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PREFACE

The purpose of the *Population Bulletin of the United Nations*, as stipulated by the Population Commission, is to publish population studies carried out by the United Nations, its specialized agencies and other organizations with a view to promoting scientific understanding of population questions. The studies are expected to provide a global perspective of demographic issues and to weigh the direct and indirect implications of population policy. The *Bulletin* is intended to be useful to Governments, international organizations, research and training institutions and other bodies that deal with questions relating to population and development.

The *Bulletin* is prepared by the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat and published semi-annually in three languages—English, French and Spanish. Copies are distributed widely to users in all member countries of the United Nations.

Although the primary source of the material appearing in the *Bulletin* is the research carried out by the United Nations Secretariat, officials of governmental and non-governmental organizations and individual scholars are occasionally invited to contribute articles.

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Explanatory notes

Symbols of United Nations documents are composed of capital letters combined with figures. Mention of such a symbol indicates a reference to a United Nations document.

Reference to "dollars" ($) indicates United States dollars, unless otherwise stated.

The term "billion" signifies a thousand million.

Annual rates of growth or change refer to annual compound rates, unless otherwise stated.

A hyphen between years (e.g., 1984-1985) indicates the full period involved, including the beginning and end years; a slash (e.g., 1984/85) indicates a financial year, school year or crop year.

A point (.) is used to indicate decimals.

The following symbols have been used in the tables:

Two dots (. .) indicate that data are not available or are not separately reported.
A dash (—) indicates that the amount is nil or negligible.
A hyphen (-) indicates that the item is not applicable.
A minus sign (–) before a number indicates a deficit or decrease, except as indicated.

Details and percentages in tables do not necessarily add to totals because of rounding.

Population Bulletin of the United Nations, No. 30—1991

THE USE OF NEW MODEL LIFE TABLES AT VERY LOW MORTALITY IN POPULATION PROJECTIONS

*Ansley Coale and Guang Guo**

SUMMARY

New model life tables have been calculated at Princeton, modifying the regional model life tables by Coale, Demeny and Vaughan published in 1983. The new tables are published in *Population Index* (Coale and Guo, 1989) and are reproduced in this issue of the *Bulletin*. The purpose of the present article is not to repeat the description of the construction of the tables but to provide a systematic procedure for their use in projecting mortality into the future in low-mortality populations. The new model life tables incorporate age patterns of mortality that conform quite closely to the patterns in the lowest mortality populations yet on record and present extrapolated schedules that include systematic extensions of recent changes in age-specific mortality rates. To use the tables for estimating future mortality schedules, it is sufficient to estimate the future time path of the expectation of life at birth; the estimated life expectancy serves to select (by interpolation) a model life table. A linear relation between the annual increase in life expectancy and the level of expectation of life provides a single-time path for the evolution of expectation of life at birth for the female population. This curve has an upper limit determined by the recent history of low-mortality experience. To estimate the future course of e_0, it is sufficient to locate the current value on the single-time path and then follow the path into the future. This simple procedure provides quite a close match to the actual values of e_0 in eight countries from the early 1950s to the 1980s, beginning in each instance with a single expectation of life in the 1930s.

INTRODUCTION

The regional model life tables of Coale, Demeny and Vaughan have recently been revised and extended to very low levels of mortality (female expectation of life at birth as high as 85 years). The new tables cover e_0 for females from 67.5 (or 70.0) to 85.0 for the West, East and South

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"families" of model life tables. (In fact, at an e_0 of 80 and above, the families' mortality schedules converge.)

Perhaps the most frequent use of these very-low-mortality model life tables is in projecting mortality to very low rates, including rates lower than those yet attained. For that purpose the essential ingredient in addition to the tables themselves is estimates of the future course of the expectation of life at birth for each sex. With an estimate of e_0 at a future date, the analyst can make a life table by interpolation between the printed model life tables with e_0s 2.5 years apart. Thus, an estimated female e_0 of 81.8 is 1.8/2.5, or 72 per cent, of the distance between level 25 ($e_0 = 80.0$) and 26 ($e_0 = 82.5$). Mortality rates can be estimated as .72 × (rates at level 26) plus .38 × (rates at level 25).

A SINGLE-TIME PATH FOR RECENT INCREASES IN LIFE EXPECTANCY

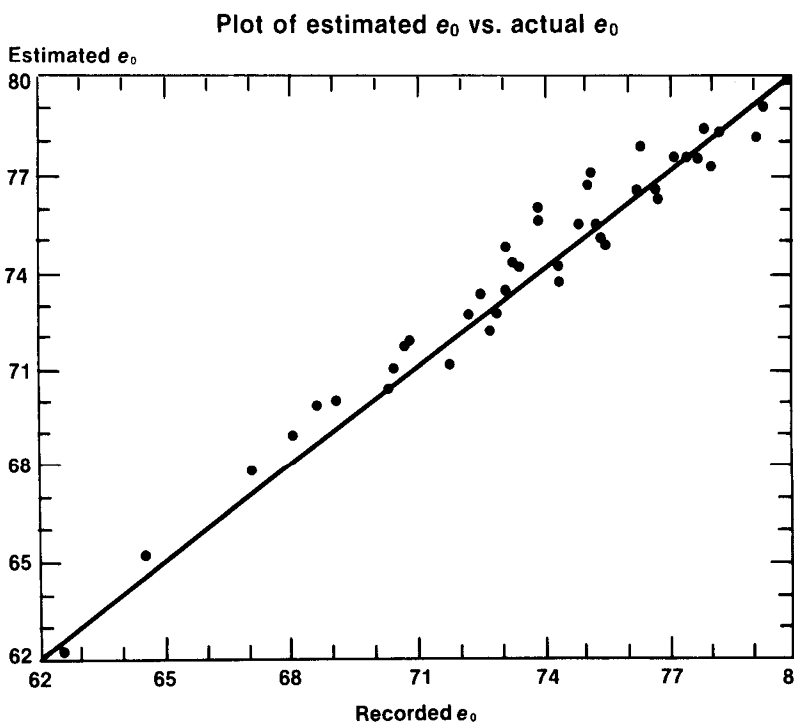
In 1955, in developing a procedure for projecting mortality in Mexico, Coale estimated a single-time path for the increase in expectation of life at birth for each sex. Evidently, this estimated time-path for females and males has been followed in the evolution of mortality in many populations (although not in all) (Coale and Hoover, 1958; Coale, 1981). These paths were derived from a linear relation within 22 populations between the average annual increase in expectation of life at birth and the level of e_0. Time-paths were derived from pairs of male and female life tables in populations with a wide range of mortality levels, pairs of tables separated in time by at least five years. In most instances, the first table was taken from before the Second World War, and the second from after the War. The average annual increase in e_0 was calculated for each pair, as was the mean e_0 for the two points. A linear regression of the annual increase in e_0 on the level of e_0 was calculated. The linear fit was fairly close (linear correlations of $-.91$ for males, and 0.85 for females). The linear relation ($de_0/dt = a - be_0$) leads to a simple equation for the expectation of life at birth at times t after an initial moment t_0.

$$e_0(t) = \bar{e}_0 - (\bar{e}_0 - e_0(t_0)) \exp(-b(t - t_0)) \tag{1}$$

where $\bar{e}_0$ is the upper limit of $e_0(t)$, (reached as t approaches infinity, when d_0/dt becomes 0), $e_0(t_0)$ is the expectation of life in the initial year, and b is the decline in the annual change of $e_0 (de_0/dt)$ with each increase of one year in e_0. Since $\bar{e}_0$ is the expectation of life when e_0 no longer increases, it can be calculated as e_0 when $de_0/dt = 0$, or $a - b\bar{e}_0 = 0$. Hence, $\bar{e}_0 = a/b$. When calculated from the original 26 pairs of life tables used for projecting mortality in Mexico starting with 1955, the asymptotic limit for female e_0 was 84.2 years, and the decrease in de_0/dt with each additional year of expectation of life at birth was 0.0301 years. For Mexico, the estimated e_0 for 1982 was 71.2 years, compared to 72.1 in the Mexican life table for that year published by the World Health Organization (WHO, 1986). Both the estimated and recorded e_0s were more than 16 years above the e_0 of 54.8 in 1955.

The basis of this composite equation for increasing e_0 as a function of date has been buttressed by determining the average level of e_0 and the average annual increase from the 1950s to the 1970s and the 1970s to the 1980s in 18 low-mortality populations with higher-quality data. Combining the data on e_0 and de_0/dt from the three periods, we find an estimated ultimate $\bar{e}_0$ for females of 83.25 years, an average decline in de_0/dt as e_0 increases by one year of -0.03099, and correlation between de_0/dt and e_0 of -0.906. In figure I, we compare estimated and recorded female e_0s in eight countries in seven five-year time periods from 1950-1954 to 1980-1984. The estimates are made by applying equation (1), incorporating an asymptote of 83.25 and a value of b of 0.03099, to an initial e_0 for each population at some date in the 1930s. In other words, the estimates make use of a single datum (e_0 at some date in the 1930s) and a single equation for the time-path of increasing e_0. (The recorded e_0s are from WHO, 1986). The countries are Austria, France, Italy, Portugal, Sweden and the United States, and include populations that were, at early dates, part of the West, East and South model life tables. The points cluster very closely about a line equating the estimated and actual value of the expectation of

Figure I. Actual female e_0 from 1950-1954 to 1980-1984 in eight countries compared to estimated e_0



life, confirming that these populations did, by and large, follow a single time-path in the evolution of increasing expectation of life at birth for females, and that expectation of life at birth 50 years after the 1930s can be closely approximated by assuming congruence in each population with this single time path.

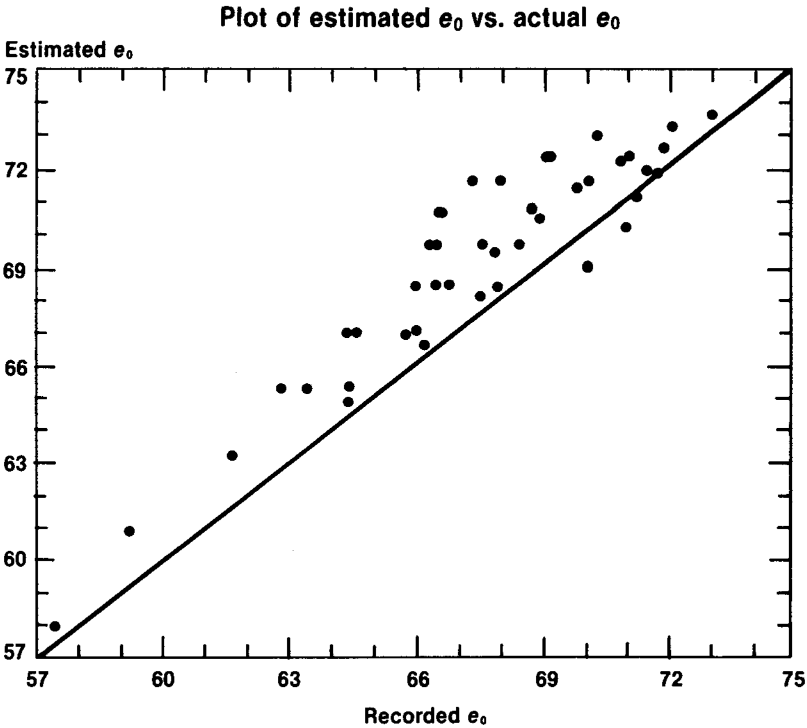
As was noted in an earlier paper, only selected developing countries seem to adhere to this "standard" curve (countries including, for example, Taiwan Province of China, Cuba and Costa Rica); in most developing countries expectation of life generally follows a slower pattern of increase than indicated by the standard curve. Similarly, in Eastern European populations in which e_0 stayed close to the time-path until the 1960s or 1970s, e_0 has fallen substantially short of the path in later years (Coale, 1981). We think of the populations that follow the standard curve as belonging to what might be called the "main sequence" of mortality schedules, in analogy to the main sequence of stars in their evolution. The main sequence stars are formed by contraction of an interstellar cloud of gas, followed by the initiation of nuclear reactions that yield a steady source of radiation for billions of years. A main sequence star moves through characteristic changes in luminosity and spectral structure, ending with conversion into a red giant and then contraction into a white dwarf. The sun is a main sequence star. Many stars do not belong to this sequence, and follow some other, quite different, evolution. We are suggesting that there are populations whose expectation of life at birth after 1930 or 1940 follows the "main sequence" of a standard curve of increasing e_0 and that, for various reasons, mortality of other populations does not follow such a sequence.

Evolution of male mortality in different populations has been much more irregular than female mortality. During the period from the 1950s until 1970, expectation of life at birth for males in many populations failed to increase or increased very slightly. The result is that when expectation of life at birth at five-year intervals from 1950-1954 to 1980-1982 is estimated for the male population from a value of expectation of life at birth in the 1930s and the "universal" path of male life expectancy, the estimates generally exceed the actual expectation of life, and the scatter of the estimates around the actual values is substantially larger than for the female estimates (see fig. II).

USING A SINGLE TIME-PATH TO ESTIMATE FUTURE LIFE EXPECTANCIES

The standard path of e_0 provides a simple rationale for projecting the future level of mortality from any given initial mortality schedule. One begins with female expectation of life at birth at the most recent date for which reliable determination of e_0 has been made. This is the value of $e_0(t_0)$ to be substituted in equation (1); $\bar{e}_0$ can be taken as 83.25 and b as -0.03099, as determined by a least squares fit to the data described earlier. A more generous estimate of the upper limit of female expectation of life can also be used, based on the apparently somewhat accelerated increase in e_0 since the mid-1970s. The mean value of e_0 is 77.28 years

Figure II. Actual male e_0 from 1950-1954 to 1980-1984 in eight countries compared to estimated e_0



and of de_0/dt .236 per year in 18 low-mortality populations when moving from the 1970s to the 1980s. If these values are combined with a value of b in equation (1) of 0.03099, they yield an $\bar{e}_0$ for females of 84.9 years.

The estimation of future values of female e_0 is illustrated by employing equation (1) to estimate the future life expectancy for females in Switzerland from e_0 in 1986. The value of e_0 at a later date (t) is calculated by the following adaptation of equation (1):

$$e_0(t) = 83.25 - (83.25 - 80.6)\exp(-0.03099(t - 1986))$$

To allow for the recent slightly accelerated increase in e_0, the asymptote 84.9 can be substituted for 83.25. When the dates 2000, 2010 and 2025 are inserted in this equation, the estimates of e_0 are 81.53, 81.99 and 82.46, when the asymptote is 83.25; and 82.11, 82.86 and 83.62, with an asymptote of 84.9. These estimated values of e_0 make possible the calculation of future age-specific mortality rates and other life-table functions by interpolation between two model tables incorporating e_0s that straddle the estimated e_0.

Because of the lesser regularity in the time-path of e_0 for males, we favour calculating the time sequence of male e_0s from the uniform time-path of female expectation of life at birth. To select male model life tables for mortality projections in a main sequence population, we suggest the following steps:

One begins with the male expectation of life listed for the same year as the point of departure for females. The level of mortality in the new male model life tables corresponding to this expectation of life is then determined. (For example, in Switzerland in 1986 expectation of life for males is listed by WHO at 73.8 years, corresponding to a level of 24.89 in the new West model life tables.) The next step is to find the female expectation of life at level 24.89—namely, 79.89 years. With this assigned initial expectation of life for females, the future female life expectancies presumed to be congruent with future male life expectancies are calculated from equation (1). This hypothetical set of future female expectations of life at birth is translated into West model levels, and the male expectation of life at each of these projected female levels is determined. When these steps are used beginning with the Swiss life tables of 1986, the projected female e_0 rises from 79.89 in 1986 to 82.75 (or 83.96) in 2040, according to an upper asymptote of 83.25 or 84.9 years. Male e_0 rises from 73.8 to 76.1 or 77.8 years, with the given estimates of the female upper asymptote. From these estimates of male and female expectation of life in the future, other mortality functions can be found from the model tables by interpolation.

Is it plausible to postulate an upper limit of female expectation of life of less than 85 years and of males less than 79 years? An ultimate increase in female life expectancy of five years or less above what has already been attained may seem a small allowance for future mortality improvements; but in fact the mortality in the listed life table with an expectation of life of 85 years is extraordinarily low. Infant mortality is less than 3 per 1,000. The proportion surviving to the mean age of child-bearing is well over 99 per cent, so that there would be no consequential difference between the net reproduction rate and the gross reproduction rate. The proportion surviving to 70 years is over 90 per cent and the expectation of life at age 70 is nearly 18 years. Annual mortality rates are well below 1 per 1,000 from age one to over age 40. It is not difficult to accept these very low risks of death as close to the ultimate. The only change that might increase expectation of life much above this presumed limit is an extension of the human life span. It remains true that even in populations with expectations of life of 80 years, the highest age of death does not surpass 110, except extremely rarely. The oldest age of death is at most one or two years greater than it was several decades ago in many low-mortality populations. To go much above 85 years in average duration of life would require, then, basic biological discoveries to overcome the incapacity of cells in the human body to sustain their viability.

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ANNEX

West tables

LEVEL 21

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	32.19	31.38	974.90	70 000	70.00
1	968.621	1.99	7.90	3 847.71	69 025	71.26
5	960.972	0.69	3.45	4 795.74	65 177	67.82
10	957.656	0.56	2.77	4 781.91	60 382	63.05
15	955.001	0.89	4.43	4 764.86	55 600	58.22
20	950.774	1.23	6.13	4 739.88	50 835	53.47
25	944.945	1.50	7.47	4 707.79	46 095	48.78
30	937.889	1.83	9.12	4 668.91	41 387	44.13
35	929.334	2.36	11.75	4 620.46	36 718	39.51
40	918.412	3.22	15.97	4 556.87	32 098	34.95
45	903.748	4.69	23.17	4 468.49	27 541	30.47
50	882.811	7.10	34.93	4 340.06	23 072	26.14
55	851.978	10.41	50.77	4 156.08	18 732	21.99
60	808.724	16.49	79.33	3 889.65	14 576	18.02
65	744.571	27.26	127.93	3 494.25	10 687	14.35
70	649.320	46.18	207.84	2 922.70	7 192	11.08
75	514.362	78.03	326.47	2 152.00	4 270	8.30
80	346.439	125.59	473.61	1 306.40	2 118	6.11
85	182.363	194.11	634.20	595.83	811	4.45
90	66.708	288.06	782.29	181.16	215	3.23
95	14.523	410.50	892.70	31.58	34	2.36
100	1.558	567.78	1 000.00	2.74	3	1.76

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	43.03	41.60	966.72	66 009	66.01
1	958.398	2.47	9.80	3 800.72	65 042	67.87
5	949.007	0.97	4.85	4 732.37	61 241	64.53
10	944.403	0.78	3.89	4 713.20	56 509	59.84
15	940.728	1.38	6.88	4 688.11	51 796	55.06
20	934.257	1.87	9.32	4 650.40	47 108	50.42
25	925.553	1.92	9.57	4 606.51	42 457	45.87
30	916.695	2.23	11.08	4 559.10	37 851	41.29
35	906.537	2.87	14.24	4 501.71	33 292	36.72
40	893.629	4.11	20.35	4 424.51	28 790	32.22
45	875.447	6.32	31.13	4 311.83	24 365	27.83
50	848.193	10.04	49.01	4 141.20	20 054	23.64
55	806.622	15.46	74.51	3 888.87	15 912	19.73
60	746.520	24.03	113.60	3 529.06	12 024	16.11
65	661.714	37.32	171.25	3 036.60	8 494	12.84
70	548.394	58.78	257.57	2 402.97	5 458	9.95
75	407.143	93.40	378.61	1 650.35	3 055	7.50
80	252.995	145.56	527.75	917.25	1 405	5.55
85	119.477	218.33	682.12	373.28	487	4.08
90	37.980	315.16	817.46	98.51	114	3.00
95	6.933	437.86	913.62	14.47	15	2.23
100	0.599	590.53	1 000.00	1.01	1	1.69

LEVEL 22

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	23.24	22.82	981.74	72 500	72.50
1	977.181	1.18	4.69	3 892.67	71 518	73.19
5	972.595	0.45	2.23	4 857.01	67 625	69.53
10	970.427	0.36	1.82	4 847.90	62 768	64.68
15	968.664	0.59	2.96	4 836.43	57 920	59.79
20	965.794	0.83	4.16	4 819.33	53 084	54.96
25	961.775	1.04	5.16	4 796.96	48 265	50.18
30	956.809	1.29	6.45	4 769.23	43 468	45.43
35	950.636	1.73	8.64	4 733.48	38 698	40.71
40	942.427	2.48	12.33	4 684.24	33 965	36.04
45	930.802	3.82	18.92	4 611.74	29 281	31.46
50	913.191	5.81	28.67	4 503.11	24 669	27.01
55	887.008	8.92	43.68	4 342.06	20 166	22.73
60	848.267	14.34	69.29	4 100.26	15 824	18.65
65	789.488	24.40	115.24	3 729.09	11 724	14.85
70	698.509	42.22	191.66	3 171.24	7 995	11.45
75	564.633	72.87	308.19	2 388.13	4 823	8.54
80	390.620	120.77	459.74	1 486.94	2 435	6.23
85	211.037	190.75	626.71	693.35	948	4.49
90	78.777	287.10	779.84	213.98	255	3.24
95	17.344	411.76	891.84	37.57	41	2.36
100	1.876	568.62	1 000.00	3.30	3	1.76

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	31.80	31.01	975.19	68 565	68.57
1	968.991	1.55	6.18	3 855.00	67 590	69.75
5	963.002	0.69	3.45	4 805.87	63 735	66.18
10	959.678	0.57	2.84	4 791.85	58 929	61.41
15	956.953	1.06	5.27	4 772.67	54 137	56.57
20	951.912	1.47	7.35	4 742.78	49 365	51.86
25	944.920	1.48	7.39	4 707.85	44 622	47.22
30	937.941	1.68	8.35	4 670.91	39 914	42.55
35	930.111	2.16	10.77	4 626.52	35 243	37.89
40	920.097	3.21	15.91	4 565.36	30 617	33.28
45	905.460	5.18	25.59	4 471.68	26 051	28.77
50	882.286	8.36	40.98	4 324.65	21 580	24.46
55	846.128	13.62	65.96	4 096.68	17 255	20.39
60	790.313	21.56	102.50	3 757.15	13 158	16.65
65	709.307	34.23	158.15	3 277.30	9 401	13.25
70	597.128	54.80	242.13	2 638.63	6 124	10.26
75	452.542	88.09	360.97	1 854.32	3 485	7.70
80	289.186	141.18	516.04	1 057.00	1 631	5.64
85	139.954	216.08	677.16	438.58	574	4.10
90	45.183	315.83	816.76	116.85	135	2.99
95	8.279	440.85	913.43	17.15	18	2.22
100	0.717	592.49	1 000.00	1.21	1	1.69

LEVEL 23

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	15.52	15.33	987.74	75 000	75.00
1	984.671	0.79	3.16	3 927.81	74 013	75.16
5	981.564	0.34	1.70	4 903.24	70 085	71.40
10	979.898	0.22	1.11	4 896.87	65 182	66.52
15	978.806	0.40	1.99	4 889.36	60 285	61.59
20	976.859	0.55	2.75	4 877.85	55 395	56.71
25	974.173	0.67	3.34	4 863.05	50 518	51.86
30	970.916	0.86	4.29	4 844.58	45 654	47.02
35	966.751	1.29	6.41	4 818.88	40 810	42.21
40	960.554	1.91	9.53	4 780.81	35 991	37.47
45	951.402	3.11	15.46	4 721.72	31 210	32.80
50	936.697	4.74	23.43	4 630.80	26 488	28.28
55	914.746	7.33	36.03	4 494.63	21 858	23.89
60	881.789	11.82	57.48	4 287.30	17 363	19.69
65	831.103	20.08	95.79	3 964.45	13 076	15.73
70	751.493	36.02	165.78	3 458.47	9 111	12.12
75	626.912	64.13	276.33	2 701.47	5 653	9.02
80	453.677	111.35	431.97	1 759.94	2 951	6.51
85	257.702	182.26	608.05	859.75	1 191	4.62
90	101.006	281.18	770.36	276.72	332	3.28
95	23.195	408.90	887.29	50.33	55	2.37
100	2.614	566.27	1 000.00	4.62	5	1.77

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	22.05	21.67	982.66	70 955	70.96
1	978.329	1.12	4.48	3 897.97	69 972	71.52
5	973.945	0.60	2.97	4 861.77	66 074	67.84
10	971.051	0.40	2.00	4 850.59	61 213	63.04
15	969.109	0.90	4.49	4 835.10	56 362	58.16
20	964.757	1.30	6.48	4 808.78	51 527	53.41
25	958.506	1.30	6.48	4 777.62	46 718	48.74
30	952.295	1.38	6.88	4 745.76	41 941	44.04
35	945.746	1.80	8.96	4 708.39	37 195	39.33
40	937.270	2.51	12.48	4 658.27	32 486	34.66
45	925.570	4.26	21.09	4 581.01	27 828	30.07
50	906.053	6.96	34.24	4 455.80	23 247	25.66
55	875.026	11.60	56.43	4 256.63	18 791	21.48
60	825.649	18.54	88.74	3 952.40	14 535	17.60
65	752.381	29.02	135.64	3 516.98	10 582	14.07
70	650.328	47.87	214.70	2 916.55	7 065	10.86
75	510.705	78.43	327.85	2 134.94	4 149	8.12
80	343.269	131.51	489.70	1 278.25	2 014	5.87
85	175.169	208.24	661.22	556.21	736	4.20
90	59.343	311.40	809.42	154.25	179	3.02
95	11.310	439.73	909.63	23.40	25	2.22
100	1.022	591.06	1 000.00	1.73	2	1.69

LEVEL 24

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	9.53	9.46	992.44	77 500	77.50
1	990.544	0.56	2.23	3 954.46	76 507	77.24
5	988.338	0.27	1.35	4 938.01	72 553	73.41
10	986.999	0.18	0.90	4 932.88	67 615	68.51
15	986.116	0.35	1.76	4 926.42	62 682	63.56
20	984.384	0.47	2.33	4 916.42	57 755	58.67
25	982.094	0.52	2.61	4 904.31	52 839	53.80
30	979.528	0.68	3.42	4 889.61	47 935	48.94
35	976.180	1.06	5.26	4 868.57	43 045	44.10
40	971.042	1.56	7.76	4 837.13	38 177	39.32
45	963.507	2.54	12.63	4 788.33	33 339	34.60
50	951.340	3.86	19.13	4 713.01	28 551	30.01
55	933.136	5.91	29.13	4 600.44	23 838	25.55
60	905.952	9.45	46.21	4 429.28	19 238	21.23
65	864.085	15.83	76.28	4 162.24	14 808	17.14
70	798.176	28.48	133.30	3 735.54	10 646	13.34
75	691.783	51.93	229.80	3 061.49	6 911	9.99
80	532.814	93.14	375.41	2 147.52	3 849	7.22
85	332.789	157.47	551.80	1 166.10	1 702	5.11
90	149.157	250.94	726.42	431.78	536	3.59
95	40.806	376.90	861.60	93.28	104	2.54
100	5.648	540.84	1 000.00	10.44	10	1.85

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	13.21	13.07	989.54	72 152	72.15
1	986.928	0.78	3.13	3 936.90	71 162	72.10
5	983.838	0.48	2.42	4 912.64	67 225	68.33
10	981.457	0.34	1.70	4 903.28	62 313	63.49
15	979.788	0.82	4.09	4 889.32	57 409	58.59
20	975.779	1.10	5.49	4 866.05	52 520	53.82
25	970.426	1.20	5.98	4 838.20	47 654	49.11
30	964.620	1.21	6.04	4 809.12	42 816	44.39
35	958.794	1.68	8.38	4 774.69	38 007	39.64
40	950.761	2.48	12.34	4 725.64	33 232	34.95
45	939.025	4.17	20.62	4 648.65	28 506	30.36
50	919.661	6.86	33.74	4 523.84	23 858	25.94
55	888.634	11.18	54.44	4 327.07	19 334	21.76
60	840.257	17.89	85.78	4 028.29	15 007	17.86
65	768.178	28.53	133.53	3 594.72	10 979	14.29
70	665.606	46.75	210.18	2 992.28	7 384	11.09
75	525.712	76.73	321.90	2 205.50	4 392	8.35
80	356.488	124.62	470.81	1 346.77	2 186	6.13
85	188.650	193.85	633.49	616.49	839	4.45
90	69.143	288.79	782.86	187.43	223	3.22
95	15.014	412.02	893.23	32.55	35	2.36
100	1.603	568.95	1 000.00	2.82	3	1.76

LEVEL 25

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	6.04	6.01	995.19	80 000	80.00
1	993.989	0.44	1.76	3 969.85	79 005	79.48
5	992.244	0.18	0.90	4 958.76	75 035	75.62
10	991.350	0.15	0.75	4 954.96	70 077	70.69
15	990.605	0.30	1.50	4 949.45	65 122	65.74
20	989.117	0.36	1.79	4 941.32	60 172	60.83
25	987.341	0.44	2.22	4 931.44	55 231	55.94
30	985.147	0.55	2.77	4 919.19	50 299	51.06
35	982.422	0.80	4.01	4 902.65	45 380	46.19
40	978.480	1.25	6.24	4 877.75	40 478	41.37
45	972.374	1.97	9.80	4 839.00	35 600	36.61
50	962.845	3.06	15.19	4 779.12	30 761	31.95
55	948.220	4.60	22.73	4 689.36	25 982	27.40
60	926.664	7.28	35.79	4 553.72	21 292	22.98
65	893.498	11.90	57.86	4 343.41	16 739	18.73
70	841.797	21.45	102.02	4 002.88	12 395	14.72
75	755.921	40.57	184.17	3 431.56	8 392	11.10
80	616.702	76.10	318.23	2 578.93	4 961	8.04
85	420.447	134.21	493.17	1 545.03	2 382	5.67
90	213.096	222.52	679.55	650.76	837	3.93
95	68.287	346.88	833.97	164.17	186	2.73
100	11.338	517.35	1 000.00	21.92	22	1.93

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	7.72	7.67	993.86	73 880	73.88
1	992.327	0.55	2.18	3 961.72	72 886	73.45
5	990.159	0.27	1.37	4 947.07	68 924	69.61
10	988.806	0.25	1.24	4 941.09	63 977	64.70
15	987.582	0.77	3.82	4 928.85	59 036	59.78
20	983.807	1.04	5.18	4 906.81	54 107	55.00
25	978.712	1.14	5.68	4 880.21	49 200	50.27
30	973.149	1.13	5.64	4 852.57	44 320	45.54
35	967.658	1.48	7.37	4 821.18	39 468	40.79
40	960.531	2.32	11.52	4 776.10	34 646	36.07
45	949.468	3.73	18.50	4 705.18	29 870	31.46
50	931.900	6.25	30.76	4 590.69	25 165	27.00
55	903.231	9.78	47.75	4 412.63	20 575	22.78
60	860.097	15.52	74.82	4 146.03	16 162	18.79
65	795.741	25.01	117.98	3 753.39	12 016	15.10
70	701.859	41.05	186.83	3 194.59	8 262	11.77
75	570.731	69.52	296.13	2 431.12	5 068	8.88
80	401.719	112.71	436.36	1 555.21	2 637	6.56
85	226.424	176.18	596.00	765.96	1 082	4.78
90	91.475	265.50	750.92	258.73	316	3.45
95	22.784	385.72	873.37	51.59	57	2.50
100	2.885	547.55	1 000.00	5.27	5	1.83

LEVEL 26

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	4.05	4.03	996.77	82 503	82.50
1	995.967	0.37	1.46	3 978.78	81 506	81.84
5	994.513	0.09	0.46	4 971.30	77 527	77.95
10	994.054	0.08	0.38	4 969.35	72 556	72.99
15	993.671	0.20	1.01	4 965.95	67 587	68.02
20	992.668	0.29	1.44	4 959.90	62 621	63.08
25	991.235	0.37	1.84	4 951.80	57 661	58.17
30	989.410	0.44	2.19	4 941.84	52 709	53.27
35	987.240	0.61	3.07	4 928.94	47 767	48.38
40	984.213	0.96	4.79	4 909.76	42 838	43.53
45	979.502	1.50	7.46	4 879.97	37 928	38.72
50	972.193	2.38	11.85	4 833.31	33 048	33.99
55	960.670	3.43	17.00	4 764.15	28 215	29.37
60	944.337	5.34	26.35	4 661.95	23 451	24.83
65	919.449	8.42	41.26	4 506.20	18 789	20.44
70	881.515	15.26	73.62	4 251.82	14 283	16.20
75	816.618	30.55	141.93	3 793.33	10 031	12.28
80	700.715	61.15	264.34	3 029.17	6 238	8.90
85	515.487	114.14	437.51	1 975.91	3 208	6.22
90	289.956	198.72	635.45	927.18	1 233	4.25
95	105.703	322.71	808.66	264.87	305	2.89
100	20.225	499.20	1 000.00	40.51	41	2.00

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	5.13	5.10	995.92	76 187	76.19
1	994.896	0.45	1.78	3 973.37	75 191	75.58
5	993.121	0.14	0.72	4 963.65	71 217	71.71
10	992.409	0.13	0.66	4 960.47	66 254	66.76
15	991.754	0.55	2.74	4 952.24	61 293	61.80
20	989.032	0.90	4.49	4 934.50	56 341	56.97
25	984.590	1.02	5.09	4 910.91	51 406	52.21
30	979.574	0.97	4.83	4 886.51	46 495	47.47
35	974.840	1.20	5.99	4 860.19	41 609	42.68
40	969.001	1.87	9.32	4 823.33	36 749	37.92
45	959.970	3.00	14.87	4 765.58	31 925	33.26
50	945.691	5.15	25.43	4 670.75	27 160	28.72
55	921.646	7.68	37.72	4 524.79	22 489	24.40
60	886.880	12.24	59.47	4 307.83	17 964	20.26
65	834.141	19.63	93.75	3 983.03	13 657	16.37
70	755.943	32.56	151.00	3 505.75	9 673	12.80
75	641.793	58.34	254.58	2 800.49	6 168	9.61
80	478.405	99.28	395.12	1 903.98	3 367	7.04
85	289.379	161.46	561.82	1 006.92	1 463	5.06
90	126.800	250.97	727.82	367.73	456	3.60
95	34.512	372.84	860.42	79.65	89	2.57
100	4.817	537.15	1 000.00	8.97	9	1.86

Level 27

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	2.84	2.83	997.73	84 997	85.00
1	997.169	0.30	1.20	3 984.50	83 999	84.24
5	995.975	0.03	0.15	4 979.48	80 015	80.34
10	995.830	0.03	0.13	4 978.84	75 035	75.35
15	995.702	0.11	0.57	4 977.14	70 056	70.36
20	995.131	0.23	1.13	4 972.95	65 079	65.40
25	994.005	0.30	1.50	4 966.44	60 106	60.47
30	992.510	0.34	1.69	4 958.54	55 140	55.56
35	990.837	0.45	2.23	4 948.89	50 181	50.65
40	988.632	0.70	3.50	4 934.86	45 232	45.75
45	985.174	1.08	5.39	4 913.13	40 297	40.90
50	979.868	1.90	9.47	4 877.08	35 384	36.11
55	970.592	2.39	11.90	4 825.24	30 507	31.43
60	959.043	3.62	17.92	4 753.96	25 682	26.78
65	941.853	5.33	26.32	4 649.78	20 928	22.22
70	917.066	9.78	47.78	4 480.18	16 278	17.75
75	873.253	21.68	102.83	4 141.76	11 798	13.51
80	783.452	47.90	213.42	3 490.49	7 656	9.77
85	616.249	97.00	385.79	2 451.08	4 166	6.76
90	378.507	180.00	597.21	1 255.84	1 715	4.53
95	152.458	306.13	789.21	393.04	459	3.01
100	32.137	488.43	1 000.00	65.80	66	2.05

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	3.57	3.56	997.15	78 979	78.98
1	996.436	0.37	1.46	3 980.64	77 982	78.26
5	994.977	0.05	0.23	4 974.26	74 002	74.38
10	994.749	0.05	0.23	4 973.20	69 027	69.39
15	994.523	0.33	1.64	4 968.69	64 054	64.41
20	992.889	0.75	3.75	4 955.50	59 085	59.51
25	989.163	0.90	4.51	4 935.12	54 130	54.72
30	984.707	0.80	3.98	4 914.13	49 195	49.96
35	980.789	0.93	4.63	4 893.04	44 281	45.15
40	976.244	1.45	7.22	4 864.29	39 388	40.35
45	969.191	2.28	11.33	4 819.60	34 523	35.62
50	958.208	4.32	21.39	4 741.85	29 704	31.00
55	937.714	5.62	27.74	4 626.13	24 962	26.62
60	911.698	8.78	43.02	4 464.36	20 336	22.31
65	872.479	13.37	64.78	4 226.75	15 871	18.19
70	815.961	22.48	106.65	3 870.96	11 645	14.27
75	728.943	44.49	200.18	3 279.90	7 774	10.66
80	583.019	82.42	339.93	2 404.60	4 494	7.71
85	384.833	143.44	517.09	1 387.30	2 089	5.43
90	185.840	234.51	699.87	554.61	702	3.78
95	55.776	360.19	846.35	131.06	147	2.64
100	8.570	527.85	1 000.00	16.24	16	1.89

