Projection Methods for Integrating Population Variables into Development Planning

Volume I

Methods for Comprehensive Planning

Module Two

Methods for preparing school enrolment, labour force and employment projections

United Nations

ST/ESA/SER.R/90/Adc

Department of International Economic and Social Affairs

Projection Methods for Integrating Population Variables into Development Planning

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EXPLANATORY NOTES

The designations employed and the presentation of the material in this publication do not imply the expression of any opinion whatsoever on the part of the Secretariat of the United Nations concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries.

The term "country" and "area" as used in the text of this report also refer, as appropriate, to territories, cities or areas.

The present study has been edited and consolidated in accordance with United Nations practice and requirements.

The following symbols have been used in the tables throughout the report:

A blank in a table indicates that the item is not applicable.

A minus sign (—) indicates a deficit or decrease, except as indicated.

A full stop (.) is used to indicate decimals.

Details and percentages in tables do not necessarily add to totals, because of rounding.

ST/ESA/SER.R/90/Add.1

PREFACE

This is the second of three projected modules of the first volume of a manual on projection methods for integrating population concerns into development planning, which is being prepared by the Population Division of the Department of International Economic and Social Affairs of the United Nations Secretariat, in collaboration with a group of consultants and the Latin American Demographic Centre of the Economic Commission for Latin America and the Caribbean. Module Two describes methods for making projections on school enrolment, labour force and employment, which are part of a larger set of techniques described in the volume. These techniques can be used to make a series of interrelated projections of demographic and socio-economic variables for comprehensive planning that take into account key linkages between population and socio-economic change.

Contained in Module Two are some of the materials on methods for projecting school enrolment, labour force and employment that were originally published in the Population Division's working paper, "Proceedings of the United Nations Ad Hoc Expert Group Meeting on the Manual on Integrating Population Variables into Development Planning, New York, 11-14 December 1984" (ESA/P/WP/87). However, the materials selected for the present publication have been substantially expanded and modified to render them more accessible to planners who may lack formal training in demography or economic theory. They emphasize those methods included in the original working paper that are less demanding of data and, therefore, are more readily applicable in developing countries.

The module describes a method for making enrolment projections that utilizes assumed enrolment ratios but omits a technique for making enrolment projections using assumed enrolment and drop out rates. In the previous working paper, these two methods were, respectively, referred to as the enrolment-ratio method and the age-grade technique. The exclusion of the latter method follows from an omission of the extended cohort component method for making projections of population by age, sex and level of educational attainment from Module One, "Conceptual issues and methods for preparing demographic projections" (ST/ESA/SER.R/90).

Also included in Module Two is a method for making labour force projections employing assumptions on labour force participation rates. The description of the method differs from that contained in the working paper, primarily in that the method included here is for projecting total labour force. The version described in the working paper was for projecting the civilian labour force.

This module also describes three of four original methods for preparing projections of employment but omits the technique presented in the working paper utilizing fixed employment-value added ratios. The reason for the omission is that medium- and especially long-term projections based on fixed ratios are often unrealistic. The three methods included in the module make projections, respectively, by (a) assuming constant rate of change of labour productivity; (b) employing employment-value added functions; and (c) utilizing employment functions that are the inverse of Cobb-Douglas production functions. These methods can be used to project overall employment, but not employment classified by level of skill or educational attainment. The method described in the working papers, which projects employment using the inverse of constant elasticity of substitution production functions, is not presented in the module as it may be difficult to apply in many developing countries.

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IV. MAKING SCHOOL ENROLMENT PROJECTIONS USING ENROLMENT RATIOS

A. Introduction

School enrolment projections may be very relevant to comprehensive planning, especially in countries where the educational sector claims a substantial part of the nation's resources. The number of students obtained as part of a school enrolment projection can be used to project financial, manpower and related resource requirements for the education sector; that is, projections of the number of students in public (government) educational institutions can serve as inputs for projecting government consumption and investment in the education sector.

A variety of methods can be used to make school enrolment projections. Some are rather simple and do not require much data, while others are complex and data demanding. Among the simpler techniques is a method that uses assumed enrolment ratios (box 1)* along with the projected school-age population. It is much less complex than the grade-transition method, which makes assumptions on three types of educational rates--promotion rates repetition rates and drop-out rates.

This chapter describes the enrolment ratio method that can be used to project school enrolment for various school levels and the entire educational system.1/ The method uses enrolment ratios specified for different school levels and the projected school-age population classified by age and sex. It can be used to make a projection for the entire country or for urban and rural areas. The method can also be used to project enrolment in public and private schools.2/

The enrolment ratio method is readily applicable in developing countries because it is relatively easy to calculate and the data it requires are widely available in those countries.

The technique cannot, however, precisely represent the actual process of schooling over time by tracing the progress of students as they enter different school levels, move from one grade to another or drop out or repeat grades. As a result, assumptions on enrolment ratios underlying a projection have to be formulated with great care in order to minimize internal inconsistencies. Unless rough consistency is achieved, the results are likely to be misleading.

This chapter will initially set forth procedures that make up the enrolment ratio method. Next, the types of inputs required by the method will

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* Terms defined in glossary boxes are underlined where they appear for the first time.

Box 1

Glossary

Enrolment ratio
The number of students attending school, divided by the number of persons of appropriate years of age. The ratio may refer to the entire educational system or to a given school level.

Drop-out rate
The proportion of students entering a given grade who withdraw from school before completing the grade.

Promotion rate
The proportion of students entering a given grade who upon completing it, enter the subsequent grade.

Repetition rate
The proportion of students entering a given grade who fail to complete it and then repeat the grade.

School-age population
The population of the school-age period, conventionally defined as ages 5 through 24 year, inclusive.

be discussed along with the ways of preparing those inputs. Following this, examples of school enrolment projections will be presented to illustrate the preparation of projections at the national and urban-rural levels.

B. The technique

1. Overview

This overview will list the inputs required to apply the method and the types of outputs that it can generate. The overview will also outline the steps involved in applying the method.

(a) Inputs

In order to prepare a national projection using the enrolment ratio method, the following inputs would be needed:

(i) Projected single-year age and sex structures of the school-age population;
(ii) Assumptions on enrolment ratios by school level.

If the educational system consists of both private and public schools, the inputs may include:

(iii) Assumptions on the proportions of students attending each type of school, specified by school level.

If an urban-rural projection of school enrolment is sought, the requisite inputs would include those listed under (i) and (ii) for each area--urban and rural. Also, they may include inputs indicated under (iii) for both areas. In addition, where required, the inputs may include assumptions on the proportions of students residing in rural areas but attending urban schools. Those assumptions must be specified by school level. Box 2 presents the inputs required to make a national or urban-rural projection.

Since the enrolment ratio method is described as a technique for preparing quinquennial projections, the inputs should be for dates five years apart.

Box 2

Inputs for making enrolment projections using enrolment ratios

1. Single-age and sex structures of the school-age population (national or urban and rural)

2. Assumptions on enrolment ratios (national or urban and rural)

   Level-specific enrolment ratios, by sex

3. Assumptions on proportions of students in public and private schools (national or urban and rural; if public and private schools enrolment projections are desired)

   Proportions of students in public and private schools, by school level

4. Assumptions on proportions of rural students attending urban schools (for rural areas; if urban-rural projection is being made)

   Proportions of students living in rural areas but attending urban schools, by school level.

(b) Outputs

Where the enrolment ratio method is employed at the national level, it can be used to project the following:

(i) Selected school enrolment aggregates, such as the number of students by school level and the growth in those numbers,

(ii) Indicators of the composition of students by school level or sex, examples of which are proportions of students by school level and sex ratios of students (box 3);

(iii) Rates of growth in school enrolment, such as rates of growth in the number of students at different school levels.

These results can be obtained by projecting overall enrolment or by projecting enrolment in public and private schools.

Box 3

Glossary

De jure population
A population enumerated on the basis of normal residence, excluding temporary visitors and including residents temporarily absent.

Sex ratio of students
The number of male students for each female student, conventionally multiplied by 100.

Sprague interpolation
A type of non-linear interpolation.

Where the technique is used to make a projection for urban and rural areas, the results can include the variables listed above plus indicators of the distribution of enrolment by location, such as proportions of students in urban and rural schools. The types of outputs that can be prepared using the enrolment ratio method are shown in box 4.

The method as described here would produce results for dates five years apart or for the intervening five-year projection intervals.

Box 4

Types of outputs prepared by making enrolment projections using enrolment ratios

A. Overall enrolment

1. Enrolment aggregates (national or urban, rural and national)

Total number of students and numbers of students disaggregated by school level and sex

Growth in the total number of students and in numbers of students, by school level and sex

2. Indicators of enrolment structure (national or urban, rural and national)

Proportions of students, by school level

Sex ratios of all students and students by school level (can only be obtained in a national projection)

3. Indicators of enrolment distribution (national; if urban-rural projection is being prepared)

Proportions of all students and students, by school level, that are urban

Proportions of all students and students, by school level, that are rural

4. Rates of growth of enrolment (national or urban, rural and national)

Rates of growth of all students and of students by school level and sex

B. Enrolment in public and private schools

1. Enrolment aggregates (national or urban, rural and national)

Total number of students and numbers of students, by school level

(continued)

Box 4 (continued)

Growth in the total number of students and in numbers of students, by school level

2. Indicators of enrolment structure (national or urban, rural and national)

Proportions of students, by school level

3. Indicators of enrolment distribution (national; if urban-rural projection is being prepared)

Proportions of all students and students, by school level, that are urban

Proportions of all students and students, by school level, that are rural

4. Rates of growth of enrolment (national or urban, rural and national)

Rates of growth of all students and of students by school level

(c) Computational steps

To project enrolment for a given date, one initially calculates the numbers of persons in the relevant age groups within the school-age period using the age and sex structure of the school-age population. Then, level-specific enrolment ratios are applied to those numbers of persons to arrive at the number of students at different school levels. Additional steps are performed to derive enrolment aggregates, indicators of the structure of enrolment and rates of change in enrolment. If projections of enrolment in public and private schools are desired, they can be obtained using additional steps which yield enrolment aggregates and indicators of the structure and change in enrolment for these two types of schools.

The steps involved in making a projection will depend, in part, on whether one projects enrolment for different types of schools, such as public and private. In addition, in the case of an urban-rural projection, the steps involved will also depend on whether or not one wishes to allow for the possibility that some students residing in rural areas will attend urban schools.

2. National level

This description of the enrolment ratio method will initially show the steps used to project school enrolment at the national level, first for overall enrolment and then for enrolment in public and private schools. The steps are summarized in box 5. A subset of those steps, which are used to project the total number of students by school level and the number of students in public and private schools, is shown in figure I. The procedures used to prepare an enrolment projection for urban and rural areas will be discussed in a later section.

(a) Overall enrolment

In order to project overall enrolment, it is first necessary to calculate the number of persons in special age groups within the school-age period.

Box 5

Computational steps to project enrolment at the national level

The steps used to project enrolment at the national level over a five-year projection interval are:

(1) For each sex, calculate the numbers of persons in the special age groups at the end of the interval on the basis of the single-year age and sex structure of the school-age population for that date.

(2) Apply assumed enrolment ratios to the numbers of persons in the special age groups to obtain the numbers of students at each school level by sex at the end of the projection interval.

(3) From the projected numbers of students classified by school level and sex, derive enrolment aggregates, such as the total number of students and the numbers of students by school level. Also calculate other enrolment aggregates, such as increases in the total number of students and the numbers of students by school level.

(4) For the end of the interval, derive indicators of the structure of students such as proportions of students by school level and sex ratios of students.

(continued)

Box 5 (continued)

(5) Calculate rates of growth during the interval, of the total number of students and the numbers of students by school level.

(6) If the enrolment projection is to be extended to public and private segments of the educational system, multiply the numbers of students at different school levels by the assumed proportions of students attending public and private schools at different school levels. This yields the numbers of students in public and private schools, by school level, at the end of the interval.

(7) Calculate other enrolment aggregates for private and public schools, such as total numbers of students in those schools. Also calculate increases in the total numbers of students in public and private schools and in the numbers of students in those schools by school level.

(8) Derive indicators of the structure of students in public and private schools for the end of the interval, such as proportions of students by school level.

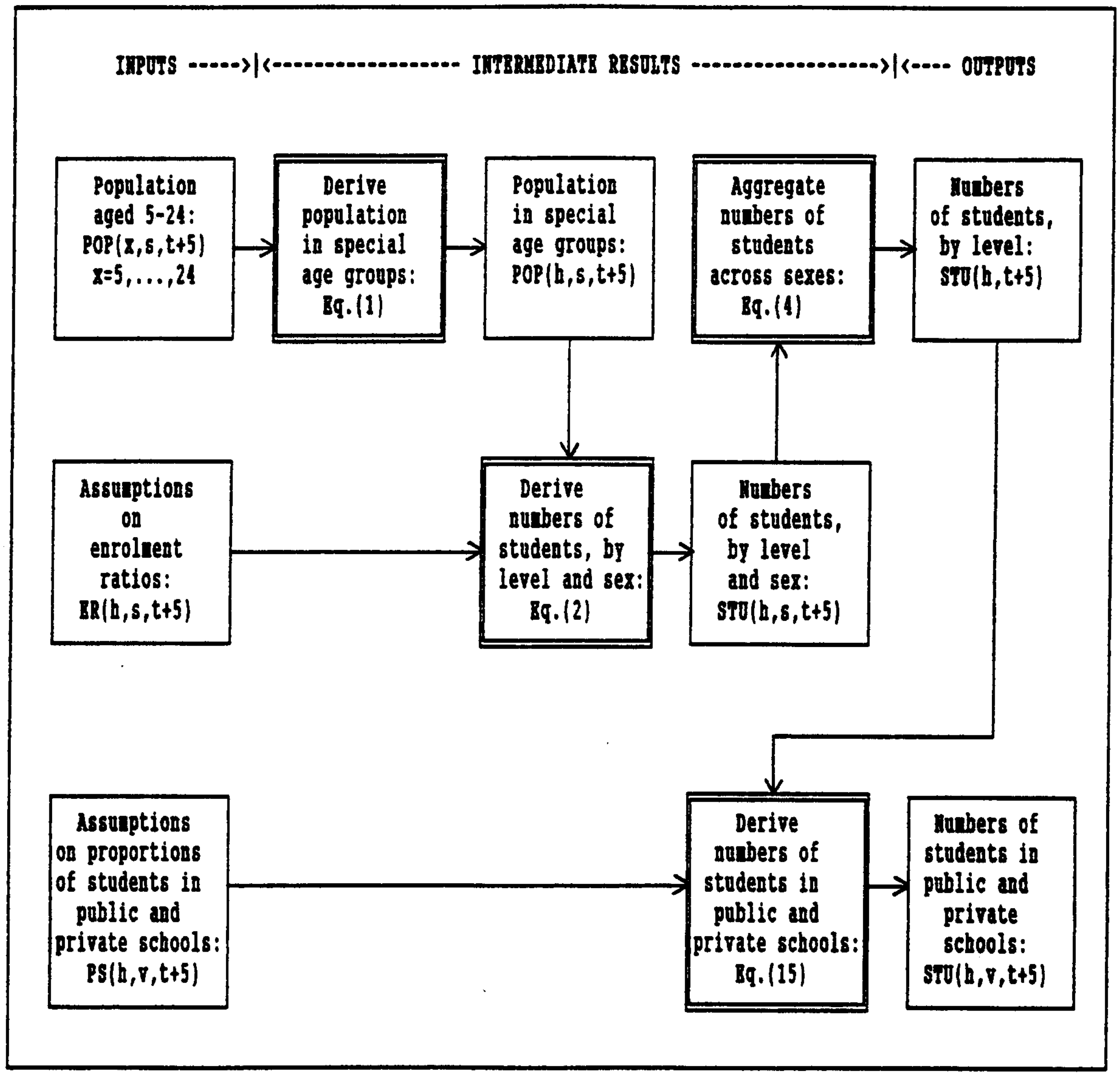
(9) Calculate rates of growth in the total number of students and the numbers of students, by school level, in public and private schools.

(i) Persons in special age groups

The numbers of persons in special age groups within the school age period are calculated for each given projection date, such as the end of the five-year projection interval (t to t+5) using the projected number of persons of school-age for that date, classified by single year of age and sex. The resultant numbers of persons, which are typically disaggregated by sex, are for the special age groups corresponding to different school levels.

The projected number of persons of the school-age period used in these calculations may be taken directly from a population projection, prepared by single years of age. Alternatively, those numbers can be derived by interpolation from a population projection by five-year age groups. Annex I describes such an interpolation based on the Sprague interpolation formula,3/ which is commonly used for such purposes.

Figure I. Steps to derive the total numbers of students, by school level and the numbers of students in public and private schools, by school level



The number of special age groups for which the numbers of persons are derived depends on the country's educational system. Thus, if the educational system consists of the primary, secondary and tertiary levels, there would be three such special age groups. The boundaries of the age groups will depend on the assumed age-ranges corresponding to the school levels. For example, the assumed ages corresponding to the three school levels mentioned might be 7 to 14, 15 to 18 and 19 to 22.

The number of persons of each sex in each special age group at a given date can be derived by aggregating the age and sex structure of the school-age population. For the end of the projection interval (t to t+5), these aggregations can be represented as follows:

$$POP(h,s,t+5) = \sum_{x=x'_h}^{x''_h} POP(x,s,t+5) \qquad (1)$$

$$h = 1,\ldots,H;$$

$$x = 5,\ldots,24;$$

$$s = 1,2,$$

where:

$h = 1,\ldots,H$	are special age groups within the school-age period or different school levels of the country's educational system,
t	is the year of the projection period,
H	is the number of special age groups within the school-age period, or the number of school levels in the country's educational system,
x'_h	is the youngest age of the special age group h,
x''_h	is the oldest age of the special age group h,
$x = 5,\ldots,24$	are single years of age 5,...,24,
$s = 1,2$	are male and female sexes,
$POP(h,s,t+5)$	is the population of special age group h and sex s at the end of the interval, and
$POP(x,s,t+5)$	is the population of single year of age x and sex s at the end of the interval.

The details of the aggregation described by equation (1) may differ from one application of the method to another depending on the years of age assumed to correspond to different school levels. This aggregation will be illustrated in the first of the two projection examples in section D.

(ii) <u>Enrolment by school level and sex</u>

In order to project the numbers of students at the various school levels, it is initially necessary to calculate the number of students of each sex in each school level. This is done by multiplying the number of persons of each sex in each special age group by the assumed level- and sex-specific enrolment ratio. Thus, for the end of the projection interval (t to t+5) the numbers of students classified by school level and sex are obtained as:

$$STU(h,s,t+5) = POP(h,s,t+5) \cdot ER(h,s,t+5); \quad h = 1,\ldots,H; \quad s = 1,2, \qquad (2)$$

where:

STU(h,s,t+5) is the number of students of school level h and sex s at the end of the interval, and

ER(h,s,t+5) is the enrolment ratio for school level h among persons of sex s at the end of the interval.

(iii) <u>Other results</u>

a. <u>Enrolment aggregates</u>

Once the number of students of each sex is calculated for each school level, those numbers can be aggregated in order to obtain the total number of students, the number of students at each school level, and the number of female and male students. Once obtained for different dates, these numbers can be used to calculate the projected growth in the total number of students and in the number of students classified by school level and sex during the intervening projection intervals.

i. <u>Total number of students</u>

The total number of students in the educational system at the end of the projection interval can be found by adding the number of students of the two sexes at different school levels at that date:

$$STU(t+5) = \sum_{h=1}^{H} \sum_{s=1}^{2} STU(s,t+5), \tag{3}$$

where:

STU(t+5) is the total number of students in the educational system at the end of the interval.

ii. Number of students by school level

The number of students by school level at the end of a projection interval can be obtained by adding the number of male and female students at each level for that date:

$$STU(h,t+5) = \sum_{s=1}^{2} STU(h,s,t+5); \tag{4}$$

$$h = 1,\ldots,H,$$

where:

STU(h,t+5) is the number of students at school level h at the end of the interval.

iii. Number of students by sex

The number of students of each sex can be calculated by adding the number of students of each sex across different school levels:

$$STU(s,t+5) = \sum_{h=1}^{H} STU(h,s,t+5); \tag{5}$$

$$s = 1,2,$$

where:

STU(s,t+5) is the number of students of sex s at the end of the interval.

iv. Growth in the total number of students

The growth in the total number of students is calculated as the difference between the total number of students at the end and the beginning of the interval:

$$SGR = STU(t+5) - STU(t), \tag{6}$$

where:

SGR is the growth in the total number of students during the interval.

v. Growth in the number of students by school level

For each school level, the growth in the number of students over a projection interval is the difference between the number of students at the end and the beginning of the interval:

$$SGR(h) = STU(h,t+5) - STU(h,t); \tag{7}$$

$$h = 1,\dots,H,$$

where:

SGR(h) is the growth in the number of students at school level h during the interval.

vi. Growth in the number of students by sex

The growth in the number of students of a given sex is the number of students of that sex at the end of the interval less the number at the beginning of the interval:

$$SGR(s) = STU(s,t+5) - STU(s,t); \tag{8}$$

$$s = 1,2,$$

where:

SGR(s) is the growth in the number of students of sex s during the interval.

b. Indicators of enrolment structure

Using the projected levels of enrolment for a particular date, it is possible to calculate different indicators of the composition of enrolment for that date. These indicators may include proportions of students in the educational system attending different school levels. In addition, they may include the sex ratio of all students in the educational system as well as sex ratios of students at different school levels.

i. Proportions of students by school level

The proportions of students by school level at the end of the given projection interval (t to t+5) can be calculated by dividing the number of students at each level by the total number of students at that date:

$$PSTU(h,t+5) = STU(h,t+5) \; / \; STU(t+5); \qquad (9)$$

$$h = 1,\ldots,H,$$

where:

PSTU(h,t+5) is the proportion of students at school level h at the end of the interval.

ii. Sex ratios of students

The sex ratio for all students can be calculated as the total number of male students (s=1) divided by the total number of female students (s=2), multiplied by one hundred:

$$SRS(t+5) = [\; STU(1,t+5) \; / \; STU(2,t+5) \;] \cdot 100, \qquad (10)$$

where:

SRS(t+5) is the sex ratio of all students at the end of the interval.

Sex ratios of students at different school levels can be obtained by dividing the number of male students by the number of female students at those levels and multiplying by one hundred:

$$SRS(h,t+5) = [\; STU(h,1,t+5) \; / \; STU(h,2,t+5) \;] \cdot 100; \qquad (11)$$

$$h = 1,\ldots,H,$$

where:

SRS(h,t+5) is the sex ratio of students at school level h at the end of the interval.

c. Rates of growth of enrolment

Using the projected levels of enrolment at adjacent dates it is also possible to calculate various rates of change in the number of students over the five-year projection intervals. Those rates can be calculated for the total number of students and for the numbers of students of different school levels and sexes.

i. Rate of growth in the total number of students

The average annual rate of growth in the total number of students in the educational system for the five-year projection interval (t to t+5) can be obtained by using the formula for calculating the exponential growth rate:

$$GRS = [(\ln(STU(t+5) / STU(t))) / 5] \cdot 100, \qquad (12)$$

where:

GRS	is the average annual exponential rate of growth of the total number of students during the interval, and
ln	is the natural logarithm.

ii. Rates of growth in the number of students by school level

The average annual rates of growth in the number of students by school level can be obtained using the same type of formula:

$$GRS(h) = [(\ln(STU(h,t+5) / STU(h,t))) / 5] \cdot 100; \qquad (13)$$

$$h = 1,\ldots,H,$$

where:

GRS(h)	is the average annual exponential rate of growth of the number of students at school level h during the interval.

iii. Rates of growth in the number of students by sex

The average annual rate of growth in the number of students of each sex can be also obtained using the formula for calculating the exponential growth rate:

$$GRS(s) = [(\ln(STU(s,t+5) / STU(s,t))) / 5] \cdot 100; \qquad (14)$$

$$s = 1,2,$$

where:

GRS(s) is the average annual exponential rate of growth of the number of students of sex s during the interval.

If the entire educational system is operated either by the public sector or the private sector, this last step completes the enrolment projection. Moreover, if the educational system consists of public and private parts but the available data cannot support separate enrolment projections for public and private schools, this step also completes the projection. Where the requisite data are available, however, the projection process can be taken one step further in order to obtain separate projections for public and private schools.

(b) Enrolment in public and private schools

In order to make separate enrolment projections for public and private schools, it is necessary to use the projected number of students by school level, obtained as indicated by equation (4) and assumptions on the proportions of students attending public and private schools by school level. The use of these inputs yields the number of students in public and private schools, which are further used to obtain other enrolment aggregates for the two types of schools, along with indicators of enrolment structure and change.

(i) Enrolment aggregates

As in the case of projecting overall school enrolment, the aggregates obtained in the course of projecting enrolment for public and private schools include the projected number of students by school level and the total number of students. They frequently do not include, however, the number of students classified by sex. Enrolment aggregates also include projected growth in the number of students during the projection intervals.

a. Number of students by school level

The number of students in public and private schools at each school level can be found by applying the assumed proportions of students attending those schools at that level. Thus, for the end of the projection interval the number of students in each type of school is:

$$STU(h,v,t+5) = STU(h,t+5) \cdot PS(h,v,t+5); \qquad (15)$$

$$h = 1,\ldots,I;$$

$$v = 1,2,$$

where:

$v = 1,2$	are public and private schools,
STU(h,v,t+5)	is the number of students at school level h attending schools of type v at the end of the interval, and
PS(h,v,t+5)	is the proportion of students at school level h attending schools of type v at the end of the interval.

b. <u>Total number of students</u>

The total number of students in public and the private schools can be calculated by aggregating the number of students in public and private schools across the different school levels:

$$STU(v,t+5) = \sum_{h=1}^{H} STU(h,v,t+5); \quad v = 1,2, \tag{16}$$

where:

STU(v,t+5)	is the total number of students in schools of type v at the end of the interval.

c. <u>Growth in the total number of students</u>

The growth in the total number of students in public and private schools can be derived as the total number of students in each type of school at the end of the interval less the total at the beginning:

$$SGR(v) = STU(v,t+5) - STU(v,t); \quad v = 1,2, \tag{17}$$

where:

SGR(v)	is the growth in the total number of students attending schools of type v during the interval.

d. <u>Growth in the number of students by school level</u>

The growth in the numbers of students in public and private schools at each school level for the given projection interval can be obtained as the number of students in those schools at that level at the end of the interval less the numbers at the beginning :

$$SGR(h,v) = STU(h,v,t+5) - STU(h,v,t), \qquad h = 1,\ldots,H; \quad v = 1,2, \tag{18}$$

where:

SGR(h,v) is the growth in the number of students at school level h attending schools of type v during the interval.

(ii) <u>Indicators of enrolment structure</u>

Among the indicators of the structure of enrolment that can be calculated from the projected number of students in public and private schools are the proportions of students at different school levels who attend each type of school. They do not include however sex ratios of students, as the projected numbers of students in public and private schools are generally not available by sex.

a. <u>Proportions of students by school level</u>

The proportions of students in public or private schools at a given school level can be obtained by dividing the number of students at individual levels by the total number of students in those schools:

$$PSTU(h,v,t+5) = STU(h,v,t+5) \; / \; STU(v,t+5); \qquad h = 1,\ldots,H; \quad v = 1,2, \tag{19}$$

where:

PSTU(h,v,t+5) is the proportion of students at school level h among those attending schools of type v at the end of the interval.

(iii) Rates of growth of enrolment

Using the projected number of students in public and private schools at dates five years apart it is also possible to compute average annual rates of growth of the number of students over the intervening intervals.

a. Rates of growth in the total number of students

The rates of growth in the total number of students in public and private schools respectively during the projection interval (t to t+5) can be obtained as:

$$GRS(v) = [(\ln(STU(v,t+5) / STU(v,t))) / 5] \cdot 100; \quad (20)$$

$$v = 1,2,$$

where:

GRS(v) is the average annual exponential rate of growth of the total number of students in schools of type v during the interval.

b. Rates of growth in the number of students by school level

The rates of growth in the number of students in public and private schools, by school level, during the interval can be obtained as:

$$GRS(h,v) = [(\ln(STU(h,v,t+5) / STU(h,v,t))) / 5] \cdot 100; \quad (21)$$

$$h = 1,\ldots,H;$$

$$v = 1,2,$$

where:

GRS(h,v) is the average annual exponential rate of growth of the number of students at school level h in schools of type v during the interval.

3. Urban-rural level

This section will describe a procedure to prepare projections of enrolment in urban and rural areas. Most of the steps in this type of projection are the urban-rural equivalents of the steps used to project enrolment at the national level. In addition, the procedure includes steps to project enrolment of students whose location of residence may differ from the location of school attendance. The procedure also includes steps to calculate indicators of the locational distribution of enrolment. The steps used to project the number of all students and the number of students in public and private schools for a given date are shown schematically in figure II.

(a) Overall enrolment

To project overall enrolment it is initially necessary to calculate the number of students for urban and rural areas respectively, using urban-rural counterparts of the steps described for making national projections. These steps may be supplemented by calculations which could deal with differences that may exist for some students between their location of residence and their location of school attendance. Enrolment for the entire country is projected by aggregating urban and rural projection results. The discussion that follows will focus on the way urban and rural overall enrolment is projected. In this discussion, we shall assume that for some students, the location of residence differs from the location of school attendance.

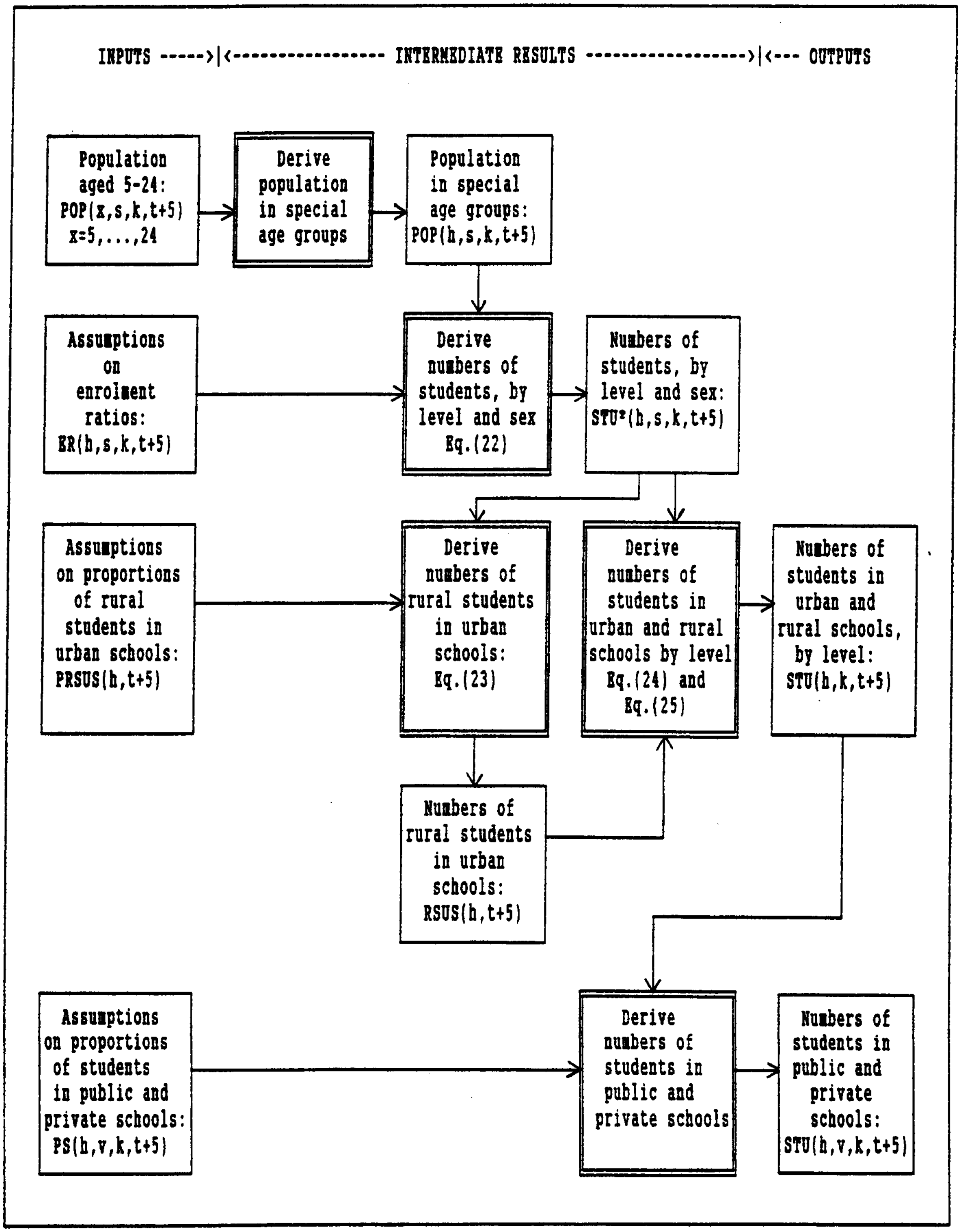
(i) Persons in special age groups

In both locations the number of persons in special age groups within the school-age period are initially calculated using steps similar to those described by equation (1). These calculations use the urban and rural structures of the school-age population disaggregated by single year of age and sex. If the number of students by single year of age are not already available, they can be derived by interpolation, as described in annex I, using urban-rural population structures by five-year age group and sex. Where possible, the projected urban and rural populations and the number of persons in special age groups should be those that permanently reside in each location.

(ii) Enrolment by school level and sex and by location of residence

The numbers of students by school level and sex are next calculated for each location using a step analagous to that indicated by equation (2). Where the number of persons in special age groups reside permanently in urban and rural areas, the enrolment ratios employed should be those applying to those persons.4/ In other words, the enrolment ratios used should represent ratios of the number of students at different school levels who reside in urban and rural areas to the number of persons at special age groups living in those areas. Where enrolment ratios of this type are used, the number of students obtained by school level and sex are those residing in urban and rural locations.

Figure II. Steps to derive the numbers of students attending urban and rural schools, by school level and the numbers of students in public and private urban and rural schools, by school level



In particular, the numbers of students by school level and sex in each location are calculated as follows:

$$STU^*(h,s,k,t+5) = POP(h,s,k,t+5) \cdot ER(h,s,k,t+5); \quad (22)$$

$$h = 1,\ldots,H;$$

$$s = 1,2;$$

$$k = 1,2,$$

where:

k = 1,2	are urban and rural locations,
STU*(h,s,k,t+5)	is the number of students at school level h of sex s who reside in location k at the end of the interval,
POP(h,s,k,t+5)	is the population of special age group h and sex s who reside in location k at the end of the interval, and
ER(h,s,k,t+5)	is the enrolment ratio for school level h among persons of sex s who reside in location k at the end of the interval.

If for all students the location of residence were the same as the location of school attendance, the number of students residing in urban and rural areas would, respectively, equal the number of students attending urban and rural schools. In such a case, the numbers of students obtained as indicated by equation (22) would be accepted as the numbers of students attending urban and rural schools. Furthermore, those numbers would be directly used to obtain other enrolment aggregates along with the indicators of enrolment structure and growth for urban and rural areas.

For some students, however, the location of permanent residence may be different from the location of school attendance. Students who permanently reside in the rural areas may attend schools in the urban areas. This could be fairly common among rural students in secondary and tertiary schools, where some or the majority of those schools may be located in urban centres. Where this is the case, one would need to calculate the number of students living in rural places but attending urban schools.

(iii) Enrolment in urban schools among rural students

The number of students residing in rural areas (k=2) but attending urban schools can be obtained from the numbers of students at each school level who reside in rural areas and the assumed proportions of those students who attend urban schools. For the end of the projection interval (t to t+5) these numbers for each school level can be obtained as follows:

$$RSUS(h,t+5) = [\sum_{s=1}^{2} STU^{*}(h,s,2,t+5)] \cdot PRSUS(h,t+5); \qquad (23)$$

$$h = 1,\ldots,H,$$

where:

RSUS(h,t+5) is the number of rural students at school level h who attend urban schools at the end of the interval, and

PRSUS(h,t+5) is the proportion of rural students at school level h who attend urban schools at the end of the interval.

(iv) Other results

a. Enrolment aggregates

Once the numbers of students residing in the two locations and the number of rural students attending urban schools are calculated, it is possible to derive the number of students attending schools in urban and rural areas and to calculate other enrolment aggregates.

i. Number of students by school level

The number of students who attend urban schools (k=1) at each school level can be obtained as the sum of the number of students of both sexes at that school level who live in urban areas and the number of students at the same level who reside in rural areas but attend school in urban places:

$$STU(h,1,t+5) = [\sum_{s=1}^{2} STU^{*}(h,s,1,t+5)] + RSUS(h,t+5); \qquad (24)$$

$$h = 1,\ldots,H,$$

where:

STU(h,k,t+5) is the number of students at school level h attending schools in location k at the end of the interval.

The number of students who attend rural schools (k=2) at each school level is obtained as the difference between the number of students of both sexes at that level who reside in rural areas and the number of students at the same level who live in rural areas but attend school in urban places:

$$STU(h,2,t+5) = [\sum_{s=1}^{2} STU^{*}(h,s,2,t+5)] - RSUS(h,t+5); \qquad (25)$$

$$h = 1,\ldots,H.$$

In addition to the number of students attending schools in urban and rural areas by school level, this type of projection can produce other enrolment aggregates: for example, the total number of students attending schools in the two areas, which can be obtained using the urban-rural equivalent of the step described by equation (3). The growth in the total number of students, and in the numbers of students by school level in urban and rural schools, can be derived using urban-rural counterparts of the steps corresponding to equations (6) and (7). Since the numbers of students attending urban and rural schools by school level are not disaggregated by sex, enrolment aggregates would not include separate projections of the number of male and female students in urban and rural schools along with the growth in those numbers and the sex ratio of students.

The total number of students and the number of students by school level for the entire country, along with the growth in those numbers over the intervening projection intervals, can be obtained by a straightforward aggregation of urban and rural results.

b. Indicators of enrolment structure

The proportions of enrolment in urban and rural schools by school level can be obtained using an urban-rural counterpart of the step indicated in equation (9). Sex ratios cannot be calculated in urban-rural projections since the number of male and female students are usually not tracked separately by location of school attendance.

c. Indicators of enrolment distribution

Indicators of the distribution of enrolment by location--proportions of enrolment that are urban and rural--can be calculated for the total enrolment and enrolment by school level.

i. Proportions urban and rural of total enrolment

The proportion of total enrolment that is urban (k=1) can be obtained by dividing the number of students attending schools in urban areas by the total number of students in the educational system:

$$PSURB(t+5) = STU(1,t+5) \ / \ STU(t+5), \qquad (26)$$

where:

PSURB(t+5)	is the proportion of all students who attend urban schools at the end of the interval, and
STU(k,t+5)	is the total number of students attending schools in location k at the end of the interval.

The proportion rural (k=2) for the entire educational system can be obtained as the complement of the relevant proportion urban:

$$PSRUR(t+5) = 1 - PSURB(t+5), \qquad (27)$$

where:

PSRUR(t+5)	is the proportion of all students who attend rural schools at the end of the interval.

ii. Proportions urban and rural of enrolment by school level

For each school level the proportion of enrolment that is urban (k=1) can be obtained by dividing the number of students attending urban schools at that level by the total number of students at the same level:

$$PSURB(h,t+5) = STU(h,1,t+5) \ / \ STU(h,t+5); \qquad (28)$$

$$h = 1,\ldots,H,$$

where:

PSURB(h,t+5) is the proportion of students at school level h who attend schools in urban areas at the end of the interval.

The proportion of enrolment that is rural (k=2) at each school level can be obtained as a complement of the relevant proportion of enrolment that is urban:

$$PSRUR(h,t+5) = 1 - PSURB(h,t+5); \quad h = 1,\ldots,H, \tag{29}$$

where:

PSRUR(h,t+5) is the proportion of students at school level h who attend schools in rural areas at the end of the interval.

d. Rates of growth of enrolment

Rates of growth in the total number of students and in the numbers of students by school level who attend schools in urban and rural areas can be calculated as part of an urban-rural projection, using urban-rural counterparts of the steps indicated by equations (12) and (13). Since the number of students attending schools in urban and rural areas are not disaggregated by sex in this projection, one cannot derive sex-specific rates of growth in the numbers of students for each location.

(b) Enrolment in public and private schools

If the educational system of the country for which the urban-rural projection is being prepared consists of a mixture of public and private institutions, the projection can be extended to derive separate results for public and private parts of the educational system. Such an extension would yield results that are fully analogous to the results obtained for public and private parts of the educational system in the national projection. Thus, the results would include enrolment aggregates, indicators of enrolment structure and rates of growth of enrolment. In addition, they would include indicators of the locational distribution of enrolment in public and private schools.

(i) Enrolment aggregates

The total number of students in the private and public parts of the educational system in urban and rural schools, along with the numbers of students in public and private schools in the two locations by school level, can be obtained using urban-rural equivalents of the steps described by equations (15) and (16). The growth in the total number of students and in the number of students by school level could be obtained for urban and rural areas by means of steps analogous to those indicated by equations (17) and (18), respectively.

(ii) Indicators of enrolment structure

The proportions of enrolment in public and private schools in urban and rural areas by school level can be derived using an urban-rural counterpart of the step described by equation (19).

(iii) Indicators of enrolment distribution

The proportions of students in public and private schools that are urban and rural can be derived for all students as well as for students at different school levels.

a. Proportions urban and rural of total enrolment

The proportions of students in either public or private schools that are urban (k=1) can be obtained by dividing the number of students attending those schools in urban areas by the total number of students in the corresponding type of school:

$$PSURB(v,t+5) = STU(v,1,t+5) \;/\; STU(v,t+5); \qquad (30)$$

$$v = 1,2,$$

where:

PSURB(v,t+5) is the proportion of all students in schools of type v who attend school in urban areas at the end of the interval, and

STU(v,k,t+5) is the total number of students attending school of type v in location k at the end of the interval.

The proportions of students in all public and private schools that are rural (k=2) can be calculated as the complements of the respective proportions of all students in those schools that are urban:

$$PSRUR(v,t+5) = 1 - PSURB(v,t+5); \qquad (31)$$

$$v = 1,2,$$

where:

PSRUR(v,t+5) is the proportion of all students in schools of type v who attend school in rural areas at the end of the interval.

b. Proportions urban and rural of enrolment by school level

The proportions of students in public and private schools, respectively, that are urban (k=1) can be obtained for each school level by dividing the number of students at a given school level attending a given type of school in urban areas by the total number of students at that level in that type of school:

$$PSURB(h,v,t+5) = STU(h,v,1,t+5) / STU(h,v,t+5); \quad h = 1,...,H; \quad v = 1,2, \tag{32}$$

where:

PSURB(h,v,t+5) is the proportion of all students in schools of type v at school level h attending schools in urban areas at the end of the interval, and

STU(h,v,1,t+5) is the number of students at school level h who attend schools of type v in urban areas at the end of the interval.

The proportions of students in a given type of school, by school level, that are rural (k=2) can be calculated as complements of the corresponding proportions urban:

$$PSRUR(h,v,t+5) = 1 - PSURB(h,v,t+5); \quad h = 1,...,H; \quad v = 1,2, \tag{33}$$

where:

PSRUR(h,v,t+5) is the proportion of all students in schools of type v at school level h attending schools in rural areas at the end of the interval.

(iv) Rates of growth of enrolment

Rates of growth of the total number of students and the number of students by school level can be calculated as part of the projection of students in public and private schools by location, using urban-rural equivalents of the steps indicated by equations (20) and (21).

C. The inputs

This section will initially discuss the types of inputs needed to prepare an enrolment projection by this method. Then, it will describe how those inputs can be prepared.

1. Types of inputs required

To project enrolment by this method, it would be necessary to use the following inputs:

(i) Projected structures of the school-age population disaggregated by single year of age and sex;
(ii) Assumed enrolment ratios disaggregated by sex.

If the educational system of the country for which the projection is being prepared consists of public and private schools the inputs should also include:

(iii) Assumed proportions of students attending public and private schools.

Depending on whether one wishes to make a national projection or a projection for urban and rural areas, these inputs will be required for the entire country or for urban and rural areas. If an urban-rural projection is to be prepared, the inputs may also include:

(iv) Assumptions on attendance of urban schools by rural students.

These additional assumptions would be required if the location of residence and the location of school attendance differ for at least some students and if the results should include the number of students by location of school attendance.

The projected age and sex structures along with various assumptions should refer to individual dates five years apart, starting with the initial year of projection. The assumptions listed under (ii) through (iv) would need to be specified by school level for each of those dates.

2. Preparation of inputs

Preparation of the inputs will involve projecting age and sex structures of the school-age population for dates five years apart. It will also entail formulating assumptions on future enrolment ratios and, where necessary, assumptions on future proportions of students in public and private schools and/or proportions of rural students in urban schools.

(a) Age and sex structures of the school-age population

The future structures of the school-age population disaggregated by single year of age and sex at different dates are taken from a population projection. If the population projection is disaggregated by single year of age and sex, these structures can be obtained directly. Alternatively, if the population projection is disaggregated by five-year age groups, the single-year age structures of the school-age population need to be derived by interpolation from five-year age structures. Annex I describes one such interpolation procedure.

If enrolment projections are needed for urban and rural places, projections of urban and rural populations will, of course, be needed. In order to ensure that the projections are of persons usually residing in the two locations, such projections should be based on census (or survey) data using the concept of de jure population, that is a population enumerated on the basis of normal residence, excluding temporary visitors and including residents temporarily absent.

(b) Needed data and observations

To formulate the required assumptions, especially for the initial year of the plan, it is necessary to have recent observations of the relevant measures. Preferably, these observations should pertain to two or more dates that span a time interval close to the initial year of the plan. Where such observations do not already exist, it will be necessary to prepare them.

Thus, where observations on enrolment ratios are unavailable, they can be derived from data on school enrolment along with estimates of the school-age population. The school enrolment data may come from a population census or from education statistics, while the data on the school-age population would normally come from a population census. These two types of data should refer to the same year. In addition, to derive level-specific enrolment ratios, the enrolment data should enable calculation of the numbers of students classified by school level and by sex.

If enrolment is to be projected for public and private schools separately, data on proportions of students by type of school will be required. As a rule, the observations can be derived from the numbers of students classified by school level and by type of school, public or private.

Where separate projections are needed for urban and rural areas, the data should refer to the number of students and the school-age population having permanent residence in those areas. Such information would usually come from a population census. If such projections are to include enrolment by location of school attendance, recent observations on the proportions of rural students attending urban schools will also be required. These observations can be obtained from population censuses and educational statistics. The data from the two sources should refer to the same year or years. They should refer to enrolment by location of residence and enrolment by location of school attendance. In addition, the data should be classified by school level.

(c) Assumptions for the plan projection

Given recent observations on the relevant measures, the assumed values for the initial year of the projection period can be obtained by extrapolation.

In formulating the enrolment assumptions, it is necessary to consider the Government's objectives and policies concerning admission and the repetition of grades at different school levels, preferably separately by sex. If required by the type of projection sought, this should be done separately for urban and rural areas. Further, in countries where both public and private sectors provide educational services, the proportions of students in public schools would have to be selected by considering how the Government perceives its role in providing education services vis-à-vis that of the private sector. Where they are required, assumptions on the proportions of rural students going to urban schools could be chosen by taking into account government policies, if any, relating to the urban-rural distribution of enrolment.

In assessing the likely demand for education services, it is necessary, to look into those factors that bear on the decisions of parents. Thus, for example, it may often prove necessary to consider likely future trends in average household income, as a proxy for the ability of parents to support children through school, as well as trends in the ratio of children to adults, as an approximation of the number of children likely to compete for the parental support. Similarly, factors such as the prevalence of child labour and early marriage, both of which are likely to impede child schooling, must be taken into consideration and their impact on enrolment assessed. Moreover, likely future expectations of parents regarding the level of education required for their children to find good jobs need to be considered.

To ensure internal consistency among the assumptions, the changes in enrolment ratios over time for the upper school levels, as specified by assumption, should be compatible with the changes in enrolment ratios at the lower levels. This is required since in most education systems only students completing the lower levels are eligible to enrol in the upper levels.

D. Illustrative examples of projections

Two examples are presented in this section to illustrate the preparation of school enrolment projections using the enrolment ratio technique. The first example illustrates the preparation of a national projection while the second first illustrates an urban-rural projection. The examples will, among other things, indicate how to project enrolment in a country having an educational system that consists of both public and private schools. The second of the two examples will assume that for some students the location of residence is different from that of school attendance and will demonstrate the projection of enrolment by location of school attendance as well as by location of residence. Both examples will emphasize the calculations for the projection interval 0-5 and will present complete projection results for a 20-year projection period.

In these examples, it will be assumed that the educational system for which the projection is being prepared consists of four school levels--lower-primary, higher-primary, secondary and tertiary--and that each of these levels takes four years to complete. It will also be assumed that the regular years of age of students corresponding to the four school levels in this hypothetical educational system are, respectively, 7 through 10, 11 through 14, 15 through 18 and 19 through 22. Therefore, the special age groups in these examples for which the number of persons need to be calculated are age groups 7-10, 11-14, 15-18 and 19-22.

