Session II: Overview of projection methods

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Cheryl Sawyer, Lina Bassarsky
Population Estimates and Projections Section

www.unpopulation.org
Outline

I. Choosing a projection method
II. Trend extrapolation methods
III. Cohort-component projections
IV. Country presentations on national experience with population projections
I. Choosing a projection method
Criteria for choosing a projection method

1. Desired outputs
   - Length of time horizon → short-term (e.g. next 5 years), medium-term, long-term, etc.
   - Amount of subject detail → total population, broad age groups, five-year age groups, single years of age, subgroups

2. Input data
   - Data availability
   - Data quality
   - Missing data → plausible assumptions

3. Projection tools
II. Trend extrapolation methods
**Trend extrapolation methods**

- Based on the continuation of observable historical trends. Mathematical models are fitted to historical data, then used to project population values.
- Historical data may be “borrowed” from population growth observed in other populations.
- **Usually applied to the figure of total population only.**
Trend extrapolation methods

- Most appropriate when the total population growth is fairly regular, or when the past is a good predictor of the future trends
  - when vital rates and age structure can be assumed to be constant
- ...or if the only thing one knows about the population is its total size.

- Not appropriate
  - where the present age structure has marked peculiarities
  - where important or rapid socio/economic changes are foreseen or expected on the grounds of current conditions of experiences of other populations

- Usually not useful for policy-related purposes.
- It can be used for making quick estimates or for illustrative or comparison purposes, but is not recommended for making official population projections.
Trend extrapolation methods

- Methods that require data for 2 dates → base period (an initial year and the launch year)
  - Linear growth
  - Geometrical growth
  - Exponential growth

- Methods that require data for more dates
  - Logistic curve
  - Other (linear trend, polynomial curve, ARIMA)

- Ratio extrapolation methods → population of a smaller area is expressed as a proportion of its “parent” larger population area
  - Constant share, shift share, share of growth

[Terminology for trend extrapolation methods section]

initial population \( P_b \)

population in launch year \( P_0 \)

target population \( P_t \)

base period \( y = \) number of years in the base period

projection period \( t = \) number of years in the projection period
Linear growth

Assumptions:
- Population will change by the same (constant) absolute amount of change per unit of time (e.g. a year) or a decreasing relative change.
- Change is experienced at the end of unit of time

Steps:
- Compute the average absolute change $\Delta$ during the base period, apply the same amount for each unit of time through the target year.
Trend extrapolation methods

Linear growth

○ Computation:

\[ \Delta = \text{annual amount of population change:} \]

\[ \Delta = \frac{(P_0 - P_b)}{y} \]

\[ P_t = \text{population in the target year:} \]

\[ P_t = P_0 + \Delta t \]

Where:

\( P_0 = \text{population in the launch year} \)

\( P_b = \text{initial population (base period)} \)

\( y = \text{number of years in the base period} \)

\( t = \text{number of years in the projection (between the launch and the target years)} \)
Geometric growth

- Assumptions:
  - Population will change by the same percentage rate per unit of time as during the base period
  - Change is occurring at discrete intervals

- Steps:
  - Compute the average annual geometric rate of change \( r \); apply the rate to the launch population.
Geometric growth

Computation:

\[ r = \text{average annual geometric rate of change} \]

\[ r = \left( \left( \frac{P_0}{P_b} \right)^{1/y} \right) - 1 \]

\[ P_t = \text{population in the target year} \]

\[ P_t = P_0 \times (1 + r)^t \]

Where:

- \( P_0 \) = population in the launch year
- \( P_b \) = initial population (base period)
- \( y \) = number of years in the base period
- \( t \) = number of years in the projection (between the launch and the target years)
Exponential growth

- **Assumptions:**
  - Yields similar results to geometric growth, BUT change occurs continuously.