LEVEL 21

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	40.46	39.20	968.64	70 000	70.00
1	960.805	1.98	7.88	3 816.73	69 032	71.85
5	953.235	0.59	2.94	4 758.48	65 215	68.41
10	950.435	0.46	2.30	4 746.92	60 456	63.61
15	948.245	0.72	3.61	4 733.00	55 709	58.75
20	944.817	1.02	5.07	4 712.60	50 976	53.95
25	940.030	1.21	6.02	4 686.56	46 264	49.22
30	934.369	1.49	7.42	4 655.21	41 577	44.50
35	927.437	1.98	9.83	4 615.30	36 922	39.81
40	918.319	2.69	13.38	4 562.11	32 307	35.18
45	906.033	4.07	20.15	4 486.34	27 745	30.62
50	887.774	6.04	29.75	4 375.49	23 258	26.20
55	861.366	9.19	44.94	4 213.93	18 883	21.92
60	822.656	15.17	73.21	3 968.75	14 669	17.83
65	762.433	26.39	124.08	3 585.11	10 700	14.03
70	667.827	47.27	212.27	2 998.92	7 115	10.65
75	526.069	82.81	343.02	2 179.22	4 116	7.82
80	345.618	140.07	512.61	1 264.88	1 937	5.60
85	168.449	222.24	686.96	520.68	672	3.99
90	52.731	330.77	830.68	132.43	151	2.87
95	8.929	461.78	921.92	17.83	19	2.12
100	0.697	608.61	1 000.00	1.15	1	1.64

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	51.27	49.25	960.60	65 415	65.42
1	950.752	2.29	9.07	3 772.81	64 455	67.79
5	942.125	0.81	4.02	4 700.20	60 682	64.41
10	938.335	0.70	3.51	4 683.76	55 982	59.66
15	935.038	1.35	6.72	4 660.11	51 298	54.86
20	928.754	1.94	9.66	4 622.24	46 638	50.22
25	919.785	1.97	9.81	4 577.27	42 016	45.68
30	910.761	2.17	10.78	4 530.25	37 438	41.11
35	900.946	2.71	13.47	4 475.60	32 908	36.53
40	888.807	3.86	19.13	4 403.22	28 433	31.99
45	871.803	6.21	30.60	4 294.99	24 029	27.56
50	845.127	10.10	49.32	4 125.60	19 734	23.35
55	803.445	15.95	76.80	3 869.14	15 609	19.43
60	741.743	24.43	115.39	3 503.30	11 740	15.83
65	656.155	37.87	173.56	3 007.45	8 236	12.55
70	542.270	60.93	265.79	2 365.44	5 229	9.64
75	398.141	98.21	394.24	1 598.30	2 863	7.19
80	241.178	161.22	566.33	847.21	1 265	5.25
85	104.592	222.24	689.88	324.67	418	4.00
90	32.436	330.77	834.95	81.88	93	2.88
95	5.353	461.78	927.18	10.75	11	2.13
100	0.390	608.61	1 000.00	0.64	1	1.64

LEVEL 22

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	26.85	26.29	978.97	72 500	72.50
1	973.712	1.36	5.40	3 876.45	71 521	73.45
5	968.456	0.42	2.11	4 836.67	67 645	69.85
10	966.415	0.34	1.71	4 828.11	62 808	64.99
15	964.763	0.54	2.70	4 817.57	57 980	60.10
20	962.161	0.77	3.84	4 801.94	53 163	55.25
25	958.468	0.92	4.60	4 781.75	48 361	50.46
30	954.058	1.17	5.82	4 756.97	43 579	45.68
35	948.508	1.60	7.96	4 724.43	38 822	40.93
40	940.962	2.26	11.22	4 679.47	34 098	36.24
45	930.402	3.58	17.73	4 612.43	29 418	31.62
50	913.910	5.33	26.30	4 511.86	24 806	27.14
55	889.872	8.16	40.01	4 363.91	20 294	22.81
60	854.267	13.61	65.90	4 136.22	15 930	18.65
65	797.969	23.50	111.22	3 776.85	11 794	14.78
70	709.223	42.09	191.14	3 220.77	8 017	11.30
75	573.664	73.94	312.03	2 420.81	4 796	8.36
80	394.662	126.01	474.37	1 485.78	2 375	6.02
85	207.446	202.18	649.13	666.04	889	4.29
90	72.786	305.45	801.68	191.04	223	3.07
95	14.435	434.50	905.37	30.08	32	2.24
100	1.366	586.77	1 000.00	2.33	2	1.70

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	34.37	33.45	973.24	67 789	67.79
1	966.548	1.60	6.37	3 844.63	66 815	69.13
5	960.387	0.62	3.08	4 793.79	62 971	65.57
10	957.425	0.58	2.91	4 780.43	58 177	60.76
15	954.636	1.14	5.68	4 760.17	53 396	55.93
20	949.214	1.64	8.17	4 727.46	48 636	51.24
25	941.458	1.67	8.34	4 688.45	43 909	46.64
30	933.606	1.84	9.18	4 647.47	39 220	42.01
35	925.040	2.31	11.49	4 599.69	34 573	37.37
40	914.409	3.34	16.58	4 535.66	29 973	32.78
45	899.250	5.49	27.10	4 437.76	25 438	28.29
50	874.878	9.31	45.55	4 278.76	21 000	24.00
55	835.031	15.07	72.72	4 029.42	16 721	20.02
60	774.309	23.32	110.40	3 666.38	12 692	16.39
65	688.825	35.43	163.28	3 174.18	9 025	13.10
70	576.350	56.19	247.56	2 539.32	5 851	10.15
75	433.669	89.77	366.57	1 770.92	3 312	7.64
80	274.699	147.93	533.26	990.24	1 541	5.61
85	128.213	202.18	651.81	413.35	551	4.29
90	44.642	305.45	805.77	117.77	137	3.07
95	8.671	434.50	910.59	18.17	19	2.25
100	0.775	586.77	1 000.00	1.32	1	1.70

Level 23

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	15.48	15.29	987.77	75 000	75.00
1	984.707	0.94	3.73	3 925.98	74 012	75.16
5	981.035	0.34	1.69	4 900.62	70 086	71.44
10	979.380	0.25	1.27	4 893.92	65 185	66.56
15	978.139	0.41	2.02	4 885.94	60 291	61.64
20	976.159	0.59	2.93	4 873.93	55 405	56.76
25	973.297	0.70	3.49	4 858.33	50 532	51.92
30	969.900	0.90	4.50	4 839.03	45 673	47.09
35	965.538	1.28	6.38	4 812.91	40 834	42.29
40	959.382	1.90	9.48	4 775.09	36 021	37.55
45	950.290	3.10	15.37	4 716.39	31 246	32.88
50	935.680	4.62	22.86	4 627.06	26 530	28.35
55	914.286	7.13	35.05	4 494.53	21 903	23.96
60	882.243	11.75	57.15	4 290.19	17 408	19.73
65	831.819	19.96	95.22	3 968.99	13 118	15.77
70	752.609	35.80	164.85	3 465.29	9 149	12.16
75	628.545	63.77	275.01	2 710.59	5 684	9.04
80	455.690	110.82	430.38	1 769.69	2 973	6.52
85	259.569	181.54	606.50	867.20	1 203	4.64
90	102.142	280.30	769.16	280.28	336	3.29
95	23.579	407.97	886.59	51.24	56	2.37
100	2.674	565.52	1 000.00	4.73	5	1.77

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	20.05	19.73	984.21	70 069	70.07
1	980.267	1.14	4.55	3 905.47	69 085	70.48
5	975.810	0.54	2.68	4 871.86	65 180	66.80
10	973.194	0.50	2.47	4 860.20	60 308	61.97
15	970.788	1.00	4.97	4 842.36	55 448	57.12
20	965.962	1.43	7.15	4 813.24	50 605	52.39
25	959.057	1.45	7.22	4 778.68	45 792	47.75
30	952.137	1.58	7.86	4 742.73	41 013	43.08
35	944.655	1.97	9.82	4 701.01	36 271	38.40
40	935.377	2.93	14.57	4 644.18	31 570	33.75
45	921.751	4.99	24.64	4 554.24	26 925	29.21
50	899.038	8.53	41.78	4 405.05	22 371	24.88
55	861.480	14.20	68.67	4 165.42	17 966	20.85
60	802.323	21.73	103.27	3 812.75	13 801	17.20
65	719.465	31.97	148.47	3 340.97	9 988	13.88
70	612.649	49.91	222.85	2 735.58	6 647	10.85
75	476.122	79.65	332.11	1 985.29	3 911	8.22
80	317.995	133.32	494.65	1 179.85	1 926	6.06
85	160.699	181.54	608.84	538.96	746	4.64
90	62.859	280.30	772.93	173.33	207	3.30
95	14.273	407.97	891.61	31.19	34	2.38
100	1.547	565.52	1 000.00	2.74	3	1.77

LEVEL 20

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	65.80	62.51	950.00	67 500	67.50
1	937.494	5.36	21.05	3 680.92	66 550	70.99
5	917.763	0.86	4.28	4 578.02	62 869	68.50
10	913.838	0.59	2.96	4 562.71	58 291	63.79
15	911.136	0.87	4.35	4 546.18	53 728	58.97
20	907.176	1.21	6.05	4 522.72	49 182	54.21
25	901.691	1.42	7.06	4 493.19	44 660	49.53
30	895.330	1.68	8.34	4 458.72	40 166	44.86
35	887.861	2.08	10.36	4 417.24	35 708	40.22
40	878.667	2.80	13.91	4 363.99	31 290	35.61
45	866.441	3.84	19.00	4 292.69	26 926	31.08
50	849.976	5.74	28.32	4 192.11	22 634	26.63
55	825.907	8.46	41.47	4 047.34	18 442	22.33
60	791.660	13.96	67.55	3 829.96	14 394	18.18
65	738.185	24.00	113.45	3 489.93	10 564	14.31
70	654.437	43.95	198.79	2 959.96	7 074	10.81
75	524.341	79.70	332.28	2 186.13	4 114	7.85
80	350.112	141.49	515.94	1 276.64	1 928	5.51
85	169.476	231.64	702.16	513.72	652	3.84
90	50.477	349.70	847.51	122.33	138	2.73
95	7.697	486.80	931.37	14.73	16	2.02
100	0.528	627.88	1 000.00	0.84	1	1.59

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	74.06	69.91	944.07	63 695	63.69
1	930.086	5.51	21.64	3 649.90	62 751	67.47
5	909.960	1.03	5.15	4 536.92	59 101	64.95
10	905.275	0.78	3.91	4 517.88	54 564	60.27
15	901.737	1.16	5.79	4 496.15	50 046	55.50
20	896.514	1.65	8.21	4 464.90	45 550	50.81
25	889.151	1.75	8.73	4 427.13	41 085	46.21
30	881.390	2.25	11.20	4 383.25	36 658	41.59
35	871.515	2.78	13.82	4 328.67	32 275	37.03
40	859.472	4.04	19.99	4 256.12	27 946	32.52
45	842.288	6.04	29.75	4 151.30	23 690	28.13
50	817.229	9.32	45.60	3 996.72	19 538	23.91
55	779.967	14.31	69.19	3 770.32	15 542	19.93
60	726.003	21.78	103.49	3 449.69	11 771	16.21
65	650.867	33.79	156.28	3 010.21	8 322	12.79
70	549.147	56.51	248.81	2 417.82	5 311	9.67
75	412.514	97.49	391.93	1 658.38	2 894	7.01
80	250.838	167.16	579.94	870.24	1 235	4.92
85	105.367	264.85	756.38	300.91	365	3.46
90	25.669	387.76	883.48	58.48	64	2.50
95	2.991	524.58	950.49	5.42	6	1.89
100	0.148	658.05	1 000.00	0.23	0	1.52

LEVEL 21

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	50.83	48.85	960.92	70 000	70.00
1	951.154	3.67	14.50	3 756.34	69 039	72.58
5	937.360	0.63	3.16	4 678.66	65 283	69.65
10	934.401	0.44	2.20	4 667.06	60 604	64.86
15	932.342	0.67	3.32	4 654.28	55 937	60.00
20	929.246	0.93	4.66	4 635.83	51 283	55.19
25	924.913	1.09	5.44	4 612.49	46 647	50.43
30	919.881	1.33	6.62	4 584.79	42 034	45.70
35	913.792	1.73	8.61	4 550.08	37 450	40.98
40	905.924	2.41	11.98	4 503.57	32 899	36.32
45	895.069	3.46	17.15	4 438.50	28 396	31.72
50	879.719	5.24	25.86	4 344.00	23 957	27.23
55	856.971	7.82	38.39	4 205.89	19 613	22.89
60	824.069	12.87	62.40	3 996.93	15 407	18.70
65	772.645	22.19	105.35	3 667.87	11 411	14.77
70	691.246	40.78	185.72	3 148.13	7 743	11.20
75	562.871	74.27	313.19	2 373.64	4 595	8.16
80	386.584	132.79	492.72	1 434.43	2 221	5.75
85	196.106	219.24	680.16	608.40	787	4.01
90	62.723	334.24	831.70	156.07	178	2.84
95	10.556	470.54	922.89	20.70	22	2.09
100	0.814	615.08	1 000.00	1.32	1	1.63

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	57.29	54.78	956.18	66 053	66.05
1	945.222	3.79	14.97	3 731.35	65 097	68.87
5	931.068	0.80	3.97	4 645.17	61 366	65.91
10	927.369	0.63	3.14	4 629.85	56 721	61.16
15	924.454	0.95	4.71	4 611.81	52 091	56.35
20	920.095	1.32	6.56	4 585.99	47 479	51.60
25	914.060	1.40	6.96	4 555.03	42 893	46.93
30	907.699	1.86	9.24	4 518.37	38 338	42.24
35	899.316	2.40	11.94	4 470.81	33 820	37.61
40	888.578	3.56	17.64	4 405.28	29 349	33.03
45	872.905	5.59	27.57	4 306.76	24 943	28.58
50	848.837	8.76	42.88	4 156.83	20 637	24.31
55	812.440	13.77	66.64	3 932.26	16 480	20.28
60	758.300	20.91	99.54	3 610.35	12 548	16.55
65	682.819	32.31	149.93	3 168.39	8 937	13.09
70	580.441	53.79	238.19	2 570.40	5 769	9.94
75	442.188	92.69	376.25	1 795.00	3 198	7.23
80	275.814	159.61	561.86	970.90	1 403	5.09
85	120.845	254.29	740.12	351.72	433	3.58
90	31.405	374.82	872.26	73.08	81	2.57
95	4.012	511.13	944.58	7.41	8	1.93
100	0.222	647.39	1 000.00	0.34	0	1.54

Level 22

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	36.70	35.65	971.48	72 500	72.50
1	964.346	2.38	9.43	3 825.54	71 528	74.17
5	955.249	0.46	2.32	4 770.15	67 703	70.87
10	953.032	0.33	1.62	4 761.44	62 933	66.03
15	951.484	0.51	2.56	4 751.57	58 171	61.14
20	949.047	0.72	3.60	4 737.04	53 420	56.29
25	945.632	0.84	4.18	4 718.66	48 683	51.48
30	941.675	1.05	5.23	4 696.55	43 964	46.69
35	936.749	1.43	7.14	4 667.69	39 267	41.92
40	930.057	2.06	10.24	4 627.43	34 600	37.20
45	920.535	3.08	15.30	4 568.88	29 972	32.56
50	906.453	4.70	23.26	4 481.66	25 403	28.03
55	885.370	7.10	34.89	4 352.70	20 922	23.63
60	854.476	11.62	56.50	4 156.51	16 569	19.39
65	806.198	20.02	95.50	3 846.22	12 412	15.40
70	729.209	36.82	169.14	3 350.03	8 566	11.75
75	605.868	67.42	288.46	2 592.42	5 216	8.61
80	431.098	121.55	461.44	1 636.64	2 624	6.09
85	232.171	202.86	649.32	743.13	987	4.25
90	81.418	313.42	808.72	210.08	244	3.00
95	15.573	448.27	910.33	31.63	34	2.18
100	1.396	597.57	1 000.00	2.34	2	1.67

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	41.44	40.11	967.91	68 238	68.24
1	959.894	2.47	9.79	3 806.69	67 270	70.08
5	950.498	0.61	3.06	4 744.50	63 464	66.77
10	947.592	0.50	2.52	4 732.23	58 719	61.97
15	945.207	0.92	4.59	4 715.62	53 987	57.12
20	940.868	1.20	5.98	4 690.83	49 271	52.37
25	935.239	1.35	6.73	4 661.10	44 580	47.67
30	928.947	1.64	8.17	4 626.52	39 919	42.97
35	921.359	2.17	10.79	4 582.93	35 293	38.31
40	911.415	3.12	15.50	4 523.16	30 710	33.69
45	897.285	5.12	25.30	4 431.94	26 187	29.18
50	874.582	8.15	39.96	4 289.04	21 755	24.87
55	839.636	13.07	63.38	4 070.46	17 466	20.80
60	786.420	19.78	94.42	3 753.89	13 395	17.03
65	712.166	30.31	141.25	3 319.40	9 641	13.54
70	611.570	50.04	223.36	2 730.02	6 322	10.34
75	474.972	86.09	354.21	1 954.26	3 592	7.56
80	306.730	149.02	535.53	1 102.32	1 638	5.34
85	142.467	239.18	715.60	426.25	535	3.76
90	40.518	355.98	854.86	97.30	109	2.69
95	5.881	491.29	935.30	11.20	12	2.01
100	0.380	631.69	1 000.00	0.60	1	1.58

LEVEL 23

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	24.11	23.65	981.08	75 000	75.00
1	976.348	1.42	5.67	3 886.02	74 019	75.81
5	970.812	0.34	1.72	4 849.46	70 133	72.24
10	969.140	0.24	1.20	4 842.92	65 283	67.36
15	967.981	0.41	2.04	4 835.17	60 440	62.44
20	966.009	0.56	2.82	4 823.51	55 605	57.56
25	963.288	0.65	3.25	4 808.93	50 782	52.72
30	960.157	0.83	4.15	4 791.22	45 973	47.88
35	956.173	1.19	5.94	4 767.24	41 182	43.07
40	950.495	1.74	8.68	4 732.68	36 414	38.31
45	942.246	2.71	13.47	4 680.77	31 682	33.62
50	929.554	4.15	20.56	4 601.91	27 001	29.05
55	910.445	6.30	31.03	4 484.42	22 399	24.60
60	882.191	10.23	49.94	4 305.22	17 915	20.31
65	838.136	17.51	84.03	4 021.66	13 609	16.24
70	767.710	32.15	149.22	3 563.61	9 588	12.49
75	653.151	59.26	258.08	2 844.33	6 024	9.22
80	484.583	107.92	421.46	1 892.46	3 180	6.56
85	280.349	182.63	608.10	933.49	1 287	4.59
90	109.870	287.19	776.66	297.12	354	3.22
95	24.539	419.69	892.29	52.17	57	2.31
100	2.643	575.05	1 000.00	4.60	5	1.74

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	24.10	23.64	981.09	70 334	70.33
1	976.357	1.32	5.24	3 887.52	69 353	71.03
5	971.240	0.45	2.26	4 850.17	65 466	67.40
10	969.049	0.36	1.80	4 841.06	60 615	62.55
15	967.307	0.90	4.49	4 826.11	55 774	57.66
20	962.964	1.18	5.88	4 801.22	50 948	52.91
25	957.298	1.18	5.87	4 772.99	46 147	48.21
30	951.674	1.37	6.85	4 742.72	41 374	43.47
35	945.153	1.87	9.31	4 704.66	36 631	38.76
40	936.358	2.80	13.90	4 650.55	31 927	34.10
45	923.341	4.66	23.03	4 565.67	27 276	29.54
50	902.075	7.59	37.26	4 429.71	22 710	25.18
55	868.464	12.20	59.25	4 218.82	18 281	21.05
60	817.005	19.06	91.13	3 906.34	14 062	17.21
65	742.554	30.43	141.81	3 460.04	10 156	13.68
70	637.250	50.25	224.21	2 843.34	6 696	10.51
75	494.370	84.70	349.50	2 039.89	3 852	7.79
80	321.587	139.98	512.58	1 177.62	1 812	5.64
85	156.749	218.72	681.19	488.18	635	4.05
90	49.973	323.15	823.53	127.35	146	2.93
95	8.819	451.43	917.66	17.93	19	2.17
100	0.726	600.60	1 000.00	1.21	1	1.66

LEVEL 24

Female

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1,000.000	13.64	13.49	989.21	77 500	77.50
1	986.512	0.77	3.06	3 935.47	76 511	77.56
5	983.491	0.26	1.30	4 913.93	72 575	73.79
10	982.211	0.21	1.07	4 908.52	67 661	68.89
15	981.156	0.38	1.89	4 901.34	62 753	63.96
20	979.305	0.43	2.16	4 891.44	57 852	59.07
25	977.185	0.53	2.64	4 879.74	52 960	54.20
30	974.609	0.67	3.34	4 865.23	48 080	49.33
35	971.352	0.97	4.81	4 845.53	43 215	44.49
40	966.675	1.49	7.41	4 816.17	38 370	39.69
45	959.508	2.32	11.53	4 770.98	33 553	34.97
50	948.440	3.61	17.88	4 701.51	28 782	30.35
55	931.484	5.46	26.96	4 597.14	24 081	25.85
60	906.367	8.79	43.03	4 438.23	19 484	21.50
65	867.364	14.77	71.32	4 188.36	15 046	17.35
70	805.506	26.91	126.39	3 783.20	10 857	13.48
75	703.701	50.09	222.59	3 126.91	7 074	10.05
80	547.064	92.28	372.57	2 208.58	3 947	7.22
85	343.246	158.90	554.87	1 198.64	1 738	5.06
90	152.788	255.69	732.91	437.94	540	3.53
95	40.808	384.55	866.65	91.97	102	2.50
100	5.442	547.26	1 000.00	9.94	10	1.83

Male

Age	l_x	$_nm_x$	$_nq_x$	$_nL_x$	T_x	e_x^0
0	1 000.000	16.84	16.62	986.70	71 528	71.53
1	983.380	0.91	3.63	3 921.02	70 542	71.73
5	979.810	0.39	1.96	4 893.76	66 621	67.99
10	977.888	0.37	1.85	4 885.09	61 727	63.12
15	976.076	0.88	4.40	4 870.08	56 842	58.24
20	971.785	1.08	5.40	4 846.32	51 972	53.48
25	966.535	1.18	5.87	4 819.05	47 125	48.76
30	960.860	1.29	6.42	4 789.48	42 306	44.03
35	954.687	1.71	8.50	4 753.96	37 517	39.30
40	946.571	2.66	13.20	4 702.88	32 763	34.61
45	934.081	4.32	21.39	4 622.45	28 060	30.04
50	914.101	7.20	35.40	4 492.84	23 438	25.64
55	881.742	11.53	56.10	4 289.99	18 945	21.49
60	832.276	18.17	87.06	3 987.47	14 655	17.61
65	759.815	29.12	136.10	3 550.88	10 667	14.04
70	656.401	47.86	214.64	2 943.87	7 116	10.84
75	515.511	80.42	334.79	2 146.08	4 173	8.09
80	342.923	131.28	489.30	1 278.07	2 026	5.91
85	175.132	204.27	654.17	560.87	748	4.27
90	60.566	302.91	800.48	160.05	188	3.10
95	12.084	428.12	903.97	25.52	28	2.28
100	1.160	581.96	1 000.00	1.99	2	1.72

OLD-AGE MORTALITY PATTERNS IN LOW-MORTALITY COUNTRIES: AN EVALUATION OF POPULATION AND DEATH DATA AT ADVANCED AGES, 1950 TO THE PRESENT

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SUMMARY

Age misreporting has impeded the development of model patterns of old-age mortality. An intercensal cohort method was used to evaluate the consistency of population age distributions and death registration data for 18 low-mortality countries from 1950 to 1985. The evaluation has focused on a comparison of the consistency of census and death registration data for open-ended cohorts for intercensal periods, using census and registration data classified by single years of age. In order to evaluate the effects of varying patterns of age misstatement on population age distributions, simulated errors were introduced into data from the Netherlands. Data from the countries examined appear consistent until advanced ages, where most English-speaking countries show a pattern of inconsistency, suggesting that similar age misreporting occurred in both the census population age distributions and the death registration data. Several other countries examined show patterns of error suggestive of more misreporting in population age distributions than in deaths.

INTRODUCTION

In the United States life tables for 1986, 56 per cent of females and 30 per cent of males will survive to age 80 (United States, National Center for Health Statistics, 1988). In 1985–1986 life tables for the Japanese population, the corresponding figures are 63 per cent and 43 per cent

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(Japan, 1987). Interest in the levels and age patterns of mortality at the very old ages has grown with the proportions of many national populations surviving to old ages. However, the patterns of mortality at the older ages have been elusive because of uncertainties about data reliability, especially with regard to the accuracy of age reports. Demographers and statisticians have, therefore, generally used empirical schedules of mortality that stop at an early open-ended age group (e.g., 80+). Beyond that point, the age pattern of mortality is often assumed to follow one of several functional forms, such as a Gompertz or Makeham curve, according to which the logarithms or logits of the age-specific death rates vary linearly with age. The idea that populations are comprised of heterogeneous subpopulations with exponentially rising but different mortality risks (Woodbury and Manton, 1977; Vaupel, Manton and Stallard, 1979) has led to modifications of the Gompertz and related curves to reflect a decline in the increase in the risks of mortality at advanced ages. Data from a number of countries have supported these modifications of the Gompertz curve (Humphrey, 1970; Heligman and Pollard, 1980; Vincent, 1951; Horiuchi and Coale, 1990).

However, empirical verification of these models has been somewhat superficial, because the data used in verifying the patterns have not been adequately checked for accuracy and only a small number of data sets have been used in the verifications. The former is a problem because similar patterns of data error may produce spurious conformity to a pattern of mortality change with age; the latter because it is risky to generalize on the basis of little experience.

The authors are engaged in a project that seeks to establish new models of old-age mortality patterns based upon data of certified quality. This article presents the results of an evaluation of the quality of the population and vital registration data that are needed to estimate old-age mortality levels from 1950 to 1985. The article begins with a general description of the evaluation procedures used and some important modifications of the procedures needed to apply them to data from many countries. Secondly, it describes how different hypothetical patterns of data error will manifest themselves in the results of the evaluation. Third, the data problems revealed by the evaluation are discussed. The final section of the article contains a description of the data that, having met our quality standards, are available for the calculation of intercensal mortality rates.

Table 1 contains a list of the countries and time periods for which census and death registration data are suitable to the evaluation procedures. In all cases, data on population and deaths by single year of age are required. The list specifies whether the population data are from censuses or population registers. In the latter case, the evaluation procedures were still used, although they were not a check on two independent sources of data. In almost all cases, census data are 100 per cent counts rather than sample data. For all time periods listed in table 1, the data used were classified by single year of age or year of birth. In many cases, unpublished data were obtained from statistical offices. There are a number of surprising omissions from the list. In the United States of America,

TABLE 1. COUNTRIES AND TIME PERIODS IN THE DATA EVALUATION STUDY

Country	Time period (source of population data)[a]	Ending age[b]	Classification of deaths
Australia	30 July 1954 (C)–30 June 1961 (C)	100	Age
	30 June 1961 (C)–30 June 1966 (C)	100	Age
	30 June 1966 (C)–20 June 1971 (C)	100	Age
	30 June 1971 (C)–30 June 1976 (C)	99	Age
	30 June 1976 (C)–30 June 1981 (C)	104	Age
Austria	21 March 1961 (C)–12 May 1971 (C)	99	Age
	12 May 1971 (C)–12 October 1981 (C)	99	Age
Belgium	31 December 1947 (C)–31 December 1961 (C)	99	Age, year of birth
	31 December 1961 (C)–31 December 1970 (C)	100	Age, year of birth
	31 December 1970 (C)–1 March 1981 (C)	99	Age, year of birth
Canada	1 June 1951 (C)–1 June 1956 (C)	90	Age
	1 June 1956 (C)–1 June 1961 (C)	90	Age
	1 June 1961 (C)–1 June 1966 (C)	90	Age
	1 June 1966 (C)–1 June 1971 (C)	90	Age
	1 June 1971 (C)–1 June 1976 (C)	90	Age
	1 June 1976 (C)–3 June 1981 (C)	90	Age
Denmark	7 November 1950 (C)–26 September 1960 (C)	99	Age, year of birth
	26 September 1960 (C)–27 September 1965 (C)	99	Age, year of birth
	27 September 1965 (C)–9 November 1970 (C)	88	Age, year of birth
	9 November 1970 (C)–31 December 1975 (R)	88	Age, year of birth
	31 December 1975 (R)–31 December 1980 (R)	93	Age, year of birth
	31 December 1980 (R)–31 December 1985 (R)	99	Age, year of birth
England and Wales	8 April 1951 (C)–23 April 1961 (C)	[c]	Age
	23 April 1961 (C)–25 April 1971 (C)	[c]	Age
	25 April 1971 (C)–5 April 1981 (C)	95	Age
Finland	31 December 1950 (C)–31 December 1960 (C)	90	Age, year of birth
	31 December 1960 (C)–31 December 1970 (C)	90	Age, year of birth
	31 December 1970 (C)–31 December 1975 (R)	95	Age, year of birth
	31 December 1975 (R)–31 December 1980 (R)	100	Age, year of birth
Hungary	1 January 1960 (C)–1 January 1970 (C)	85	Age
	1 January 1970 (C)–1 January 1980 (C)	85	Age
Ireland	9 April 1961 (C)–17 April 1966 (C)	99	Age
	17 April 1966 (C)–18 April 1971 (C)	100	Age
	18 April 1971 (C)–1 April 1979 (C)	100	Age
	1 April 1979 (C)–5 April 1981 (C)	100	Age
Italy	4 November 1951 (C)–15 October 1961 (C)	100	Age, year of birth[d]
	15 October 1961 (C)–24 October 1971 (C)	100	Age, year of birth
	24 October 1971 (C)–25 October 1981 (C)	100	Age, year of birth
Japan	1 October 1955 (C)–1 October 1960 (C)	100	Age
	1 October 1960 (C)–1 October 1965 (C)	100	Age
	1 October 1965 (C)–1 October 1970 (C)	100	Age
	1 October 1970 (C)–1 October 1975 (C)	100	Age
	1 October 1975 (C)–1 October 1980 (C)	100	Age
	1 October 1980 (C)–1 October 1985 (C)	100	Age
Netherlands	31 December 1955 (R)–31 December 1960 (R)	[c]	Age
	31 December 1960 (R)–31 December 1965 (R)	[c]	Age
	31 December 1965 (R)–31 December 1970 (R)	[c]	Age

TABLE 1 (*continued*)

Country	Time period (source of population data)[a]	Ending age[b]	Classification of deaths
	31 December 1970 (R)–31 December 1975 (R)	c	Age
	31 December 1975 (R)–31 December 1980 (R)	c	Age
New Zealand	17 April 1951 (C)–17 April 1956 (C)	c	Age
	17 April 1956 (C)–18 April 1961 (C)	c	Age
	18 April 1961 (C)–22 March 1966 (C)	c	Age
	22 March 1966 (C)–23 March 1971 (C)	c	Age
	23 March 1971 (C)–23 March 1976 (C)	c	Age
	23 March 1976 (C)–24 March 1981 (C)	c	Age
Northern Ireland.....	23 April 1961 (C)–9 October 1966 (C)	99	Age
	9 October 1966 (C)–25 April 1971 (C)	95	Age
Norway................	1 November 1960 (C)–1 November 1970 (C)	c	Age, year of birth
	1 November 1970 (C)–1 November 1980 (C)	c	Age, year of birth
Portugal...............	15 December 1950 (C)–15 December 1960 (C)	c	Age
Scotland...............	8 April 1951 (C)–23 April 1961 (C)	98	Age
	23 April 1961 (C)–25 April 1971 (C)	100	Age
	25 April 1971 (C)–5 April 1981 (C)	95	Age
Spain....................	31 December 1950 (C)–31 December 1960 (C)	80	Age
	31 December 1960 (C)–31 December 1970 (C)	85	Age
Sweden.................	31 December 1950 (R)–31 December 1955 (R)	95	Age, year of birth
	31 December 1955 (R)–31 December 1960 (R)	95	Age
	31 December 1960 (R)–31 December 1965 (R)	95	Age
	31 December 1965 (R)–31 December 1970 (R)	95	Age, year of birth
	31 December 1970 (R)–31 December 1975 (R)	100	Age, year of birth
	31 December 1975 (R)–31 December 1980 (R)	100	Age, year of birth
	31 December 1980 (R)–31 December 1985 (R)	100	Age, year of birth

[a] (C) = census; (R) = population register.
[b] Age at the end of the time period.
[c] Both population age distributions and deaths were published to the oldest recorded age.
[d] From 1951 to 1954, deaths were by age only. From 1954 to 1961, deaths were cross-classified by age and year of birth.

there is no intercensal period for which the required data are available by single year of age. France does not appear in table 1 because its census data are published by age as of 1 January rather than at the time of the census.

INTERCENSAL COHORT METHOD OF DATA EVALUATION

The major method of data evaluation used, the intercensal cohort method, checks the consistency between reported intercensal changes in the size of cohorts and the number of deaths occurring between the two censuses that was reported by vital registration. The *expected* size of an open-ended age cohort in the second census can be estimated from its size at the first census and the intercensal deaths occurring to that cohort, after adjustment for migration. The equation for the predicted population at time $t + j$ is:

$$\hat{N}_{i+j}(t + j) = N_i(t) - D_i$$

where:

$N_i(t)$ = the enumerated population age i and above at the first census

D_i = the intercensal deaths to the cohort age i and above at the first census

$\hat{N}_{i+j}(t + j)$ = the predicted population aged $i + j$ and above at the second census, taken j years after the first.

The ratio of the actual to the expected populations, $(N_{i+j}(t + j) / \hat{N}_{i+j}(t + j))$, can then be calculated. A ratio of 1,000 indicates perfect consistency between sources. It is recognized that consistency is a necessary but not sufficient condition for accuracy, because sources with the same patterns of error may produce consistency. However, for the large majority of countries it is not only the best but the only available test of the quality of the data that are needed to calculate mortality rates at older ages.

Application of the intercensal cohort method to empirical data has not been straightforward. Three problems arise. First, both population data and registered deaths are usually reported by age at last birthday rather than year of birth; therefore, data must be adjusted so that censuses and deaths refer to identically defined cohorts. Secondly, death register data generally refer to calendar years, while censuses are typically not taken at the beginning of the year. Therefore, deaths in the intercensal period must be adjusted for the timing of the census. Finally, in a number of countries successive censuses are not taken on the same month and day. In these cases, a final adjustment must be made for intercensal periods that include a non-integer number of years.

The simplest way to handle both the problem of relating deaths to the appropriate census cohort and that of censuses that are taken past the beginning of the calendar year is to assume that death and birth dates of the deceased were spread evenly over the year. That is, one could assume that the lexis surface of deaths between ages i and $i + 1$ during period t to $t + 1$ is flat in both dimensions. It was possible to test the effects of this assumption by using two data sets containing more detail than usual. Deaths recorded in the Swedish population register were classified by both year of birth and age. Population estimates refer to the beginning of the year, so that age can be mapped in one-to-one fashion into birth cohort and deaths during an interval can be attributed precisely to the birth cohort to which they occurred. Table 2 shows selected ratios of actual to expected populations for Swedish male birth cohorts defined on 1 January 1970 and survived five years forward by subtraction of registered deaths and net migrations to the cohort. Deaths and migrations are assigned directly to birth cohorts. As expected in these ideal circumstances, the ratio of actual to expected populations is very close to 1.0 even at advanced ages.

TABLE 2. EXAMPLE OF INTERCENSAL COHORT METHOD APPLIED TO SWEDISH REGISTER DATA, MALES 1970 TO 1975: COMPARISON OF RESULTS USING DEATHS REPORTED BY YEAR OF BIRTH AND BY AGE

Year of birth	Population born in year x or before, beginning of interval (1)	Population born in year x or before, end of interval (2)	Net migration to birth cohort during interval (3)
1920	1 243 362	1 035 008	−513.0
1915	982 276	785 269	−221.5
1910	729 803	550 120	36.5
1905	495 640	341 814	282.0
1900	308 123	187 079	227.5
1895	168 269	85 064	111.5
1890	76 930	30 027	39.5
1885	26 513	7 238	10.5
1880	5 991	973	3.0

Deaths reported by year of birth

Year of birth	Deaths to birth cohort during interval (4)	Expected population at the end of interval (1) + (3) − (4) (5)	Ratio of actual to expected population (2)/(5) (6)
1920	207 751	1 035 098.0	0.9999
1915	196 729	785 325.5	0.9999
1910	179 701	550 138.5	1.0000
1905	154 081	341 841.0	0.9999
1900	121 230	187 120.5	0.9998
1895	83 277	85 103.5	0.9995
1890	46 921	30 048.5	0.9993
1885	19 264	7 259.0	0.9970
1880	5 006	988.0	0.9848

Deaths reported by age[a]

Year of birth	Estimated deaths in birth cohort during interval (7)	Expected population at the end of interval (1) + (3) − (7) (8)	Ratio of actual to expected population (2)/(8) (9)
1920	207 731.5	1 035 117.5	0.9999
1915	196 761.0	785 293.5	1.0000
1910	179 745.0	550 094.5	1.0000
1905	154 034.0	379 365.5	0.9998
1900	121 249.5	187 101.0	0.9999
1895	83 250.0	85 130.5	0.9992
1890	47 039.0	29 930.5	1.0032
1885	19 354.5	7 169.0	1.0096
1880	5 063.5	930.5	1.0457

[a] Assuming that the lexis surface in one-year wide intervals is flat.