1. National level

This section will present an example of a national projection using the inputs shown in tables 1 and 2. These inputs include the structures of the school-age population by single year of age and sex along with assumptions on enrolment ratios and proportions of students in public and private schools, by school level. The example will initially describe the projection of overall enrolment. Later it will describe the projection of enrolment in public and private parts of the educational system.

(a) Overall enrolment

The first step in projecting overall enrolment is to derive the numbers of persons in special age groups.

(i) Persons at special age groups

The numbers of persons of each sex in special age groups at the given date are derived from the structure of the school-age population by single year of age by aggregating the number of persons within the school-age period,

Table 1. Inputs for projecting school enrolment for the entire country; school-age population by single year of age and sex

(Thousands)

Age	Year 0	5	10	15	20
			Male		
5	154.3	149.3	175.9	192.9	202.5
6	151.8	147.3	169.8	188.9	201.2
7	148.6	146.6	164.0	184.8	199.4
8	144.8	146.7	158.6	180.5	197.1
9	140.5	147.1	153.9	176.2	194.3
10	135.5	149.1	149.5	171.8	191.0
11	130.4	148.5	146.7	167.4	187.5
12	125.0	147.1	144.8	163.1	183.7
13	119.4	144.6	144.0	158.8	179.7
14	113.6	140.6	144.3	154.4	175.6
15	107.1	134.7	147.4	149.4	171.0
16	101.7	129.3	147.5	146.2	166.6
17	96.8	123.8	146.2	144.1	162.3
18	92.4	118.2	143.7	143.1	158.1
19	88.6	112.6	140.1	142.7	154.0
20	85.2	106.4	134.8	144.4	149.7
21	82.4	101.0	129.2	144.7	146.4
22	80.0	95.9	123.0	144.7	143.6
23	78.1	91.2	116.5	144.1	141.5
24	76.4	87.0	109.9	142.2	140.2
			Female		
5	145.6	143.7	168.7	184.0	192.3
6	142.8	141.4	163.0	180.4	191.2
7	139.4	140.3	157.5	176.6	189.6
8	135.6	139.8	152.4	172.8	187.6
9	131.3	139.6	148.0	168.8	185.1
10	126.4	140.8	143.7	164.8	182.1
11	121.5	139.8	140.7	160.8	179.0
12	116.3	138.0	138.6	156.7	175.6
13	110.9	135.3	137.3	152.6	172.0
14	105.4	131.3	137.1	148.4	168.2
15	99.4	125.6	139.2	143.4	164.0
16	94.2	120.3	138.8	140.2	160.0
17	89.3	115.1	137.1	137.9	156.0
18	84.8	109.7	134.4	136.4	151.9
19	80.7	104.2	130.7	135.6	148.0
20	76.8	98.4	125.5	136.5	143.7
21	73.8	93.1	120.1	136.2	140.3
22	71.3	88.1	114.2	135.7	137.4
23	69.5	83.5	107.9	134.6	135.0
24	68.4	79.3	101.7	132.4	133.2

Table 2. Inputs for projecting school enrolment for the entire country; assumptions on enrolment ratios and proportions of students attending public and private schools

School level	Year 0	5	10	15	20
	Enrolment ratios				
Male					
Lower-primary	0.625	0.680	0.723	0.768	0.814
Higher-primary	0.458	0.508	0.550	0.593	0.628
Secondary	0.196	0.225	0.251	0.272	0.305
Tertiary	0.093	0.105	0.119	0.132	0.143
Female					
Lower-primary	0.595	0.637	0.669	0.705	0.741
Higher-primary	0.277	0.322	0.353	0.386	0.421
Secondary	0.100	0.126	0.150	0.170	0.193
Tertiary	0.052	0.068	0.082	0.104	0.114
	Proportions of students attending:				
Public schools					
Lower-primary	0.91	0.88	0.86	0.83	0.80
Higher-primary	0.85	0.82	0.80	0.77	0.74
Secondary	0.78	0.75	0.72	0.70	0.68
Tertiary	0.47	0.49	0.50	0.54	0.56
Private schools					
Lower-primary	0.09	0.12	0.14	0.17	0.20
Higher-primary	0.15	0.18	0.20	0.23	0.26
Secondary	0.22	0.25	0.28	0.30	0.32
Tertiary	0.53	0.51	0.50	0.46	0.44

classified by single year of age and sex for that date. Thus, the numbers of males in special age groups at the end of the projection interval 0-5 are shown in table 3.

The number of males aged 7 through 10 years, 589.5, which is shown in column 2 of panel B, is obtained as follows:

$$589.5 = 146.6 + 146.7 + 147.1 + 149.1, \qquad (1)$$

where 146.6, 146.7, 147.1 and 149.1 are the numbers of males aged 7, 8, 9 and 10 years, respectively (indicated in column 2 of panel A). The numbers of other persons at special age groups are obtained in an analogous way.

(ii) Enrolment by school level and sex

Projections of the numbers of students of each sex at various school levels are obtained by multiplying the number of persons in each special age-sex group by the appropriate enrolment ratio. The calculation of those numbers of students for year 5 is illustrated in table 4. The number of students at any given school level (column 5) is obtained as the product of the number of persons in the special age group corresponding to that level (column 2) and the enrolment ratio pertaining to the level (column 4).

For example, the number of male students at the lower-primary school level at the end of interval 0-5, 400.9, is obtained as:

$$400.9 = (589.5)\ (0.680), \qquad (2)$$

where 589.5 is the number of males aged 7-10, and 0.680 is the enrolment rate for males for the lower-primary school level.

When calculations illustrated for year 5 are performed for each projection date over the entire projection period, one obtains projected levels of enrolment by school level and sex. The projected levels of enrolment for the 20-year projection interval are presented in table 5.

(iii) Other results

a. Enrolment aggregates

After the numbers of students of each sex are calculated for each school level, they can be aggregated in different ways to obtain the total number of students and the numbers of students by school level and sex. The numbers obtained through this aggregation for dates five years apart can be further used to calculate the increase in those numbers.

Table 3. Calculating the numbers of males at special age groups: entire country, year 5

(Thousands)

PANEL A: Single year of age	Age (1)	Number of males a/ (2)
	7	146.6
	8	146.7
	9	147.1
	10	149.1
	11	148.5
	12	147.1
	13	144.6
	14	140.6
	15	134.7
	16	129.3
	17	123.8
	18	118.2
	19	112.6
	20	106.4
	21	101.0
	22	95.9
PANEL B: Special age groups	7-10	589.5
	11-14	580.8
	15-18	505.9
	19-22	415.8

a/ Numbers in panel A are from table 1.

Table 4. Calculating the numbers of students, by school level and sex: entire country, year 5

Special age group	Number of persons a/ (thousands)	School level	Enrolment ratio b/ (thousands)	Number of students c/
(1)	(2)	(3)	(4)	(5)
		Male		
7-10	589.5	Lower-primary	0.680	400.9
11-14	580.8	Higher-primary	0.508	295.1
15-18	505.9	Secondary	0.225	113.8
19-22	415.8	Tertiary	0.105	43.7
		Female		
7-10	560.5	Lower-primary	0.637	357.0
11-14	544.4	Higher-primary	0.322	175.3
15-18	470.7	Secondary	0.126	59.3
19-22	383.8	Tertiary	0.068	26.1

a/ Males from table 3, panel B, col. 2.
b/ From table 2.
c/ (Col. 2) . (Col. 4).

Table 5. Projected numbers of students, by school level and sex: entire country

(Thousands)

School level	Year				
	0	5	10	15	20
			Male		
Lower-primary	355.9	400.9	452.6	547.8	636.4
Higher-primary	223.7	295.0	318.9	381.7	456.2
Secondary	78.0	113.9	146.8	158.5	200.7
Tertiary	31.3	43.7	62.7	76.1	84.9
			Female		
Lower-primary	317.0	357.0	402.5	481.5	551.6
Higher-primary	125.8	175.3	195.5	238.7	292.5
Secondary	36.8	59.3	82.4	94.8	122.0
Tertiary	15.7	26.1	40.2	56.6	64.9

i. Total number of students

The total number of students in the education system at the end of a given projection interval is equal to the sum of the male and female students at different school levels at that date. The total number of students in year 5, 1,471.2, as shown in table 6, is the sum of the male and female students in year 5 at the lower-primary level through the tertiary level (table 5). The increase in the total enrolment over the 20-year projection period is indicated in figure III.

ii. Number of students by school level

At any school level the number of students can be obtained as the sum of the male and female students. For example, the number of students at the lower-primary school level, 757.9, is obtained as follows:

$$757.9 = 400.9 + 357.0, \tag{4}$$

where 400.9 and 357.0 are, respectively, the number of male and female students at the lower-primary school level. The number is shown in table 6 (in the column corresponding to year 5).

The projected numbers of students, by school level, over the 20-year period are shown in figure IV.

iii. Number of students by sex

The total number of students of a given sex can be obtained as the sum of the number of students of that sex at various school levels. Thus, the number of male students in the educational system at the end of the 0-5 interval, 853.4, is obtained as:

$$853.4 = 400.9 + 295.1 + 113.8 + 43.7, \tag{5}$$

where 400.9, 295.1, 113.8 and 43.7 are, respectively, the numbers of male students at the lower-primary through the tertiary school level (in table 5).

The projected numbers of male and female students at various dates over the 20-year projection period are indicated in figure III.

iv. Growth in the total number of students

The increase in the total number of students over a given projection interval can be derived as the difference between the total number of students

Table 6. Enrolment aggregates, structure and rates of growth: entire country

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	1184.1	1471.2	1701.6	2035.8	2409.2
Lower-primary	672.8	757.9	855.1	1029.3	1188.0
Higher-primary	349.5	470.3	514.3	620.5	748.8
Secondary	114.8	173.2	229.2	253.4	322.6
Tertiary	47.0	69.8	102.9	132.7	149.8
Male	688.8	853.4	981.0	1164.1	1378.2
Female	495.2	617.7	720.6	871.7	1031.0
Growth in number of students					
Total		287.1	230.4	334.3	373.4
Lower-primary		85.1	97.2	174.3	158.7
Higher-primary		120.9	44.0	106.1	128.3
Secondary		58.4	56.1	24.2	69.3
Tertiary		22.8	33.2	29.7	17.1
Male		164.6	127.6	183.2	214.1
Female		122.5	102.8	151.1	159.3
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.57	0.52	0.50	0.51	0.49
Higher-primary	0.30	0.32	0.30	0.30	0.31
Secondary	0.10	0.12	0.13	0.12	0.13
Tertiary	0.04	0.05	0.06	0.07	0.06
Sex ratios of students					
Total	139	138	136	134	134
Lower-primary	112	112	112	114	115
Higher-primary	178	168	163	160	156
Secondary	212	192	178	167	165
Tertiary	199	167	156	135	131
Rates of growth of enrolment (percentage)					
Total		4.34	2.91	3.59	3.37
Lower-primary		2.38	2.41	3.71	2.87
Higher-primary		5.94	1.79	3.75	3.76
Secondary		8.22	5.61	2.00	4.83
Tertiary		7.90	7.78	5.07	2.43
Male		4.29	2.79	3.42	3.38
Female		4.42	3.08	3.81	3.36

Figure III. Total school enrolment and male and female school enrolment

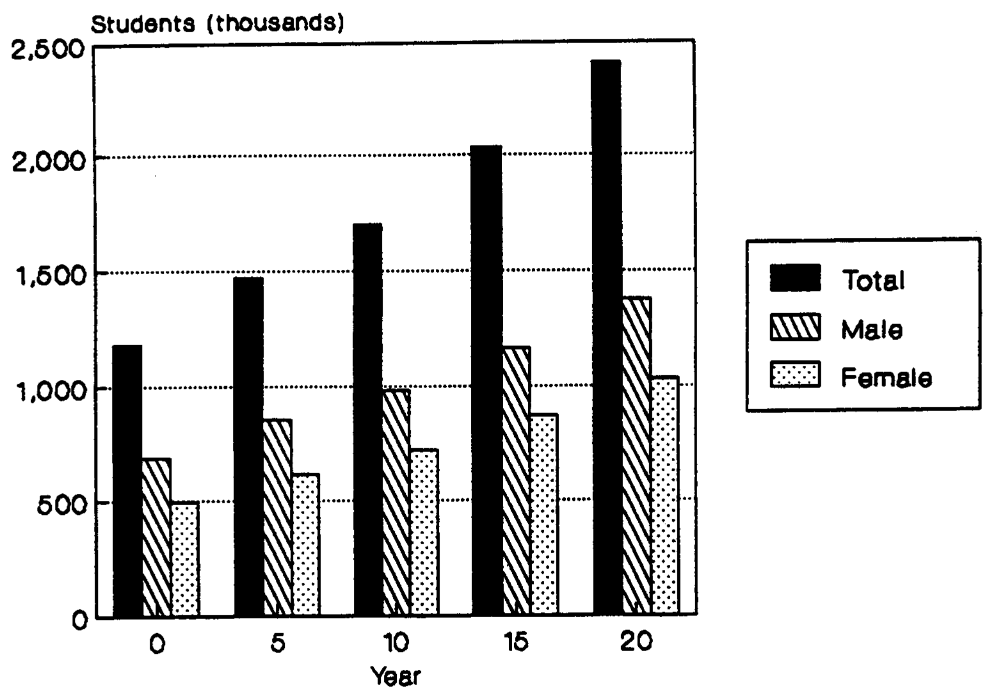
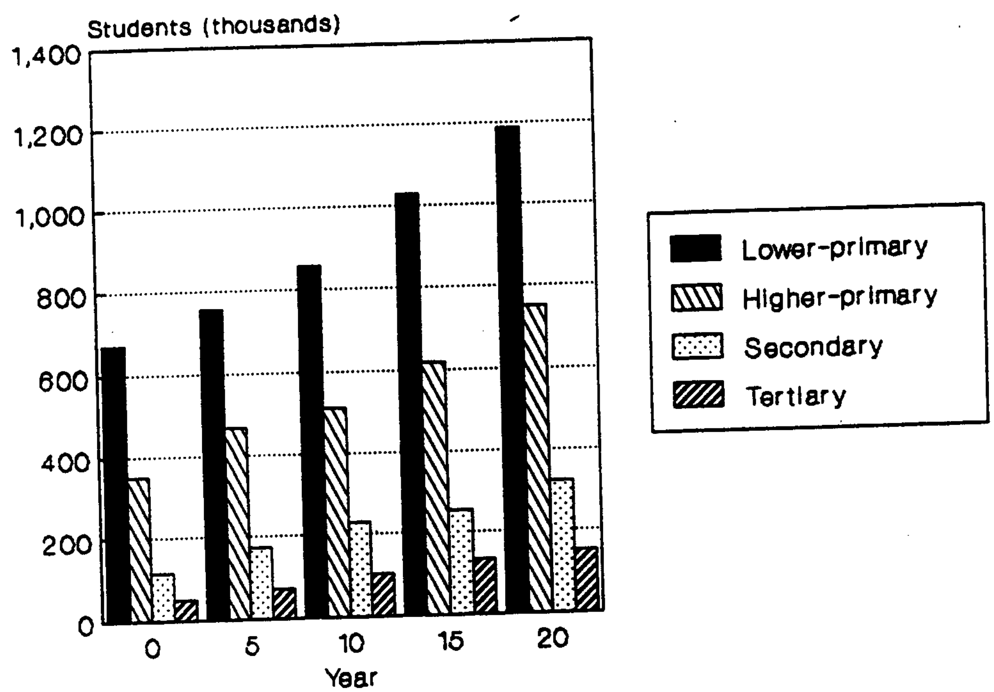


Figure IV. School enrolment: lower-primary, higher-primary, secondary and tertiary levels



at the end and the beginning of the interval. For the interval 0-5, the growth in the total number of students, 287.1 is obtained as:

$$287.1 = 1{,}471.2 - 1{,}184.1, \qquad (6)$$

where 1,184.1 and 1,471.2 are, respectively, the total number of students at the beginning and the end of the interval (shown in columns corresponding to years 0 and 5, respectively).

The rates of growth of the numbers of students in the entire educational system for the 20-year projection period are presented in figure V.

v. Growth in the number of students by school level

The growth in the number of students at each school level over a given projection interval is calculated as the difference between the number of students at the end and at the beginning of the interval at that level. Thus, the increase in the number of students at the lower-primary school level over the interval 0-5, 85.1, is obtained as:

$$85.1 = 757.9 - 672.8, \qquad (7)$$

where 672.8 and 757.9 are the numbers of students at the lower-primary school level in years 0 and 5.

The rates of growth in the number of students at different school levels are shown in figure VI.

vi. Growth in the number of students by sex

The growth in the number of male (or female) students over a projection interval can be obtained as the difference between the number of male (or female) students at the end and at the beginning of the interval. Thus the growth in the number of male students over the interval 0-5, 164.6, is obtained as:

$$164.6 = 853.4 - 688.8, \qquad (8)$$

where 688.8 and 853.4 are the numbers of male students in years 0 and 5, respectively.

Figure V. Rates of growth of total school enrolment and in male and female school enrolment

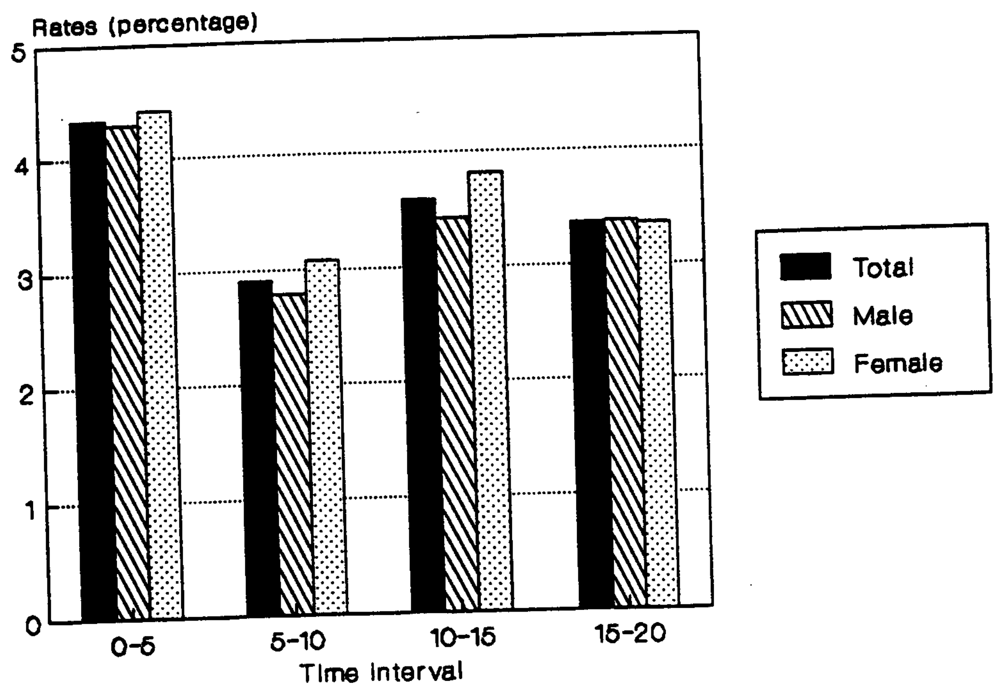
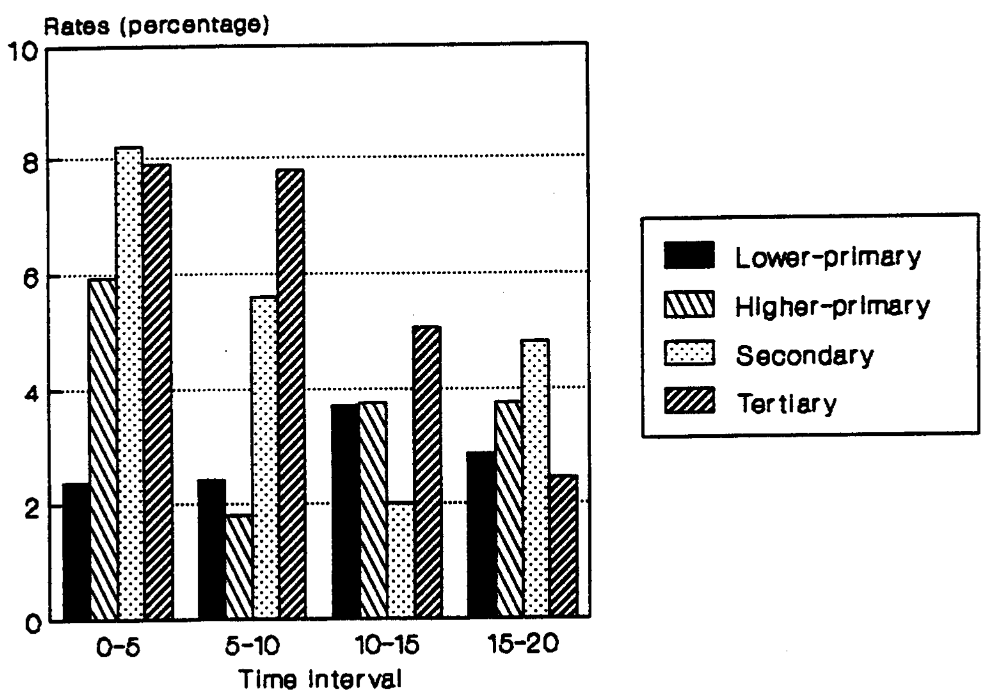


Figure VI. Rates of growth of school enrolment: lower-primary, higher-primary, secondary and tertiary levels



b. Indicators of enrolment structure

The projections of students at different school levels and the sex ratios of students can be obtained as part of this projection.

i. Proportions of students by school level

The proportions of students by school level at the end of the given projection interval can be obtained by dividing the number of students at each school level by the total number of students in the educational system. For example, the proportion of students at the lower-primary school level, 0.52, is calculated as:

$$0.52 = 757.9 / 1{,}471.2, \qquad (9)$$

where 757.9 and 1,471.2 are the numbers of students at the lower-primary level and the total number of students in year 5.

ii. Sex ratios of students

Sex ratios can be calculated for all students in the educational system and for students at different school levels.

The sex ratio for all students in year 5, 138, is calculated as:

$$138 = (853.4 / 617.7) \cdot 100, \qquad (10)$$

where 853.4 and 617.7 are the total number of male students and the total number of female students respectively in year 5.

For each school level, sex ratios are calculated in an analogous fashion. For example, at the lower-primary school level, the sex ratio for year 5, 112, is obtained as:

$$112 = (400.9 / 357.0) \cdot 100, \qquad (11)$$

where 400.9 and 357.0 are the number of male and female students, respectively, at the the lower-primary school level in year 5 (shown in table 5).

c. Rates of growth of enrolment

Rates of growth of enrolment can be calculated for the total number of students and the numbers of students at each school level and of each sex.

i. Rate of growth in the total number of students

The rate of growth of the total number of students in the educational system can be obtained using the exponential growth rate formula. For the projection interval, 0-5, the growth rate of the total number of students, 4.34, which is shown in table 6, can be obtained as:

$$4.34 = [(\ln(1,471.2 / 1,184.1)) / 5] \cdot 100, \quad (12)$$

where 1,184.1 and 1,471.2 are the total number of students in years 0 and 5.

ii. Rates of growth in the number of students by school level

Rates of growth of the number of students at each school level can be obtained in a way analogous to that used to calculate the rate of growth of the total number of students. Thus, for example, the rate of growth of the number of students at the lower-primary school level for the interval 0-5, 2.38, is obtained as follows:

$$2.38 = [(\ln(757.9 / 672.8)) / 5] \cdot 100, \quad (13)$$

where 672.8 and 757.9 are the numbers of students at the lower-primary school level in years 0 and 5, respectively.

iii. Rates of growth in the number of students by sex

Rates of growth of the numbers of male and female students can be obtained in the same way as other growth rates. For example, the rate of growth of the number of male students for the interval 0-5, 4.29, is:

$$4.29 = [(\ln(853.4 / 688.8)) / 5] \cdot 100, \quad (14)$$

where 688.8 and 853.4 are the numbers of male students in years 0 and 5, respectively.

(b) Enrolment in public and private schools

The numbers of students in public and private schools, respectively, are calculated using the projected number of students and assumed proportions of students attending public and private schools.

(i) Enrolment aggregates

The first results obtained in this part of the projection are the numbers of students in each type of school at each school level.

a. Number of students by school level

For each school level the numbers of students in public and private schools, respectively, are obtained as illustrated in table 7. In particular, the number of students in public schools at each school level (column 5) is obtained as the product of the number of students at that level (column 2) and the proportion of students in public schools assumed for the level (column 3). The number of students in private schools at each level (column 6) is derived as the product of the number of students (column 2) and the assumed proportion of students in private schools at the level (column 4).

For example, the number of students in public schools at the lower-primary school level in year 5, 667.0, is calculated as:

$$667.0 = (757.9)\ (0.88), \tag{15}$$

where 757.9 and 0.88 are, respectively, the number of students and the proportion in public schools at the lower-primary level in year 5.

Similarly, the number of students in private schools at the lower-primary school level, 90.9, is obtained as:

$$90.9 = (757.9)\ (0.12), \tag{15}$$

where 757.9 and 0.12 are, respectively, the number of students and the proportion in private schools at this level.

Performing these calculations for different dates over the entire projection period yields projected levels of enrolment in public and private schools by school level. Projected levels of enrolment in public and private schools for the 20-year projection interval are presented in tables 8 and 9. Also shown in these tables are other results obtained for public and private schools as part of this illustrative projection.

Table 7. Calculating the numbers of students in public and private schools: entire country, year 5

School level	Students a/ (thousands)	Proportions of students attending: Public schools b/	Proportions of students attending: Private schools b/	Numbers of students in: Public schools c/ (thousands)	Numbers of students in: Private schools d/ (thousands)
(1)	(2)	(3)	(4)	(5)	(6)
Lower-primary	757.9	0.88	0.12	667.0	90.9
Higher-primary	470.3	0.82	0.18	385.7	84.7
Secondary	173.2	0.75	0.25	129.9	43.3
Tertiary	69.8	0.49	0.51	34.2	35.6

a/ From table 6, column for year 5.
b/ From table 2.
c/ (Col. 2) . (Col. 3).
d/ (Col. 2) . (Col. 4).

Table 8. Enrolment aggregates, structure and rates of growth: results for public schools for the entire country

Indicators	Year 0	Year 5	Year 10	Year 15	Year 20
Enrolment aggregates (thousands)					
Numbers of students					
Total	1020.9	1216.7	1363.3	1581.1	1807.8
Lower-primary	612.3	667.0	735.4	854.3	950.4
Higher-primary	297.1	385.7	411.5	477.8	554.1
Secondary	89.5	129.9	165.0	177.4	219.4
Tertiary	22.1	34.2	51.5	71.6	83.9
Growth in number of students					
Total		195.7	146.7	217.8	226.7
Lower-primary		54.7	68.4	119.0	96.0
Higher-primary		88.6	25.8	66.3	76.3
Secondary		40.3	35.2	12.3	42.0
Tertiary		12.1	17.3	20.2	12.3
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.60	0.55	0.54	0.54	0.53
Higher-primary	0.29	0.32	0.30	0.30	0.31
Secondary	0.09	0.11	0.12	0.11	0.12
Tertiary	0.02	0.03	0.04	0.05	0.05
Rates of growth of enrolment (percentage)					
Total		3.51	2.28	2.96	2.68
Lower-primary		1.71	1.95	3.00	2.13
Higher-primary		5.22	1.29	2.99	2.96
Secondary		7.44	4.79	1.44	4.25
Tertiary		8.73	8.18	6.61	3.16

Table 9. Enrolment aggregates, structure and rates of growth: results for private schools for the entire country

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	163.1	254.5	338.2	454.7	601.4
Lower-primary	60.6	90.9	119.7	175.0	237.6
Higher-primary	52.4	84.7	102.9	142.7	194.7
Secondary	25.3	43.3	64.2	76.0	103.2
Tertiary	24.9	35.6	51.5	61.0	65.9
Growth in number of students					
Total		91.3	83.7	116.5	146.7
Lower-primary		30.4	28.8	55.3	62.6
Higher-primary		32.2	18.2	39.8	52.0
Secondary		18.0	20.9	11.8	27.2
Tertiary		10.7	15.9	9.6	4.9
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.37	0.36	0.35	0.38	0.40
Higher-primary	0.32	0.33	0.30	0.31	0.32
Secondary	0.15	0.17	0.19	0.17	0.17
Tertiary	0.15	0.14	0.15	0.13	0.11
Rates of growth of enrolment (percentage)					
Total		8.89	5.69	5.92	5.59
Lower-primary		8.13	5.50	7.59	6.12
Higher-primary		9.59	3.90	6.55	6.21
Secondary		10.78	7.88	3.38	6.13
Tertiary		7.13	7.38	3.41	1.54

b. <u>Total number of students</u>

The total number of students in public (or private) schools can be obtained by adding the numbers of students in those schools at each level.

Thus, the total number of students in public schools at the end of the interval 0-5, 1,216.7, which is shown in table 8 (in the column corresponding to year 5), is obtained as:

$$1,216.7 = 667.0 + 385.7 + 129.9 + 34.2, \tag{16}$$

where 667.0, 385.7, 129.9 and 34.2 are, respectively, the numbers of students in public schools at the lower-primary through the tertiary school level. The total number of students in private schools in year 5, 254.5, is obtained in an analogous way (table 9).

The total number of students in public and private schools over the 20-year period is shown in figure VII.

c. <u>Growth in the total number of students</u>

The increase in the total number of students in public or private schools over an interval is obtained as the difference between the total number of students in those schools at the end and the beginning of the interval. Thus, for the interval 0-5, the growth in the total number of students in public schools, 197.2 , which is shown in table 8, is obtained as:

$$195.7 = 1,216.7 - 1,020.9, \tag{17}$$

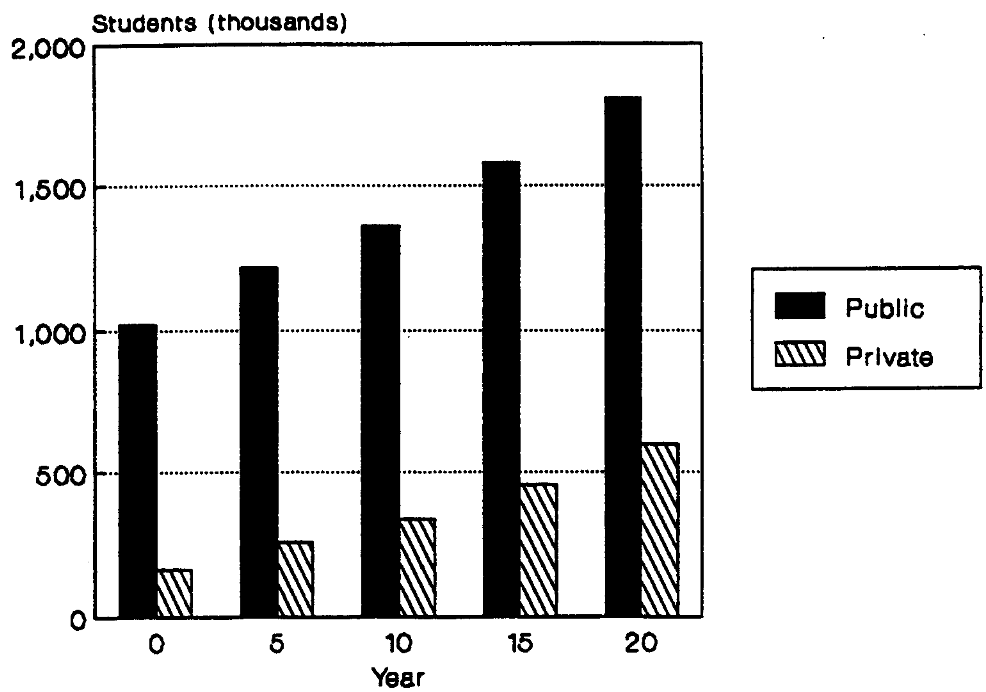
where 1,020.9 and 1,216.7 are, respectively, the total numbers of students in public schools at the beginning and the end of the interval (shown in table 8 in columns corresponding to years 0 and 5).

d. <u>Growth in the number of students by school level</u>

Increases in the number of students in public (or private) schools at different school levels over a given projection interval are obtained in a similar way. For example, the increase in the number of students in public schools at the lower-primary school level over the interval 0-5, 54.7, is calculated as:

$$54.7 = 667.0 - 612.3, \tag{18}$$

Figure VII. Total school enrolment: public and private schools



where 612.3 and 667.0 are the numbers of students in public schools at the lower-primary school level in years 0 and 5.

(ii) Indicators of enrolment structure

The number of students in public and private schools can be used to derive indicators of the structure of enrolment in public and private parts of the educational system, which consist of proportions of students by school level.

(iii) Proportions of students by school level

The proportion of students in the public and private schools, respectively, at any given school level is obtained by dividing the number of students in those schools at that level by the total number of students in public or private schools. Thus, for example, the proportion of students in public schools at the lower-primary level in year 5, 0.55, which is shown in table 8 (column corresponding to year 5), is obtained as:

$$0.55 = 667.0 / 1{,}216.7, \qquad (19)$$

where 667.0 is the number of students in public schools at the lower-primary school level and 1,216.7 is the total number of students in public schools at that date.

(iv) Rates of growth of enrolment

The rates of growth in the number of students in public and private schools, respectively, can be calculated for all students and for students at each school level.

a. Rates of growth in the total number of students

The rates of growth of the number of students in public (or private) schools can be obtained using the exponential growth rate formula. Thus, the rate of growth of the number of students in all private schools for the interval 0-5, 8.89, would be:

$$8.89 = [\ (\ \ln(\ 254.5 / 163.1\ )\ )\ / 5\ ] \cdot 100, \qquad (20)$$

where 163.1 and 254.5 are the total numbers of students in private schools in years 0 and 5, respectively. This growth rate is shown in table 9, corresponding to the interval 0-5.

b. Rates of growth in the numbers of students by school level

The rates of growth of the number of students in public (or private) school at each school level can be obtained using the same exponential growth rate formula. For example, the rate of growth of the number of students in private schools at the lower-primary school level for the interval 0-5, 8.13, can be obtained as:

$$8.13 = [(\ln(90.9 / 60.6)) / 5] \cdot 100, \tag{21}$$

where 60.6 and 90.9 are the numbers of students in private schools at the lower-primary level in years 0 and 5, respectively. This growth rate is also shown in table 9.

2. Urban-rural level

An urban-rural projection of school enrolment can be prepared using the inputs shown in tables 10 through 13. Tables 10 and 11 show the structures of the urban and rural school-age populations, respectively, classified by single year of age and sex. These structures reflect the fertility, mortality and internal migration conditions assumed in the course of projecting urban and rural populations. Tables 12 and 13 indicate assumptions on enrolment ratios applying to urban and rural school-age populations along with the proportions of students in urban and rural schools who attend public and private schools. In addition, table 13 also indicates assumptions on proportions of rural students attending urban schools.

The example presented below will emphasize the steps that are unique to the urban-rural projection.

(a) Overall enrolment

The first step in projecting overall enrolment for urban and rural areas is the derivation for each area of the number of persons in special age groups.

(i) Persons in special age groups

The number of persons in special age groups in urban and rural areas, respectively, are calculated, using urban and rural school-age population structures disaggregated by single year of age and sex (tables 10 and 11). The aggregation procedure employed is identical to that described in the example of a national projection. This procedure yields the number of males and females in urban and rural areas in special age groups in year 5 (column 2 of table 14).

Table 10. Inputs for projecting school enrolment in urban areas; school-age population by single year of age and sex

(Thousands)

Age	Year 0	5	10	15	20
			Male		
5	37.5	36.8	55.2	72.5	86.3
6	37.0	37.5	52.7	70.4	85.5
7	36.4	39.4	51.0	68.9	85.0
8	35.7	42.1	50.2	67.8	84.9
9	35.1	45.2	50.4	67.3	85.0
10	34.3	49.9	51.6	67.4	85.5
11	34.0	52.8	53.5	67.7	85.9
12	33.8	55.2	56.2	68.4	86.5
13	33.8	56.9	59.8	69.4	87.1
14	34.1	57.7	64.1	70.7	87.8
15	34.6	57.3	70.9	72.2	88.7
16	35.3	56.9	74.7	74.2	89.2
17	36.2	56.2	77.1	76.6	89.6
18	37.3	55.3	78.1	79.4	89.8
19	38.3	54.0	78.0	82.2	90.0
20	40.0	52.2	75.8	86.1	89.7
21	40.7	50.7	73.4	88.4	89.8
22	40.9	49.1	70.2	90.2	90.0
23	40.5	47.7	66.4	91.0	90.3
24	39.2	46.5	62.6	90.5	90.9
			Female		
5	35.7	34.6	50.8	67.7	80.5
6	35.3	34.5	48.0	65.0	79.1
7	34.7	35.3	45.8	62.6	77.8
8	34.1	36.7	44.2	60.4	76.5
9	33.3	38.3	43.3	58.5	75.3
10	32.4	41.1	43.1	57.0	74.1
11	31.7	42.5	43.6	55.8	72.9
12	30.9	43.6	44.7	55.0	71.8
13	30.2	44.2	46.5	54.5	70.7
14	29.6	44.1	48.9	54.2	69.6
15	29.2	43.0	52.8	54.2	68.4
16	28.5	42.3	55.2	55.0	67.7
17	27.6	41.6	56.8	56.4	67.2
18	26.6	40.8	57.6	58.4	67.0
19	25.5	40.0	57.9	60.6	67.1
20	24.2	39.2	57.1	64.3	67.5
21	23.1	38.2	56.0	66.7	68.1
22	22.1	37.2	54.4	68.9	69.0
23	21.1	36.0	52.5	70.4	70.2
24	20.4	34.6	50.3	71.0	71.6

Table 11. Inputs for projecting school enrolment in rural areas; school-age population by single year of age and sex

(Thousands)

Age	Year 0	5	10	15	20
			Male		
5	116.8	112.1	118.8	119.3	116.4
6	114.7	109.5	115.4	117.1	115.7
7	112.2	107.0	111.4	114.3	114.1
8	109.1	104.4	107.1	111.0	111.7
9	105.5	101.7	102.5	107.2	108.5
10	101.2	99.0	97.3	102.7	104.3
11	96.5	95.6	92.8	98.0	100.2
12	91.2	91.9	88.3	93.1	95.7
13	85.6	87.6	84.0	88.0	90.9
14	79.5	82.9	80.0	82.6	86.0
15	72.5	77.3	76.3	76.3	80.5
16	66.4	72.4	72.6	71.4	75.7
17	60.6	67.6	69.1	67.2	71.1
18	55.2	63.0	65.6	63.6	66.8
19	50.3	58.6	62.2	60.5	62.9
20	45.1	54.2	58.9	58.3	59.1
21	41.7	50.3	55.8	56.2	56.0
22	39.2	46.8	52.9	54.5	53.4
23	37.6	43.5	50.0	53.0	51.2
24	37.3	40.6	47.3	51.6	49.4
			Female		
5	110.0	108.8	116.4	115.5	112.1
6	107.5	106.6	113.7	114.3	112.3
7	104.7	104.8	110.6	112.8	111.8
8	101.5	103.0	107.3	111.0	110.8
9	98.0	101.2	103.9	108.9	109.4
10	94.0	99.7	100.1	106.5	107.2
11	89.8	97.2	96.8	103.7	105.1
12	85.4	94.4	93.7	100.5	102.6
13	80.7	91.0	90.7	97.0	100.0
14	75.8	87.2	88.2	93.2	97.2
15	70.2	82.5	86.3	88.7	94.2
16	65.7	78.0	83.5	84.8	91.0
17	61.7	73.5	80.3	81.3	87.5
18	58.2	68.8	76.7	78.0	83.8
19	55.1	64.2	72.9	74.9	79.9
20	52.6	59.1	68.3	72.2	75.6
21	50.7	54.9	64.1	69.4	71.7
22	49.3	51.0	59.7	66.7	68.1
23	48.4	47.6	55.5	64.1	64.7
24	48.0	44.8	51.4	61.3	61.7

Table 12. Inputs for projecting school enrolment in urban areas; assumptions on enrolment ratios and proportions of students attending public and private schools

School level	Year 0	5	10	15	20
			Enrolment ratios		
Male					
Lower-primary	0.700	0.750	0.790	0.830	0.870
Higher-primary	0.530	0.570	0.610	0.650	0.680
Secondary	0.260	0.280	0.300	0.320	0.350
Tertiary	0.130	0.140	0.150	0.160	0.170
Female					
Lower-primary	0.670	0.710	0.740	0.770	0.800
Higher-primary	0.350	0.390	0.420	0.450	0.480
Secondary	0.170	0.190	0.210	0.230	0.250
Tertiary	0.100	0.110	0.120	0.140	0.150
			Proportions of students attending:		
Public schools					
Lower-primary	0.80	0.78	0.76	0.74	0.72
Higher-primary	0.75	0.73	0.71	0.69	0.67
Secondary	0.70	0.68	0.66	0.64	0.62
Tertiary	0.40	0.42	0.44	0.48	0.50
Private schools					
Lower-primary	0.20	0.22	0.24	0.26	0.28
Higher-primary	0.25	0.27	0.29	0.31	0.33
Secondary	0.30	0.32	0.34	0.36	0.38
Tertiary	0.60	0.58	0.56	0.52	0.50

Table 13. Inputs for projecting school enrolment in rural areas; assumptions on enrolment ratios, proportions of students attending public and private schools and proportions of rural students attending urban schools

School level	Year 0	5	10	15	20
	Enrolment ratios				
Male					
Lower-primary	0.600	0.650	0.690	0.730	0.770
Higher-primary	0.430	0.470	0.510	0.550	0.580
Secondary	0.160	0.180	0.200	0.220	0.250
Tertiary	0.060	0.070	0.080	0.090	0.100
Female					
Lower-primary	0.570	0.610	0.640	0.670	0.700
Higher-primary	0.250	0.290	0.320	0.350	0.380
Secondary	0.070	0.090	0.110	0.130	0.150
Tertiary	0.030	0.040	0.050	0.070	0.080
	Proportions of students attending:				
Public schools					
Lower-primary	0.95	0.93	0.91	0.89	0.87
Higher-primary	0.90	0.88	0.86	0.84	0.82
Secondary	0.85	0.83	0.81	0.79	0.77
Tertiary	0.60	0.62	0.64	0.68	0.70
Private schools					
Lower-primary	0.05	0.07	0.09	0.11	0.13
Higher-primary	0.10	0.12	0.14	0.16	0.18
Secondary	0.15	0.17	0.19	0.21	0.23
Tertiary	0.40	0.38	0.36	0.32	0.30
	Proportions of rural students attending urban schools				
Lower-primary	0.05	0.04	0.03	0.02	0.01
Higher-primary	0.10	0.09	0.08	0.07	0.06
Secondary	0.80	0.75	0.70	0.65	0.60
Tertiary	0.90	0.88	0.86	0.84	0.82

(ii) Enrolment by school level and sex and by location of residence

The numbers of students classified by school level and sex who reside in urban and rural areas are obtained by multiplying the number of persons in special age groups in each area by the relevant enrolment ratios. These calculations are illustrated in table 14 for year 5. The number of students (column 5) are derived as products of the number of persons in special age groups (column 2) and enrolment ratios (column 4).

Thus, the number of male students at the lower-primary school level who reside in urban areas in year 5, 132.5, is obtained as follows:

$$132.5 = (176.7) (0.750), \qquad (22)$$

where 176.7 is the number of males aged 7-10 who reside in urban areas and 0.750 is the enrolment ratio for the lower-primary school level.

If some of the students who reside in rural areas attend urban schools, as is assumed in this example, the number of students can be calculated by location of school attendance. To do so, it is necessary to derive the number of rural students attending urban schools by school level.

(iii) Enrolment in urban schools among rural students

The number of students who reside in rural areas but attend urban schools can be derived for year 5 as illustrated in table 15. The number of such students at each school level (column 5) is obtained by multiplying the sum of the number of male and female students at that level living in rural areas (column 2 and 3) by the proportion of those students attending urban schools (column 4).

For example, the number of rural students attending urban schools at the lower-primary school level in year 5, 20.7, is obtained as:

$$20.7 = (267.9 + 249.3) (0.04), \qquad (23)$$

where 267.9 and 249.3 are the number of male and female students at the lower-primary school level who reside in rural areas and 0.04 is the proportion of those students who attend urban schools.

(iv) Other results

After calculating the number of rural students attending urban schools, it is further possible to derive various enrolment aggregates.

Table 14. Calculating the numbers of students by school level and sex: urban and rural areas, year 5

Special age group	Number of persons a/ (thousands)	School level	Enrolment ratio b/	Number of students c/ (thousands)
(1)	(2)	(3)	(4)	(5)
		Urban		
Male				
7-10	176.6	Lower-primary	0.750	132.5
11-14	222.6	Higher-primary	0.570	126.9
15-18	225.7	Secondary	0.280	63.2
19-22	206.0	Tertiary	0.140	28.8
Female				
7-10	151.4	Lower-primary	0.710	107.5
11-14	174.4	Higher-primary	0.390	68.0
15-18	167.7	Secondary	0.190	31.9
19-22	154.6	Tertiary	0.110	17.0
		Rural		
Male				
7-10	412.1	Lower-primary	0.650	267.9
11-14	358.0	Higher-primary	0.470	168.3
15-18	280.3	Secondary	0.180	50.5
19-22	209.9	Tertiary	0.070	14.7
Female				
7-10	408.7	Lower-primary	0.610	249.3
11-14	369.8	Higher-primary	0.290	107.2
15-18	302.8	Secondary	0.090	27.3
19-22	229.2	Tertiary	0.040	9.2

a/ Calculations not illustrated.
b/ From tables 12 and 13.
c/ (Col. 2) . (Col. 4).

Table 15. Calculating the numbers of rural students attending schools in urban areas: year 5

School level	Students residing in rural areas a/ (thousands)		Proportions of rural students attending urban schools b/	Rural students attending urban schools c/ (thousands)
	Male	Female		
(1)	(2)	(3)	(4)	(5)
Lower-primary	267.9	249.3	0.04	20.7
Higher-primary	168.3	107.2	0.09	24.8
Secondary	50.5	27.3	0.75	58.3
Tertiary	14.7	9.2	0.88	21.0

a/ From table 14, col. 5.
b/ From table 13.
c/ ((Col. 2) + (Col. 3)) . (Col. 4).

a. Enrolment aggregates

The numbers of students attending schools in urban and rural areas, classified by school level, are obtained as indicated in table 16. Hence, the number of students in urban schools at any school level (column 5) is equal to the sum of the number of male and female students living in urban areas (columns 2 and 3) plus the number of rural students attending urban schools (column 4). Thus, for the lower-primary school level in year 5, the number of students attending urban schools, 260.6, is obtained as:

$$260.6 = (132.5 + 107.4) + 20.7, \tag{24}$$

where 132.5 and 107.4 are the numbers of male and female students at the lower-primary school level living in urban areas. The number of rural students at this level attending urban schools is 20.7.

As illustrated in table 17, the number of students in rural schools at any school level (column 5) is obtained by adding the number of male and female students living in rural areas (columns 2 and 3) and subtracting from this sum the number of rural students attending urban schools (column 4). Thus, for the lower-primary school level the number of students attending rural schools in year 5, 496.5, is obtained as:

$$496.5 = (267.9 + 249.3) - 20.7, \tag{25}$$

where 267.9 and 249.3 are the numbers of male and female students at the lower-primary school level living in rural areas and 20.7 is the number of rural students at the same level attending urban schools.

When the calculations illustrated for urban and rural areas for year 5 are performed for each projection date over the entire projection period, one obtains the projected number of students in urban and rural schools, respectively, by school level. The projected numbers of students for the 20-year projection period are presented in tables 18 and 19. These results can be aggregated across the two locations to obtain the projected number of students by school level for the entire country. The number of students for the 20-year projection period for the entire country, which was obtained by aggregating the relevant results in tables 18 and 19, are shown in table 20.

Using the projected numbers of students in urban and rural schools and in the entire country by school level, one can obtain the total number of students in urban and rural schools and in the country as a whole. One can also derive the growth in the total number of students and the numbers of students at different school levels, both in urban and rural schools and nationally. These aggregates, which can be derived using steps that are analogous to those illustrated in the national enrolment projection are shown for the 20-year projection period in tables 18 through 20.

Table 16. Calculating the numbers of students attending schools in urban areas: year 5

(Thousands)

School level	Students residing in urban areas a/		Rural students attending urban schools b/	Students attending urban schools c/
	Male	Female		
(1)	(2)	(3)	(4)	(5)
Lower-primary	132.5	107.5	20.7	260.6
Higher-primary	126.9	68.0	24.8	219.7
Secondary	63.2	31.9	58.3	153.3
Tertiary	28.8	17.0	21.0	66.8

a/ From table 14, col. 5.
b/ From table 15, col. 5.
c/ ((Col. 2) + (Col. 3)) + (Col. 4).

Table 17. Calculating the numbers of students attending schools in rural areas: year 5

(Thousands)

School level	Students residing in urban areas a/		Rural students attending urban schools b/	Students attending rural schools c/
	Male	Female		
(1)	(2)	(3)	(4)	(5)
Lower-primary	267.9	249.3	20.7	496.5
Higher-primary	168.3	107.2	24.8	250.7
Secondary	50.5	27.3	58.3	19.4
Tertiary	14.7	9.2	21.0	2.9

a/ From table 14, col. 5.
b/ From table 15, col. 5.
c/ ((Col. 2) + (Col. 3)) - (Col. 4).