- **Steps:**
  - Compute the average annual exponential rate of change $r$, apply the rate to the launch population.
Exponential growth

○ Computation:

\[ r = \text{average annual exponential rate of change} \]

\[ r = \left( \ln \left( \frac{P_0}{P_b} \right) \right) / y \]

\[ P_t = \text{population in the target year} \]

\[ P_t = P_0 \times e^{rt} \]

Where:

\( P_0 = \text{population in the launch year} \)

\( P_b = \text{initial population (base period)} \)

\( y = \text{number of years in the base period} \)

\( e = \text{base of the system of natural logarithms} \)

\( t = \text{number of years in the projection (between the launch and the target years)} \)

○ It can be observed that:

\[ (1 + rt) \leq (1 + r)^t \leq e^{rt} \]
Exponential growth

- Time required for one population \((P_0)\) to double \((2P_0)\), given a known growth rate

\[
2P_0 = P_0 e^{rt} \\
2 = e^{rt} \\
\ln 2 = rt \\
\therefore t = \frac{\ln 2}{r}
\]
The logistic function: exhibits an S-shape and describes a diffusion process where new behavior is gradually being adopted by people.

It can be used as a model representing the behaviour of various populations and demographic phenomena, such as population proportions and projection of small areas.

Population growth: trend of growth with increasing annual increments until a maximum. Beyond that point, growth diminishes until a minimum or becomes negligible.

Currently mostly used in projection of life expectancy at birth and total fertility (growing from an initial level to an upper or lower asymptote).
Logistic growth

Computation:

$$P_t = \frac{a}{1 + b(e^{-ct})}$$

Where

- $a, b, c \rightarrow$ three parameters
- $a$ is the upper asymptote
- $b$ & $c$ define the shape of the logistic curve
- $t$ is the time

Determine the magnitude of the upper asymptote ($a$) and the time ($t$) required to reach it.
The general form of a logistic function can be expressed as:

1. \[ P(t) = \frac{k}{1 + \exp[-\alpha(t - \beta)]} \]
   - \( k \) = asymptote (saturation level)
   - \( \alpha \) = growth rate of the curve
   - \( \beta = t_m \) = length of time to midpoint

2. \[ P(t) = \frac{k}{1 + \exp[-\frac{\ln(81)}{\Delta t}(t - t_m)]} \]
   - Duration for the growth process to proceed from 10% to 90% of \( k \)
   
   where: \( \Delta t = \frac{\ln(81)}{\alpha} \)
Trend extrapolation methods

Arithmetic or linear growth

Arithmetic or linear decline

Exponential growth

Exponential decline or decay

Logistic growth

Logistic decline

Source:
http://papp.iussp.org
III. Cohort-component projections
Cohort-component method (CCM)

- Most commonly used method that accounts for age distribution
- Benefit of taking into account the age distribution
  
a) Population accounting concept
b) Data required
c) Model implementation, example
Population accounting concept

\[
\begin{align*}
    P_{t+n} & = P_t + B_{t,t+n} - D_{t,t+n} + I_{t,t+n} - E_{t,t+n} \\
    P_{t+n} - P_t & = B_{t,t+n} - D_{t,t+n} + I_{t,t+n} - E_{t,t+n}
\end{align*}
\]

- \( P_t \) is the population at time \( t \)
- \( B_{t,t+n} \) and \( D_{t,t+n} \) are number of births and deaths occurring between \( t \) and \( t + n \).
- \( I_{t,t+n} \) and \( E_{t,t+n} \) are number of immigrants and emigrants from the country during the same period.
The Cohort component method (CCM) for population estimation

The population enumerated in the first census is projected to the reference date of the second census, based on intercensal estimates. The expected population from the projection is then compared with the actual population enumerated in the second census (e.g. 10 years after).

Data required:

1. Population enumerated in two censuses by age and sex
2. Age specific fertility rates for women aged 15 to 49 (in 5-year age groups), assumed to represent the level and age structure of fertility during the intercensal period
3. Life table survival ratios for males and females, assumed to be representative of mortality conditions during the intercensal period
4. An estimate of sex ratio at birth
5. Estimates of the level and age pattern of net international migration during the intercensal period if the level of net migration is substantial
The Cohort component method (CCM) for population estimation projection at the launch year of the projection

Assumptions about trends during the projection span

Data required:

1. Population enumerated in two censuses by age and sex
2. Age specific fertility rates for women aged 15 to 49 (in 5-year age groups), assumed to represent the level and age structure of fertility during the intercensal period
3. Life table survival ratios for males and females, assumed to be representative of mortality conditions during the intercensal period
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5. Estimates of the level and age pattern of net international migration during the intercensal period if the level of net migration is substantial
Computational steps