Table 2 also presents an equivalent calculation using deaths recorded by age at death rather than by year of birth. Deaths are attributed to a cohort by assuming that deaths during a year to a person aged i are evenly divided between persons aged i and aged $i - 1$ at the beginning of the year. As shown by comparing the two sets of ratios in table 2, the ratios are only slightly further from 1.0 than when deaths are recorded by year of birth until the highest age interval (90+). There, the divergence has increased to the point where the flat lexis surface assumption produces misleading results. At these ages, ratios above or below 1.0 could not be interpreted, inasmuch as they could result either from errors in the data or from problems in the procedures used to evaluate the data. Consequently, it was decided to abandon the simple assumption of a lexis surface that is flat over age and use available data to estimate separation factors that would more accurately reflect the distribution of deaths at age i to appropriate birth cohorts. This procedure is described below.

Only a very small error resulted because censuses were not taken at the beginning of the year. Table 3 presents data for the Japanese population of Japan from censuses taken on 1 October 1970 and 1 October 1975. In this example, the population is enumerated in a census independently of the register of deaths. Cohorts are survived forward using deaths and migrations classified by age and assuming that the lexis surface of deaths is flat in both dimensions. However, the Japanese data system also records deaths by age and month of occurrence, so that the assumption of a lexis surface that is flat over time can be avoided. The two sets of ratios are compared in the final two panels of table 3. The assumption of a surface that is flat in time clearly has very little effect on the ratios, relative to calculations based on the less frequently available data by month of death. Of course, when the more precise data are available, they have been used.

Estimating separation factors by age

The data set contained a number of countries for which deaths were recorded by age at death and year of birth. For those countries, deaths occurring to a particular age group could be divided directly between the two birth cohorts to which they belonged. The larger countries having such data—Belgium, France, Italy and Sweden—were used to estimate an equation where the separation factor at each age is expressed as a function of age and of the relative size at birth of the two cohorts contributing to that age group. The model used all data above age 40 for the four countries combined, and observations were weighted by the number of deaths. Age was defined as the distance from age 40 and entered into the equation in linear, squared, and cubed terms to reflect the pattern of separation factors across age. The resulting equation is of the form:

$$\text{Sepfact}(i) = B(0) + B(1)\text{Age} + B(2)\text{Age}^2 + B(3)\text{Age}^3 + B(4)\text{Sepbirth}(i)$$

where:

Sepfact(i) = the proportion of deaths at age i to the earlier (i.e., older) birth cohort

Age = age at death minus 40

Sepbirth(i) = the number of births in the earlier cohort divided by the sum of the births in the two cohorts contributing to age i.

TABLE 3. EXAMPLE OF THE INTERCENSAL COHORT METHOD APPLIED TO JAPANESE CENSUS AND REGISTRATION DATA, MALES, 1970 TO 1975: COMPARISON OF RESULTS USING DEATHS REPORTED BY MONTH OF DEATH AND BY CALENDAR YEAR

Age at start of open-ended interval at second census (percentage)	*Population aged x and above at the beginning of interval (1)*	*Population aged x and above at the end of interval (2)*	*Net migration in age group during interval (3)*
55	9 126 810	7 804 929	2 073.5
60	6 986 919	5 747 348	1 285.5
65	4 958 219	3 823 030	513.5
70	3 212 180	2 259 359	104.5
75	1 818 920	1 115 811	−135.0
80	860 590	429 588	−65.0
85	329 827	122 409	−38.0
90	88 910	21 667	0.0
	Deaths reported by month of death		
	Deaths to birth cohort during interval (4)	*Expected population at the end of interval (1) + (3) − (4) (5)*	*Ratio of actual to expected population (2)/(5) (6)*
55	1 442 168.0	7 686 716.0	1.0154
60	1 347 656.0	5 640 549.0	1.0189
65	1 201 328.0	3 757 405.0	1.0175
70	994 247.5	2 218 037.0	1.0186
75	727 794.5	1 090 991.0	1.0228
80	443 137.0	417 388.0	1.0292
85	208 882.0	120 907.0	1.0124
90	65 692.5	23 217.5	0.9332
	Deaths reported by calendar year		
	Estimated deaths in birth cohort during interval (7)	*Expected population at the end of interval (1) + (3) − (7) (8)*	*Ratio of actual to expected population (2)/(8) (9)*
55	1 444 387.0	7 684 497.0	1.0157
60	1 349 635.0	5 638 570.0	1.0193
65	1 203 066.0	3 755 667.0	1.0179
70	995 815.9	2 216 469.0	1.0194
75	729 151.6	1 089 633.0	1.0240
80	444 031.5	416 493.5	1.0314
85	209 115.1	120 673.9	1.0144
90	65 992.9	22 917.1	0.9455

The values of the regression coefficients estimated on the pooled data via weighted least squares regression are:

	Females	*Males*
Intercept	0.15244	0.15697
Age	−0.00117	−0.00105
Age^2	0.00836	0.000065
Age^3	$-1.1976e^{-6}$	$-8.9111e^{-7}$
Sepbirth	0.69763	0.68191
R^2	0.228	0.260

All coefficients are significant at a 1 per cent level.

Figures I and II show for females and males the values of the separation factors at each age predicted from the equations, assuming a relative birth cohort size of 0.5 at birth for ages 40 and above. Plotted with the estimated values are the actual separation factors at each age averaged across all countries and time periods. For females, predicted separation factors fit the data quite well until very advanced ages. For males the fit is less good, and at very advanced ages the actual separation factors derived from the small numbers of deaths at these ages fluctuate widely. The shape of the curve is very similar for the two sexes across most of the age range. The estimated values fall below 0.5 for men and women in their forties, begin to climb and peak at 0.5063 for men and 0.5116 for women at age 78, and then turn down quickly at more advanced ages.

In countries where death registration data cross-classified by age at death and year of birth were available, they were used in the data evaluation. When only deaths by age were available, separation factors derived from the equation above were used to distribute deaths at an age to the two birth cohorts contributing those deaths. As input to the equation, the actual size at birth of the two contributing birth cohorts, derived from vital statistics, was used.

Relating deaths to single-year age groups in the census

The change in the assumption about the lexis surface complicates the application of the intercensal cohort method. When both censuses occur at the beginning of the year, application of the estimated separation factors to the deaths in two squares (1,3) of the lexis diagram produces the appropriate division of deaths for assignments to cohorts (see fig. III). When censuses do not occur at the beginning of the year, the procedure is more complicated. Appropriate proportions of the deaths from four squares (1,2,3,4) of the lexis diagram must be used to project the population aged i forward to age $i + 1$. In terms of figure III, deaths in each of the areas labelled (A) to (D) must be summed to estimate the number of deaths to persons who were aged 65 at the time of the census, and to project that group forward by one year to the point when they are aged 66. The estimation of the deaths in each of the areas used the following equations:

$$A1 = (1-x)^2 \cdot M_1 \cdot S_1$$

$$A2 = 2(x-x^2) \cdot M_1 \cdot S_1$$

$$B = x^2 \cdot M_2 \cdot S_2$$

$$C1 = 2(x-x^2) \cdot M_4 \cdot S_4$$

$$C2 = x^2 \cdot M_4 \cdot (1-S_4)$$

$$D = (1-x)^2 \cdot M_3 \cdot (1-S_3)$$

Figure I. Proportion of deaths in an age occurring to older cohort: comparison of actual and predicted values: females

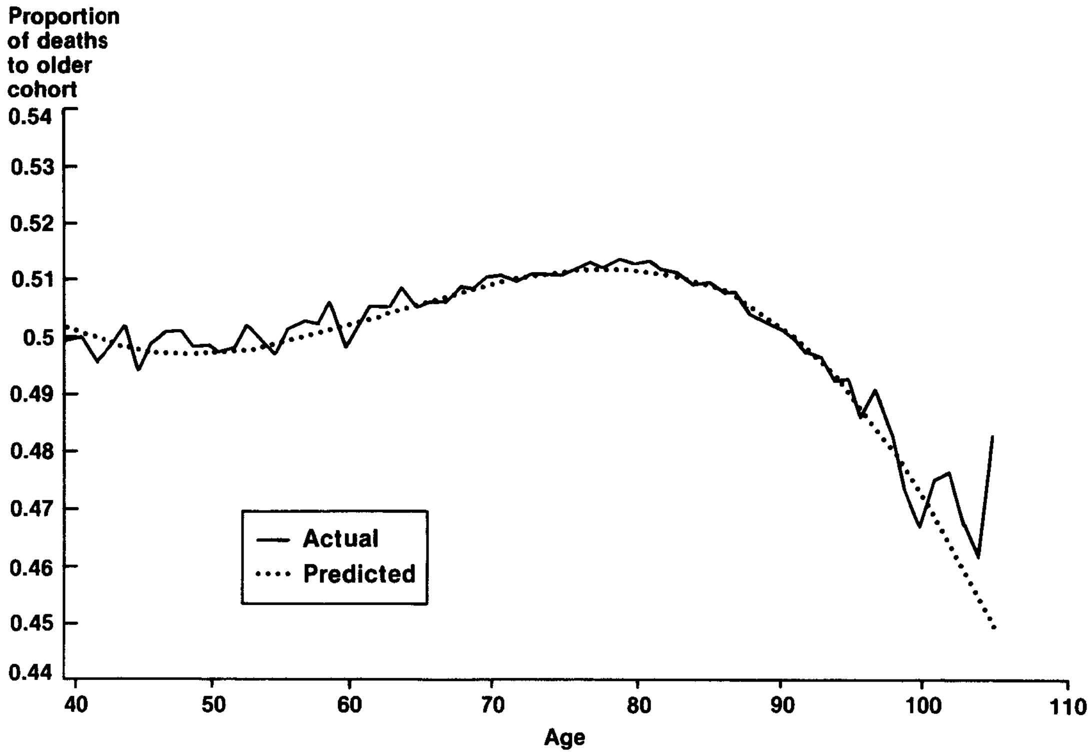


Figure II. Proportion of deaths in an age occurring to older cohort: comparison of actual and predicted values: males

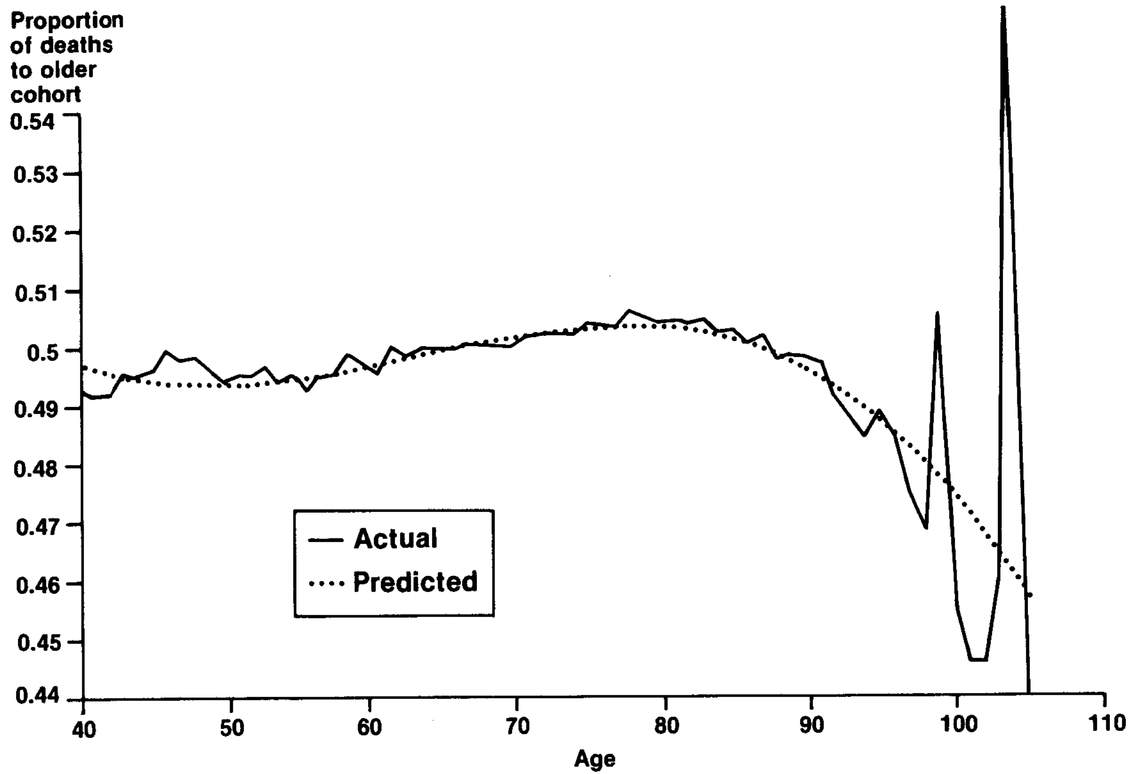
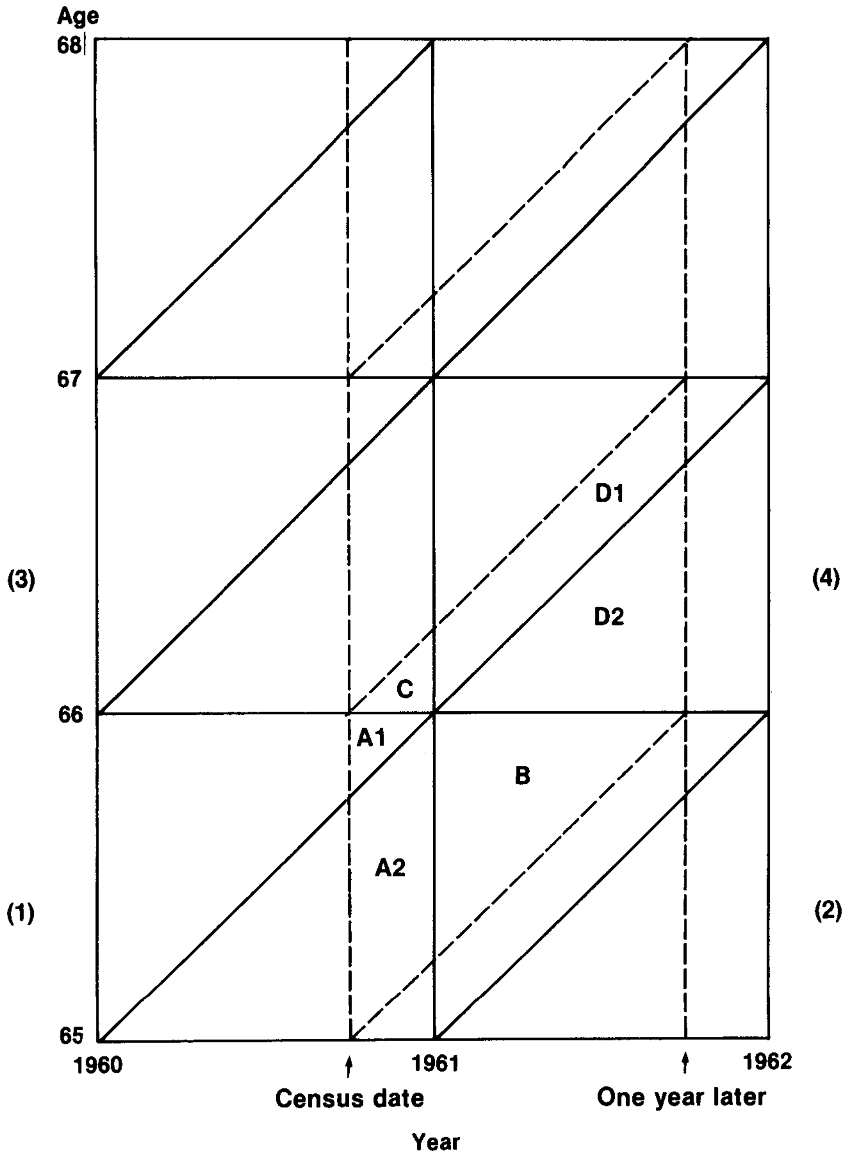


Figure III. Lexis diagram showing deaths occurring in one year to persons aged 65 at the first census



where:

M = the deaths in each of the squares numbered (1) to (4)

S_i = the proportion of deaths in age interval i attributed to the earlier birth cohort (i.e., the separation factor, estimated by the procedure described above)

x = the proportion of the calendar year elapsed between 1 January and the date of the census (t).

In deriving these equations, it was assumed that each birth cohort *triangle* on the lexis surface is level, rather than each square. The separation factor provides the relative level of the two triangles within a square. Using these equations, the populations were projected forward by yearly intervals over the period until the next census, producing a population estimate for each year on the date of the census.

Adjusting for varying census dates

In a number of countries, the months and/or days of the census change in successive censuses, and therefore the calculation of the expected population does not correspond to the age-at-last-birthday population numbers reported in the second census. In those countries, the population at the second census is first estimated in such a way that it corresponds to the date (day/month) of the earlier census. If the second census was later in the year than the first, deaths occurring after the date of the first census are subtracted from this estimate to produce the estimated population at the actual date of the second census. If the second census occurred earlier in the year than the first census, estimates of deaths occurring after the date of the first census are added to move the projected population backward in time. A final adjustment is required so that the age groups projected at the second census correspond to the birth cohorts identified at the first census.

Interpretation of ratios deviating from 1.0

Observed inconsistencies in the data (i.e., ratios deviating from a value of 1.0) could result from underreporting in either or both censuses, from underreporting in the death registration data, and/or from misreporting of age in any or all of the data sources. For the countries included in this project, the completeness of coverage in census and registration data is less of a problem than age misreporting. However, attributing inconsistencies between sources to misreporting requires that coverage errors can be discounted. As will be shown, there is typically a high degree of consistency at young ages (e.g., 40 or 50), which strongly suggests that the completeness of enumeration in the two censuses is similar to that in death registration; any divergences at the higher ages are then plausibly ascribed to age misreporting. Published evaluation studies from the data collection agencies in a number of countries (Heldal, 1984; Westlund, 1984; Fellegi, 1980; United Kingdom, 1983, 1985; Nilsson, 1979; Morris and Compton, 1985; Britton and Birch, 1985) confirm our results that age misstatement is a far more serious problem than undercoverage, especially at older ages.

Previous studies of age reporting in census and death registration data

There has been considerable work documenting age overstatement in censuses. Linking and comparing individual records from different sources reveal that the number of centenarians reported in the censuses of many countries is substantially enlarged by age exaggeration (Myers, 1966; Rosenwaike, 1968, 1979; Rosenwaike and Logue, 1983; Thatcher, 1981, 1984; Bennett and Garson, 1983; Siegel and Passel, 1976; Bayo and Faber, 1983). There is evidence from these direct checks that inconsistency between records increases with age and that individuals' ages are generally more likely to be misreported in censuses than in death registration data (Hambright, 1969; Rosenwaike and Logue, 1983).

In the United States, published evaluation studies linking individual records have been conducted on the 1950, 1960 and 1970 censuses and have produced estimates of inconsistency between sources in age misreporting, some of which are shown in table 4. In general, the results are not published in a form to make comparison easy or in detail necessary to analyse the quality of data for single years of age.

A number of generalizations about patterns of age misreporting in censuses can be made, however, from the studies cited in table 4. First and most importantly for the problem confronting us, until fairly advanced ages, age as reported in the censuses is not consistently older than is implied in the other source. It is important to note, however, that because cohort sizes drop off rapidly at older ages, even a misreporting pattern in which equal proportions overstate and understate their ages will have greater impact on the older than on the younger age category; more people will transfer into an older age category than will transfer out of it, and the disparity will increase with age. Secondly, disagreement between sources increases with age, as does the likelihood that the census age will be higher rather than lower than the source to which it is being compared. Thirdly, the patterns of inconsistency could be represented by a gravity model—i.e., the closer two ages or age groups are to each other, the more likely that people will "migrate" between them.

All of the results reported in table 4 are from studies that matched individual records and reported gross transfers of persons from one age group to another. Some results consistent with these are also found in studies that do not match individuals but that rely on comparisons of marginal age distributions. Those studies do not produce estimates of the gross transfers of individuals from one age to the other but only of the net effects of those transfers on the age distributions. For example, extinct generation techniques have been used extensively to reconstruct old-age populations from deaths alone to be used in the denominators of death rates but also as a comparison to census data (Salvini, 1980; Vincent, 1951; Despoid, 1973; Rosenwaike, 1968, 1979, 1981; Kimura, 1971). This method accumulates deaths over time to individuals in the cohort aged x at some initial census until the cohort has completely expired. The accumulated deaths are then compared to the initial census count at age x. The difference between the enumerated population and the population

TABLE 4. PROPORTION OF CENSUS AGE GROUP REPORTING DIFFERENT AGE GROUPS IN VARIOUS OTHER SOURCES, WHITE MALES, AGE 50 AND OVER

	Age in census									
Age in census evaluation survey	*50-54*	*55-59*	*60-64*	*65-69*	*70-74*	*75-79*	*80-84*	*85-89*	*90-94*	*95-99*
	1960 United States census and 1960 census evaluation study									
0-4										
5-9										
10-14										
15-19										
20-24										
25-29										
30-34		.0042								
35-39										
40-44	.0314									
45-49	.0185	.0173								
50-54	.9224	.0257	.0134							
55-59		.9186	.0127	.0202						
60-64	.0277	.0204	.9186	.0242	.0406					
65-69		.0139	.0265	.8984	.0091					
70-74			.0214	.0572	.9504	.0160				
75-79			.0073			.9840				
80-84							1.0000			
85+								1.0000		

TABLE 4 (*continued*)

Age in current population survey	Age in census 50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95-99
	1960 United States census and current population survey									
0-4										
5-9										
10-14			.0019							
15-19										
20-24	.0016	.0024								
25-29										
30-34										
35-39			.0019							
40-44	.0109									
45-49	.0120	.0146								
50-54	.9200	.0354	.0174							
55-59	.0399	.9209	.0265	.0027						
60-64	.0106	.0149	.9277	.0358	.0557					
65-69	.0016	.0100	.0152	.9270	.0022	.0035				
70-74		.0019	.0094	.0221	.9091	.0227				
75+	.0020			.0126	.0329	.9739				

1960 United States census and death registration

Age in death register										
0-4								.0001		
5-9						.0001	.0001			
10-14	.0001	.0001			.0001	.0004				
15-19	.0003	.0002						.0001		
20-24	.0004	.0002		.0002						
25-29	.0002	.0001					.0001			
30-34	.0003	.0002		.0001			.0001			
35-39	.0002	.0005	.0003	.0001	.0001		.0001			
40-44	.0029	.0005	.0003	.0003	.0003		.0002		.0003	
45-49	.0177	.0038	.0003	.0001	.0001	.0001	.0001			
50-54	.9092	.0270	.0034	.0007	.0002	.0002	.0002	.0001		
55-59	.0464	.9175	.0225	.0054	.0009	.0003	.0002	.0003	.0003	.0013
60-64	.0102	.0338	.8997	.0352	.0112	.0011	.0014	.0005	.0015	.0025
65-69	.0040	.0079	.0399	.9038	.0349	.0134	.0017	.0019	.0020	.0051
70-74	.0021	.0027	.0196	.0352	.8826	.0540	.0123	.0048	.0056	
75-79	.0024	.0028	.0042	.0137	.0454	.8867	.0423	.0167	.0051	.0190
80-84	.0020	.0012	.0069	.0018	.0183	.0347	.8991	.0640	.0193	.0063
85-89	.0012	.0012	.0016	.0028	.0051	.0081	.0355	.8875	.0502	.0317
90-94	.0004	.0000	.0092	.0004	.0008	.0011	.0069	.0231	.9017	.1077
95-99		.0003	.0002					.0009	.0142	.8264

Table 4 (*continued*)

Age in post-enumeration survey	Age in census 50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95-99
1950 United States census and post-enumeration survey										
0-4	.0014		.0037							
5-9										
10-14										
15-19										
20-24			.0018		.0034	.0057				
25-29		.0015								
30-34										
35-39		.0031	.0018							
40-44		.0025	.0018		.0068					
45-49	.0351			.0023		.0045				
50-54	.9038	.0297		.0023						
55-59	.0520	.9391	.0355	.0065		.0045				
60-64	.0041	.0182	.9217	.0397						
65-69	.0014	.0059	.0337	.9095	.0441	.0045				
70-74				.0397	.9186	.0255				
75+	.0014				.0271	.9553				

Sources: Calculated from the United States Bureau of the Census, 1954, 1964, 1965; National Center for Health Statistics, 1968.

estimated from accumulated deaths represents the degree of age misreporting in the census, if age at death is accurately recorded and deaths are completely registered. Extinct generation studies generally show that, at ages over 90, population estimates derived from death-register data are lower than those reported in censuses, although the results are by no means uniform.

The accuracy of the age reported on death records has also been evaluated. In general, reporting of age has been considered better in death registration than in the census (Hambright, 1969; Rosenwaike and Logue, 1983). Many incentives for misreporting age, such as the social status that may accrue to the very old, are likely to disappear at death. While age at death is never self-reported, any errors due to lack of knowledge of the decedent's age are probably outweighed by the declining incentives deliberately to misreport. An outstanding study of reports of age at death was done by Rosenwaike and Logue (1983), who linked a sample of individual death records of persons who died at ages 85 and over in New Jersey and Pennsylvania from 1968 to 1972 to the 1900 manuscript census of population. The resulting errors reported in table 5 show a greater tendency for death-certificate age to be older than age at death calculated from age in the 1900 census, but that pattern does not begin until age 90. Before age 90, age on the death certificate is more likely to be younger than the presumably more accurate age derived from the 1900 census. The discrepancy between sources increases with age. The increase in misreporting with age is confirmed in a study checking age reported on a sample of death records being processed by the British National Health Service Central Register during April 1966 against the dates of birth reported to the same agency in 1939 (or subsequently, if the birth occurred after 1939). The dates of birth from this registration are considered highly accurate. Although that study confirmed some of Rosenwaike and Logue's (1983) findings, it showed that age was almost twice as likely to be overstated as to be understated beginning at age 80 (Canada, 1970).

TABLE 5. AGE AGREEMENT OF 1968-1972 DEATH CERTIFICATES TO MATCH 1900 CENSUS RECORDS: UNITED STATES WHITE POPULATION AGED 85 AND OVER

	Age on death certificate			
Death certificate age relative to census age	*85-89 years*	*90-94 years*	*95-99 years*	*100+ years*
5 or more years younger	0.4	1.1	0.4	0.0
2-4 years younger	1.6	2.9	2.3	2.4
1 year younger	9.1	7.6	6.1	4.8
0-11 months difference	82.6	72.0	68.2	63.1
1 year older	4.3	10.5	15.5	14.9
2-4 years older	1.6	4.3	5.7	12.5
5 or more years older	0.4	1.5	1.9	2.4

Source: Ira Rosenwaike and Barbara Logue, "Accuracy of death certificate ages for the extreme aged", *Demography* (Washington, D.C.), vol. 20, No. 4 (November 1983).

While previous studies of age misreporting are generally difficult to compare and do not produce net rates of overstatement—and understatement—of age with the precision that would be desirable for the analysis, they provided some general notions about how age is misreported. The information has been used to estimate the effects of age misreporting on the ratios of actual to expected populations derived from the intercensal cohort method.

Simulated effects of errors in age reporting on ratios of actual to expected population

Patterns of age misstatement based on generalizations from the studies of age misreporting cited above were introduced into Netherlands data that were constrained to have perfect consistency between two population age distributions five years apart and the intervening deaths. In a base-line simulation of misreporting, it was assumed that the misreporting pattern was identical in both deaths and population and did not change with age. In this case, it can be shown (see annex) that all ratios of actual to expected population will be unity. Here is a clear example that demonstrates that consistency need not imply accuracy (although inconsistency necessarily implies inaccuracy). However, the example is highly unrealistic, since empirical misreporting patterns typically demonstrate increasing misreporting with age.

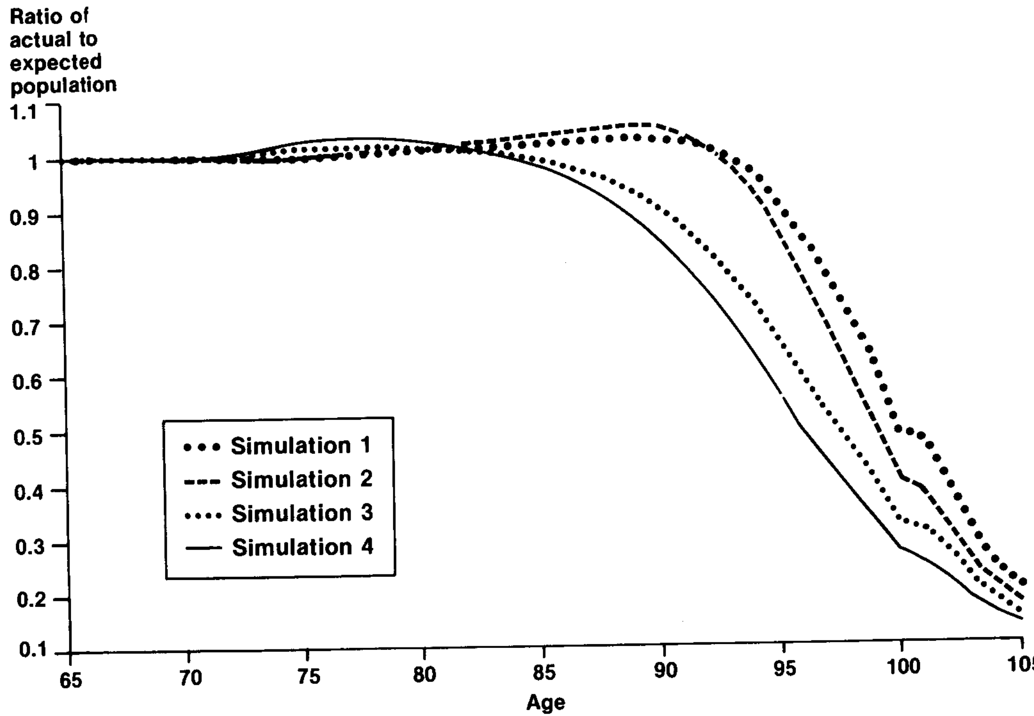
In all of the remaining simulations, it was assumed that age was correctly reported until age 70, at which age hypothesized patterns of age misreporting were introduced. The distribution of reported ages relative to actual age was assumed to decrease with increasing distance from the correct age and to be within five years of the actual age. In all of these simulations, the proportion misreporting age was assumed to increase with age at a constant rate. The simulations show the effects of varying the amount of age misstatement and the proportion reporting themselves older than their true ages. The four simulated patterns of age misreporting were as follows:

Misreporting pattern	*Proportion misreporting increases at a rate of:*	*Proportion of those misreporting who reported older ages*
1	0.01 at each age	Increases from 0.3 to 0.7 with age
2	0.02 at each age	Increases from 0.3 to 0.7 with age
3	0.01 at each age	Is 1.0 at every age
4	0.02 at each age	Is 1.0 at every age

The basic pattern of misreporting on which these age trends are superimposed is one in which the proportion misreporting into a particular age group is inversely related to the distance of that age group from an individual's true age. Thus, among those overstating their age, 5/15 are assumed to overstate by one year, 4/15 by two years, 3/15 by 3 years, 2/15 by 4 years, and 1/15 by 5 years. The same distribution is used when age understatement occurs.

Figure IV shows the results of the first set of simulations. In these, the four patterns of age misreporting were introduced only into the two

Figure IV. Ratio of actual to expected population at the time of the second census, based on simulated errors in the two censuses only



population age distributions. Results of a second set of simulations, in which the deaths were assumed to have the same pattern of error as the population age distributions, are shown in figure V. Obviously, the age pattern of ratios differed dramatically according to whether error was also introduced into deaths or was limited to the population data. In the latter case, the ratios of actual to expected population declined with advancing age (sometimes after a mild increase) to a value between 0.1 and 0.2 at age 105 and over. However, the ratios changed in the opposite direction when age misstatement was introduced into *both* the deaths and the population age distributions. In that case, the ratios rose with age—i.e., actual populations were increasingly larger than expected populations as age advanced. Regardless of which of the four patterns of age misstatement was introduced, simulations with identical errors in both deaths and population always produced ratios rising well above 1.0 and those having errors only in the age distributions produced declining ratios.

Varying the patterns of age misstatement produced much smaller differences in the ratios than varying which sources contained error. When errors were introduced into the population age distributions only, ratios deteriorated earlier and more rapidly when all misreporters were overstating their ages (misreporting patterns 3 and 4) than when the proportions overstating increased with age (misreporting patterns 1 and 2). The level of misreporting made less difference in the ratios. When misreporting increased at a rate of 2 per cent from age to age (patterns 2 and 4), ratios deviated from 1.0 only slightly more and at a somewhat earlier age than when misreporting increased at a rate of 1 per cent.

Variations in age-reporting patterns were even less important in determining the ratios when the errors were introduced into the deaths as well as into the populations (figure V). Higher rates of misreporting produced higher ratios, but variations in proportions reporting themselves older did not affect the ratios, as they did in figure IV.

A fifth simulation, in which a lower level of age misreporting was introduced into the deaths than into the populations, also produced declining ratios (as in the population-only distortions). In that simulation, shown in figure VI, age misreporting in the deaths increased at a rate of 1 per cent (pattern 1) while the rate of misreporting increased at a rate of 2 per cent (pattern 2) in the population age distributions. The ratios deteriorate less than when no errors are introduced into the deaths, but a definite decline in ratios occurred when reporting was better in the deaths than in the populations.

A final simulation tested the effects of introducing errors into a baseline population that had a less steep decline in cohort size at the older ages than in the actual Netherlands data. We were interested in testing whether the effects of age misreporting on the ratios were dependent on the age structure of the initial population. The age structure of the initial population of the Netherlands was adjusted so that the log of the ratio age_i/age_{i+1} increased at a rate from age to age that was .01 lower than in the original data. Probabilities of death were kept the same, and a new set of interven-

Figure V. Ratio of actual to expected population at the time of the second census, based on simulated errors in the two censuses and in the death registration data

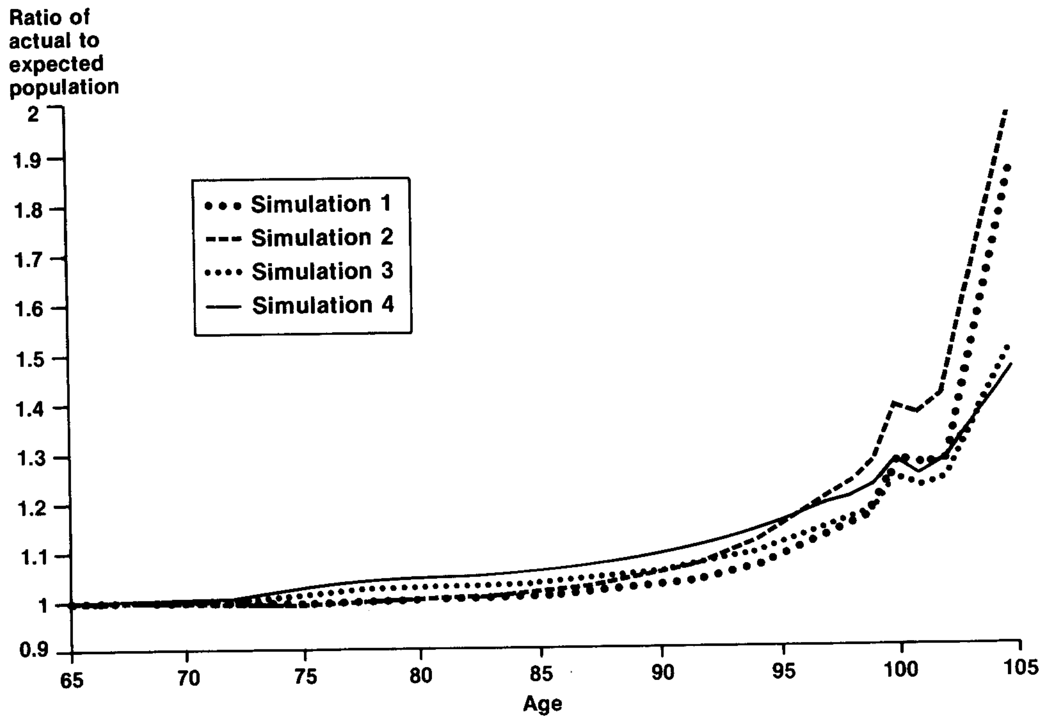
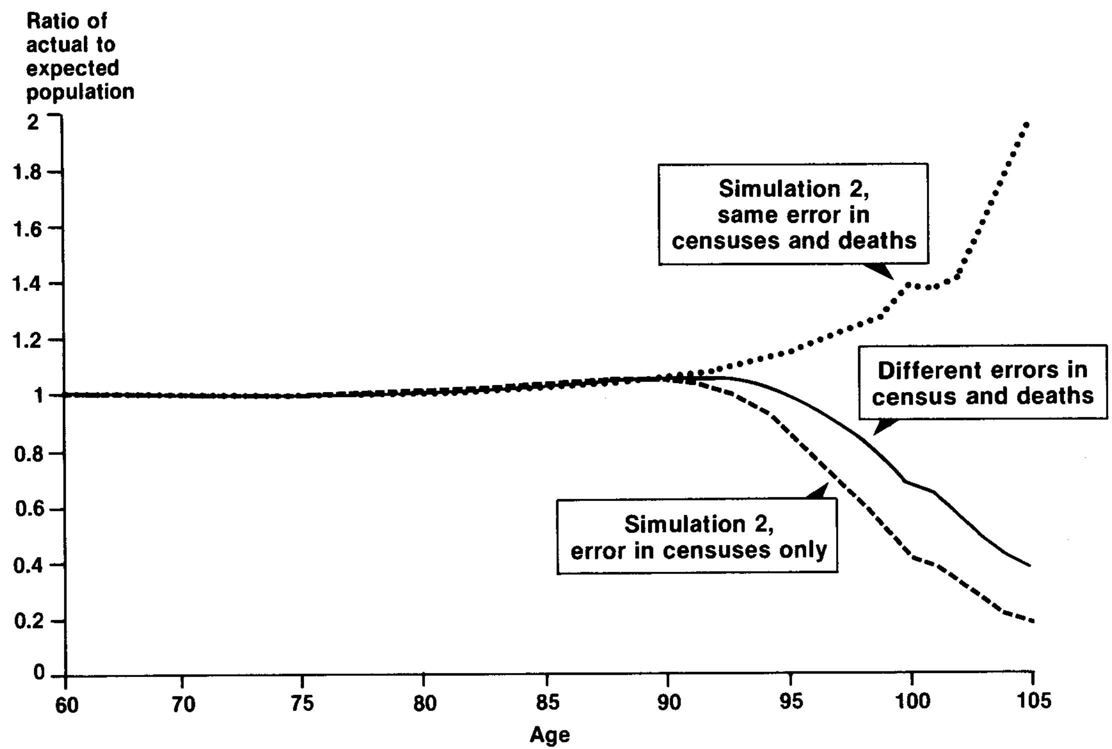


Figure VI. Comparison of ratios of actual to expected population at the time of the second census, based on simulated errors which differ between the two censuses and the death registration data



ing deaths and a second population age distribution were generated. Figure VII shows a simulation using the newly calculated age structures with pattern-1 error introduced first into the population age distributions only and then into the deaths as well. Changing the age structure of the initial population had almost no effect on the pattern of ratios produced. Using other patterns of age misreporting did not alter the finding that the slope of the age structure had little effect on the ratios.

