.9210	2.1385	6.0890
45-49	7	5.9135	6.1201	1.1361	6.3903

Column (6) of table 33 shows the hypothetical-cohort average parities for the period 1960-1970.

Step 3: interpolation between the hypothetical-cohort parities to estimate cumulated fertility. Using equations (C.11)-(C.13), cumulated fertility is estimated from the average parities obtained in step 2. It is assumed that $\phi(0,s)=0.0$, so that for $i=1$:

$$\phi(1,s)=0.9283(0.1309)+0.4547(0.9940)-0.0585(2.4682)=0.4291.$$

For other values of i, previously estimated ϕ values are used as input in equation (C.11). For example:

$$\phi(3,s)=0.9283(2.4682)+0.4547(3.9505)-0.0585(5.2542)-0.3245(1.6747)=3.2367.$$

When $i=6$, equation (C.12) is used, as shown below:

$$\phi(6,s)=0.0209(3.9505)-0.5574(5.2542)+1.0478(6.0890)+0.2869(6.3903)+0.2018(4.6498)=6.3056.$$

Lastly, for $i=7$,

$$\phi(7,s)=1.007(6.3903)=6.4350$$

Column (4) of table 34 displays all the $\phi(i,s)$ values.

Step 4: calculation of intersurvey age-specific fertility rates. Following equation (C.14), age-specific fertility rates are obtained from the cumulated fertility values, $\phi(i,s)$, by finding the difference between successive values of $\phi(i,s)$ and dividing the difference by five to obtain an annual rate; $\phi(1,s)$ is cumulated fertility by age 20, so

$$f(1,s)=\phi(1,s)/5,$$

but thereafter,

$$f(i,s)=(\phi(i,s)-\phi(i-1,s))/5.$$

To give two examples:

$$f(1,s)=0.4291/5=0.0858$$

and

$$f(6,s)=(\phi(6,s)-\phi(5,s))/5=(6.3056-5.7635)/5=0.1084.$$

Column (5) of table 34 shows the complete set of $f(i,s)$ values.

Step 5: calculation of total fertility and the birth rate. Total fertility estimated for the intersurvey period is immediately available, being equal to $\phi(7,s)$, or in this case 6.44. An approximate estimate of the birth rate can be obtained by estimating the mid-period female population of each age group by adding the 1960 and 1970 populations and dividing by two, then estimating an average number of annual births by multiplying the female population by the age-specific fertility rate shown in table 34, summing the births for all age groups of women, and dividing the total by the average of the 1960 and 1970 total populations. This method can be used because censuses are the sources of the basic data. As an example, the average female population for age group 25-29, $N(3,s)$, is found as

$$N(3,s)=(N(3,1)+N(3,2))/2.0=(1{,}046{,}464+1{,}143{,}377)/2.0=1{,}094{,}921.$$

The annual number of births to women in this age group is then found by multiplying by the estimated $f(3,s)$. So, if the number of births to age group i is denoted by $B(i,s)$,

$$B(3,s)=N(3,s)f(3,s)=(1{,}094{,}921)(0.3124)=342{,}053.$$

Repeating these steps for other values of i and summing over all i values produces a total number of births of 1,355,416; given that the total population in 1960 was 26,257,916 and in 1970 it was 34,397,374, the average mid-period population is 30,327,645, and the birth rate b can be estimated as

$$b=1{,}355{,}416/30{,}327{,}645=0.0447.$$

As a commentary on the results obtained, it may be

TABLE 34. AVERAGE PARITIES FOR THE HYPOTHETICAL COHORT, ESTIMATED SCHEDULE OF CUMULATED FERTILITY AND AGE-SPECIFIC FERTILITY RATES. THAILAND, 1960-1970

Age group *(1)*	*Index* *i* *(2)*	*Hypothetical-cohort parity* *P(i, s)* *(3)*	*Cumulated fertility* *φ (i, s)* *(4)*	*Intersurvey age-specific fertility rate* *f(i, s)* *(5)*	*Age-specific fertility rate from registered births* *(6)*
15-19	1	0.1309	0.4291	0.0858	0.0606
20-24	2	0.9940	1.6747	0.2491	0.2781
25-29	3	2.4682	3.2367	0.3124	0.3153
30-34	4	3.9505	4.6498	0.2826	0.2724
35-39	5	5.2542	5.7635	0.2227	0.2248
40-44	6	6.0890	6.3056	0.1084	0.1132
45-49	7	6.3903	6.4350	0.0259	0.0285

noted that the hypothetical-cohort average parities are higher than either of the observed values, confirming the fact that the hypothetical-cohort parities are very sensitive to changes in the level of fertility. However, they are also very vulnerable to error, especially to errors that affect one of the observed sets of average parities more than the other. Yet, when the estimated age-specific fertility rates derived from the hypothetical-cohort parities are compared with those obtained from births registered in Thailand during the intercensal period and adjusted for level as described in subsection B.4 (d) (the figures shown in column (6) of table 34 were copied from table 19), the similarity is reassuring, although differences for the first two and the last age groups are fairly marked.

In general, whenever the additional data required exist, the procedure outlined in subsection B.4 (c) for comparing cumulated intersurvey fertility rates with hypothetical-cohort average parities is to be preferred to the method described here, since the former method is less sensitive to the omission of children ever born from the reports of older women.

Next to be discussed is the case of the five-year interval. As steps 1, 3 and 4 are identical to those just described, merely the results obtained are given; step 2A, however, is described in detail.

Step 1: calculation of average parities for both surveys. Columns (3) and (4) of table 35 show the average parities for 1970 and 1975, respectively.