Table 18. Enrolment aggregates, structure and rates of growth: urban areas

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	390.5	575.7	719.3	927.5	1206.6
Lower-primary	189.2	239.9	291.1	408.9	539.1
Higher-primary	114.8	194.9	219.7	278.3	373.0
Secondary	56.3	95.1	136.9	148.3	192.6
Tertiary	30.3	45.8	71.7	92.0	101.9
Growth in number of students					
Total		185.2	143.6	208.2	279.1
Lower-primary		50.8	51.1	117.8	130.2
Higher-primary		80.1	24.8	58.7	94.7
Secondary		38.8	41.9	11.3	44.3
Tertiary		15.6	25.8	20.3	9.9
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.48	0.42	0.40	0.44	0.45
Higher-primary	0.29	0.34	0.31	0.30	0.31
Secondary	0.14	0.17	0.19	0.16	0.16
Tertiary	0.08	0.08	0.10	0.10	0.08
Rates of growth of enrolment (percentage)					
Total		7.76	4.45	5.08	5.26
Lower-primary		4.76	3.86	6.80	5.53
Higher-primary		10.59	2.39	4.73	5.86
Secondary		10.47	7.30	1.59	5.23
Tertiary		8.30	8.93	4.99	2.04

Table 19. Enrolment aggregates, structure and rates of growth: rural areas

	Year				
	0	5	10	15	20
<u>Enrolment aggregates (thousands)</u>					
Numbers of students					
Total	793.9	894.2	977.2	1093.9	1189.0
Lower-primary	483.8	517.2	558.6	612.0	645.2
Higher-primary	234.6	275.5	294.2	337.0	370.1
Secondary	58.7	77.7	92.7	104.5	127.0
Tertiary	16.8	23.9	31.6	40.5	46.8
Growth in number of students					
Total	100.4	82.9	116.8	95.1	
Lower-primary	33.4	41.5	53.3	33.2	
Higher-primary	40.9	18.7	42.8	33.1	
Secondary	19.0	15.0	11.9	22.5	
Tertiary	7.1	7.8	8.8	6.3	
<u>Indicators of enrolment structure</u>					
Proportions by school levels					
Lower-primary	0.61	0.58	0.57	0.56	0.54
Higher-primary	0.30	0.31	0.30	0.31	0.31
Secondary	0.07	0.09	0.09	0.10	0.11
Tertiary	0.02	0.03	0.03	0.04	0.04
<u>Rates of growth of enrolment (percentage)</u>					
Total	2.38	1.77	2.26	1.67	
Lower-primary	1.34	1.54	1.82	1.06	
Higher-primary	3.21	1.31	2.71	1.87	
Secondary	5.62	3.52	2.41	3.89	
Tertiary	7.01	5.64	4.93	2.89	

Table 20. Enrolment aggregates, structure, distribution and rates of growth: entire country

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	1184.4	1470.0	1696.5	2021.4	2395.6
Lower-primary	672.9	757.1	849.7	1020.9	1184.3
Higher-primary	349.4	470.4	513.9	615.3	743.1
Secondary	115.0	172.8	229.6	252.8	319.6
Tertiary	47.1	69.7	103.3	132.5	148.6
Growth in number of students					
Total		285.6	226.5	325.0	374.2
Lower-primary		84.2	92.6	171.2	163.4
Higher-primary		121.0	43.5	101.4	127.8
Secondary		57.8	56.8	23.2	66.8
Tertiary		22.6	33.6	29.2	16.2
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.57	0.52	0.50	0.51	0.49
Higher-primary	0.29	0.32	0.30	0.30	0.31
Secondary	0.10	0.12	0.14	0.13	0.13
Tertiary	0.04	0.05	0.06	0.07	0.06
Indicators of enrolment distribution					
Proportions of total enrolment					
Urban	0.33	0.39	0.42	0.46	0.50
Rural	0.67	0.61	0.58	0.54	0.50
Proportions by school level					
Urban					
Lower-primary	0.28	0.32	0.34	0.40	0.46
Higher-primary	0.33	0.41	0.43	0.45	0.50
Secondary	0.49	0.55	0.60	0.59	0.60
Tertiary	0.64	0.66	0.69	0.69	0.69
Rural					
Lower-primary	0.72	0.68	0.66	0.60	0.54
Higher-primary	0.67	0.59	0.57	0.55	0.50
Secondary	0.51	0.45	0.40	0.41	0.40
Tertiary	0.36	0.34	0.31	0.31	0.31
Rates of growth of enrolment (percentage)					
Total		4.32	2.87	3.51	3.40
Lower-primary		2.36	2.31	3.67	2.97
Higher-primary		5.95	1.77	3.60	3.77
Secondary		8.15	5.69	1.93	4.69
Tertiary		7.85	7.87	4.97	2.31

The increase in the numbers of students attending urban and rural schools and in the number of students for the entire country is indicated in figure VIII. Projections of all students who attend urban and rural schools are shown in figure IX.

b. Indicators of enrolment structure

The proportions of students in urban and rural schools by school level and the proportions of students in the entire country by school level can be calculated as part of projecting overall enrolment for urban and rural areas. These proportions, calculated for the 20-year period, are shown in tables 18 through 20.

c. Indicators of enrolment distribution

The proportions of all students that are urban and rural, respectively, along with the proportions of students at different school levels that are urban and rural can be obtained as part of this projection.

i. Proportions urban and rural of total enrolment

The proportion urban among all students at the given projection date can be derived as a ratio of the total number of students in urban schools to the total number of students for the entire country. The proportion urban for all students in year 5, 0.39, is calculated as:

$$0.39 = 575.7 \,/\, 1{,}470.0, \tag{26}$$

where 575.7 is the total number of students in urban schools and 1,470.0 is the total number of students for the entire country.

The proportion rural for all students can be obtained as the complement of the proportion urban. Thus, the proportion rural in year 5, 0.61, is:

$$0.61 = 1 - 0.39, \tag{27}$$

where 0.39 is the proportion urban.

Figure VIII. Total school enrolment: urban, rural and national

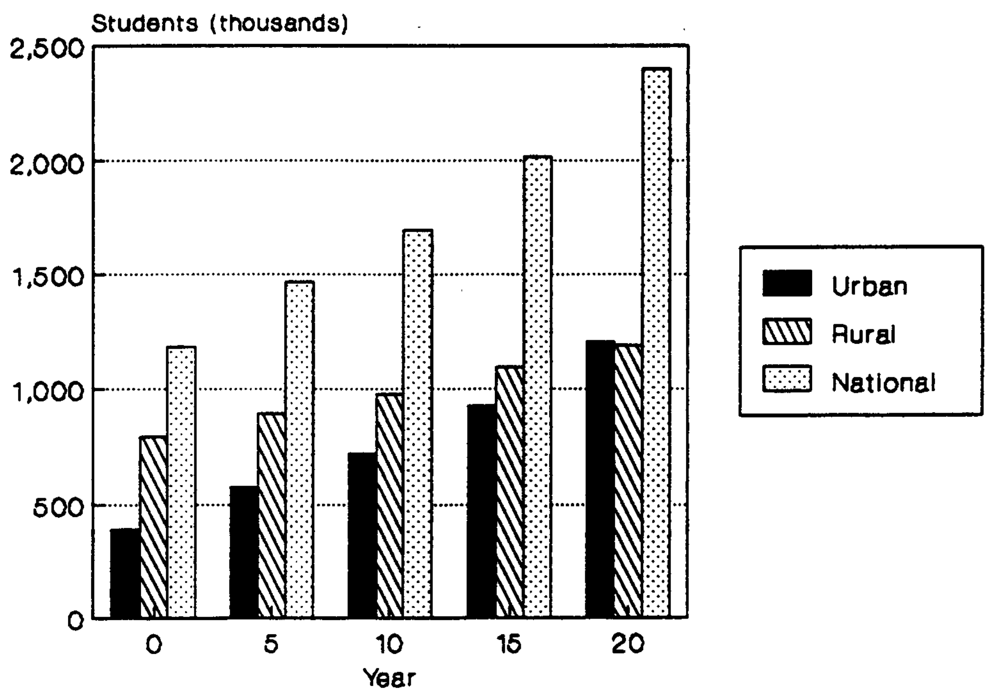
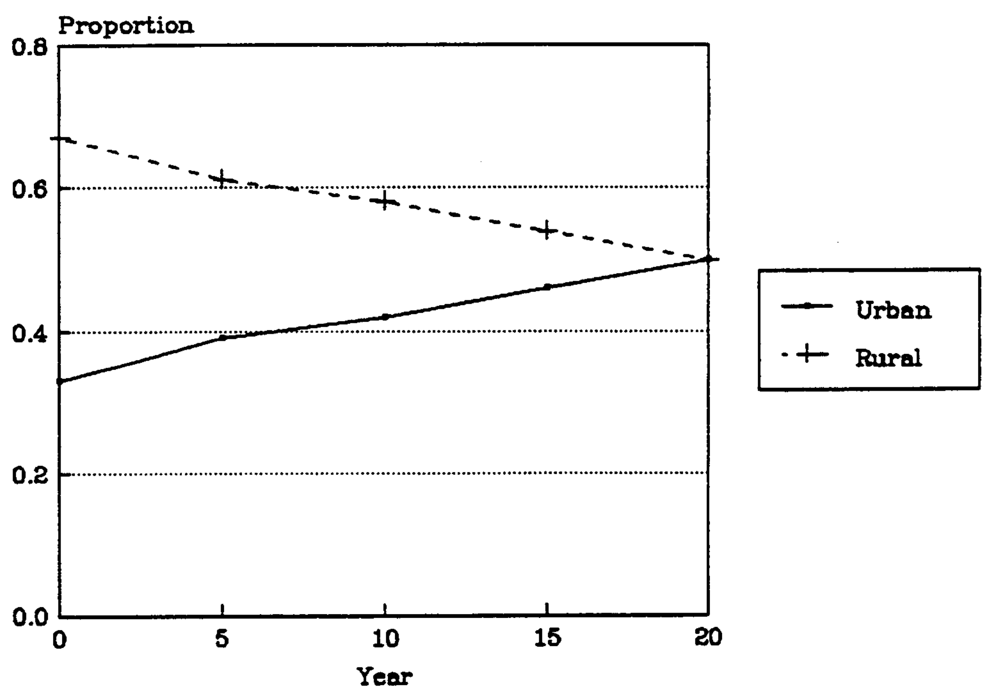


Figure IX. Proportions of school enrolment: urban and rural



ii. Proportions urban and rural of enrolment by school level

Among students at any given school level the proportion urban at the end of a given projection interval can be derived as a ratio of the number of students in urban schools at that level to the number of students in the entire country for the same level. For example, the proportion urban among students at the lower-primary school level in year 5, 0.32, is calculated as:

$$0.32 = 239.9 / 757.1, \qquad (28)$$

where 239.9 is the number of students in urban schools at the lower-primary school level and 757.1 is the number of students in all lower-primary schools in the country.

For students at a given school level the proportion rural can be calculated as the complement of the proportion urban. Thus, proportion rural among students at the lower-primary school level in year 5, 0.68, is:

$$0.68 = 1 - 0.32, \qquad (29)$$

where 0.32 is the proportion urban.

d. Rates of growth of enrolment

The projected number of students for various dates five years apart can be used to calculate rates of growth of total enrolment and enrolment by school level. The rates, which can be obtained using the formula employed with the national projection, would be for urban and rural areas as well as for the entire country. The rates obtained as part of the illustrative calculation over the 20-year projection period are shown in tables 18 through 20.

(b) Enrolment in public and private schools

Where the educational system consists of public and private schools, the number of students in each type of school in urban and rural areas can be calculated and other results derived.

(i) Enrolment aggregates

The number of students in public (or private) schools can be calculated using the projected number of students in urban and rural schools and the assumed proportions of students in those schools who are in public (or private) schools. The calculations are analogous to those illustrated as part of the national projection. Once urban and rural results are obtained, they can be used to derive results at the national level. The numbers of studentsOin public and private schools at urban, rural and national levels are displayed in tables 21 through 26. Also shown in the tables are increases (or decreases) in the number of students over various projection intervals.

(ii) Indicators of enrolment structure

The proportions of students in public (or private) schools at different school levels can be obtained using steps similar to those illustrated in the national projection. The proportions, which can be derived for urban and rural areas and the entire country, that were obtained as part of the illustrative projection, are presented in tables 21 through 26.

(iii) Indicators of enrolment distribution

Indicators of enrolment distribution that are analogous to indicators of enrolment distribution obtained in the portion of the projection referring to overall enrolment can also be calculated. The indicators include proportions of all students in public (or private) schools that are urban (or rural), along with proportions of students in public (or private) schools at different school levels that are urban (or rural).

a. Proportions urban and rural of total enrolment

The proportion urban among all students in public or private schools at the end of a given projection interval can be derived as a ratio of the total number of students in urban public (or private) schools to the total number of students in all public (or private) schools in the entire country. For example, the proportion urban for all students in public schools in year 5, 0.34, is calculated as:

$$0.34 = 413.3 \ / \ 1{,}216.0, \qquad (30)$$

where 413.3 is the total number of students in urban public schools and 1,216.0 is the total number of students in all public schools in the country.

The proportion rural for all students in public (or private) schools can be obtained as the complement of the relevant proportion urban for all students. Thus, the proportion rural among all students in public schools, in year 5, 0.66, is:

$$0.66 = 1 - 0.34, \qquad (31)$$

Table 21. Enrolment aggregates, structure and rates of growth: results for public schools in urban areas

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	288.9	413.3	499.1	633.7	808.4
Lower-primary	151.3	187.2	221.2	302.6	388.2
Higher-primary	86.1	142.3	156.0	192.0	249.9
Secondary	39.4	64.6	90.4	94.9	119.4
Tertiary	12.1	19.3	31.5	44.1	50.9
Growth in number of students					
Total	124.4	85.7	134.6	174.7	
Lower-primary	35.8	34.1	81.4	85.6	
Higher-primary	56.2	13.7	36.1	57.9	
Secondary	25.2	25.7	4.5	24.5	
Tertiary	7.1	12.3	12.6	6.8	
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.52	0.45	0.44	0.48	0.48
Higher-primary	0.30	0.34	0.31	0.30	0.31
Secondary	0.14	0.16	0.18	0.15	0.15
Tertiary	0.04	0.05	0.06	0.07	0.06
Rates of growth of enrolment (percentage)					
Total	7.16	3.77	4.78	4.87	
Lower-primary	4.25	3.34	6.27	4.98	
Higher-primary	10.05	1.84	4.16	5.27	
Secondary	9.89	6.70	0.98	4.60	
Tertiary	9.27	9.86	6.73	2.86	

Note: growth values are printed between the year columns (0–5, 5–10, 10–15, 15–20).

Table 22. Enrolment aggregates, structure and rates of growth: results for private schools in urban areas

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	101.6	162.4	220.2	293.8	398.2
Lower-primary	37.8	52.8	69.9	106.3	151.0
Higher-primary	28.7	52.6	63.7	86.3	123.1
Secondary	16.9	30.4	46.6	53.4	73.2
Tertiary	18.2	26.6	40.1	47.8	50.9
Growth in number of students					
Total		60.8	57.8	73.6	104.4
Lower-primary		15.0	17.1	36.5	44.6
Higher-primary		23.9	11.1	22.6	36.8
Secondary		13.5	16.1	6.8	19.8
Tertiary		8.4	13.5	7.7	3.1
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.37	0.33	0.32	0.36	0.38
Higher-primary	0.28	0.32	0.29	0.29	0.31
Secondary	0.17	0.19	0.21	0.18	0.18
Tertiary	0.18	0.16	0.18	0.16	0.13
Rates of growth of enrolment (percentage)					
Total		9.39	6.09	5.76	6.08
Lower-primary		6.66	5.60	8.40	7.01
Higher-primary		12.13	3.82	6.07	7.11
Secondary		11.76	8.51	2.73	6.31
Tertiary		7.62	8.23	3.51	1.26

Table 23. Enrolment aggregates, structure and rates of growth: results for public schools in rural areas

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	730.7	802.7	856.7	937.8	995.3
Lower-primary	459.6	481.0	508.4	544.6	561.3
Higher-primary	211.2	242.4	253.0	283.1	303.5
Secondary	49.9	64.5	75.1	82.6	97.8
Tertiary	10.1	14.8	20.2	27.5	32.7
Growth in number of students					
Total		72.0	54.0	81.1	57.5
Lower-primary		21.4	27.4	36.3	16.6
Higher-primary		31.3	10.6	30.0	20.4
Secondary		14.6	10.6	7.5	15.2
Tertiary		4.7	5.5	7.3	5.2
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.63	0.60	0.59	0.58	0.56
Higher-primary	0.29	0.30	0.30	0.30	0.30
Secondary	0.07	0.08	0.09	0.09	0.10
Tertiary	0.01	0.02	0.02	0.03	0.03
Rates of growth of enrolment (percentage)					
Total		1.88	1.30	1.81	1.19
Lower-primary		0.91	1.11	1.38	0.60
Higher-primary		2.76	0.85	2.24	1.39
Secondary		5.15	3.03	1.91	3.38
Tertiary		7.66	6.27	6.14	3.47

Table 24. Enrolment aggregates, structure and rates of growth: results for private schools in rural areas

Indicators	Year 0	5	10	15	20
<u>Enrolment aggregates (thousands)</u>					
Numbers of students					
Total	63.2	91.5	120.5	156.1	193.7
Lower-primary	24.2	36.2	50.3	67.3	83.9
Higher-primary	23.5	33.1	41.2	53.9	66.6
Secondary	8.8	13.2	17.6	22.0	29.2
Tertiary	6.7	9.1	11.4	13.0	14.0
Growth in number of students					
Total	28.4	28.9	35.7	37.6	
Lower-primary	12.0	14.1	17.0	16.6	
Higher-primary	9.6	8.1	12.7	12.7	
Secondary	4.4	4.4	4.3	7.3	
Tertiary	2.3	2.3	1.6	1.1	
<u>Indicators of enrolment structure</u>					
Proportions by school levels					
Lower-primary	0.38	0.40	0.42	0.43	0.43
Higher-primary	0.37	0.36	0.34	0.35	0.34
Secondary	0.14	0.14	0.15	0.14	0.15
Tertiary	0.11	0.10	0.09	0.08	0.07
<u>Rates of growth of enrolment (percentage)</u>					
Total	7.42	5.49	5.19	4.31	
Lower-primary	8.06	6.57	5.84	4.40	
Higher-primary	6.86	4.40	5.38	4.23	
Secondary	8.13	5.75	4.41	5.71	
Tertiary	5.98	4.56	2.58	1.60	

Table 25. Enrolment aggregates, structure, distribution and rates of growth: results for public schools for the entire country

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	1019.6	1216.0	1355.8	1571.5	1803.7
Lower-primary	610.9	668.1	729.6	847.2	949.4
Higher-primary	297.2	384.7	409.0	475.1	553.4
Secondary	89.3	129.1	165.4	177.5	217.2
Tertiary	22.2	34.0	51.8	71.7	83.7
Growth in number of students					
Total		196.4	139.7	215.7	232.2
Lower-primary		57.2	61.4	117.7	102.2
Higher-primary		87.5	24.3	66.1	78.3
Secondary		39.9	36.3	12.0	39.7
Tertiary		11.9	17.7	19.9	12.0
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.60	0.55	0.54	0.54	0.53
Higher-primary	0.29	0.32	0.30	0.30	0.31
Secondary	0.09	0.11	0.12	0.11	0.12
Tertiary	0.02	0.03	0.04	0.05	0.05
Indicators of enrolment distribution					
Proportions of total enrolment					
Urban	0.28	0.34	0.37	0.40	0.45
Rural	0.72	0.66	0.63	0.60	0.55
Proportions by school levels					
Urban					
Lower-primary	0.25	0.28	0.30	0.36	0.41
Higher-primary	0.29	0.37	0.38	0.40	0.45
Secondary	0.44	0.50	0.55	0.53	0.55
Tertiary	0.55	0.57	0.61	0.62	0.61
Rural					
Lower-primary	0.75	0.72	0.70	0.64	0.59
Higher-primary	0.71	0.63	0.62	0.60	0.55
Secondary	0.56	0.50	0.45	0.47	0.45
Tertiary	0.45	0.43	0.39	0.38	0.39
Rates of growth of enrolment (percentage)					
Total		3.52	2.18	2.95	2.76
Lower-primary		1.79	1.76	2.99	2.28
Higher-primary		5.16	1.22	3.00	3.05
Secondary		7.38	4.96	1.41	4.04
Tertiary		8.56	8.38	6.50	3.10

Table 26. Enrolment aggregates, structure, distribution and rates of growth: results for private schools for the entire country

Indicators	Year 0	5	10	15	20
Enrolment aggregates (thousands)					
Numbers of students					
Total	164.8	254.0	340.7	449.9	591.9
Lower-primary	62.0	89.0	120.1	173.6	234.8
Higher-primary	52.2	85.7	104.9	140.2	189.7
Secondary	25.7	43.6	64.2	75.3	102.4
Tertiary	24.9	35.7	51.5	60.8	65.0
Growth in number of students					
Total		89.2	86.7	109.2	142.0
Lower-primary		27.0	31.1	53.5	61.2
Higher-primary		33.5	19.2	35.3	49.5
Secondary		17.9	20.5	11.2	27.1
Tertiary		10.8	15.9	9.3	4.2
Indicators of enrolment structure					
Proportions by school levels					
Lower-primary	0.38	0.35	0.35	0.39	0.40
Higher-primary	0.32	0.34	0.31	0.31	0.32
Secondary	0.16	0.17	0.19	0.17	0.17
Tertiary	0.15	0.14	0.15	0.14	0.11
Indicators of enrolment distribution					
Proportions of total enrolment					
Urban	0.62	0.64	0.65	0.65	0.67
Rural	0.38	0.36	0.35	0.35	0.33
Proportions by school levels					
Urban					
Lower-primary	0.61	0.59	0.58	0.61	0.64
Higher-primary	0.55	0.61	0.61	0.62	0.65
Secondary	0.66	0.70	0.73	0.71	0.71
Tertiary	0.73	0.75	0.78	0.79	0.78
Rural					
Lower-primary	0.39	0.41	0.42	0.39	0.36
Higher-primary	0.45	0.39	0.39	0.38	0.35
Secondary	0.34	0.30	0.27	0.29	0.29
Tertiary	0.27	0.25	0.22	0.21	0.22
Rates of growth of enrolment (percentage)					
Total		8.65	5.88	5.56	5.48
Lower-primary		7.22	6.00	7.37	6.04
Higher-primary		9.93	4.04	5.80	6.05
Secondary		10.59	7.72	3.21	6.14
Tertiary		7.19	7.36	3.31	1.33

where 0.34 is the proportion urban.

b. Proportions urban and rural of enrolment by school level

The proportion urban among students in public (or private) schools at any given school level can be derived as a ratio of the number of students in urban public (or private) schools at that level to the number of students in public (or private) schools at the same level in the entire country. For example, the proportion urban among students in public schools at the lower-primary school level in year 5, 0.28, is calculated as:

$$0.28 = 187.2 / 668.1, \qquad (32)$$

where 187.2 is the number of students in urban public schools at the lower-primary school level and 668.1 is the number of students in all public schools in the entire country at the lower-primary level.

The proportion rural among students in public schools at a given school level can be calculated as the complement of the relevant proportion urban for that level. Thus, the proportion rural among students in public schools at the lower-primary school level in year 5, 0.72, is:

$$0.72 = 1 - 0.28, \qquad (33)$$

where 0.28 is the proportion urban among students in public schools at the lower-primary school level.

Various proportions urban and rural for students in public and private schools derived in the course of projecting enrolment in public and private schools over the 20-year period are shown in tables 25 and 26.

(iv) Rates of growth of enrolment

The projected numbers of students in public and private schools for various dates five years apart can be used to calculate rates of growth in those numbers of students over the intervening projection intervals. The rates, which can be obtained using calculations illustrated earlier as part of the national projection, would be for students in public and private schools in urban and rural areas and in the entire country. The rates obtained as part of the illustrative calculation over the 20-year projection period are shown in tables 21 through 26.

E. Summary

The foregoing chapter has described the enrolment ratio method for making school enrolment projections at the national and at the urban-rural level. These projections can be made for overall enrolment as well as for enrolment in public and private schools. When used to prepare an urban-rural projection, the method can deal with the fact that the area of residence and the area of school attendance may not be the same for rural students. It projects the number of rural students attending urban schools and uses that figure to calculate the number of students attending schools in urban and rural areas.

The procedures used to make national and urban-rural projections were presented. The types of inputs required for the technique were discussed and the methods for their preparation described.

Examples of a national projection and of an urban-rural projection were presented and discussed. They included illustrations relating to the calculations of enrolment in public and private schools. The latter example also indicated how to allow for the fact that a portion of students residing in rural areas attend urban schools. A complete listing of the output that can be generated by the method is shown in box 6.

Box 6

Outputs derived from making enrolment projections using enrolment ratios

A. Overall enrolment

1. Enrolment aggregates (national or urban, rural and national)

Number of students:

Total

Lower-primary
Higher-primary
Secondary
Tertiary

Male*
Female*

(continued)

Box 6 (continued)

Growth in the number of students:

Total

Lower-primary
Higher-primary
Secondary
Tertiary

Male*
Female*

2. Indicators of enrolment structure (national or urban, rural and national)

Proportions of students by school level:

Lower-primary
Higher-primary
Secondary
Tertiary

Sex ratios of students*:

Total

Lower-primary
Higher-primary
Secondary
Tertiary

3. Indicators of enrolment distribution (national; if urban-rural projection is being prepared)

Proportions of total enrolment:

Urban
Rural

Proportions of enrolment by school level:

Urban
- Lower-primary
- Higher-primary
- Secondary
- Tertiary

(continued)

Box 6 (continued)

Rural
- Lower-primary
- Higher-primary
- Secondary
- Tertiary

4. Rates of growth of enrolment (national or urban, rural and national)

Rates of growth of students:

Total

Lower-primary
Higher-primary

Secondary
Tertiary

Male*
Female*

B. Enrolment in public and private schools

1. Enrolment aggregates (national or urban, rural and national)

Number of students:

Total

Lower-primary
Higher-primary
Secondary
Tertiary

Growth in the number of students:

Total

Lower-primary
Higher-primary
Secondary
Tertiary

(continued)

Box 6 (continued)

2. Indicators of enrolment structure (national or urban, rural and national)

Proportions of students by school level:

Lower-primary
Higher-primary
Secondary
Tertiary

3. Indicators of enrolment distribution (national; if urban-rural projection is being prepared)

Proportions of total enrolment:

Urban
Rural

Proportions of enrolment, by school level:

Urban
- Lower-primary
- Higher-primary
- Secondary
- Tertiary

Rural
- Lower-primary
- Higher-primary
- Secondary
- Tertiary

4. Rates of growth of enrolment (national or urban, rural and national)

Rates of growth of students:

Total

Lower-primary
Higher-primary
Secondary
Tertiary

* Results by sex can be obtained only in a national projection.

F. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

h = 1,...,H	are special age groups within the school-age period or different school levels of the country's educational system
k = 1,2	are urban and rural locations
s = 1,2	are male and female sexes
t	is the year of the projection period
v = 1,2	are public and private types of schools
x = 5,...,24	are single years of age 5,...,24
x'_h	is the youngest age of the special age group h
x''_h	is the oldest age of the special age group h

(b) List of variables

ER(h,s,k,t+5)	is the enrolment ratio for school level h among persons of sex s who reside in location k at the end of the interval
ER(h,s,t+5)	is the enrolment ratio for school level h among persons of sex s at the end of the interval
GRS	is the average annual exponential rate of growth of the total number of students during the interval
GRS(h)	is the average annual exponential rate of growth of the number of students at school level h during the interval
GRS(h,v)	is the average annual exponential rate of growth of the number of students at school level h in schools of type v during the interval
GRS(s)	is the average annual exponential rate of growth of the number of students of sex s during the interval

GRS(v)	is the average annual exponential rate of growth of the total number of students in schools of type v during the interval
POP(h,s,k,t+5)	is the population of special age group h and sex s who reside in location k at the end of the interval
POP(h,s,t+5)	is the population of special age group h and sex s at the end of the interval
POP(x,s,t+5)	is the population of single year of age x and sex s at the end of the interval
PRSUS(h,t+5)	is the proportion of rural students at school level h who attend urban schools at the end of the interval
PS(h,v,t+5)	is the proportion of students at school level h attending schools of type v at the end of the interval
PSRUR(h,t+5)	is the proportion of students at school level h who attend schools in rural areas at the end of the interval
PSRUR(h,v,t+5)	is the proportion of students in schools of type v at school level h attending schools in rural areas at the end of the interval
PSRUR(t+5)	is the proportion of all students who attend rural schools at the end of the interval
PSRUR(v,t+5)	is the proportion of students in schools of type v attending schools in rural areas at the end of the interval
PSTU(h,t+5)	is the proportion of students at school level h at the end of the interval
PSTU(h,v,t+5)	is the proportion of students at school level h among those attending schools of type v at the end of the interval
PSURB(h,t+5)	is the proportion of students at school level h who attend schools in urban areas at the end of the interval
PSURB(h,v,t+5)	is the proportion of students in schools of type v at school level h attending schools in urban areas at the end of the interval

PSURB(t+5)	is the proportion of all students who attend urban schools at the end of the interval
RSUS(h,t+5)	is the number of rural students at school level h who attend urban schools at the end of the interval
SGR	is the growth in the total number of students during the interval
SGR(h)	is the growth in the number of students at school level h during the interval
SGR(h,v)	is the growth in the number of students at school level h attending schools of type v during the interval
SGR(s)	is the growth in the number of students of sex s during the interval
SGR(v)	is the growth in the total number of students attending schools of type v during the interval
SRS(h,t+5)	is the sex ratio of students at school level h at the end of the interval
SRS(t+5)	is the sex ratio of all students at the end of the interval
STU(h,k,t+5)	is the number of students at school level h attending schools in location k at the end of the interval
STU(h,s,t+5)	is the number of students at school level h of sex s at the end of the interval
STU(h,t+5)	is the number of students at school level h at the end of the interval
STU(h,v,k,t+5)	is the number of students at school level h who attend schools of type v in location k at the end of the interval
STU(h,v,t+5)	is the total number of students at school level h attending schools of type v at the end of the interval
STU(k,t+5)	is the number of students attending schools in location k at the end of the interval

STU(s,t+5)	is the number of students of sex s at the end of the interval
STU(t+5)	is the total number of students in the educational system at the end of the interval
STU(v,k,t+5)	is the number of students attending school of type v in location k at the end of the interval
STU(v,t+5)	is the total number of students in schools of type v at the end of the interval
STU*(h,s,k,t+5)	is the number of students at school level h of sex s who reside in location k at the end of the interval
SURB(v,t+5)	is the proportion of students in schools of type v attending schools in urban areas at the end of the interval

(c) List of special symbols

H	is the number of special age groups within the school-age period or the number of school levels in the country's educational system
ln	is the natural logarithm

2. Equations

1. National level

(a) Overall enrolment

(i) Persons in special age groups

$$POP(h,s,t+5) = \sum_{x=x'_h}^{x''_h} POP(x,s,t+5); \qquad (1)$$

$h = 1,\ldots,H;$

$x = 5,\ldots,24;$

$s = 1,2$

(ii) <u>Enrolment by school level and sex</u>

$$STU(h,s,t+5) = POP(h,s,t+5) \cdot ER(h,s,t+5); \quad (2)$$

$$h = 1,\ldots,H;$$

$$s = 1,2$$

(iii) <u>Other results</u>

a. <u>Enrolment aggregates</u>

i. <u>Total number of students</u>

$$STU(t+5) = \sum_{h=1}^{H} \sum_{s=1}^{2} STU(s,t+5) \quad (3)$$

ii. <u>Number of students by school level</u>

$$STU(h,t+5) = \sum_{s=1}^{2} STU(h,s,t+5); \quad (4)$$

$$h = 1,\ldots,H$$

iii. <u>Number of students by sex</u>

$$STU(s,t+5) = \sum_{h=1}^{H} STU(h,s,t+5); \quad (5)$$

$$s = 1,2$$

iv. <u>Growth in the total number of students</u>

$$SGR = STU(t+5) - STU(t) \quad (6)$$

v. <u>Growth in the number of students by school level</u>

$$SGR(h) = STU(h,t+5) - STU(h,t); \quad (7)$$

$$h = 1,\ldots,H$$

vi. <u>Growth in the number of students by sex</u>

$$SGR(s) = STU(s,t+5) - STU(s,t); \qquad (8)$$

$$s = 1,2$$

b. <u>Indicators of enrolment structure</u>

i. <u>Proportions of students by school level</u>

$$PSTU(h,t+5) = STU(h,t+5) \;/\; STU(t+5); \qquad (9)$$

$$h = 1,...,H$$

ii. <u>Sex ratios of students</u>

$$SRS(t+5) = [\; STU(1,t+5) \;/\; STU(2,t+5) \;] \cdot 100 \qquad (10)$$

$$SRS(h,t+5) = [\; STU(h,1,t+5) \;/\; STU(h,2,t+5) \;] \cdot 100; \qquad (11)$$

$$h = 1,...,H$$

c. <u>Rates of growth of enrolment</u>

i. <u>Rate of growth in the total number of students</u>

$$GRS = [\; (\; \ln(\; STU(t+5) \;/\; STU(t) \;) \;) \;/\; 5 \;] \cdot 100 \qquad (12)$$

ii. <u>Rates of growth in the number of students by school level</u>

$$GRS(h) = [\; (\; \ln(\; STU(h,t+5) \;/\; STU(h,t) \;) \;) \;/\; 5 \;] \cdot 100; \qquad (13)$$

$$h = 1,...,H$$

iii. <u>Rates of growth in the number of students by sex</u>

$$GRS(s) = [\; (\; \ln(\; STU(s,t+5) \;/\; STU(s,t) \;) \;) \;/\; 5 \;] \cdot 100; \qquad (14)$$

$$s = 1,2$$

(b) Enrolment in public and private schools

(i) Enrolment aggregates

a. Number of students by school level

$$STU(h,v,t+5) = STU(h,t+5) \cdot PS(h,v,t+5); \quad (15)$$

$h = 1,...,H;$

$v = 1,2$

b. Total number of students

$$STU(v,t+5) = \sum_{h=1}^{H} STU(h,v,t+5); \quad (16)$$

$v = 1,2$

c. Growth in the total number of students

$$SGR(v) = STU(v,t+5) - STU(v,t); \quad (17)$$

$v = 1,2$

d. Growth in the number of students by school level

$$SGR(h,v) = STU(h,v,t+5) - STU(h,v,t), \quad (18)$$

$h = 1,...,H;$

$v = 1,2$

(ii) Indicators of enrolment structure

a. Proportions of students by school level

$$PSTU(h,v,t+5) = STU(h,v,t+5) / STU(v,t+5); \quad (19)$$

$h = 1,...,H;$

$v = 1,2$

(iii) Rates of growth of enrolment

a. Rates of growth in the total number of students

$$GRS(v) = [(\ln(STU(v,t+5) / STU(v,t))) / 5] \cdot 100; \qquad (20)$$

$$v = 1,2$$

b. Rates of growth in the number of students by school level

$$GRS(h,v) = [(\ln(STU(h,v,t+5) / STU(h,v,t))) / 5] \cdot 100; \qquad (21)$$

$$h = 1,\ldots,H;$$

$$v = 1,2$$

3. Urban-rural level

(a) Overall enrolment

(i) Persons in special age groups

(ii) Enrolment by school level and sex and by location of residence

$$STU^*(h,s,k,t+5) = POP(h,s,k,t+5) \cdot ER(h,s,k,t+5); \qquad (22)$$

$$h = 1,\ldots,H;$$

$$s = 1,2;$$

$$k = 1,2$$

(iii) Enrolment in urban schools among rural students

$$RSUS(h,t+5) = [\sum_{s=1}^{2} STU^*(h,s,2,t+5)] \cdot PRSUS(h,t+5); \qquad (23)$$

$$h = 1,\ldots,H$$

(iv) Other results

a. Enrolment aggregates

i. Number of students by school level

$$STU(h,1,t+5) = \left[\sum_{s=1}^{2} STU^{*}(h,s,2,t+5) \right] + RSUS(h,t+5); \quad h = 1,\ldots,H \qquad (24)$$

$$STU(h,2,t+5) = \left[\sum_{s=1}^{2} STU^{*}(h,s,2,t+5) \right] - RSUS(h,t+5); \quad h = 1,\ldots,H \qquad (25)$$

b. Indicators of enrolment structure

c. Indicators of enrolment distribution

i. Proportions urban and rural of total enrolment

$$PSURB(t+5) = STU(1,t+5) / STU(t+5) \qquad (26)$$

$$PSRUR(t+5) = 1 - PSURB(t+5) \qquad (27)$$

ii. Proportions urban and rural of enrolment by school level

$$PSURB(h,t+5) = STU(h,1,t+5) / STU(h,t+5); \quad h = 1,\ldots,H \qquad (28)$$

$$PSRUR(h,t+5) = 1 - PSURB(h,t+5); \quad h = 1,\ldots,H \qquad (29)$$

d. Rates of growth of enrolment

(b) Enrolment in public and private schools

(i) Enrolment aggregates

(ii) Indicators of enrolment structure

(iii) Indicators of enrolment distribution

a. Proportions urban and rural of total enrolment

PSURB(v,t+5) = STU(v,1,t+5) / STU(v,t+5); (30)

v = 1,2

PSRUR(v,t+5) = 1 - PSURB(v,t+5); (31)

v = 1,2

b. Proportions urban and rural of enrolment by school level

PSURB(h,v,t+5) = STU(h,v,1,t+5) / STU(h,v,t+5); (32)

h = 1,...,H;

v = 1,2

PSRUR(h,v,t+5) = 1 - PSURB(h,v,t+5); (33)

h = 1,...,H;

v = 1,2

Notes

1/ The grade-transition method of educational projection will be described in the second volume of this manual, dealing with techniques relevant to sectoral planning.

2/ For a description of an alternative version of the enrolment ratio method, see UNESCO (1980).

3/ For a description of the Sprague interpolation, see United States Bureau of the Census, The Methods and Materials of Demography, vol. 2, chap. 22 (Washington, D.C.: 1973).

4/ In particular, the level-specific enrolment ratios used in the projection should be defined as ratios of the number of students at various school levels residing in urban and rural areas to the number of persons belonging to special age groups corresponding to those school levels who reside in urban and rural areas.

Annex I

CALCULATING THE NUMBERS OF PERSONS BY SINGLE YEAR OF AGE WITHIN THE SCHOOL-AGE PERIOD

Where the population is projected by five-year age groups and sex over five-year intervals, one could not directly use the projected age and sex structures as an input for projecting school enrolment employing enrolment ratios. Those structures would first need to be converted into single-year age and sex structures of the population by distributing the numbers of persons belonging to five-year age groups among numbers of persons at single years of age. A procedure for doing this, using the Sprague interpolation formula, is presented and illustrated in this annex.

A. Procedure

This procedure utilizes a projection of the numbers of persons classified by five-year age group and sex below age 30 to derive a projection of the numbers of persons distributed by single year of age and sex within the school-age period, 5-24. The numbers of persons by five-year age group and sex are initially used along with the Sprague coefficients to compute preliminary numbers of persons by single years of age and sex from age 2 to age 26. For the period t+5 these preliminary numbers are obtained as follows:

$$\mathrm{POP}^*(x,s,t+5) = \sum_{a'=1}^{6} m(y,a,a') \cdot [\ \mathrm{POP}(a',s,t+5)\ /\ 5\ ]; \qquad (1)$$

$$y = 1,\ldots,5;$$

$$a = 1,\ldots,5;$$

$$s = 1,2,$$

where:

$$x = 5 \cdot (a - 1) + y + 1,$$

and where:

$a = 1,\ldots,5$	are five-year age groups 0-4,...,20-24,
$a' = 1,\ldots,6$	are five-year age groups 0-4,...,25-29,
$x = 2,\ldots,26$	are single years of age 2,...,26,
$s = 1,2$	are male and female sexes,

$y = 1,\dots,5$	are consecutive single years of age within any given five-year age group,
POP*(x,s,t+5)	is the preliminary number of persons of single year of age x and sex s at the end of the interval,
POP(a',s,t+5)	is the number of persons of age group a' and sex s at the end of the interval, and
m(y,a,a')	is the Sprague coefficient that corresponds to single year of age y within five-year age group a and applies to the population of five-year age group a'.

As indicated in equation (1), the preliminary number of persons at any given single year of age is obtained as the sum of the products of the numbers of persons at age groups 0-4 through 25-29, divided by five, and the relevant Sprague coefficients.

Normally, when the preliminary numbers of persons are summed up within the standard five-year age groups, the numbers of persons within those age groups will differ from the original numbers of persons in each five-year age group. Although the differences are usually minor, it is desirable to remove them through an adjustment.

Thus, in order to remove the differences, one may initially calculate the preliminary numbers of persons within standard five-year age groups as follows:

$$POP^*(a,s,t+5) = \sum_{x=x'}^{x'+4} POP^*(x,s,t+5); \qquad (2)$$

$$a = 2,\dots,5;$$

$$s = 1,2,$$

where:

$$x' = 5 \cdot (a - 1),$$

and where:

x'	is the earliest single year of age of a given five-year age group, and
POP*(a,s,t+5)	is the preliminary number of persons of age group a and sex s at the end of the interval.

The preliminary numbers of persons in the five-year age groups between 5-9 and 20-24 can be further used along with the original numbers of persons within those same five-year age groups to compute adjustment factors, as follows:

$$AF(a,s,t+5) = POP(a,s,t+5) \ / \ POP^{*}(a,s,t+5); \qquad (3)$$

$$a = 2,\ldots,5;$$

$$s = 1,2,$$

where:

AF(a,s,t+5) is the adjustment factor applying to the numbers of persons within five-year age group a and sex s at the end of the interval.

The adjustment factors can be further used to derive the numbers of persons at single years of age and sex within the school age period as follows:

$$POP(x,s,t+5) = POP^{*}(x,s,t+5) \cdot AF(a,s,t+5); \qquad (4)$$

$$y = 1,\ldots,5;$$

$$a = 2,\ldots,5;$$

$$s = 1,2,$$

where:

$$x = 5 \cdot (a - 1) + y - 1,$$

and where:

POP(x,s,t+5) is the numbers of persons of single year of age x and sex s at the end of the interval.

As indicated by equation (4), the numbers of persons at single years of age within different five-year age groups are derived by assuming that the same adjustment factor applies to each preliminary number of persons within the age group.

B. Illustrative application of the procedure

This illustrative application will be based on the projected structures of the population, by five-year age group and sex below age 30 for a 20-year projection interval, which are shown in table 27. It will also use the Sprague coefficients displayed in table 28. The example will illustrate how the numbers of persons within the school age period can be calculated for the initial year of projection (year 0). It will also present the calculated numbers at single years of age for the entire 20-year projection period.

To derive the numbers of persons at single years of age, one would first calculate the preliminary number of persons at each relevant single year of age as the sum of the products of the numbers of persons at five-year age groups, divided by 5, and the appropriate Sprague coefficients. The derivation of the preliminary numbers of males for the initial year of the projection (year 0) is illustrated with help of table 29, where the numbers of males in different five-year age groups in the initial year (column 2) are shown along with the preliminary numbers of males at single years of age in the same year (column 4). The latter numbers are derived as illustrated below for a particular single year of age.

Illustrated below is the derivation of the preliminary number of males at age 11, to which the following values of indices a and y in the table of Sprague coefficients (table 28) correspond: a = 2 and y = 5. Thus, the preliminary number of males at age 11, 130.2, which is shown in table 29, is obtained as:

$$\begin{aligned} 130.2 = & - 0.0176 \cdot 784.5/5 \\ & + 0.1584 \cdot 740.0/5 \\ & + 0.9504 \cdot 624.0/5 \\ & - 0.1056 \cdot 486.6/5 \\ & + 0.0144 \cdot 402.2/5 \\ & + 0.0 \cdot 361.5/5, \end{aligned} \qquad (1)$$

where -0.0176, 0.1584, 0.9504, -0.1056, 0.0144 and 0.0 are the relevant Sprague coefficients, which correspond to a = 2 and y = 5 in table 28. 784.5, 740.0, 624.0, 486.6, 402.2 and 361.5 are the numbers of males at five-year age groups 0-4 through 25-29 in year 0 (column 2, table 29). The 5 is the width of the five-year age group.

Once the preliminary numbers at single years of age are calculated, it is necessary to aggregate them in order to obtain preliminary numbers of persons in five-year age groups 5-9 through 20-24. The preliminary numbers of males for the initial year, derived from the preliminary numbers of males at single years of age for the same date, are shown in table 30 (column 2). Among these numbers, for example, that for the age group 5-9, 737.1, is obtained as:

Table 27. Projected population below age 30 by five-year age group and sex: entire country a/

(Thousands)

Age group	Year 0	5	10	15	20
			Male		
0-4	784.5	862.2	957.3	1022.5	1012.7
5-9	740.0	737.0	822.2	923.3	994.4
10-14	624.0	729.9	729.3	815.6	917.5
15-19	486.6	618.5	724.9	725.4	812.0
20-24	402.2	481.5	613.4	720.1	721.5
25-29	361.5	397.1	476.7	608.5	715.5
			Female		
0-4	750.7	828.9	916.3	974.5	961.3
5-9	694.8	704.7	789.6	882.6	945.8
10-14	580.5	685.2	697.4	783.3	877.0
15-19	448.3	574.9	680.2	693.5	779.9
20-24	359.8	442.5	569.3	675.3	689.6
25-29	348.3	354.5	437.5	564.5	670.7

a/ From module 1, table 8.

Table 28. Sorague's coefficients

a	y	a'					
		1	2	3	4	5	6
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	1	1.0000	0.0	0.0	0.0	0.0	0.0
	2	0.6384	0.638	-0.426	0.182	-0.034	0.0
	3	0.3744	0.998	-0.562	0.230	-0.042	0.0
	4	0.1904	1.142	-0.490	0.190	-0.034	0.0
	5	0.0704	1.126	-0.282	0.102	-0.018	0.0
2	1	0.0	1.000	0.0	0.0	0.0	0.0
	2	-0.0336	0.806	0.302	-0.090	0.014	0.0
	3	-0.0416	0.582	0.582	-0.146	0.022	0.0
	4	-0.0336	0.358	0.806	-0.154	0.022	0.0
	5	-0.0176	0.158	0.950	-0.106	0.014	0.0
3	1	0.0	0.0	1.000	0.0	0.0	0.0
	2	0.0128	-0.098	0.934	0.174	-0.026	0.002
	3	0.0144	-0.114	0.726	0.438	-0.074	0.008
	4	0.0080	-0.074	0.438	0.726	-0.114	0.014
	5	0.0016	-0.026	0.174	0.934	-0.098	0.013
4	1	0.0	0.0	0.0	1.000	0.0	0.0
	2	0.0	0.014	-0.106	0.950	0.158	-0.018
	3	0.0	0.022	-0.154	0.806	0.358	-0.034
	4	0.0	0.022	-0.146	0.582	0.582	-0.042
	5	0.0	0.014	-0.090	0.302	0.806	-0.034
5	1	0.0	0.0	0.0	0.0	1.000	0.0
	2	0.0	-0.018	0.102	-0.282	1.126	0.070
	3	0.0	-0.034	0.190	-0.490	1.142	0.190
	4	0.0	-0.042	0.230	-0.562	0.998	0.374
	5	0.0	-0.034	0.182	-0.426	0.638	0.638

$$737.1 = 153.7 + 151.2 + 148.0 + 144.3 + 140.0, \tag{2}$$

where 153.7, 151.2, 148.0, 144.3 and 140.0 are the preliminary numbers at single years of age 5 through 9 (column 4, table 29).

The resultant preliminary numbers or persons at five-year age groups would normally differ slightly from the original numbers at those age groups. This is the case of the preliminary numbers at five-year age groups 5-9 through 20-24, which differ from the original numbers at those same age groups (columns 2 and 3, table 30). These minor inconsistencies, can be removed by dividing the original numbers by the preliminary numbers at different five-year age groups to obtain adjustment factors (column 4). The adjustment factor for age group 5-9, 1.004, is calculated as follows:

$$1.004 = 740.0 / 737.1, \tag{3}$$

where 740.0 and 737.1 are the original and the preliminary numbers of males aged 5-9 in the initial year.

These factors are used as illustrated in table 31. The number of persons at each single year of age within every five-year age group (column 2), which is obtained by the Sprague interpolation, is multiplied by the adjustment factor computed for that age group (column 3). The result is the adjusted number of persons at the single year of age in question (column 4). The adjusted number of persons at age 5, 154.3, is obtained as:

$$154.3 = (153.7)\ (1.004), \tag{4}$$

where 153.7 is the preliminary number of males aged 5 and 1.004 is the adjustment factor for the age group 5-9. Other numbers of males at single years of age are calculated in the same way.

The calculations which were illustrated for the initial year of the projection need to be performed for both sexes for the entire projection period to obtain the structures of the school-age population by single year of age and sex for various dates over the projection period. These structures, which are derived from the numbers of persons shown in table 27, are presented in table 32.