To summarize: ratios systemically rising above 1.0 were produced only by introducing equal or nearly equal errors into both deaths and population age distributions. When deaths had either no error or a substantially lower level of error, the ratios of actual to expected population declined with age to values well below 1.0. The age-pattern of ratios is robust to the type of age-misreporting pattern introduced (among those simulated) and to the slope of the underlying true age distribution. Changing the patterns of misreporting had much less effect on the ratios, and the small changes that occurred could be anticipated. However, as expected, ratios deteriorated more rapidly with age when misreporting was more extensive and more consistent in direction.

Results: the consistency between death registration and census data

The intercensal cohort method described above was used to evaluate the consistency between population data reported by single year of age at two censuses and intervening deaths, also reported by single year of age, for the countries listed in table 1. Data for two countries, the Netherlands and Sweden, were entirely drawn from population registers, and for Denmark and Finland all data from 1975 on were from population registers. Because population estimates and deaths come from the same source in these countries, we expected and found greater consistency in register countries than when census population estimates were independent of the report of deaths. In the population register data, each registration event is recorded by using an identifier which includes the person's date of birth, so that deaths can be assigned directly to the appropriate birth cohort, and age at death can be derived from the date of birth. The intercensal cohort method was applied to these data in five-year time blocks and, despite the register procedures, found inconsistencies that increased with age and were greater than 1 per cent at advanced ages. data for Sweden are shown in figure VIII. Sweden acknowledges errors in its register at very old ages by printing a column of such inconsistencies in its official statistical series.

For all other countries and/or time periods in table 1, data were from death registration systems and censuses, sources of data that are independent of each other. The general pattern of the results is reported here with illustrative results shown in figures IX through XI. In these figures, age on the *x*-axis refers to age at the beginning of the open-ended interval in the second census.

The cohort comparisons for the countries and time periods listed in table 1 in general display age patterns of ratios that we have come to expect from the simulations. The two basic patterns of change are evident:

Figure VII. Comparison of ratios of actual to expected population at the time of the second census, based on simulated errors in censuses and deaths and on simulated changes in the age distribution of the population

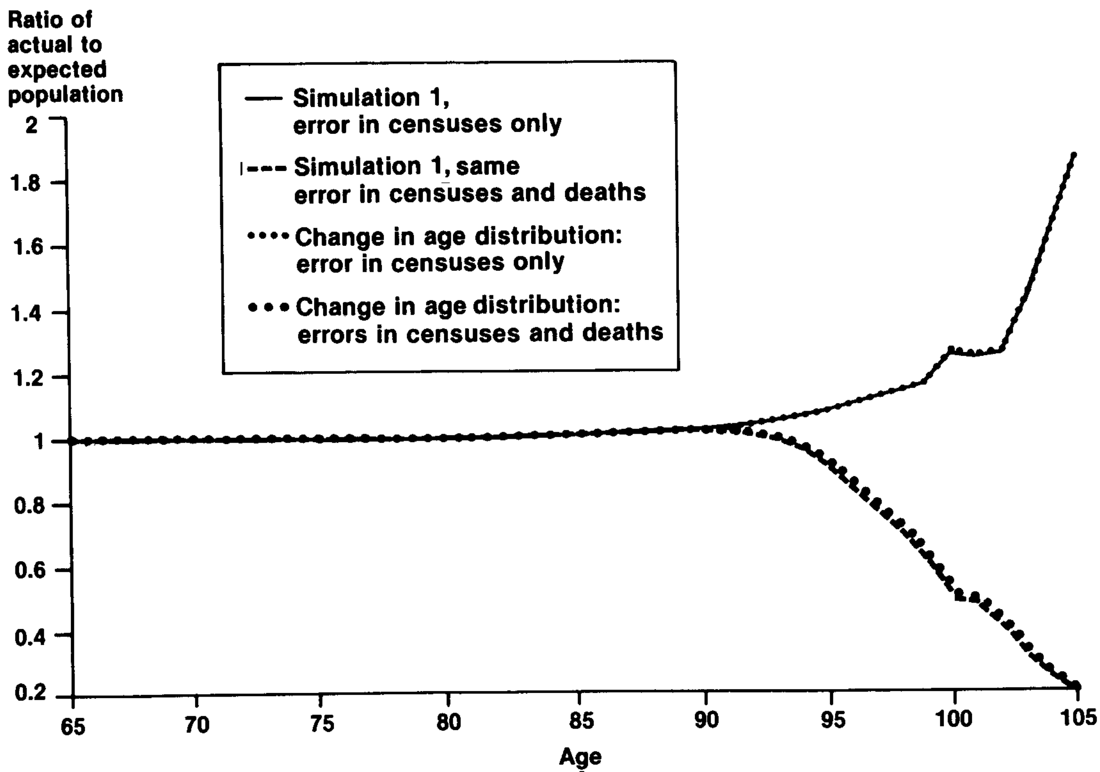


Figure VIII. Ratio of actual to expected population aged $x+$ at the time of the second census: Sweden, females

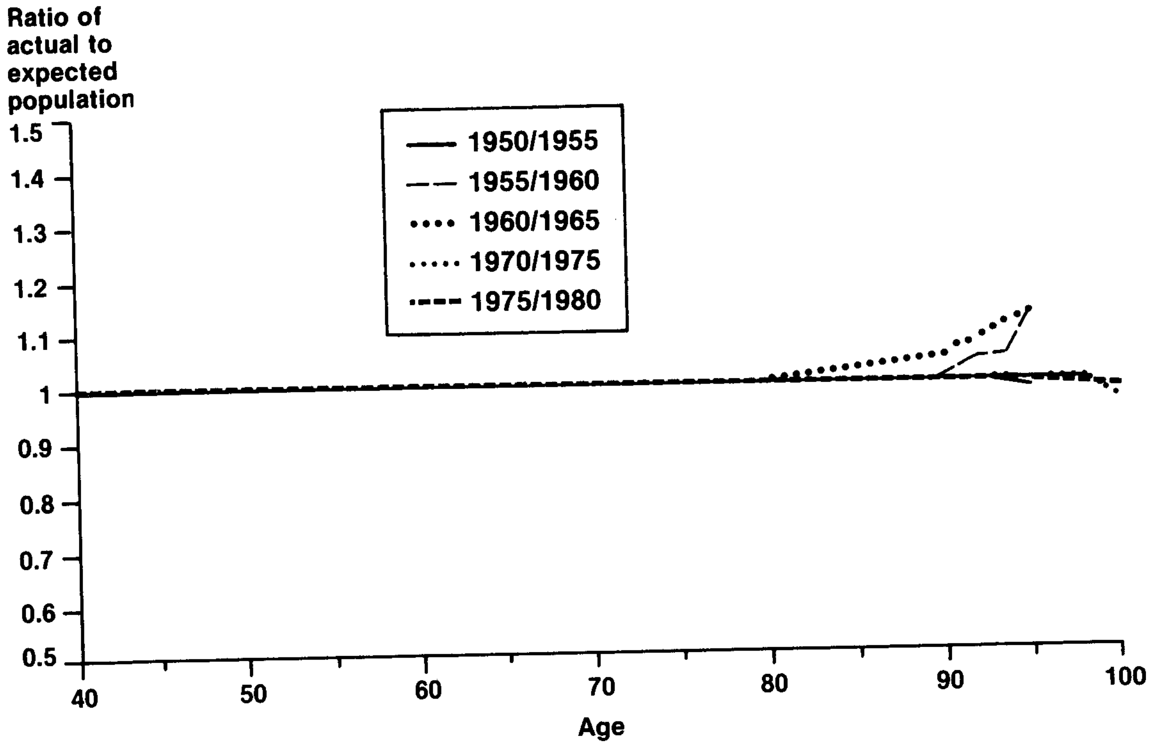
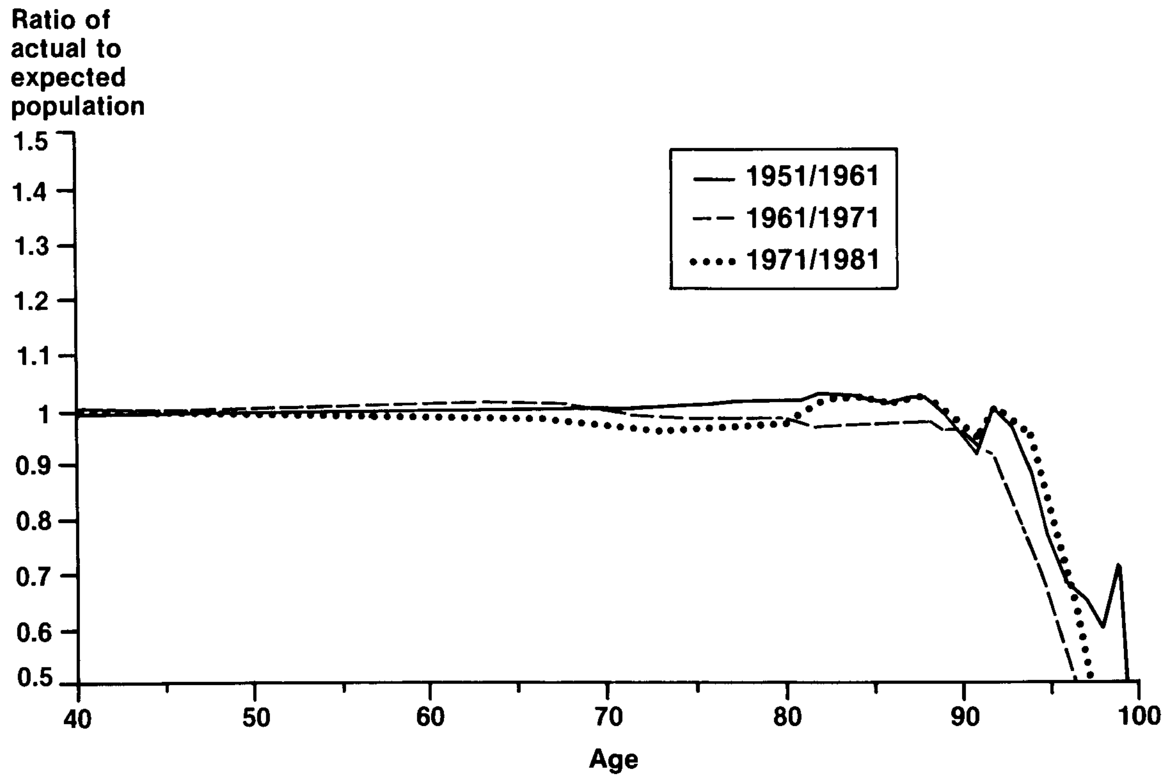


Figure IX. Ratio of actual to expected population aged $x+$ at the time of the second census: Italy, females



declining ratios with age, where reported populations fell increasingly below expected populations, and rising ratios, indicating actual populations that were increasingly larger than expected population with advancing age. For females, the age patterns of changing ratios in the countries examined are as follows:

Generally declining ratios	*Generally rising ratios*	*Slightly rising (overall more consistent)*
Austria	England and Wales	Sweden
Belgium	Scotland	Netherlands
Denmark (non-register)	Ireland	Finland (register)
Italy	Canada	Denmark (register)
Japan	Australia[a]	
Portugal	New Zealand[a]	
Spain	Finland (non-register)	
Norway (1970–1980)	Norway (1960–1970)	

[a]For Australia and New Zealand the patterns are more variable over time and/or across the age range than for the other countries. However, they were more likely to have ratios that rise to fairly high values than declining values. Hungary has not been classified because the age at the start of the open-ended interval (80) is too low to allow an assessment of the age pattern.

The grouping of countries is striking. As expected, countries operating a population register show quite good consistency between deaths and population counts. (An exception is Sweden from 1965 to 1970, which shows a considerable downturn in ratios after age 90.) Non-register countries form two clusters. English-speaking countries show generally rising ratios, while all other countries display falling ratios (though some have mild increases before the fall). Finland is an exception to this grouping during the period when it was not operating a population register.

The ratios for Italy shown in figure IX stay close to 1.0 until age 90 or a few years older and then drop off rapidly. The patterns for Austria and Belgium are very similar to that for Italy, while the Japanese data produced ratios that were more erratic after age 90. In Japan, for some time periods, values dipped very low and then rose again at advanced ages. Denmark's declining ratios begin at various ages, as early as 75 in the periods 1965-1970 and 1970-1975, but as late as age 98 in the 1960-1965 period.

Rising ratios characterize the United Kingdom, Canada, Australia and New Zealand. The latter two countries are included in this list somewhat tentatively, because, as noted above, they have an erratic pattern of change of ratios at advanced ages. The values for England and Wales, which show a much steadier rise, are graphed in figure X, while those for Australia are shown in figure XI. The ratios for England and Wales are very high and become negative at very advanced ages. Negative values occur when the number of deaths to a cohort exceeds the number of people left in the cohort, producing a negative expected population. Negative values occur

Figure X. Ratio of actual to expected population aged $x+$ at the time of the second census: England and Wales, females

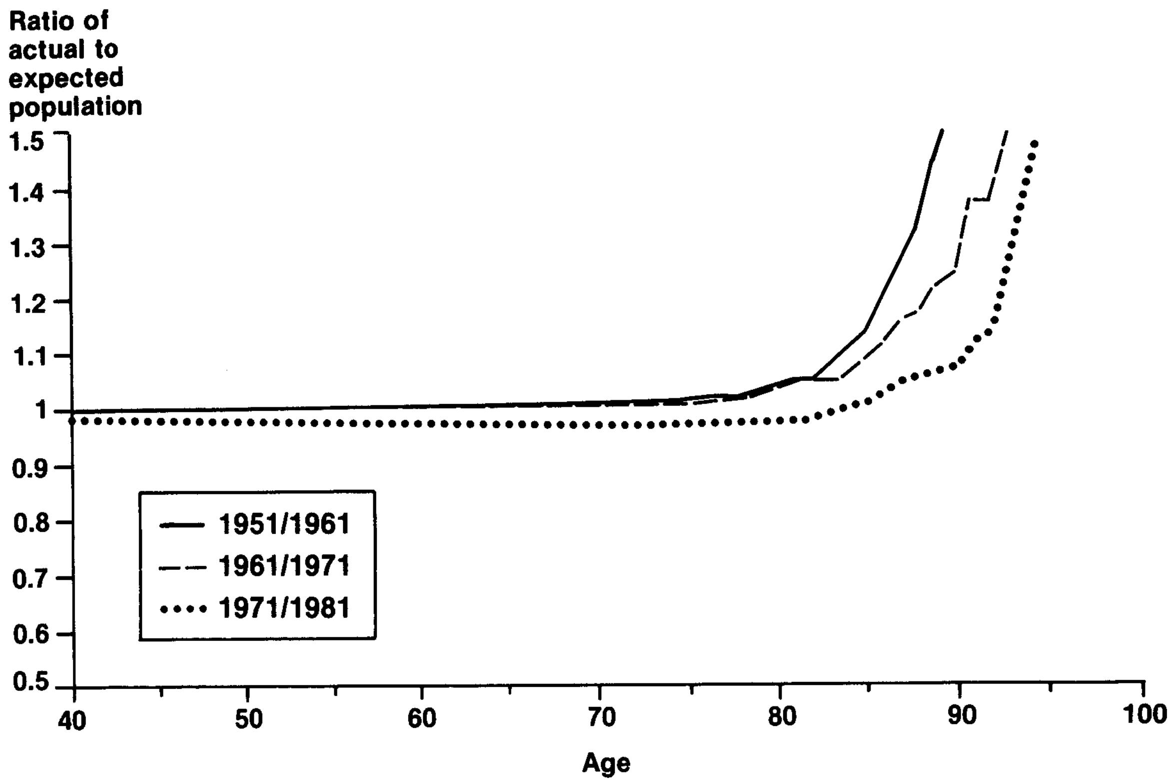
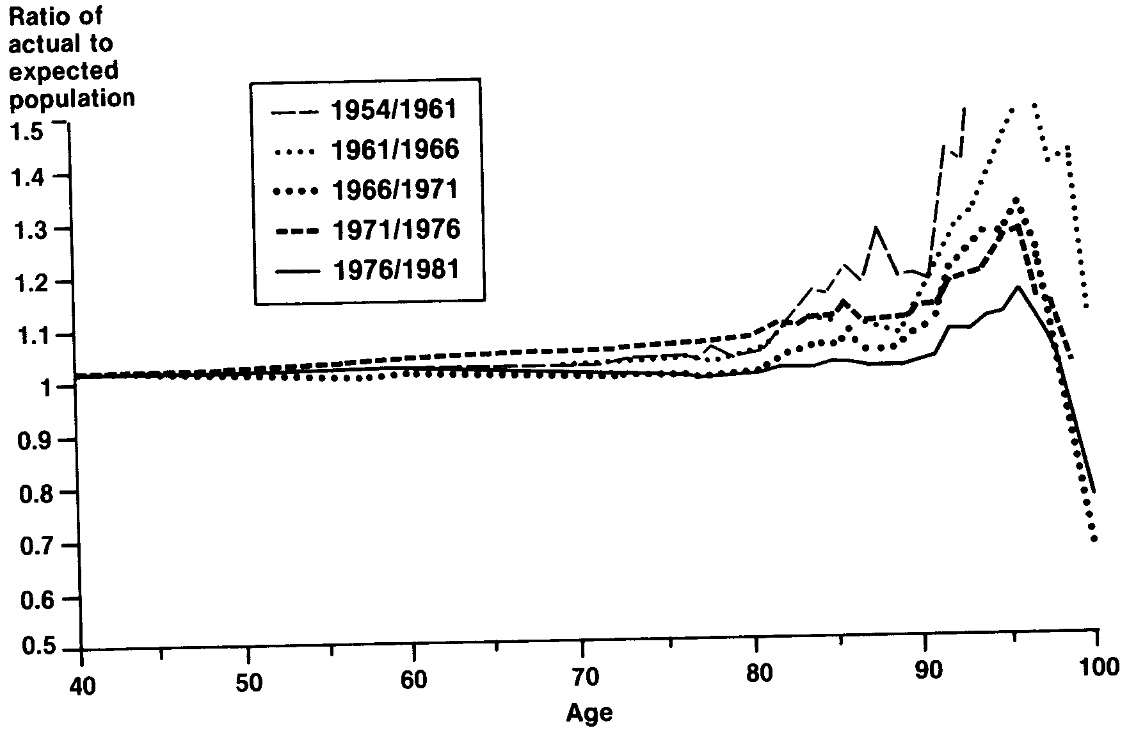


Figure XI. Ratio of actual to expected population ages $x+$ at the time of the second census: Australia, females



only after age 97 in the English and Welsh data; however, the inconsistency before that age is substantial. For other countries in this group, the data are rarely as inconsistent as for England and Wales; however, the inconsistency between sources revealed by the ratios is quite large above age 85.

A pattern of rising ratios has been found in previous research on a number of Latin American countries (Rosenwaike and Preston, 1984; Bhat, Dechter and Preston, 1988). Ratios for Latin American countries begin to rise at a younger age than for the European countries being analysed here and reach higher levels.

For one country, Portugal, the ratios of actual to expected population fluctuate widely from age to age, rising well above 1.0 and then dropping well below. The data for Portugal show substantial age heaping, which probably explains the unusual pattern of ratios. Spain also shows a great deal of age heaping, but lack of detailed data after age 80 makes it impossible to know whether age-heaping would produce the same fluctuations in the ratios there.

For most countries and time intervals, ratios of actual to expected populations calculated for males showed remarkably similar patterns of change to those for females. There are a few exceptions. Ratios for males in Austria from 1961 to 1971 rise with age, in contrast to the falling ratios for Austrian females during that time period. For males, ratios rise in all three time periods in Belgium, while ratios fall for Belgian females. Finally, the ratios for Italy rise above 1.0 and then fall for males. Female ratios in Italian data show a slight rise but one that is much smaller than that for males. In general, the differences between males and females are not great and the disparities are not regular. There appears, therefore, to be little difference between males and females in either the levels or patterns of age misreporting.

Table 6 presents the age at the beginning of the open-ended interval at the second census at which the level of error first exceeds 1 per cent and 5 per cent for each country. The age at which inconsistency exceeds these percentages varies a great deal from country to country. There is some tendency for data to improve over time, although there are exceptions, such as Japanese females. Finally, there is no regular sex difference in the age at which the ratios deviate from 1.0. The relative inconsistency for males and females varies in an unstructured way from time to time and from place to place.

One per cent and 5 per cent are, of course, arbitrary values. The effects of inconsistencies of these magnitudes on calculated mortality levels are currently being investigated. The size of the errors that can be tolerated will be determined by extending our simulation procedures to the effects of various patterns of age misreporting on the mortality levels that would be observed.

Table 7 summarizes the information presented in table 6 by presenting the number of data sets (out of 66 investigated) in which the inconsistency between actual and expected populations is less than 5 per cent. In

TABLE 6. AGES AT WHICH THE RATIO OF ACTUAL TO EXPECTED POPULATIONS DERIVED FROM INTERCENSAL COHORT ANALYSIS FIRST DEVIATES FROM 1.0 BY MORE THAN 1 PER CENT AND 5 PER CENT

		Age at which ratio deviates by more than				
		1%	*5%*	*1%*	*5%*	
Country	*Time period*	*Males*		*Females*		
Australia	30 July 1954–30 June 1961	a	82	a	82	
	30 June 1961–30 June 1966	a	82	a	80	
	30 June 1966–20 June 1971	a	83	a	84	
	30 June 1971–30 June 1976	a	68	a	70	
	30 June 1976–30 June 1981	a	92	a	91	
Austria..................	21 March 1961–12 May 1971	68	76	81	90	
	12 May 1971–12 October 1981	69	78	78	87	
Belgium	31 December 1947–31 December 1961	69	83	89	91	
	31 December 1961–31 December 1970	87	99	71	90	
	31 December 1970–1 March 1981	80	87	83	97	
Canada..................	1 June 1951–1 June 1956	a	83	60	81	
	1 June 1956–1 June 1961	a	80	a	79	
	1 June 1961–1 June 1966	62	85	a	82	
	1 June 1966–1 June 1971	a	81	a	78	
	1 June 1971–1 June 1976	83	90	a	89	
	1 June 1976–1 June 1981	b		b		
Denmark	7 November 1950–26 September 1960	63	88	62	89	
	26 September 1960–27 September 1965	90	97	90	97	
	27 September 1965–9 November 1970	76	81	76	82	
	9 November 1970–31 December 1975	74	80	67	80	
	31 December 1975–31 December 1980	82	92	80	90	R[c]
	31 December 1980–31 December 1985	84	98	81	91	R[c]
England and Wales	8 April 1951–23 April 1961	63	79	70	81	
	23 April 1961–25 April 1971	71	80	76	81	
	25 April 1971–5 April 1981	a	87	a	88	
Finland	31 December 1950–31 December 1960	66	81	70	83	
	31 December 1960–31 December 1970	69	87	67	83	
	31 December 1970–31 December 1975	75	90	76	91	
	31 December 1975–31 December 1980	91	94	92	97	R[c]
Hungary................	1 January 1960–1 January 1970	80	d	67	83	
	1 January 1970–1 January 1980	61	d	66	d	
Ireland	9 April 1961–17 April 1966	63	82	69	82	
	17 April 1966–18 April 1971	56	83	71	83	
	18 April 1971–1 April 1979	a	81	a	81	
	1 April 1979–5 April 1981	79	89	79	89	
Italy	4 November 1951–15 October 1961	68	87	76	91	
	15 October 1961–24 October 1971	90	93	60	81	
	24 October 1971–25 October 1981	53	69	a	95	
Japan	1 October 1955–1 October 1960	77	83	81	87	
	1 October 1960–1 October 1965	76	89	76	87	
	1 October 1965–1 October 1970	68	81	73	85	
	1 October 1970–1 October 1975	77	84	80	86	
	1 October 1975–1 October 1980	72	83	78	87	

TABLE 6 (*continued*)

		Age at which ratio deviates by more than				
		1%	*5%*	*1%*	*5%*	
Country	*Time period*	*Males*		*Females*		
Netherlands	31 December 1955–31 December 1960	89	93	85	93	R[c]
	31 December 1960–31 December 1965	96	96	87	96	R[c]
	31 December 1965–31 December 1970	96	99	75	85	R[c]
	31 December 1970–31 December 1975	81	88	81	87	R[c]
	31 December 1975–31 December 1980	82	90	86	98	R[c]
New Zealand	17 April 1951–17 April 1956	72	82	a	82	
	17 April 1956–18 April 1961	77	86	72	82	
	18 April 1961–22 March 1966	[a]	82	[a]	82	
	22 March 1966–23 March 1971	[a]	84	[a]	83	
	23 March 1971–23 March 1976	[a]	82	[a]	82	
	23 March 1976–23 March 1981	75	90	81	90	
Norway.................	1 November 1960–1 November 1970	78	87	[a]	85	
	1 November 1970–1 November 1980	82	90	81	90	
Portugal	15 December 1950–15 December 1960	[a]	71	[a]	74	
Scotland	8 April 1951–23 April 1961	65	81	71	81	
	23 April 1961–25 April 1971	75	84	76	81	
	25 April 1971–5 April 1981	83	90	93	93	
Spain	31 December 1950–31 December 1960	[a]	77	[a]	77	
	31 December 1960–31 December 1970	71	79	68	78	
Sweden	31 December 1950–31 December 1955	91	95	[e]		R[c]
	31 December 1955–31 December 1960	82	92	90	94	R[c]
	31 December 1960–31 December 1965	79	88	81	91	R[c]
	31 December 1965–31 December 1970	82	89	83	89	R[c]
	31 December 1970–31 December 1975	96	98	[a]	100	R[c]
	31 December 1975–31 December 1980	98	[d]	[a]	99	R[c]

[a] Ratio deviates from 1.0 by more than 1 per cent at all ages.

[b] Ratio period deviates from 1.0 by more than 1 per cent at all ages but never deviates by more than 5 per cent.

[c] Data from population registers.

[d] Ratio never deviates from 1.0 by more than 5 per cent.

[e] Ratio period never deviates from 1.0 by more than 1 per cent.

all 66 instances, inconsistencies are less than 5 per cent below age 65 for both males and females. At higher ages, inconsistencies become more frequent, to the point where only eight male and nine female data sets show less than 5 per cent inconsistency at all ages below 95. Clearly, estimation of mortality for the extremely old will have to emphasize these cases.

CONCLUSIONS

Although accurate estimates of mortality are crucial to planning for the elderly populations of low-mortality countries, knowledge of the age

TABLE 7. NUMBER OF DATA SETS WITH LESS THAN 5 PER CENT INCONSISTENCY BETWEEN ACTUAL AND EXPECTED POPULATION BELOW SPECIFIED AGES

Age	*Males*	*Females*
60	66	66
65	66	66
70	64	66
75	63	64
80	58	60
85	34	38
90	21	24
95	8	9
100	1	1

patterns of mortality at advanced ages has been severely hampered by the poor information available about data quality. In an effort to evaluate the population age distributions and death registration data needed to calculate mortality levels, data from 16 low-mortality countries, covering 67 intercensal periods from 1950 to 1985, were checked for consistency between census age distributions and recorded deaths. Ratios of reported population at the second census to the expected population showed two distinct patterns of inconsistency. In English-speaking countries and Finland, ratios tended to rise with age, while in continental Europe and Japan the predominant pattern was ratios that declined with age. Population-register countries showed a high degree of consistency and rates near unity at all but the highest ages.

Simulations carried out by introducing plausible patterns of age misreporting into perfect data indicate that declining ratios result when overstatement of age increases with age in censuses but not (or to a much lower extent) in deaths. On the other hand, when overstatement of age increases with age to the same extent in both deaths and censuses, an age pattern of rising ratios of actual to expected population is produced. On the basis of these simulations, it would appear that English-speaking countries and Finland tend to have patterns of age misstatement that are similar in censuses and death registration, whereas continental European populations have less overstatement of age in deaths than in censuses. This latter group, along with population-register countries, would appear to hold out the most promise for establishing sound estimates of mortality for the extremely old. For example, extinct generation methods are likely to prove more accurate there, since the quality of age reporting in deaths appears good, relative to age reporting in censuses.

ANNEX

Demonstration of one age-misreporting pattern that does not produce inconsistencies between actual and expected populations

There is at least one circumstance in which a pattern of age misreporting can result in a ratio of exactly 1.0 at all ages. If the pattern of age misreporting is identical in all three sources of data—the two population distributions and the death registration system—and if the pattern does not vary by age, then the population expected at the time of the second census will be exactly equal to the enumerated population at every age.

If deaths and populations are completely accurate, then

$$N_{i+j} = N_i - D_i \qquad (1)$$

where N_i = the enumerated population age i at the time of the first census,

D_i = the intercensal deaths to persons age i at the time of the first census,

N_{i+j} = the enumerated population age $i+j$ at the time of the second census, j years later.

Suppose that individuals can misreport their ages, but only into adjacent age intervals. Introducing error into each of the data sources gives new estimates of the populations and intercensal deaths; N_i', D_i', and N_{i+j}', which can be expressed in terms of the original data:

$$N \cdot i = xN_i + yN_{i+1} + zN_{i-1}$$

$$D_i' = xD_i + yD_{i+1} + zD_{i-1}$$

$$N_{i+j}' = xN_{i+j} + yN_{i+j+1} + zN_{i+j-1}$$

where $y + z = 1 - x$.

The expected population at the time of second census is now $\hat{N}_{i+j}' = N_i' - D_i'$, and each of these terms can also be expressed in terms of the original true population:

$$\hat{N}_{i+j}' = (xN_i + yN_{i+1} + zN_{i-1}) - (xD_i + yD_{i+1} + zD_{i-1})$$

Similarly, the enumerated population N_{i+j}' is a function of the underlying "true" population:

$$N_{i+j}' = x(N_i - D_i) + y(N_{i+1} - D_{i+1}) + z(N_{i-1} - D_{i-1})$$

Rearranging terms shows that the ratio of the enumerated population to the expected population, $N_{i+j}' / \hat{N}_{i+j}'$ is equal to 1.0.

This result does not depend on the values of $x, y,$ and z. The result is also not dependent upon the distribution of misstated ages, so long as the distribution is constant across ages. This result can be generalized to misreporting patterns that extend beyond adjacent age intervals.

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THE DEMOGRAPHY OF DISABILITY

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SUMMARY

Many countries have a long history of collecting disability data through population censuses, sample surveys and disability registration. Such disability data are usually cross-classified by various social, economic and demographic characteristics and are therefore important for identifying the size, the type and the various social, economic and demographic attributes of the disabled population.

This article examines selected demographic characteristics of disabled persons. It analyses the general levels of disablement in various countries and compares their age and sex compositions. Since variations in methods of data collection and classification among countries have contributed greatly to the differences in the measurement of levels of disablement, this paper discusses the need for common approaches to collection, classification, tabulation and analysis so as to improve understanding of the issues of disability.

THE UNITED NATIONS DISABILITY STATISTICS DATA BASE

The United Nations Statistical Office has compiled national disability data and constructed a statistical data base called the United Nations Disability Statistics Data Base (DISTAT).[1] The data base is designed to "inform policy-makers, programme planners and researchers of the availability of statistics on disabled persons and to offer examples of their potential uses". Since DISTAT is a microcomputer data base, it permits "rapid and efficient retrieval of data to identify demographic and socio-economic trends concerning the prevalence of various impairments and disabilities and to address policy-related requirements, such as estimating programme service needs" (United Nations, 1988a). Through DISTAT, it is also hoped that the development of national disability statistics can be improved and consolidated on a common conceptual framework so that international and national comparison of disability data will be facilitated.

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DISTAT contains information on the availability of statistics on disability from population censuses and surveys for 95 countries or areas for various years between 1960 and 1986 and detailed statistics for 55 of those countries and areas. However, it is important to observe that since there have been no international recommendations on how disability statistics should be collected and compiled, each country has tended to develop its own system of collection, classification and tabulation not necessarily comparable to any other system. Therefore, the data contained in DISTAT have various kinds of classifications. Many may not be comparable to the International Classification of Impairments, Disabilities and Handicaps (ICIDH) (World Health Organization, 1980). Nevertheless, despite the difficulties in making international comparisons on levels and patterns of disablement, DISTAT is valuable for conceptualizing and assessing various characteristics of the disabled, impaired and handicapped population.

Characteristics of disability statistics

The statistics contained in DISTAT are compiled from population censuses, disability or household surveys and a variety of disability registration systems. Strengths and weaknesses of various data collection systems are discussed in detail in a number of United Nations reports (United Nations, 1986, 1988b and 1989). When disability information is collected from population censuses, a question concerning the presence of disabled persons may be asked for every household or for a selected sample of households. The information may also be obtained from just the sub-population that is not economically active or from the child and youth sub-populations who are not attending school because of disability. However, information obtained from such sub-populations does not cover all the disabled persons in those sub-populations, because persons who are disabled but economically active or attending school would not be counted.

Sample surveys—whether a specially designed disability survey or a special disability module attached to a general survey—have been used to collect disability data. Good survey data can provide detailed characteristics of the disabled population for a country as a whole or for major political divisions, but are limited by small cell sizes when an estimate of disability by small local areas is attempted.

Disability information can also be obtained from various administrative recording systems, such as population registration, civil registration, social security systems, health and family planning systems, industrial recording of occupational injuries etc. Each of these can provide information on certain characteristics of the disabled persons covered by the systems. A comprehensive disability registration system closely linked with the population registration or civil registration system may be developed. However, the reliability and validity of the data will depend upon how well the disability data system is developed and maintained.

Using a standard demographic approach, a simple indicator may be used to measure the level of disability. This is the disability rate, the proportion of population that is disabled in the total population. The rate is called the "crude disability rate" when it relates all disabilities to the population as a whole. Other rates, such as age/sex-specific disability rate and the type-specific disability rate, can also be computed when the required data are available. The crude disability rate and the attribute-specific rate are generally expressed in units per 100 but may also be expressed in units per 1,000 or per 100,000 population, depending upon the frequency of occurrence of the attributes of the disabled population. In this article, units per 100 are used to express the crude disability rate and the age-specific disability rates in most tables. Units per 1,000 are also used in some tables for purposes of illustration.

Different methods of data collection generate disability rates that are not strictly comparable among countries or even between two surveys in a given country. Data obtained from sample surveys or registration tend to give higher estimates of disability rates than do those from population censuses, because surveys tend to use disability or functional limitation questions in addition to impairment questions for identifying disabled persons. A single question assessing functional limitation, or disability, typically embraces symptoms associated with a broad range of impairment conditions (United Nations, 1989).