Step 2A: calculation of average parities for a hypothetical cohort: five-year intersurvey period. Following equation (C.1), cohort parity increments are calculated by subtracting from each recorded parity in 1975 the parity of the corresponding cohort, five years younger, in 1970. Thus, in the case of age group 30-34 in 1975,

$$\Delta P(4)=P(4,2)-P(3,1)=3.4851-2.4090=1.0761.$$

TABLE 35. AVERAGE PARITIES FOR 1970 AND 1975, COHORT PARITY INCREMENTS AND HYPOTHETICAL-COHORT AVERAGE PARITIES, THAILAND

Age group (1)	Index i (2)	Average parity 1970 (3)	Average parity 1975 (4)	Cohort parity increment (5)	Hypothetical cohort parity (6)
15-19	1	0.1309	0.1274	0.1274	0.1274
20-24	2	0.9940	0.8839	0.7530	0.8804
25-29	3	2.4090	2.1262	1.1322	2.0126
30-34	4	3.8186	3.4851	1.0761	3.0887
35-39	5	5.0819	4.6595	0.8409	3.9296
40-44	6	5.9210	5.6969	0.6150	4.5446
45-49	7	6.1201	6.3121	0.3911	4.9357

Values of $\Delta P(i)$ for all cases are shown in column (5) of table 35. It should be noted that $\Delta P(1)$ is put equal to $P(1,2)$.

Average parities for hypothetical cohorts are obtained by cumulating successive cohort parity increments. According to equation (C.2),

$$P(i,s)=\sum_{j=1}^{i}\Delta P(j).$$

Thus, $P(4,s)$ is obtained by summing the values of $\Delta P(i)$ from 1 to 4:

$$P(4,s)=\Delta P(1)+\Delta P(2)+\Delta P(3)+\Delta P(4)$$
$$=0.1274+0.7530+1.1322+1.0761$$
$$=3.0887.$$

Other values are obtained in a similar fashion, and the complete set of average parities for the hypothetical cohort is shown in column (6) of table 35. Note that in the case of a five-year intersurvey interval, there is only one chain of summation.

Step 3: interpolation between the hypothetical-cohort parities to estimate cumulated fertility. As in the case of a 10-year interval, cumulated fertility is estimated by using equations (C.11)-(C.13). When $i=1, \phi(0,s)$ is assumed to equal zero. In other cases, the application is straightforward. The complete set of $\phi(i,s)$ estimates is shown in column (4) of table 36.

TABLE 36. HYPOTHETICAL-COHORT AVERAGE PARITIES, ESTIMATED SCHEDULE OF CUMULATED FERTILITY AND AGE-SPECIFIC FERTILITY RATES, THAILAND, 1970-1975

Age group (1)	Index i (2)	Hypothetical cohort parity $P(i,s)$ (3)	Intersurvey cumulated fertility $\phi(i,s)$ (4)	Age-specific fertility rate $f(i,s)$ (5)
15-19	1	0.1274	0.4008	0.0802
20-24	2	0.8804	1.4217	0.2042
25-29	3	2.0126	2.5815	0.2320
30-34	4	3.0887	3.5505	0.1938
35-39	5	3.9296	4.2734	0.1446
40-44	6	4.5446	4.7686	0.0990
45-49	7	4.9357	4.9702	0.0403

Step 4: calculation of intersurvey age-specific fertility rates. Age-specific fertility rates are obtained by subtracting fertility cumulated to the lower boundary of each age group from that cumulated to the upper boundary and dividing the result by five, as shown in equation (C.14). For example,

$$f(2,s)=(\phi(2,s)-\phi(1,s))/5$$
$$=(1.4217-0.4008)/5$$
$$=0.2042.$$

Column (5) of table 36 shows the full set of intersurvey fertility rates $f(i, s)$.

Step 5: calculation of total fertility and the birth rate. Total fertility is equal to the value of $phi(7, s)$, that is, 4.97. It is more difficult in this case to estimate the birth rate, since the 1975 population comes from a sample survey, and the mid-period population cannot be obtained by averaging the 1970 and 1975 populations. However, an approximate estimate of the birth rate during the period between the surveys can be obtained by calculating the birth rates for 1970 and for 1975 implied by the intersurvey fertility rates and averaging them. The births in 1970 (not an estimate of the true number of births in 1970, but rather the number of births that would have occured in 1970 given the intersurvey fertility schedule) are obtained by summing the products of the enumerated female population appearing in column (5) of table 32 and the estimated age-specific fertility rates given in column (5) of table 36; then the birth rate is found by dividing this sum by the total 1970 population. The total births are 1,141,687, and the total enumerated population is 34,397,374, so the estimated birth rate is 0.0332. The births for 1975 are estimated by summing the products of the 1975 female population (column (7) of table 32) and the estimated fertility rates (column (5) of table 36) to give 7,959 births; since the total survey population in 1975 was 230,060, the estimated birth rate is 0.0346. Therefore, an estimate of the intersurvey birth rate is obtained by averaging these two, giving a final value of 0.0339.

To conclude, it may be mentioned that whereas between 1960 and 1970 every average parity for the hypothetical cohort was higher than the recorded parities for the beginning-point and end-point, between 1970 and 1975 the hypothetical-cohort average parities were lower than the parities recorded for the beginning-point or end point. The suitability of this method for situations in which fertility is changing is thus clearly demonstrated; it must be remembered, however, that the results will be seriously distorted if children ever born tend to be omitted from the reports provided by their mothers or if the extent of such omission changes from one survey to the next.

D. Estimation of fertility from information on children ever born classified by duration of marriage

1. *Basis of methods and their rationale*

Most of the methods proposed for estimating fertility schedules from data on children ever born are not robust to the presence of substantial age-misreporting. If in a given population there is less distortion in the reporting of marriage duration than in the reporting of age, a method that estimates fertility schedules by using data on children ever born classified by duration of marriage may yield better fertility estimates than a method based on data classified by age. An additional advantage of a duration-based method derives from the fact that fertility schedules by duration of marriage are more uniform than schedules by age, making the method more robust to recent changes in marriage patterns.

Coale, Hill and Trussell[10] propose one such method of estimation, which is called here the "P/P^* ratio method". Its applicability is, however, somewhat limited by the fact that it can only be used in populations in which there is and has been very little voluntary control of fertility and in which only a small proportion of all births occurs outside marriage. On the other hand, this method does not require an essentially unchanging age at marriage.