C. Notation and equations

1. Indices and variables

(a) List of indices

$a = 1,\ldots,5$ are five-year age groups 0-4,...,20-24

Table 29. Computing the numbers of males at single years of age within the school-age period by Sprague interpolation: entire country, year 0

(Thousands)

Age group (1)	Males a/ (2)	Single year of age (3)	Males after Sprague interpolation b/ (4)
0-4	784.5	2	156.9
5-9	740.0	3	156.6
10-14	624.0	4	155.5
15-19	486.6	5	153.7
20-24	402.2	6	151.2
25-29	361.5	7	148.0
		8	144.3
		9	140.0
		10	135.3
		11	130.2
		12	124.8
		13	119.2
		14	113.4
		15	107.7
		16	102.2
		17	97.3
		18	92.9
		19	89.0
		20	85.7
		21	82.8
		22	80.4
		23	78.5
		24	76.8
		25	75.3
		26	73.9

a/ From table 27.
b/ Derivation illustrated in text.

Table 30. Computing adjustment factors, results for males: entire country, year 0

Age group	Number of males after Sprague interpolation a/	Number of males b/	Adjustment factors c/
	(thousands)		
(1)	(2)	(3)	(4)
5-9	737.1	740.0	1.003
10-14	622.9	624.0	1.001
15-19	489.2	486.6	0.994
20-24	404.2	402.2	0.995

a/ Obtained by aggregation within 5-year age groups using results in table 29, col. 4.
b/ From table 29, col. 2.
c/ (Col. 3)/(Col. 2).

Table 31. Deriving adjusted numbers of males at single year of age within the school-age period: entire country, year 0

(Thousands)

Age	Number of males after Sprague interpolation a/	Adjustment factors b/	Adjusted number of males c/
(1)	(2)	(3)	(4)
5	153.7	1.003	154.3
6	151.2		151.8
7	148.0		148.6
8	144.3		144.8
9	140.0		140.5
10	135.3	1.001	135.5
11	130.2		130.4
12	124.8		125.0
13	119.2		119.4
14	113.4		113.6
15	107.7	0.994	107.1
16	102.2		101.7
17	97.3		96.8
18	92.9		92.4
19	89.0		88.6
20	85.7	0.995	85.2
21	82.8		82.4
22	80.4		80.0
23	78.5		78.1
24	76.8		76.4

a/ From table 29, col. 4.
b/ From table 30, col. 4.
c/ (Col. 2) . (Col. 3).

Table 32. School-age population by single year of age and sex: entire country

(Thousands)

Age	Year 0	5	10	15	20
			Male		
5	154.3	149.3	175.9	192.9	202.5
6	151.8	147.3	169.8	188.9	201.2
7	148.6	146.6	164.0	184.8	199.4
8	144.8	146.7	158.6	180.5	197.1
9	140.5	147.1	153.9	176.2	194.3
10	135.5	149.1	149.5	171.8	191.0
11	130.4	148.5	146.7	167.4	187.5
12	125.0	147.1	144.8	163.1	183.7
13	119.4	144.6	144.0	158.8	179.7
14	113.6	140.6	144.3	154.4	175.6
15	107.1	134.7	147.4	149.4	171.0
16	101.7	129.3	147.5	146.2	166.6
17	96.8	123.8	146.2	144.1	162.3
18	92.4	118.2	143.7	143.1	158.1
19	88.6	112.6	140.1	142.7	154.0
20	85.2	106.4	134.8	144.4	149.7
21	82.4	101.0	129.2	144.7	146.4
22	80.0	95.9	123.0	144.7	143.6
23	78.1	91.2	116.5	144.1	141.5
24	76.4	87.0	109.9	142.2	140.2
			Female		
5	145.6	143.7	168.7	184.0	192.3
6	142.8	141.4	163.0	180.4	191.2
7	139.4	140.3	157.5	176.6	189.6
8	135.6	139.8	152.4	172.8	187.6
9	131.3	139.6	148.0	168.8	185.1
10	126.4	140.8	143.7	164.8	182.1
11	121.5	139.8	140.7	160.8	179.0
12	116.3	138.0	138.6	156.7	175.6
13	110.9	135.3	137.3	152.6	172.0
14	105.4	131.3	137.1	148.4	168.2
15	99.4	125.6	139.2	143.4	164.0
16	94.2	120.3	138.8	140.2	160.0
17	89.3	115.1	137.1	137.9	156.0
18	84.8	109.7	134.4	136.4	151.9
19	80.7	104.2	130.7	135.6	148.0
20	76.8	98.4	125.5	136.5	143.7
21	73.8	93.1	120.1	136.2	140.3
22	71.3	88.1	114.2	135.7	137.4
23	69.5	83.5	107.9	134.6	135.0
24	68.4	79.3	101.7	132.4	133.2

a' = 1,...,6	are five-year age groups 0-4,...,25-29
s = 1,2	are male and female sexes
x = 2,...,26	are single years of age 2,...,26
x'	is the earliest single year of age of a given five-year age group
y = 1,...,5	are consecutive single years of age within any given five-year age group

(b) List of variables

AF(a,s,t+5)	is the adjustment factor applying to the numbers of persons within five-year age group a and sex s at the end of the interval
m(y,a,a')	is the Sprague coefficient that corresponds to single year of age y within five-year age group a and applies to the population of five-year age group a'
POP(a,s,t+5)	is the number of persons of age group a and sex s at the end of the interval
POP(x,s,t+5)	is the number of persons of single year of age x and sex s at the end of the interval
POP*(a,s,t+5)	is the preliminary number of persons of age group a and sex s at the end of the interval
POP*(x,s,t+5)	is the preliminary number of persons of single year of age x and sex s at the end of the interval

2. Equations

$$POP^*(x,s,t+5) = \sum_{a'=1}^{6} m(y,a,a') \cdot [\ POP(a',s,t+5) / 5\ ]; \quad (1)$$

$y = 1,\ldots,5;$

$a = 1,\ldots,5;$

$s = 1,2$

where:

$$x = 5 \sum (a - 1) + y + 1$$

$$POP^*(a,s,t+5) = \sum_{x=x'}^{x'+4} POP^*(x,s,t+5); \quad a = 2,\ldots,5; \quad s = 1,2 \tag{2}$$

$$AF(a,s,t+5) = POP(a,s,t+5) \,/\, POP^*(a,s,t+5); \quad a = 2,\ldots,5; \quad s = 1,2 \tag{3}$$

$$POP(x,s,t+5) = POP^*(x,s,t+5) \cdot AF(a,s,t+5); \quad y = 1,\ldots,5; \quad a = 2,\ldots,5; \quad s = 1,2 \tag{4}$$

where:

$$x = 5 \cdot (a - 1) + y - 1$$

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V. MAKING LABOUR FORCE PROJECTIONS USING LABOUR FORCE PARTICIPATION RATES

A. Introduction

Labour force projections can be extremely useful in preparing comprehensive plans to accommodate future population change, particularly changes in the working-age population (box 7) and labour force. When used along with employment projections labour force projections can be used to analyse potential future imbalances in the labour market. Specifically, when projected levels of the labour force are compared with the projected levels of employment, it is possible to identify future surpluses or shortages of labour. The analysis can indicate whether projections of population and value added, factors influencing the supply of labour and demand for labour, are realistic and whether the policies underlying those projections will lead to a balance between the supply and demand sides of the labour market.

This chapter describes a method for preparing labour force projections that applies assumed labour force participation rates to the projected population at different dates. This method, which has been used in many planning exercises, yields the labour force, classified by age and sex along with various indicators of labour force size, structure and change. The method is suitable for making national or urban-rural projections.1/

The method would be easy to apply in most developing countries. It does not require extensive data nor does it entail laborious calculations.

When using the method to make an urban-rural projection, however, caution should be exercised if the population projection that underlies the labour force projection is based on a de jure population count. In that case, the urban and rural labour forces would correspond to the labour supplies provided to the urban and rural labour markets only if for the large majority of the labour force, the area of residence was the same as the area of work. Where a sizeable proportion of the urban labour force consisted of commuters or seasonal migrants from rural areas, it would be misleading to treat the urban and rural labour forces as equivalent to the labour supplies provided to these labour markets.

This chapter will first describe the procedure for projecting the labour force. It will then describe inputs required and discuss how assumptions on labour force participation rates can be formulated. Lastly, the chapter will present two illustrative labour force projections, one for the entire country and the other for urban and rural areas.

Box 7

Glossary

<u>Demand for labour</u>
The quantity of labour that users of labour services desire to purchase at prevailing wages and salaries.

<u>Labour force participation rate</u>
The number of persons in the labour force of a given age, sex and/or level of education, divided by the corresponding total number of persons with the same characteristics.

<u>Labour market</u>
The market in which labour services are bought and sold through a process that determines the number of persons employed as well as wages and salaries.

<u>Sex ratio of labour force</u>
The number of males in the labour force, divided by the corresponding number of females and conventionally multiplied by a hundred.

<u>Supply of labour</u>
The quantity of labour that owners of labour services desire to sell at prevailing wages and salaries.

<u>Value added</u>
For a firm or farm, the difference between its total revenue and the cost of raw materials, services and components used in production, over a specified time period; for the economy as a whole or any of its industries, the aggregate of value added of different firms or farms of which the economy or industry is composed.

<u>Working-age population</u>
The population in the working ages, conventionally defined as 15 to 59 years or 15 to 64 years.

B. The technique

1. Overview

This overview will first indicate the types of inputs required by the labour force participation rate method and list the types of outputs that it can generate. Also, the overview will outline the computational steps required to prepare a labour force projection using this method.

(a) Inputs

The inputs required to project labour forces will include:

(i) Projected age and sex structure of the population at age 10 and over;
(ii) Assumptions on labour force participation rates.

For a national projection the inputs would be for the entire country. For a urban-rural projection, they would be for urban and rural areas, respectively. The inputs are listed in box 8.

Box 8

Inputs for making labour force projections using labour force participation rates

1. Age and sex structures of the population at age 10 and over (national or urban and rural)

2. Assumptions on labour force participation rates (national or urban and rural)

Labour force participation rates, by age and sex

The technique will be described as a procedure for making quinquennial labour force projections, using inputs for dates five years apart, starting with the initial year of the plan.

(b) Outputs

The types of output that the labour force participation rate method can generate would partly depend on the type of projection being made. In the case of a national projection, the method would yield:

(i) Structure of the labour force by age and sex;
(ii) Various labour force aggregates, such as the total labour force and the growth in total labour force;
(iii) Indicators of the structure of the labour force, such as proportions of labour force in broad age groups (10-24, 25-64 and 65+);
(iv) Rates of change in the total labour force and its components.

If the technique is used to make an urban-rural projection, the results would include all those listed under (i) through (iv), which would be for urban and rural areas as well as for the entire country. In addition, the results would include indicators of the urban-rural distribution of the labour force. The types of outputs that the method can generate as part of the national or urban-rural projection are shown in box 9.

Box 9

Types of outputs obtained by projecting labour force using labour force participation rates

1. Structure of the labour force by age and sex (national or urban, rural and national)

2. Labour force aggregates (national or urban, rural and national)

   Total labour force and labour force by broad age group and sex

   Growth in total labour force and in labour force by broad age group and sex

3. Indicators of structure of the labour force (national or urban, rural and national)

   Proportions of the labour force at different broad age groups, median age of the labour force and sex ratio of the labour force

4. Indicators of the urban-rural distribution of the labour force (national only; if urban and rural labour force are being projected)

   Proportions of the labour force in different residential locations

5. Rates of growth of the labour force (national or urban, rural and national)

   Rates of growth of the total labour force and of the labour force disaggregated by broad age and sex groups

These results would be for dates five years apart or the intervening projection intervals.

(c) Computational steps

The first step of the procedure is to calculate the structure of the labour force disaggregated by age and sex. This structure is obtained from the age and sex structure of the population at age 10 and over and age- and sex-specific labour force participation rates. The procedure also entails deriving for each projection date the total labour force along with other date-specific indicators. These results can then be used to calculate the absolute growth in the labour force and the rates of growth for the intervening intervals.

2. National level

This section will initially elaborate the steps to compute the structure of the labour force. Then it will explain the steps needed to derive other results for a given projection date or interval at the national level. A summary of those steps is presented in box 10. The steps used to derive urban and rural labour force structures together with the related results will be described in a later section.

Box 10

Computational steps needed to project the labour force at the national level

The steps used to project labour force at the national level over a five-year projection interval are:

(1) Compute the structure of the labour force by age and sex by applying age-specific and sex-specific labour force participation rates to the number of persons in different age and sex groups.

(2) Derive from the structure of the labour force various labour force aggregates, such as the total labour force and the numbers of persons in the labour force in broad age groups. Also, calculate other labour force aggregates, such as the growth in total labour force and the increase in labour force in different broad age groups over the projection interval.

(3) Derive indicators of the structure of the labour force, such as proportions of labour force in broad age groups and the sex ratio of the labour force.

(4) Compute the rates of growth of the total labour force and of the labour force by different broad age group and sex over the projection interval.

(a) Labour force structure

The number of persons in the labour force, classified by age and sex, at a given date, such as the end of a five-year projection interval (t to t+5) can be obtained as follows:

$$LF(a,s,t+5) = POP(a,s,t+5) \cdot LFPR(a,s,t+5); \quad a = 3,\ldots,16; \quad s = 1,2, \tag{1}$$

where:

a = 3,...,16	are five-year age groups 10-14,...,75+,
s = 1,2	are male and female sexes,
t	is the year of the projection period,
LF(a,s,t+5)	is the number of persons of age group a and sex s in the labour force at the end of the interval,
POP(a,s,t+5)	is the population of age group a and sex s at the end of the interval, and
LFPR(a,s,t+5)	is the labour force participation rate among persons of age group a and sex s at the end of the interval.

As indicated by equation (1), the number of persons of a given age group and sex who are in the labour force is obtained as a product of the population of that age group and sex and the relevant labour force participation rate.

(b) Other results

After the structure of the labour force by age and sex is derived, it is possible to make projections for a variety of useful labour force indicators, which include different labour force aggregates, indicators of the structure of the labour force as well as rates of change in the labour force.

(i) Labour force aggregates

A key aggregate that can be projected from the labour force structure is the size of the total labour force. Also, it is possible to obtain from this structure the number of persons in the labour force who belong to different broad age groups along with the number of males and females in the labour force. Once these numbers are obtained for different dates five years apart, it is possible to calculate the increases in those numbers over the intervening five-year projection intervals.

a. Total labour force

The total labour force can be obtained by aggregating the number of persons in the labour force across the range of age groups and sexes. For the end of a projection interval (t to t+5) this number can be calculated as follows:

$$LF(t+5) = \sum_{a=3}^{16} \sum_{s=1}^{2} LF(a,s,t+5), \tag{2}$$

where:

LF(t+5) is the total labour force at the end of the interval.

b. Labour force in broad age groups

The age span 10 and over may be subdivided into broad age groups, e.g. 10-24, 25-64 and 65 and over. The first group might include younger members of the labour force, persons who are still of school-age, many of whom are just entering the labour force. The second group consists of labour force members who are in their prime working years, 25-64, and the third group, aged 65 and over, would include elderly members of the labour force.

i. Young-age labour force

The number of persons in the youngest component of the labour force can be obtained by aggregating the number of male and female members of the labour force who belong to age groups 10-14 through 20-24:

$$LFY(t+5) = \sum_{a=3}^{5} \sum_{s=1}^{2} LF(a,s,t+5), \tag{3}$$

where:

LFY(t+5) is the number of persons in the young-age labour force at the end of the interval.

ii. Prime-working-age labour force

The number of persons in the prime-working-age labour force can be derived by summing the number of males and female in the labour force at age groups 25-29 through 60-64:

$$LFP(t+5) = \sum_{a=6}^{13} \sum_{s=1}^{2} LF(a,s,t+5), \qquad (4)$$

where:

LFP(t+5) is the number of persons in the prime-working-age labour force at the end of the interval.

iii. <u>Old-age labour force</u>

The number of persons in the old-age labour force can be derived by adding up the number of males and females in the labour force who are 65 or older:

$$LFO(t+5) = \sum_{a=14}^{16} \sum_{s=1}^{2} LF(a,s,t+5), \qquad (5)$$

where:

LFO(t+5) is the number of persons in the old-age labour force at the end of the interval.

c. <u>Labour force disaggregated by sex</u>

In addition to the labour force within different broad age groups, it is also possible to compute the number of males and females in the total labour force and each of its age components. For the sake of brevity, this section will discuss only the disaggregation of the total labour force.

i. <u>Male labour force</u>

The number of males (s=1) in the labour force can be calculated by summing up males of different age groups who are in the labour force:

$$LF(s,t+5) = \sum_{a=3}^{16} LF(a,s,t+5); \qquad (6)$$

$$s = 1,2,$$

where:

LF(s,t+5) is the number of persons of sex s in the labour force at the end of the interval.

ii. Female labour force

The number of females (s=2) in the labour force can be obtained as the difference between the total labour force and the number of males in the labour force:

$$LF(2,t+5) = LF(t+5) - LF(1,t+5). \quad (7)$$

d. Growth in total labour force

The growth in the total labour force for the projection interval (t to t+5) equals the difference between the total labour force at the end and the beginning of the five-year interval:

$$LFGR = LF(t+5) - LF(t), \quad (8)$$

where:

LFGR is the growth in the total labour force over the interval.

e. Growth in young-age, prime-working-age and old-age labour force

Increases in labour force within different broad age groups over the projection interval are respectively obtained by subtracting the size of the labour force component at the beginning of the interval from the size of the corresponding component at the end of the interval.

Growth in the young-age component of the labour force is:

$$LFYGR = LFY(t+5) - LFY(t), \quad (9)$$

Growth in the prime-working-age component of the labour force is calculated as:

$$LFPGR = LFP(t+5) - LFP(t), \quad (10)$$

Growth in the old-age component of the labour force is:

$$LFOGR = LFO(t+5) - LFO(t), \quad (11)$$

where:

LFYGR	is the growth in the young-age component of the labour force over the interval,
LFPGR	is the growth in the prime-working-age component of the labour force over the interval, and
LFOGR	is the growth in the old-age component of the labour force over the interval.

f. Growth in male and female labour force

The increase in the number of males (or females) in the labour force can be obtained as:

$$LFGR(s) = LF(s,t+5) - LF(s,t); \qquad (12)$$

$$s = 1,2,$$

where:

LFGR(s)	is the growth in the component of the labour force consisting of persons of sex s over the interval.

(ii) Indicators of the structure of the labour force

Once the various labour force aggregates are obtained, it is possible to derive proportions of labour force found in different broad age groups. It is also possible to calculate the median age of labour force and the sex ratio of the labour force.

a. Proportions by broad age groups

The proportions of labour force found in the broad age groups identified above can be obtained as follows:

The proportion of young-age persons in the labour force is equal to the young labour force divided by the total labour force:

$$PLFY(t+5) = LFY(t+5) \;/\; LF(t+5), \qquad (13)$$

The proportion of prime-working-age persons in the labour force is equal to the prime working age labour force divided by the total labour force:

$$PLFP(t+5) = LFP(t+5) / LF(t+5), \quad (14)$$

The proportion of old-age persons in the labour force is equal to the old age labour force divided by the total labour force:

$$PLFO(t+5) = LFO(t+5) / LF(t+5), \quad (15)$$

where:

PLFY(t+5) is the proportion of young persons in the labour force at the end of the interval,

PLFP(t+5) is the proportion of prime-working-age persons in the labour force at the end of the interval, and

PLFO(t+5) is the proportion of old-age persons in the labour force at the end of the interval.

b. Median age of labour force

The median age of the labour force can be derived using the standard formula for computing the median age from grouped data.2/ If applied to the age structure of the labour force, this formula is:

$$MALF(t+5) = (a' - 1) \cdot 5 + \left[\left(LF(t+5)/2 - \sum_{a=3}^{a'-1} \sum_{s=1}^{2} LF(a,s,t+5) \right) / \sum_{s=1}^{2} LF(a',s,t+5) \right] \cdot 5, \quad (16)$$

where:

MALF(t+5) is the median age of the labour force at the end of the interval, and

a' is the number that stands for the five-year age group containing the member of the labour force who is older than one half of the total labour force and younger than the other half.

In equation (16), the first term on the right-hand side, (a'-1).5, represents the lower limit of the five-year age group containing the middle member of the population. The term, $\sum_{a=3,a'-1}\sum_{s=1,2}LF(a,s,t+5)$, stands for the number of persons in all five-year age groups preceding the age group containing the middle member, and the term, $\sum_{s=1,2}LF(a',s,t+5)$ is the number of persons in that latter age group.

c. Sex ratio of labour force

The sex ratio of the labour force is equal to the number of males in the labour force, divided by the number of females s and multiplied by a hundred:

$$SRLF(t+5) = [(LF(1,t+5)) / (LF(2,t+5))] \cdot 100, \qquad (17)$$

where:

SRLF(t+5) is the sex ratio of the labour force at the end of the five-year interval.

(iii) Rates of growth of labour force

As part of a labour force projection, it is possible to compute average annual rates of growth in the total labour force and in the labour force of different broad age groups or sexes.

a. Rate of growth in the total labour force

The average annual growth rate in the total labour force for a given projection interval can be computed from the size of the total labour force at the beginning and at the end of the five-year interval, using the formula for calculating an exponential growth rate:

$$GRLF = [(\ln (LF(t+5) / LF(t))) / 5] \cdot 100, \qquad (18)$$

where:

GRLF is the average annual exponential growth rate of the total labour force for the interval, and

ln is the natural logarithm.

b. Rates of growth in young-age, prime-working-age and old-age labour force

The average annual rates of growth of the labour force disaggregated into broad age groups can be obtained as follows:

The growth rate of the young-age component of the labour force is calculated as:

$$GRLFY = [(\ln (LFY(t+5) / LFY(t))) / 5] \cdot 100, \quad (19)$$

The growth rate of the prime-working-age component of the labour force is calculated as:

$$GRLFP = [(\ln (LFP(t+5) / LFP(t))) / 5] \cdot 100, \quad (20)$$

The growth rate of the old-age component of the labour force is calculated as:

$$GRLFO = [(\ln (LFO(t+5) / LFO(t))) / 5] \cdot 100, \quad (21)$$

where:

GRLFY	is the average annual exponential growth rate of the young-age component of the labour force for the interval,
GRLFP	is the average annual exponential growth rate of the prime-working-age component of the labour force for the interval, and
GRLFO	is the average annual exponential growth rate of the old-age component of the labour force for the interval.

c. Rates of growth in male and female components of the labour force

The growth rates of the labour force of either sex can be obtained as:

$$GRLF(s) = [(\ln (LF(s,t+5) / LF(s,t))) / 5] \cdot 100; \quad (22)$$

$$s = 1,2,$$

where:

GRLF(s) is the average annual exponential growth rate of the number of persons in the labour force of sex s over the interval.

3. Urban-rural level

This section will discuss a procedure that can be used to make an urban-rural projection of the labour force which is similar to that employed to make a national projection. The procedure consists of steps used to project the structures of labour force by age and sex along with those needed to derive a variety of other results.

(a) Labour force structures

Urban and rural structures of labour force for the end of a given projection interval (t to t+5) can be calculated using an urban-rural equivalent of the step described by equation (1):

$$LF(a,s,k,t+5) = POP(a,s,k,t+5) \cdot LFPR(a,s,k,t+5); \qquad (23)$$

$$a = 3,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2,$$

where:

k = 1,2 are urban and rural locations,

LF(a,s,k,t+5) is the number of persons of age group a and sex s in location k in the labour force at the end of the interval,

POP(a,s,k,t+5) is the population of age group a and sex s in location k at the end of the interval, and

LFPR(a,s,k,t+5) is the labour force participation rate among persons of age group a and sex s in location k at the end of the interval.

(b) Other results

The indicators discussed in connection with the national projection can also be computed as part of an urban-rural projection. Those indicators are, however, calculated for urban and rural areas and for the entire country, using steps analogous to those indicated by equations (2) through (22). In addition, indicators of the distribution of labour force by residential location--proportions urban and rural--can be calculated.

(i) Proportions urban and rural

The proportion of labour force that is urban (k=1) at the end of a projection interval is computed by dividing the total labour force in urban areas by the total national labour force:

$$PLFURB(t+5) = LF(1,t+5) / LF(t+5), \tag{24}$$

where:

PLFURB(t+5) is the proportion of the total labour force that is urban at the end of the interval, and

LF(k,t+5) is the labour force in location k at the end of the interval.

The proportion of labour force that is rural (k=2) can be found as a complement of the proportion urban:

$$PLFRUR(t+5) = 1 - PLFURB(t+5), \tag{25}$$

where:

PLFRUR(t+5) is the proportion of the total labour force that is rural at the end of the interval.

This completes the description of the technique for making labour force projections.

C. The inputs

This section will discuss the inputs required by the labour force participation rate method. Specifically, it will list those inputs and then describe how they can be prepared.

1. Types of inputs required

The following inputs are required to apply the labour force participation rate method:

(i) Projected age and sex structure of the population;
(ii) Assumptions on labour force participation rates, by age and sex.

Depending on whether one wishes to make a national projection or a projection for urban and rural areas, those inputs will be required for the nation as a whole or for urban and rural areas.

2. Preparation of the inputs

The projected population structures can be prepared by making a population projection with the cohort component method (see module one, chapter II). Assumptions on labour force participation rates can be prepared as discussed below, relying among other things, on the observations of those rates.

(a) Observed labour force participation rates

In order to prepare assumptions on labour force participation rates, observations on these rates for a recent date or a few such dates are needed. These observations are needed in order to formulate assumptions for the initial year of the projection, which would normally be derived by extrapolating recent empirical rates. The extrapolation will reflect the planner's judgement regarding changes in the rates over the time interval prior to the initial year of the projection.

Where observations on labour force participation rates are not available, they can be derived from information on labour force and population, classified by age and sex. The data may come from population censuses, demographic surveys and/or other specialized surveys, such as labour force or employment surveys. Where data from two or more statistical sources are to be used, it is necessary to ensure that labour force information from different sources is based on the same definition of the labour force. Depending on the type of projection sought, the requisite data should be available at the national or urban-rural level.

(b) Assumptions on future labour force participation rates

To formulate assumptions on labour force participation rates for dates beyond the initial year of the projection, it is normally necessary to consider socio-economic and demographic changes expected to occur during the plan horizon and to estimate their effects on the participation rates. In particular, as part of formulating assumptions on labour force participation rates for young-age persons of either sex, special attention needs to be paid to future trends in schooling. As school attendance is generally incompatible with participation in the labour force, future trends in proportions of children and adolescents enrolled in schools are likely to have a strong effect on the incidence of labour force participation among these groups. Assumptions on labour force participation rates at the upper end of the working-age span must reflect likely future changes in old-age security arrangements, both at the family and societal levels, since those arrangements are likely to have an important influence on withdrawal from the labour force at advanced years of age.

Assumptions concerning male and female labour force participation rates in the prime working ages are, typically, formulated differently from each other. In many countries male rates are assumed to remain fixed over time or are expected to change only slightly, since those rates are generally close to unity and show only minor variations over time. On the other hand, rates for females in the prime working ages are changing rapidly in many societies and so the assumptions must be based on a consideration of likely developments in areas such as childbearing and childrearing practices, female education, employment opportunities, household income and urbanization.

Developments in some areas will tend to raise and in others to reduce female participation rates. For example, if fertility is expected to decline over the plan horizon, female labour force participation may well increase. Similarly, where female schooling is expanding assumptions on the prime working-age female participation rates are likely to increase over time.

These rates may also be assumed to increase if employment is expanding rapidly, since improved employment opportunities are likely to draw an increasing number of women into the work force. On the other hand, an increase in household income can have a depressing effect on female participation rates, especially if the major source of the increase is the rising earnings of males. Under these conditions, the incentives for women to enter and/or remain in the labour market and contribute to household income may decline.

These developments and their effects on female labour force participation may not be the same at the national and at the urban-rural levels. Therefore, where the assumptions on female participation rates are formulated for a national projection, the effects of the expected trends in urbanization should also be taken into account. This is necessary wherever there are sizeable urban-rural differentials in female labour force participation rates. Thus, if participation in the labour force is more prevalent among rural women than urban women, and if rapid urbanization is likely to occur over the plan horizon as a result of the rapid employment expansion in the urban-based sectors of the economy, urbanization will tend to reduce the national female participation rates.

D. Illustrative examples of projections

This section will present examples to illustrate the use of the labour force participation rates to prepare a national and an urban-rural labour force projection. These examples will indicate how the relevant calculations are made by focusing on the projection interval 0-5. In addition, they will provide complete projection results for the 20-year period.

1. National projection

The calculations presented in this example will be based on the inputs contained in tables 33 and 34, which respectively show population structures and assumptions on future labour force participation rates. The inputs are specified for dates five years apart, starting with the initial year of the plan, which is denoted as year 0. The age and sex structures are those at age 10 and over, and the labour force participation rates are for various age and sex groups over the same age range.

(a) Labour force structure

For any given date, the number of persons in the civilian labour force in various age and sex groups are calculated as the products of the number of persons in those groups and the corresponding labour force participation rates. These calculations are illustrated in table 35 for the end of the projection interval 0-5. The number of persons of either sex in the civilian labour force, classified by age (column 4), are obtained as the number of persons of that sex classified by age (column 2), multiplied by the corresponding labour force participation rates (column 3).3/

For example, the number of males aged 10-14 in the civilian labour force at the end of the interval 0-5, 136.5, is obtained as:

$$136.5 = (729.9)\ (0.187), \qquad (1)$$

where 729.9 is the number of males aged 10-14 and 0.187 is the male labour force participation rate for age group 10-14 at that date.

Performing the calculations illustrated for the end of the interval 0-5 for each five-year interval of the entire projection period produces the labour force structures for the entire period. The labour force structures for the 20-year projection interval are shown in table 36.

(b) Other results

Other results that can be obtained as part of a labour force projection at the national level include various labour force aggregates, indicators of the structure of the labour force and rates of change of the labour force.

Table 33. Population structures at age 10 and over: entire country

(Thousands)

Age group	Year 0	5	10	15	20
			Male		
10-14	624.0	729.9	729.3	815.6	917.5
15-19	486.6	618.5	724.9	725.4	812.0
20-24	402.2	481.5	613.4	720.1	721.5
25-29	361.5	397.1	476.7	608.5	715.5
30-34	358.7	355.9	392.2	472.0	603.6
35-39	312.4	351.5	350.2	387.1	466.9
40-44	223.6	303.6	343.4	343.5	381.0
45-49	152.1	214.5	293.2	333.5	335.1
50-54	192.5	142.8	203.2	279.9	320.3
55-59	157.0	175.3	131.5	189.0	262.4
60-64	128.1	136.3	154.4	117.4	170.5
65-69	90.1	103.6	112.4	129.5	99.9
70-74	44.2	65.9	77.7	86.2	101.4
75+	38.6	45.6	63.0	81.4	99.1
			Female		
10-14	580.5	685.2	697.4	783.3	877.0
15-19	448.3	574.9	680.2	693.5	779.9
20-24	359.8	442.5	569.3	675.3	689.6
25-29	348.3	354.5	437.5	564.5	670.7
30-34	352.2	342.4	349.9	433.2	560.0
35-39	300.9	345.1	337.0	345.5	428.7
40-44	206.7	293.5	338.2	331.6	340.8
45-49	158.5	200.2	285.7	330.7	325.2
50-54	174.6	151.2	192.3	276.1	320.9
55-59	164.0	162.2	141.9	181.9	262.8
60-64	126.1	145.9	146.3	129.5	167.6
65-69	105.6	105.1	124.0	126.4	113.4
70-74	63.5	80.0	81.7	98.5	102.3
75+	69.4	72.9	86.4	97.5	116.1

Table 34. Labour force particapation rates: entire country

Age group	Year 0	5	10	15	20
			Male		
10–14	0.239	0.186	0.160	0.129	0.099
15–19	0.492	0.447	0.386	0.344	0.295
20–24	0.807	0.819	0.825	0.823	0.827
25–29	0.929	0.923	0.928	0.930	0.935
30–34	0.941	0.939	0.939	0.942	0.947
35–39	0.933	0.933	0.935	0.937	0.944
40–44	0.926	0.926	0.929	0.934	0.940
45–49	0.912	0.915	0.919	0.922	0.928
50–54	0.887	0.889	0.894	0.899	0.905
55–59	0.837	0.834	0.835	0.835	0.838
60–64	0.738	0.729	0.722	0.723	0.731
65–69	0.395	0.385	0.382	0.381	0.383
70–74	0.099	0.102	0.098	0.103	0.103
75+	0.049	0.049	0.043	0.043	0.037
			Female		
10–14	0.153	0.132	0.117	0.098	0.078
15–19	0.344	0.334	0.319	0.313	0.298
20–24	0.348	0.357	0.367	0.378	0.391
25–29	0.357	0.375	0.392	0.409	0.427
30–34	0.303	0.316	0.334	0.352	0.369
35–39	0.247	0.257	0.267	0.283	0.294
40–44	0.196	0.200	0.204	0.211	0.220
45–49	0.145	0.149	0.154	0.158	0.166
50–54	0.097	0.101	0.105	0.110	0.116
55–59	0.077	0.080	0.081	0.082	0.083
60–64	0.068	0.071	0.071	0.072	0.072
65–69	0.065	0.067	0.062	0.062	0.063
70–74	0.062	0.058	0.058	0.052	0.052
75+	0.052	0.054	0.047	0.047	0.042

Table 35. Calculating labour force structure, by age and sex: entire country, year 5

Age group	Population at age 10 and over a/ (thousands)	Labour force participation rates b/	Labour force c/ (thousands)
(1)	(2)	(3)	(4)
		Male	
10-14	729.9	0.187	136.5
15-19	618.5	0.448	277.1
20-24	481.5	0.820	394.8
25-29	397.1	0.924	366.9
30-34	355.9	0.940	334.5
35-39	351.5	0.934	328.3
40-44	303.6	0.927	281.4
45-49	214.5	0.915	196.3
50-54	142.8	0.890	127.1
55-59	175.3	0.834	146.2
60-64	136.3	0.729	99.4
65-69	103.6	0.386	40.0
70-74	65.9	0.102	6.7
75+	45.6	0.049	2.2
		Female	
10-14	685.2	0.132	90.4
15-19	574.9	0.335	192.6
20-24	442.5	0.358	158.4
25-29	354.5	0.375	132.9
30-34	342.4	0.317	108.5
35-39	345.1	0.257	88.7
40-44	293.5	0.200	58.7
45-49	200.2	0.149	29.8
50-54	151.2	0.101	15.3
55-59	162.2	0.080	13.0
60-64	145.9	0.072	10.5
65-69	105.1	0.068	7.1
70-74	80.0	0.058	4.6
75+	72.9	0.055	4.0

a/ From table 33.
b/ From table 34.
c/ (Col. 2) . (Col. 3).

Table 36. Projected labour force, by age and sex: entire country

(Thousands)

Age group	Year 0	5	10	15	20
			Male		
10-14	149.1	136.5	116.7	106.0	90.8
15-19	239.4	277.1	280.5	249.5	240.4
20-24	324.6	394.8	506.7	593.4	597.4
25-29	335.8	366.9	442.9	566.5	669.7
30-34	337.5	334.5	368.7	445.1	572.2
35-39	291.5	328.3	327.8	363.1	441.2
40-44	207.1	281.4	319.4	321.2	358.5
45-49	138.7	196.3	269.5	307.5	311.3
50-54	170.7	127.1	181.7	251.6	289.9
55-59	131.4	146.2	109.8	158.0	219.9
60-64	94.5	99.4	111.5	84.9	124.6
65-69	35.6	40.0	43.0	49.3	38.3
70-74	4.4	6.7	7.6	8.9	10.4
75+	1.9	2.2	2.7	3.5	3.7
			Female		
10-14	88.8	90.4	82.3	76.8	68.4
15-19	154.2	192.6	217.7	217.8	232.4
20-24	125.2	158.4	208.9	255.9	270.3
25-29	124.3	132.9	171.5	230.9	286.4
30-34	106.7	108.5	117.2	152.9	206.6
35-39	74.3	88.7	90.0	98.1	126.0
40-44	40.5	58.7	69.3	70.0	75.3
45-49	23.0	29.8	44.0	52.6	54.0
50-54	16.9	15.3	20.2	30.6	37.5
55-59	12.6	13.0	11.5	14.9	21.8
60-64	8.6	10.5	10.5	9.5	12.2
65-69	6.9	7.1	7.7	7.8	7.1
70-74	3.9	4.6	4.7	5.2	5.4
75+	3.6	4.0	4.1	4.7	4.9

(i) Labour force aggregates

Labour force aggregates include the total labour force, the number of persons in the labour force who belong to broad age groups and the number of males and females in the labour force. Other labour force aggregates include increases in the total labour force, in the number of persons in the labour force in broad age groups and increases in the number of males and females in the labour force.

a. The total labour force

The total labour force at the end of a given projection interval is obtained as the sum of the number of persons in the labour force in different age and sex groups. At the end of the interval 0-5, the total labour force, 3,652.2, is computed by adding the number of male and female members of the labour force in various age groups. This number (3,652.2) is shown in table 37 (in the column corresponding to year 5) along with other results derived for the entire 20-year projection period. The total labour force and its male and female components over this period is illustrated in figure X.

b. Labour force in broad age groups

The number of persons in the young-age, prime-working-age, and old-age labour force can be derived by aggregating the number of persons in the labour force within broad age groups 10-24, 25-64 and 65+, respectively.

i. Young-age labour force

The number of persons in the young-age component of the labour force at the end of the projection interval 0-5, 1,249.9, is obtained by aggregating the number of males and females in the labour force in age groups 10-14 through 20-24. The number is shown in table 37 (column corresponding to year 5).

ii. Prime-working-age labour force

The number of persons in prime-working-age component of the labour force, 2,337.6, is found by summing the number of persons of both sexes in labour force in age groups 25-29 through 60-64.

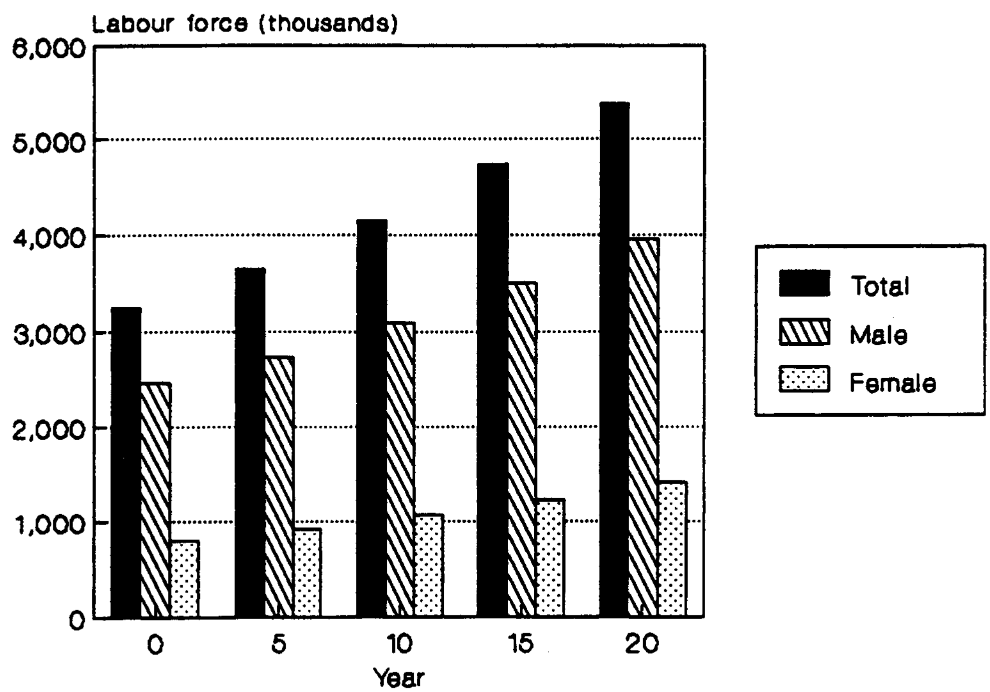
iii. Old-age labour force

The number of persons in old-age component of the labour force, 64.7, equals the sum of males and females in the labour force at age 65 and over.

Table 37. Labour force aggregates, structure and rates of growth: entire country

Indicators	Year 0	5	10	15	20
Labour force aggregates (thousands)					
Labour force					
Total	3251.9	3652.2	4148.0	4736.2	5376.9
Young-age	1081.4	1249.9	1412.8	1499.4	1499.7
Prime-working-age	2114.3	2337.6	2665.3	3157.4	3807.3
Old-age	56.3	64.7	69.9	79.5	69.8
Male	2462.3	2737.5	3088.3	3508.5	3968.3
Female	789.7	914.7	1059.7	1227.7	1408.5
Growth in labour force					
Total		400.2	495.9	588.2	640.7
Young-age		168.5	162.9	86.6	0.3
Prime-working-age		223.3	327.7	492.1	650.0
Old-age		8.5	5.2	9.5	-9.6
Male		275.2	350.8	420.2	459.8
Female		125.0	145.0	168.0	180.9
Indicators of labour force structure					
Proportions by broad age groups					
Young	0.33	0.34	0.34	0.32	0.28
Prime-working-age	0.65	0.64	0.64	0.67	0.71
Old-age	0.02	0.02	0.02	0.02	0.01
Median age of labour force	31.0	30.9	30.5	30.6	31.5
Sex ratio of labour force	312	299	291	286	282
Rates of growth in labour force (percentage)					
Total		2.32	2.55	2.65	2.54
Young-age		2.90	2.45	1.19	0.00
Prime-working-age		2.01	2.62	3.39	3.74
Old-age		2.81	1.55	2.55	-2.59
Male		2.12	2.41	2.55	2.46
Female		2.94	2.94	2.94	2.75

Figure X. Total labour force and numbers of males and females in labour force



The projected levels of young-age, prime-working-age and old-age components of the labour force for the entire projection period are shown in figure XI.

c. Labour force disaggregated by sex

In addition to labour force within broad age groups, this method can be used to derive the number of males and females in the labour force.

i. Male labour force

The number of males in the labour force at the end of the interval 0-5, 2,737.5, is obtained as the sum of males in the labour force of different age groups from 10-14 onward. The number is shown in table 37.

ii. Female labour force

The number of females in the labour force, 914.7, can be calculated by subtracting the number of males in the labour force, 2,737.5, from the total labour force, 3,652.2:

$$914.7 = 3{,}652.2 - 2{,}737.5. \quad (7)$$

The numbers of males and females in the labour force at different dates during the projection period are illustrated in figure X.

d. Growth in the total labour force

The increase in the total labour force over a given projection interval equals the difference between the total labour force at the end and the beginning of the interval. For the interval 0-5, the growth in the total labour force, 400.2, is obtained as:

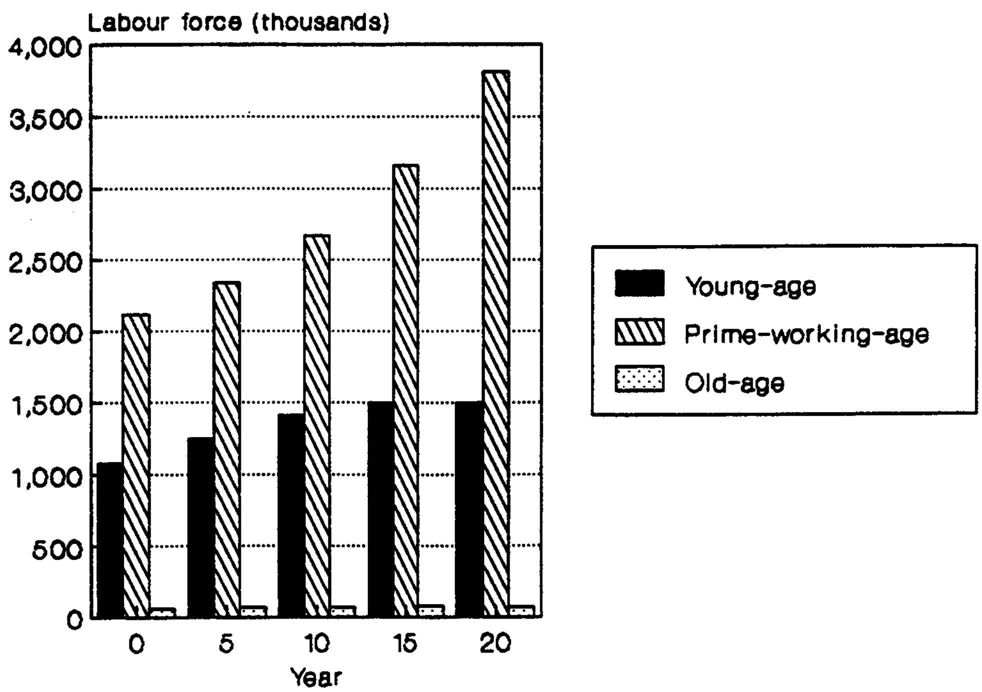
$$400.2 = 3{,}652.2 - 3{,}251.9, \quad (8)$$

where 3,251.9 and 3,652.2 are, respectively, the total labour force at the beginning and the end of the interval (shown in columns corresponding to years 0 and 5, respectively).

e. Growth in young-age, prime-working-age and old-age components of the labour force

The increase in labour force components corresponding to the different broad age groups over the interval 0-5 is obtained as follows:

Figure XI. Young-age, prime-working-age and old-age labour force



Growth in the young-age component of the labour force, 168.5, is calculated as:

$$168.5 = 1{,}249.9 - 1{,}081.4, \tag{9}$$

where 1,081.4 and 1,249.9 are the numbers of persons in the young-age labour force in years 0 and 5, respectively;

Growth in the prime-working-age component of the labour force, 223.3, is calculated as:

$$223.3 = 2{,}337.6 - 2{,}114.3, \tag{10}$$

where 2,114.3 and 2,337.6 are the numbers of persons in the prime-working-age labour force in years 0 and 5;

Growth in the old-age component of the labour force, 8.5, is calculated as:

$$8.5 = 64.7 - 56.3, \tag{11}$$

where 56.3 and 64.7 are the numbers of persons in the old-age component of the labour force in years 0 and 5.

f. Growth in male and female labour force

The increase in the number of males in the labour force over the interval 0-5, 275.2, is calculated as:

$$275.2 = 2{,}737.5 - 2{,}462.3, \qquad (12)$$

where 2,462.3 and 2,737.5 are the numbers of males in the labour force in years 0 and 5, respectively.

Growth in the number of females in the labour force, 125.0, can be calculated in an anologous way:

$$125.0 = 914.7 - 789.7, \qquad (12)$$

where 789.7 and 914.7 are the numbers of females in the labour force in years 0 and 5, respectively.

(ii) Indicators of the structure of the labour force

Indicators of the structure of the labour force that can be calculated as part of a labour force projection include the proportions of labour force found in broad age groups, the median age of labour force and the sex ratio of the labour force.

a. Proportions by broad age groups

The proportions of the labour force in each broad age group in year 5 can be calculated as follows:

The proportion of the total labour force in the young-age labour force, 0.34, is:

$$0.34 = 1{,}249.9 \,/\, 3{,}652.2, \qquad (13)$$

where 1,249.9 and 3,652.2 are, respectively, the number of persons in the young-age component of the labour force and the total labour force;

The proportion of the total labour force in the prime-working-age labour force, 0.64, is:

$$0.64 = 2{,}337.6 \,/\, 3{,}652.2, \qquad (14)$$

where 2,337.6 is the number of persons in the prime-working-age component of the labour force;

The proportion of the total labour force in the old-age labour force, 0.02, is:

$$0.02 = 64.7 \,/\, 3{,}652.2, \qquad (15)$$

where 64.7 is the number of persons in the old-age component of the labour force.

b. Median age of labour force

The median age of the labour force, which is the age that divides the labour force into two groups of equal size, is computed using the formula for calculating the median age from the grouped data. For the end of the 0-5 interval, the median age of the labour force, 30.9, is computed as follows:

$$30.9 = (7 - 1)\,(5) + [\,(3{,}652.2 \,/\, 2 - 1{,}749.6) \,/\, 443.0\,]\,(5). \qquad (16)$$

For the end of the interval 0-5, the median-age divides the total labour force, 3,652.2, into one half, 1,826.1 (3,652.2 / 2), that is older than the median, and one half, 1826.1, that is younger. By cumulating the number of persons of both sexes in the labour force in the various five-year age groups starting with group 10-14, it can be verified that one half the total labour force, 1,826.1, falls in the seventh five-year age group. This can be seen by comparing the cumulative sum of the number of persons in the four age groups 10-14 through 25-29, 1,749.6, with one half the labour force, 1,826.1. Therefore, among the numbers used, the 7 in the first parenthesis stands for the seventh five-year age group, 30-34, which contains the middle member of the labour force; the term in the parenthesis is multiplied by 5, the length of the five-year age group. The result, 30, is the lower limit of the five-year age group containing the middle member.