CRUDE DISABILITY RATES

The overall level of disability among the 30 censuses or surveys given in table 1 varies between 0.2 and 20.9 per cent. Two thirds of the data sources indicated a rate of 5 per cent or under; half are under 2 per cent. In the six cases that showed a rate of over 10 per cent, all obtained their high rates on the basis of sample surveys using a disability question to identify the disabled population. Among the 10 low-disability populations, all but three based their estimates on population censuses using impairment questions to identify the disabled population.

In table 1, there are three countries (Canada, Spain and Tunisia) with disability rates for two different dates. Tunisia obtained its low disability rates from its population censuses carried out in 1975 and 1984, which show very similar levels of disability—0.8-0.9 per cent—in the 1975-1984 period. Canada obtained its estimated high disability rates—11.2 per cent for 1983 and 13.2 per cent for 1986—through specially designed disability surveys. In Spain, the 1981 census shows a crude disability rate of 5.1 per cent, but the 1986 survey gives a high disability rate of 15.0 per cent. Generally, the level of disability in a country would not change so rapidly in such a relatively short time. The large difference between the two rates is clearly a result of using different concepts of disability in the two data-collection activities.

TABLE 1. CRUDE DISABILITY RATES BY SEX

Country or area	*Data source*	*Year*	*Total*	*Rate (percentage)* *Males*	*Females*
Peru	Census	1981	0.16	..	..
Egypt	Census	1976	0.30	0.44	0.16
Kuwait	Census	1980	0.44	0.50	0.35
Pakistan	Census	1981	0.45	0.38	0.53
Sri Lanka	Census	1981	0.49	0.58	0.40
Thailand	Survey	1981	0.77	0.88	0.66
Tunisia	Census	1975	0.78	0.95	0.61
Hong Kong	Census	1981	0.84	..	..
Tunisia	Census	1984	0.87	1.07	0.66
Fiji	Survey	1982	0.90	1.70	0.50
Bahrain	Census	1981	0.99	1.08	0.87
Indonesia	Census	1981	1.14	..	..
Turkey	Census	1975	1.46	1.71	1.19
Egypt	Survey	1979-1981	1.51	1.84	1.17
St. Helena	Census	1976	1.61	1.47	1.75
Comoros	Census	1980	1.70	1.94	1.46
Swaziland	Survey	1983	2.51	..	..
Netherlands Antilles	Census	1981	2.86	3.26	2.49
Nepal	Survey	1980	3.00	..	..
Mali	Census	1976	3.02	3.08	2.96
Philippines	Census	1980	4.42	5.10	3.75
China	Survey	1987	4.90	..	..
Spain	Census	1981	5.10	6.30	4.80
Ethiopia	Survey	1979-1981	5.49	(rural)	..
Poland	Census	1978	7.10	..	..
Germany, Federal Republic of[a]	Survey	1983	10.80	11.80	9.80
Canada	Survey	1983	11.24	10.60	11.80
Canada	Survey	1986	13.20	..	..
Australia	Survey	1981	13.24	..	..
Spain	Survey	1986	15.00	..	..
Austria	Survey	1976	20.88	19.89	21.76

Source: United Nations Disability Statistics Data Base, 1975-1986: Technical Manual (United Nations publication, Sales No. 88.XVII.12).

[a] With effect from 3 October 1990, the Federal Republic of Germany and the German Democratic Republic united to form one sovereign State under the designation "Germany".

Table 1 also shows that the crude disability rate of Austria in 1978 (20.9 per cent) was extremely high, suggesting that one in every five Austrians was a disabled person. One should bear in mind that the concept of disability is not the same across countries: many of the "disabled" in one country would not be considered disabled in another. Physical and severe mental disabilities can be identified easily and clearly, but minor aural and ocular disabilities tend to be ignored in either survey designs or in field investigations, or both. The high disability rate of Austria may be attributed to the use of a very broad definition of disability which included large groups of population who had minor hearing and visual impairments. (Similar high levels of hearing and visual impairments have also been found in Denmark.)

Disability by age and sex

Disability is both a health issue and a social issue. It is also demographically related: the levels of disablement differ according to age, sex and other sub-groupings. Table 2 gives disability rates by age and sex for 18 countries or areas (19 censuses or surveys). Most countries classify their age data by five-year age groups, some by 10-year age groups, a few by much broader age groups or other types of groupings. For the oldest age group, some countries provide data for ages 60 years and over. A few have 65, 70, 75 or 80 years old and over. One country (Mali) reports data available for those aged 95 years and over. In a few countries, the data are limited to the population aged 10 or 15 years and over. In four countries, the data are not cross-classified by sex but are available only for both sexes combined.

It should be noted that data presented in table 2 are selected from DISTAT. There are disability data by age and sex for other countries available in DISTAT. Some countries or areas, through either their surveys or registration, have obtained the number of disabled population; however, the disability rates by age and sex cannot be computed because the corresponding total populations are not available.

Towards demographic interpretation of disability

Some special features of the disabled population can be seen from table 2 and figure I. First, the level of disability increases with the age of the population. Secondly, the levels of disability of children and of the elderly differ significantly from those of other population groups. Thirdly, the level of disability of the male population differs from that of the female. These features will be discussed below.

The increase of disability with age is gradual and slow until after about age 50, when the disability rates increase much faster. In countries where the general level of disability is high, the increase of disability from one age group to the next is faster than in countries where the general level is low.

In figure I, the crude disability rates of Austria, Mali, the Philippines and Turkey are plotted along with the age-specific rates. The increase of disability as the age of population increases is clearly seen. It is important to point out that the patterns of increase in these four countries are very similar despite the difference in the levels of their disability. Conceptually, disabilities may be broadly classified into two types: those that are directly age- (or time-) related; and those that are not. The frequency of the former increases as a result of the deterioration of the condition of health of the population because of individual aging (see fig. II). The increase is very slow at younger ages but very rapid at older ages. The faster increase is the result of various kinds of morbidity, accidents, military conflicts and others, which may be reduced through the expansion of public health facilities and other preventive measures. These rates increase very rapidly dur-

TABLE 2. DISABILITY RATES (PER CENT) BY AGE AND SEX

Age	Males	Females	Both sexes
	Australia, 1981		
Total			13.24
0-4			3.50
5-14			6.10
15-24			5.77
25-34			7.98
35-44			10.82
45-54			16.87
55-64			27.45
65-74			35.46
75+			53.14
	Austria, 1976		
Total	19.89	21.76	20.88
0–5			1.61
6–9			3.38
10–14			4.36
15–19			5.10
20–29			7.29
30–39			12.78
40–49			18.73
50–59			33.97
60–69			45.34
70–79			59.96
80+			74.11
	Bahrain, 1981		
Total	1.08	0.87	
0-4	0.19	0.12	
5-9	0.62	0.37	
10-14	0.78	0.61	
15-19	1.09	0.63	
20-24	0.64	0.43	
25-29	0.38	0.27	
30-34	0.46	0.43	
35-39	0.57	0.61	
40-44	1.02	0.70	
45-49	1.67	0.97	
50-54	2.69	2.54	
55-59	3.79	2.81	
60-64	7.07	4.90	
65-69	8.75	5.79	
70+	13.17	12.60	
	Canada, 1983		
Total, 0+	10.61	11.85	11.24
Total, 15+	11.84	13.69	12.79
0-4	4.80	3.91	4.36
5-9	7.05	4.72	5.92
10-14	7.37	6.05	6.73
15-34	4.29	4.82	4.56
35-54	9.22	10.68	9.95
55-64	24.68	24.77	24.71
65+	37.86	39.21	38.62

Age	Males	Females	Both sexes
	Cape Verde, 1980		
Total, 10+	3.46	4.91	
10-14	0.44	0.40	
15-19	0.55	0.49	
20-24	0.95	0.81	
25-29	1.11	0.82	
30-34	1.66	0.74	
35-39	2.12	1.45	
40-44	1.38	1.23	
45-49	1.45	1.29	
50-54	2.64	2.16	
55-59	3.94	3.77	
60-64	6.67	10.75	
65-69	12.99	19.35	
70-74	24.86	30.48	
75+	43.96	50.75	
Not stated	5.43	12.73	
	Comoros, 1980		
Total	1.94	1.46	
0-4	0.63	0.46	
5-9	1.16	0.69	
10-14	1.35	0.96	
15-19	1.70	1.17	
20-24	1.91	1.19	
25-29	1.96	1.24	
30-34	2.28	1.64	
35-39	2.46	2.02	
40-44	2.94	2.23	
45-49	3.13	2.26	
50-54	2.99	3.02	
55-59	3.62	2.95	
60-64	4.20	3.75	
65-69	5.23	4.00	
70-74	5.88	4.46	
75-79	7.21	6.54	
80-84	8.57	8.02	
85+	11.61	11.71	
Not stated	2.40	1.38	
	Egypt, 1976		
Total	0.44	0.16	
0-4	0.02	0.01	
5-9	0.17	0.11	
10-14	0.32	0.15	
15-19	0.36	0.15	
20-24	0.45	0.13	
25-29	0.51	0.11	
30-34	0.61	0.13	
35-39	0.65	0.13	
40-44	0.70	0.16	
45-49	0.73	0.16	
50-54	0.81	0.23	
55-59	0.90	0.26	
60-64	0.98	0.42	

TABLE 2 (*continued*)

Age	*Males*	*Females*	*Both sexes*
65-69	1.03	0.54	
70-74	1.22	0.81	
75+	1.53	1.18	
Not stated	2.37	0.54	
	Ethiopia (rural), 1979-1981		
Total			5.49
0-4			1.42
5-9			2.14
10-14			3.03
15-19			3.89
20-24			5.11
25-29			5.88
30-34			6.91
35-39			7.26
40-44			9.93
45-49			10.33
50-54			12.63
55-59			13.58
60-64			15.00
65+			20.73
Not stated			13.55
	Kiribati, 1978		
Total 15	0.68	0.39	
15-19	0.33	0.27	
20-24	0.43	0.31	
25-29	0.61	0.23	
30-34	0.31	0.19	
35-39	0.42	0.00	
40-44	0.87	0.26	
45-49	1.53	0.62	
50-54	0.82	0.41	
55-59	0.88	0.81	
60-64	1.65	1.17	
65-69	2.56	1.24	
70+	1.06	1.18	
	Kuwait, 1980		
Total	0.50	0.35	
0-4	0.07	0.08	
5-9	0.47	0.38	
10-14	0.91	0.56	
15-19	1.04	0.63	
20-24	0.44	0.33	
25-29	0.24	0.18	
30-34	0.23	0.15	
35-39	0.27	0.16	
40-44	0.32	0.27	
45-49	0.40	0.27	
50-54	0.62	0.47	
55-59	1.30	0.56	
60-64	1.86	0.95	
65-69	3.62	1.47	

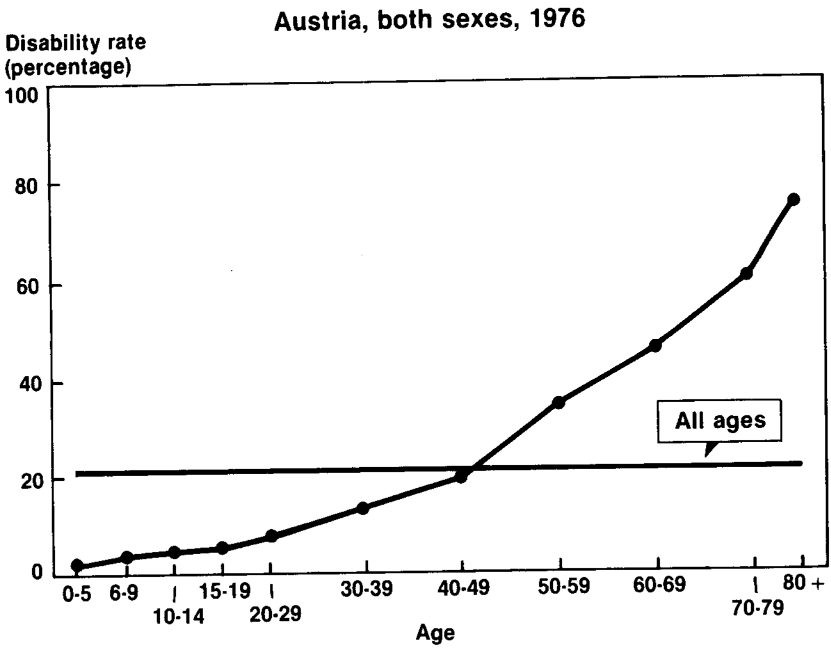
Age	*Males*	*Females*	*Both sexes*
70-74	4.60	2.28	
75-79	6.12	3.05	
80-84	7.18	3.85	
85+	7.20	6.27	
	Mali, 1976		
Total	3.08	2.96	
0-4	0.20	0.19	
5-9	1.13	0.69	
10-14	1.32	1.11	
15-19	1.46	1.54	
20-24	2.19	2.16	
25-29	2.54	2.68	
30-34	3.52	3.54	
35-39	4.31	4.18	
40-44	5.40	5.35	
45-59	6.42	6.42	
50-54	7.62	7.89	
55-59	8.96	9.09	
60-64	11.03	11.00	
65-69	13.08	12.90	
70-74	15.88	14.54	
75-79	18.62	16.29	
80-84	18.56	17.04	
85-89	20.94	18.92	
90-94	20.76	17.33	
95+	21.94	19.85	
Not stated	3.78	4.28	
	Netherlands Antilles, 1981		
Total	3.26	2.49	
0-4	0.70	0.92	
5-9	1.92	1.41	
10-14	3.23	2.15	
15-19	2.96	1.78	
20-24	2.86	1.66	
25-29	2.49	1.71	
30-34	2.45	1.35	
35-39	2.31	1.80	
40-44	3.10	1.50	
45-49	3.22	2.14	
50-54	3.89	2.46	
55-59	5.08	2.95	
60-64	6.68	5.08	
65-69	6.74	4.97	
70-74	10.87	8.06	
75-79	14.42	12.74	
80-84	20.43	17.60	
85+	29.33	25.39	
	Pakistan, 1981		
Total	0.38	0.53	
0-4	0.11	0.09	
5-9	0.28	0.18	
10-14	0.29	0.27	
15-19	0.26	0.39	
20-24	0.26	0.50	

TABLE 2 (*continued*)

Age	Males	Females	Both sexes
25-29	0.24	0.63	
30-34	0.27	0.66	
35-39	0.27	0.56	
40-44	0.31	0.54	
45-49	0.30	0.56	
50-54	0.46	0.78	
55-59	0.66	0.88	
60+	1.81	2.85	
Philippines, 1980			
Total	5.10	3.75	
Under 1	0.00	0.00	
1-4	1.67	1.02	
5-9	2.72	2.33	
10-14	3.53	2.89	
15-19	2.78	1.84	
20-24	3.66	2.58	
25-29	4.58	2.35	
30-34	4.42	3.28	
35-39	5.03	3.69	
40-44	11.46	5.27	
45-49	9.01	6.67	
50-54	9.87	7.96	
55-59	15.27	8.80	
60-64	17.30	12.23	
65-69	19.92	10.93	
70-74	26.67	20.75	
75+	25.15	32.48	
Thailand, 1981			
Total	0.88	0.66	
0-6	0.37	0.51	
7-10	0.65	0.44	
11-14	0.82	0.62	2.51
15-19	0.87	0.81	
20-24	1.04	0.65	
25-29	0.66	0.25	
30-34	0.80	0.56	
35-39	1.03	0.50	
40-49	1.26	0.41	
50-59	1.51	1.03	
60+	2.38	2.22	
Trinidad and Tobago, 1980			
Total 15+	1.31	0.92	
15-19	0.63	0.45	
20-24	0.72	0.46	
25-29	0.81	0.44	
30-34	1.01	0.50	
35-39	1.06	0.58	
40-44	1.28	0.74	
45-49	1.72	0.94	
50-54	2.33	1.73	
55-59	3.19	2.42	
60-64	4.56	3.65	
65+	1.99	1.96	
Tunisia, 1975			
Total	0.95	0.61	
0-4	0.07	0.09	
5-9	0.34	0.22	
10-14	0.58	0.39	
15-19	0.74	0.42	
20-29	0.89	0.47	
30-39	0.89	0.48	
40-49	1.29	0.65	
50-59	2.05	1.11	
60+	4.38	4.34	
Unstated	0.93	1.06	
Tunisia, 1984			
Total	1.07	0.66	
<5	0.07	0.08	
5-14	0.44	0.31	
15-59	1.22	0.69	
60+	4.15	3.20	
Turkey, 1975			
Total	1.71	1.19	
0-4	0.61	2.08	
5-9	0.75	0.63	
10-14	1.07	0.89	
15-19	0.97	0.68	
20-24	1.20	0.67	
25-29	1.25	0.66	
30-34	2.14	0.80	
35-39	1.87	0.86	
40-44	2.06	0.95	
45-49	2.28	1.01	
50-54	2.64	1.35	
55-59	3.38	1.70	
60-64	3.91	2.16	
65+	5.41	3.46	
Not stated	75.93	15.14	

Source: United Nations Disability Statistics Data Base, 1975-1986: Technical Manual (United Nations publication. Sales No. 88.XVII.12).

Figure I. Age-specific disability rates



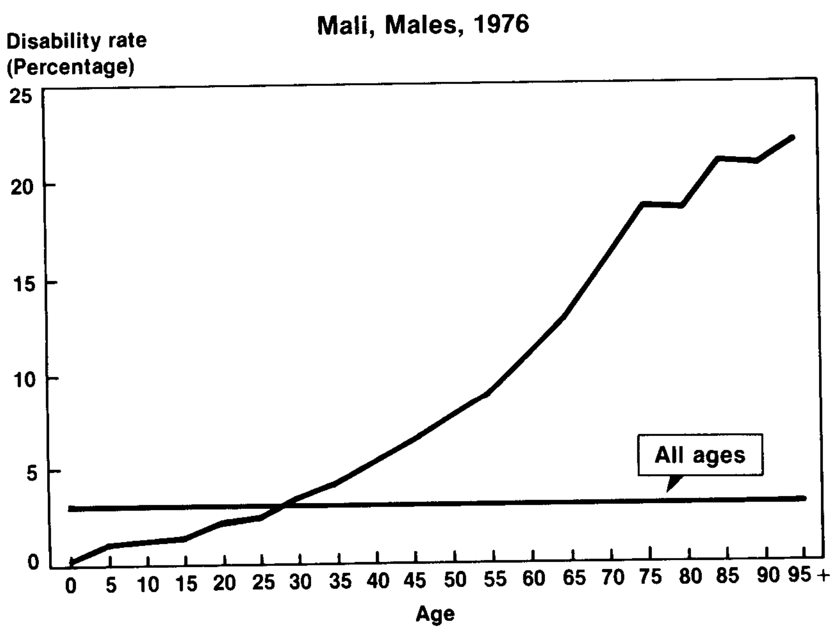
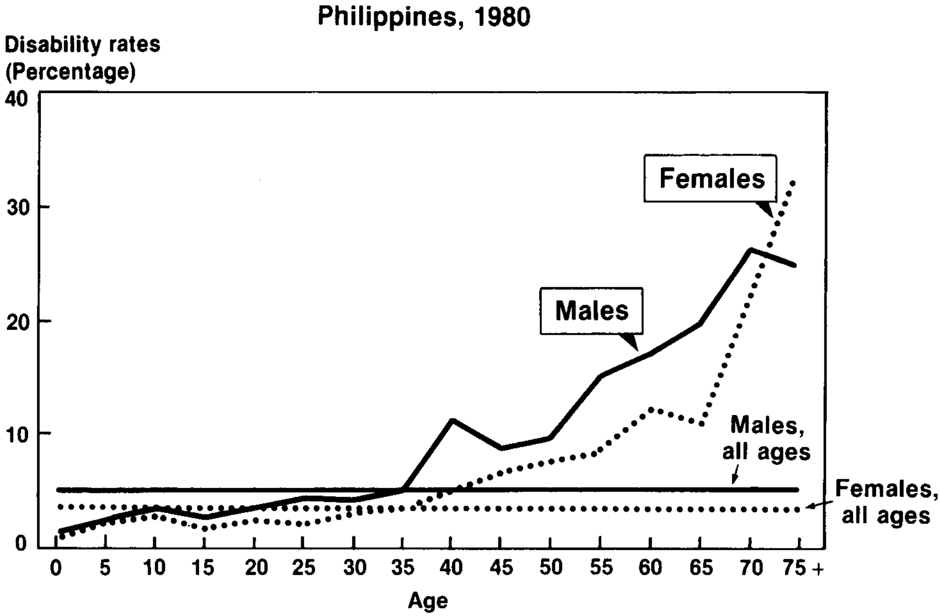
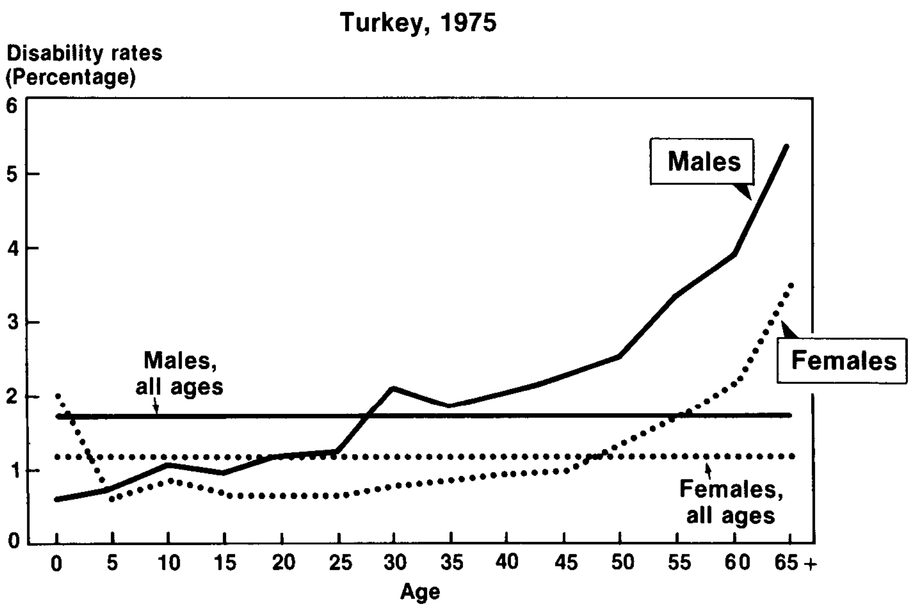


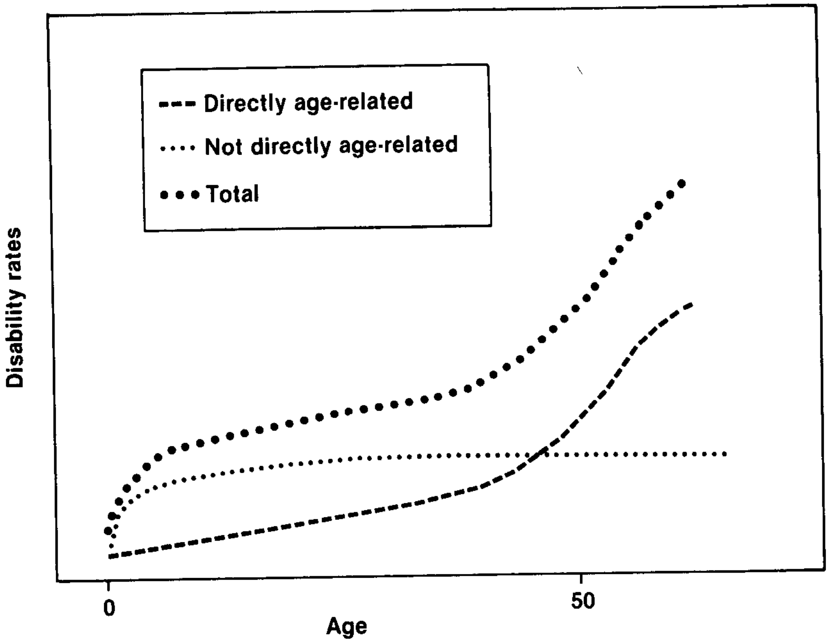
Figure I (*continued*)





Source: *United Nations Disability Statistics Data Base, 1975-1986: Technical Manual* (United Nations publication, Sales No. 88.XVII.12).

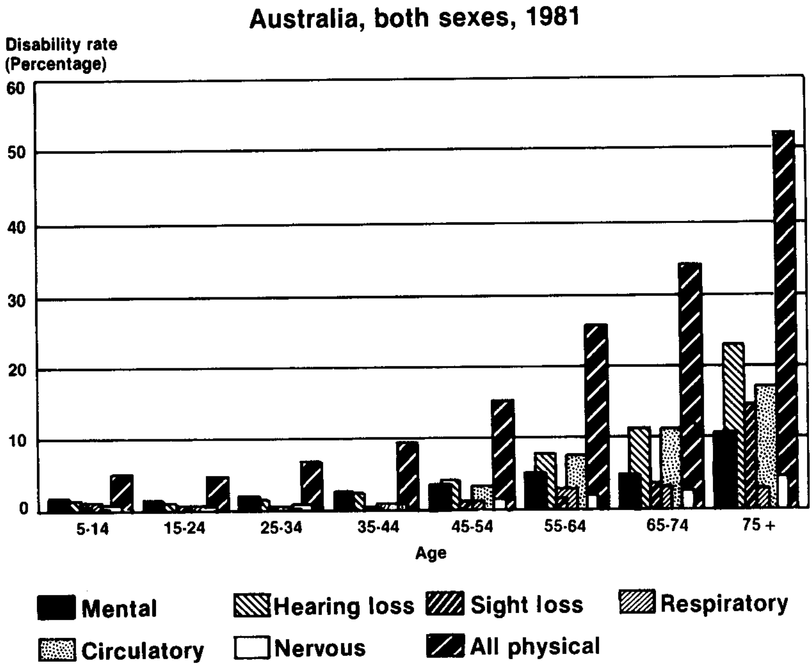
Figure II. Hypothetical curves of disability rates



ing the young ages but very slowly at the adult ages. The two types of increases can be conceptually shown in figure I but it would be difficult to show them statistically because the disability data are not collected according to those types of classifications.

The age effect on the level of disability can be seen from another perspective when the data are cross-classified by type of disability. Data in DISTAT show that the change of disability rates by age resulting from deafness, blindness and restrictions on physical movement are clearly linked with the deterioration of health as people age. Other types of disability, such as those related to mental and intellectual ability, may not be as directly related to age, but nevertheless increase with old age. Figure III gives disability rates by age and type of disability for Australia, Canada, Mali and Turkey. It is clear, in the case of Mali, that blindness is primarily responsible for the increase of disability as age increases. The proportions of other types of disability, such as leprosy, tuberculosis etc., increase only at the middle ages and either remain unchanged or decline as people become older. In Turkey (males), seeing disabilities (blindness and loss of sight in one eye) are also the main cause for the increase of disability as age of population increases. Other types of disabilities (lameness, crippling etc.) also increase with age, but much less markedly. The increases of old-age disability related to physical movement, agility etc. in Australia and Canada are quite drastic. However, part of the reason for the

Figure III. Disability rates by age, of selected types[a]



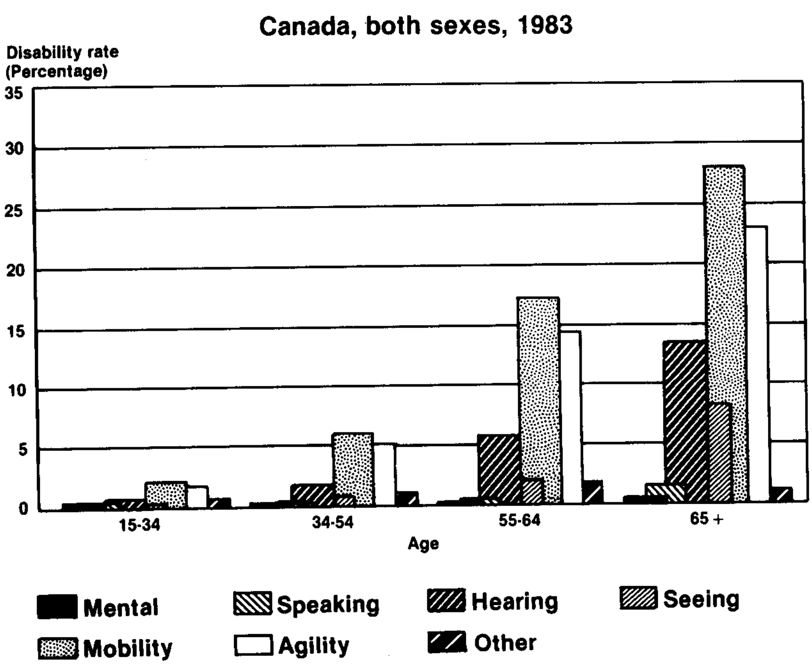


Figure III (*continued*)

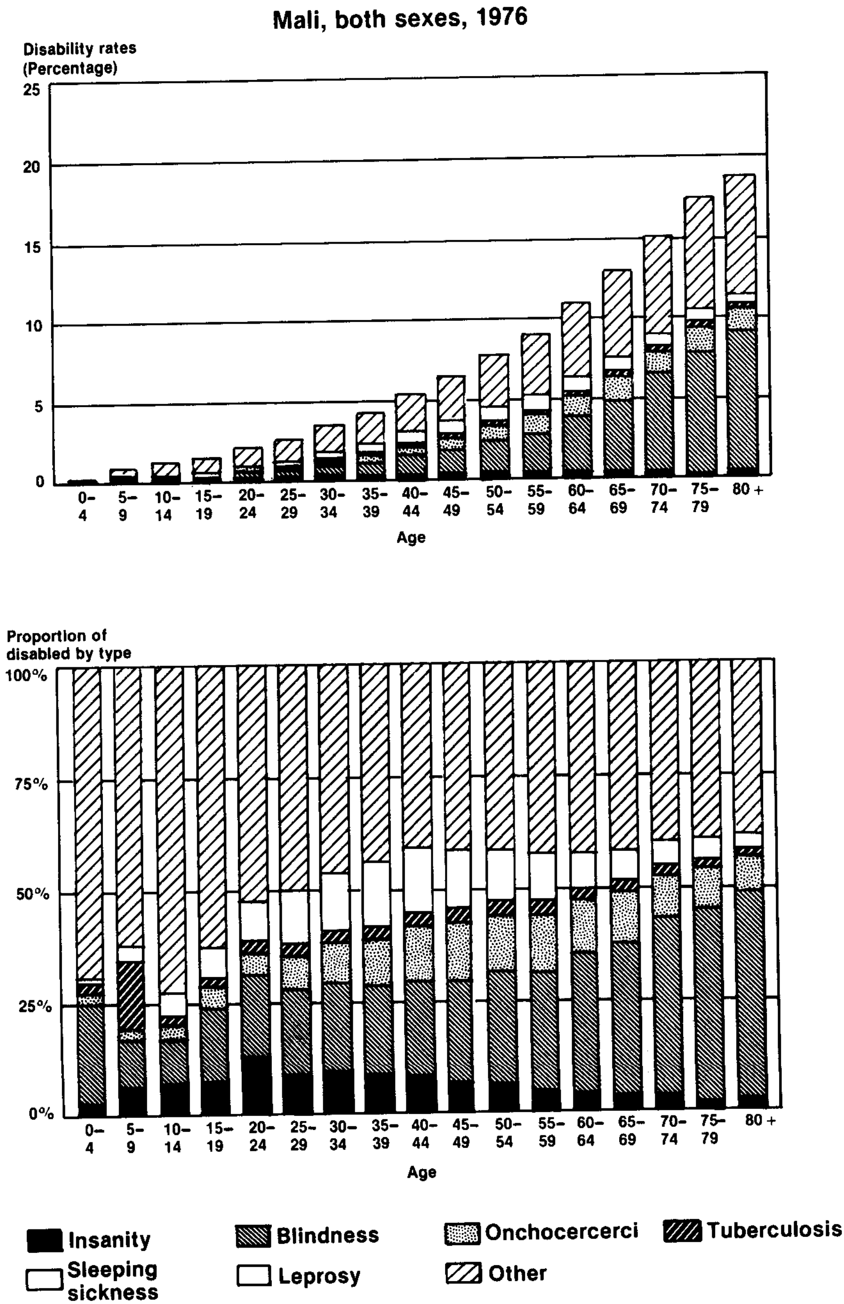
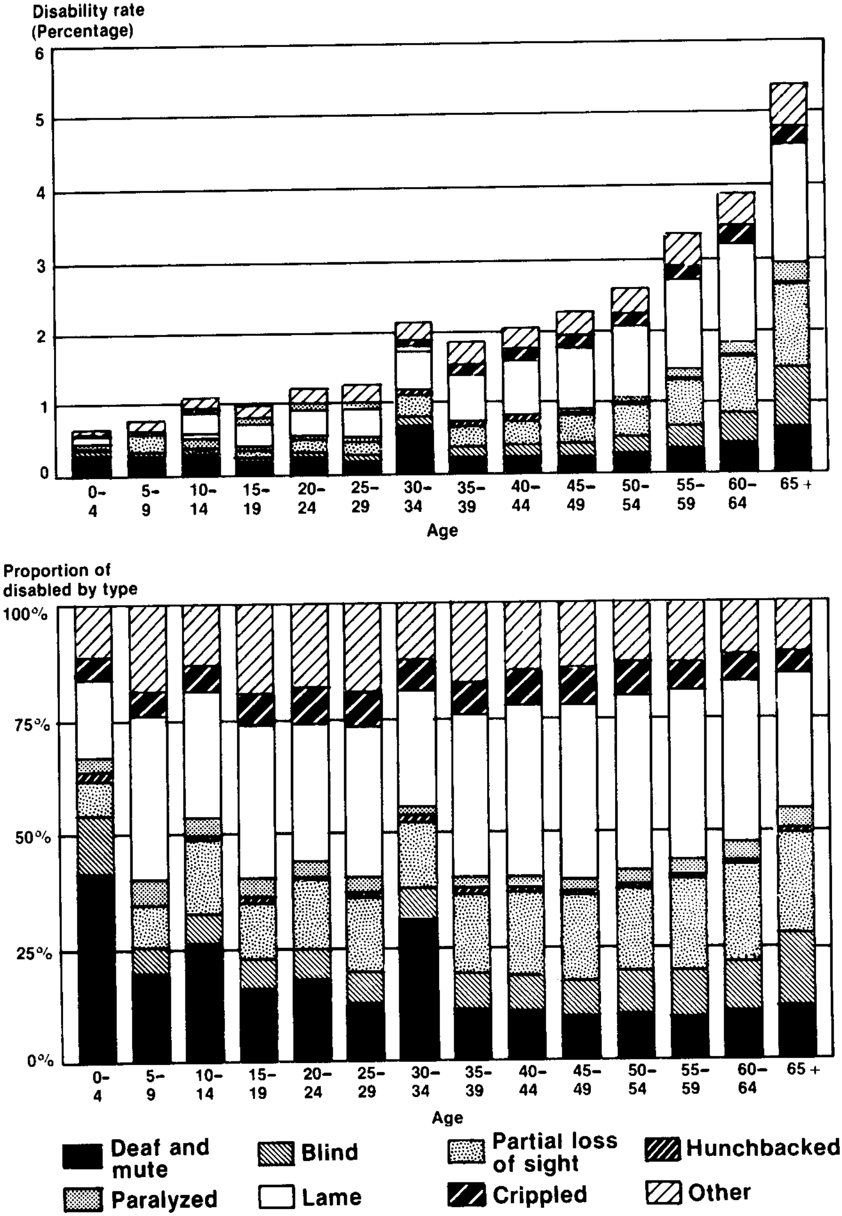


Figure III *(continued)*

Turkey, Male, 1975



Source: *United Nations Disability Statistics Data Base, 1975-1986*: *Technical Manual* (United Nations publication, Sales No. 88.XVII.12).

[a] Vertical bars show age-specific rates for each type of disablility. Types shown are not necessarily mutually exclusive (see text).

increase is that a person may have multiple disabling conditions and therefore be counted in more than one category of disability

The high disability rates in the population aged 60 and over point to the need for special attention to that sub-group of the population. The increases in rates from ages 60-64 to 65-69 and over are very rapid, and are found in almost every type of disability, whether or not they are directly age-related. Presumably, as the age of the population increases, the health conditions of the older members deteriorate and they acquire certain disabilities. The majority of those who survive to very old ages will be disabled.

Despite the great variations in the disability rates among countries, the differences in the rates between those under age 15 and those aged 60 and over are quite consistent among countries. The data in table 2 show that there is a high correlation between the rates of the two age groups (fig. IV). A country that has low disability rates for children tends to have low old-age rates and vice versa. If that relationship can be generalized to other populations, one may, in the absence of the actual data, estimate the level of disability of the old ages from that of the young and vice versa, or may verify the quality of the age data. The relationship can also be used to investigate the implication of the level of old-age disability from that of children or youth. However, more data and analyses are needed before any reliable generalization can be made.