1. Survive the base population in $t$ by age and sex to $t+n$ in projection period
   Multiply each age group population by life table survival ratios
   (open-ended interval requires special handling)
2. Make any necessary adjustments for migration
3. Calculate the total number of births (total, by sex) during $t$, $t+n$
4. Apply survivorship to these births to determine number that survive to $t+n$
5. Obtain population projection at $t+n$
6. Repeat steps 1-5 for the next projection period
CCM

Base population

Estimated Population by sex and age

Fertility hypothesis

Mortality hypothesis

Sex ratio at birth

International migration hypothesis

Projection period \([t - t+5]\)

TFR \( \xrightarrow{f_{x, x+4}} \)

\( e^0 \)

\( S_{x, x+4} \)

SR at birth

“net migrants” by sex and age

TFR

\( e^0 \)

\( S_{x, x+4} \)

SR at birth

Estimated Births \( 5 * B \)

Pop 5+

Pop 0-4

Projected population by sex and age (closed)

Projected Population by sex and age (open to migration)

a. input data

b. intermediate indicators

c. output data

in t

in t+5

in t+5

in t+5
Survival ratios measure how many people in the younger age group will survive into the older age group based on the age-specific mortality rates used to construct the life table.

Computed from two adjoining $5L_x$ values of a life table

In other words, $S_{x,x+n}$ is the survivorship ratio of persons who are aged $x$ to $x+n$ at the start of projection interval $t$ to $t+n$.

Used to project forward the population in each subgroup at the beginning of the time interval in order to estimate the number still alive at the beginning of the next interval accounting for those who survive each specific age interval

$$P_{x+n}(t+n) = P_x(t) * S_{x,x+n}$$
Survival ratios \( S_{x,x+n} \)

\[
S_{x,x+n} = \frac{nL_{x+n}}{nL_x}
\]

for all age groups except the last and the first

\[
S_{b,0} = \frac{nL_0}{n \times l_0}
\]

for the first age group

\[
\infty P_{85} (t+n) = [\infty P_{80} (t) + \infty P_{85} (t)] \times \frac{T_{85}}{T_{80}}
\]

for the last age group
International migration hypothesis
(for open populations)

- If net international migration is substantial, the “survived” cohort population must be adjusted to reflect the effects of migration.
- Assumptions are more often formulated in terms of *absolute numbers* and not by rates (related to policies).
- Migrants are also exposed to giving births and dying:
  - = migrants have the same fertility and mortality level as the non-migrant population.
- The introduction of net migrants by age group at the midpoint of the projection period and the survival of net migrants to the end of the period:
  - = an equal distribution of net migrants across time interval.
CCM Model implementation – Example
A simplified Cohort-component method projection of the female population of "Westlands", from 2010 to 2015

Population values are expressed in thousands

<table>
<thead>
<tr>
<th>Age group</th>
<th>Population in 2010</th>
<th>Person-years lived (-sL_x) ((e_0=64))</th>
<th>Net migrant population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4</td>
<td>1,084</td>
<td>469097</td>
<td>-2.1</td>
</tr>
<tr>
<td>5-9</td>
<td>922</td>
<td>459092</td>
<td>-0.9</td>
</tr>
<tr>
<td>10-14</td>
<td>793</td>
<td>454840</td>
<td>-0.8</td>
</tr>
<tr>
<td>15-19</td>
<td>706</td>
<td>450962</td>
<td>-4.0</td>
</tr>
<tr>
<td>20-24</td>
<td>623</td>
<td>445347</td>
<td>-6.9</td>
</tr>
<tr>
<td>25-29</td>
<td>528</td>
<td>438622</td>
<td>-6.4</td>
</tr>
<tr>
<td>30-34</td>
<td>442</td>
<td>431379</td>
<td>-4.7</td>
</tr>
<tr>
<td>35-39</td>
<td>355</td>
<td>423529</td>
<td>-3.1</td>
</tr>
<tr>
<td>40-44</td>
<td>277</td>
<td>414509</td>
<td>-2.0</td>
</tr>
<tr>
<td>45-49</td>
<td>223</td>
<td>403997</td>
<td>-1.3</td>
</tr>
<tr>
<td>50-54</td>
<td>181</td>
<td>390632</td>
<td>-0.8</td>
</tr>
<tr>
<td>55-59</td>
<td>145</td>
<td>372490</td>
<td>-0.5</td>
</tr>
<tr>
<td>60-64</td>
<td>111</td>
<td>346186</td>
<td>-0.3</td>
</tr>
<tr>
<td>65-69</td>
<td>92</td>
<td>306649</td>
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</tr>
<tr>
<td>70-74</td>
<td>67</td>
<td>248840</td>
<td>-0.1</td>
</tr>
<tr>
<td>75-79</td>
<td>41</td>
<td>172444</td>
<td>-0.1</td>
</tr>
<tr>
<td>80-84</td>
<td>18</td>
<td>91395</td>
<td>0.0</td>
</tr>
<tr>
<td>85+</td>
<td>6</td>
<td>39221</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td>6,613</td>
<td></td>
<td>-34.4</td>
</tr>
</tbody>
</table>