The number 1,749.6, in the second parenthesis, stands for the number of persons in the labour force below age 30 (prior to the seventh age group). The number 443.0, which divides that parenthesis, is the sum of the numbers of males and females in the labour force in the age group 30-34. The 5 which multiplies the brackets stands for the length of the five-year age group. The result of this multiplication, 0.9, is the number which is added to the value indicating the lower limit of the five-year age group containing the middle member of the labour force, 30, in order to obtain the value of the median age sought, 30.9.

c. Sex ratio of labour force

The sex ratio of the labour force is the number of males in the labour force to the number of females, multiplied by a hundred. The sex ratio at the end of the 0-5 interval, 299, is obtained as follows:

$$299 = (2{,}737.5 / 914.7) \cdot 100 \qquad (17)$$

where 2,737.5 and 914.7 are, respectively, the numbers of males and females in the labour force at that date.

(iii) Rates of growth of the labour force

The rates of growth of the labour force that can be calculated include those of the total labour force, the labour force disaggregated into broad age groups and labour force disaggregated according to sex.

a. Rate of growth in the total labour force

The average annual growth rate of the total labour force for a given interval can be obtained by the exponential growth rate formula. For the projection interval 0-5, this growth rate, which equals 2.32 per cent (table 37), is obtained as follows:

$$2.32 = [(\ln (3,652.2 / 3,251.9)) / 5] \cdot 100, \qquad (18)$$

where 3,251.9 and 3,652.2 are the total labour force in years 0 and 5, respectively, and 5 is the length of the interval.

Rates of growth in the total labour force over the 20-year projection period are shown in figure XII.

b. Rates of growth in the young-age, prime-working-age and old-age labour force

The average annual rates of increase in the number of persons in labour force disaggregated by broad age groups for the interval 0-5 are calculated as follows:

The rate of growth in the number of persons in the young-age component of the labour force, 2.90 per cent, is:

$$2.90 = [(\ln (1,249.9 / 1,081.4)) / 5] \cdot 100, \qquad (19)$$

where 1,081.4 and 1,249.9 are the numbers of persons in the young-age component of the labour force in years 0 and 5, respectively;

The rate of growth in the number of persons in the prime-working-age component of the labour force, 2.01 per cent, is:

$$2.01 = [(\ln (2,337.6 / 2,114.3)) / 5] \cdot 100, \qquad (20)$$

where 2,114.3 and 2,337.6 are the numbers of persons in the prime-working-age component of the labour force in years 0 and 5;

The rate of growth in the number of persons in the old-age component of the labour force, 2.81 per cent, is:

$$2.81 = [(\ln (64.7 / 56.3)) / 5] \cdot 100, \qquad (21)$$

where 56.3 and 64.7 are the numbers of persons in the old-age component of the labour force in years 0 and 5.

The rates of growth in the numbers of persons in the young-age, prime-working-age and old-age components of the labour force are shown in figure XIII.

Figure XII. Rates of growth of total labour force and numbers of males and females in labour force

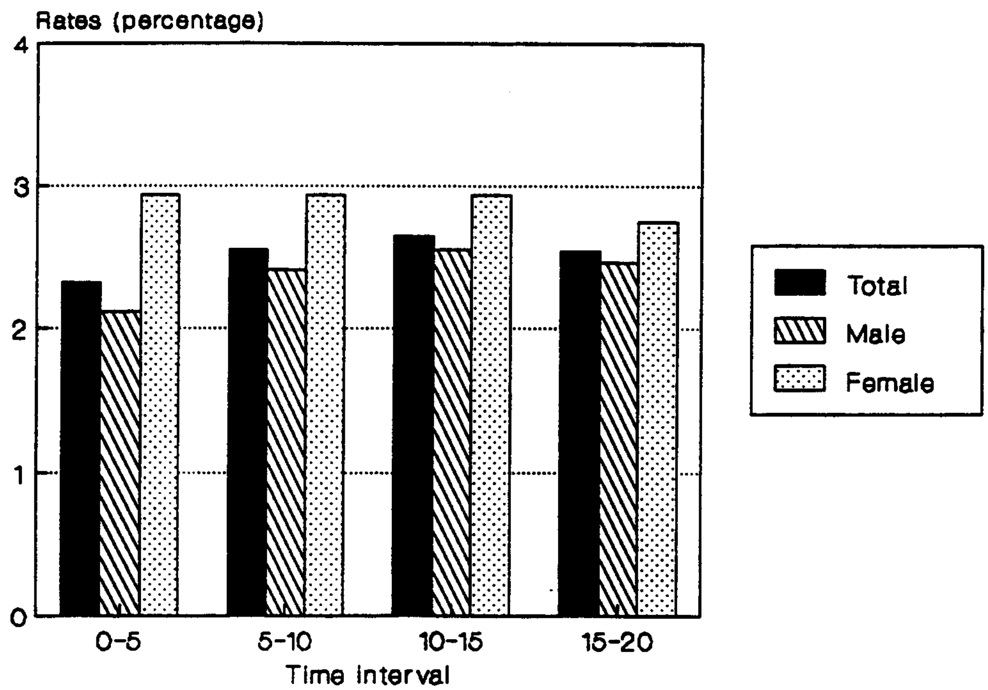
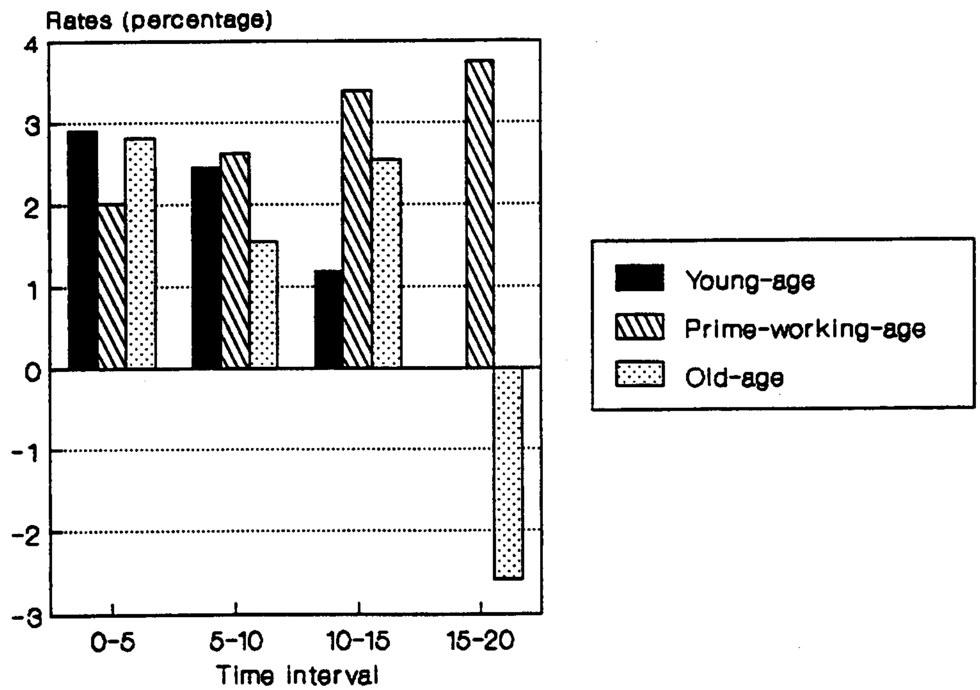


Figure XIII. Rates of growth of young-age, prime-working-age and old-age labour force



c. Rates of growth in male and female components of the labour force

The average annual percentage rate of growth in the number of males in the labour force for the interval 0.5, 2.12 per cent, can be calculated as follows:

$$2.12 = [\ (\ \ln (2{,}737.5\ /\ 2{,}462.3)\ )\ /\ 5\ ] \cdot 100, \qquad (22)$$

where 2,462.3 and 2,737.5 are the numbers of males in the labour force in years 0 and 5;

The average annual rate of growth in the number of females in the labour force, 2.94 per cent, can be computed in an analogous way:

$$2.94 = [\ (\ \ln (914.7\ /\ 789.7)\ )\ /\ 5\ ] \cdot 100, \qquad (22)$$

where 789.7 and 914.7 are the numbers of females in the labour force in years 0 and 5.

The rates of growth in the male and female components of the labour force are indicated in figure XII.

2. Projection for urban and rural areas

The technique for projecting labour force for urban and rural areas is similar to that for the country as a whole. This example will show how such a projection can be prepared by utilizing the inputs shown in tables 38 through 41. Table 38 shows the structure of the urban population at age 10 and over. Table 39 presents assumptions on the future labour force participation rates for urban areas. Table 40 shows the structure of the rural population at age 10 and over. Table 41 presents assumptions on the future labour force participation for rural areas.

The example will emphasize calculations that are unique to an urban-rural projection of the labour force.

(a) Labour force structures

For any given date, such as the end of the projection interval, the structures of the labour force for urban and rural areas are obtained by calculations identical to those used to make a national projection. In the urban-rural projection, however, those calculations are performed for either area.

Table 38. Population structures at age 10 and over: urban areas

(Thousands)

Age group	Year 0	5	10	15	20
			Male		
10-14	170.0	272.6	285.2	343.5	432.8
15-19	181.7	279.7	378.8	384.6	447.3
20-24	201.3	246.1	348.4	446.3	450.7
25-29	134.5	228.4	277.0	381.7	480.5
30-34	126.2	162.5	247.3	298.9	406.5
35-39	109.9	149.9	180.9	259.4	313.8
40-44	77.7	124.7	163.5	191.3	265.8
45-49	51.6	85.1	132.9	170.9	196.7
50-54	59.6	54.7	88.3	136.1	173.3
55-59	47.1	61.6	55.0	87.9	134.8
60-64	31.6	47.2	60.5	53.1	84.7
65-69	26.1	29.8	42.8	54.7	47.9
70-74	12.8	21.7	25.2	35.5	45.6
75+	11.2	14.7	22.3	29.4	40.5
			Female		
10-14	154.8	215.6	226.8	276.4	359.1
15-19	137.4	207.7	280.3	284.6	337.4
20-24	110.9	185.2	270.3	341.3	346.4
25-29	103.2	145.2	224.4	310.7	386.0
30-34	102.4	130.8	171.1	247.6	338.0
35-39	96.7	124.8	151.7	186.3	262.8
40-44	63.0	111.3	142.4	164.4	196.7
45-49	50.0	71.5	123.2	153.3	173.2
50-54	49.2	54.4	77.4	128.8	159.3
55-59	52.4	52.7	56.9	79.5	130.6
60-64	33.3	53.4	54.7	56.9	79.1
65-69	32.6	32.2	50.3	51.5	53.2
70-74	19.6	27.9	28.4	43.1	44.7
75+	21.4	24.9	32.7	37.9	50.7

Table 39. Labour force participation rates: urban areas

Age group	Year 0	5	10	15	20
			Male		
10–14	0.017	0.019	0.013	0.009	0.006
15–19	0.250	0.245	0.207	0.159	0.118
20–24	0.737	0.749	0.768	0.772	0.776
25–29	0.901	0.908	0.915	0.922	0.931
30–34	0.914	0.921	0.929	0.936	0.944
35–39	0.908	0.916	0.926	0.934	0.943
40–44	0.905	0.914	0.922	0.934	0.942
45–49	0.904	0.911	0.920	0.929	0.937
50–54	0.885	0.894	0.904	0.912	0.921
55–59	0.838	0.847	0.857	0.866	0.876
60–64	0.736	0.746	0.755	0.764	0.773
65–69	0.393	0.410	0.427	0.442	0.455
70–74	0.099	0.108	0.116	0.123	0.130
75+	0.049	0.049	0.049	0.049	0.049
			Female		
10–14	0.022	0.018	0.012	0.008	0.008
15–19	0.190	0.186	0.174	0.158	0.141
20–24	0.313	0.332	0.348	0.365	0.384
25–29	0.362	0.388	0.408	0.428	0.453
30–34	0.306	0.326	0.349	0.369	0.387
35–39	0.248	0.257	0.268	0.282	0.293
40–44	0.195	0.205	0.215	0.225	0.239
45–49	0.144	0.154	0.165	0.175	0.185
50–54	0.096	0.106	0.117	0.126	0.136
55–59	0.077	0.086	0.086	0.087	0.089
60–64	0.067	0.077	0.077	0.078	0.078
65–69	0.058	0.067	0.067	0.067	0.068
70–74	0.048	0.058	0.058	0.058	0.058
75+	0.038	0.047	0.047	0.048	0.048

Table 40. Population structures at age 10 and over: rural areas

(Thousands)

Age group	Year 0	5	10	15	20
			Male		
10–14	454.0	457.1	442.4	464.4	477.1
15–19	304.9	338.8	345.8	339.1	357.0
20–24	200.9	235.5	265.0	273.6	269.1
25–29	227.0	169.1	200.0	226.9	234.8
30–34	232.5	193.5	145.5	173.5	197.3
35–39	202.5	201.7	169.5	128.5	153.7
40–44	145.9	179.1	180.1	152.6	116.1
45–49	100.5	129.5	160.5	162.8	138.8
50–54	132.9	88.3	115.1	144.1	147.4
55–59	109.9	113.8	76.7	101.2	127.8
60–64	96.5	89.2	93.9	64.3	85.9
65–69	64.0	73.7	69.6	74.7	52.0
70–74	31.4	44.2	52.3	50.7	55.6
75+	27.4	31.0	40.7	51.7	58.3
			Female		
10–14	425.7	469.5	469.4	500.9	512.0
15–19	310.9	367.1	399.7	407.6	436.5
20–24	248.9	257.3	299.0	333.7	341.7
25–29	245.1	209.3	213.3	253.7	284.4
30–34	249.8	211.6	178.9	185.8	222.0
35–39	204.2	220.3	185.4	159.3	166.2
40–44	143.7	182.2	195.8	167.2	144.3
45–49	108.5	128.7	162.6	177.4	152.0
50–54	125.4	96.9	115.0	147.4	161.6
55–59	111.6	109.5	85.0	102.5	132.3
60–64	92.8	92.6	91.6	72.6	88.5
65–69	73.0	72.9	73.8	74.8	60.2
70–74	43.9	52.2	53.3	55.6	57.5
75+	48.0	48.1	53.9	59.6	65.5

Table 41. Labour force participation rates: rural areas

Age group	Year 0	5	10	15	20
			Male		
10–14	0.322	0.288	0.257	0.222	0.186
15–19	0.636	0.615	0.586	0.557	0.523
20–24	0.878	0.893	0.902	0.908	0.917
25–29	0.946	0.945	0.947	0.946	0.947
30–34	0.956	0.955	0.954	0.955	0.955
35–39	0.947	0.946	0.946	0.944	0.945
40–44	0.937	0.937	0.937	0.935	0.935
45–49	0.917	0.917	0.917	0.916	0.916
50–54	0.887	0.887	0.887	0.887	0.887
55–59	0.838	0.828	0.819	0.809	0.799
60–64	0.740	0.721	0.701	0.690	0.690
65–69	0.395	0.376	0.356	0.337	0.317
70–74	0.100	0.100	0.090	0.090	0.080
75+	0.049	0.049	0.039	0.039	0.030
			Female		
10–14	0.200	0.185	0.169	0.150	0.130
15–19	0.412	0.421	0.423	0.425	0.423
20–24	0.362	0.375	0.385	0.393	0.401
25–29	0.354	0.364	0.374	0.383	0.392
30–34	0.301	0.311	0.321	0.330	0.339
35–39	0.246	0.257	0.266	0.284	0.294
40–44	0.195	0.196	0.196	0.196	0.197
45–49	0.146	0.146	0.146	0.146	0.146
50–54	0.098	0.098	0.098	0.098	0.098
55–59	0.078	0.078	0.078	0.078	0.078
60–64	0.068	0.068	0.068	0.068	0.068
65–69	0.068	0.068	0.059	0.059	0.059
70–74	0.068	0.059	0.059	0.049	0.049
75+	0.057	0.058	0.048	0.048	0.038

The calculation of the structure of the labour force in the urban areas for year 5 is shown in table 42.

The structure of the urban labour force at dates five years apart can be found by performing those calculations for all relevant dates over a period starting with the initial date of the projection. Table 43 displays the urban structures.

The structure of the rural labour force at dates five years apart can be projected in the same way for all relevant dates over a period starting with the initial date of the projection. Table 44 shows such rural structures.

The rural and urban labour force structures of the labour force at dates five years apart can be aggregated across the two locations to obtain the structures of the national labour force, which are shown in table 45.

(b) Other results

Projections of the labour force structures for urban (or rural) areas can be used to calculate all those additional results that can be obtained as part of the national projection. Those results referring to urban (or rural) areas as well as to the entire country can be calculated by means of the steps illustrated above. In addition, in the course of the projection, it is possible to calculate proportions of the total labour force in urban and rural areas, respectively. This section will illustrate how those proportions can be obtained.

(i) Proportions of the labour force that are urban and rural

The proportion of the labour force that is urban is calculated for the end of a given projection interval as a ratio of the total labour force in the urban areas to the total labour force for the entire country. The proportion of the labour force that is urban in year 5, 0.38, is obtained as:

$$0.38 = 1{,}383.7 / 3{,}652.7, \qquad (24)$$

where 1,383.7 is the total labour force in the urban areas and 3,652.7 is the total labour force for the entire country.

The proportion of the labour force that is rural, 0.62, is calculated as a complement of the proportion that is urban:

$$0.62 = 1 - 0.38, \qquad (25)$$

where 0.38 is the proportion of the labour force that is urban.

Table 42. Calculating labour force structure, by age and sex: urban areas, year 5

Age group	Population at age 10 and over a/ (thousands)	Labour force participation rates b/	Labour force c/ (thousands)
(1)	(2)	(3)	(4)
		Male	
10-14	272.6	0.019	5.2
15-19	279.7	0.245	68.5
20-24	246.1	0.749	184.3
25-29	228.4	0.908	207.4
30-34	162.5	0.921	149.7
35-39	149.9	0.916	137.3
40-44	124.7	0.914	114.0
45-49	85.1	0.911	77.5
50-54	54.7	0.894	48.9
55-59	61.6	0.847	52.2
60-64	47.2	0.746	35.2
65-69	29.8	0.410	12.2
70-74	21.7	0.108	2.3
75+	14.7	0.049	0.7
		Female	
10-14	215.6	0.018	3.9
15-19	207.7	0.186	38.6
20-24	185.2	0.332	61.5
25-29	145.2	0.388	56.3
30-34	130.8	0.326	42.6
35-39	124.8	0.257	32.1
40-44	111.3	0.205	22.8
45-49	71.5	0.154	11.0
50-54	54.4	0.106	5.8
55-59	52.7	0.086	4.5
60-64	53.4	0.077	4.1
65-69	32.2	0.067	2.2
70-74	27.9	0.058	1.6
75+	24.9	0.047	1.2

a/ From table 38.
b/ From table 39.
c/ (Col. 2) . (Col. 3).

Table 43. Projected labour force, by age and sex: urban areas

(Thousands)

Age group	Year 0	5	10	15	20
			Male		
10–14	2.9	5.2	3.7	3.1	2.6
15–19	45.4	68.5	78.4	61.2	52.8
20–24	148.4	184.3	267.6	344.5	349.7
25–29	121.2	207.4	253.5	351.9	447.3
30–34	115.3	149.7	229.7	279.8	383.7
35–39	99.8	137.3	167.5	242.3	295.9
40–44	70.3	114.0	150.7	178.7	250.4
45–49	46.6	77.5	122.3	158.8	184.3
50–54	52.7	48.9	79.8	124.1	159.6
55–59	39.5	52.2	47.1	76.1	118.1
60–64	23.3	35.2	45.7	40.6	65.5
65–69	10.3	12.2	18.3	24.2	21.8
70–74	1.3	2.3	2.9	4.4	5.9
75+	0.5	0.7	1.1	1.4	2.0
			Female		
10–14	3.4	3.9	2.7	2.2	2.9
15–19	26.1	38.6	48.8	45.0	47.6
20–24	34.7	61.5	94.1	124.6	133.0
25–29	37.4	56.3	91.6	133.0	174.9
30–34	31.3	42.6	59.7	91.4	130.8
35–39	24.0	32.1	40.7	52.5	77.0
40–44	12.3	22.8	30.6	37.0	47.0
45–49	7.2	11.0	20.3	26.8	32.0
50–54	4.7	5.8	9.1	16.2	21.7
55–59	4.0	4.5	4.9	6.9	11.6
60–64	2.2	4.1	4.2	4.4	6.2
65–69	1.9	2.2	3.4	3.5	3.6
70–74	0.9	1.6	1.6	2.5	2.6
75+	0.8	1.2	1.5	1.8	2.4

Table 44. Projected labour force, by age and sex: rural areas

(Thousands)

Age group	Year 0	5	10	15	20
			Male		
10-14	146.2	131.6	113.7	103.1	88.7
15-19	193.9	208.4	202.6	188.9	186.7
20-24	176.4	210.3	239.0	248.4	246.8
25-29	214.7	159.8	189.4	214.6	222.4
30-34	222.3	184.8	138.8	165.7	188.4
35-39	191.8	190.8	160.3	121.3	145.2
40-44	136.7	167.8	168.8	142.7	108.6
45-49	92.2	118.8	147.2	149.1	127.1
50-54	117.9	78.3	102.1	127.8	130.7
55-59	92.1	94.2	62.8	81.9	102.1
60-64	71.4	64.3	65.8	44.4	59.3
65-69	25.3	27.7	24.8	25.2	16.5
70-74	3.1	4.4	4.7	4.6	4.4
75+	1.3	1.5	1.6	2.0	1.7
			Female		
10-14	85.1	86.9	79.3	75.1	66.6
15-19	128.1	154.5	169.1	173.2	184.6
20-24	90.1	96.5	115.1	131.1	137.0
25-29	86.8	76.2	79.8	97.2	111.5
30-34	75.2	65.8	57.4	61.3	75.3
35-39	50.2	56.6	49.3	45.2	48.9
40-44	28.0	35.7	38.4	32.8	28.4
45-49	15.8	18.8	23.7	25.9	22.2
50-54	12.3	9.5	11.3	14.4	15.8
55-59	8.7	8.5	6.6	8.0	10.3
60-64	6.3	6.3	6.2	4.9	6.0
65-69	5.0	5.0	4.4	4.4	3.6
70-74	3.0	3.1	3.1	2.7	2.8
75+	2.7	2.8	2.6	2.9	2.5

Table 45. Projected labour force, by age and sex: entire country

(Thousands)

Age group	Year 0	5	10	15	20
			Male		
10–14	149.1	136.8	117.4	106.2	91.3
15–19	239.3	276.9	281.1	250.0	239.5
20–24	324.7	394.6	506.6	593.0	596.5
25–29	335.9	367.2	442.9	566.6	669.7
30–34	337.6	334.5	368.5	445.5	572.2
35–39	291.6	328.1	327.9	363.6	441.2
40–44	207.0	281.8	319.5	321.4	358.9
45–49	138.8	196.3	269.4	307.9	311.4
50–54	170.6	127.2	181.9	251.9	290.4
55–59	131.6	146.4	110.0	158.0	220.2
60–64	94.7	99.5	111.5	84.9	124.7
65–69	35.5	39.9	43.1	49.4	38.3
70–74	4.4	6.8	7.6	8.9	10.4
75+	1.9	2.2	2.7	3.5	3.7
			Female		
10–14	88.5	90.7	82.1	77.3	69.4
15–19	154.2	193.2	217.8	218.2	232.2
20–24	124.8	158.0	209.2	255.7	270.0
25–29	124.1	132.5	171.3	230.1	286.3
30–34	106.5	108.4	117.1	152.7	206.1
35–39	74.2	88.7	90.0	97.8	125.9
40–44	40.3	58.5	69.0	69.8	75.4
45–49	23.0	29.8	44.1	52.7	54.2
50–54	17.0	15.3	20.3	30.7	37.5
55–59	12.7	13.1	11.5	14.9	21.9
60–64	8.5	10.4	10.4	9.4	12.2
65–69	6.9	7.1	7.7	7.9	7.2
70–74	3.9	4.7	4.8	5.2	5.4
75+	3.5	4.0	4.1	4.7	4.9

These proportions along with all other results for the entire projection interval are shown in tables 46 through 48, which respectively refer to the urban and rural areas and the entire country. The proportions are also depicted in figure XIV.

E. Summary

The foregoing chapter has discussed the uses of projections of the labour force in development planning and has described the labour force participation rate method for preparing such projections at the national or urban-rural level. As part of the description of the method, the procedures used in making national and urban-rural projections were presented. In addition, the types of inputs required by the method were described along with a discussion relating to the preparation of the inputs. Lastly, two examples of projections--national and urban-rural--were discussed. A complete listing of the outputs that can be generated by the method is shown in box 11.

Box 11

Outputs of a method for projecting labour force with labour force participation rates

1. Structure of the labour force by age and sex (national or urban, rural and national)

2. Labour force aggregates (national or urban, rural and national)

Labour force size

Total labour force

Young-age labour force
Prime-working-age labour force
Old-age labour force

Male labour force
Female labour force

Growth in the labour force

Total labour force

Young-age labour force
Prime-working-age labour force
Old-age labour force

(continued)

Box 11 (continued)

Male labour force
Female labour force

3. Indicators of structure of the labour force (national or urban, rural and national)

Proportions of persons in:

Young-age labour force
Prime-working-age labour force
Old-age labour force

Median age of the labour force

Sex ratio of the labour force

4. Indicators of the urban-rural distribution of the labour force (national only; if urban and rural labour force are being projected)

Proportions of the national labour force:

Urban
Rural

5. Rates of growth in the labour force (national or urban, rural and national)

Rate of growth in the labour force:

Total labour force:

Young-age labour force
Prime-working-age labour force
Old-age labour force

Male labour force
Female labour force

Table 46. Labour force aggregates, structure and rates of growth: urban areas

Indicators	Year 0	5	10	15	20
Labour force aggregates (thousands)					
Labour force					
Total	968.5	1383.7	1881.5	2438.8	3033.0
Young-age	260.9	362.0	495.2	580.5	588.6
Prime-working-age	691.9	1001.4	1357.4	1820.5	2406.0
Old-age	15.7	20.2	28.8	37.8	38.4
Male	777.5	1095.5	1468.3	1891.0	2339.7
Female	191.0	288.2	413.1	547.8	693.3
Growth in labour force					
Total		415.2	497.8	557.3	594.2
Young-age		101.1	133.2	85.3	8.0
Prime-working-age		309.5	356.0	463.1	585.5
Old-age		4.5	8.6	8.9	0.6
Male		318.0	372.9	422.7	448.7
Female		97.2	124.9	134.7	145.5
Indicators of labour force structure					
Proportions by broad age group					
Young-age	0.27	0.26	0.26	0.24	0.19
Prime-working-age	0.71	0.72	0.72	0.75	0.79
Old-age	0.02	0.01	0.02	0.02	0.01
Median age of labour force	32.2	31.7	31.7	32.1	33.0
Sex ratio of labour force	407	380	355	345	337
Rates of growth in labour force (percentage)					
Total		7.13	6.15	5.19	4.36
Young-age		6.55	6.27	3.18	0.28
Prime-working-age		7.39	6.08	5.87	5.58
Old-age		5.04	7.10	5.38	0.31
Male		6.86	5.86	5.06	4.26
Female		8.23	7.20	5.64	4.71

Table 47. Labour force aggregates, structure and rates of growth: rural areas

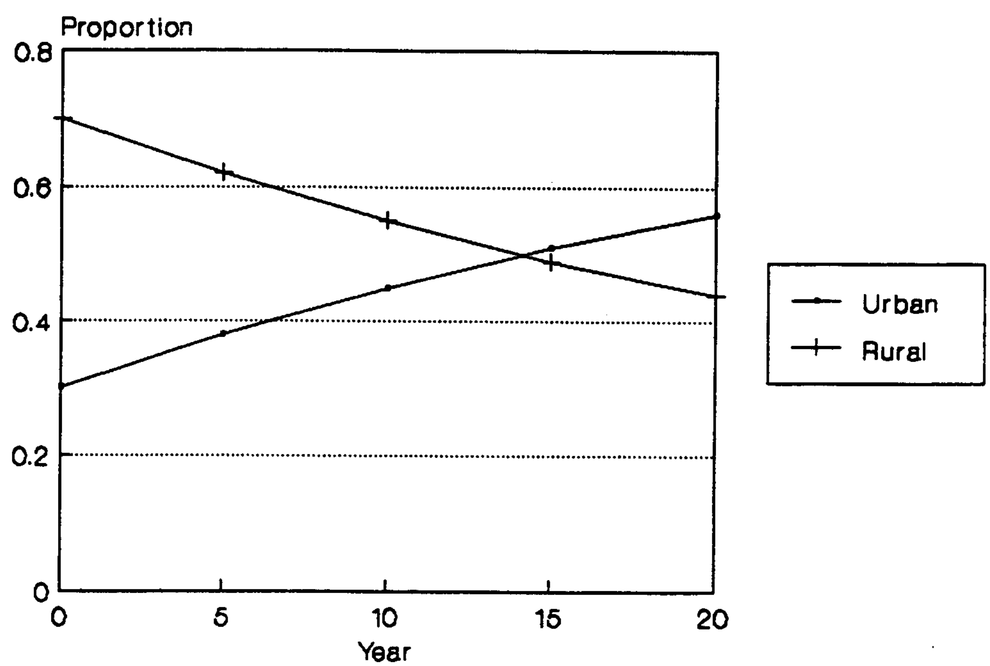
Indicators	Year 0	5	10	15	20
<u>Labour force aggregates (thousands)</u>					
Labour force					
Total	2282.7	2269.0	2268.0	2298.9	2344.2
Young-age	819.8	888.2	918.9	919.9	910.4
Prime-working-age	1422.4	1336.3	1308.0	1337.3	1402.2
Old-age	40.4	44.5	41.2	41.8	31.5
Male	1685.3	1642.8	1621.7	1619.7	1628.7
Female	597.4	626.2	646.4	679.3	715.5
Growth in labour force					
Total	-13.7	-0.9	30.9	45.3	
Young-age	68.4	30.7	1.0	-9.5	
Prime-working-age	-86.1	-28.3	29.3	65.0	
Old-age	4.0	-3.3	0.6	-10.2	
Male	-42.5	-21.1	-2.0	9.1	
Female	28.8	20.2	32.9	36.2	
<u>Indicators of labour force structure</u>					
Proportions by broad age groups					
Young-age	0.36	0.39	0.41	0.40	0.39
Prime-working-age	0.62	0.59	0.58	0.58	0.60
Old-age	0.02	0.02	0.02	0.02	0.01
Median age of labour force	30.3	30.2	29.0	28.7	28.9
Sex ratio of labour force	282	262	251	238	228
<u>Rates of growth in labour force (percentage)</u>					
Total	-0.12	-0.01	0.27	0.39	
Young-age	1.60	0.68	0.02	-0.21	
Prime-working-age	-1.25	-0.43	0.44	0.95	
Old-age	1.90	-1.55	0.29	-5.61	
Male	-0.51	-0.26	-0.02	0.11	
Female	0.94	0.63	0.99	1.04	

Note: growth figures are printed between the year columns (for the intervals 0–5, 5–10, 10–15, 15–20).

Table 48. Labour force aggregates, structure, distribution and rates of growth: entire country

Indicators	Year 0	5	10	15	20
<u>Labour force aggregates (thousands)</u>					
Labour force					
Total	3251.2	3652.7	4149.5	4737.7	5377.2
Young-age	1080.7	1250.2	1414.1	1500.5	1499.0
Prime-working-age	2114.3	2337.7	2665.4	3157.8	3808.3
Old-age	56.2	64.7	70.0	79.5	69.9
Male	2462.8	2738.3	3090.0	3510.7	3968.4
Female	788.4	914.4	1059.5	1227.1	1408.8
Growth in labour force					
Total		401.5	496.9	588.2	639.4
Young-age		169.5	163.9	86.3	-1.4
Prime-working-age		223.4	327.7	492.4	650.5
Old-age		8.5	5.3	9.5	-9.6
Male		275.5	351.7	420.7	457.8
Female		126.0	145.1	167.6	181.7
<u>Indicators of labour force structure</u>					
Proportions by broad age groups					
Young-age	0.33	0.34	0.34	0.32	0.28
Prime-working-age	0.65	0.64	0.64	0.67	0.71
Old-age	0.02	0.02	0.02	0.02	0.01
Median age of labour force	31.0	30.9	30.5	30.6	31.5
Sex ratio of labour force	312	299	292	286	282
Indicators of labour force distribution					
Proportions of labour force					
Urban	0.30	0.38	0.45	0.51	0.56
Rural	0.70	0.62	0.55	0.49	0.44
<u>Rates of growth in labour force (percentage)</u>					
Total		2.33	2.55	2.65	2.53
Young-age		2.91	2.46	1.19	-0.02
Prime-working-age		2.01	2.62	3.39	3.75
Old-age		2.83	1.57	2.55	-2.58
Male		2.12	2.42	2.55	2.45
Female		2.97	2.95	2.94	2.76

Figure XIV. Proportions of total labour force: urban and rural



F. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

a = 3,...,16	are five-year age groups 10-14,...,75+
a'	is the number that stands for the five-year age group containing the member of the labour force who is older than one half of the total labour force and younger than the other half
k = 1,2	are urban and rural locations
s = 1,2	are male and female sexes
t	is the year of the projection period

(b) List of variables

GRLF	is the average annual exponential growth rate of the total labour force for the interval
GRLF(s)	is the average annual exponential growth rate of the number of persons in the labour force of sex s over the interval
GRLFO	is the average annual exponential growth rate of the old-age component of the labour force for the interval
GRLFP	is the average annual exponential growth rate of the prime-working-age component of the labour force for the interval
GRLFY	is the average annual exponential growth rate of the young-age component of the labour force for the interval
LF(a,s,k,t+5)	is the number of persons of age group a and sex s in location k in the labour force at the end of the interval
LF(a,s,t+5)	is the number of persons of age group a and sex s in the labour force at the end of the interval

LF(k,t+5)	is the total labour force in location k at the end of the interval
LF(s,t+5)	is the number of persons of sex s in the labour force at the end of the interval
LF(t+5)	is the total labour force at the end of the interval
LFGR	is the growth in the total labour force over the interval
LFGR(s)	is the growth in the labour force consisting of persons of sex s over the interval
LFO(t+5)	is the number of persons in the old-age labour force at the end of the interval
LFOGR	is the growth in the old-age component of the labour force over the interval
LFP(t+5)	is the number of persons in the prime-working-age labour force at the end of the interval
LFPGR	is the growth in the prime-working-age component of the labour force over the interval
LFPR(a,s,k,t+5)	is the labour force participation rate among persons of age group a and sex s in location k at the end of the interval
LFPR(a,s,t+5)	is the labour force participation rate among persons of age group a and sex s at the end of the interval
LFY(t+5)	is the number of persons in the young-age labour force at the end of the interval
LFYGR	is the growth in the young-age component of the labour force over the interval
MALF(t+5)	is the median age of the labour force at the end of the interval
PLFO(t+5)	is the proportion of the old-age persons in the labour force at the end of the interval
PLFP(t+5)	is the proportion of the prime-working-age persons in the labour force at the end of the interval
PLFRUR(t+5)	is the proportion of the total labour force which is rural at the end of the interval

PLFURB(t+5)	is the proportion of the total labour force which is urban at the end of the interval
PLFY(t+5)	is the proportion of the young-age persons in the labour force at the end of the interval
POP(a,s,k,t+5)	is the population of age group a and sex s in location k at the end of the interval
POP(a,s,t+5)	is the population of age group a and sex s at the end of the interval
SRLF(t+5)	is the sex ratio of the labour force at the end of the interval

(c) Special symbols

ln	is the natural logarithm

2. Equations

1. National level

(a) Labour force structure

$$LF(a,s,t+5) = POP(a,s,t+5) \cdot LFPR(a,s,t+5); \quad (1)$$

$$a = 3,\ldots,16;$$

$$s = 1,2$$

(b) Other results

(i) Labour force aggregates

a. Total labour force

$$LF(t+5) = \sum_{a=3}^{16} \sum_{s=1}^{2} LF(a,s,t+5) \quad (2)$$

b. Labour force in broad age groups

i. Young-age labour force

$$LFY(t+5) = \sum_{a=3}^{5} \sum_{s=1}^{2} LF(a,s,t+5) \qquad (3)$$

ii. Prime-working-age labour force

$$LFP(t+5) = \sum_{a=6}^{13} \sum_{s=1}^{2} LF(a,s,t+5) \qquad (4)$$

iii. Old-age labour force

$$LFO(t+5) = \sum_{a=14}^{16} \sum_{s=1}^{2} LF(a,s,t+5) \qquad (5)$$

c. Labour force disaggregated by sex

i. Male labour force

$$LF(s,t+5) = \sum_{a=3}^{16} LF(a,s,t+5); \qquad (6)$$

$$s = 1,2$$

ii. Female labour force

$$LF(2,t+5) = LF(t+5) - LF(1,t+5) \qquad (7)$$

d. Growth in total labour force

$$LFGR = LF(t+5) - LF(t) \qquad (8)$$

e. Growth in young-age, prime-working-age and old-age labour force

$$LFYGR = LFY(t+5) - LFY(t) \quad (9)$$

$$LFPGR = LFP(t+5) - LFP(t) \quad (10)$$

$$LFOGR = LFO(t+5) - LFO(t) \quad (11)$$

f. Growth in male and female labour force

$$LFGR(s) = LF(s,t+5) - LF(s,t); \quad s = 1,2 \quad (12)$$

(ii) Indicators of the structure of labour force

a. Proportions by broad age groups

$$PLFY(t+5) = LFY(t+5) \ / \ LF(t+5) \quad (13)$$

$$PLFP(t+5) = LFP(t+5) \ / \ LF(t+5) \quad (14)$$

$$PLFO(t+5) = LFO(t+5) \ / \ LF(t+5) \quad (15)$$

b. Median age of labour force

$$MALF(t+5) = (a' - 1) \cdot 5 + \left[\left(LF(t+5)/2 - \sum_{a=3}^{a'-1} \sum_{s=1}^{2} LF(a,s,t+5) \right) \Big/ \sum_{s=1}^{2} LF(a',s,t+5) \right] \cdot 5 \quad (16)$$

c. Sex ratio of labour force

$$SRLF(t+5) = [\ (LF(1,t+5)\ )\ /\ (LF(2,t+5)\ )\ ] \cdot 100 \qquad (17)$$

(iii) Rates of growth of the labour force

a. Rate of growth in the total labour force

$$GRLF = [\ (\ \ln\ (\ LF(t+5)\ /\ LF(t)\ )\ )\ ] \cdot 100 \qquad (18)$$

b. Rates of growth in young-age, prime-working-age and old-age labour force

$$GRLFY = [\ (\ \ln\ (\ LFY(t+5)\ /\ LFY(t)\ )\ )\ /\ 5\ ] \cdot 100 \qquad (19)$$

$$GRLFP = [\ (\ \ln\ (\ LFP(t+5)\ /\ LFP(t)\ )\ )\ /\ 5\ ] \cdot 100 \qquad (20)$$

$$GRLFO = [\ (\ \ln\ (\ LFO(t+5)\ /\ LFO(t)\ )\ )\ /\ 5\ ] \cdot 100 \qquad (21)$$

c. Rates of growth of male and female components of the labour force

$$GRLF(s) = [\ (\ \ln\ (\ LF(s,t+5)\ /\ LF(s,t)\ )\ )\ /\ 5\ ] \cdot 100; \qquad (22)$$

$$s = 1,2$$

2. Urban-rural level

(a) Labour force structures

$$LF(a,s,k,t+5) = POP(a,s,k,t+5) \cdot LFPR(a,s,k,t+5); \qquad (23)$$

$$a = 3,\ldots,16;$$

$$s = 1,2;$$

$$k = 1,2$$

(b) Other results

(i) Proportions of the labour force that are urban and rural

PLFURB(t+5) = LF(t+5) / LF(t+5) (24)

PLFRUR(t+5) = 1 - PLFURB(t+5) (25)

Notes

1/ The elements of the method described in this chapter were originally discussed in United Nations, (1971).

2/ For a discussion of the calculation of the median age from grouped data, see Shryock and others (1973).

3/ The age and sex structures of the population used in this example are expressed in units of one thousand. Therefore, the projected numbers of labour force are also given in thousands.

References

Shryock, Henry S., Jacob S. Siegel and Associates (1973). The Methods and Materials of Demography, Washington D.C.: U.S. Department of Commerce.

United Nations (1971). Manual V: Methods of Projecting the Economically Active Population. Population Studies, No. 46. Sales No. E.70.XIII.2.

VI. MAKING EMPLOYMENT PROJECTIONS BY ASSUMING A CONSTANT RATE OF CHANGE IN LABOUR PRODUCTIVITY

A. Introduction

Employment projections are an indispensable input to those comprehensive planning exercises that are designed to accommodate future growth in the working-age population and the labour force. Employment projections can be compared to labour force projections in order assess future surpluses or shortages in the labour market. The comparison should indicate whether projections of factors influencing the supply of labour, such as population, and projections of factors influencing the demand for labour, such as value added, are realistic. Such comparisons should also indicate whether the policies underlying those projections are conducive to balancing the supply and demand sides of the labour market.1/.

A variety of methods can be used to project total employment as well as employment for the various industries comprising a nation's economy. The methods, which vary in complexity, have different data requirements and use different assumptions. One of the simplest techniques derives employment projections for each industry by multiplying projected levels of value added by constant average employment-value added ratios (box 12).2/ That method is easy to apply because it requires very little information and assumes a fixed relationship between employment and value added over time.3/ However, the projections that the employment-value added ratio method yields may not be valid, since it recognizes neither the possibility of factor substitution (e.g. capital for labour) nor the possibility of technical progress.

This method is a special case of the labour productivity method described in this chapter. In particular, if it is assumed that the rates of change of labour productivity by industry equal zero, the labour productivity method would be mathematically equivalent to the constant employment-value added ratio method. The employment-value added ratio method is not described in this volume owing to its limited value in medium-term planning and especially in long-term planning.

To ensure that development plans are based on realistic projections of future trends, many planners prefer to use methods of employment projections that implicitly or explicitly allow for factor substitution or technical change. The simplest among these methods projects employment for each industry by dividing projected levels of value added by the projected levels of average labour productivity, which are assumed to grow at constant rates. The assumptions concerning the rates of change in labour productivity used with this method, which is capable of making national or urban-rural projections, can be based on historical experience or on expected labour productivity trends.

Box 12

Glossary

<u>Average employment-value added ratio</u>
For a given time period, the quantity of labour employed, divided by the valued added produced.

Average labour productivity
The level of output per unit of labour input, usually measured as value added per man-hour or man-year.

Elasticity of employment with respect to value added
For a given time period, the proportionate change in the quantity of labour employed, divided by the proportionate change in the value added.

Factor substitution
The process by which one factor of production (e.g. labour) is replaced in production by some other factor of production (e.g. capital).

Marginal-employment-value added ratio
For a given time period, the change in the quantity of labour employed, divided by the change in the value added.

Technical progress
The application of new scientific knowledge in the form of inventions and innovations to capital, both physical and human, usually leading to lower costs or increased output.

The labour productivity method is less rigid in its basic assumptions than the employment-value added ratio method because it allows, albeit implicitly, for capital-labour substitution and technical change. However, it is more restrictive in its assumptions than the more complex methods for making employment projections which will be described in chapters VII and VIII. Thus, it makes the implicit assumption that the elasticity of employment with respect to value added is equal to one.

Furthermore, the labour productivity method makes it possible to utilize actual historical experience as a basis for preparing projections, since it utilizes time series data on employment and value added. In addition, since it may require a relatively limited amount of data (time series containing observations for as few as two points in time) the labour productivity method can be more readily applied at the urban-rural level than the more complex methods.

However, the fact that the labour productivity method utilizes time series data, calls for caution. First, labour productivity for any given year is very sensitive to fluctuations in economic activity and shifts in government policy. Consequently, estimates of past trends in labour productivity can be heavily influenced by past developments or policies which may not continue in the future. Such estimates may be, therefore, inappropriate for preparing future projections of employment.

Secondly, in developing countries, value added and employment data may often have different coverage. Thus, time series data on employment may be limited to modern establishments in various industries, while the data on value added, obtained from the national accounts, may refer (at least in principle) to entire industries. Since historical periods often involve significant increases in the shares of modern at the expense of traditional establishments in different industries and since the two types of establishments have very different levels of labour productivity, the resulting time series data may be inappropriate as the basis for applying the labour productivity method.

B. The technique

1. Overview

This overview lists the inputs required, indicates the type of results that can be generated and outlines the computational steps involved in making employment projections with this method.

(a) Inputs

To project employment at the national level, the following inputs are required:

(i) Projected levels of value added, by industry;
(ii) Assumed levels of labour productivity, by industry, for the initial year of the projection;
(iii) Assumed constant percentage rates of change of labour productivity, by industry.

If, in addition to employment, shortages and/or surpluses in the labour market are to be projected, the inputs should also include:

(iv) Projected total labour force; and
(v) Projected non-civilian employment.

For a national projection, the inputs should refer to the entire country. For an urban-rural projection, they should refer to urban and rural areas. The inputs are listed in box 13.

Box 13

Inputs for making employment projections assuming a constant rate of change in labour productivity

1. Value added, by industry (national or urban and rural)

2. Assumptions on initial-year levels of labour productivity, by industry (national or urban and rural)

3. Assumptions on constant rates of change in labour productivity, by industry (national or urban and rural)

4. Total labour force (national or urban and rural; if projection of labour market balances is desired)

5. Non-civilian employment (national or urban and rural; if projection of labour market balances is desired)

Since the labour productivity method will be described in the context of making quinquennial projections, the projected levels of value added would be for dates five years apart, starting with the initial year of the projection. Projected total labour force and projected non-civilian employment would be for the same dates. Given the appropriate annual inputs, however, the method could be used for making annual projections.

(b) Outputs

In the case of a national projection, the method could be used to generate the following outputs:

(i) Levels of employment by industry;
(ii) Various employment aggregates, such as total employment and the growth in total employment;
(iii) Indicators of the structure of employment, such as the proportions of total employment accounted for by major sectors (e.g. primary, secondary and tertiary), defined in (box 14);
(iv) Rates of change in employment, including that of total employment or employment by major sector.

If the inputs include projected total labour force and projected non-civilian employment, the outputs could also include:

(v) Absolute and relative levels of excess supply of labour and/or excess demand for labour.

Box 14

Glossary

Excess demand for labour
: The amount by which the quantity of labour demanded exceeds the quantity of labour available at the prevailing level of wages and salaries.

Excess supply of labour
: The amount by which the quantity of labour available exceeds the quantity of labour demanded at the prevailing level of wages and salaries.

Primary sector
: The part of the economy that specializes in the production of agricultural products and the extraction of raw materials. Major industries in the sector generally include: agriculture, forestry, fishing and mining.

Secondary sector
: The part of the economy that uses raw materials and intermediate products to produce final goods and other intermediate products. Major industries comprising the sector generally include: manufacturing, construction and utilities.

Tertiary sector
: The part of the economy that provides various services to businesses and households. Major industries of the sector generally include: banking and insurance, public administration, health and education.

If the method is used to prepare an urban-rural projection, the results would include all those listed under (i) through (v), which would be for urban and rural areas as well as for the entire country. In addition, they would include indicators of the urban-rural distribution of employment. The outputs that the technique can generate are shown in box 15.

(c) Computational steps

The first step in projecting employment with this method is to project the levels of labour productivity, by industry, for a given projection date.

Box 15

Types of outputs that can be derived from employment projections made by assuming a constant rate of change in labour productivity

1. Employment by industry (national or urban, rural and national)

2. Employment aggregates (national or urban, rural and national)

   Total employment and employment by sector (e.g. primary, secondary and tertiary)

   Growth in total employment and employment by sector

3. Indicators of the structure of employment (national or urban, rural and national)

   Proportions of total employment found in different sectors

4. Indicators of the urban-rural distribution of employment (national only; if urban and rural employment is being projected)

   Proportions of total employment and of employment by sector in different locations

5. Rates of growth of employment (national or urban, rural and national)

   The rates of growth in total employment and employment by sector

6. Labour market balances (national or urban, rural and national)

   Absolute and relative levels of excess supply of and/or excess demand for labour

After this is done, the projected levels of employment by industry are obtained by dividing the projected levels of value added, by industry, for the date by the projected levels of labour productivity. The method can also be used to derive levels of total employment and employment in major sectors, such as primary, secondary and tertiary, along with other date-specific indicators. If the employment projection is accompanied by a labour force projection, the projected total employment along with the projected total labour force and non-civilian employment can be used to calculate the surplus or shortage of labour.

2. National level

This section will introduce the steps needed to compute the levels of employment by industry and to compute other results for a given projection date or interval at the national level. A summary of those steps is presented in box 16, while a subset of the steps needed to project levels of employment by industry is depicted in the form of a flow diagram (figure XV).

(a) Labour productivity, by industry

In order to implicitly allow for the effect of factor substitution or technical change, the levels of labour productivity, by industry, for any future year will be normally computed using assumptions on positive rates of change of labour productivity. This method requires that constant rates of change in labour productivity by industry be used along with the levels of labour productivity for the initial year of the projection.