The underlying rationale for the procedure can be stated simply. The age pattern of natural fertility, defined as marital fertility in the absence of voluntary fertility control,[11] is very similar in different populations, although their levels of fertility may differ. Moreover, natural fertility does not vary much from the beginning of reproductive maturity (around age 20) until the early thirties, when a fairly steep decline begins. Therefore, populations that marry early and do not practise voluntary fertility control have rather similar patterns (though not necessarily similar levels) of fertility by duration of marriage. Hence, except for a scale factor, the sequence of average parities by duration of marriage among different populations subject to natural fertility should be similar. It should be possible, therefore, to compare the reported sequence of average parities (for durations of marriage under 5 years, of 5-9 years and of 10-14 years) with a standard embodying the natural fertility pattern and, thus, to determine the level of natural fertility in the population in question. Marital fertility could then be estimated as equal to standard natural fertility multiplied by the estimated level in the population in question, and an estimate of the overall fertility schedule could be obtained as the product of the proportion married and the estimated marital fertility for each age group. In practice, in selecting a standard natural fertility schedule, some allowance is made for the population's distribution by age of entry into marriage and the effect this distribution has through known, though small, variations of natural fertility with age.

When using this method, a problem arises as to which women to consider. The standard schedule of average parity by duration of marriage, $P^*(i)$, refers only to women who are still in their first union. Thus, when estimating the level at which natural fertility is experienced, only reports of women still in their first union should be used. Of course, the estimate of fertility obtained from this type of data would not refer to all currently married women. Besides, fertility data are not commonly tabulated according to these specifications (for women still in their first union separately from the rest). Therefore, it is often necessary to use an alternative type of data. The average parities for currently married women by time elapsed since first union,

[10] Ansley J. Coale, Allan G. Hill and T. James Trussell, "A new method of estimating standard fertility measures from incomplete data", *Population Index*, vol. 41, No. 2 (April 1975), pp. 182-212.

[11] Louis Henry, "Some data on natural fertility", *Eugenics Quarterly*, vol. XIII, No. 2 (June 1961), pp. 81-91.

regardless of the number of unions, provides such an alternative. Use of these data makes it possible to estimate the level of natural fertility experienced by all currently married women, but the effect of "dead time" between unions will affect the ratios of the observed average parities by duration groups, $P(i)$, to the corresponding parities derived from the standard, $P^*(i)$. These ratios are denoted by $R(i)$, where i determines the duration of marriage group, 1 denoting duration group 0-4, 2 duration group 5-9 and so on. The "dead time" effect will increase with marriage duration, since the probability of marital dissolution, at least through widowhood, increases with the time elapsed since first union, while the rapidity of remarriage may decrease. If marriages are fairly stable, however, this effect (a tendency for the $R(i)$ ratios to decline with i) will be small; and the use of data for all currently married women is probably the best option if they are available. The third possibility is to use average parities for ever-married women. In this case, the problem of "dead time" between or after unions will be more serious, and the tendency for the $R(i)$ ratios to decline with i will be more marked. The use of a level (or adjustment factor) of natural fertility based on the first three duration groups to estimate the fertility of ever-married women of all age groups will overestimate the fertility of the older age groups, where the proportion currently married is lower. This third possibility should not be used unless the data are only tabulated for ever-married women. In cases of this type, however, although parity information may only be available for ever-married women, the data on marital status may show currently married women classified by five-year age group. Then the expected biases in the final estimates of age-specific fertility rates may be minimized by a combination of two strategies: selecting an adjustment factor for natural fertility on the basis of the first three duration groups; and using the observed proportions of currently married women to weight the adjusted natural fertility schedule.

As in the case of the method that used data on children ever born classified by age, the different adjustment factors obtained by evaluating the ratios of the observed over the expected parities corresponding to a given duration group should be approximately equal for the different duration groups. Large differences between the ratios corresponding to different duration groups reveal either that the assumptions are not satisfied (for example, the population is not subject to natural fertility) or that the data may be affected by substantial misreporting.

Because the P/P^* method is applicable only to populations in which there is little practice of voluntary birth control and in which childbearing by women not currently married is infrequent, it should not be used (at least not without major *ad hoc* adjustments) for parts of Africa and Latin America where several forms of cohabitation other than recognized marriage are prevalent and where a significant proportion of all births are illegitimate.

Furthermore, this method is not applicable without modification to populations, such as that of India, in which many marriages occur before the age of menarche and where there is no close relation between the age at marriage and the age at which exposure to the risk of conception begins. In some Muslim populations to which this method is potentially applicable (with natural fertility and very little illegitimacy), the date of marriage for many couples precedes the beginning of cohabitation. As a result, the observed average parities by duration of marriage are lower than would be expected, especially during the first five-year duration interval, in which the proportion of childless women is higher than would be expected in a population not practising contraception. A procedure for adjusting estimates of fertility to allow for the effects of delay in cohabitation after marriage is described by Coale, Hill and Trussell.[12]

The converse problem is that in countries where consensual unions form a sizeable proportion of all unions, the duration of marriage reported may be that of the current union or the time elapsed since the beginning of the first stable union, rather than that elapsed since the first union. In such cases, the average parity for each duration group, and for the first group in particular, will be too high. A procedure for estimating the average extent of any non-correspondence between reported duration of marriage and apparent duration of cohabitation from the proportions of women with at least one child in duration groups 1 and 2 is also described by Coale, Hill and Trussell.[13]

None of the adjustment procedures that may be applied to the basic data before using the P/P^* method are described in this *Manual*. The main reason for this omission is that most of today's populations are unlikely to be subject to natural fertility within marriage and to lack, at the same time, acceptable information on the pattern of marital fertility they experience.

If data allowing the estimation of the pattern of marital fertility are available in the form of births occurring during a given year classified by the mother's duration of marriage, another method, analogous to that described in subsection B.2, can be used to compare period fertility rates with reported average parities and to adjust the level of the former rates by that of the latter. Once again the cumulated period fertility rates will refer to the end-point of each duration group and will therefore not be strictly comparable to average parities, but parity equivalents, $G(i)$, may be obtained by interpolating between consecutive cumulated fertility values. Indeed, the interpolation process will be much more satisfactory than in the case of data classified by age, since the cumulated duration-specific fertility schedule is likely to be more regular than the cumulated age-specific fertility schedule. Furthermore, this method will be less affected by changing age at marriage than will the method based on data classified by age; and it does not require that marital fertility follow the pattern

[12] A. J. Coale, A. G. Hill and T. J. Trussell, *loc. cit.*

[13] *Ibid.*

of natural fertility. This method is called the "*P*/*G* method".