Figure IV. Disability rates (per 1,000) for persons aged under 15 and persons aged 60 and over

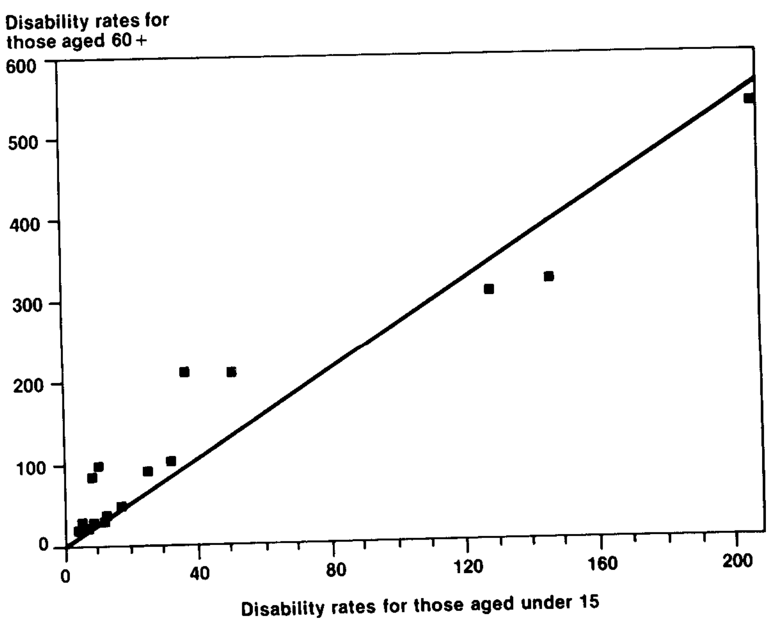


Table 1 shows that males generally have higher crude disability rates than females for most countries, and table 2 shows that this is true for most age groups. The countries where male rates are significantly greater than female rates include Egypt, the Philippines, Thailand and Tunisia. However, in Austria, Canada and Pakistan, the female rates are higher than the male rates for the population as a whole and in the population aged 15 years old and over. There is no self-evident reason why males should become disabled more easily than females, or vice versa. The cases in Egypt, the Philippines, Thailand, Tunisia and Pakistan, where sex differentials appear to be too large, suggest the possibility of certain biases in data collection and should be examined more carefully in order to determine the validity of the data. In general, further research on type-specific disability rates by age as well as sex will be required so as to provide a better understanding of expected differences between men and women's disability experience.

Disability, old age and the aging of population

The high disability rates of those aged 60 and over is clearly important from a public-policy perspective. In some countries, disability rates begin to rise very rapidly in the population aged 50-54 or 55-59. Since disability is a health issue, when individuals become older and health deteriorates, the level of disability increases. Moreover, the process of demographic aging is well established in most countries and is projected to continue, and indeed accelerate, during the first quarter of the next century.

Even if the rate of disability of the older ages remains unchanged in the future, the absolute size of the old-age disabled population will, of course, increase. This increase in the numbers of elderly disabled is likely to be faster than the growth of the total population, because the increase in the size of the older population will be greater than the increase in the size of the total population.

Despite the fact that the proportion of the young population will decline in an aging population, their absolute numbers may still increase, if the population is growing. Unless the level of disability can be reduced through effective programmes of prevention and rehabilitation, the numbers of disabled children and youth will also increase, even when the population as a whole is aging.

In the process of aging, the level of disability, particularly old-age disability, will increase. It may be conjectured that the majority of those old people who survive to very old ages will have certain types of disability, such as visual and hearing problems. Many will also have problems related to physical movement or communication. Those who now survive to become disabled persons would have died when the life expectancy at old ages was lower. Therefore, the disabled old-age population is likely to grow faster than the total old-age population in an aging society unless medical progress can delay the average age of the onset of disability. For those types of disability which are directly age-related, the disabilities are expected to increase at old ages but not at younger ages. This is an issue that deserves special attention in governmental policy and programmes on disability.

One must pay special attention to the increase in old disabled women in an aging society, because their increase will be greater and faster than that of old men. Generally, there are more old women in a population than old men, and they survive longer than men. The expectation of life at birth is still increasing in all countries, and the increase in female life expectancy is more than that of male. It can be anticipated that the female disabled population will be more than the male in the future, even if females may have slightly lower disability rates.

The need for better statistics and analyses

Table 2 and data in DISTAT have shown that disability statistics collected by many countries use very different classifications and age grouping. Some have detailed characteristics, including marital status, occupation, education, type of housing, income etc.; others are very simple and do not even classify the data by sex. It is not uncommon for detailed characteristics to have been collected but not tabulated. It should be recognized that very detailed cross-classified data may not be useful for certain countries in which the disabled population is small or in which the programmes or projects for improving the welfare of the disabled population are carried out on a limited scale. However, there are certain basic requirements for disability data. For example, the information for children under age 10 is very important and should be collected in all disability surveys. Detailed information for the older population is important, since a large proportion of the disabled population is elderly.

Many countries have not tabulated their data according to the International Classification of Impairments, Disabilities and Handicaps (ICIDH). However, there is a clear advantage to adopting this international classification. It not only facilitates international comparison of disability statistics but also can guide the development of national data in a comprehensive and systematic manner. It is recognized that for statistical compilation and analysis purposes, a summary if ICIDH is needed so that the disability data can be used more easily and efficiently (Chamie, 1989).

The demography of disability emerges as a new and important research subject as fertility starts to decline and population is aging at the global level. A considerable amount of disability data has been collected, but few disability studies have been made. There are formidable conceptual and analytical challenges to be dealt with. Since a person may acquire more than one kind of disability or handicap, or may not become disabled at all in life, the force of disability will not operate on the population in the same manner as the force of mortality. Recently, the study of disability-free life expectancy as an index for measuring the health progress of a society has been attempted (Robine and Michel, 1990), but more research is needed in order to make the index practical and useful. Furthermore, depending upon the concept that is adopted, a disability in one country may not be recognized as such in another. Nevertheless, data in DISTAT have shown common characteristics of disability in various populations. This is a new field that calls for further attention from the research community and from policy-makers.

NOTE

[1] DISTAT can be obtained, at a nominal cost, by writing to the Director, Statistical Office, United Nations, New York, NY 10017, U.S.A. The data base is stored in microcomputer diskettes and can be retrieved and manipulated using Lotus 1-2-3.

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APPLICATIONS OF THE HELIGMAN/POLLARD MODEL MORTALITY SCHEDULE

Andrei Rogers and Kathy Gard***

SUMMARY

The United Nations working manual for MORTPAK-LITE, a software package for demographic measurement, includes among its 16 computer programs a routine (UNABR) that graduates a set of age-specific probabilities of dying ($_nq_x$) for the standard set of five-year age groups into a set of single-year probabilities of dying. The graduation is effected with an eight-parameter "law of mortality" known as the Heligman/Pollard model mortality schedule. Model mortality schedules such as the Heligman/Pollard curve are useful in applications other than graduation. For example, their compact and transparent representations of mortality patterns are useful in mortality analysis and forecasting. Several such applications are illustrated in this article: a historical time-series analysis, a forecasting application, studies of causes of death, spatial differences and sex-specific mortality disaggregated by race.

The United Nations working manual for MORTPAK-LITE (United Nations, 1988), a software package for demographic measurement, includes among its 16 computer programs a routine (UNABR) that graduates a set of age-specific probabilities of dying ($_nq_x$) for the standard set of "five-year" age groups—0-1, 1-5, 5-10, 10-15,... 80+—into a smooth set of single-year probabilities of dying.[1] The graduation is effected with an eight-parameter "law of mortality" known as the Heligman/Pollard model mortality schedule (Heligman and Pollard, 1980):

$$_1q_x = A^{(x+B)^C} + D\exp[-E(ln\,x - ln\,F)^2] + \frac{GH^x}{1 + GH^x} \qquad (1)$$

where $_1q_x$ is the probability of an individual at exact age x dying before reaching exact age $x + 1$. The capital letters $A, B, \ldots H$ denote the eight parameters defining the age curve of mortality. These are estimated using

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a least squares fitting criterion and a non-linear estimation algorithm. The result is a smoothed set of ${}_nq_x$ and single-year ${}_1q_x$ values which aggregate to the smoothed ${}_nq_x$ values.

Model mortality schedules such as the Heligman/Pollard curve are also useful in applications other than graduation. For example, their compact and transparent representations of mortality patterns are useful in mortality analysis and forecasting. Several such applications are illustrated in this article. We begin with a brief review of the Heligman/Pollard model, apply it to a historical time series, move on to examine a forecasting application, extend the discussion to causes of death, turn to a study of spatial differences, and then consider descriptions of sex-specific mortality schedules that are disaggregated by race. Finally, we propose an alternative model to the Heligman/Pollard specification and offer a few conclusions.

THE HELIGMAN/POLLARD MODEL MORTALITY SCHEDULE

Equation (1) is defined for $x > 0$; for $x = 0$, one could either set x to a very small number, say 1×10^{-10}, or simply set

$$ {}_1q_x = A^{B^C} + \frac{G}{1 + G} \quad (2) $$

The eight parameters have the following interpretation. The first three describe the fall in mortality during the early childhood years: A reflects the level of mortality during these years and is nearly equal to ${}_1q_1$, B is an age displacement variable that indicates how much higher ${}_1q_0$ is above ${}_1q_1$ (as B increases, ${}_1q_0$ decreases below 0.5 and approaches ${}_1q_1$), and C indicates the rate at which mortality declines during the early-childhood years (as C increases, mortality declines more rapidly with age).

The second three parameters refer mostly to the "accident hump" among males (with a peak around age 20 or thereabouts) and to maternal mortality among females: D defines the level, E describes the spread and F locates the position.

Finally, the last two parameters describe the near geometric increase in mortality at the older adult ages: G measures the base level of late-life mortality (that is, at $x = 0$) and H defines its rate of increase with age.

Although the above influences of the eight parameters on the shape of the model mortality schedule are readily established, their interpretation is of a mechanical nature:

> "In other words, while the interpretations of the parameters at hand indicate how they affect the model-age pattern of mortality, they cannot be extended to also explain the nature of the biological and environmental circumstances yielding the empirical age pattern of mortality" (Hartmann, 1983, p. 46).

In applying the UNABR routine of MORTPAK-LITE to appropriate aggregations of the single-year-of-age Australian data presented in Heligman and Pollard (1980), we found the values for the eight parameters that are set out in panel 1 of table 1. In panel 2 we present estimates of the same eight parameters obtained using another program provided to us by Larry Heligman—one which requires mortality data inputs by single years of age instead of the standard set of five-year age groups. Differences between the estimates in panel 1 and those in panel 2 are a consequence of aggregation bias. Panel 3 gives the values of the corresponding parameters that appear in the original Heligman and Pollard article, which used a slightly different stopping rule (Heligman, 1989). Although these were obtained with a slightly modified specification of the model mortality schedule, Heligman and Pollard (1980) state that the estimated values are almost identical to those obtained using the specification in equation (1).

An examination of the parameter values for Australia in table 1 reveals that the three specifications yield estimates that are roughly the same (with the exception of the E parameter in the latter two time periods). Thus, we may conclude that the UNABR routine gives "correct" estimates and that the aggregation bias that arises from using five-year instead of single-year age groups is probably acceptably small in most instances, with the exception of the E parameter, which measures the spread of the "accident hump". Here the aggregation hides important characteristics of the hump.

Panels 4, 5 and 6 of table 1 present parameter estimates for three other countries at roughly the same time periods for purposes of comparison. Although different in several particular respects, in general the parameters for all four countries exhibit remarkable similarities, particularly in their patterns of behaviour over time. For example, the A parameter declines over time in all countries, indicating a drop in infant mortality. The D and E parameters increase in Australia and in the United States of America; the E parameter also increases in England. Apparently the importance and concentration of the accident hump increased in all three countries, but because the influence of the G and H parameters is measured by their product, their trends over time become intertwined—with low estimates of one parameter generating higher compensating estimates for the other. Nevertheless, the common pattern over time is a decline in G and an increase in H. Australia, England and Sweden all exhibit this pattern; so does the United States in strictly *pairwise comparisons* of the 1950 data with either of the two subsequent observations.

Panel 7 sets out the eight parameters that define the well-known Brass standard mortality schedule. They are included in table 1 in order to underscore the fact that the Brass logit model mortality schedule is not merely a two-parameter model schedule, as is sometimes claimed by its exponents; it includes a hundred observations on single-year survival probabilities, or at least an additional eight parameters with which the standard age pattern of mortality can be constituted, to be reshaped thereafter by the two-parameter logit linear regression equation.

Finally, to illuminate the influences wielded by the eight parameters on the shape and level of the model mortality schedule, we illustrate in figure I a selection of the mortality schedules described in table 1.

TABLE 1. HELIGMAN/POLLARD PARAMETERS FOR NATIONAL MALE MORTALITY SCHEDULES: AUSTRALIA, ENGLAND, SWEDEN AND THE UNITED STATES

	Parameter							
Country	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
1. Australia[a]								
1946-48	.00336	.0191	.1239	.00093	9.52	20.42	.00008	1.0977
1960-62	.00188	.0143	.1196	.00111	11.56	20.49	.00007	1.1007
1970-72	.00163	.0148	.1178	.00160	15.96	19.93	.00006	1.1028
2. 1946-48	.00340	.0226	.1289	.00093	9.72	20.16	.00009	1.0963
1960-62	.00184	.0142	.1201	.00109	13.83	20.39	.00007	1.0985
1970-72	.00163	.0166	.1210	.00163	19.28	19.81	.00007	1.0992
3. 1946-48	.00341	.0208	.1284	.00094	9.49	20.22	.00009	1.0970
1960-62	.00184	.0189	.1189	.00110	13.55	20.43	.00007	1.0992
1970-72	.00163	.0144	.1182	.00164	18.49	19.88	.00006	1.1013
4. England[b]								
1951	.00227	.0055	.1125	.00071	8.86	21.77	.00005	1.1052
1961	.00151	.0037	.1012	.00074	14.79	19.93	.00004	1.1090
1971	.00121	.0035	.0958	.00070	17.25	19.36	.00004	1.1093
5. Sweden[c]								
1941-50	.00292	.0063	.1029	.00159	12.18	22.77	.00007	1.0951
1951-60	.00177	.0065	.0972	.00076	13.16	22.13	.00004	1.1045
1961-70	.00103	.0015	.0768	.00067	12.05	22.14	.00004	1.1055
6. United States[d]								
1950	.00236	.0110	.1336	.00096	13.80	20.32	.00016	1.0871
1960	.00190	.0119	.1312	.00105	15.16	20.43	.00012	1.0910
1970	.00142	.0142	.1317	.00155	15.72	20.69	.00014	1.0884
7. Brass standard[e]								
No date	.06008	.3109	.3443	.00698	1.99	26.71	.00022	1.0880

[a] Panel 1 was obtained using the standard set of "five-year" age groups: 0-1, 1-5, 5-10, . . . ,80+ and the routine UNABR in MORTPAK-LITE. Panel 2 was obtained using single-year age groups and a program made available by

[c] M. Hartmann, "Past and recent experiments in modeling mortality at all ages", Demography Section Working Paper No. 13 (Stockholm, University of Stockholm, 1983), p. 36. The parameter values for E in the source apparently need to be multiplied by 100.

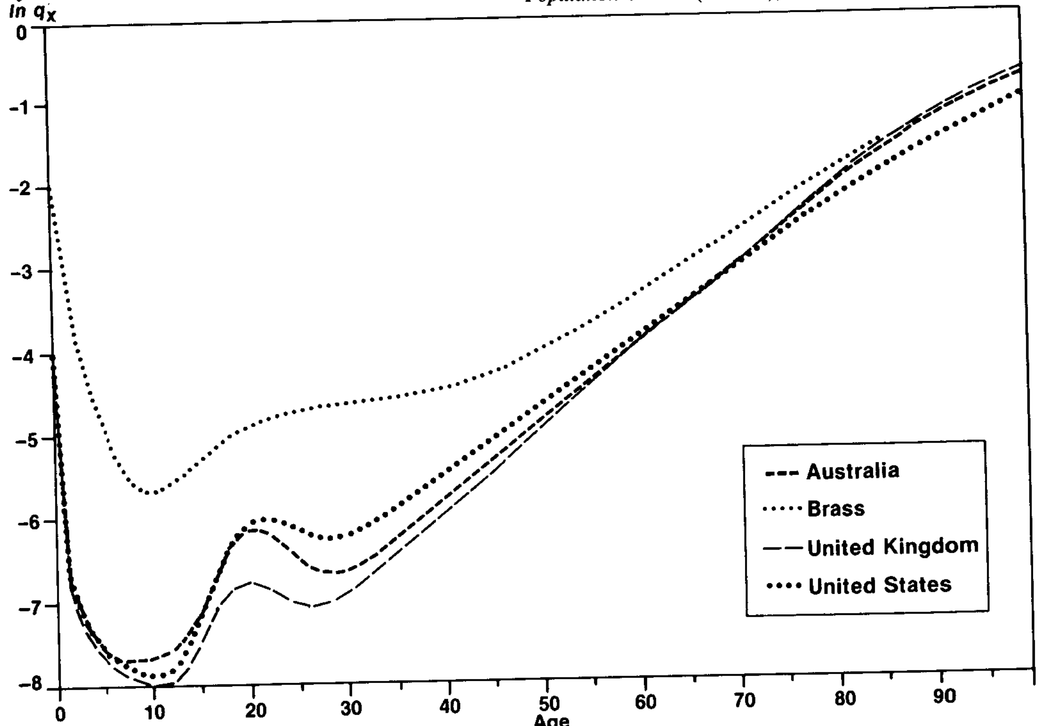
Larry Heligman. Panel 3 was obtained from Heligman and Pollard (1980), p. 58.

[b] D. O. Forfar and D. M. Smith, "The changing shape of English life tables", *Transactions of the Faculty of Actuaries*, vol. 40, No. 270 (1987), p. 99.

[d] Calculated by the authors using UNABR and data provided by Alice Wade of the Office of the Actuary, Social Security Administration, United States Department of Health and Human Services (1983).

[e] Calculated by the authors using the Heligman single-year-of-age program and the data set out in B. Zaba, "The four parameter logit life table system", *Population Studies* (London), vol. 33, No. 1 (March), p. 96.

Figure I. National Heligman/Pollard male model mortality schedules: Australia (1970-1972), the Brass standard, England (1971), and the United States (1970)



Source: Table 1.

The history of mortality decline in the United States during this century can be compactly described by the time series of parameter estimates for the eight parameters of the Heligman/Pollard model schedule (McNown and Rogers, 1988a and 1988b). Figure II illustrates the trajectories of four of the eight parameters for males: A, D, E and F. The first two measure the levels of childhood and middle-life mortality, respectively. Both exhibit sharp declines over the first half of this century and a levelling off thereafter (the two sharp peaks in the evolution of D reflect the impacts of the influenza epidemic of 1918 and of the two World Wars). Parameter F also declines until 1950 and then levels off and, in more recent years, begins a gradual increase. Its evolution indicates that the modal age of the accident hump shifted to a much younger age during the first half of this century and then began to increase. Finally, parameter E, the measure of concentration of the accident hump, exhibits an unusually sudden and dramatic upturn around 1940, climbing from a value of 6.4 in 1940 to a value of 20.0 in 1944. This pattern, of course, is a direct consequence of the relative increase in deaths among males in their early twenties that occurred during the Second World War. But, somewhat surprisingly, the post-war years do not show a return to pre-war values, although the parameter E does decline to almost half of its peak value by 1985.

Of the four parameters not shown in figure II, B and C exhibit patterns of decline similar to parameter A, and G and H show a mixed pattern. While parameter G was generally increasing during the first four decades of this century, parameter H was generally decreasing. During the second four decades the pattern was reversed. Since the two parameters jointly determine the level of late-life mortality, it is difficult to unconfound their individual contributions. What is clear, however, is that a "break with the past" occurred around 1940, as happened with a number of other parameters.

Using only post-1940 data, because of the "break", McNown and Rogers developed univariate time-series models for each of the parameters, applying the methods of Box and Jenkins (1976). An example of such a model is the following expression for the temporal evolution of parameter A (McNown and Rogers, 1988b):

$$Y(t) = e(t) - 1.11e(t-1) + .30e(t-2) \tag{3}$$

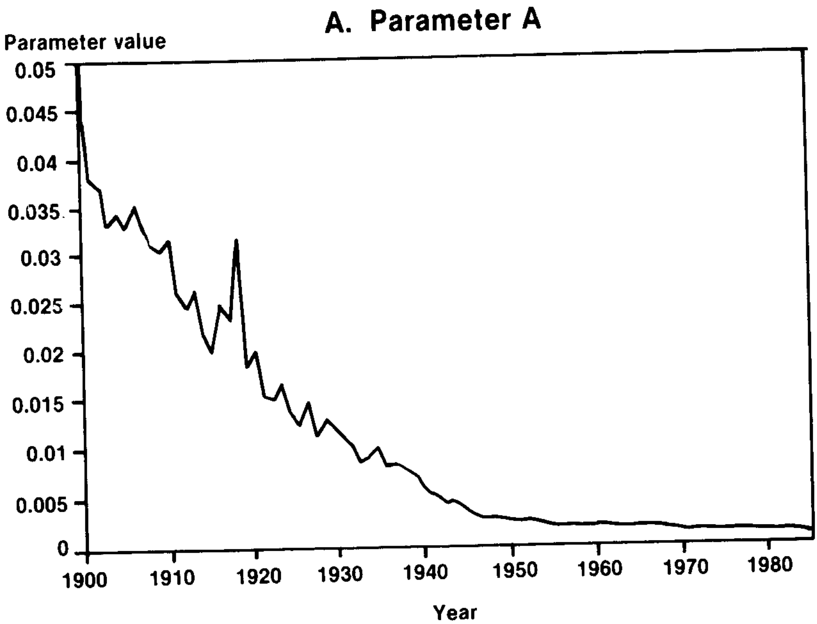
where $Y(t) = \Delta^2 Z(t)$ and $Z(t) = \ln A(t)$.

Notice that the parameter A is first transformed into logarithmic form and then differenced twice to define $Y(t)$. Thus

$$Y(t) = [\ln A(t) - \ln A(t-1)] - [\ln A(t-1) - \ln A(t-2)]$$

Equation (3) defines an integrated moving average (IMA) model for the A parameter. It is a member of the more general class of so-called Autoregressive Integrated Moving Average (ARIMA) models. The vari-

Figure II. Trajectories of four Heligman/Pollard parameters over time for United States male mortality schedules, 1900-1985



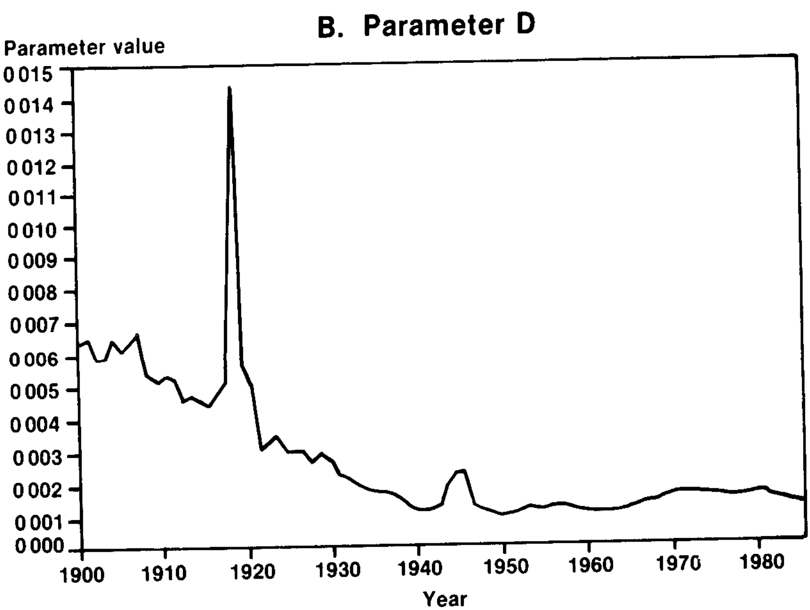
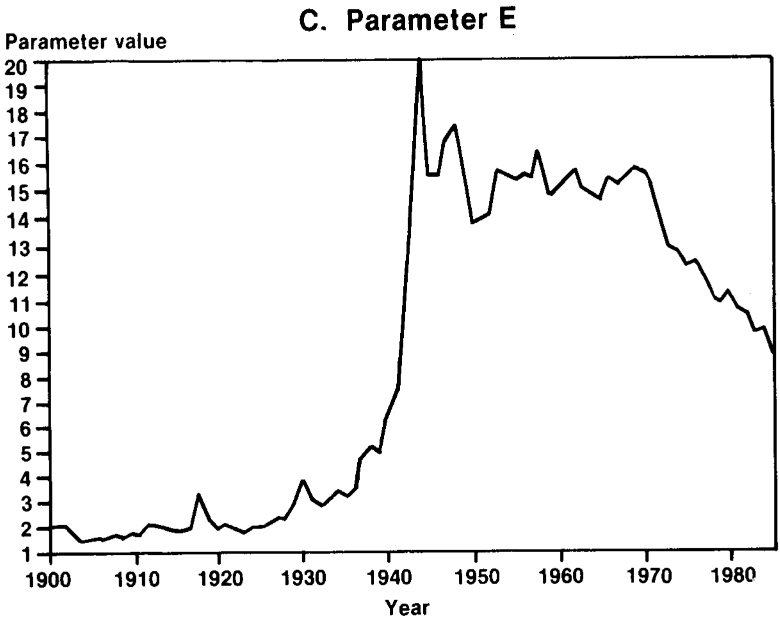
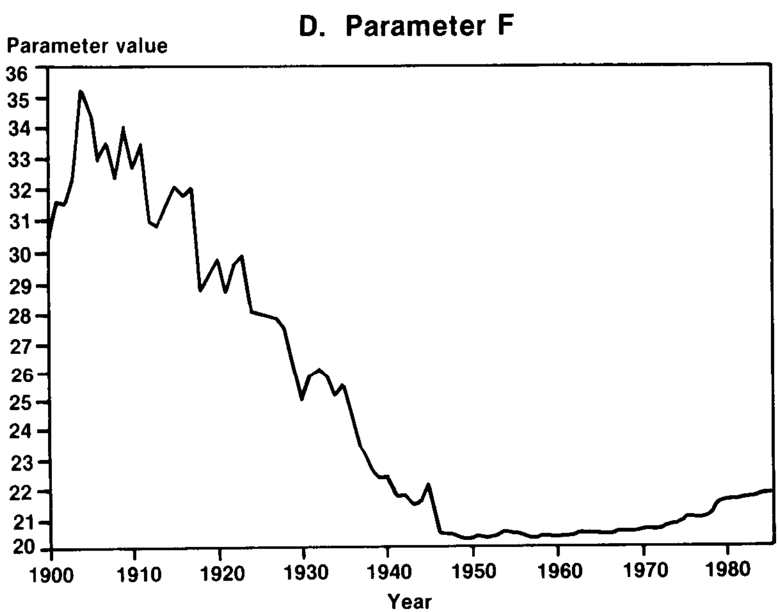


Figure II (*continued*)





ables $e(t)$ are assumed to be identically and independently distributed error terms ("white noise" processes in the jargon of time-series analysts) that measure the deviation of predicted versus observed variables:

$$e(t) = Y(t) - \hat{Y}(t) .$$

APPLICATION 2: FORECASTING

Univariate time-series models, such as the one set out in equation (3), can be used for forecasting purposes. The logic is straightforward:

> "First, the expected value of the time-series process is calculated and, secondly, the expected value is extrapolated into the future. . . . If the current time-series observation is Y_t, then we are interested in predicting the values of $Y_{t+1}, Y_{t+2}, \ldots, Y_{t+n}$." (McCleary and Hay, 1980, p. 206)

The expected values of all $e(t)$ variables beyond the currently observed one are 0. thus, for example if $t = T$ denotes the last observation for $A(t)$ in equation (3), then a one-step-ahead forecast is

$$Y(T+1) = 0 - 1.11e(T) + .30e(T-1)$$

and a two-step-ahead forecast is

$$Y(T+2) = 0 - 0 + .30e(T) .$$

Clearly, all forecasts of $A(t)$ more than two steps ahead of the last observation are simply trend extrapolations, for example,

$$[\ln A(T+3) - \ln A(T+2)] = [\ln A(T+2) - \ln A(T+1)] .$$

Because forecasts from univariate time-series models tend to converge rapidly to the long-run trends incorporated in the models, a proper representation of trend is a crucial component of time-series analysis. In the McNown and Rogers (1988b) analysis, all male parameters, for example, with the exception of C and D, were modelled with trend components. All models exhibited satisfactory diagnostics, and a number of statistically insignificant coefficients were retained because they improved forecasting performance.

Table 2 presents the forecasts of the eight parameters produced by the McNown/Rogers univariate models and compares them with those implied by the mortality forecasts developed by the Social Security Administration (United States, 1983). Table 3 contrasts the associated expectations of life at birth for the year 2000. Both tables indicate that the two sets of forecasts are quite similar, even though the McNown/Rogers forecasts did not incorporate a disaggregation by cause of death.

APPLICATION 3: CAUSE-SPECIFIC

The mortality forecasts produced by the Social Security Administration are founded on judgements made about expected trends in future

TABLE 2. ALTERNATIVE FORECASTS OF THE HELIGMAN/POLLARD PARAMETERS FOR NATIONAL UNITED STATES MALE AND FEMALE MORTALITY SCHEDULES: 1990 AND 2000

Mortality schedule	*Parameter* A	B	C	D	E	F	G	H
Males								
1990 (M/R)[a]	.00075	.0414	.1386	.00118	6.74	22.39	.00008	1.0936
1990 (SSA)[b]	.00093	.0583	.1432	.00135	8.98	22.00	.00006	1.0949
2000 (M/R)	.00052	.0683	.1386	.00118	3.99	23.32	.00006	1.0957
2000 (SSA)	.00087	.0781	.1508	.00136	8.81	22.23	.00005	1.0970
Females								
1990 (M/R)	.00049	.0134	.0972	.00028	14.81	19.53	.00004	1.0957
1990 (SSA)	.00068	.0350	.1205	.00036	10.06	20.05	.00003	1.0992
2000 (M/R)	.00028	.0134	.0880	.00028	14.81	19.53	.00003	1.0957
2000 (SSA)	.00061	.0451	.1250	.00038	10.41	19.99	.00002	1.1007

[a] Forecast by McNown and Rogers, in R. F. McNown and A. Rogers, *Forecasting Mortality: A Parameterized Time Series Approach*. Institute of Behavioral Science, Population Program Working Paper 88-10 (Boulder, Colorado, University of Colorado, 1988).

[b] Forecast by the Social Security Administration, in United States, "Life tables for the United States: 1900-2050", *Actuarial Study*, No. 89 (Washington, D. C., Office of the Actuary, Social Security Administration, United States Department of Health and Human Services, 1983).

Note: The Social Security Administration projections of single-year mortality probabilities were fitted with the program made available by Larry Heligman of the United Nations.

TABLE 3. ALTERNATIVE FORECASTS OF UNITED STATES MALE AND FEMALE LIFE EXPECTANCIES, 2000

Age	*Males* M/R[a]	*Males* SSA[b]	*Females* M/R[a]	*Females* SSA[b]
0	72.79	73.65	80.77	81.11
20	53.77	54.87	61.34	62.00
40	35.45	36.32	41.95	42.58
60	18.66	19.24	23.91	24.57
70	11.97	12.62	16.13	17.05

[a] Forecast by McNown and Rogers, in R. F. McNown and A. Rogers, *Forecasting Mortality: A Parameterized Time Series Approach*. Institute of Behavioral Science, Population Program Working Paper 88-10 (Boulder, Colorado, University of Colorado, 1988).

[b] Forecast by the Social Security Administration, in United States, "Life tables for the United States:1900-2050", *Actuarial Study*, No. 89 (Washington, D. C., Office of the Actuary, Social Security Administration, United States Department of Health and Human Services, 1983).

cause-specific death rates. Thus, a natural extension of the time-series analysis and forecasting methodology described above is to data on mortality by cause. But there are also other reasons for considering cause-specific patterns.

Policy-makers are, of course, interested in forecasts of mortality. But they are also interested in expected changing patterns in causes of death and how such changes are likely to affect the demands for various health-care services. Policy-makers are also interested in the impact that particular intervention programs might have on the cause structure of mortality. And, as scholars and professionals extend their understanding of the linkages between morbidity and causes of death, the analysis and forecasting of cause-specific mortality will play an important role in the study of active-life expectancy among the elderly.

Model schedules offer the student of mortality a compact and powerful means for describing the cause-specific structures of different mortality régimes in a way that fosters comparisons over time and space or across population subgroups. Consolidation of cause-specific mortality régimes should lead to more accurate aggregate mortality probability forecasts, because schedules of cause-specific mortality are likely to exhibit more stable and, therefore, more predictable age profiles over time. However, levels of cause-specific mortality would have to be forecast for each cause, thereby inviting complications that arise because of the interdependence of the various causes of death (Heligman, 1989).

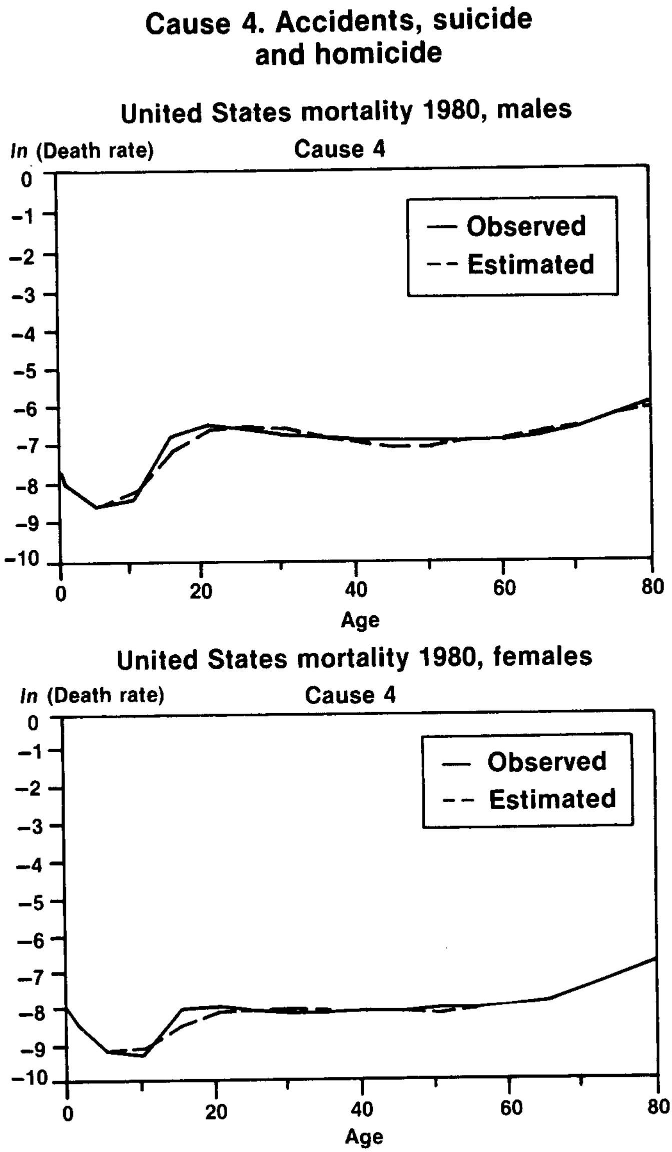
Although the Heligman/Pollard model schedule is difficult to fit to certain particular cause-specific age patterns of mortality, a number of such schedules can nevertheless be accurately described by that specification. Two are illustrated in figure III; their parameters are set out in table 4.

The age profile of mortality probabilities due to accidents, suicide and homicide first declines to a low point during adolescence, then rises to a high point in the early twenties, and then declines slightly until the elderly ages, at which point it starts to increase once again to the last years of life. This age pattern is reflected in the respective values taken on by the parameters set out in table 4. All indicators of level (A, D and G) are, of course, much lower than in the case of the aggregate schedule; the "accident hump" is located at a somewhat older age and with a sharply reduced concentration; and the slope parameter H is, as expected, considerably lower.

Deaths due to congenital malformations and diseases of early infancy significantly affect mortality during the childhood years, as is to be expected. But they apparently also rise in importance at the elderly ages. The level of mortality at the young adult ages is relatively low, with both level parameters D and G being 0-5 decimal places. But with increasing age, the slope parameter H elevates late-life mortality due to this cause into relative importance once again.