(i) Discrete growth

If it is assumed that labour productivity grows in discrete intervals, the levels of labour productivity at the end of the projection interval (t to t+5) can be calculated as follows:

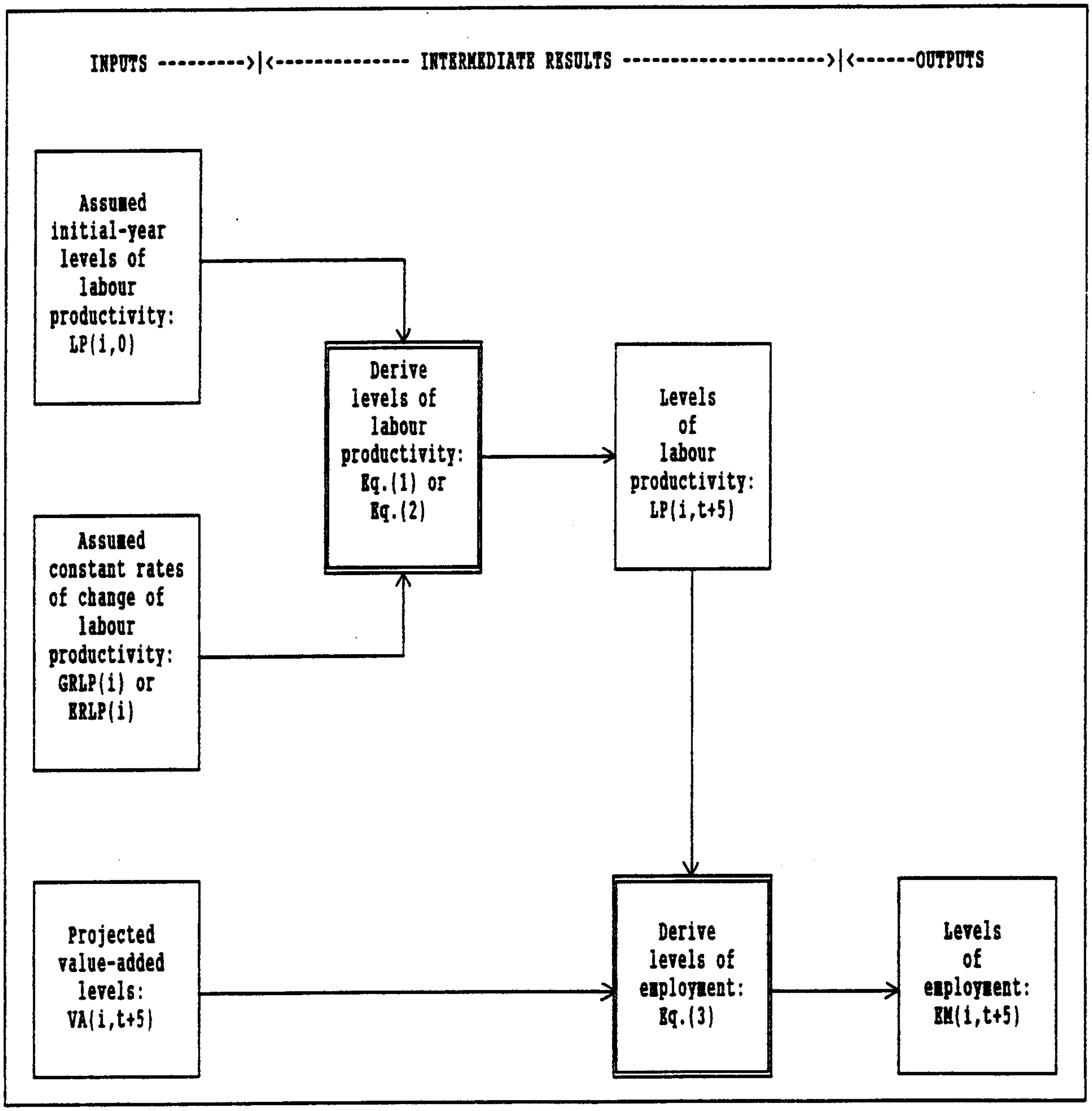
$$LP(i,t+5) = LP(i,0) \cdot (1 + GRLP(i)/100)^{t+5}; \qquad (1)$$

$$i = 1,...,I,$$

where:

$i = 1,...,I$	are industries of the nation's economy,
I	is the number of industries,
t	is the year of the projection period,4/

Figure XV. Steps to derive the levels of employment, by industry



LP(i,t+5) is the labour productivity in industry i at the end of the interval,

LP(i,0) is the labour productivity of industry i in the initial year of the projection period, and

Box 16

Computational steps to project employment at the national level, assuming a constant rate of change in labour productivity

The steps used to project employment at the national level over a five-year projection interval are:

(1) By means of suitable formulae, derive levels of labour productivity, by industry, for the end of the interval using the assumed initial-year levels of labour productivity and the assumed constant rates of change of labour productivity by industry.

(2) For each industry, divide the assumed levels of value added for the end of the interval by the levels of labour productivity by industry for the same date to obtain projected levels of employment.

(3) Calculate various aggregates, such as total employment and the increase in total employment.

(4) Derive indicators of the structure of employment, such as the proportions of total employment accounted for by each sector.

(5) Obtain rates of growth of employment, such as the rate of growth of total employment.

(6) If the labour force projection is available, calculate the absolute and percentage levels of excess supply of or excess demand for labour.

GRLP(i) is the annual geometric rate of change of labour productivity in industry i expressed in percentage.

(ii) Continuous growth

If one assumes that labour productivity grows continuously, then the levels of labour productivity at the end of the projection interval (t to t+5) are obtained as follows:

$$LP(i,t+5) = LP(i,0) \cdot e^{[(ERLP(i)/100) \cdot (t+5)]}; \quad i = 1,\dots,I, \qquad (2)$$

where:

ERLP(i) is the annual exponential rate of change of labour productivity in industry i expressed in percentage, and

e is the base of the natural logarithm.5/

(b) Employment by industry

Having calculated the levels of labour productivity by industry, the levels of employment for each industry can be obtained by dividing the projected levels of value added by the levels of labour productivity:

$$EM(i,t+5) = VA(i,t+5)/LP(i,t+5); \quad i = 1,\dots,I, \qquad (3)$$

where:

EM(i,t+5) is employment in industry i at the end of the interval, and

VA(i,t+5) is the value added in industry i at the end of the interval.

(c) Other results

Once the levels of employment by industry are derived for the end of a given projection interval, several derivative indicators can be calculated, which would be useful in planning. These indicators include aggregates, indicators of the structure and rates of change of employment.

(i) Employment aggregates

A key aggregate that can be calculated from the projected levels of employment by industry is the level of total employment. Also, using the industry level results, it is possible to obtain the levels of employment in major sectors, such as the primary, secondary and tertiary sectors. Once total and sectoral levels are obtained for different dates five years apart, increases in total and sectoral employment over the intervening projection interval can be calculated.

a. Total employment

Total employment can be obtained by aggregating the levels of employment across industries. For the end of a projection interval (t to t+5) this number could be calculated as:

$$EM(t+5) = \sum_{i=1}^{I} EM(i,t+5), \qquad (4)$$

where:

EM(t+5) is the total employment at the end of the interval.

b. Employment by sector

A variety of criteria can be used to aggregate industries into sectors. Thus, one could aggregate industries into primary, secondary and tertiary sectors or into agricultural, industrial and service sectors. For illustrative purposes, the primary-secondary-tertiary-sector mode of aggregation will be used. In addition, it will be assumed that the numbering of industries for which the levels of employment are being projected lists industries of the primary, secondary and tertiary sectors one after another.

i. Employment in the primary sector

Using these aggregation and classification rules, employment in the primary sector for the end of the projection interval (t to t+5) can be obtained as:

$$EMP(t+5) = \sum_{i=1}^{Ip} EM(i,t+5), \qquad (5)$$

where:

Ip	is the number of industries in the primary sector, and
EMP(t+5)	is the employment in the primary sector at the end of the interval.

ii. Employment in the secondary sector

Employment in the secondary sector can be obtained as:

$$EMS(t+5) = \sum_{i=Ip+1}^{Ip+Is} EM(i,t+5), \qquad (6)$$

where:

Is	is the number of industries in the secondary sector, and
EMS(t+5)	is the employment in the secondary sector at the end of the interval.

iii. Employment in the tertiary sector

Employment in the tertiary sector can be calculated as follows:

$$EMT(t+5) = \sum_{i=Ip+Is+1}^{I} EM(i,t+5), \qquad (7)$$

where:

EMT(t+5)	is the employment in the tertiary sector at the end of the interval.

c. Growth in total employment

The growth in total employment over the projection interval (t to t+5) equals the difference between total employment at the end and total employment at the beginning of the interval:

$$EMGR = EM(t+5) - EM(t), \qquad (8)$$

where:

EMGR is the growth of total employment during the interval.

d. Growth in employment by sector

The increase of employment over the projection interval in the primary, secondary and tertiary sectors, respectively, is obtained as follows:

Growth of employment in the primary sector is calculated as:

$$EMPGR = EMP(t+5) - EMP(t), \quad (9)$$

Growth of employment in the secondary sector is calculated as:

$$EMSGR = EMS(t+5) - EMS(t), \quad (10)$$

Growth of employment in the tertiary sector is calculated as:

$$EMTGR = EMT(t+5) - EMT(t), \quad (11)$$

where:

EMPGR is the growth of employment in the primary sector during the interval,

EMSGR is the growth of employment in the secondary sector during the interval, and

EMTGR is the growth of employment in the tertiary sector during the interval.

(ii) Indicators of the structure of employment

Once the various employment aggregates are obtained, it is possible to derive the proportions of total employment accounted for by each sector.

a. Proportions by sector

The proportions of total employment accounted for by each sector (e.g., primary, secondary and tertiary) can be obtained as follows:

The proportion of employment in the primary sector is calculated as:

$$PEMP(t+5) = EMP(t+5) / EM(t+5), \quad (12)$$

The proportion of employment in the secondary sector is calculated as:

$$PEMS(t+5) = EMS(t+5) / EM(t+5), \quad (13)$$

The proportion of employment in the tertiary sector is calculated as:

$$PEMT(t+5) = EMT(t+5) / EM(t+5), \quad (14)$$

where:

PEMP(t+5)	is the proportion of total employment accounted for by the primary sector at the end of the interval,
PEMS(t+5)	is the proportion of total employment accounted for by the secondary sector at the end of the interval, and
PEMT(t+5)	is the proportion of total employment accounted for by the tertiary sector at the end of the interval.

(iii) Rates of growth of employment

As part of an employment projection, it is also possible to compute average annual rates of growth in employment, for the total employment and for employment by sectors.

a. Rate of growth of total employment

The average annual rate of growth of total employment for a given projection interval can be computed from the total employment at the beginning and the end of the interval. If, as part of the projection process, it is assumed that growth occurs over discrete intervals, then the percentage rate of growth can be obtained using the formula for calculating a geometric growth rate:

$$GGREM = [(EM(t+5) / EM(t))^{1/5} - 1] \cdot 100, \quad (15)$$

where:

GGREM	is the average annual geometric growth rate of total employment for the interval.

Alternatively, if it is assumed that growth is continuous, then the growth rate of total employment can be calculated using the formula for calculating an exponential growth rate:

$$EGREM = [(\ln (EM(t+5) / EM(t))) / 5] \cdot 100, \qquad (16)$$

where:

ln	is the natural logarithm, and
EGREM	is the average annual exponential growth rate of total employment for the interval.

b. Rates of growth of employment by sector

Assuming discrete growth, the percentage rates of growth of employment for sectors can be obtained as follows:

The geometric growth rate for the primary sector is calculated as:

$$GGREMP = [(EMP(t+5) / EMP(t))^{1/5} - 1] \cdot 100, \qquad (17)$$

The geometric growth rate for the secondary sector is calculated as:

$$GGREMS = [(EMS(t+5) / EMS(t))^{1/5} - 1] \cdot 100, \qquad (18)$$

The geometric growth rate for the tertiary sector is calculated as:

$$GGREMT = [(EMT(t+5) / EMT(t))^{1/5} - 1] \cdot 100, \qquad (19)$$

where:

GGREMP	is the average annual geometric growth rate of employment in the primary sector for the interval,
GGREMS	is the average annual geometric growth rate of employment in the secondary sector for the interval, and
GGREMT	is the average annual geometric growth rate of employment in the tertiary sector for the interval.

If the projections were based on the assumption of continuous growth, then the rates of growth of employment by major sector would be calculated using the formula for obtaining the exponential growth rate. The calculations would be as follows:

The exponential growth rate for the primary sector is calculated as:

$$EGREMP = [(\ln (EMP(t+5) / EMP(t))) / 5] \cdot 100, \quad (20)$$

The exponential growth rate for the secondary sector is calculated as:

$$EGREMS = [(\ln (EMS(t+5) / EMS(t))) / 5] \cdot 100, \quad (21)$$

The exponential growth rate for the tertiary sector is calculated as:

$$EGREMT = [(\ln (EMT(t+5) / EMT(t))) / 5] \cdot 100, \quad (22)$$

where:

EGREMP is the average annual exponential growth rate of employment in the primary sector for the interval,

EGREMS is the average annual exponential growth rate of employment in the secondary sector for the interval, and

EGREMT is the average annual exponential growth rate of employment in the tertiary sector for the intervals.

(iv) Labour market balances

Once various projection results are obtained, it is possible to calculate the excess demand for labour or excess supply of labour using projections of labour force and employment as indicators of the future supply of and demand for labour, respectively. Also, it is possible to calculate the excess demand or excess supply as a percentage of the total labour force.

In countries where there is a sizeable non-civilian employment which may include military or internal security personnel, the projected labour force to be used in these calculations should not be the projected total labour force, which can be obtained as described in chapter V. The projected labour force to be used is the projected civilian labour force, which can be calculated as the difference between the projected total labour force and projected non-civilian employment, where the latter projection is an additional input.

The reason for this is related to the fact that in projections regarding the labour market, the projections of demand for labour (or employment) will normally apply to the civilian segment of the labour market. Therefore, projections of the supply of labour (or labour force) used to compute excess supply or demand, must also be those for the civilian segment of the market.

To calculate excess supply or excess demand, therefore, the civilian labour force may first have to be calculated; for the end of the time interval (t to t+5), this can be obtained as:

$$CLF(t+5) = LF(t+5) - NEM(t+5), \qquad (23)$$

where:

CLF(t+5) is the civilian labour force at the end of the interval,

LF(t+5) is the total labour force at the end of the interval, and

NEM(t+5) is the non-civilian employment at the end of the interval.

The excess supply of (or demand for) labour for the end of the interval can be obtained as the difference between the projected civilian labour force and the projected employment for that date:

$$EXL(t+5) = CLF(t+5) - EM(t+5), \qquad (24)$$

where:

EXL(t+5) is the excess supply of labour (if positive) or excess demand for labour (if negative) for the end of the interval.

The excess demand or excess supply as a percentage of the civilian labour supply (civilian labour force) can be calculated as:

$$PEXL(t+5) = [\ EXL(t+5) / CLF(t+5)\ ] \cdot 100, \qquad (25)$$

where:

PEXL(t+5) is the excess supply of labour or excess demand for labour as a percentage of the civilian labour force at the end of the interval.

3. Urban-rural level

This section will describe a procedure to calculate an urban-rural projection of employment. The procedure, which is similar to that used in the national projection, projects the levels of labour productivity and employment by industry and derives a variety of other results.

(a) Labour productivity by industry

Labour productivity in urban and rural areas can be calculated by industry for a given projection date using an urban-rural equivalent of the step shown by equation (1) or equation (2).

(i) Discrete growth

If it is assumed that the growth in labour productivity over time occurs in discrete intervals, the levels of labour productivity at the end of the projection interval (t to t+5) would be calculated as:

$$LP(i,k,t+5) = LP(i,k,0) \cdot (1 + GRLP(i,k)/100)^{t+5}; \qquad (26)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

k = 1,2	are urban and rural locations,
LP(i,k,t+5)	is the labour productivity in industry i in location k at the end of the interval,
LP(i,k,0)	is the labour productivity of industry i in location k in the initial year of the projection period, and
GRLP(i,k)	is the annual geometric rate of change of labour productivity in industry i in location k expressed in percentage.

(ii) Continuous growth

If it is assumed that the growth in labour productivity is continuous, the levels of labour productivity at the end of any projection interval would be obtained by means of an urban-rural equivalent of the step indicated by equation (2).

(b) Employment by industry

Given the levels of labour productivity by industry in each area, the levels of employment by sector can be obtained using an urban-rural equivalent of equation (3):

$$EM(i,k,t+5) = VA(i,k,t+5)/LP(i,k,t+5); \qquad (27)$$

$$i = 1,...,I;$$

$$k = 1,2,$$

where:

EM(i,k,t+5) is the employment in industry i in location k at the end of the interval, and

VA(i,k,t+5) is the value added in industry i in location k at the end of the interval.

(c) Other results

The indicators described in relation to the national projection can also be computed as part of an urban-rural projection. Those indicators are, however, calculated for urban and rural areas and for the entire country, using steps analogous to those indicated by equations (4) through (22). The projection would also permit the calculation of the excess supply of (or excess demand for) labour using calculations analogous to those described by equations (23) through (25). These calculations would refer to the urban and rural areas as well as the entire country. In addition, indicators of the distribution of employment by location--proportions urban and rural--can be calculated.

(i) Proportions of employment that are urban and rural

The proportions of employment occurring in urban and rural areas, respectively, can be derived both for total employment and employment by sector.

a. Proportions of total employment

The proportion of total employment that is urban (k=1) is obtained as a ratio of total employment in urban areas to total employment in the entire country:

$$PEMURB(t+5) = EM(1,t+5)/EM(t+5), \qquad (28)$$

where:

PEMURB(t+5) is the proportion of total employment that is urban at the end of the interval, and

EM(k,t+5) is total employment in location k at the end of the interval.

The proportion of total employment that is rural (k=2) can be obtained as the complement of the proportion urban:

$$PEMRUR(t+5) = 1 - PEMURB(t+5), \qquad (29)$$

where:

PEMRUR(t+5) is the proportion of total employment that is rural at the end of the interval.

b. Proportions of employment by sector

Proportions of employment in the primary, secondary and tertiary sectors that are urban (k=1) can be calculated as ratios of urban employment in those sectors to national employment in those sectors. In particular, the proportions can be calculated as follows:

The proportion of employment in the primary sector that is urban is:

$$PEMPURB(t+5) = EMP(1,t+5)/EMP(t+5), \qquad (30)$$

The proportion of employment in the secondary sector that is urban is:

$$PEMSURB(t+5) = EMS(1,t+5)/EMS(t+5), \qquad (31)$$

The proportion of employment in the tertiary sector that is urban is:

$$PEMTURB(t+5) = EMT(1,t+5)/EMT(t+5), \qquad (32)$$

where:

PEMPURB(t+5) is the proportion of employment in the primary sector that is urban at the end of the interval,

PEMSURB(t+5) is the proportion of employment in the secondary sector that is urban at the end of the interval, PEMTURB(t+5) is the proportion of employment in the tertiary sector that is urban at the end of the interval,

EMP(k,t+5) is the employment in the primary sector in location k at the end of the interval,

EMS(k,t+5) is the employment in the secondary sector in location k at the end of the interval, and

EMT(k,t+5) is the employment in the tertiary sector in location k at the end of the interval.

For each sector, the proportions of employment that are rural (k=2) can be obtained as complements of proportions urban:

The proportion of employment in the primary sector that is rural is:

$$PEMPRUR(t+5) = 1 - PEMPURB(t+5), \tag{33}$$

The proportion of employment in the secondary sector that is rural is:

$$PEMSRUR(t+5) = 1 - PEMSURB(t+5), \tag{34}$$

The proportion of employment in the tertiary sector that is rural is:

$$PEMTRUR(t+5) = 1 - PEMTURB(t+5), \tag{35}$$

where:

PEMPRUR(t+5) is the proportion of employment in the primary sector that is rural at the end of the interval,

PEMSRUR(t+5) is the proportion of employment in the secondary sector that is rural at the end of the interval, and

PEMTRUR(t+5) is the proportion of employment in the tertiary sector that is rural at the end of the interval.

This completes the discussion of the technique for making employment projections. The following section will discuss the preparation of the inputs needed for making the projection.

C. The inputs

This section will first list the inputs used by the labour productivity method of employment projection and then describe how they can be prepared.

1. Types of inputs required

The following types of inputs are required in order to apply the labour productivity method:

(i) Projected levels of value added, by industry;
(ii) Assumptions on initial-year levels of labour productivity, by industry;
(iii) Assumptions on constant rates of change of labour productivity, by industry.

If projections of labour surpluses and/or shortages are also to be prepared, the inputs should include:

(iv) Projected total labour force;
(v) Projected non-civilian employment.

Depending on whether one wishes to make a national projection or a projection for urban and rural areas, those inputs will refer to the entire country or to urban and rural areas.

2. Preparation of the inputs

The projected levels of value added, by industry, can be obtained directly from value added projections, which are part of a typical development planning exercise. Procedures for making such projections are not discussed in this module or elsewhere in this volume, but a brief description of the procedure that has been often used by planners to project value added by industry is given in box 17. Projections of the total labour force can be prepared as described in chapter V, while those of non-civilian employment can be obtained by considering likely future developments in the non-civilian sector of the economy. Assumptions on the initial-year levels of labour productivity and on constant rates of change of labour productivity would need to be prepared by the user of the method. Typically, one would first prepare assumptions on the rates of change of labour productivity.

(a) Assumptions on rates of change of labour productivity

Assumptions on rates of change of labour productivity can be derived from time series data on value added and employment, by industry, for a recent time period. These rates of change can be obtained from such time series, which can be fairly long (referring to 15 or more years) or very short (referring to as few as two years). Alternatively, assumptions on the rates of change of labour productivity can be selected on the basis of anticipated trends in factors influencing labour productivity, including government policies. Of course, assumptions that are not based on empirical data are very likely to be less reliable, since there is always the danger of using hoped-for trends in place of plausibly anticipated trends.

Wherever possible, 10 or more observations on labour productivity, covering a period of 10 years or longer should be used if the assumptions on rates of change of labour productivity are derived from historical data. The reason is that labour productivity tends to be quite sensitive to the business cycle, declining during periods of economic down-turn and increasing during periods of economic expansion. In addition, historical data on labour productivity may reflect specific government policies (e.g., a wage freeze, pressure to expand employment, land reform), which may not apply over long time periods. As a result, unless a longer time series is used, estimated rates of change of labour productivity may reflect conditions prevailing over short, a typical periods and may not be suitable for medium-range and especially long-term projections.

Before describing and illustrating how rates of change of labour productivity can be estimated from both longer time series as well as very short time series, we shall briefly discuss this type of information and sources from which they are typically obtained. Some problems typical to this kind of data will also be briefly described.

(i) <u>Time series data</u>

Time series data on employment, by industry, at the national or urban-rural level generally can be obtained from annual surveys of establishments or from periodic labour force surveys of households. Unfortunately, in many countries those data may refer only to the "modern" establishment in various industries. However, where data on "traditional" establishments are available only for a few years, they may be used as a basis for inflating the employment in modern establishments in order to estimate total employment levels, by industry, over time.

Time series on value added, by industry, would normally be obtained from the national accounts, and they would refer (at least in principle) to entire industries. Unfortunately, national accounts (or other relevant data sources) will rarely include value added information for industries broken down by urban-rural location. Therefore, the use of the method with rates of change in labour productivity derived from relatively long time series would be normally possible only at the national level. Where one wishes to make projections of employment by location using rates of productivity change grounded in historical data, it would be often necessary to find estimates of those rates using relatively short time series.

Box 17

A procedure to project value added

A common procedure to project value added by industry makes use of an input-output model along with projected levels of final demand, by industry, for selected future dates.

The input-output model, which is normally constructed around a fixed input-output matrix can show the implications of a specific change in one part of the economy for the rest of the economy. Each row of the input-output matrix indicates the way in which the output of the industry corresponding to that row is used to satisfy final demand or as inputs to other industries. Each column of the matrix shows the origins of the inputs used by the given industry, including those of factors of production (e.g. labour). The projected final demand indicates projected levels of final use of goods and services produced in various industries at various future dates. Examples of final use are household consumption and government consumption.

Once the planner has projected levels of final demand for each industry, projected levels of value added by industry can be calculated by substituting the projected levels of final demand, by industry, into the input-output model. In particular, the projected level of value added for any given industry can be obtained as the difference between the derived value of total production for the industry, and the derived value of intermediate inputs purchased from other industries.

Examples of longer time series data on employment and value added by industry at the national level are shown in tables 49 and 50, respectively. Such time series would be required to derive time series of labour productivity by industry, such as those presented in table 51, which were obtained by dividing the time series of value added in table 50 by those of employment in table 49. Such time series of labour productivity would be used to estimate constant rates of change of labour productivity using OLS or some similar regression analysis technique.

Tables 52 and 53 provide examples of very short time series, which include observations on employment and value added for urban and rural areas, by industry, for just two years. Such data would be required to derive observations of labour productivity by location and industry, such as those shown in table 54. These observations were derived by dividing for each industry the value added (table 53) by the employment (table 52). Information on labour productivity such as that presented in table 54 would make it possible to estimate constant rates of change of labour productivity by industry and location for the time period between the dates to which the data refer.

Table 49. Employment for the entire country, by industry: 1968-1978

(Thousands of employed persons)

Year	Agri- culture	Mining	Manufac- turing	Utilities	Cons- truction	Trade	Transport	Ser- vices
1968	1656.5	5.0	124.5	9.6	57.6	79.3	68.5	312.8
1969	1688.5	5.4	127.7	9.2	53.8	79.7	73.9	330.1
1970	1739.8	6.3	137.8	8.9	55.8	77.4	84.4	336.8
1971	1766.1	7.2	155.4	9.0	59.8	85.4	82.1	365.9
1972	1764.8	7.1	145.5	11.3	64.3	83.9	86.2	375.7
1973	1843.5	7.4	159.8	9.6	69.4	81.1	82.5	391.4
1974	1846.3	9.4	171.8	10.0	76.0	100.5	89.2	442.1
1975	1887.0	8.3	170.3	13.7	71.0	94.1	83.7	468.5
1976	1917.8	10.3	184.0	14.4	81.8	104.9	88.4	483.3
1977	1955.6	9.5	198.2	15.6	85.4	108.5	89.5	503.3
1978	1994.0	6.7	217.3	15.3	93.8	109.4	95.1	516.2

Table 50. Value added for the entire country, by industry: 1968-1978

(Millions of local currency units; constant 1968 prices)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
1968	155.2	2.2	44.6	8.2	18.4	41.2	38.1	119.2
1969	165.4	2.0	48.6	9.1	18.6	44.0	38.6	128.1
1970	172.4	2.6	52.5	9.8	19.0	44.6	41.2	139.0
1971	175.9	2.7	59.3	10.6	20.2	47.1	43.1	152.9
1972	189.3	2.5	63.6	11.6	23.1	42.6	42.4	172.2
1973	199.4	3.6	70.8	11.9	23.8	45.5	45.2	185.7
1974	202.6	3.9	74.9	12.7	22.2	46.4	44.7	207.1
1975	237.1	3.5	75.5	13.7	21.2	49.5	42.0	223.0
1976	235.2	3.8	89.6	15.4	20.7	51.8	46.4	237.5
1977	256.5	4.0	103.9	16.5	22.3	50.5	47.7	252.6
1978	260.3	2.6	118.9	18.0	23.5	50.9	50.3	266.9

Table 51. Average labour productivity for the entire country, by industry, 1968-1978

(Local currency units per person-years of employment)

Year	Agri- culture	Mining	Manufac- turing	Utilities	Cons- truction	Trade	Transport	Ser- vices
1968	0.093	0.440	0.358	0.854	0.319	0.519	0.556	0.381
1969	0.097	0.370	0.380	0.989	0.345	0.552	0.522	0.388
1970	0.099	0.412	0.380	1.101	0.340	0.576	0.488	0.413
1971	0.099	0.375	0.381	1.177	0.337	0.551	0.524	0.418
1972	0.107	0.352	0.437	1.026	0.359	0.507	0.491	0.458
1973	0.108	0.486	0.443	1.239	0.342	0.561	0.547	0.474
1974	0.109	0.414	0.435	1.270	0.292	0.461	0.501	0.468
1975	0.125	0.421	0.443	1.000	0.298	0.526	0.501	0.476
1976	0.122	0.368	0.486	1.069	0.253	0.493	0.524	0.491
1977	0.131	0.421	0.524	1.057	0.261	0.465	0.532	0.502
1978	0.130	0.388	0.547	1.176	0.250	0.465	0.528	0.517

Table 52. Employment for urban and rural areas, by industry: 1973 and 1978

(Thousands of employed persons)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
				Urban				
1973	16.8	5.9	137.9	8.9	59.4	72.9	81.2	274.9
1978	22.2	5.9	184.2	12.7	81.4	98.8	92.2	334.3
				Rural				
1973	1826.7	1.5	21.9	0.7	10.0	8.2	1.4	116.5
1978	1971.8	0.8	33.1	2.6	12.4	10.6	2.9	181.9

Table 53. Value added for urban and rural areas, by industry: 1973 and 1978

(Millions of local currency units; constant 1968 prices)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
				Urban				
1973	6.9	3.1	66.9	10.3	21.2	43.6	44.8	146.8
1978	12.9	2.4	111.8	15.7	21.2	48.9	49.3	200.2
				Rural				
1973	192.5	0.5	3.9	1.6	2.6	1.9	0.4	38.9
1978	247.3	0.2	7.1	2.3	2.3	2.0	1.0	66.7

Table 54. Average labour productivity for urban and rural areas, by industry: 1973 and 1978

(Local currency units per person-years of employment)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
				Urban				
1973	0.412	0.524	0.485	1.159	0.357	0.597	0.552	0.534
1978	0.581	0.406	0.606	1.236	0.260	0.494	0.534	0.599
				Rural				
1973	0.105	0.336	0.177	2.260	0.258	0.237	0.257	0.334
1978	0.125	0.250	0.214	0.884	0.185	0.188	0.344	0.367

(ii) Rates of change based on time series consisting of several observations

Estimating constant rates of change will differ depending on the number of observations in the time series being utilised. It will be shown initially how to obtain estimates of these rates from relatively long time series.

a. Estimation procedures

The procedure used to estimate constant rates of change in labour productivity will differ depending on whether the assumption of discrete growth or the assumption of continuous growth is used.

i. Discrete growth

If it is assumed that labour productivity grows in discrete intervals, estimates of constant rates of change in labour productivity, by industry, at the national level can be obtained from time series data on labour productivity by estimating the coefficients of a set of functions that can be derived from the following functions:

$$LP(i,t') = LP(i,0) \cdot (1 + GRLP(i)/100)^{t'}; \qquad (36)$$

$$i = 1,\ldots,I,$$

where:

t'	is the calendar year,
LP(i,t')	is the labour productivity in industry i in year t',
LP(i,0)	is the labour productivity of industry i in the initial year of the time series.

In order to arrive at the functions to be estimated, it would be necessary to take natural logarithms of the functions by industry shown in equation (36) and to add a random disturbance term (box 18) to each:

$$\ln LP(i,t') = a(i) + b(i) \cdot t' + u(i,t'); \qquad (37)$$

$$i = 1,\ldots,I$$

where:

$$a(i) = \ln LP(i,0),$$

$$b(i) = \ln[\,1 + GRLP(i)/100\,],$$

Box 18

Glossary

Asymptotically unbiased
An estimator, such as the coefficient in a regression equation, is said to be asymptotically unbiased if the probability that the estimator is different from the true value of the parameter it purports to assess approaches zero as the size of the sample approaches infinity.

Coefficient of determination, R^2
The measure of the goodness of fit of a regression equation, which denotes the proportion of the variance in the dependent variable associated with independent variable(s) included in the regression. The coefficient may lie between 0 and 1: when it is close to 0, it suggests a weak relationship; when it is close to 1, a strong one.

Random disturbance term
The term added to a regression equation as an average relationship between dependent and independent variables, which ensures equality between the left and the right hand side of the equation for each observation. The disturbance or error term may represent random disturbances in an observation or it may reflect errors of measurement.

Statistically significant
An estimate of a particular statistic, such as a partial regression coefficient, is said to be statistically significant if the probability that it could have occurred by chance is less than, say, 5 per cent.

t-statistic
In regression analysis, a statistic calculated for each partial coefficient which makes it possible for the analyst to determine whether or not the coefficient is statistically significant.

and where:

a(i) is the intercept coefficient of the function for industry i,

b(i) is the partial coefficient of the function for industry i, and u(i,t') is the random disturbance term in the function for industry i in year t'.

The functions indicated in equation (37) can be estimated from time series data on labour productivity by industry using such standard methods of regression analysis as OLS.

Once the coefficients of the functions shown in equation (37) are estimated, the estimates of constant geometric rates of change of labour productivity, GRLP(i)'s, can be obtained as follows:

$$GRLP(i) = [\text{antiln } b^*(i) - 1] \cdot 100; \qquad (38)$$

$$i = 1,\ldots,I,$$

where:

b*(i) is the estimate of the partial coefficient of the function for industry i, and

antiln is the antilogarithm of the natural logarithm.

This estimate of GRLP(i) is <u>asymptotically unbiased</u> and has other desirable properties.

Estimates of labour productivity for urban and rural areas can also be obtained using longer time series of labour productivity, given the assumption of discrete growth in labour productivity. Initially, one would transform urban-rural equivalents of the functions shown in equation (36) and add a random disturbance term to each. The result would be urban-rural counterparts of functions indicated in equation (37), which, when estimated, would provide the basis for deriving the estimates of the rates of change of labour productivity for the two areas. In particular, those estimates would be derived from the estimates of the partial coefficients using the urban-rural equivalent of the step described by equation (38).

ii. <u>Continuous growth</u>

If it is assumed that labour productivity grows continuously over time, estimates of constant rates of change of labour productivity, by industry, for the entire country can be obtained from time series data on labour productivity by estimating parameters of the functions that can be derived from the following set of functions:

$$LP(i,t') = LP(i,0) \cdot e^{[(ERLP(i)/100) \cdot t']}; \qquad (39)$$

$$i = 1,\ldots,I.$$

In order to derive the functions to be estimated, it is necessary to take natural logarithms of the functions indicated in equation (39) and add a random disturbance term to each:

$$\ln LP(i,t') = a(i) + b(i) \cdot t' + u(i,t'); \qquad (40)$$

$$i = 1,\ldots,I,$$

where:

$$a(i) = \ln LP(i,0),$$

$$b(i) = ERLP(i)/100.$$

These functions can be estimated for each industry from time series data on labour productivity using OLS regression technique. The functions shown in equation (40) are identical to those indicated by equation (37), except for the interpretation of their partial coefficients, b(i)'s, in terms of constant rates of change of labour productivity.

Once the estimates of the coefficients are obtained, the ERLP(i)'s can be estimated by multiplying the corresponding estimates of partial coefficients, b*(i)'s, by 100:

$$ERLP(i) = b^*(i) \cdot 100; \qquad (41)$$

$$i = 1,\ldots,I.$$

An analogous procedure could be used to obtain estimates of rates of change of labour productivity from longer urban and rural time series of labour productivity. First, one would transform urban-rural equivalents of the functions shown in equation (39) and add random disturbance terms to each. This would yield urban-rural counterparts of functions shown in equation (40), which, when estimated, would provide the basis for deriving estimates of rates of change in labour productivity in urban and rural areas. Those rates would be derived from the estimates of the partial coefficients by means of the urban-rural equivalent of the step indicated in equation (41).

b. Illustrative estimations

This section will illustrate the estimation of constant rates of change of labour productivity for the entire country using the time series on labour productivity, by industry, shown in table 51.

i. Discrete growth

If discrete growth in labour productivity is assumed, it will first be necessary to estimate the coefficients of the functions shown in equation (37). Estimates of those functions, obtained from the time series data of table 51, using OLS, are shown in table 55.

Those results are fairly typical of those that could be obtained from time series data in a developing country. Coefficients of determination, R^2's (column 4), for agriculture, manufacturing, construction, trade and services range between 0.502 (trade) to 0.945 (agriculture), and according to t-statistics (shown in parenthesis in column 3), the estimates of coefficients of the time variable are all statistically significant.6/ On the other hand, for mining, utilities and transport, the R^2's are very low and the coefficients of the time variable are not statistically significant.

The estimates of the functions need further to be used to derive the estimates of the rates of change of labour productivity as illustrated in table 56. In particular, the estimate of the rate of change for each industry (column 3) is obtained by taking the antilogarithm of the estimate of the partial coefficient for the industry (column 2), subtracting 1 from the result and multiplying by 100. For example, the estimate of the annual percentage rate of change in labour productivity in agriculture, 3.66, is obtained as:

$$3.66 = [\text{antiln}\ (0.0359) - 1] \cdot 100, \tag{38}$$

where 0.0359 is the estimate of the partial coefficient for agriculture.

ii. Continuous growth

If the growth in labour productivity is assumed to be continuous, then the coefficients of the functions indicated in equation (40) would need to be estimated. Estimates of those functions, based on the data shown in table 51, are identical to the estimates of the functions indicated by equation (37) and presented in table 55. The use of those estimates to derive estimates of the constant rates of change in labour productivity is different from that based on the assumption of discrete growth in labour productivity.

The derivation of the estimates of rates of change of labour productivity is illustrated in table 57, where the estimate of the rate of change for any industry (column 3) is obtained by multiplying the estimate of the partial coefficients for the industry (column 2) by 100. For example, the estimate of the annual percentage rate of change in labour productivity in agriculture, 3.59, is obtained as:

$$3.59 = 0.0359 \cdot 100, \tag{41}$$

Table 55. Estimates of the coefficients of functions relating the logarithm of average labour productivity to time, by industry, for the entire country: lnLP(i,t')= a(i) + b(i)*t'+ u(i,t') a/

Industry	Coefficients		R-square
	Intercept	Time variable b/	
(1)	(2)	(3)	(4)
Agriculture	-73.106	0.036 (12.448)	0.945
Mining	0.035	-0.000 (-0.051)	0.000
Manufacturing	-80.332	0.040 (11.310)	0.934
Utilities	-29.818	0.015 (1.458)	0.190
Construction	64.852	-0.033 (-4.396)	0.682
Trade	33.188	-0.017 (-3.014)	0.502
Transport	-0.207	-0.000 (-0.053)	0.000
Services	-61.068	0.031 (11.780)	0.939

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

Table 56. Computing estimates of constant rates of change in average labour productivity, by industry, assuming discrete growth in labour productivity: entire country

Industry	Coefficient of time variable a/	Rate of change of labour productivity b/
(1)	(2)	(3)
Agriculture	0.036	3.66
Mining	-0.000	-0.05
Manufacturing	0.040	4.11
Utilities	0.015	1.53
Construction	-0.033	-3.29
Trade	-0.017	-1.70
Transport	-0.000	-0.02
Services	0.031	3.10

a/ From table 55, col. 3.
b/ [Antiln(Col.2) - 1] . (100).

Table 57. Computing estimates of constant rates of change in average labour productivity, by industry, assuming continuous growth in labour productivity: entire country

Industry	Coefficient of time variable a/	Rate of change of labour productivity b/
(1)	(2)	(3)
Agriculture	0.036	3.59
Mining	-0.000	-0.05
Manufacturing	0.040	4.03
Utilities	0.015	1.52
Construction	-0.033	-3.35
Trade	-0.017	-1.72
Transport	-0.000	-0.02
Services	0.031	3.05

a/ From table 55, col. 3.
b/ (Col.2) . (100).

where 0.0359 is the estimate of the partial coefficient for agriculture.

(iii) Rates of change based on time series consisting of two observations

Rates of change of labour productivity (both discrete and continuous) can be estimated from time series including as few as two observations on labour productivity, such as those shown in table 54. Since such estimates may be more sensitive to swings in business activity or government policy than those obtained from time series data consisting of several observations, they should be used with considerably more caution for making medium-term and especially long-run employment projections.

a. Estimation procedures

The procedure used to estimate constant rates of change in labour productivity, using observations referring to two dates, will vary depending on the type of assumption on the labour productivity growth used.

i. Discrete growth

If it is assumed that the labour productivity grows in discrete intervals, estimates of the rates of change of labour productivity, by industry, for the entire country based on observations for two points in time can be obtained by using the following formulae, obtained by solving equation (36) for the growth rate in labour productivity:

$$GRLP(i) = [\ (\ LP(i,t')/LP(i,0)\ )^{1/t'} - 1\ ] \cdot 100; \qquad (42)$$

$$i = 1,\ldots,I,$$

where:

$LP(i,0)$ is the labour productivity of industry i in year 0, and

$LP(i,t')$ is the labour productivity in industry i in year t'.

Where labour productivity data are available for urban and rural areas for two points in time, estimates of rates of change of labour productivity by industry for the two areas can be obtained using urban-rural equivalents of the formulae shown in equation (42).

ii. Continuous growth

If labour productivity growth is assumed to be continuous, estimates of the rates of change of labour productivity by industry at the national level can be obtained by using the following formulae, derived from equation (39):

$$ERLP(i) = [(\ln (LP(i,t')/LP(i,0))) / t'] \cdot 100; \qquad (43)$$

$$i = 1,\ldots,I.$$

The urban-rural counterparts of the formulae shown in equation (43) can be used to estimate rates of change of labour productivity for the two locations, by industry, assuming continuous growth.

b. Illustrative estimations

This section will illustrate the procedures used to estimate constant rates of change of labour productivity by location and industry, using the observations of labour productivity shown in table 54.

i. Discrete growth

Table 58 illustrates the estimation of annual rates of change of labour productivity, by industry, for urban and rural areas using data such as those shown in table 57, assuming discrete growth. For each industry, the level of labour productivity corresponding to the later year (column 3) would be divided by the level of labour productivity referring to the earlier year (column 2). The ratio of the two levels (column 4) would then be raised to the power that equals 1 over the number of years between the two years. The estimate of the rate of change for the industry (column 5) would be obtained by subtracting 1 from this result and multiplying by 100.

For example, the annual percentage rate of growth of labour productivity for rural agriculture, 3.55, is obtained as follows:

$$3.55 = [(0.125/0.105)^{1/5} - 1] \cdot 100, \qquad (42)$$

where 0.105 and 0.125 are the levels of labour productivity in rural agriculture in 1973 and 1978 and 5 is the number of years in the 1973-1978 time period.

ii. Continuous growth

Table 59 illustrates how the rates of change of labour productivity for urban and rural areas can be estimated, assuming continuous growth and using the productivity levels shown in table 54. For any given industry, the level

Table 58. Computing estimates of constant rates of change in labour productivity during 1973-1978, by industry, assuming discrete growth in labour productivity: urban and rural areas

Industry	Levels of labour productivity in		Ratio of labour productivity, 1973 to 1978 c/	Estimate of rate of change in labour productivity, 1973 to 1978 d/
	1973 a/	1978 b/		
(1)	(2)	(3)	(4)	(5)
Urban				
Agriculture	0.412	0.581	1.41	7.12
Mining	0.524	0.406	0.78	-4.96
Manufacturing	0.485	0.606	1.25	4.58
Utilities	1.159	1.236	1.07	1.29
Construction	0.357	0.260	0.73	-6.12
Trade	0.597	0.494	0.83	-3.69
Transport	0.552	0.534	0.97	-0.64
Services	0.533	0.598	1.12	2.32
Rural				
Agriculture	0.105	0.125	1.19	3.55
Mining	0.336	0.250	0.74	-5.75
Manufacturing	0.177	0.214	1.21	3.83
Utilities	2.260	0.884	0.39	-17.11
Construction	0.258	0.185	0.72	-6.44
Trade	0.237	0.188	0.79	-4.50
Transport	0.257	0.344	1.34	5.98
Services	0.334	0.366	1.10	1.88

a/ From table 54, year 1973.
b/ From table 54, year 1978.
c/ (Col. 3)/(Col. 2).
d/ $[(\text{Col. } 4)^{1/5} - 1] \cdot (100)$.

Table 59. Computing estimates of constant rates of change in labour productivity during 1973-1978, by industry, assuming continuous growth in labour productivity: urban and rural areas

Industry	Levels of labour productivity in 1973 a/	Levels of labour productivity in 1978 b/	Ratio of labour productivity, 1973 to 1978 c/	Estimate of rate of change in labour productivity, 1973 to 1978 d/
(1)	(2)	(3)	(4)	(5)
		Urban		
Agriculture	0.412	0.581	1.41	6.88
Mining	0.524	0.406	0.78	-5.09
Manufacturing	0.485	0.606	1.25	4.48
Utilities	1.159	1.236	1.07	1.28
Construction	0.357	0.260	0.73	-6.31
Trade	0.597	0.494	0.83	-3.76
Transport	0.552	0.534	0.97	-0.64
Services	0.533	0.598	1.12	2.29
		Rural		
Agriculture	0.105	0.125	1.19	3.48
Mining	0.336	0.250	0.74	-5.92
Manufacturing	0.177	0.214	1.21	3.76
Utilities	2.260	0.884	0.39	-18.76
Construction	0.258	0.185	0.72	-6.65
Trade	0.237	0.188	0.79	-4.61
Transport	0.257	0.344	1.34	5.81
Services	0.334	0.366	1.10	1.87

a/ From table 54, year 1973.
b/ From table 54, year 1978.
c/ (Col. 3)/(Col. 2).
d/ [ln(Col. 4)/5] . (100).

of labour productivity referring to the later year (column 3) is divided by the level of labour productivity corresponding to the earlier year (column 2). The result is the ratio of the two productivity levels (column 4). Then, the logarithm of this ratio is taken and the result is divided by the number of years between those two years. The estimate of the rate of change of labour productivity (column 5) is obtained by multiplying this result by 100.

For example, the rate of change for rural agriculture, 3.48, is derived as follows:

$$3.48 = [(\ln (0.125/0.105)) / 5] \cdot 100; \qquad (43)$$

where 0.105 and 0.125 are the levels of labour productivity in rural agriculture in 1973 and 1978, while 5 is the number of years in the 1973-1978 interval.

(iv) Rates of change based on the assessment of factors influencing labour productivity trends

Another approach to determining rates of change of labour productivity calls for selecting these rates on the basis of the likely future trends in government policies and other relevant factors influencing productivity trends. Owing to its *ad hoc* nature, this approach should be used with caution. In many situations, however, it may prove necessary to combine this approach with the empirical estimation, so that the rates of labour productivity change used in the projection reflect both historical trends in labour productivity and the planner's judgements regarding the likely impact of possible future policies or other factors.

(b) Assumptions on initial-year levels of labour productivity

In most countries, the levels of labour productivity for the initial year would be obtained through some kind of forward extrapolation from the productivity levels for recent year(s). Recent productivity levels can be combined with estimates of the rates of change of labour productivity to obtain the levels of labour productivity for the initial year by using suitable procedures to project labour productivity.

Described below are procedures for deriving levels of labour productivity, classified by industry, using alternative assumptions on the growth of labour productivity. Use of the procedures will be illustrated in a later section.

(i) Procedures to derive initial-year levels of labour productivity

The procedures used for deriving the initial-year levels of labour productivity vary depending on whether that labour productivity is assumed to grow discretely or continuously.

a. Discrete growth

Where discrete growth in labour productivity is assumed, the initial-year levels of labour productivity for the entire country can be obtained using the following version of the functions indicated by equation (1):

$$LP(i,0) = LP(i,-t^*) \cdot (1 + GRLP(i)/100)^{t^*}; \qquad i = 1,\ldots,I, \tag{44}$$

where:

t^*	is the number of years between the year to which the observed levels of labour productivity used refer and the initial year of the projection period (t=0),
$-t^*$	is the year preceding the initial year of projection (t=0) in which the levels of labour productivity were observed,
LP(i,0)	is the level of labour productivity of industry i in the initial year of the projection period, and
LP(i,-t*)	is the level of labour productivity of industry i observed in year -t*.

To derive initial-year levels of labour productivity, it would be necessary to use observations on the levels of labour productivity for a recent year (-t* in equation (44)) and estimates of the rates of change in labour productivity that assume discrete growth in labour productivity.

Where labour productivity has fluctuated widely during the period to which the data refer, it would be preferable to use the mean levels of labour productivity, by industry, for several years rather than the levels pertaining to any single year. This would reduce the likelihood that conditions peculiar to any particular year would unduly influence the initial-year levels of labour productivity. In such a case, -t* will refer to the central point of the time period to which the mean levels of labour productivity refer.

Initial-year productivity levels for urban and rural areas, can be obtained by a procedure analogous to that applying to the national level (equation (44)). In this case, it would be necessary to use the levels of productivity for urban and rural areas for a recent year or mean levels for several such years along with the rates of change of labour productivity for the two areas.

(b) Continuous growth

If it is assumed that the labour productivity growth is continuous, the initial-year levels of labour productivity can be obtained using the following versions of the functions indicated by equation (2):

$$LP(i,0) = LP(i,-t^*) \cdot e^{[(ERLP(i)/100) \cdot t^*]}; \qquad (45)$$

$$i = 1,\ldots,I.$$

To compute initial-year levels of labour productivity, it would be necessary to use levels of labour productivity for a recent year or mean levels of productivity for several such years along with estimates of the rates of change of labour productivity assuming continuous growth in labour productivity.

To derive initial-year levels of labour productivity for urban and rural areas, it would be necessary to use the urban-rural equivalents of the formulae shown in equation (45). This would require data on recent levels of labour productivity for urban and rural areas along with the estimates of rates of change in labour productivity for those areas.