However, the *P*/*G* method is not immune to the effects of errors frequently present in the basic data. Two types of errors are common:

(*a*) Women with no children are incorrectly recorded as being among those whose parity is not stated. The presence of this type of error is revealed by a sharp decline in the proportion of women of unknown parity with increasing age or duration of marriage, paralleling the genuine decline of the proportion of childless women. If the cases classified as "parity unknown" are omitted from the calculations, average parity is overestimated, particularly at duration 0-4, whereas if all such women are assigned a parity of zero, average parity is underestimated. El-Badry[14] proposed a method of adjustment, described in annex II;

(*b*) Misstatement of the time elapsed since first union (duration of marriage). This error biases the values of average parity in two duration intervals (both the true interval and that to which the woman is mistakenly assigned), because the transfer of women from one interval to another is usually selective with respect to their parity. For example, the women who falsely report a duration of first marriage as 5-9 years rather than 0-4 very probably belong near the upper boundary of the lower interval and are of generally higher parity than the average for their true interval but of lower parity than the average for the interval in which they are reported. An upward transfer—an overstatement of duration—thus introduces a downward bias in the recorded average parity for both duration groups. Similarly, a downward transfer is likely to inflate the recorded average parity for both, and some mixture of these two types of transfer will tend to reduce the differential in average parity between the two duration groups.

Errors in stated duration of marriage are analogous to errors of reference period in the reporting of events during a fixed interval, such as the past year, and to misstatements of age or of a child's date of birth. The extent and direction of the bias probably depend upon the wording of the question by which duration is ascertained, as well as upon the education of the enumerator and respondent, and upon the general cultural context. It is not clear which way of collecting data on duration—a question on date of first marriage, time elapsed since first marriage or age at first marriage—is best, but the first method seems likely to be least affected by rounding errors.

Lastly, when data on current fertility and parity information are obtained from different sources it is important to verify that duration of marriage is measured in the same way in both. If, for example, the first source records duration of current marriage while the second measures time elapsed since first marriage, the basic data would be incompatible and should not be used as input for the *P*/*G* method.

[14] M. A. El-Badry, "Failure of enumerators to make entries of zero: errors in recording childless cases in population censuses", *Journal of the American Statistical Association*, vol. 56, No. 296 (December 1961), pp. 909-924.

2. *Estimation of level of natural fertility from reported parity by duration of marriage*

(a) *Data required*

The data listed below are required for this method:

(*a*) A value of a_0, the earliest age at which a significant number of marriages take place;

(*b*) The female population classified by five-year age group and marital status (single, married, widowed and divorced);

(*c*) The number of children ever born classified by the duration of first marriage and current marital status of their mothers;

(*d*) The total population.

(b) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities by duration of marriage. In the calculation of the reported average parities, $P(i)$, the index i refers to the different duration groups. Thus, $i = 1$ indicates duration group 0-4; $i = 2$, duration group 5-9; and $i = 3$, duration group 10-14. $P(i)$ is defined as the ratio of the number of children ever born to currently married women in duration group i to the number of currently married women belonging to the same group. If data for currently married women are not available, data for ever-married women may be used. However, as stated above, use of the latter data may introduce some undesirable biases (see subsection D.1). Furthermore, if the proportion of currently married (or ever-married) women whose parity was not stated is non-trivial, the question arises as to the way in which these cases should be treated. For guidelines about the treatment of women whose parity is not stated, refer to subsection A.2 and annex II (El-Badry's correction).

Step 2: calculation of the female singulate mean age at marriage. The female singulate mean age at marriage, *SMAM*, is calculated from the proportions single, classified by age group, as described in detail in annex I.

Step 3: calculation of expected average parities by duration of marriage. To calculate the expected average parities, $P^*(i)$, one needs an estimate of the female singulate mean age at marriage, obtained in step 2, and an estimate of the youngest age at which a significant number of women marry, denoted by a_0. As a rule, a_0 is not known exactly; but it can be guessed with fair accuracy from what is known about the population being studied. Its values generally fall within the range from 12 to 15 years. Therefore, if early marriages are known to take place in the given population, a_0 can be chosen to have a value of 12 or 13. For a population in which early marriages are not common, a value of 14 or even 15 may be appropriate for a_0.

Expected average parities by duration of marriage are obtained from table 37, beginning with a round value of

TABLE 37. EXPECTED AVERAGE PARITIES FOR SELECTED VALUES OF THE YOUNGEST AGE AT WHICH A SIGNIFICANT NUMBER OF WOMEN MARRY AND OF THE FEMALE SINGULATE MEAN AGE AT MARRIAGE WHEN MARITAL FERTILITY IS EXPERIENCED AT THE LEVEL INDICATED IN TABLE 38

Youngest age at marriage a_0 (1)	Index i (2)	Duration group (3)	Average parities, $P^*(i)$, for singulate mean age at marriage equal to: 18 (4)	19 (5)	20 (6)	21 (7)	22 (8)	23 (9)	24 (10)	25 (11)	26 (12)	27 (13)
12............	1	0-4	1.072	1.090	1.097	1.097	1.091	1.080	1.066	1.050	1.032	1.012
	2	5-9	3.338	3.337	3.316	3.279	3.230	3.171	3.106	3.036	2.963	2.888
	3	10-14	5.500	5.445	5.364	5.261	5.141	5.009	4.868	4.722	4.575	4.427
13............	1	0-4	1.097	1.113	1.119	1.117	1.109	1.097	1.081	1.063	1.043	1.021
	2	5-9	3.385	3.381	3.357	3.318	3.266	3.205	3.136	3.062	2.984	2.904
	3	10-14	5.555	5.501	5.419	5.315	5.192	5.054	4.907	4.754	4.598	4.442
14............	1	0-4	1.120	1.135	1.138	1.134	1.125	1.111	1.093	1.074	1.052	1.029
	2	5-9	3.424	3.417	3.392	3.351	3.298	3.234	3.163	3.085	3.004	2.920
	3	10-14	5.601	5.548	5.467	5.362	5.237	5.097	4.944	4.784	4.621	4.456
15............	1	0-4	1.140	1.152	1.154	1.148	1.137	1.122	1.104	1.083	1.060	1.036
	2	5-9	3.455	3.446	3.419	3.377	3.324	3.260	3.187	3.107	3.022	2.935
	3	10-14	5.638	5.585	5.505	5.402	5.278	5.137	4.981	4.816	4.644	4.472

a_0, and then estimating $P^*(1)$, $P^*(2)$ and $P^*(3)$ by interpolation between columns, using the observed value of the female singulate mean age at marriage.