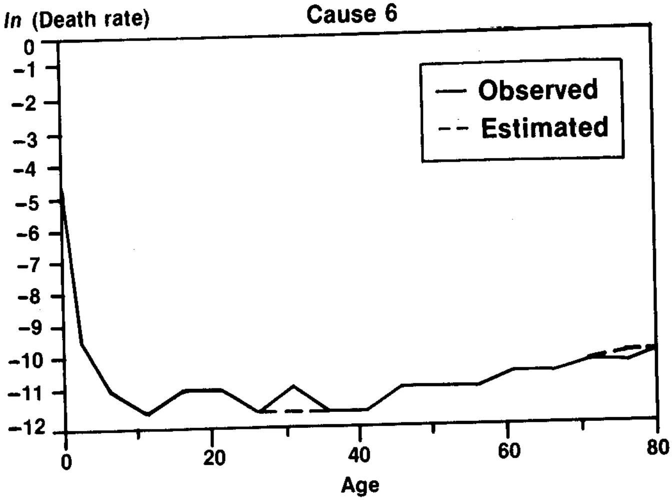
The representation of each cause-specific mortality régime over time by an appropriately specified model schedule would yield a time-series of observations of the model parameters. These time-series could then in turn be modelled as univariate processes, just as the aggregate series were modelled earlier in this article. Forecasts of the parameters would produce

Figure III. Fit of the Heligman/Pollard model to two United States male and female cause-specific mortality schedules, 1980



Cause 6. Congenital malformations and diseases of early infancy

United States mortality 1980, males



United States mortality 1980, females

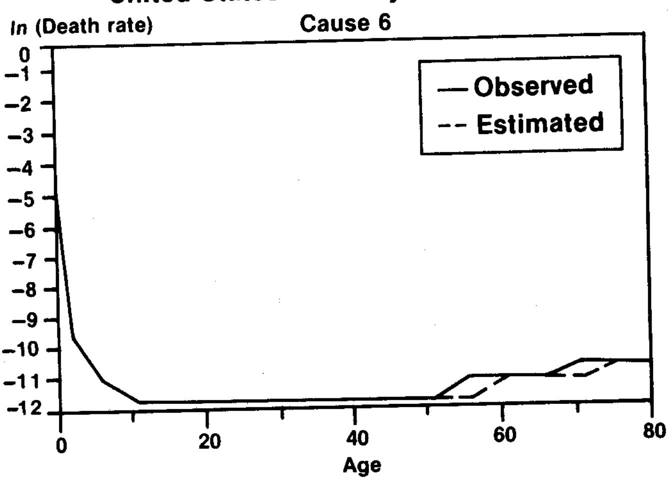


TABLE 4. HELIGMAN/POLLARD PARAMETERS FOR TWO UNITED STATES CAUSE-SPECIFIC MALE AND FEMALE MORTALITY SCHEDULES, 1980

	Parameter[a]							
Sex	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
				Accidents, suicide and homicide				
Males	.00027	.6939	.1577	.00027	4.23	26.04	.00002	1.0410
Females	.00014	.3482	.1049	.00007	1.93	28.92	.00000	1.0778
				Congenital malformations and diseases of early infancy				
Males	.00005	.0028	.1298	.00000	22.40	19.36	.00000	1.0380
Females	.00004	.0020	.1184	.00000	4.88	24.00	.00000	1.0369

[a] Calculated using the Unabridged program.

forecasts of cause-specific probabilities of dying from each cause, and summations of these probabilities over all causes would yield the corresponding aggregate mortality probabilities at each age. A comparison of the accuracy of these forecasts against corresponding ones obtained without the introduction of cause specificity would measure the gains in forecasting accuracy contributed by the disaggregation into causes. Work is currently under way to carry out such an experiment (McNown and Rogers, 1988b).

APPLICATION 4: SPATIAL ANALYSIS

Age-specific probabilities of dying vary across space as well as over time, defining a national "geography of death" that has important consequences for population age distributions and health-care demands, for example. Significant regional differentials in mortality régimes can be found even in highly industrialized countries such as the United States. For example, the infant mortality rate in the state of Louisiana was 14.6 per 1,000 in 1979-1981; in Hawaii it was only 11.2 per 1,000 (Rogers and Gard, 1988). The expectation of life at birth for males was 72.52 years in Minnesota and only 64.55 years in the District of Columbia (United States, 1987, p. 5). Such interstate comparisons, however, only identify variations in single measures of mortality; for more detailed contrasts, one must consider the age-specific probabilities of dying for each state—a data set with about 5,000 elements if single-year age probabilities are examined. An effective data reduction device is offered by model mortality schedules, for example, by the Heligman/Pollard model.

Table 5 presents a succinct description of the state geography of death in the United States during 1979-1981. The 51 × 8 = 408 parameters compactly summarize the 51 state mortality regimes, replacing the over 5,000 observations that constituted the original data set (Washington, D.C. is treated as the fifty-first state).

TABLE 5. HELIGMAN/POLLARD PARAMETERS FOR UNITED STATES MORTALITY SCHEDULES; 1979–1981

	Parameter[a]							
State	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Alabama	0.00127	0.05402	0.15290	0.00082	8.67142	21.51325	0.00009999	1.08731
Alaska	0.00169	0.07266	0.14858	0.00162	7.81826	21.63189	0.00007650	1.09118
Arizona	0.00191	0.14366	0.18056	0.00122	7.69353	22.33182	0.00006476	1.09173
Arkansas	0.00103	0.02365	0.12090	0.00077	8.08558	20.74850	0.00008226	1.08913
California	0.00122	0.07788	0.15307	0.00103	8.41167	21.91822	0.00005900	1.09343
Colorado	0.00118	0.07023	0.14448	0.00080	6.07175	22.89458	0.00005065	1.09505
Connecticut	0.00068	0.01003	0.10752	0.00079	9.71352	21.10103	0.00004726	1.09646
Delaware	0.00113	0.02295	0.12531	0.00071	10.29700	20.54648	0.00006574	1.09314
District of Columbia	0.00293	0.02496	0.10470	0.00071	2.37951	20.99593	0.00008900	1.08918
Florida	0.00140	0.05643	0.15094	0.00112	8.18964	21.82725	0.00008331	1.08795
Georgia	0.00115	0.03273	0.13635	0.00075	9.74408	20.69110	0.00010874	1.08624
Hawaii	0.00103	0.05479	0.13774	0.00077	9.03341	20.64624	0.00004496	1.09480
Idaho	0.00149	0.07267	0.13064	0.00114	6.54282	21.27552	0.00004480	1.09663
Illinois	0.00103	0.03116	0.13991	0.00091	9.98781	21.42407	0.00006762	1.09261
Indiana	0.00099	0.02921	0.12680	0.00086	9.99552	20.95375	0.00005663	1.09500
Iowa	0.00089	0.01775	0.10881	0.00082	10.00984	19.97452	0.00004052	1.09793
Kansas	0.00120	0.05333	0.13553	0.00094	8.00740	22.01973	0.00004911	1.09536
Kentucky	0.00106	0.04245	0.13814	0.00073	9.83401	20.93885	0.00008178	1.09025
Louisiana	0.00116	0.03599	0.14037	0.00107	9.49933	21.72346	0.00010885	1.08660
Maine	0.00065	0.00425	0.08507	0.00084	12.76119	20.56008	0.00004928	1.09642
Maryland	0.00098	0.02061	0.12335	0.00076	16.76257	20.98812	0.00008800	1.08909
Massachusetts	0.00068	0.01742	0.11505	0.00054	9.90901	22.03519	0.00005072	1.09579
Michigan	0.00093	0.02484	0.12822	0.00072	8.82641	21.46765	0.00006680	1.09270
Minnesota	0.00082	0.01386	0.10318	0.00082	10.17072	20.46872	0.00003712	1.09889
Mississippi	0.00125	0.02185	0.12787	0.00093	8.35026	22.61549	0.00010927	1.08606
Missouri	0.00086	0.01734	0.11933	0.00095	9.11417	21.02075	0.00006463	1.09280
Montana	0.00106	0.03705	0.12777	0.00140	9.11540	20.69451	0.00006665	1.09191
Nebraska	0.00102	0.03997	0.13085	0.00092	10.27551	20.37116	0.00004694	1.09601
Nevada	0.00151	0.09753	0.15862	0.00150	9.33375	21.20697	0.00008394	1.08974

TABLE 5 (*continued*)

State	Parameter[a]							
	A	B	C	D	E	F	G	H
New Hampshire	0.00082	0.01849	0.10854	0.00075	10.94748	20.18323	0.00004336	1.09802
New Jersey	0.00082	0.01491	0.11290	0.00071	9.57272	21.68993	0.00005627	1.09513
New Mexico	0.00127	0.04311	0.12844	0.00162	6.89681	22.64253	0.00006699	1.09113
New York	0.00099	0.02101	0.11972	0.00070	11.90164	20.98604	0.00008800	1.08917
North Carolina	0.00103	0.03488	0.14254	0.00067	9.85720	21.26633	0.00009437	1.08781
North Dakota	0.00094	0.02669	0.12363	0.00100	10.06139	20.08463	0.00004135	1.09757
Ohio	0.00087	0.01453	0.11263	0.00077	11.10597	21.99614	0.00006449	1.09341
Oklahoma	0.00115	0.03865	0.13249	0.00111	8.15183	22.54592	0.00006875	1.09176
Oregon	0.00097	0.02951	0.12357	0.00100	9.94835	20.75782	0.00005014	1.09547
Pennsylvania	0.00196	0.01321	0.08455	0.00095	16.59017	21.05061	0.00009800	1.09033
Rhode Island	0.00088	0.01153	0.10820	0.00089	18.35858	20.99379	0.00010500	1.08929
South Carolina	0.00116	0.03800	0.15068	0.00069	12.39519	22.30984	0.00011192	1.08608
South Dakota	0.00124	0.05748	0.13824	0.00115	10.32983	20.60110	0.00006377	1.09147
Tennessee	0.00086	0.01831	0.12200	0.00079	8.93031	21.80218	0.00008217	1.08964
Texas	0.00123	0.04842	0.13824	0.00111	8.19793	22.41467	0.00007416	1.09063
Utah	0.00095	0.02165	0.10903	0.00078	7.29175	21.59060	0.00004197	1.09748
Vermont	0.00099	0.01382	0.08958	0.00069	10.93689	19.03173	0.00004681	1.09697
Virginia	0.00101	0.02161	0.12290	0.00065	17.57906	20.96588	0.00008700	1.08907
Washington	0.00105	0.04453	0.13658	0.00085	7.51984	21.45783	0.00005056	1.09521
West Virginia	0.00117	0.05473	0.14818	0.00080	11.15572	21.35246	0.00008360	1.09005
Wisconsin	0.00085	0.02359	0.11716	0.00071	9.53521	20.89670	0.00004524	1.09701
Wyoming	0.00114	0.04162	0.12914	0.00143	8.76442	20.88086	0.00006959	1.09129

[a] Calculated using single-year age groups.

Despite the relative economy of representation provided by the model-schedule parameterization, the interpretation of 408 parameters still poses a challenge. Further consolidation of information is desirable; clustering algorithms offer a possible avenue.

Figure IV presents the results of the clustering algorithm provided by the SPSS program package (Statistical Programs for the Social Sciences, 1986). The procedure operates in four steps. First, it computes "distances" between the observations (the 51 states in this case) in the parameter space. Next, it joins observations that are the closest to each other ("nearest neighbours") to form clusters. Proceeding in this manner to consolidate the observations, the algorithm continues the clustering procedure until all of the original observations have been collected into a single cluster. The results illustrated in figure IV reveal four clusters of mortality régimes.

Although a number of "outlier" observations make it difficult to identify distinct regional families of mortality régimes, three major "mortality belts" running from the north-west to the south-east seem to stand out. The first includes much of what is called by the United States Census the West region and the West South Central division plus Florida. The second includes most of the Mid-west region and parts of the East-South Central and South Atlantic divisions. The third (combining two of the four clusters) contains several of the states in the North-eastern region. Finally, the District of Columbia is a single-observation cluster, because its mortality régime is so different from those of the 50 states.

Application 5: race-specific analysis

Why is the mortality régime of Washington, D.C. so distinctly different from those of the 50 states? A natural direction for seeking an answer to this question is to examine the relative contributions of different causes of death to mortality patterns in Washington, D.C. and in other parts of the United States. Another direction is a disaggregation by race.

Over 70 per cent of the population of Washington, D.C. is black, about twice the percentage of the runner-up state, Mississippi. Thus, a natural question to ask is whether, in addition to the well-known differential in mortality levels, there are in addition significant differences in race-specific age patterns of mortality. Rogers and Gard (1988) demonstrate that to be the case.

The principal difference between the age profiles of black and white mortality probabilities is the virtual total absence of the accident hump in the former. This, of course, does not imply that the black population has no deaths due to accidents and violence in the young adult ages. It simply is an indication that the mortality probabilities at ages immediately beyond the accident-hump ages are high enough to hide the hump itself. The result is an age profile such as is exhibited by Alabama's black population, illustrated in figure V and in table 6. Note the contrast to that pattern which is

Figure IV. Interstate geography of death: a regionalization

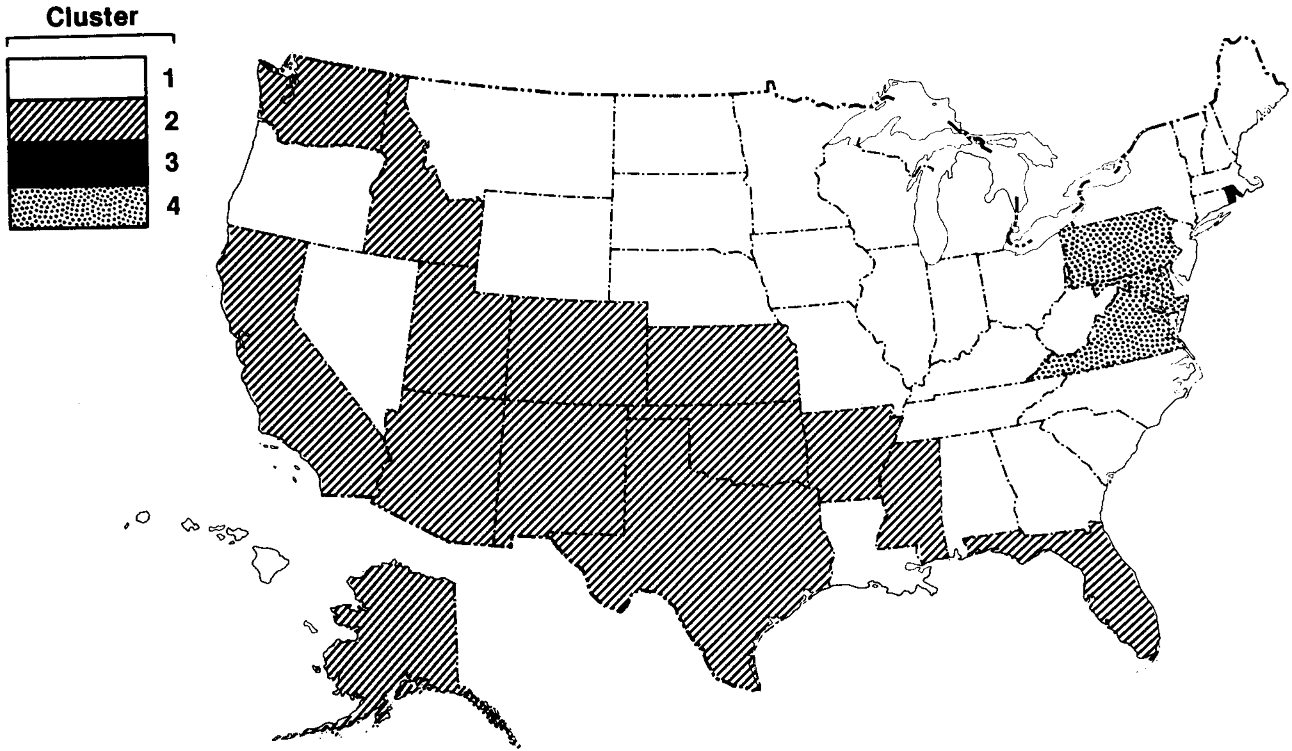


TABLE 6. HELIGMAN/POLLARD PARAMETERS FOR REPRESENTATIVE BLACK, WHITE, AND OTHER UNITED STATES MALE MORTALITY SCHEDULES, 1979-1981

State	Parameter[a]							
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
				Black males				
Alabama								
(1-year)	.00190	.0319	.1421	.03198	.60	208.45	.00010	1.0874
(5-year)	.00190	.0357	.1472	.01278	.84	110.19	.00012	1.0872
Georgia								
(1-year)	.00206	.0177	.1344	.00000	−.09	.00	.00007	1.0850
(5-year)	.00232	.0697	.1782	.00000	−.08	.00	.00013	1.0854
				White males				
Minnesota								
(1-year)	.00095	.0222	.1147	.00138	10.21	20.74	.00004	1.1007
				Other males				
Hawaii								
(1-year)	.00126	.0776	.1515	.00123	9.68	20.78	.00006	1.0932

[a] In addition to the fits to single-year-of-age data, the fits for five-year data in the case of the black populations of Alabama and Georgia are also included for purposes of comparison.

Figure V. Representative race-specific United States male mortality schedules in selected states: whites, blacks and others, 1979-1981

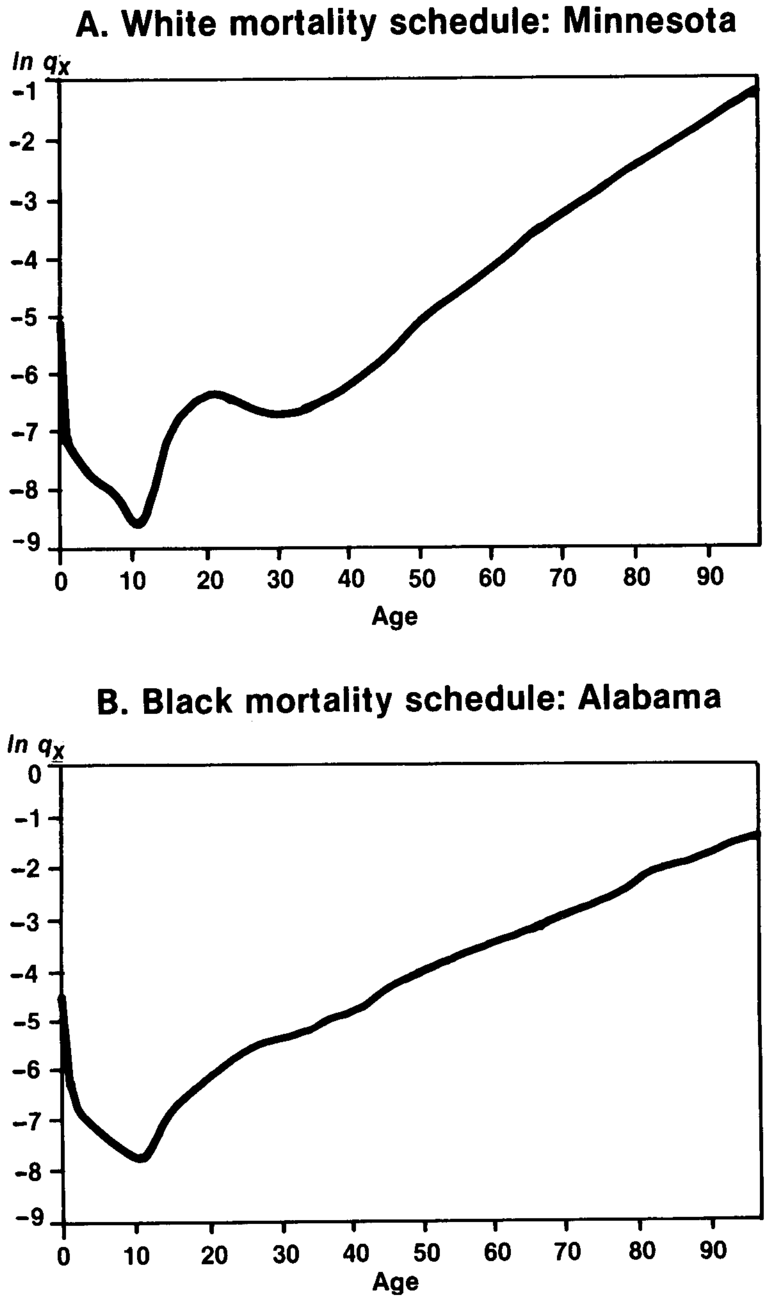
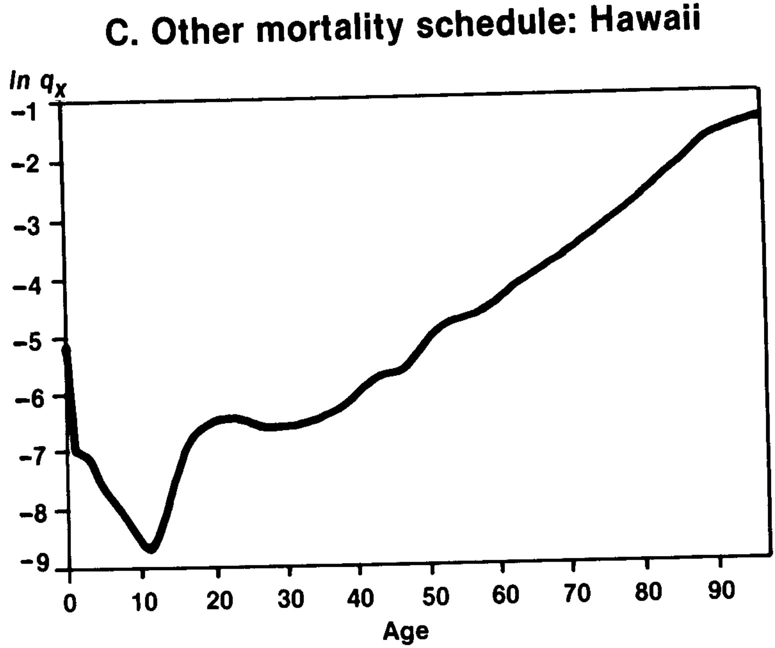


Figure V (*continued*)



offered by the mortality régime of Minnesota's white population or of Hawaii's Asian (other) population.

Table 6 indicates that the absence of an accident hump in age patterns of mortality among the black population creates problems in fitting the Heligman/Pollard model to the observed data. The level of the accident hump as measured by parameter D is either essentially 0 (as in Georgia) or about two times higher than that of the rest of the population (as in Alabama). The spread parameter, E, is essentially 0, allowing the location parameter F to take on unreasonably high or low values. Deleting the middle-life mortality component, however, gives rise to problems in fitting the model to the childhood years. An alternative model mortality schedule seems to be needed for the analysis of black mortality régimes. Heligman and Pollard (1980) suggest a nine-parameter version of their formula; we propose yet another alternative formulation in the section below.

An alternative specification: a multi-exponential model

The Heligman/Pollard model schedule divides the age profile of mortality into three components: early-life (childhood), middle-life and late-life mortality. Symbolically,

$$f(x) = f_1(x) + f_2(x) + f_3(x), \tag{4}$$

with $f(x)$ denoting the mortality probability at age x.

Following the pioneering work of Thiele (1872), the first and third components are often represented as negative and positive exponentials, respectively:

$$f_{(x)} = a_1 \exp(-\alpha_1 x) \tag{5}$$

$$f_3(x) = a_3 \exp(\alpha_3 x). \tag{6}$$

Rogers and Planck (1983) introduced a model that is similar to Thiele's, with the middle-life mortality component specified by the double exponential distribution of Coale and McNeil (1972):

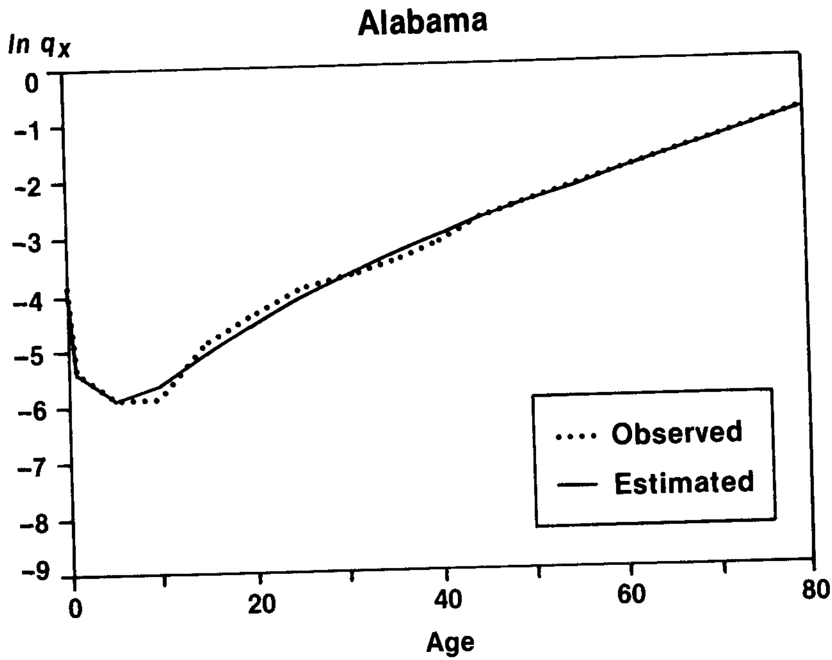
$$f_2(x) = a_2 \exp\{-\alpha_2(x - \mu_2) - \exp[-\lambda_2(x - \mu_2)]\}. \tag{7}$$

Explanatory work with this multi-exponential model has produced promising results when compared to alternative specifications, but usually at the cost of including an additional parameter—a constant term—c, say—that is added to the expression in equation (4). Rogers and Watkins (1986) experimented with an alternative "extra term" specification by setting ${}_1q_0 = a_0$, designating the infant mortality rate as the ninth parameter. This discontinuity at $x = 0$ was grafted on to the schedule in order to capture the dramatic decline between ${}_1q_0$ and ${}_1q_1$ in most mortality schedules—a decline that is difficult to represent with equation (5) alone. It is this latter reformulation that we propose as a possible alternative to the Heligman/Pollard specification—particularly for the description of black mortality régimes and other problematic age profiles. We call this particular nine-parameter (a_0, a_1, α_1, a_2, α_2, λ_2, μ_2, a_3, α_3) schedule the *multi-exponential* model, and in figure VI illustrate its fit to the black mortality régimes discussed in the section above. Table 7 presents the corresponding parameter values, including those found for several of the national male mortality régimes set out earlier in table 1.

Our experiments with the multi-exponential model reveal that it does a good job of fitting curves that exhibit a lower accident component than the typical mortality curve, for example, the mortality schedules of Alabama's and Georgia's black populations. For example, table 7 shows that our goodness-of-fit index for the nine-parameter multi-exponential model is lower than the corresponding index for the eight-parameter Heligman/Pollard model for the black male populations of Alabama and Georgia, but the index for the multi-exponential model is higher for the general male populations of Australia and the United States. (The goodness-of-fit index in both models is the mean absolute percentage error, which is not the criterion function being minimized.) We have not yet tried to fit the nine-parameter Heligman/Pollard model, which would lead to a fairer test of the two alternatives.

The fitting of the multi-exponential model was carried out by fixing the a_0 parameter equal to the probability of dying in the first age interval. All other parameters were allowed to iterate between 0 and 1, except μ_2, which was allowed to range between 0 and 100. Computing standard errors (not shown), one can determine approximately how "close" the estimated parameter value is to the "true" parameter value. Of all model

Figure VI. Fit of the multi-exponential model to representative black United States male mortality schedules in Alabama and Georgia: 1979-1981



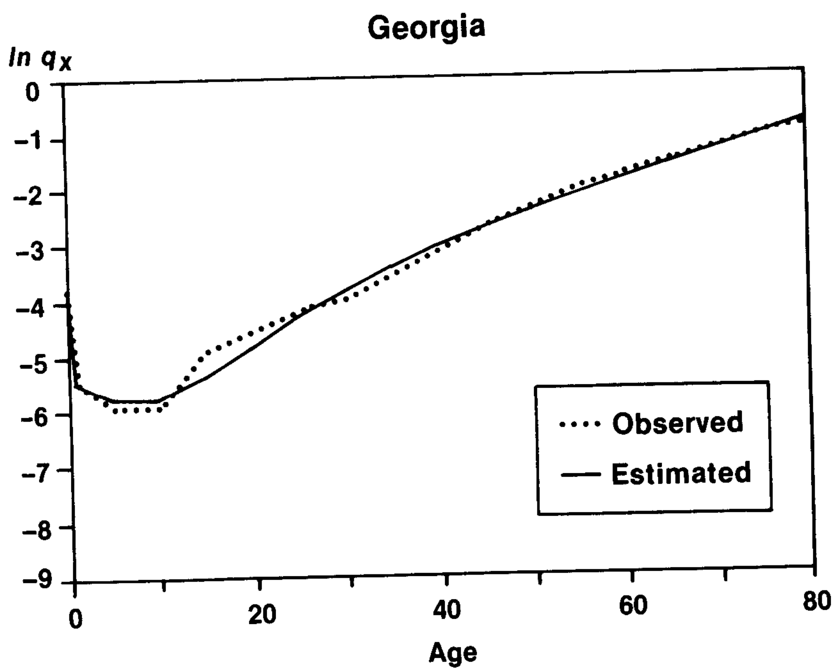


Table 7. Multi-exponential model parameters for several male mortality schedules

Schedule	Parameter[a]									Goodness-of-fit index	
	a_0	a_1	α_1	a_2	α_2	λ_2	μ_2	a_3	α_3	Multi-exponential model	Heligman/Pollard model
Black population, 1979-1981											
Alabama	0.02168	0.00183	0.3460	0.02154	0.0094	0.0410	49.24	0.00014	0.0800	3.50	5.79
Georgia	0.02273	0.00145	0.2203	0.02856	0.0095	0.0420	54.80	0.00016	0.0780	7.14	11.67
Australia, 1970-1972	0.02015	0.00164	0.2400	0.00421	0.2422	0.2900	18.33	0.00007	0.0949	6.16	4.02
England, 1971	0.02013	0.00118	0.2010	0.00176	0.3196	0.2226	20.15	0.00005	0.0986	-	-
Sweden, 1961-1970	0.01542	0.00099	0.1647	0.00171	0.1775	0.2045	20.17	0.00006	0.0937	-	-
United States, 1970	0.02381	0.00137	0.2521	0.00368	0.1863	0.2927	18.45	0.00014	0.0842	6.40	3.41

[a] Calculated using the standard set of five-year age groups. No goodness-of-fit indices are set out for the English and Swedish data because the raw data inputs were not available for those two countries and the fits were made to "data" produced by the Heligman/Pollard model with the parameters presented in D. O. Forfar and D. M. Smith, "The changing shape of English life tables", *Transactions of the Faculty of Actuaries*, vol. 40, No. 270 (1987), p. 99; and M. Hartmann, "Past and recent experiments in modeling mortality at all ages", Demography Section Working Paper No. 13 (Stockholm, University of Stockholm, 1983).

schedule parameter estimates, a_2 consistently receives the "worst" estimate. (The "best" estimates are for λ_2, a_3 and α_3.)

As figure VI shows, the multi-exponential model generally preserves the age profile of the original data. The parameters in table 7 are all approximately the same for the six fitted schedules, except for the accident component parameters (a_2, α_2, λ_2, and μ_2) of the black mortality schedules. Since there is almost no accident hump in those two schedules, the best fit of the model to the data is obtained by moving the accident component to around age 50. And because there is very little decline after the peak around age 50, α_2 becomes small and a_2 increases in value to place the peak at a higher level on the vertical axis.

A comparison of the parameters for the male schedules of the selected countries reveals that Sweden has the least pronounced accident hump (a_2, α_2, and λ_2 in table 7 are all relatively small); Australia and England have accident humps that are more pronounced, and the latter country has the sharpest rise from childhood mortality to the accident peak.

To conclude, our experiments have demonstrated that the multi-exponential model is a useful alternative to the Heligman/Pollard model. But it is not clear which model performs better in general. The Heligman/Pollard model seems to work best with more "normal" mortality curves, and the multi-exponential model seems to work well with the more unusual (e.g., black) mortality régimes. However, further experiments are needed (in particular, with the nine-parameter Heligman/Pollard model schedule) before that conjecture can be validated.

Conclusion

The Heligman/Pollard model mortality schedule and its computer implementation using the UNABR routine of the United Nations MORTPAK-LITE software package for mortality measurement have been the central focus of this paper. Five applications have been described, testimony to the usefulness of both the model and the software. In the process of developing these applications, a few problems were encountered. Problems with fitting the model to certain data sets were discussed and led to the introduction of an alternative specification, called the multi-exponential model schedule. (Yet another alternative specification is the nine-parameter Heligman/Pollard model, which we did not attempt to fit to the data.) Problems that arose out of the particular needs of our applications required a few modifications of the UNABR program.

The principal purpose of the UNABR routine is to provide users of the MORTPAK software package with a computerized procedure for graduating a set of age-specific probabilities of dying (${}_nq_x$ values) in age groups 0-1, 1-5, 10-15, . . . , to produce a smooth set of ${}_nq_x$ values and estimated single-year probabilities of dying and surviving (United Nations, 1988, p. 111). It accomplishes this specific purpose admirably. However, the reader should be aware that because the UNABR routine was not designed to carry out some of the applications outlined in this paper, it does require additional programming enhancements in order, for example, to produce goodness-of-fit measures, to fit models other than the eight-parameter Heligman/Pollard specification, to permit a user-designated set of initial parameter estimates with which to start the iterative algorithm, or to fix or constrain a particular parameter value.

NOTE

[1] Although the first two and the last age periods are not five years in length, for convenience they will be referred to in this particular age disaggregation as the standard set of five-year age groups.

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MEASUREMENT AND ANALYSIS OF COHORT-SIZE VARIATIONS*

*Shiro Horiuchi***

SUMMARY

This article discusses two measures of cohort-size variations: the age-standardized cohort size, and the growth rate of the number of births. The age-standardized cohort size is derived by reverse-surviving the current cohort size to birth, using the current life table. It adjusts observed cohort size for mortality, thereby making the size of cohorts at different ages comparable. The growth rate of the number of births reflects the impact of changes in fertility behaviour on population growth in a more sensitive manner than the growth rate of total population. The growth rate of the number of births can be decomposed into the population growth rate, fertility changes and age structure changes.

The usefulness of the two measures in analysing recent demographic trends is demonstrated. Variations in the age-standardized cohort size derived from the 1980 round of censuses in selected Asian countries lend support to Keyfitz's argument that the growing trend of cohort size was accelerated world-wide around the mid-twentieth century. Decomposition of the growth rate of the number of births shows that the number of births in the world decreased in the 1970s due to the decline of fertility in developing countries, particularly in China, and it returned to a positive growth later because of a slowdown in the fertility decline and changes in the age structure.

Annex II to this article discusses meanings and usefulness of the ratio of the crude birth rate (CBR)/total fertility rate (TFR) in studying the effects of age structure changes on population growth.

INTRODUCTION

Recently there has been increasing interest in the formal demography of cohort-size variations, with a focus on implications of cohort-size varia-

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tions for total population dynamics (such as population growth and age composition) and economic and social policies. The formal demography of cohort-size variations should not be confused with economic and social demography of cohorts in which the effects of particular historical experiences of cohorts on their demographic behaviour are studied (e.g., Easterlin, 1961; Ryder, 1965; Horiuchi, 1983). The following several reasons seem to underlie the growing interest in cohort-size variations.

(*a*) It has been widely recognized that cohort-size variations have important policy implications. As cohorts of different size enter and leave particular age groups, the demand for goods and services related to those life-cycle stages changes accordingly. Policy-makers should take into consideration changes in needs for pediatric care, school education, entry-level jobs, different types of housing, family-planning services and obstetric care, geriatric care and other services for the elderly and so on. These needs are related to particular life-cycle stages, and the amount of the needs is closely related to the number of persons at the various stages.

(*b*) Studies in population aging shed light on cohort-size variations. It is shown that a large part of population aging in the near future has already been built in the current age structure, because the current cohort-size variations will simply shift to older ages (United Nations, 1988). For example, the current relative difference in cohort size between middle-age cohorts and young-adult cohorts will be echoed in the relative difference a few decades hence between old-age cohorts and middle-age cohorts. The increase in the elderly population in developed countries will be accelerated when the cohorts born in the period of so-called "post-war baby boom" reach old age.

(*c*) It has been recognized that recent trends in world population growth are closely related to the cohort-size variations after the Second World War. The decline in the growth rate of world population which started around 1970 stagnated in the 1980s, mainly because of the effects of cohort-size variations on the number of births and, in turn, the population growth rate. In developed countries, large cohorts, called "baby-boomers", are now in child-bearing ages, which has positive effects on the number of births. The rapid increase in the number of young children in developing countries during the 1950s and 1960s was called a "population explosion". The increase in the number of children is currently echoed as an increase in the number of young adults in ages around the peak of child-bearing. This caused a slowdown in the 1980s of the decline in growth rate which started in many developed countries during the 1970s.

(*d*) Keyfitz (1988) has recently shown that estimates of the world population by age and sex in the 1950s, 1960s, 1970s and 1980s indicate a trace of accelerated increase in population size between some successive cohorts. The turning point appears to be located at the cohort born in the late 1940s. The acceleration is called "the demographic discontinuity in the mid-twentieth century".

(*e*) Keyfitz's study of demographic discontinuities seems to suggest that the existing theory of age structure changes in the context of demographic transition should be more elaborated in consideration of cohort-size variations related to the timing of transition. The existing theory simply states that the transition from high to low levels of fertility and mortality produces population aging. The study by Keyfitz, however, seems to lead to the realization that the initiation of significant mortality decline and of significant fertility decline may leave traces on the population pyramid as two points at which the slope of the pyramid changes drastically. A corollary of the empirical generalization that mortality decline typically precedes fertility decline in demographic transition is that the population pyramid during the transition period may have three different sections: a steeply sloped section comprising cohorts born before the initiation of significant mortality decline; an even more steeply sloped section comprising cohorts born between the initiation of mortality decline and that of fertility decline; and a moderately sloped section comprising cohorts born after the initiation of significant fertility decline. It will be seen in the figure below that four selected Asian populations fit this pattern quite well.