(ii) Illustrative derivations of initial-year levels of labour productivity

This section will illustrate the derivation of the levels of labour productivity for the initial year of the projection period assuming discrete growth of labour productivity. The illustrations, which will be for the entire country and for urban and rural areas separately, will take 1980 as the initial year of the projection period and 1978 as the year to which the recent levels of labour productivity refer. Hence, $-t^*$ will equal -2 and t^* will equal 2.

The illustrative examples will use the data and estimates presented earlier in illustrating the procedures for estimating constant rates of change of labour productivity. In particular, the example for the entire country will be based on the levels of labour productivity for 1978, shown in table 51, and the estimates of the rates of change in labour productivity indicated in table 56 (column 3). The example for urban and rural areas will use levels of labour productivity for 1978 displayed in table 54, along with the estimates of rates of change in labour productivity shown in table 58. The national and urban-rural estimates of the rates of change of labour productivity will be used on the assumption that the estimated rates of change would prevail between 1978 and 1980.

Table 60 shows how initial-year levels of labour productivity are derived for the entire country. For each industry, the level of labour productivity in the initial year (column 5) is obtained by multiplying the observed level of labour productivity for the industry in a chosen year (column 2), which is year 1978, by a factor (column 4) indicating the growth in labour productivity for the industry until the initial year, which is year 1980. This factor is calculated using the rate of change of labour productivity previously estimated for that industry (column 3).

For example, the initial-year level of labour productivity in agriculture, 0.140, is obtained as follows:

$$0.140 = 0.131 \cdot (1 + 3.66/100)^2, \qquad (44)$$

where 0.131 is the level of labour productivity in agriculture in 1978; 3.66 is the estimate of the historical annual percentage rate of change of labour productivity in agriculture; while 2 is the number of years between 1978 and 1980.

Tables 61 and 62 illustrate the derivation of the initial-year levels of labour productivity for urban and rural areas by industry. As shown in either table, the level of labour productivity for any given industry (column 5) is found as the product of the level of labour productivity in a chosen year (column 2) and the factor (column 4) indicating the growth in labour productivity between the selected year and the initial year of the projection. This factor is obtained from the rate of change of labour productivity used for the industry (column 3).

Thus, the level of labour productivity in rural agriculture for the initial year, 0.134, is calculated as follows:

$$0.134 = 0.125 \cdot (1 + 3.55/100)^2, \qquad (45)$$

where 0.125 is the labour productivity in rural agriculture in 1978 and 3.55 is the estimate of the rate of change in labour productivity in rural agriculture.

D. Illustrative examples of projections

The examples presented below will illustrate the use of the labour productivity method to prepare a national projection and an urban-rural projection, respectively. These examples will indicate how the relevant calculations are made for the projection interval 0-5. In addition, they will provide complete projection results for the 20-year period. The examples will be based on the assumption that the growth in labour productivity occurs over discrete intervals. However, they will also illustrate steps to be used when continuous growth of labour productivity is assumed.

Table 60. Calculating initial-year levels of labour productivity, by industry, assuming discrete growth in labour productivity: entire country

Industry (1)	Levels of labour productivity in year -2 a/ (2)	Constant rate of change of labour productivity b/ (3)	Factor indicating increase in labour productivity between year -2 and year 0 c/ (4)	Level of labour productivity in year 0 d/ (5)
Agriculture	0.130	3.66	1.074	0.140
Mining	0.388	-0.05	0.999	0.388
Manufacturing	0.547	4.11	1.083	0.593
Utilities	1.176	1.53	1.030	1.213
Construction	0.250	-3.29	0.935	0.234
Trade	0.465	-1.70	0.966	0.450
Transport	0.528	-0.02	0.999	0.529
Services	0.517	3.10	1.063	0.550

a/ From table 51, year 1978.
b/ From table 56, col. 3.
c/ $[1 + ((\text{Col. } 3)/100)]^2$.
d/ (Col. 2) . (Col. 4).

Table 61. Calculating initial-year levels of labour productivity, by industry, assuming discrete growth in labour productivity: urban areas

Industry	Levels of labour productivity in year -2 a/	Constant rate of change of labour productivity b/	Factor indicating increase in labour productivity between year -2 and year 0 c/	Level of labour productivity in year 0 d/
(1)	(2)	(3)	(4)	(5)
Agriculture	0.581	7.12	1.147	0.667
Mining	0.406	-4.96	0.903	0.367
Manufacturing	0.606	4.58	1.093	0.664
Utilities	1.236	1.29	1.026	1.268
Construction	0.260	-6.12	0.881	0.230
Trade	0.494	-3.69	0.927	0.459
Transport	0.534	-0.64	0.987	0.528
Services	0.598	2.32	1.046	0.627

a/ From table 54, year 1978, urban.
b/ From table 58, col. 5, urban.
c/ $[1 + ((\text{Col. } 3)/100)]^2$.
d/ (Col. 2) . (Col. 4).

Table 62. Calculating initial-year levels of labour productivity, by industry, assuming discrete growth in labour productivity: rural areas

Industry (1)	Levels of labour productivity in year -2 a/ (2)	Constant rate of change of labour productivity b/ (3)	Factor indicating increase in labour productivity between year -2 and year 0 c/ (4)	Level of labour productivity in year 0 d/ (5)
Agriculture	0.125	3.55	1.072	0.134
Mining	0.250	-5.75	0.888	0.222
Manufacturing	0.214	3.83	1.078	0.231
Utilities	0.884	-17.11	0.687	0.608
Construction	0.185	-6.44	0.875	0.162
Trade	0.188	-4.50	0.911	0.172
Transport	0.344	5.98	1.123	0.387
Services	0.366	1.88	1.038	0.381

a/ From table 54, year 1978, rural.
b/ From table 58, col. 5, rural.

c/ $[1 + ((\text{Col. } 3)/100)]^2$.
d/ (Col. 2) . (Col. 4).

1. National level

The calculations involved in projecting employment presented in this example will be based on the inputs contained in table 63. These include projected levels of value added, by industry, along with the assumed levels of labour productivity for the initial-year and the assumed rates of change in labour productivity, by industry. The value added levels are given for dates five years apart, starting with the initial year of the plan, which is denoted as year 0. The assumed initial-year levels of labour productivity are those which were derived as part of the example shown in the section on inputs (table 60). The assumed rates of change of labour productivity are the rates derived by the illustrative estimation of such rates from time series data based on discrete growth assumption (table 56).

(a) Labour productivity by industry

For any given year during the projection period, the levels of labour productivity can be calculated for each industry from the initial-year levels and the assumed constant rates of change of labour productivity by industry. The calculations for the end of the projection interval 0-5 (year 5), assuming discrete growth in labour productivity over time are illustrated in table 64. The level of productivity in year 5 for each industry (column 4) is obtained by substituting values into equation (1). Thus, labour productivity is the product of the initial-year level of labour productivity (column 2) and the factor indicating the increase in labour productivity between the initial year and year 5. This factor is obtained using the assumed rate of change of labour productivity for the industry (column 3).

(i) Discrete growth

For example, the level of labour productivity in agriculture at the end of the interval 0-5, 0.168, is obtained as:

$$0.168 = (0.140)\ (\ 1 + 3.66/100\ )^5, \qquad (1)$$

where 0.140 is the initial-year level of labour productivity in agriculture (column 2); 3.66 is the annual geometric growth rate of labour productivity in this industry (column 3); and 5 is the number of years between the initial year (year 0) and year 5.

(ii) Continuous growth

If the growth of labour productivity is assumed to be continuous, then the same inputs as above would produce the same result for labour productivity in agriculture in year 5, 0.168:

Table 63. Inputs for projecting employment, by industry: entire country

Industry	Value added in year (millions of LCUs a/)					Initial-year labour productivity (thousands of LCUs a/ per person)	Rate of change of labour productivity (percentage)
	0	5	10	15	20		
Agriculture	273.1	308.2	347.8	392.5	443.1	0.140	3.66
Mining	2.9	4.0	5.5	7.6	10.4	0.387	-0.05
Manufacturing	140.4	212.7	322.5	489.3	742.7	0.593	4.11
Utilities	21.2	31.9	48.0	72.5	109.4	1.212	1.53
Construction	25.6	31.5	38.9	48.0	59.3	0.234	-3.29
Trade	59.0	85.1	122.9	177.6	256.7	0.449	-1.70
Transport	56.0	73.0	95.3	124.4	162.5	0.528	-0.02
Services	312.5	464.3	691.2	1031.2	1541.4	0.549	3.10

a/ Local currency units.

Table 64. Calculating labour productivity, by industry: entire country, year 5

Industry	Initial-year labour productivity a/ (thousands of LCUs d/ per person)	Rate of change of labour productivity b/ (percentage)	Labour productivity c/ (thousands of LCUs d/ per person)
(1)	(2)	(3)	(4)
Agriculture	0.140	3.66	0.167
Mining	0.387	-0.05	0.386
Manufacturing	0.593	4.11	0.725
Utilities	1.212	1.53	1.308
Construction	0.234	-3.29	0.198
Trade	0.449	-1.70	0.412
Transport	0.528	-0.02	0.528
Services	0.549	3.10	0.640

a/ From table 63.
b/ From table 63.
c/ $(\text{Col. } 2) \cdot (1 + (\text{Col. } 3)/100)^5$.
d/ Local currency units.

$$0.168 = (0.140) \cdot e^{[(3.66/100) \cdot 5]}. \qquad (2)$$

For relatively low annual rates of change in labour productivity, such as 3.66 per cent, the results obtained with different assumptions on the type of growth of labour productivity (discrete or continuous) would be very similar.

(b) Employment by industry

Once the levels of labour productivity by industry are computed for a given date, the levels of employment for that date can be obtained from the assumed levels of value added and the projected productivity levels, as illustrated in table 65. The employment level for each industry (column 4) is obtained by dividing the value added for the industry (column 2) by the level of labour productivity for the industry (column 3).7/

Thus, the level of employment in agriculture in year 5, 1,835.5, is obtained as:

$$1{,}835.5 = 308.2/0.168, \qquad (3)$$

where 308.2 is the projected value added in agriculture and 0.168 is the projected level of labour productivity in agriculture in year 5.

Performing the calculations illustrated above for each five-year interval of the entire projection period produces the projected levels of employment by industry for the entire period. The projected levels for the 20-year projection interval are shown in table 66.

(c) Other results

Other results that are useful in planning can be obtained as part of a projection at the national level. These include various employment aggregates, indicators of the structure of employment and the rates of growth of employment.

(i) Employment aggregates

The employment aggregates that can be derived from the projections by industry include total employment and employment in various sectors at dates five years apart. They also include increases in total employment and employment by sector over the intervening projection intervals.

a. Total employment

Total employment at the end of a given projection interval is obtained by aggregating the projected levels of employment by industry. Total employment

Table 65. Deriving employment, by industry: entire country, year 5

Industry	Value added a/ (LCUs d/)	Labour productivity b/ (thousands of LCUs d/ per person)	Projected employment c/ (thousands of persons)
(1)	(2)	(3)	(4)
Agriculture	308.2	0.167	1835.5
Mining	4.0	0.386	10.4
Manufacturing	212.7	0.725	293.2
Utilities	31.9	1.308	24.4
Construction	31.5	0.198	159.1
Trade	85.1	0.412	206.3
Transport	73.0	0.528	138.3
Services	464.3	0.640	725.1

a/ From table 63.
b/ From table 64, col. 4.
c/ (Col. 2)/(Col. 3).
d/ Local currency units.

Table 66. Projected employment, by industry: entire country

(Thousands of persons)

Industry	Year 0	5	10	15	20
Agriculture	1946.8	1835.5	1730.8	1632.2	1539.4
Mining	7.6	10.4	14.3	19.6	27.0
Manufacturing	236.7	293.2	363.5	450.8	559.4
Utilities	17.5	24.4	34.0	47.6	66.6
Construction	109.1	159.1	232.0	338.5	494.0
Trade	131.1	206.3	324.7	511.1	804.7
Transport	105.8	138.3	180.7	236.2	308.7
Services	568.6	725.1	926.6	1186.6	1522.3

in year 5, 3,392.3, is computed by adding the projected levels of employment by industry. Total employment is shown in table 67 for the entire 20-year projection period. The increase in total employment over this period is indicated in figure XVI.

b. Employment by sector

Employment in the primary, secondary and the tertiary sector can be obtained by aggregating employment projected for various industries, using appropriate aggregation rules. For illustrative purposes, we shall assume that the primary sector consists of agriculture and mining, the secondary sector of manufacturing, utilities and construction and the tertiary sector of trade, transport and services.

i. Employment in the primary sector

Employment in the primary sector in year 5, 1,846.0, is obtained as:

$$1{,}846.0 = 1{,}835.5 + 10.4, \tag{5}$$

where 1,835.5 and 10.4 are, respectively, projected levels of employment in agriculture and mining.

ii. Employment in the secondary sector

Employment in the secondary sector in year 5, 476.7, is obtained as:

$$476.7 = 293.2 + 24.4 + 159.1, \tag{6}$$

where 293.2, 24.4 and 159.1 are, respectively, projected levels of employment in manufacturing, utilities and construction.

iii. Employment in the tertiary sector

Employment in the tertiary sector in year 5, 1,069.6, is obtained as:

$$1{,}069.6 = 206.3 + 138.3 + 725.1, \tag{7}$$

where 206.3, 138.3 and 725.1 are, respectively, projected levels of employment in trade, transportation and services.

Employment by sector obtained for different dates over the projection period is presented in figure XVII.

Table 67. Employment aggregates, structure and rates of growth: entire country

Indicators	Year 0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	3123.2	3392.3	3806.6	4422.6	5322.1
Primary	1954.4	1846.0	1745.1	1651.8	1566.3
Secondary	363.2	476.7	629.5	836.9	1120.1
Tertiary	805.5	1069.6	1432.0	1933.8	2635.8
Growth in employment					
Total		269.1	414.3	616.0	899.6
Primary		-108.4	-100.9	-93.3	-85.5
Secondary		113.4	152.9	207.4	283.1
Tertiary		264.1	362.3	501.8	701.9
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.63	0.54	0.46	0.37	0.29
Secondary	0.12	0.14	0.17	0.19	0.21
Tertiary	0.26	0.32	0.38	0.44	0.50
Rates of growth of employment (percentage)					
Total		1.67	2.33	3.05	3.77
Primary		-1.14	-1.12	-1.09	-1.06
Secondary		5.59	5.72	5.86	6.00
Tertiary		5.84	6.01	6.19	6.39

Figure XVI. Total employment

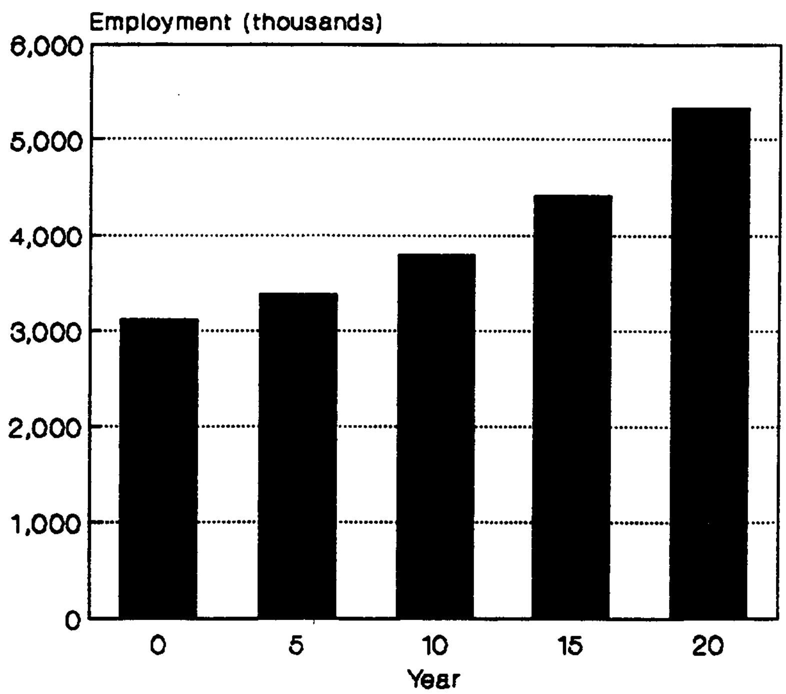
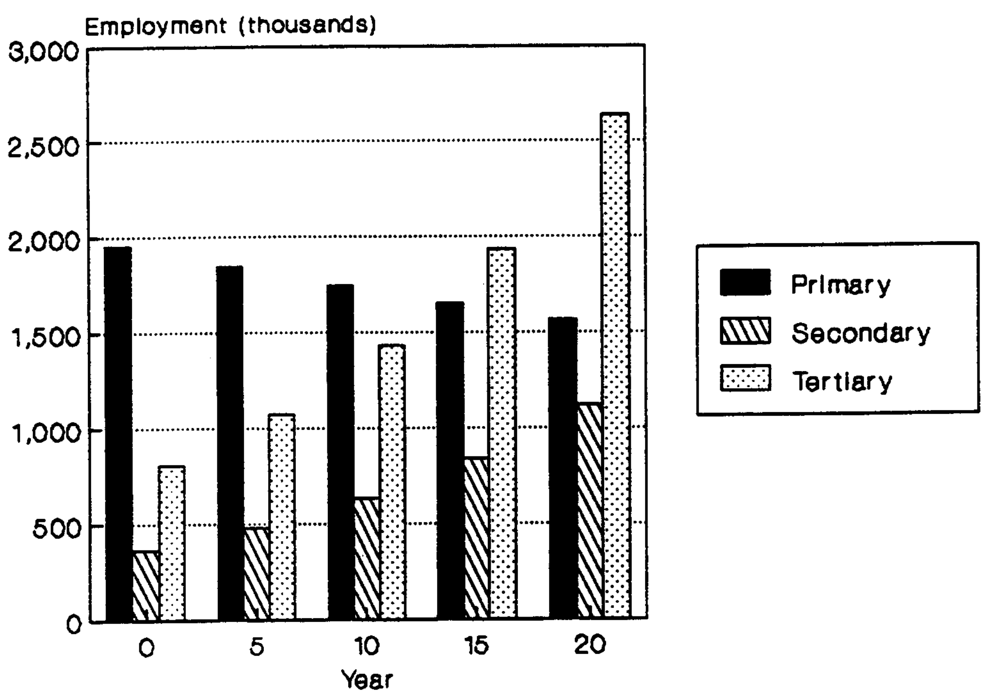


Figure XVII. Employment: primary, secondary and tertiary sectors



c. Growth in total employment

The growth in total employment over a given projection interval equals the difference between total employment at the end of the interval and total employment at its beginning. For the interval 0-5, the growth in total employment 269.1, is obtained as:

$$269.1 = 3{,}392.3 - 3{,}123.2, \qquad (8)$$

where 3,123.2 and 3,392.3 are, respectively, total employment at the beginning and the end of the interval (shown in columns corresponding to years 0 and 5) .

d. Growth in employment by sector

The change in employment over the interval 0-5 in various sectors is obtained as follows:

Decline of employment in the primary sector, -108.4, is:

$$-108.4 = 1{,}846.0 - 1{,}954.4, \qquad (9)$$

where 1,954.4 and 1,846.0 are, respectively, the levels of employment in the primary sector in years 0 and 5;

Growth of employment in the secondary sector, 113.4, is:

$$113.4 = 476.7 - 363.2, \qquad (10)$$

where 363.2 and 476.7 are the levels of employment in the secondary sector in years 0 and 5;

Growth of employment in the tertiary sector, 264.1, is:

$$264.1 = 1{,}069.6 - 805.5, \qquad (11)$$

where 805.5 and 1,069.6 are the levels of employment in the tertiary sector in years 0 and 5.

(ii) Indicators of the structure of employment

Indicators of the structure of employment that can be calculated as part of an employment projection include proportions of total employment found in each sector.

a. Proportions by sector

For the end of the interval 0-5, these proportions are obtained as follows:

The proportion of total employment found in the primary sector, 0.54, is:

$$0.54 = 1{,}846.0 / 3{,}392.3, \qquad (12)$$

where 1,846.0 and 3,392.3 are, respectively, employment in the primary sector and the total employment;

The proportion of total employment found in the secondary sector, 0.14, is:

$$0.14 = 476.7 / 3{,}392.3, \qquad (13)$$

where 476.7 is employment in the secondary sector;

The proportion of total employment found in the tertiary sector is 0.32:

$$0.32 = 1{,}069.6 / 3{,}392.3, \qquad (14)$$

where 1,069.6 is employment in the tertiary sector.

(iii) Rates of growth of employment

The rates of growth of employment can be calculated for total employment and for employment in each sector.

a. Rate of growth of total employment

If growth in employment is assumed to occur over discrete intervals, the average annual growth rate of total employment for a given interval is obtained using the geometric growth rate formula. For the projection interval 0-5, this annual growth rate, 1.67 per cent (table 67), is obtained as follows:

$$1.67 = [(3{,}392.3 / 3{,}123.2)^{1/5} - 1] \cdot 100, \qquad (15)$$

where 3,123.2 and 3,392.3 are the levels of total employment in years 0 and 5, respectively, and 5 is the length of the interval.

Rates of growth of total employment over the 20-year projection period, which were computed using the geometric growth rate formula, are shown in figure XVIII.

If it is assumed that growth in employment is continuous, the average annual growth rate of total employment for a given interval is obtained by substituting the same data as above in the exponential growth rate formula. For the projection interval 0-5, this annual growth rate, 1.65 per cent, is obtained as follows:

$$1.65 = [\ (\ \ln (3{,}392.5\ /\ 3{,}123.2)\ )\ /\ 5\ ] \cdot 100. \qquad (16)$$

b. Rates of growth of employment by sector

Assuming discrete growth, the rates of increase in employment by sector for the interval 0-5 are calculated as follows:

The annual rate of growth of employment in the primary sector which is negative, -1.4 per cent, is obtained as follows:

$$-1.14 = [\ (1{,}846.0\ /\ 1{,}954.4)^{1/5} - 1\ ] \cdot 100, \qquad (17)$$

where 1,954.4 and 1,846.0 are the levels of employment in the primary sector in years 0 and 5, respectively;

The annual rate of growth of employment in the secondary sector, 5.59 per cent, is obtained as follows:

$$5.59 = [\ (476.7\ /\ 363.2)^{1/5} - 1\ ] \cdot 100, \qquad (18)$$

where 363.2 and 476.7 are the levels of employment in the secondary sector in years 0 and 5, respectively;

The annual rate of growth of employment in the tertiary sector, 5.84 per cent, is obtained as:

$$5.84 = [\ (1{,}069.6\ /\ 805.5)^{1/5} - 1\ ] \cdot 100, \qquad (19)$$

where 805.5 and 1,069.6 are the levels of employment in the tertiary sector in years 0 and 5.

The rates of growth of employment in primary, secondary and tertiary sectors over the 20-year projection interval are shown in figure XIX.

Figure XVIII. Rate of growth in total employment

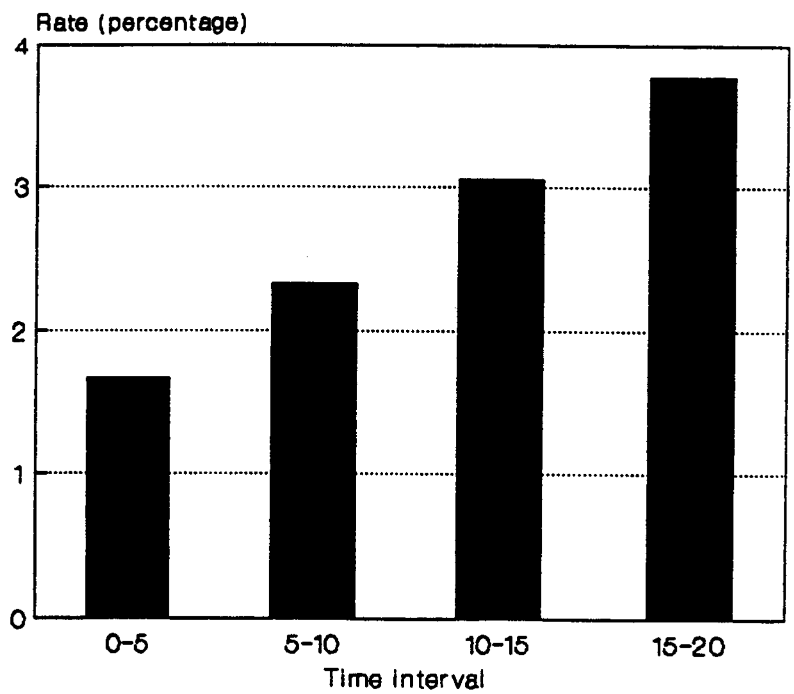
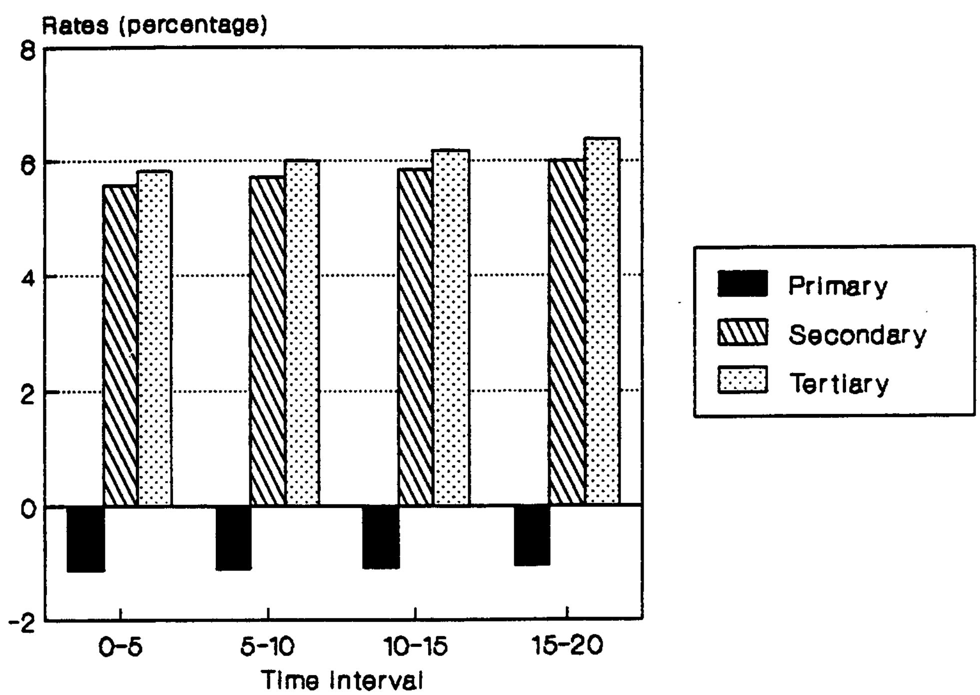


Figure XIX. Rates of growth of employment: primary, secondary and tertiary sectors



If continuous growth is assumed, rates of employment by sector would be calculated using the exponential growth rate formula. The calculations would be analogous to that indicated by equation (16) for total employment.

(iv) Labour market balances

If labour force projections are available for the same years as the employment projection, it is possible to calculate the levels of excess demand for or excess supply of labour. Also, it is possible to calculate the excess demand for or excess supply of labour as a percentage of the level of labour supply.

The calculations can be based on the projected total labour force, diminished where necessary by the size of non-civilian employment, and the projected total employment as indicators of the labour supply and the labour demand.

In order to illustrate these calculations, we shall use projections of the total labour force and the total employment (shown, respectively, in tables 37 and 67) along with the illustrative projections of non-civilian employment, which are shown in table 68. These calculations are illustrated in table 69.

For example, the civilian labour force in year 5 interval, 3,615.4, can be calculated as follows:

$$3,615.4 = 3,651.9 - 36.5, \qquad (23)$$

where 3,651.9 and 36.5 are the projected total labour force and the projected non-civilian employment for year 5, shown in columns 2 and 3. The calculated level of civilian labour force for year 5 is shown in column 4.

The excess supply of labour for the same date, 223.1, is calculated as follows:

$$223.1 = 3,615.4 - 3,392.3, \qquad (24)$$

where 3,615.4 is the civilian labour force and 3,392.3 is the total employment, shown in columns 4 and 5, respectively.

The excess supply expressed as a percentage of the civilian labour force in year 5, 6.17 per cent, is calculated as follows:

$$6.17 = (223.1 / 3,615.4) \cdot 100. \qquad (25)$$

This percentage is shown in column 7.

Table 68. Projected non-civilian employment: entire country

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	32.5
5	36.5
10	41.5
15	47.4
20	53.8

Table 69. Labour market balances: entire country

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	3251.5	32.5	3219.0	3123.2	95.8	2.98
5	3651.9	36.5	3615.4	3392.3	223.1	6.17
10	4147.8	41.5	4106.3	3806.6	299.7	7.30
15	4735.4	47.4	4688.0	4422.6	265.4	5.66
20	5377.4	53.8	5323.6	5322.1	1.5	0.03

a/ From table 37, "Labour force (Total)".
b/ From table 68.
c/ (Col. 2) - (Col. 3).
d/ From table 67, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

2. Urban-rural level

This example will illustrate the use of the labour productivity method to project employment for urban and rural areas, which will be similar to the example for the country as a whole. The inputs for urban and rural areas are shown in tables 70 and 71, respectively. Among the inputs are projected levels of value added along with the assumed initial-year levels of labour productivity and the assumed constant rates of change of labour productivity. The assumed initial-year levels of labour productivity are those shown in tables 61 and 62. The assumed rates of change of labour productivity are similar to those shown in table 58. The differences could be interpreted as modifications based on judgements as to how policies over the projection period may influence the growth of labour productivity.

The example will focus on those calculations which are unique to an urban-rural projection of employment. Like the previous example, it will be based on the assumption that growth in labour productivity and employment occurs over discrete time intervals.

(a) Labour productivity

For any given date, such as year 5, levels of labour productivity and levels of employment, by industry, are obtained by means of calculations that are identical to those used to make the national projection. In the urban-rural projection, however, these calculations are performed for either area. The calculations of the levels of labour productivity and employment for year 5 for the urban areas are illustrated in tables 72 and 73, respectively.

In table 72, the levels of labour productivity, by industry, for the urban areas in year 5 (column 4) are derived from the initial-year levels of labour productivity (column 2) and the constant rates of change of labour productivity (column 3). In table 73, the projected levels of employment, by industry, for the urban areas in year 5 (column 4) are obtained using the projected levels of value added (column 2) and the projected levels of labour productivity (column 3) for the date.

Projected levels of employment, by industry, for urban and rural areas at dates five years apart can be found by performing these calculations for the relevant dates over the projection period, starting with the initial year of the projection. Those levels can be aggregated across the two locations to obtain the levels of employment by sector for the entire country. Tables 74 through 76 display urban, rural and national projected levels of employment, by industry, respectively.

Table 70. Inputs for projecting employment, by industry: urban areas

	Value added in year					Initial-year labour productivity	Rate of change of labour productivity
	0	5	10	15	20	(thousands of LCUs a/ per person)	(percentage)
	(millions of LCUs a/)						
Agriculture	13.8	16.2	19.0	22.4	26.4	0.667	5.000
Mining	2.7	3.8	5.2	7.1	9.8	0.367	0.500
Manufacturing	132.3	201.8	307.6	469.0	715.0	0.664	5.000
Utilities	18.6	28.3	43.2	65.9	100.4	1.268	1.500
Construction	23.1	28.7	35.5	44.1	54.7	0.230	-1.000
Trade	56.7	82.2	119.1	172.6	250.1	0.459	0.500
Transport	54.9	71.7	93.7	122.5	160.1	0.528	1.000
Services	237.0	361.3	550.8	839.8	1280.3	0.627	2.000

a/ Local currency units.

Table 71. Inputs for projecting employment, by industry: rural areas

	Value added in year (millions of LCUs a/)					Initial-year labour productivity (thousands of LCUs a/ per person)	Rate of change of labour productivity (percentage)
	0	5	10	15	20		
Agriculture	259.3	292.0	328.7	370.1	416.7	0.134	3.500
Mining	0.2	0.3	0.4	0.4	0.6	0.222	0.000
Manufacturing	8.0	11.0	14.9	20.4	27.8	0.231	4.000
Utilities	2.6	3.6	4.8	6.6	9.0	0.607	0.000
Construction	2.4	2.9	3.4	3.9	4.6	0.162	-1.000
Trade	2.2	2.9	3.8	5.1	6.6	0.172	0.000
Transport	1.1	1.3	1.6	1.9	2.4	0.387	2.000
Services	75.5	103.0	140.4	191.5	261.1	0.380	1.500

a/ Local currency units.

Table 72. Calculating labour productivity, by industry: urban areas, year 5

Industry	Initial-year labour productivity a/ (thousands of LCUs d/ per person)	Rate of change of labour productivity b/ (percentage)	Labour productivity c/ (thousands of LCUs d/ per person)
(1)	(2)	(3)	(4)
Agriculture	0.666	5.0	0.850
Mining	0.367	0.5	0.376
Manufacturing	0.663	5.0	0.847
Utilities	1.268	1.5	1.366
Construction	0.229	-1.0	0.218
Trade	0.459	0.5	0.470
Transport	0.527	1.0	0.554
Services	0.626	2.0	0.692

a/ From table 70.
b/ From table 70.
c/ $(\text{Col. } 2) \cdot (1+(\text{Col. } 3)/100)^{5}$.
d/ Local currency units.

Table 73. Deriving employment, by industry; urban areas, year 5

Industry	Value added a/ (LCUs d/)	Labour productivity b/ (thousands of LCUs d/ per person)	Projected employment c/ (thousands of persons)
(1)	(2)	(3)	(4)
Agriculture	16.2	0.850	19.0
Mining	3.8	0.376	10.0
Manufacturing	201.8	0.847	238.1
Utilities	28.3	1.366	20.7
Construction	28.7	0.218	131.3
Trade	82.2	0.470	174.6
Transport	71.7	0.554	129.3
Services	361.3	0.692	521.9

a/ From table 70.
b/ From table 72, col. 4.
c/ (Col. 2)/(Col. 3).
d/ Local currency units.

Table 74. Projected employment, by industry: urban areas

(Thousands of persons)

Industry	Year 0	5	10	15	20
Agriculture	20.6	19.0	17.5	16.2	14.9
Mining	7.4	10.0	13.4	18.0	24.1
Manufacturing	199.4	238.1	284.5	339.8	405.9
Utilities	14.7	20.7	29.3	41.5	58.8
Construction	100.7	131.3	171.2	223.3	291.2
Trade	123.6	174.6	246.8	348.8	493.0
Transport	104.0	129.3	160.7	199.9	248.6
Services	378.0	521.9	720.7	995.2	1374.2

Table 75. Projected employment, by industry: rural areas

(Thousands of persons)

Industry	Year 0	5	10	15	20
Agriculture	1928.4	1828.1	1733.0	1642.9	1557.4
Mining	1.0	1.3	1.6	2.0	2.5
Manufacturing	34.8	39.0	43.7	48.9	54.9
Utilities	4.3	5.8	8.0	10.9	14.8
Construction	15.1	18.6	22.9	28.1	34.6
Trade	13.0	17.0	22.4	29.3	38.5
Transport	2.8	3.1	3.4	3.7	4.1
Services	198.4	251.1	317.9	402.4	509.3

Table 76. Projected employment, by industry: entire country

(Thousands of persons)

Industry	Year				
	0	5	10	15	20
Agriculture	1949.1	1847.1	1750.5	1659.0	1572.3
Mining	8.4	11.2	15.0	20.0	26.6
Manufacturing	234.1	277.1	328.1	388.8	460.8
Utilities	18.9	26.6	37.3	52.4	73.6
Construction	115.7	149.8	194.1	251.4	325.8
Trade	136.5	191.7	269.2	378.2	531.6
Transport	106.7	132.3	164.1	203.6	252.7
Services	576.4	773.1	1038.6	1397.6	1883.5

(b) Other results

An urban-rural projection of employment permits the calculation of all those additional results that can be obtained as part of the national projections. Those results, which refer to urban and rural areas and to the entire country include various aggregates, indicators of structure and rates of growth of employment, as well as labour market balances. They can be calculated by means of the steps illustrated in connection with the national projection. The results also include proportions of employment which are urban and rural.

Figure XX indicates projected levels of total employment for urban and rural areas and for the entire country, which were obtained in this illustrative projection.

(i) Proportions of employment that are urban and rural

The proportions of employment found in urban and rural areas, respectively, can be obtained for total employment and for employment by sector.

a. Proportions of total employment

The proportion of total employment that is urban at the end of the projection interval is calculated as a ratio of total employment in the urban areas to the total employment in the country as a whole for that date. At the end of the interval 0-5, the proportion of total employment that is urban, 0.37, is obtained as:

$$0.37 = 1{,}245.0 \;/\; 3{,}408.9, \qquad (28)$$

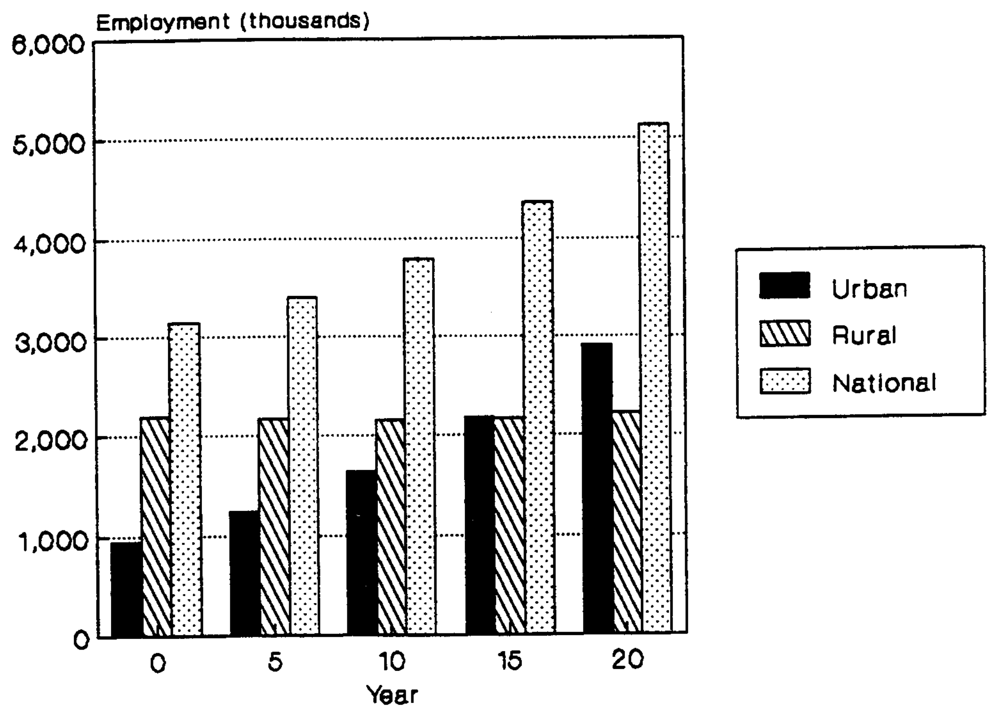
where 1,245.0 is total employment in the urban areas and 3,408.9 is total employment for the entire country.

The proportion of total employment that is rural, 0.63, is calculated as a complement of the proportion urban:

$$0.63 = 1 - 0.37, \qquad (29)$$

where 0.37 is the proportion urban.

Figure XX. Total employment: urban, rural and national



These proportions along with all other results for the entire country, excepting the labour market balances, are shown in table 79; similar results for urban and rural areas are shown in tables 77 and 78. Labour market balances for urban and rural areas and the entire country were calculated using, among other things, illustrative projected levels of non-civilian employment, which are shown in tables 80 through 82. The results relating to the labour market balances are shown in tables 83 through 85.

Proportions of total employment over the 20-year period which are urban and rural are shown in figure XXI.

b. Proportions of employment by sector

For each sector, the proportions of employment in year 5 that are urban can be calculated as follows:

The proportion of employment in the primary sector that is urban, 0.02, is:

$$0.02 = 29.0 \;/\; 1{,}858.4, \tag{30}$$

where 29.0 is employment in the primary sector in the urban areas and 1,858.4 is employment in the primary sector in the entire country;

The proportion of employment in the secondary sector that is urban, 0.86, is:

$$0.86 = 390.2 \;/\; 453.5, \tag{31}$$

where 390.2 is employment in the secondary sector in the urban areas and 453.5 is employment in the secondary sector in the entire country;

The proportion of employment in the tertiary sector that is urban, 0.75, is:

$$0.75 = 825.8 \;/\; 1{,}097.0, \tag{32}$$

where 825.8 is employment in the tertiary sector in the urban areas and 1,097.0 is employment in the tertiary sector in the entire country;

The proportions of employment, by sector, that are rural can be obtained as complements of proportions of employment by sector which are urban:

The proportion of employment in the primary sector that is rural, 0.98, is:

$$0.98 = 1 - 0.02, \tag{33}$$

Table 77. Employment aggregates, structure and rates of growth: urban areas

Indicators	Year 0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	948.2	1245.0	1644.2	2182.7	2910.7
Primary	28.1	29.0	30.9	34.1	39.0
Secondary	314.7	390.2	485.0	604.6	755.9
Tertiary	605.5	825.8	1128.3	1543.9	2115.8
Growth in employment					
Total		296.7	399.2	538.4	728.0
Primary		0.9	1.9	3.2	4.9
Secondary		75.5	94.9	119.6	151.3
Tertiary		220.3	302.4	415.6	571.9
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.03	0.02	0.02	0.02	0.01
Secondary	0.33	0.31	0.29	0.28	0.26
Tertiary	0.64	0.66	0.69	0.71	0.73
Rates of growth of employment (percentage)					
Total		5.60	5.72	5.83	5.93
Primary		0.65	1.29	2.00	2.71
Secondary		4.39	4.45	4.51	4.57
Tertiary		6.40	6.44	6.47	6.50

Table 78. Employment aggregates, structure and rates of growth: rural areas

Indicators	Year 0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	2197.7	2163.9	2152.7	2168.3	2216.2
Primary	1929.4	1829.4	1734.6	1644.8	1559.9
Secondary	54.1	63.4	74.5	87.9	104.3
Tertiary	214.1	271.2	343.6	435.5	552.0
Growth in employment					
Total		-33.7	-11.3	15.6	48.0
Primary		-100.1	-94.8	-89.7	-84.9
Secondary		9.2	11.1	13.5	16.4
Tertiary		57.1	72.4	91.8	116.5
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.88	0.85	0.81	0.76	0.70
Secondary	0.02	0.03	0.03	0.04	0.05
Tertiary	0.10	0.13	0.16	0.20	0.25
Rates of growth of employment (percentage)					
Total		-0.31	-0.10	0.14	0.44
Primary		-1.06	-1.06	-1.06	-1.05
Secondary		3.20	3.29	3.38	3.47
Tertiary		4.84	4.85	4.85	4.86

Table 79. Employment aggregates, structure and rates of growth: entire country

Indicators	Year 0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	3145.9	3408.9	3796.9	4350.9	5126.9
Primary	1957.5	1858.4	1765.5	1679.0	1598.9
Secondary	368.8	453.5	559.5	692.6	860.2
Tertiary	819.6	1097.0	1471.9	1979.4	2667.8
Growth in employment					
Total		263.0	388.0	554.0	776.0
Primary		-99.1	-92.8	-86.5	-80.0
Secondary		84.7	106.0	133.0	167.6
Tertiary		277.4	374.8	507.5	688.4
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.62	0.55	0.46	0.39	0.31
Secondary	0.12	0.13	0.15	0.16	0.17
Tertiary	0.26	0.32	0.39	0.45	0.52
Indicators of employment distribution					
Proportions urban					
Total	0.30	0.37	0.43	0.50	0.57
Primary	0.01	0.02	0.02	0.02	0.02
Secondary	0.85	0.86	0.87	0.87	0.88
Tertiary	0.74	0.75	0.77	0.78	0.79
Proportions rural					
Total	0.70	0.63	0.57	0.50	0.43
Primary	0.99	0.98	0.98	0.98	0.98
Secondary	0.15	0.14	0.13	0.13	0.12
Tertiary	0.26	0.25	0.23	0.22	0.21
Rates of growth of employment (percentage)					
Total		1.62	2.18	2.76	3.34
Primary		-1.03	-1.02	-1.00	-0.97
Secondary		4.22	4.29	4.36	4.43
Tertiary		6.00	6.05	6.10	6.15

Table 80. Projected non-civilian employment: urban areas

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	29.3
5	32.9
10	37.4
15	42.6
20	48.4

Table 81. Projected non-civilian employment: rural areas

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	3.3
5	3.7
10	4.2
15	4.7
20	5.4

Table 82. Projected non-civilian employment: entire country

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	32.5
5	36.5
10	41.5
15	47.4
20	53.8

Table 83. Labour market balances: urban areas

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	968.3	29.3	939.0	948.2	-9.2	-0.98
5	1383.8	32.9	1350.9	1245.0	105.9	7.84
10	1881.1	37.4	1843.7	1644.2	199.5	10.82
15	2438.7	42.6	2396.1	2182.7	213.4	8.91
20	3032.4	48.4	2984.0	2910.7	73.3	2.46

a/ From table 46, "Labour force (Total)".
b/ From table 80.
c/ (Col. 2) - (Col. 3).
d/ From table 77, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Table 84. Labour market balances: rural areas

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	2282.6	3.3	2279.3	2197.7	81.6	3.58
5	2269.1	3.7	2265.4	2163.9	101.5	4.48
10	2268.5	4.2	2264.3	2152.7	111.6	4.93
15	2298.6	4.7	2293.9	2168.3	125.6	5.48
20	2344.1	5.4	2338.7	2216.2	122.5	5.24

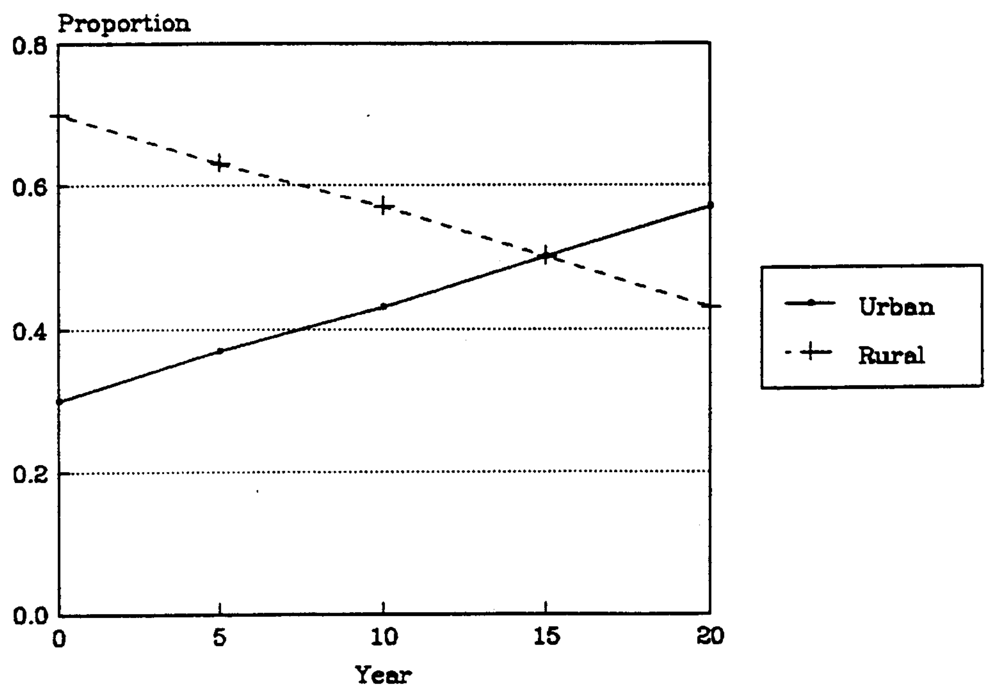
a/ From table 47, "Labour force (Total)".
b/ From table 81.
c/ (Col. 2) - (Col. 3).
d/ From table 78, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Table 85. Labour market balances: entire country

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	3250.9	32.5	3218.4	3145.9	72.5	2.25
5	3652.9	36.5	3616.4	3408.9	207.5	5.74
10	4150.2	41.5	4108.7	3796.9	311.8	7.59
15	4737.3	47.4	4689.9	4350.9	339.0	7.23
20	5376.5	53.8	5322.7	5126.9	195.8	3.68

a/ From table 48, "Labour force (Total)".
b/ From table 82.
c/ (Col. 2) - (Col. 3).
d/ From table 79, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Figure XXI. Proportions of total employment: urban and rural



where 0.02 is the proportion of employment in the primary sector that is urban;

The proportion of employment in the secondary sector that is rural, 0.14, is:

$$0.14 = 1 - 0.86, \tag{34}$$

where 0.86 is the proportion of employment in the secondary sector that is urban;

The proportion of employment in the tertiary sector that is rural, 0.25, is:

$$0.25 = 1 - 0.75, \tag{35}$$

where 0.75 is the proportion of employment in the tertiary sector that is urban.

The proportions of employment, by sector, that are urban and rural for the entire projection interval are shown in table 79.

E. Summary

This chapter has described the method for preparing employment projections, by industry, using constant rates of change of labour productivity, which may be discrete or continuous. As part of the description of the method, procedures that can be used in making both national and urban-rural projections were presented. In addition, types of inputs required by the method were described, and the preparation of the inputs discussed. Lastly, two examples of projections--national and urban-rural-- were described. A complete listing of the outputs that can be generated by the method is shown in box 19.