Step 4: selection of an adjustment factor. To select the adjustment factor, K, the values of $R(i)$ are calculated as the ratios of the reported parities to the expected parities for each duration group i. Thus,

$$R(i) = P(i)/P^*(i). \qquad \text{(D.1)}$$

The $R(i)$ values thus indicate the level of the reported average parities in relation to the level of the expected parities based on a natural fertility schedule. Three values of $R(i)$ are computed in each case, one for each of the duration categories under consideration. Ideally, all three values should be similar; and if they are, an average of the three may be selected as a final adjustment factor, K, to allow for the level of natural fertility in the population being studied. When the values of $R(i)$ are not similar or show a clear increasing and decreasing trend with i, the method in its simplest form should not be used. In such a case, it may be necessary to correct the original data by using some of the procedures mentioned in subsection D.1.

Step 5: calculation of adjusted age-specific marital fertility schedule. Once an adjustment factor, K, has been successfully identified, an adjusted age-specific marital fertility schedule, $g(j)$, for the population in question is calculated by multiplying the model natural fertility schedule appearing in table 38 by the adjustment factor K.

TABLE 38. AGE-SPECIFIC MARITAL FERTILITY RATES WHEN NATURAL FERTILITY IS EXPERIENCED AT THE STANDARD LEVEL

Age group (1)	Index j (2)	Marital fertility rate $v(j)$ (3)
10-14	0	0.1350
15-19	1	0.4112
20-24	2	0.4694
25-29	3	0.4418
30-34	4	0.3988
35-39	5	0.3226
40-44	6	0.1668
45-49	7	0.0252
Total marital fertility		11.8540

Step 6: calculation of adjusted age-specific fertility schedule. The adjusted age-specific marital fertility schedule estimated in step 5 may be transformed into an adjusted age-specific fertility schedule, $f(j)$, referring to the entire female population (and not just to those currently married), under the assumption that all childbearing occurs within marriage, by multiplying each marital fertility rate, $g(j)$, by the proportion of currently married women in the age group being considered. The sum of the age-specific fertility rates, $f(j)$, thus obtained, multiplied by five, provides an estimate of total fertility.

Step 7: calculation of adjusted birth rate. An adjusted birth rate is obtained by adding the products of the estimated age-specific marital fertility schedule obtained in step 5 and the number of currently married women in different age groups and dividing this total (which is an estimate of the total number of births) by the total population.

(c) *A detailed example*

Tables 39 and 40 show data obtained from the 1976 census of Egypt. Because the incidence of illegitimate births is very low in the Egyptian population and because there is little evidence substantiating the existence of a fertility decline within marriage, the method described below may be used to estimate age-specific fertility rates.

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities by dura-

TABLE 39. CHILDREN EVER BORN AND EVER-MARRIED WOMEN, BY TIME ELAPSED SINCE FIRST MARRIAGE, EGYPT, 1976

Duration group (1)	Index i (2)	Ever-married women (3)	Children ever born (4)
0-4	1	1 249 606	964 962
5-9	2	1 121 552	2 615 104
10-14	3	1 078 492	3 908 481

TABLE 40. NEVER-MARRIED WOMEN AND TOTAL NUMBER OF WOMEN, BY AGE GROUP, EGYPT, 1976

Age group (1)	Index i (2)	Never married women (3)	Total number of women (4)	Proportion of single women (5)	Proportion of currently married women (6)
15-19	1	1 540 388	1 849 952	0.8327	0.1617
20-24	2	605 528	1 556 231	0.3891	0.5899
25-29	3	190 214	1 359 752	0.1399	0.8268
30-34	4	76 995	1 088 690	0.0707	0.8774
35-39	5	48 737	1 027 457	0.0474	0.8756
40-44	6	46 062	944 083	0.0488	0.8096
45-49	7	28 491	736 495	0.0387	0.7681
50-54	8	33 259	739 863	0.0450	0.6217

tion of marriage. The values of the reported average parities, $P(i)$, are obtained by dividing each of the entries in column (4) of table 39 (number of children ever born) by the corresponding entries in column (3) of that table (number of women ever married in each duration group). Column (3) of table 41 shows the final set of parities. As an example, $P(2)$ is calculated here:

$$P(2)=2{,}615{,}104/1{,}121{,}552=2.3317.$$

TABLE 41. REPORTED AND EXPECTED AVERAGE PARITIES, BY DURATION OF MARRIAGE, EGYPT 1976

Duration group (1)	Index i (2)	Reported average parity $P(i)$ (3)	Expected average parity $P^*(i)$ (4)	Ratio of reported to expected parity $R(i)=P(i)/P^*(i)$ (5)
0-4	1	0.772	1.127	0.69
5-9	2	2.332	3.312	0.70
10-14	3	3.624	5.270	0.69

It should be noted that in this case data for ever-married women are used because the data for children ever born by duration of marriage were not tabulated by current marital status.

Step 2: calculation of female singulate mean age at marriage. The singulate mean age at marriage for females is calculated from the proportion of females single in each age group, as is explained in annex I. The proportions single are shown in column (5) of table 40. They are calculated by dividing the entries in column (3) by those listed in column (4) of that table. As an example, the proportion single in age group 30-34, denoted by $U(4)$, is calculated below:

$$U(4)=76{,}995/1{,}088{,}690=0.0707.$$

Following the procedure described in detail in annex I, with $RN=U(7)$, the value of the singulate mean age at marriage for females is calculated to be 21.74 years.