(*f*) The last reason is methodological. Since the inception of the age-specific-growth-rate approach in the study by Bennett and Horiuchi (1981), there has been an increasing number of studies in which age-specific growth rates are used. Those studies are based on an equation that expresses the age distribution as a function of the crude birth rate, age-specific death rates, age-specific rates of net migration and age-specific growth rates (Bennett and Horiuchi, 1981, appendix; Preston and Coale, 1982). It has been shown that the key roles played by age-specific growth rates in the general equation of age distribution are closely related to the cohort-size variations (Horiuchi and Preston, 1988). It will be shown below that cumulated age-specific growth rates can be interpreted as cohort-size effects on the age distribution.

Although increasing attention is being drawn to cohort-size variations, the demographic methodology for analysing them does not seem to have been fully developed. In this article two demographic measures that can be used for studying cohort-size variations will be discussed. They are the age-standardized cohort size and the growth rate of the number of births. The former is derived from "stock data" from censuses and the latter is computed from "flow data" from civil registration. Their usefulness in analysing post-war demographic trends will be demonstrated.

Standardized measure of cohort size

When demographers compare relatively large cohorts and relatively small cohorts, their discussion is usually based on either bumps and hollows on the slope of population pyramids or changes in the number of registered births. In the former case, base data are population at a given time classified by age and sex. The numbers of persons in different cohorts at a given time, however, are not directly comparable with each other, because the attrition of cohorts due to mortality causes a tendency for older cohorts to be smaller. Therefore, in order to compare the size of cohorts at a given time, it is necessary to adjust the current cohort size for age.

No adjustment is needed if a certain age is selected as the reference age and the actual size of different cohorts at that age is compared. If the exact age 0 is chosen, the number of births for different years is compared. The size of cohorts at age 5 may be examined if effects of child mortality decline on population growth are within the scope of the study. Age 65 may be chosen for studying cohort-size variations in relation to population aging.

This approach, however, has a few limitations. First, there may be some cohorts that have not reached the reference age. Secondly, reliable estimates of the cohort size at birth and young ages may not be available for many old cohorts in a number of developing countries. Demographic data may be lacking or very defective, particularly at the time when those who are currently old were young children. Thirdly, patterns of variation in size among the same group of cohorts may change with age, because some cohorts may increase or decrease due to mortality and migration in ways significantly different from those of other cohorts. For example, a cohort that was relatively large at birth may later be considered relatively small as a result of heavy war casualties or out-migration.

A simple way to adjust the observed cohort size for age differences is to reverse-survive the cohort to birth, using the life table prevailing in the population at the time of observation. The birth equivalent of current cohort size is thus given by,

$$ {}_nu_x = {}_nN_x / {}_nL_x, $$

where ${}_nN_x$ is the number of persons aged x to $x + n$ and ${}_nL_x$ is the number of person-years lived in the life table. The life table l-function, the number of survivors, is set to be 1 at age 0. It is also useful to compute

$$ {}_nv_x = {}_nu_x / B, $$

where B is the actual annual number of births at the time of observation. In the Lexis-surface framework in which population size is treated as a continuous and differentiable function of age and time (Arthur and Vaupel, 1984), u's and v's can be defined at exact age x by

$$ u(x) = N(x)/l(x) $$

and

$$ v(x) = u(x)/B, $$

where $N(x)$ is the density of population size at age x and B is the density of births. The u-value results from standardization of cohort size with respect to a given mortality schedule and is directly comparable to the annual number of births, whether the original data on population size are classified by one-year age groups, five-year age groups, other age groups or their mixtures. The v function is the ratio of the birth equivalent of cohort size to the actual number of births at the time of observation.

Table 1 reveals values of these two standardized cohort-size functions for female populations in selected Asian countries (China, Indonesia, the Philippines and Thailand) at the 1980 round of censuses. For example, it is shown in the table that, although the Indonesian female cohort aged 65–69 in 1980 is about one fifth (918/4,336) of the cohort aged 35–39, the older cohort could be considered to be about one third (405/1,188) of the younger cohort when their age difference is taken into account. The Thai female cohort aged 0–4 in 1980 is slightly larger than the cohort aged 5–9, but the order is reversed when observed figures are converted into the birth-equivalent values. Information on the life tables and the estimates of the number of births used in the computation are given in annex I.

The standardized measure of cohort size has a few interesting characteristics:

(*a*) The u's are a special kind of *backward* projections; $u(x)$ shows what the annual number of births x years ago would have been if mortality had been constant at the current level during the last x years. In that sense, u's may be considered hypothetical reverse-survival estimates of the number of births in the past.

(*b*) The $u(x)$ function contains all information on constant-mortality forward projections as far as the cohorts born before the base year of projection are concerned. If mortality remains unchanged for t years after the base date, the population aged x at that time can be projected by

$$N(x, t)/N(x, 0) = u(x - t, 0)/u(x, 0),$$

where N's and u's are as previously defined except that here they have the second subscript indicating time.

In constant-mortality projections, the following statement holds true for any age and any combination of two cohorts: the ratio of u-values of two cohorts at a given time (base year of projection) is equal to the ratio of the projected population size of one of the cohorts at a certain age to that of the other cohort at the same age. For example, u values for Filipino females in table 1 imply that if mortality remains constant in the future, the age group 70–74 will double (194/99) in about 20 years, triple (290/99) in about 30 years, and in 70 years, become eight times (798/99) as large as its size at the 1980 census. (Such a sharp increase in the elderly population is implied for all of the four countries presented in table 1. Expected mortality improvement in the future will make the increase even steeper.)

In brief, for any age group selected, the sequence of current u-values indicates the future trajectory of population size of the age group in constant-mortality projections. Although population projections by age are usually produced as a large (age-by-period) matrix containing many numbers, we do not need such a large matrix for constant-mortality projections. The u-vector provides a concise summary of constant-mortality projections.

It can be inferred from the above-mentioned two characteristics of the standardized cohort-size functions that, assuming the continuation of mor-

TABLE 1. AGE-STANDARDIZED COHORT SIZE FOR SELECTED FEMALE POPULATIONS IN ASIA[a]

Age group	China, 1982			Indonesia, 1980			Philippines, 1980			Thailand, 1980		
	N	*u*	*v*	*N*	*u*	*v*	*N*	*u*	*v*	*N*	*u*	*v*
0-4	45,724	9,628	0.937	10,289	2,354	0.967	3,656	798	0.944	3,113	667	1.023
5-9	53,691	11,464	1.115	10,429	2,521	1.035	3,154	709	0.838	3,053	681	1.029
10-14	63,941	13,704	1.333	8,508	2,088	0.858	2,864	649	0.767	2,992	672	1.016
15-19	61,564	13,245	1.289	7,828	1,944	0.799	2,654	605	0.715	2,601	588	0.888
20-24	36,457	7,889	0.768	6,966	1,761	0.723	2,361	543	0.642	2,278	519	0.784
25-29	44,810	9,766	0.950	5,658	1,462	0.601	1,933	449	0.531	1,873	431	0.652
30-34	35,051	7,699	0.749	4,126	1,095	0.450	1,485	350	0.414	1,429	333	0.504
35-39	25,657	5,690	0.554	4,336	1,188	0.488	1,206	290	0.342	1,179	280	0.422
40-44	22,589	5,073	0.494	3,744	1,064	0.437	1,055	259	0.306	1,111	269	0.406
45-49	22,317	5,100	0.496	3,150	935	0.384	864	218	0.258	936	233	0.352
50-54	19,290	4,528	0.441	2,726	855	0.351	740	194	0.229	765	197	0.298
55-59	16,410	4,014	0.391	1,685	569	0.234	600	166	0.196	577	157	0.236
60-64	13,668	3,577	0.348	1,710	643	0.264	496	149	0.175	459	134	0.203
65-69	11,092	3,235	0.315	918	405	0.166	395	134	0.158	359	119	0.179
70-74	7,910	2,762	0.269	877	495	0.204	241	99	0.117	282	113	0.170
75-79[b]	5,111	2,380	0.232	860	356	0.146	152	84	0.099	283	71	0.106
80-84	2,357	1,765	0.172	..	..	..	61	54	0.063	..	..	..
85+	930	1,013	0.099	..	..	..	51	60	0.070	..	..	..

Sources: See annex I.

[a] *N*'s and *u*'s are given in 1,000s.

[b] 75+ for Indonesia and Thailand.

tality improvement in the recent past and near future, the standardized cohort-size functions provide the most conservative picture of the population growth of age groups. For example, table 1 shows that the v value at the 1980 Indonesian census is 0.234 for females aged 55-59 and 0.264 for females aged 60-64. Thus, we can estimate that $v(60)$ is in the neighbourhood of 0.25. This implies that the number of births in 1920 (60 years ago) was at most about one quarter of the number of births in 1980 and that the population aged 60 in 2040 (60 years later) will be about four times larger (at least) than the population aged 60 in 1980.

(*c*) The $v(x)$ function represents deviations of the actual age distribution from the corresponding stationary and stable distributions. If the population is stationary, all values of $v(x)$ will be 1. If the population is stable, $v(x)$ values will follow the exponential curve $\exp(rx)$ where r is the stable growth rate, so that they will fall on the straight line with a slope r on a semi-log graph paper.

In this connection, it can easily be shown that, in a population closed to migration, $v(x)$ is equal to the exponential of the negative of the integral of age-specific growth rate function from age 0 to x–that is:

$$v(x) = \exp\left(-\int_0^x r(a)da\right),$$

where $r(a)$ is the growth rate at age a. It has previously been shown that cumulated age-specific growth rates indicate deviations of the actual from stationary age distribution (Horiuchi and Preston, 1988).

(*d*) The $v(x)$ function can be used for representing the net reproduction rate (NRR) as the ratio of cohort size of babies to cohort size of their mothers. Preston and Coale (1982) have shown that

$$\text{NRR} = \int_\alpha^\beta h(x) \exp\left(\int_0^x r(a)da\right) dx,$$

where $h(x)$ is the density function representing the proportional distribution of births by the age of mother, and α and β are the lowest and highest ages of child-bearing, respectively. Making use of the previously given expression of $v(x)$ in terms of the age-specific growth rate function, NRR is given by

$$\text{NRR} = \int_\alpha^\beta h(x)/v(x)\, dx.$$

Since $1/v(x)$ is the ratio of size of cohort aged 0 to size of cohort aged x, the above equation indicates that NRR can be interpreted as the average cohort-size ratio of new-born babies to their mothers.

(*e*) The $v(x)$ function is related to the concept of population momentum. Keyfitz (1971, 1977) has derived an equation for calculating the ratio of the size of the ultimate stationary population to the current size of actual population under the assumption of an immediate fall in the net production rate to 1 and the constancy of fertility and mortality thereafter. Preston (1986) has developed another expression under the same assumption.

A different approach to the population momentum is to assume that the growth rate of the number of births becomes 0 immediately so that the number of births will remain constant thereafter. It is also assumed that mortality will remain unchanged as well. Because these assumed conditions will result in a stationary population, the population growth that will occur before reaching the ultimate stationary population size can be considered as a result of population momentum. It should be noted, however, that the ultimate population size resulting from the immediate fall of the growth rate of birth to 0 is different from the ultimate population size resulting from the immediate fall of the net reproduction rate to unity, except for some special cases.

If mortality remains unchanged, the population aged x will grow in the next x years by the factor of $1/v(x)$. Because of the assumed zero growth of the number of births, the age group will cease to grow thereafter, so that the reciprocal of $v(x)$ is the ratio of the ultimate to current size of the age group. Therefore, the ratio of the ultimate to current size of the total population is the average of the reciprocal of $v(x)$ weighted according to the current age distribution. It can easily be shown that the weighted average is equal to the product of the crude birth rate and the expectation of life at birth. The ratio can be expressed in terms of age-specific growth rates as well. In brief, the ratio of the ultimate to current population size is given by

$$b\,e(o) = \int_0^\infty c(x)/v(x)\,dx = \int_0^\infty c(x)\exp\left(\int_0^\infty r(a)da\right)dx,$$

where b is the crude birth rate, $e(0)$ is the expectation of life at birth and $c(x)$ is the density function of proportional age distribution. It is interesting to note that the product of the crude birth rate and the expectation of life at birth is a term included in Keyfitz's formula of population momentum.

It should be noted that the pattern of $v(x)$ indicates the age structure of population momentum. The smaller value of $v(x)$ implies the greater proportional growth of the age group in the future. Table 1 shows that for all of the four countries, $v(x)$ tends to be lower at older ages. Thus, the current variations of cohort size in those countries seem to suggest considerable prospective growth of the elderly population there.

The above discussion of population momentum is based on the assumption of constant mortality. The assumption, however, can be waived for generalization. Instead, it is assumed that the number of births will remain constant in the future but that mortality will change, reach a minimum level and remain unchanged at that level. Then the ratio of ultimate to current population size is the product of the current crude birth rate and the ultimate life expectancy at birth. Ratios of the ultimate to latest-census population size for both sexes together in China, Indonesia, the Philippines and Thailand are 1.43, 1.77, 2.20 and 1.80, respectively, under the assumption of constant mortality, and 1.68, 2.72, 2.88 and 2.32 if the maximum expectation of life at birth for both sexes is assumed to be 80. A very simple calculation, the crude birth rate times 80, shows that the

in ultimate pop., pop size is $B \times T_0$ (since stationary)

so ratio ult/current is $\frac{B(\infty)}{P(0)} \times T_0(\infty) = \frac{B(\infty)}{P(0)} \times e$

female populations of Indonesia and the Philippines will nearly triple even if the number of births ceases to grow immediately.

(*f*) Lastly, it is possible to adjust the cohort size for both the age schedule of mortality and the age schedule of migration. Insofar as data on age-specific migration rates are available, we can calculate life-table functions for migration and "reverse-migrate" cohorts. It may be reasonable to make such an adjustment for migration if the direction, size and age-pattern of migration are expected to be stable for the next several decades. The expectation may not be met in many cases. Furthermore, unlike mortality, which causes only attrition of cohorts, migration could either increase or decrease the cohort size. It thus seems reasonable to age-standardize the cohort size with respect to mortality only.

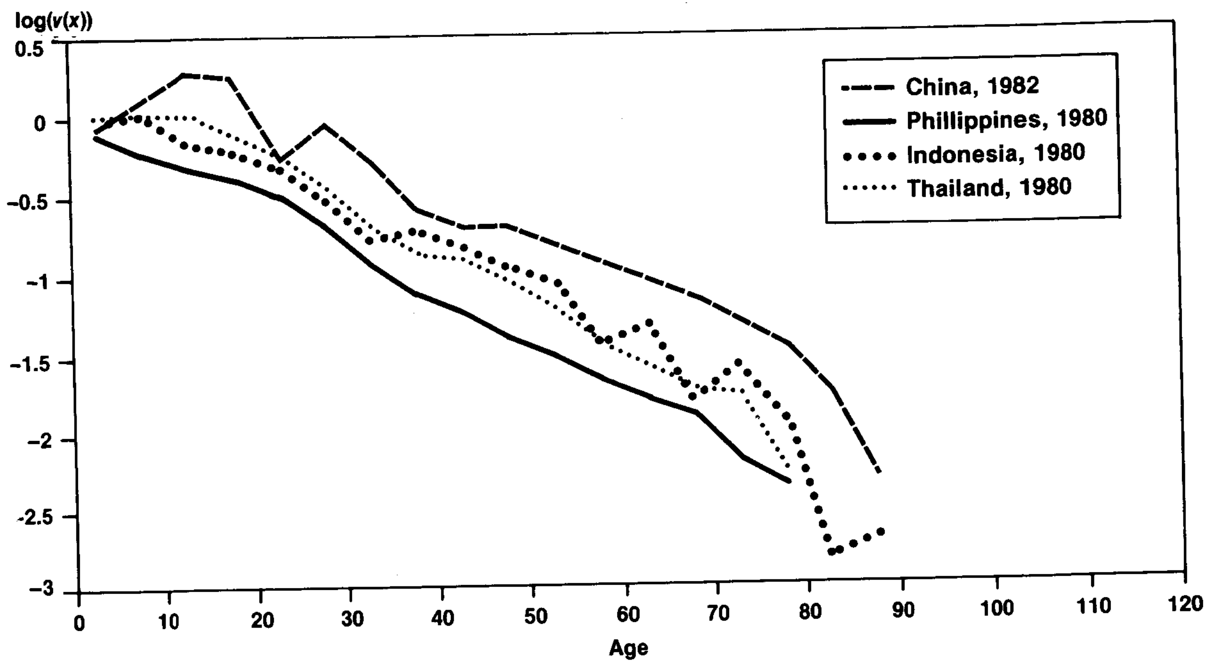
The $u(x)$ and $v(x)$ values for four Asian female populations presented in table 1 exhibit interesting patterns. The $v(x)$ values in table 1 are displayed in the figure. In order to follow the standardized cohort size function from earlier-born cohorts to later-born cohorts, it is necessary to look at the curves in the figure from right to left, which is a reversal of the usual direction. The standardized cohort size grows gradually in both the Philippines and Thailand from the cohort born in 1910–1915 (aged 65–69 at the 1980 census) to the cohort born in 1940–1945 (aged 35–39), then increases sharply from the birth cohort of 1940–1945 to the cohort of 1945–1950 (aged 30–34). The steep increase continues to the cohort of 1955–1960 (aged 20–24), then slows down. In particular, the cohort size in Thailand remains at the same level for the cohorts born from 1965 to 1980, indicating the sharp fertility decline during the period.

Indonesia follows a similar pattern, with a few notable differences. The fluctuations at old ages seem to indicate a popular pattern of digit preference in the census data. The cohort size decreases from the cohort of 1940-1945 (aged 35-39) to that of 1945-1950 (aged 30-34), probably reflecting effects of the Independence War. The acceleration of cohort-size growth is located between the cohort of 1945-1950 (aged 30-34) and that of 1950-1955 (aged 25-29), which is later than in the Philippines and Thailand by five years. For China, the acceleration is located between the cohort of 1942-1947 (aged 35-39 at the 1982 census) and that of 1947-1952 (aged 30-34). The trough at the cohort of 1957-1962 (aged 25-29) is due to the crisis around 1960. The decrease in cohort size among young cohorts indicates the fertility decline in the 1970s.

For each of the four populations, acceleration in the growth of cohort size is noticeable. The timing of acceleration, in terms of the birth year of cohort, ranges from around 1945 (the Philippines and Thailand) to around 1950 (Indonesia). This lends support to Keyfitz's argument that the growing trend of cohort size was accelerated world-wide around the mid-twentieth century.

Similarly, deceleration of the growth of cohort size can be seen among young cohorts in each of the four populations. This seems to reflect the recent fertility decline in those countries. It can be stated, therefore, that each of those populations has three generations: pre-acceleration (old

Figure. Age-standardized cohort size for four Asian female populations



cohorts); post-deceleration (young cohorts); and in-between. This three-segment pattern can be visually recognized in the figure, although the smoothness of the curves is perturbed to some extent by the trace of special events such as the 1960 crisis in China and the Independence War of Indonesia and the trace of age misreporting.[1]

Before concluding the discussion on the standardized cohort-size functions, a few words of caution seem needed. The calculation of the standardized cohort-size functions does not complete the investigation of cohort-size variations in the study population. The standardized cohort-size functions provide an overall summary of current cohort-size variations, which should be a starting point of further investigation in both retrospective and prospective directions. In order to investigate how the current cohort-size variations have emerged, efforts should be made to prepare accurate estimates of the size of those cohorts at birth and different ages and accurate estimates of past fertility, mortality and migration that affected the size of the study cohorts. For drawing policy implications of current cohort-size variations, population projections with the assumption of constant mortality and no migration are probably too simple and not sufficient. Several versions of population projections with different sets of assumptions should be prepared, and projected changes in the size of each age group should be carefully examined.

Growth rate of the number of births

The usefulness of age-specific growth rates for analysing age structure changes has recently been demonstrated in various studies (Horiuchi, 1988; Horiuchi and Preston, 1988; Preston, 1986; Preston, Himes and Eggers, 1989; United Nations, 1988 and 1989). The growth rate of the number of births is a special type of age-specific growth rate—namely, the growth rate at exact age 0.

Although the growth rate of the number of births plays a key role in the theoretical demography of convergence to stability, it has seldom been used in empirical research, in contrast to the great popularity of the growth rate of total population. The growth rate of the number of births, however, deserves a place in the regular set of demographic indicators. For example, it is far more sensitive to the performance of family-planning programmes than the population growth rate. The sensitivity of the growth rate of total population to fertility changes is limited, because the growth rate of total population is a weighted average of all age-specific growth rates, including those at middle and old ages. Current family-planning programmes have no direct impact on growth rates of the elderly population and the working-age adult population in the near future, because they have already been born.

Furthermore, the growth rate of total population and the growth rate of the number of births give different pictures of demographic trends. As shown in table 2, the total population of Japan has recently been growing, though at a very low, decreasing rate. The number of births, on the other hand, has already entered the era of negative growth since the mid-1970s.

TABLE 2. GROWTH RATES OF TOTAL POPULATION (r) AND GROWTH RATES OF THE NUMBER OF BIRTHS (r_B) FOR JAPAN, 1962-1986

Year	r^a	$r_B{}^b$
1962	0.95	1.87
1963	1.02	2.50
1964	1.07	3.38
1965	1.13	(2.16)[c]
1966	0.77	(2.16)[c]
1967	1.17	(2.16)[c]
1968	1.13	(2.16)[c]
1969	1.19	0.96
1970	1.15	2.30
1971	1.37	3.41
1972	1.41	1.88
1973	1.40	2.57
1974	1.35	−3.01
1975	1.24	−6.57
1976	1.03	−3.64
1977	0.95	−4.35
1978	0.90	−2.66
1979	0.84	−3.94
1980	0.78	−4.10
1981	0.70	−3.09
1982	0.69	−0.92
1983	0.67	−0.40
1984	0.63	−1.27
1985	0.60	−3.97
1986	0.52	−3.85

Sources: *Population Projections for Japan: 1985-2025* and *Latest Demographic Statistics, 1986* (Tokyo, Institute of Population Problems, Ministry of Health and Welfare, 1987).

[a] Growth rate centred around 1 April of the year.

[b] Growth rate centred around the beginning of the year.

[c] The average annual growth rate of the number of births from 1964 to 1968. The actual annual rate showed very large fluctuations because a belief in the misfortunes of women born in the year of "the white horse" resulted in relatively few births in 1966 and many births in 1965 and 1967.

It is interesting to note that, according to the official medium-variant projections, the population of Japan will enter the era of negative growth in 2015 (Japan, 1987a). Although the current negative growth in the number of births does not necessarily imply a negative growth of total population in the future, the discrepancy between the two sequences of recent growth rates helps to understand the projected shift of population growth rate of Japan from positive to negative.

The growth rate of the number of births can be decomposed on the basis of the following equation:

$$B = N \times \text{TFR} \times (\text{CBR}/\text{TFR}),$$

where B is the number of births and N is the size of total population. The ratio of the crude birth rate to the total fertility rate indicates the effects of relationships between the age structure of population and the age pattern of fertility on the crude birth rate. A detailed discussion on the characteristics of the ratio is given in annex II. The above equation shows that the number of births is determined by three factors: the size of total population; the level of fertility (as measured by TFR); and relationships between the age structure of population and the age pattern of fertility. Accordingly, the growth rate of the number of births, r_B, is decomposed by

$$r_B = r + r_f + f_{bf},$$

where r, r_f and r_{bf} are the growth rate of the total population, the growth rate of the total fertility rate and the growth rate of the ratio of the crude birth rate to the total fertility rate, respectively.

One data set that can be used for international comparison of the growth rate of the number of births is the demographic estimates and projections for all countries in the world prepared biennially by the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat. The 1988 revision (United Nations, 1989) includes the estimated number of births for each quinquennial period from 1950 to 1985 as well as projections from 1985 to 2025. For statistically developed countries, those estimates are based on civil registration data. For statistically underdeveloped countries, the number of births is derived from the estimated population size (usually interpolated between censuses) and the birth rate estimated indirectly from census or survey data.

Growth rates of the number of births in the world in the past few decades are calculated using those United Nations estimates and shown in table 3. Because the number of births is estimated for quinquennial periods, the growth rate is computed as an average annual rate of increase from one quinquennial period to the next. The trend does not appear to be very simple: it has both a downturn and an upturn. The growth rate of the number of births fell sharply in the 1970s and then returned to the level previously observed in the mid-1960s. The negative rate for 1970-1975/1975-1980 implies that the number of births in the world actually decreased during the 1970s, when total population size kept growing at a relatively high level of about 2 per cent per year.

In order to analyse the global trend, we divide the world population into three subpopulations: China; developing countries[2] excluding China; and developed countries.[3] China is singled out not only because it has the largest population size in the world but also because in the 1970s it had achieved the fastest fertility decline ever experienced by populous countries.

Table 3 suggests that China, with about 22 per cent of the world population in 1970 and about 24 per cent of births in the world for 1965-1970, had a significant impact on the global trend, particularly in the 1970s.[4] The number of births in China decreased at the average annual rate of 5.26 per cent from the early 1970s to the late 1970s, which was caused by the sharp decline in fertility. The total fertility rate in China fell

TABLE 3. GROWTH RATE OF THE NUMBER OF BIRTHS IN THE WORLD AND ITS DECOMPOSITION, 1950-1985

	1950-1955 1955-1960	*1955-1960 1960-1965*	*1960-1965 1965-1970*	*1965-1970 1970-1975*	*1970-1975 1975-1980*	*1975-1980 1980-1985*
			World total			
r_B	0.81	1.73	1.24	0.56	−0.23	1.23
r	1.83	1.93	2.03	2.01	1.85	1.74
r_f	−0.47	0.37	−0.38	−1.83	−2.95	−1.24
r_{bf}	−0.55	−0.56	−0.41	−0.38	0.87	0.73
			China			
r_B	−2.19	2.84	1.87	−1.35	−5.26	−1.14
r	1.69	1.81	2.35	2.39	1.80	1.33
r_f	−2.91	1.89	0.21	−4.62	−9.87	−4.11
r_{bf}	0.98	−0.86	−0.69	0.87	2.82	1.65
			Developing countries, excluding China			
r_B	2.24	1.91	1.69	1.52	1.34	1.97
r	2.29	2.45	2.49	2.49	2.43	2.42
r_f	0.15	−0.18	−0.50	−1.03	−1.60	−0.98
r_{bf}	−0.20	−0.36	−0.30	0.06	0.52	0.53
			Developed countries			
r_B	0.36	−0.23	−1.32	0.47	0.63	0.24
r	1.27	1.22	1.04	0.88	0.80	0.69
r_f	−0.12	−0.94	−1.95	−2.08	−1.55	−1.04
r_{bf}	−0.78	−0.51	−0.41	0.73	0.12	0.60

Definitions:
r_B Growth rate of the number of births.
r Growth rate of the total population.
r_f Growth rate of the total fertility rate.
r_{bf} Growth rate of the ratio of the crude birth rate to the total fertility rate.
Source: *World Population Prospects, 1988* (United Nations publication, Sales No. E.88.XIII.7).

from 5.81 children per woman in 1970 to 2.24 children in 1980 (China, 1984).

The trend of r_B in the other developing countries combined, though not so spectacular as that of China, seems to have contributed significantly to the global deceleration of the growth of births in the 1970s. Table 3 reveals that r_B decreased gradually from 2.24 per cent for the mid-1950s (1950-1955/1955-1960) to 1.34 per cent for the mid-1970s (1970-1975/1975-1980). The decomposition of r_B in table 3 indicates that the gradual decline in r_B resulted mainly from the accelerated decline of fertility.

The trend of the growth rate of births in developed countries does not seem to have had a strong impact on that of the world total. The number of births in developed countries kept decreasing from the late 1950s to the late 1970s, with a fluctuating rate. The trend in the growth rate of TFR in table 3 suggests that fertility decline was faster in the late 1960s and the early 1970s than in the late 1970s. The timing of fertility decline in developed countries, therefore, does not exactly coincide with the timing of the reduction of births in the world.

In brief, the steep decline of the growth rate of the number of births in the world during the 1970s seems to be mainly attributable to the fertility decline in China, and to a lesser extent, the fertility decline in other developing countries. It has previously been indicated that there was a "turning point" of world population growth around 1970, when "significant fertility decline started in many countries in Latin America and Asia" (United Nations, 1988, p. 21).

A question remains as to why the growth rate of the number of births in the world rose from the trough of −0.23 per cent in the mid-1970s (1970-1975/1975-1980) to a relatively high level of 1.23 per cent per year for around 1980 (1975-1980/1980-1985). The decomposition of r_B in table 3 suggests that there are two major reasons for the relatively higher r_B for 1975-1980/1980-1985.

First, the global fertility decline in the 1970s had slowed down in the 1980s, particularly in China and India, which jointly had 38 per cent of the world population in 1980. The total fertility rate in China remained in the range of 2.2-2.6 from 1980 to 1987 (Jiang, 1988). The 1982 national fertility survey showed a total fertility rate of 2.24 in 1980. The crude birth rate in the 1987 national survey was 21.2 per 1,000, which suggested a total fertility rate of about 2.4-2.5, given the age structure of the population at the 1982 census and the age pattern of fertility observed in the 1982 national fertility survey. In India, the crude birth rate derived from the Sample Registration System changed very little, from 33.7 per 1,000 in 1980 to 32.9 per 1,000 in 1985.[5] Although the fertility decline in the 1970s continued in the 1980s without significant slowdown in some populous developing countries, including Brazil, Indonesia, Mexico, the Republic of Korea and Thailand, their combined effects on the global trend did not match those of China and India together. Furthermore, fertility in the majority of developed countries was already around the replacement level or even below at the beginning of the 1980s, so that further drastic decline was difficult to achieve there.

Secondly, some age structure changes were underlying a relatively high annual r_B of 1.23 per cent around 1980 (1975-1980/1980-1985). As described above and in annex II, the CBR/TFR ratio is a measure of age structure effects on the crude birth rate, and in turn, the number of births. The growth rate of CBR/TFR (r_{bf}) has been rising in China, other developing countries, and developed countries during the past few decades. The age structure has been increasingly in favour of a larger number of births relative to the population size and the level of total fertility rate.

The age structure effects seem attributable mainly to the accelerated growth of female population at peak child-bearing ages, resulting from the influx into the age range of the cohorts born in the 1950s and 1960s. They were born after Keyfitz's "demographic discontinuity in the mid-twentieth century". Those cohorts were so-called "baby-boomers" in developed countries and the generations born in the period of so-called "population explosion" in developing countries. In addition, the decreasing proportion of population who are young children, caused by the fertility decline in both developed and developing countries, has raised the proportion of the other age groups, including peak child-bearing ages.

The age structure changes do not explain the *short-term* rise of r_B for the world total from -0.23 per cent in the mid-1970s (1970-1975/1975-1980) to 1.23 per cent around 1980 (1975-1980/1980-1985), because r_{bf} actually declined during the period. The age structure changes, however, can be considered as a medium-term trend contributing significantly to the relatively high level of r_B around 1980 (1975-1980/1980-1985), which is almost the same as 1.24 per cent in the mid-1960s (1960-1965/1965-1970). By comparing the decomposition of r_B for those two periods, we notice that r_B returned to the previous level of the mid-1960s because the negative effects on r_B of the falling population growth rate and declining fertility were cancelled out by the positive effects of the age-structure changes.

Conclusion

Changes in the proportional age distribution and changes in the size of age groups have significant socio-economic implications and policy relevance. Those changes are partly due to general demographic trends: fertility decline and continued mortality decline produce population aging. It should also be noted that age structure changes are partly due to cohort-size variations caused by historically idiosyncratic events, such as the World Wars and the post-War baby boom. Furthermore, earlier cohort-size variations affect later cohort-size variations, thereby causing some "population waves" (Keyfitz, 1973), which complicate the age structure.

In this article, two measures of cohort-size variations—the age-standardized cohort size and the growth rate of the number of births—were discussed and their usefulness in studying the post-War population trends was demonstrated. Given the significance of cohort-size variations in demographic analysis, returns from further methodological efforts seem highly promising.

Notes

[1] It can be seen in the figure that the age-standardized cohort size decreases very steeply with age in the oldest age groups in China, Indonesia and Thailand. Horiuchi and Preston (1988) have shown that mortality improvement produces a tendency for the age-specific growth rate to rise sharply with age in very old age groups. The steep slopes at the right end of the curves in the figure may simply be a reflection of this tendency.

[2] The term "developing countries" in this article refers to countries in the "less developed regions" as defined by the Population Division (United Nations, 1989). They comprise Africa, Asia (excluding Japan), Latin America and the Caribbean, and Oceania (excluding Australia and New Zealand).

[3] The term "developed countries" in this article refers to countries in the "more developed regions" as defined by the Population Division (United Nations, 1989). They comprise all countries in Europe, together with Australia, Canada, Japan, New Zealand, the United States and the USSR.

[4] The growth rate of the number of births in China fluctuates considerably in the first three periods in table 3. This is due to the demographic crisis around 1960. Since the impact of the crisis on fertility is estimated to be stronger in the first half of 1960 than in the second

half, the crisis reduced the number of births in the five-year period 1955/1960 (mid-1955 to mid-1960), thereby lowering r_B for 1950-1955/1955-1960, and in turn, raising r_B for 1955-1960/1960-1965. These fluctuations are reflected in the r_B sequence for the world total, which would otherwise be smoother.

[5] Those birth rates are not adjusted for underregistration. Actual birth rates may be estimated to be greater by about one birth per 1,000 population if adjusted for the officially estimated 3 per cent of underreporting of births in the areas of the Sample Registration System.

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ANNEX I

Data source for table 1 and for computation of population momentum

Population by sex and age

For China, Indonesia and the Philippines, census data were taken from a publication of the Statistical Office of the United Nations (United Nations, 1984). Thai census data were adjusted by the National Economic and Social Development Board (NESDB) Working Group on Population Projections (Thailand, 1985), and their estimates were used here.

Life table

The United Nations Latin American pattern model life tables for females with the expectation of life at birth of 54, 63 and 64 years were used for Indonesia, the Philippines and Thailand, respectively. (Empirical life tables for the Philippines and Thailand in earlier periods belong to the Latin American pattern group.) The life tables prepared by Hao, Arriaga and Banister (1988) were used for China. The life expectancy at birth for females in Hao, Arriaga and Banister's life table is 69 years.

Crude birth rate (annual number of births per 1,000 population)

The crude birth rate was 21 for China (1982), 34 for Indonesia (1980), 36 for the Philippines (1980) and 29 for Thailand (1980), based on results of various fertility surveys in those countries.

ANNEX II

Ratio of crude birth rate to total fertility rate

It is well known that the crude birth rate (CBR) is affected by the age structure, although the total fertility rate (TFR) is not. When age-specific fertility rates are kept constant, a higher proportion of women in child-bearing ages tends to produce a higher crude birth rate. Thus the proportion of women in child-bearing ages from 15 to 49—namely,

$$\sum_{i=15}^{49} W(i)/N,$$

where N is the total population size and $W(i)$ is the number of women aged i on the last birthday—is an indicator of age-structure effects on the crude birth rate.

This indicator assigns equal weights to all women from age 15 to 49. However, since the fertility rate varies with age, women at different ages do not contribute to the crude birth rate to the same extent. The proportion of women aged 25-34, for example, has greater impact on the crude birth rate than the proportion of women aged 40-49. Therefore, it seems reasonable to assign different weights to different ages, proportionally to age-specific fertility rates. Thus, the indicator should be constructed as

$$\sum_{i=15}^{49} g(i)W(i)/N,$$

where $g(i)$ is the weight given to age i.

One idea is simply to set the weight to be equal to the age-specific fertility rate: $g(i) = f(i)$, where $f(i)$ is the fertility rate at age i. However, the value of the indicator tends to be greater for populations with high fertility levels. Since we are measuring age-structure effects on the crude birth rate, it is desirable for the indicator to be dependent on the *age pattern* of fertility but independent of the *level* of fertility. Therefore, the weight should be scaled such that

$$\sum_{i=15}^{49} g(i) = 1.$$

The proportion of women at each age, therefore, should be weighted according to the proportional age distribution of fertility rate:

$$g(i) = f(i) / \sum_{i=15}^{49} f(i) = f(i)/\text{TFR}.$$

Substituting this into the previous expression of the indicator, we obtain CBR/TFR as a measure of age structure effects on the crude birth rate.

As for the scaling of this indicator, we propose to adopt the conventional definitions of the crude birth rate as the number of live births per 1,000 population and the total fertility rate as the number of children per woman. These definitions will usually place the ratio within the range from five to nine.

The indicator has several advantages. First, it is extremely simple to calculate. Secondly, data on both the numerator and the denominator of the ratio are widely available, because they are the two most popular fertility indicators. Thirdly, the ratio has an intuitive appeal as a measure of age-structure effects, because it is the ratio of a fertility indicator that is dependent on the age structure to a fertility indicator that is independent of the age structure

ERRATA*

Lattes, Zulma Recchini de, "Women in internal and international migration, with special reference to Latin America", *Population Bulletin of the United Nations*, No. 27 (1989), pp. 95-107.

Page 96, second paragraph, line 6
For dependent *read* second importance

Page 97, third paragraph, line 3
For he *read* she

Page 98, 3rd paragraph, line 13
For he *read* she

Page 100, line 10

After destination;[11] *delete* the rest of the sentence and *substitute* and that origin and destination characteristics greatly influence migration and migrant characteristics (Arizpe, 1986).

Before it *insert* However,

*Requested by the author.

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