1. Survive base population by age and sex

First age group (0-4 → 5-9)

\[ S_{x,x+n} = \frac{nL_{x+n}}{nL_x} \]

= 1,084 \* (459092 / 469097) = 1,061

2. Adjust for migration

= 1061-0.9 = 1,060

3. Survive the open-ended age group:

\[ \propto P_{85}(t+n) = [\propto P_{80}(t) + \propto P_{85}(t)] \times \frac{T_{85}}{T_{80}} \]

= (18+6) \* 39221 / (39221 + 91395) = 7

Still need to estimate youngest cohorts based on fertility data
## CCM Model implementation – Example

A simplified Cohort-component method projection of the female population of "Westlands", from 2010 to 2015

### 4. Calculate the total number of births during interval

- By multiplying projected age-specific fertility rates (ASFRs), with the average number of women in each reproductive age group → an estimation of the surviving women who gave birth during interval

\[
B_{15-19} = \frac{(706+782)}{2} \times 0.0967 = 360
\]

- Sum up births by age of mother to obtain total births

\[
B = 2,728
\]
CCM Model implementation – Example
A simplified Cohort-component method projection of the female population of "Westlands", from 2010 to 2015

\[ P^f_{2010} = 6,613 \] \[ P^f_{2015} = 7,632 \]

<table>
<thead>
<tr>
<th>Age group (e0=64)</th>
<th>Person-years lived (-5L_x)</th>
<th>Net migrant population</th>
<th>Population in 2015</th>
<th>Age-specific fertility rates</th>
<th>Births by age of mother</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4</td>
<td>1,084</td>
<td>469097</td>
<td>-2.1</td>
<td></td>
<td></td>
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<td>5-9</td>
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<td>1,060</td>
<td></td>
</tr>
<tr>
<td>10-14</td>
<td>793</td>
<td>454840</td>
<td>-0.8</td>
<td>912</td>
<td></td>
</tr>
</tbody>
</table>

5. Apply the sex ratio at birth to obtain female births (consider sex ratio assumptions)
\[ B^f = 2,728/2.04 = 1,337 \]

6. Apply survivorship to these births to determine the surviving \(5P^f_0\) in 2015
\[ S_{b,0} = \frac{nL_0}{n \times l_0} \]

\[ 5P^f_0 = 1,337 \times (469097/500,000) \approx 1,254 \]

7. Adjust for migration of the group age (-2.1)
\[ 5P'_0 = 1,254 - 2.1 = 1,252 \]
CCM Model implementation – Example
A simplified Cohort-component method projection of the female population of "Westlands", from 2010 to 2015

“5. Apply the sex ratio at birth to obtain female births (consider sex ratio assumptions)”

From a ratio to a proportion:

Sex ratio (male/females) = 1.04 males per 1.00 females
Total males+females = 2.04
Proportion males = 1.04/2.04 = 0.51
Proportion females = 1.00/2.04 = 0.49

E.g.: $B^f = \frac{2,728}{2.04} = 2,728 \times \left(\frac{1.00}{2.04}\right) = 2,728 \times \left(1 - \frac{1.04}{2.04}\right) = 1,337$
Session II: Hands-on exercise

1. Open the file: S II Projection methods exercise.doc
2. Work with Excel file: S II Projection methods exercise.xls
3. Work with MORTPAK PROJCT application
Thank you

Questions?

>> until 11 March:

>> After 11 March: sawyerc@un.org
   bassarsky@un.org