Step 3: calculation of expected average parities by duration of marriage. Female marriage is known to take place somewhat late in Egypt, so a_0 was assumed to be 14. The expected average parities, $P^*(i)$, are obtained by interpolating between the values shown in table 37, given 'ne assumed value of a_0 and the estimated value of the singulate mean age at marriage. Table 42 illustrates the way in which this interpolation is carried out. For a description of the linear interpolation procedure, see annex IV. Since the values for $a_0=14$ are only given for

TABLE 42. CALCULATION OF EXPECTED AVERAGE PARITIES BY DURATION GROUP, EGYPT, 1976

Duration group (1)	Index i (2)	Expected average parity, $P^*(i)$, when: Singulate mean age at marriage is 21 $\theta=0.26$ (3)	Singulate mean age at marriage is 22 $1-\theta=0.74$ (4)	Singulate mean age at marriage is 21.74 (5)
0-4	1	1.134	1.125	1.127
5-9	2	3.351	3.298	3.312
10-14	3	5.362	5.237	5.270

the singulate mean ages at marriage that are equal to integer numbers of years, the two singulate mean ages that bracket the observed value identify the $P^*(i)$ values between which interpolation is to be performed. In this case, the ages are 21 and 22. Hence, the interpolation factor Θ is calculated as follows:

$$\Theta=(22-21.74)/(22-21)=0.26.$$

This interpolation factor is then used to weight adjacent expected parities, so that

$$P^*(i)=\Theta P^*_{21}(i)+(1-\Theta)P^*_{22}(i).$$

The final set of expected $P^*(i)$ values is shown in column (5) of table 42.

Step 4: selection of an adjustment factor. An adjustment factor, K, is selected from the values of $R(i)=P(i)/P^*(i)$, the ratios between the reported and the estimated parities at each duration interval. The values of $R(i)$ are shown in column (5) of table 41. They are clearly very consistent. Their trend does not invalidate the hypothesis made above about the similarity between the actual pattern of marital fertility in Egypt and that of natural fertility. Although recent evidence has shown that in the large urban centers of Egypt (Cairo and Alexandria) women are limiting their fertility by the use of contraceptives, their relative weight in the entire population and the recency of such practices do not seem to have affected the overall picture to a significant degree. In addition, the consistency of the observed $R(i)$ ratios does not suggest that the effect of delayed cohabitation is of importance. However, because the combined biases due to delayed cohabitation and to the adoption of fertility control may also produce the pattern seen in the $R(i)$ ratios, their importance cannot be adequately assessed on the basis of these results. Other information is needed to make a definitive statement about fertility in the country as a whole. But, continuing with this example, the choice of the mean of the observed $R(i)$ values as adjustment factor K is appropriate. Thus,

$$K = (0.69+0.70+0.69)/3.0 = 0.69.$$

Step 5: calculation of adjusted age-specific marital fertility schedule. In this step, an adjusted age-specific marital fertility schedule, $g(j)$, is calculated by multiplying each entry of the schedule given in table 38 by the adjustment factor K calculated in the previous step. The resulting set of $g(j)$ values is shown in column (3) of table 43. As an example, $g(5)$ is

$$g(5) = (0.69)(0.3226) = 0.2226.$$

Step 6: calculation of adjusted age-specific fertility schedule. The marital age-specific fertility schedule shown in column (3) of table 43 can be transformed into an age-specific fertility schedule referring to all women, $f(j)$, by multiplying each of the estimated marital fertility rates by the proportion of currently married women in the corresponding age group. These proportions, shown in column (6) of table 40, were obtained from a tabulation showing the female population classified by marital status and by age. In this step, the proportions currently married are used instead of the proportions of ever-married women in order to minimize the biases due to the fact that ever-married women have not, in general, been continuously in the married state. Thus, the estimated age-specific fertility rates appearing in column (4) of table 43 are obtained as is illustrated below in the case of $f(3)$:

TABLE 43. ADJUSTED MARITAL FERTILITY AND ESTIMATED AGE-SPECIFIC FERTILITY RATES AND NUMBER OF BIRTHS, EGYPT, 1976

Age group (1)	*Index* j (2)	*Adjusted marital fertility* $g(j)$ (3)	*Estimated age-specific fertility* $f(j)$ (4)	*Estimated number of births* $B(j)$ (5)
15-19	1	0.2837	0.0459	84 913
20-24	2	0.3239	0.1911	297 396
25-29	3	0.3048	0.2520	342 658
30-34	4	0.2752	0.2415	262 919
35-39	5	0.2226	0.1949	200 251
40-44	6	0.1151	0.0932	87 989
45-49	7	0.0174	0.0134	9 869
TOTAL		1.5427	1.0320	1 285 995

$$f(3) = (0.3048)(0.8268) = 0.2520.$$

Once age-specific fertility rates are available, total fertility is estimated as five times their sum, that is,

$$TF = 5.0(1.0320) = 5.16.$$

Step 7: calculation of adjusted birth rate. The number of births expected if currently married women in each age group were to experience the estimated marital fertility rates, $g(j)$, shown in table 43 and if there were no illegitimate fertility can be obtained either by multiplying those rates by the number of currently married women or, alternatively, by multiplying the age-specific fertility rates derived in the previous step, $f(j)$, by the total number of women in each age group. The resulting estimated numbers of births by age group are shown in column (5) of table 43. As an example,

$$B(5) = (0.1949)(1{,}027{,}457) = 200{,}251.$$

By adding the number of estimated births over all ages and dividing the result by the total population, in this case 36,626,204 persons, an estimate of the birth rate is obtained. Thus,

$$b = 1{,}285{,}995/36{,}626{,}204 = 0.0351.$$

3. *Comparison of period duration-specific fertility rates with average parities by duration group*

(a) *Data required*

The data required for this procedure are described below:

(*a*) The number of children ever born, classified by time elapsed since first marriage of mother (five-year duration groups);

(*b*) The ever-married female population, classified by five-year duration of marriage group (duration of marriage should be measured as the time elapsed since first marriage);

(*c*) The births in a year, classified by the duration of the mother's first marriage (this information may come either from a survey question on births in the past year or date of most recent birth, or from a vital registration system).

(b) *Computational procedure*

The steps of the computational procedure are given below.

Step 1: calculation of reported average parities by duration of marriage. Average parities by duration of marriage, $P(i)$, are calculated by dividing the number of children ever born reported by women in duration of marriage group i by the number of women in that duration group. It is important to make sure that a consistent treatment of marital status is employed: if, for example, the children ever born are reported by ever-married women, then the denominator of the average parities should also be ever-married women. If children ever born refer only to women still in their first marriage, the denominator used should be women still in their first marriage, and so on. Furthermore, if the proportion of women in the selected marital category who did not state their parity is non-trivial, a question arises as to how to treat such cases. For guide-lines about the treatment of women whose parity is not stated, refer to subsection A.2 and to annex II on El-Badry's correction.

Step 2: calculation of duration-specific period fertility rates. These rates, denoted by $g(i)$, are calculated for each duration group i by dividing the births reported to have occurred during a year to women of duration group i (the births being obtained either from a question on date of most recent birth in a survey or from a vital registration system) by the number of ever-married women in duration group i. Note once again that consistency should be maintained with regard to the women covered; and if ever-married women were used in step 1, they should be used again in this step (in such a case, the births in a year should also refer to all ever-married women).

Step 3: calculation of cumulated fertility schedule by duration. The duration-specific fertility rates, $g(i)$, are cumulated and multiplied by five in order to estimate fertility cumulated to the upper limit of age group i, $\Gamma(i)$. Thus,

$$\Gamma(i)=5\left[\sum_{j=1}^{i} g(j)\right]. \quad \text{(D.2)}$$

Step 4: estimation of average parity equivalents. Average parity equivalents, $G(i)$, are estimated by interpolation using the current fertility rates, $g(i)$, and the cumulated fertility schedule, $\Gamma(i)$. As in the case where data are classified by age, the estimation procedure uses different equations, depending upon the way in which duration of marriage has been measured. When the source of the data is a vital registration system, recorded marital duration is likely to be that at the time of the birth, and the estimation equation to be used is

$$G(i)=\Gamma(i-1)+2.917g(i)-0.417g(i+1). \quad \text{(D.3)}$$

Average parities are usually available only for four or five duration groups, that is, up to duration group 15-19 or 20-24. In order to exploit all the information available, the last or upper value of $G(i)$ is estimated by using the following equation:

$$G(i)=\Gamma(i-1)+0.417g(i-1)+2.083g(i). \quad \text{(D.4)}$$

When data on current fertility by duration of marriage come from a survey question on births in the past year, the recorded duration of marriage will almost certainly be that at the time of the survey and not the duration at the time of the birth; therefore, fertility rates calculated from these data will refer to duration groups shifted, on average, six months towards lower durations. Hence, a different equation to estimate parity equivalents is needed. The parallel with the case in which data are classified by age is obvious. However, the two cases are not quite the same, since in the present instance it is assumed that childbearing does not begin until marriage, and it is also assumed that if cohabitation begins with first marriage the burst of childbearing nine months after and in the following few months compensates for the period immediately following marriage during which no births occur. The first duration period is thus only 4.5 years; and at the beginning of it, cumulated fertility equals zero. Hence, for the first duration group, 0-4, $G(i)$ is estimated by

$$G(1)=3.137g(1)-0.324g(2). \quad \text{(D.5)}$$

For subsequent duration groups, the general interpolation equations proposed by Coale and Trussell in the case of data classified by age are also used in the duration case:

$$G(i)=\Gamma(i-1)+3.392g(i)-0.392g(i+1) \quad \text{(D.6)}$$

for $i=2, 3, \ldots, j-1$, and

$$G(j)=\Gamma(j-1)+0.392g(j-1)+2.608g(j) \quad \text{(D.7)}$$

where the index j indicates the last duration group considered. Because fertility changes more smoothly with marriage duration after the first two years or so than it does with age, there is no need to resort to more sensitive, model-based interpolation procedures. The Coale-Trussell procedure based on the fitting of a second-order polynomial to cumulated fertility is adequate.

Step 5: comparison of reported average parity and average parity equivalents for duration-of-marriage groups. When marital fertility has remained reasonably constant, a comparison of reported average parities with the parity equivalents estimated on the basis of period fertility should provide a basis for adjusting current births. Such a comparison is carried out by calculating the ratios $P(i)/G(i)$. If these ratios are reasonably consistent by duration group, at least for the first three or four values of i, the average of the first three ratios can be used as an adjustment factor for the reported number of current births, whether the latter are classified by age or by duration of marriage. The adjustment of births classified by age of mother is likely to be more useful, but it is valid only if the error in reporting births is constant across duration, as well as across age groups. If such an assumption seems tenable in a given case, births by age of mother may be adjusted, age-specific fertility rates can be calculated, and other measures of fertility may be derived. (See subsection B.2 (c) and take note that, if data on births come from a survey, the age-specific fertility rates will have to be adjusted for the fact that they probably refer to age groups shifted by six months.)

In cases where it is not possible to assume that errors in data classified by age are similar to those in data classified by duration, or where data classified by age are not available, the number of reported births classified by duration can be adjusted and an adjusted marital fertility schedule, as well as an adjusted birth rate, can be derived. The latter rate will, of course, be a valid estimate only if childbearing occurs mainly within marriage in the population in question.

(c) *A detailed example*

Data on children ever born and date of most recent

TABLE 44. EVER-MARRIED WOMEN, CHILDREN EVER BORN AND BIRTHS IN THE YEAR PRECEDING THE SURVEY, BY TIME ELAPSED SINCE FIRST MARRIAGE, 1976

Duration group (1)	*Index* i (2)	*Number of ever-married women* (3)	*Children ever born* (4)	*Births in preceding year* (5)	*Reported average parity* $P(i)$ (6)	*Duration-specific fertility rate* $g(i)$ (7)
0-4	1	725	858	324	1.1834	0.4469
5-9	2	696	2 396	317	3.4425	0.4555
10-14	3	596	3 214	215	5.3926	0.3607
15-19	4	574	3 961	167	6.9007	0.2909
20-24	5	471	3 863	88	8.2017	0.1868
25-29	6	333	3 028	28	9.0931	0.0841

birth classified by time elapsed since first marriage are available from the fertility survey of a Muslim population in 1976. Table 44 shows the reported number of ever-married women in each duration group, the total number of children ever born to them and the births they reported as having occurred during the year preceding the survey.

The steps of the computation procedure are given below.

Step 1: calculation of reported average parities by duration of marriage. Children ever born, given in column (4) of table 44, are divided by the corresponding number of ever-married women, shown in column (3). The values for reported average parities by duration of marriage, $P(i)$, thus obtained are shown in column (6). As an example, $P(3)$ is calculated below:

$$P(3)=3{,}214/596=5.3926.$$

Step 2: calculation of duration-specific period fertility rates. These rates, denoted by $g(i)$, are calculated by dividing the births that occurred in the year preceding the survey, shown in column (5) of table 44, by the corresponding number of ever-married women, shown in column (3). The resulting marital fertility rates, $g(i)$, are shown in column (7). As an example,

$$g(4)=167/574=0.2909.$$

Step 3: calculation of cumulated fertility schedule by duration. To obtain each value of the cumulated fertility schedule, $\Gamma(i)$, the duration-specific fertility rates, $g(i)$, are cumulated, up to and including group i, and then multiplied by five. Thus, for duration group 4 (15-19 years since first marriage), the calculations are

$$\Gamma(4)=5.0(g(1)+g(2)+g(3)+g(4))$$
$$=5.0(0.4469+0.4555+0.3607+0.2909)$$
$$=7.7700.$$

All values of $\Gamma(i)$ are shown in column (5) of table 45.

TABLE 45. CUMULATED FERTILITY SCHEDULE, PARITY EQUIVALENTS AND RATIOS OF AVERAGE PARITIES TO PARITY EQUIVALENTS, BY TIME ELAPSED SINCE FIRST MARRIAGE, 1976

Duration group (1)	*Index* i (2)	*Average parity* $P(i)$ (3)	*Period fertility* $g(i)$ (4)	*Cumulated fertility* $\Gamma(i)$ (5)	*Parity equivalent* $G(i)$ (6)	*Ratio of average parity to parity equivalent* $P(i)/G(i)$ (7)
0-4	1	1.1834	0.4469	2.2345	1.2543	0.944
5-9	2	3.4425	0.4555	4.5120	3.6382	0.946
10-14	3	5.3926	0.3607	6.3155	5.6215	0.959
15-19	4	6.9007	0.2909	7.7700	7.2290	0.955
20-24	5	8.2017	0.1868	8.7040	8.3707	0.980
25-29	6	9.0931	0.0841	9.1245	8.9966	1.011

Step 4: estimation of average parity equivalents. The data on births were obtained from fertility histories, and although it is possible to derive from this information data classified by marriage duration at the time of the birth, the data displayed in table 44 refer to marriage duration at the time of the interview. Therefore, the true duration groups to which the marital fertility rates, $g(i)$, refer are six months younger than shown. Values of the current average parity equivalents, $G(i)$, are therefore obtained using equations (D.5), (D.6) and (D.7). For $i=1$:

$$G(1)=3.137g(i)-0.324g(2)$$
$$=3.137(0.4469)-0.324(0.4555)$$
$$=1.2543.$$

For i equal to 2, 3, 4 and 5, equation (D.6) is used, so for duration group 3, for example,

$$G(3)=\Gamma(2)+3.392g(3)-0.392g(4)$$
$$=4.5120+(3.392)(0.3607)-(0.392)(0.2909)$$
$$=5.6215.$$

For the last value of i, namely 6, equation (D.7) is used:

$$G(6)=\Gamma(5)+0.392g(5)+2.608g(6)$$
$$=8.7040+(0.392)(0.1868)+(2.608)(0.0841)$$
$$=8.9966.$$

Column (6) of table 45 shows all six values of $G(i)$.

Step 5: comparison of reported average parity and average parity equivalents for duration-of-marriage groups. The ratios $P(i)/G(i)$ are calculated for each duration group, the $P(i)$ values being obtained from column (3) of table 45 and the $G(i)$ values coming from column (6). The complete set of P/G ratios is shown in column (7). The ratio values are fairly consistent, although they display some tendency to increase as duration increases. This tendency may be indicative of the fact that marital fertility has decreased somewhat, although it is not possible to discard entirely the possibility of it being caused by errors in the data (particularly in the parity reports for higher duration groups). According to the P/G ratios for the first four duration groups, period fertility rates would overestimate marital fertility by about 5 per cent. Multiplying them by the adjustment factor $K = 0.95$ would yield current fertility estimates consistent with the reported parities. Total marital fertility adjusted in this way amounts to 8.67 children per ever-married woman.

It is of interest to mention that although the Muslim population in question is one in which the practice of delaying cohabitation after marriage may be widespread, the trend of the P/G ratios calculated on the basis of the data available does not appear to be affected by such practice. In fact, the P/G ratios are robust to the existence of any practice that delays or postpones childbearing immediately after marriage, as long as it affects in the same way both retrospective fertility (parities) and period fertility (births in the past year). Biases are to be expected only if there have been changes in the prevalence of such practices in recent years or if there is a non-negligible incidence of pre-marital fertility in the population being studied. In the latter case, pre-marital births will increase the parities of the lower duration groups; but because only ever-married women are considered, they are not likely to be adequately reflected by period fertility rates. Thus, if the basic data were accurate in all other respects, a non-negligible incidence of pre-marital fertility would result in a series of P/G ratios that decline as duration of marriage increases.

Additional references

Brass, William *and* Hoda Rashad. Exploratory demographic analysis of imperfect maternity histories to determine levels and trends of fertility. Paper presented at the International Union for the Scientific Study of Population Seminar on the Analysis of Maternity Histories, Liege, 1980.

Carrier, Norman H. *and* John Hobcraft. Demographic estimation for developing societies; a manual of techniques for the detection and reduction of errors in demographic data. London, London School of Economics, Population Investigation Committee, 1971.

Goldman, Noreen *and* John Hobcraft. Birth histories. Voorburg, The Hague, International Statistical Institute, 1981. (World Fertility Survey comparative studies, 17)

Hobcraft, John. Illustrative analysis; evaluating fertility levels and trends in Colombia. Voorburg, The Hague, International Statistical Institute, 1980. (World Fertility Survey scientific report, 15)

Hobcraft, John, Noreen Goldman *and* V. C. Chidambaram. Advances in the P/F ratio method for the analysis of birth histories. Voorburg, The Hague, International Statistical Institute, 1981. (World Fertility Survey technical report, 1619)

Rodriguez, German *and* John N. Hobcraft. Illustrative analysis; life table analysis of birth intervals in Colombia. Voorburg, The Hague, International Statistical Institute, 1980. (World Fertility Survey scientific report, 16)

United States. Department of Commerce. Bureau of the Census. Estimating fertility from data on children ever born, by age of mother. Washington, 1983. (International research document, 11)