Box 19

Outputs of the method for making employment projections assuming a constant rate of change in labour productivity

1. Employment by industry (national or urban, rural and national)

2. Employment aggregates (national or urban, rural and national)

   Levels of employment:

      Total

      Primary sector
      Secondary sector
      Tertiary sector

   Growth in employment:

      Total

      Primary sector
      Secondary sector
      Tertiary sector

3. Indicators of the structure of employment (national or urban, rural and national)

   Proportions of employment by sector:

      Primary sector
      Secondary sector
      Tertiary sector

4. Indicators of the urban-rural distribution of employment (national only; if urban and rural employment is being projected)

   Proportions of employment urban:

      Total

      Primary sector
      Secondary sector
      Tertiary sector

(continued)

Box 19 (continued)

Proportions of employment rural:

Total

Primary sector
Secondary sector
Tertiary sector

5. Rates of growth of employment (national or urban, rural and national)

Total

Primary sector
Secondary sector
Tertiary sector

6. Labour market balances (national or urban, rural and national)

Excess supply of or excess demand for labour

Percentage excess supply of or excess demand for labour

F. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

i = 1,...,I	are industries of the nation's economy
k = 1,2	are urban and rural locations
t	is the year of the projection period
t'	is the calendar year
t*	is the number of years between the year to which the observed levels of labour productivity used refer and the initial year of the projection period (t=0)
-t*	is the year preceding the initial year of projection (t=0) in which the levels of labour productivity were observed.

(b) List of variables

CLF(t+5)	is the civilian labour force at the end of the interval
EGREM	is the average annual exponential growth rate of total employment for the interval
EGREMP	is the average annual exponential growth rate of employment in the primary sector for the interval
EGREMS	is the average annual exponential growth rate of employment in the secondary sector for the interval
EGREMT	is the average annual exponential growth rate of employment in the tertiary sector for the interval
EM(i,k,t+5)	is the employment in industry i in location k at the end of the interval
EM(i,t+5)	is the employment in industry i at the end of the interval

EM(k,t+5)	is the total employment in location k at the end of the interval
EM(t+5)	is the total employment at the end of the interval
EMGR	is the growth of total employment during the interval
EMP(k,t+5)	is the employment in the primary sector in location k at the end of the interval
EMP(t+5)	is the employment in the primary sector at the end of the interval
EMPGR	is the growth of employment in the primary sector during the interval
EMS(k,t+5)	is the employment in the secondary sector in location k at the end of the interval
EMS(t+5)	is the employment in the secondary sector at the end of the interval
EMSGR	is the growth of employment in the secondary sector during the interval
EMT(k,t+5)	is the employment in the tertiary sector in location k at the end of the interval
EMT(t+5)	is the employment in the tertiary sector at the end of the interval
EMTGR	is the growth of employment in the tertiary sector during the interval
ERLP(i)	is the annual exponential rate of change of labour productivity in industry i expressed as percentage
EXL(t+5)	is the excess supply of labour (if positive) or excess demand for labour (if negative) for the end of the interval
GGREM	is the average annual geometric growth rate of total employment for the interval
GGREMP	is the average annual geometric growth rate of employment in the primary sector for the interval
GGREMS	is the average annual geometric growth rate of employment in the secondary sector for the interval

GGREMT	is the average annual geometric growth rate of employment in the tertiary sector for the interval
GRLP(i)	is the annual geometric rate of change of labour productivity in industry i expressed as percentage
GRLP(i,k)	is the annual geometric rate of change of labour productivity in industry i in location k expressed as percentage
LF(t+5)	is the total labour force at the end of the interval
LP(i,-t*)	is the observed level of labour productivity of industry i in year -t*
LP(i,0)	is the labour productivity of industry i in the initial year of projection period or in the initial year of the time series
LP(i,k,0)	is the labour productivity of industry i in location k in the initial year of the projection period
LP(i,k,t+5)	is the labour productivity in industry i in location k at the end of the interval
LP(i,t')	is the labour productivity in industry i in year t'
LP(i,t+5)	is the labour productivity in industry i at the end of the interval
NEM(t+5)	is the non-civilian employment at the end of the interval
PEMP(t+5)	is the proportion of total employment accounted for by the primary sector at the end of the interval
PEMPRUR(t+5)	is the proportion of employment in the primary sector which is rural at the end of the interval
PEMPURB(t+5)	is the proportion of employment in the primary sector which is urban at the end of the interval
PEMRUR(t+5)	is the proportion of total employment which is rural at the end of the interval
PEMS(t+5)	is the proportion of total employment accounted for by the secondary sector at the end of the interval
PEMSRUR(t+5)	is the proportion of employment in the secondary sector which is rural at the end of the interval

PEMSURB(t+5)	is the proportion of employment in the secondary sector which is urban at the end of the interval
PEMT(t+5)	is the proportion of total employment accounted for by the tertiary sector at the end of the interval
PEMTRUR(t+5)	is the proportion of employment in the tertiary sector which is rural at the end of the interval
PEMTURB(t+5)	is the proportion of employment in the tertiary sector which is urban at the end of the interval
PEMURB(t+5)	is the proportion of total employment which is urban at the end of the interval
PEXL(t+5)	is the excess supply of labour or excess demand for labour as a percentage of the civilian labour force at the end of the interval
VA(i,k,t+5)	is the value added in industry i in location k at the end of the interval
VA(i,t+5)	is the value added in industry i at the end of the interval

(c) <u>List of special symbols</u>

a(i)	is the intercept coefficient of the function for industry i
antiln	is the antilogarithm of the natural logarithm
b(i)	is the partial coefficient of the function for industry i
b*(i)	is the estimate of the partial coefficient of the function for industry i
e	is the base of the natural logarithm
I	is the number of industries
Ip	is the number of industries in the primary sector
Is	is the number of industries in the secondary sector
ln	is the natural logarithm

u(i,t') is the random disturbance term in the function for industry i in year t'

2. Equations

A. The technique

1. National level

(a) Labour productivity by industry

(i) Discrete growth

$$LP(i,t+5) = LP(i,0) \cdot (1 + GRLP(i)/100)^{t+5}; \quad i = 1,\dots,I \tag{1}$$

(ii) Continuous growth

$$LP(i,t+5) = LP(i,0) \cdot e^{[(ERLP(i)/100) \cdot (t+5)]}; \quad i = 1,\dots,I \tag{2}$$

(b) Employment by industry

$$EM(i,t+5) = VA(i,t+5)/LP(i,t+5); \quad i = 1,\dots,I \tag{3}$$

(c) Other results

(i) Employment aggregates

a. Total employment

$$EM(t+5) = \sum_{i=1}^{I} EM(i,t+5) \tag{4}$$

b. Employment by sector

i. Employment in the primary sector

$$EMP(t+5) = \sum_{i=1}^{Ip} EM(i,t+5) \qquad (5)$$

ii. Employment in the secondary sector

$$EMS(t+5) = \sum_{i=Ip+1}^{Ip+Is} EM(i,t+5) \qquad (6)$$

iii. Employment in the tertiary sector

$$EMT(t+5) = \sum_{i=Ip+Is+1}^{I} EM(i,t+5) \qquad (7)$$

c. Growth in total employment

$$EMGR = EM(t+5) - EM(t) \qquad (8)$$

d. Growth in employment by sector

$$EMPGR = EMP(t+5) - EMP(t) \qquad (9)$$

$$EMSGR = EMS(t+5) - EMS(t) \qquad (10)$$

$$EMTGR = EMT(t+5) - EMT(t) \qquad (11)$$

(ii) Indicators of the structure of employment

a. Proportions by sector

$$PEMP(t+5) = EMP(t+5) / EM(t+5) \qquad (12)$$

$$PEMS(t+5) = EMS(t+5) / EM(t+5) \tag{13}$$

$$PEMT(t+5) = EMT(t+5) / EM(t+5) \tag{14}$$

(iii) Rates of growth of employment

a. Rate of growth of total employment

$$GGREM = [(EM(t+5) / EM(t))^{1/5} - 1] \cdot 100 \tag{15}$$

$$EGREM = [(\ln (EM(t+5) / EM(t))) / 5] \cdot 100 \tag{16}$$

b. Rates of growth of employment by sector

$$GGREMP = [(EMP(t+5) / EMP(t))^{1/5} - 1] \cdot 100 \tag{17}$$

$$GGREMS = [(EMS(t+5) / EMS(t))^{1/5} - 1] \cdot 100 \tag{18}$$

$$GGREMT = [(EMT(t+5) / EMT(t))^{1/5} - 1] \cdot 100 \tag{19}$$

$$EGREMP = [(\ln (EMP(t+5) / EMP(t))) / 5] \cdot 100 \tag{20}$$

$$EGREMS = [(\ln (EMS(t+5) / EMS(t))) / 5] \cdot 100 \tag{21}$$

$$EGREMT = [(\ln (EMT(t+5) / EMT(t))) / 5] \cdot 100 \tag{22}$$

(iv) Labour market balances

$$CLF(t+5) = LF(t+5) - NEM(t+5) \tag{23}$$

$$EXL(t+5) = CLF(t+5) - EM(t+5) \tag{24}$$

$$PEXL(t+5) = [EXL(t+5) / CLF(t+5)] \cdot 100 \tag{25}$$

3. Urban-rural level

(a) Labour productivity

(i) Discrete growth

$$LP(i,k,t+5) = LP(i,k,0) \cdot (1 + GRLP(i,k)/100)^{t+5}; \quad (26)$$

$$i = 1,\ldots,I;$$

$$k = 1,2$$

(ii) Continuous growth

(b) Employment by industry

$$EM(i,k,t+5) = VA(i,k,t+5)/LP(i,k,t+5); \quad (27)$$

$$i = 1,\ldots,I;$$

$$k = 1,2$$

(c) Other results

(i) Proportions of employment that are urban and rural

a. Proportions of total employment

$$PEMURB(t+5) = EM(1,t+5)/EM(t+5) \quad (28)$$

$$PEMRUR(t+5) = 1 - PEMURB(t+5) \quad (29)$$

b. Proportions of employment by sector

$$PEMPURB(t+5) = EMP(1,t+5)/EMP(t+5) \quad (30)$$

$$PEMSURB(t+5) = EMS(1,t+5)/EMS(t+5) \quad (31)$$

$$PEMTURB(t+5) = EMT(1,t+5)/EMT(t+5) \quad (32)$$

$$PEMPRUR(t+5) = 1 - PEMPURB(t+5) \quad (33)$$

$$PEMSRUR(t+5) = 1 - PEMSURB(t+5) \quad (34)$$

$$PEMTRUR(t+5) = 1 - PEMTURB(t+5) \quad (35)$$

B. The inputs

1. Types of inputs required

2. Preparation of the inputs

(a) Assumptions on rates of change of labour productivity

(i) Time series data

(ii) Rates of change based on time series consisting of several observations

a. Estimation procedures

i. Discrete growth

$$LP(i,t') = LP(i,0) \cdot (1 + GRLP(i)/100)^{t'}; \quad (36)$$

$$i = 1,\ldots,I$$

$$lnLP(i,t') = a(i) + b(i) \cdot t' + u(i,t'); \quad (37)$$

$$i = 1,\ldots,I,$$

where:

$$a(i) = lnLP(i,0)$$

$$b(i) = ln[\, 1 + GRLP(i)/100 \,]$$

$$GRLP(i) = [\text{antiln } b^*(i) - 1] \cdot 100; \quad i = 1,\ldots,I, \tag{38}$$

ii. Continuous growth

$$LP(i,t') = LP(i,0) \cdot e^{[(ERLP(i)/100) \cdot t']}; \quad i = 1,\ldots,I, \tag{39}$$

$$\ln LP(i,t') = a(i) + b(i) \cdot t' + u(i,t'); \quad i = 1,\ldots,I, \tag{40}$$

where:

$a(i) = \ln LP(i,0)$

$b(i) = ERLP(i) \cdot 100$

$$ERLP(i) = b^*(i) \cdot 100; \quad i = 1,\ldots,I, \tag{41}$$

b. Illustrative estimations

i. Discrete growth

ii. Continuous growth

(iii) Rates of change based on time series consisting of two observations

a. Estimation procedures

i. Discrete growth

$$GRLP(i) = [\ (\ LP(i,t')/LP(i,0)\ )^{1/t'} - 1\ ] \cdot 100; \quad i = 1,\ldots,I, \tag{42}$$

ii. Continuous growth

$$ERLP(i) = [(\ln (LP(i,t')/LP(i,0))) / t'] \cdot 100; \qquad (43)$$

$$i = 1,...,I$$

b. Illustrative estimations

i. Discrete growth

$$3.55 = [(0.125/0.105)^{1/5} - 1] \cdot 100. \qquad (42)$$

ii. Continuous growth

$$3.48 = [(\ln (0.125/0.105)] \cdot 100. \qquad (43)$$

(iv) Rates of change based on the assessment of factors influencing labour productivity trends

(b) Assumptions on initial-year levels of labour productivity

(i) Procedures to derive initial-year levels of labour productivity

a. Discrete growth

$$LP(i,0) = LP(i,-t^*) \cdot (1 + GRLP(i)/100)^{t^*}; \qquad (44)$$

$$i = 1,...,I$$

b. Continuous growth

$$LP(i,0) = LP(i,-t^*) \cdot e^{[(ERLP(i)/100) \,.\, t^*]}; \qquad (45)$$

$$i = 1,...,I$$

(ii) Illustrative derivations of initial-year levels of labour productivity

$$0.140 = 0.131 \cdot (1 + 3.66/100)^2, \tag{44}$$

$$0.134 = 0.125 \cdot (1 + 3.55/100)^2, \tag{45}$$

Notes

1/ Policies that can be considered in this context are those relating to investment allocation among sectors, modern sector wages (especially if those wages are set by the Government) and population variables.

2/ Throughout this chapter, "value added" will refer to value added measured in constant prices.

3/ This amounts to assuming equality between the marginal employment-value added ratio and the average employment-value added ratio.

4/ The initial year of the projection period is denoted by 0.

5/ Through much of this chapter, discussion of the labour productivity method is based on the alternative assumptions about growth of labour productivity--discrete and continuous. This should enable the user to make employment projections using either assumption. Making projections with both types of assumptions would, most of the time, result in very similar results.

6/ For a discussion on the use of t-statistics to assess statistical significance of regression coefficients, see Kmenta (1971).

7/ The time series data on employment which were used to obtain levels of labour productivity expressed employment in units of 1,000 employed persons. Therefore, the levels of employment in these illustrative examples will be given in thousands of employed persons.

Reference

Kmenta, Jan (1971), Elements of Econometrics. New York: Macmillan.

VII. MAKING EMPLOYMENT PROJECTIONS USING EMPLOYMENT-VALUE ADDED FUNCTIONS

A. Introduction

This chapter describes a method for projecting employment by industry, which makes use of econometrically estimated functions relating employment to value added. The method is similar to the labour productivity method described in chapter VI in that it can be used to project employment at the national or urban-rural level. Furthermore, it provides the same type of results that can be obtained by the labour productivity method. Unlike the labour productivity method, however, which can be applied even when the requisite data come only from observations for two years, the employment-value added function method can be applied only if a fairly long time series on employment and value added is available. 1/

The method can be used with several different functional forms characterizing the relationship between employment and value added. As a result, the method is considerably more flexible than simpler methods for preparing employment projections. In particular, it does not use certain restrictive assumptions which are required by those methods. Thus, unlike the employment-value added ratio technique, (see chapter VI), this method does not assume a constant average ratio of employment to value added.2/ Also, unlike the labour productivity method, it does not make an implicit assumption that increases in employment are proportional to increases in value added (unitary elasticity of employment with respect to value added).

An additional advantage of the employment-value added function method over the simpler techniques, especially the employment-value added ratio method, is that it makes better use of data sets that refer to longer time periods. By using more of the historical experience of the country the projections are less likely to be affected by unusual circumstances prevailing over short time periods, such as a down turn in economic activity or a temporary wage freeze.

However, the reliance on time series data may be one of the method's drawbacks if the coverage of employment and value added data by industry varies over time. In such situations, estimates of the employment-value added functions derived from time series data would yield misleading projections of employment.

Another potential weakness of the method arises from making an implicit assumption that the future relationship between employment and value added would be similar to that observed for the period for which the time series data are available. Depending on government wage and/or employment policies and other factors, such an assumption may not be warranted. Where this is the case, the method should not provide the only input used in preparing employment projections.

The method does not make explicit provision for capital-labour substitution, which is often manifested in the increase of the amount of capital per unit of labour (capital deepening), nor does it explicitly provide for technical change. Hence, it implicitly assumes that the average ratio of capital to labour in each industry is either constant or steadily changing over time. If the assumption is likely to be violated, say, owing to an anticipated significant increase in the rate of capital formation over the plan horizon relative to that over the period used to estimate the functions, the method may provide misleading projections.

B. The technique

1. Overview

This overview lists inputs needed by the method and describes the types of results it can generate. It also outlines the computational steps used in making an employment projection with the method.

(a) Inputs

To project employment at the national level, the following inputs are required:

(i) Projected levels of value added by industry;
(ii) Estimates of the coefficients of employment-value added functions, by industry;

If, in addition to employment, shortages and/or surpluses in the labour market are to be projected, the inputs should also include:

(iii) Projected total labour force;
(iv) Projected non-civilian emloyment.

For a national projection, the inputs ought to refer to the entire country. For an urban-rural projection, the inputs should be for urban and rural areas with the exception of projected value added levels by industry. The reasons for this exception are explained below. The requisite inputs are listed in box 20, which indicates that employment-value added functions may have different forms, examples of which are linear and non-linear forms. Those functions may include time as an additional explanatory variable.

Since the employment-value added function method is described in the context of procedures for making quinquennial projections, the projected levels of value added would be for dates five years apart, starting with the initial year of the plan. Projected total labour force and projected non-civilian employment would be for the same dates. Given the appropriate annual inputs, however, the method could be used for making annual projections.

Box 20

Inputs for making employment projections using employment-value added functions

1. Value added, by industry (national)

2. Coefficients of the employment-value added functions (national or urban and rural)

   Coefficients of linear functions without or with the time variable or

   Coefficients of non-linear or log-linear functions without or with the time variable

3. Total labour force (national or urban and rural; if projection of labour market balances is desired)

4. Non-civilian employment (national or urban and rural; if projection of labour market balances is desired)

(b) Outputs

The outputs which the method can generate would partly depend on the type of projection being prepared. In the case of a national projection, the method would yield:

(i) Levels of employment by industry;
(ii) Various employment aggregates, such as total employment and the growth in total employment;
(iii) Indicators of the structure of employment, such as proportions of employment by sector (primary, secondary and tertiary);
(iv) Rates of change in employment, including that of total employment or employment by sector.

If the inputs include projected total labour force and projected non-civilian employment, the outputs could also include:

(v) Absolute and relative levels of excess supply of labour and/or excess demand for labour.

If the method is used to prepare an urban-rural projection, the results would include all of those listed under (i) through (v), which would be for urban and rural areas as well as for the entire country. In addition, they would include indicators of the urban-rural distribution of employment. The types of outputs that the technique can generate as part of the national or urban-rural projection are shown in box 21.

Box 21

Types of outputs obtained by making employment projections using employment-value added functions

1. Employment by industry (national or urban, rural and national)

2. Employment aggregates (national or urban, rural and national)

   Total employment and employment by sector (e.g. primary, secondary and tertiary)

   Growth in total employment and employment by sector

3. Indicators of the structure of employment (national or urban, rural and national)

   Proportions of employment, by sector

4. Indicators of the urban-rural distribution of employment (national only; if urban and rural employment is being projected)

   Proportions of the total employment and of employment by sector, in different locations

5. Rates of growth of employment (national or urban, rural and national)

   The rates of growth in total employment and employment by sector

6. Labour market balances (national or urban, rural and national)

   Absolute and relative levels of excess supply of and/or excess demand for labour

The results would be for dates five years apart or the intervening projection intervals.

(c) Computational steps

To project employment for a given date using employment-value added functions, it is first necessary to evaluate the functions by using the projected levels of value added, by industry, for that date. If the functions include time as an additional explanatory variable, it is also necessary to use a value of the time variable for the date in question. This step will yield projected levels of employment, by industry, for the selected date. Like the other techniques of projecting employment described in this volume, the method can also be used to calculate other results, examples of which are total employment and employment in various sectors, such as primary, secondary and tertiary, along with other date-specific indicators. The growth in employment and growth rates of employment for the intervening projection intervals can also be calculated. If the employment projection is accompanied by a labour force projection, the projected total employment and the projected total labour force can be used to calculate the surplus or shortage of labour.

2. National level

The description of the method will initially present employment-value added functions, followed by the steps to derive the levels of employment, by industry, at the national level using those functions. It will also discuss steps to derive other results for a given projection date or interval. A summary of those steps is shown in box 22. The functions and steps used to derive urban and rural employment by industry along with the related results will be described in a later section.

(a) Employment-value added functions, by industry

This section will describe several different specifications of employment-value added functions. First, specifications which exclude time as an explanatory variable will be discussed. Then, specifications which include time as an explanatory variable will be presented.

(i) Functions without the time variable

A simple specification of employment-value added function postulates that employment is a linear function of value added. Linear employment-value added functions, by industry, are written as follows:

$$EM(i,t') = a(i) + b(i) \cdot VA(i,t'); \qquad i = 1,\ldots,I, \tag{1}$$

where:

$i = 1,\ldots,I$	are industries of the nation's economy,
I	is the number of industries,
t'	is the calendar year,

Box 22

Computational steps to project employment at the national level with the method using employment-value added functions

The steps used to project employment at the national level over a five-year projection interval are:

(1) Derive projected levels of employment, by industry, at the end of the interval by evaluating empirically estimated industry-specific employment-value added functions using the assumed levels of value added, by industry, for that date. If necessary, also use a value of the time variable to evaluate the functions.

(2) Calculate various employment aggregates, such as total employment and the increase in total employment.

(3) Derive indicators of the employment structure, such as the proportions of total employment found in each sector.

(4) Obtain rates of growth of employment, such as the rate of growth of total employment.

(5) If the labour force projection is available, calculate the absolute and percentage levels of excess supply of or excess demand for labour.

EM(i,t') is the employment in industry i in year t',

VA(i,t') is the value added in industry i in year t',

a(i) is the intercept coefficient of the linear employment-value added function for industry i, and

b(i) is the partial coefficient of the value added variable in the linear employment-value added function for industry i. (Blitzer and others (1975)

The partial coefficients in equation (1), b(i)'s, are the marginal employment-value added ratios in various industries. These ratios differ from the average employment-value added ratios as long as the intercept coefficients, a(i)'s, differ from zero. If all intercept coefficients were equal to zero, the method using such linear functions would be identical to the employment-value added ratio method of employment projections ((chapter VI).

A multiplicative specification of employment-value added functions postulates that employment is a non-linear function of value added. Such non-linear employment value added functions, by industry, are as follows:

$$EM(i,t') = a(i) \cdot VA(i,t')^{b(i)}; \qquad (2)$$

$$i = 1,\ldots,I,$$

where:

a(i)	is the intercept coefficient of the non-linear employment-value added function for industry i, and
b(i)	is the partial coefficient of the value added variable in the non-linear employment-value added function for industry i.

The non-linear functions indicated in equation (2) can be transformed into log-linear employment-value added functions by taking logarithms of their left-hand and right-hand sides:

$$\ln EM(i,t') = \ln a(i) + b(i) \cdot \ln VA(i,t'); \qquad (3)$$

$$i = 1,\ldots,I,$$

where:

ln	is the natural logarithm.

The exponents in equation (2), b(i)'s, become the partial coefficients of the functions indicated in equation (3). However, they have a different meaning from the partial coefficients in the functions shown in equation (1). They stand for elasticities of employment with respect to value added, by industry. That is, they express the percentage change in employment for a given percentage change in value added.

The non-linear or log-linear functions embody assumptions which differ from those of the linear functions. Thus, in the non-linear or log-linear functions, marginal employment-value added ratios vary with the levels of value added and employment, while the elasticities of employment with respect to value added remain fixed (they are equal to the partial coefficients, b(i)'s). In the linear functions, the marginal employment-value added ratios are fixed (they are equal to b(i)'s), while the elasticities of employment with respect to value added vary with the levels of employment and value added.

In spite of these differences, the non-linear or log-linear functions tend to yield employment projections over the medium term which are very similar to those that can be obtained using linear functions. In view of this, with the limited time series data available in most developing countries, there is often little reason to select one form over the other in making employment projections. Linear functions are often preferred, since their use simplifies the process of estimation as well as that of making projections.

(ii) Functions with the time variable

The functions indicated in equations (1) and (2) can be modified to implicitly allow for the effects of the capital-labour substitution and technical change by respectively adding to them linear and exponential time trends. Linear functions that include time as a variable are as follows:

$$EM(i,t') = a(i) + b(i) \cdot VA(i,t') + c(i) \cdot t'; \qquad (4)$$

$$i = 1,\ldots,I,$$

where:

c(i) is the partial coefficient of the time variable in the linear employment-value added function for industry i.

Non-linear functions that include a time are as follows:

$$EM(i,t') = a(i) \cdot VA(i,t') \cdot e^{[c(i)\ .\ t']}; \qquad (5)$$

$$i = 1,\ldots,I$$

where:

c(i) is the partial coefficient of the time variable in the non-linear employment-value added function for industry i, and

e is the base of the natural logarithm.

The non-linear functions shown in equation (5) can be transformed into log-linear employment-value added functions as follows:

$$\ln EM(i,t') = \ln a(i) + b(i) \cdot \ln VA(i,t') + c(i) \cdot t'; \qquad (6)$$

$$i = 1,\ldots,I$$

The coefficients of the time variable c(i)'s implicitly capture the effects of both capital-labour substitution and technical change. A positive rate of technical change, for example, would tend to make the coefficients negative. As technical progress occurs, a given increase in value added will be associated with a smaller increase in employment. Capital deepening (the use of more capital per unit of labour) would tend to have a similar effect on the coefficients.

(b) Employment by industry

The previous section described various specifications of the employment value added functions. This section will describe the steps necessary to derive levels of employment.

Projecting employment by industry with this method amounts to evaluating the empirically estimated employment-value added functions by industry for various dates over the projection period. The way that the functions are evaluated will depend on their functional form as well as on whether the time variable is used.

(i) Functions without the time variable

The levels of employment by industry for the end of any projection interval (t to t+5) could be projected using the estimates of linear functions without the time variable as follows:

$$EM(i,t+5) = a^*(i) + b^*(i) \cdot VA(i,t+5); \qquad (7)$$

$$i = 1,\ldots,I,$$

where:

t	is the year of the projection period,
EM(i,t+5)	is the employment in industry i at the end of the interval,
VA(i,t+5)	is the value added in industry i at the end of the interval,

$a^*(i)$ is the estimate of the intercept coefficient of the linear employment-value added function for industry i, and

$b^*(i)$ is the estimate of the partial coefficient of the value added variable in the linear employment-value added function for industry i.

Estimated non-linear functions without the time variable would yield projections of employment by industry as follows:

$$EM(i,t+5) = a^*(i) \cdot VA(i,t+5)^{b^*(i)}; \quad i = 1,\ldots,I, \tag{8}$$

where:

$a^*(i)$ is the estimate of the intercept coefficient of the non-linear employment-value added function for industry i, and

$b^*(i)$ is the estimate of the partial coefficient of the value added variable in the non-linear employment-value added function for industry i.

Alternatively, the logarithmic transformations of the non-linear functions without the time variable could be used to project employment. Those functions would be first used to obtain the logarithms of employment levels by industry as follows:

$$\ln EM(i,t+5) = [\ln a(i)]^* + b^*(i) \cdot \ln VA(i,t+5); \quad i = 1,\ldots,I, \tag{9}$$

where:

$[\ln a(i)]^*$ is the estimate of the logarithm of the intercept coefficient of the non-linear function for industry i.

Once the logarithm of employment levels are obtained, as indicated in equation (9), the employment levels themselves can be obtained by calculating antilogarithms of the results:

$$EM(i,t+5) = \text{antiln}[\ \ln EM(i,t+5)\ ]; \quad i = 1,\ldots,I, \tag{10}$$

where:

antiln is the antilogarithm of the natural logarithm.

(ii) <u>Functions with the time variable</u>

To prepare a projection using estimates of linear functions which include time as a variable, the levels of employment by industry for the end of the projection interval (t to t+5) would be obtained as follows:

$$EM(i,t+5) = a^*(i) + b^*(i) \cdot VA(i,t+5) + c^*(i) \cdot (\bar{t}'+t+5); \quad i = 1,\ldots,I, \tag{11}$$

where:

$\bar{t}'$ is the calendar year designated as the initial year of the projection period, and

$c^*(i)$ is the estimate of the partial coefficient of the time variable in the linear employment-value added function for industry i.

If the estimates of non-linear functions with the time variable were used, the projection for the end of the interval (t to t+5) would be made as follows:

$$EM(i,t+5) = a^*(i) \cdot VA(i,t+5)^{b^*(i)} \cdot e^{[c^*(i) \cdot (\bar{t}'+t+5)]}; \quad i = 1,\ldots,I, \tag{12}$$

where:

$c^*(i)$ is the estimate of the partial coefficient of the time variable in the non-linear employment-value added function for industry i.

If the logarithmically transformed non-linear employment-value added functions were to be used, the projection would be made as follows:

$$\ln EM(i,t+5) = [\ln a(i)]^* + b^*(i) \cdot \ln VA(i,t+5) + c^*(i) \cdot (\bar{t}'+t+5); \quad i = 1,\ldots,I. \tag{13}$$

After the logarithms of employment levels by industry are obtained, the employment levels themselves can be obtained as indicated by equation (10).

(c) Other results 3/

Once the levels of employment by industry are projected for the end of a given projection interval, several derived indicators can be calculated. These indicators include aggregates, indicators of the structure and the rates of change of employment.

(i) Employment aggregates

A key aggregate that one can calculate from the projected levels of employment by industry is the level of total employment. In addition, using the same results, it is possible to obtain the levels of employment in sectors, such as the primary, secondary and tertiary sectors. Once the total and sectoral levels are obtained for different dates five years apart, increases in total and sectoral employment over the intervening projection intervals can be calculated.

a. Total employment

Total employment can be obtained by aggregating the levels of employment across industries. For the end of a projection interval (t to t+5) this number can be obtained as follows:

$$EM(t+5) = \sum_{i=1}^{I} EM(i,t+5), \qquad (14)$$

where:

EM(t+5) is the total employment at the end of the interval.

b. Employment by sector

A variety of criteria can be used to aggregate industries into sectors. For example, one can aggregate industries into primary, secondary and tertiary sectors, or into agricultural, industrial and service sectors. For illustrative purposes the primary-secondary-tertiary-sector classification will be used. In addition, it will be assumed that the numbering of industries for which the levels of employment are being projected lists industries of the primary, secondary and tertiary sectors one after another.

i. Employment in the primary sector

Using these aggregation and classification rules, employment in the primary sector for the end of the projection interval (t to t+5) can be obtained as:

$$EMP(t+5) = \sum_{i=1}^{Ip} EM(i,t+5), \tag{15}$$

where:

Ip is the number of industries in the primary sector, and

EMP(t+5) is the employment in the primary sector at the end of the interval.

ii. Employment in the secondary sector

Employment in the secondary sector can be obtained as follows:

$$EMS(t+5) = \sum_{i=Ip+1}^{Ip+Is} EM(i,t+5), \tag{16}$$

where:

Is is the number of industries in the secondary sector, and

EMS(t+5) is the employment in the secondary sector at the end of the interval.

iii. Employment in the tertiary sector

Employment in the tertiary sector can be calculated as:

$$EMT(t+5) = \sum_{i=Ip+Is+1}^{I} EM(i,t+5), \tag{17}$$

where:

EMT(t+5) is the employment in the tertiary sector at the end of the interval.

c. Growth in total employment

The growth in total employment over the projection interval (t to t+5) equals the difference between total employment at the end and total employment at the beginning of the interval:

$$EMGR = EM(t+5) - EM(t), \tag{18}$$

where:

EMGR is the growth of total employment during the interval.

d. Growth of employment by sector

The increase in employment in the primary, secondary and the tertiary sectors over the projection interval is respectively obtained as follows:

The growth in employment in the primary sector is calculated as:

$$EMPGR = EMP(t+5) - EMP(t), \tag{19}$$

The growth in employment in the secondary sector is calculated as:

$$EMSGR = EMS(t+5) - EMS(t), \tag{20}$$

The growth in employment in the tertiary sector is calculated as:

$$EMTGR = EMT(t+5) - EMT(t), \tag{21}$$

where:

EMPGR is the growth of employment in the primary sector during the interval,

EMSGR is the growth of employment in the secondary sector during the interval, and

EMTGR is the growth of employment in the tertiary sector during the interval.

(ii) Indicators of the structure of employment

Once the various employment aggregates are obtained, it is possible to derive the proportions of employment accounted for by each sector.

a. Proportions by sector

Proportions of total employment accounted for by each sector (primary, secondary and tertiary) can be obtained as follows:

The proportion of employment found in the primary sector is calculated as:

$$PEMP(t+5) = EMP(t+5) / EM(t+5), \tag{22}$$

The proportion of employment found in the secondary sector is calculated as:

$$PEMS(t+5) = EMS(t+5) / EM(t+5), \tag{23}$$

The proportion of employment found in the tertiary sector is calculated as:

$$PEMT(t+5) = EMT(t+5) / EM(t+5), \tag{24}$$

where:

PEMP(t+5) is the proportion of employment accounted for by the primary sector at the end of the interval,

PEMS(t+5) is the proportion of employment accounted for by the secondary sector at the end of the interval, and

PEMT(t+5) is the proportion of employment accounted for by the tertiary sector at the end of the interval.

(iii) Rates of growth of employment

As part of an employment projection, it is also possible to compute average annual rates of growth of employment, for the total employment and employment by sectors.

a. Rate of growth of total employment

The average annual rate of growth of total employment for a given projection interval can be computed from the total employment at the beginning and the end of the interval. If, as part of the projection process, the planner makes assumption that growth occurs over discrete intervals, then the percentage growth rate can be obtained using the formula for calculating a geometric growth rate:

$$GGREM = [(EM(t+5) / EM(t))^{1/5} - 1] \cdot 100, \tag{25}$$

where:

GGREM is the average annual geometric growth rate of total employment for the interval.

Alternatively, if the planner assumes that growth is continuous, then the percentage growth rate of total employment can be calculated using the formula for calculating an exponential growth rate:

$$EGREM = [\ln (EM(t+5) / EM(t)) / 5] \cdot 100, \tag{26}$$

where:

EGREM is the average annual exponential growth rate of total employment for the interval.

b. Rates of growth of employment, by sector

Assuming discrete growth, the percentage rates of growth of employment for major sectors can be obtained as follows:

The geometric growth rate for the primary sector is calculated as:

$$GGREMP = [(EMP(t+5) / EMP(t))^{1/5} - 1] \cdot 100, \tag{27}$$

The geometric growth rate for the secondary sector is calculated as:

$$GGREMS = [(EMS(t+5) / EMS(t))^{1/5} - 1] \cdot 100, \tag{28}$$

The geometric growth rate for the tertiary sector is calculated as:

$$GGREMT = [(EMT(t+5) / EMT(t))^{1/5} - 1] \cdot 100, \tag{29}$$

where:

GGREMP is the average annual geometric growth rate of employment in the primary sector for the interval,

GGREMS	is the average annual geometric growth rate of employment in the secondary sector for the interval, and
GGREMT	is the average annual geometric growth rate of employment in the tertiary sector for the interval.

If the projections were based on the assumption of continuous growth, then the percentage rates of growth of employment by sector would be calculated using the formulae for obtaining the exponential growth rate. The calculations would be as follows:

The exponential growth rate for the primary sector is calculated as:

$$EGREMP = [\ \ln\ (\ EMP(t+5)\ /\ EMP(t)\ )\ /\ 5\ ] \cdot 100, \tag{30}$$

The exponential growth rate for the secondary sector is calculated as:

$$EGREMS = [\ \ln\ (\ EMS(t+5)\ /\ EMS(t)\ )\ /\ 5\ ] \cdot 100, \tag{31}$$

The exponential growth rate for the tertiary sector is calculated as:

$$EGREMT = [\ \ln\ (\ EMT(t+5)\ /\ EMT(t)\ )\ /\ 5\ ] \cdot 100, \tag{32}$$

where:

EGREMP	is the average annual exponential growth rate of employment in the primary sector for the interval,
EGREMS	is the average annual exponential growth rate of employment in the secondary sector for the interval, and
EGREMT	is the average annual exponential growth rate of employment in the tertiary sector for the interval.

(iv) <u>Labour market balances</u>

Once various projection results are obtained, it is possible to calculate the excess demand for labour or excess supply of labour using projections of labour force and employment as indicators of future supply of or demand for labour. It is also possible to calculate the excess demand or excess supply as a percentage of the total labour force.

In countries where there is sizeable non-civilian employment, which may include military or internal security personnel, the projected labour force to be used in these calculations should not be the projected total labour force obtained as described in chapter V.

The projected labour force to be used is the projected civilian labour force, obtained as the difference between the projected total labour force and the projected non-civilian employment, where the latter projection is an additional input. The reason for this is related to the fact that in projections relating to the labour market, projections of the demand for labour (or employment) will normally apply to the civilian segment of the labour market. Therefore, projections of the supply of labour (or labour force) used to compute excess supply or demand must be those for the civilian segment.

To calculate excess supply or excess demand, therefore, the civilian labour force may first have to be calculated; for the end of the time interval (t to t+5), this can be obtained as:

$$CLF(t+5) = LF(t+5) - NEM(t+5), \tag{33}$$

where:

CLF(t+5) is the civilian labour force at the end of the interval,

LF(t+5) is the total labour force at the end of the interval, and

NEM(t+5) is the non-civilian employment at the end of the interval.

The excess supply of (or demand for) labour for the end of the interval can be obtained as the difference between the projected civilian labour force and the projected employment for that date:

$$EXL(t+5) = CLF(t+5) - EM(t+5), \tag{34}$$

where:

EXL(t+5) is the excess supply of labour (if positive) or excess demand for labour (if negative) for the end of the interval.

The excess demand or excess supply as a percentage of the civilian labour supply (civilian labour force), can be calculated as:

$$PEXL(t+5) = [\ EXL(t+5)\ /\ CLF(t+5)\ ] \cdot 100, \qquad (35)$$

where:

PEXL(t+5) is the excess supply of labour or excess demand for labour as a percentage of the civilian labour force at the end of the interval.

3. Urban-rural level

This section will describe employment-value added functions along with a procedure that utilizes them to calculate an urban-rural projection of employment. The procedure, which is similar to that used in the national projection, consists of steps used to project the levels of employment by industry as well as those needed to derive a variety of other results.

(a) Employment-value added functions by industry

The procedure for making urban-rural projections could make use of estimates of employment-value added functions, which are the urban-rural equivalents of the functions indicated in equations (1) through (6). This would be the case where time series data on employment and value added, by industry, are available for urban and rural areas and where it is therefore possible to empirically estimate parameters of the employment-value added functions separately for urban and rural areas.

However, sufficiently long and reliable time series on value added by industry for urban and rural areas are not available in some countries. Accordingly, estimating the urban-rural counterparts of the functions shown in equations (1) through (6) would be often difficult, if not impossible in those countries.

Where the value added time series for urban and rural areas are short, unreliable or altogether lacking, employment projections can be made by means of empirically based functions which relate employment by industry in urban and rural areas to value added by industry for the entire country. Functions of this type are briefly reviewed below.

(i) Functions without the time variable

Linear employment-value added functions by industry and urban-rural location, having the national value added by industry as explanatory variable but omitting the time variable, are as follows:

$$EM(i,k,t') = a(i,k) + b(i,k) \cdot VA(i,t'); \qquad (36)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

$k = 1,2$	are urban and rural locations,
$EM(i,k,t')$	is the employment in industry i in location k in year t',
$a(i,k)$	is the intercept coefficient of the linear employment-value added function for industry i in location k, and
$b(i,k)$	is the partial coefficient of the value added variable in the linear employment-value added function for industry i in location k.

Non-linear functions by industry and urban-rural location, without time as a variable, are as follows:

$$EM(i,k,t') = a(i,k) \, . \, VA(i,t')^{b(i,k)}; \qquad (37)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

$a(i,k)$	is the intercept coefficient of the non-linear employment-value added function for industry i in location k, and
$b(i,k)$	is the partial coefficient of the value added variable in the non-linear employment-value added function for industry i in location k.

Log-linear functions by industry and location, which can be obtained by a logarithmic transformation of non-linear functions shown in equation (37), are as follows:

$$\ln EM(i,k,t') = \ln a(i,k) + b(i,k) \cdot \ln VA(i,t'); \quad (38)$$

$$i = 1,\ldots,I;$$

$$k = 1,2.$$

(ii) Functions with the time variable

Linear employment-value added functions by industry and location, having both national value added and time as explanatory variables, are as follows:

$$EM(i,k,t') = a(i,k) + b(i,k) \cdot VA(i,t') + c(i,k) \cdot t'; \quad (39)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

c(i,k) is the partial coefficient of the time variable in the linear employment-value added function for industry i in location k.

Non-linear functions by industry and location, having national value added and time as explanatory variables, are:

$$EM(i,k,t') = a(i,k) \cdot VA(i,t')^{b(i,k)} e^{[c(i,k) \cdot t']}; \quad (40)$$

$$i = 1,\ldots,I,$$

where:

c(i,k) is the partial coefficient of the time variable in the non-linear employment-value added function for industry i in location k.

Log-linear functions by industry and location, having national value added and time as explanatory variables, are:

$$\ln EM(i,k,t') = \ln a(i,k) + b(i,k) \cdot \ln VA(i,t') + c(i,k) \cdot t'; \quad (41)$$

$$i = 1,\ldots,I;$$

$$k = 1,2.$$

(b) Employment by industry

Projecting employment by industry for urban and rural areas using functions of this type would amount to evaluating empirically estimated functions by industry for the two areas for various dates over the projection period. As in the case of the national projection, the exact way of evaluating the functions would depend on their functional form as well as on whether the time variable is used.

(i) Functions without the time variable

The levels of employment by industry and location for the end of the projection interval (t to t+5) would be projected using estimates of linear functions without the time variable, as follows:

$$EM(i,k,t+5) = a^*(i,k) + b^*(i,k) \cdot VA(i,t+5); \quad (42)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

EM(i,k,t+5) is the employment in industry i in location k at the end of the interval,

a*(i,k) is the estimate of the intercept coefficient of the linear employment-value added function for industry i in location k, and

b*(i,k) is the estimate of the partial coefficient of the value added variable in the linear employment-value added function for industry i in location k.

Where estimates of non-linear functions without the time variable were used, the projected levels of employment would be obtained as follows:

$$EM(i,k,t+5) = a^*(i,k) \cdot VA(i,t+5)^{b^*(i,k)}; \quad (43)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

a*(i,k) is the estimate of the intercept coefficient of the non-linear employment-value added function for industry i in location k, and

b*(i,k) is the estimate of the partial coefficient of the value added variable in the non-linear employment-value added function for industry i in location k.

The logarithms of the projected levels of employment, by industry and location for the end of the interval can be derived from estimates of log-linear functions that did not include the time variable, as follows:

$$\ln EM(i,k,t+5) = [\ln a(i,k)]^{*} + b^{*}(i,k) \cdot \ln VA(i,t+5); \quad i = 1,\dots,I; \quad k = 1,2. \tag{44}$$

The levels of employment can be calculated as antilogarithms of the logs of employment levels by industry:

$$EM(i,k,t+5) = \mathrm{antiln}[\ \ln EM(i,k,t+5)\ ]; \quad i = 1,\dots,I; \quad k = 1,2. \tag{45}$$

(ii) Functions with the time variable

To make a projection using estimates of the linear functions with the time variable, the levels of employment by industry and location for the end of the projection interval (t to t+5) would be obtained as follows:

$$EM(i,k,t+5) = a^{*}(i,k) + b^{*}(i,k) \cdot VA(i,t+5) + c^{*}(i,k) \cdot (\bar{t}'+t+5); \quad i = 1,\dots,I; \quad k = 1,2, \tag{46}$$

where:

$c^*(i,k)$ is the estimate of the partial coefficient of the time variable in the linear employment-value added function for industry i in location k.

To make a projection using the non-linear functions with the time variable, the projected levels of employment would be obtained as:

$$EM(i,k,t+5) = a^*(i,k) \cdot VA(i,t+5)^{b^*(i,k)} \cdot e^{[c^*(i,k) \cdot (\bar{t}'+t+5)]}; \qquad (47)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

$c^*(i,k)$ is the estimate of the partial coefficient of the time variable in the non-linear employment-value added function for industry i in location k.

Where the projection is made with log-linear functions involving the time variable, it is first necessary to derive the logarithms of the projected levels of employment, by industry and location, for the end of the interval as:

$$\ln EM(i,k,t+5) = [\ln a(i,k)]^* + b^*(i,k) \cdot \ln VA(i,t+5) \qquad (48)$$

$$+ c^*(i,k) \cdot (\bar{t}'+t+5);$$

$$i = 1,\ldots,I;$$

$$k = 1,2.$$

Once the logarithms of employment levels by industry and location are obtained, the employment levels can be calculated by taking the antilogarithms of the results, as indicated by equation (45).

(c) Other results 4/

The indicators obtained as part of the national projection can also be computed in the course of preparing an urban-rural projection. Those indicators are calculated for urban and rural areas and for the entire country, using steps analogous to those indicated by equations (14) through (32). The projection also permits the calculation of the excess supply of (or excess demand for) labour based on steps analogous to those described by equations (33) and (35). These calculations would also refer to the urban and rural areas as well as the entire country. In addition, indicators of the distribution of employment by location--proportions urban and rural--can be calculated.

(i) Proportions of employment, urban and rural

The proportions of employment occurring in urban and rural areas can be derived both for total employment and employment by sector.

a. Proportions of total employment

The proportion of total employment that is urban (k=1) is obtained as a ratio of total employment in urban areas to total employment in the entire country:

PEMURB(t+5) = EM(1,t+5)/EM(t+5); (49)

where:

PEMURB(t+5) is the proportion of total employment that is urban at the end of the interval, and

EM(k,t+5) is the total employment in location k at the end of the interval.

The proportion of total employment that is rural (k=2) can be obtained as a complement of the proportion urban:

PEMRUR(t+5) = 1 - PEMURB(t+5); (50)

where:

PEMRUR(t+5) is the proportion of total employment that is rural at the end of the interval.

b. Proportions of employment, by sector

Proportions of total employment in the primary, secondary and tertiary sectors that are urban (k=1) can be calculated as ratios of employment in those sectors in urban areas to employment in those sectors in the entire country. In particular, urban proportions can be calculated as follows:

The proportion of employment in the primary sector that is urban:

$$PEMPURB(t+5) = EMP(1,t+5)/EMP(t+5), \tag{51}$$

The proportion of employment in the secondary sector that is urban:

$$PEMSURB(t+5) = EMS(1,t+5)/EMS(t+5), \tag{52}$$

The proportion of employment in the tertiary sector that is urban:

$$PEMTURB(t+5) = EMT(1,t+5)/EMT(t+5), \tag{53}$$

where:

PEMPURB(t+5)	is the proportion of employment in the primary sector that is urban at the end of the interval,
PEMSURB(t+5)	is the proportion of employment in the secondary sector that is urban at the end of the interval,
PEMTURB(t+5)	is the proportion of employment in the tertiary sector that is urban at the end of the interval,
EMP(k,t+5)	is the employment in the primary sector in location k at the end of the interval,
EMS(k,t+5)	is the employment in the secondary sector in location k at the end of the interval, and
EMT(k,t+5)	is the employment in the tertiary sector in location k at the end of the interval.

For each sector, the proportions of employment that are rural (k=2) can be obtained as complements of proportions urban:

The proportion of employment in the primary sector that is rural:

$$PEMPRUR(t+5) = 1 - PEMPURB(t+5), \tag{54}$$

The proportion of employment in the secondary sector that is rural:

$$PEMSRUR(t+5) = 1 - PEMSURB(t+5), \tag{55}$$

The proportion of employment in the tertiary sector that is rural:

$$PEMTRUR(t+5) = 1 - PEMTURB(t+5), \tag{56}$$

where:

PEMPRUR(t+5) is the proportion of employment in the primary sector that is rural at the end of the interval,

PEMSRUR(t+5) is the proportion of employment in the secondary sector that is rural at the end of the interval, and

PEMTRUR(t+5) is the proportion of employment in the tertiary sector that is rural at the end of the interval.

This completes the discussion of the technique for making employment projections using employment-value added functions.

C. The inputs

This section first lists the inputs required by the method of employment projection utilizing employment-value added functions and then describes how they can be prepared.

1. Types of inputs required

The following inputs are required in order to project employment by using the employment-value added functions:

(i) Projected levels of value added by industry;
(ii) Estimates of the coefficients of the employment-value added functions, by industry.

If projections of labour surpluses and/or shortages are also to be prepared, the inputs should include:

(iii) Projected total labour force;
(iv) Projected non-civilian employment.

Depending on whether it is desired to make a national projection or a projection for urban and rural areas, most inputs will be required for the entire country or for urban and rural areas. The projected levels of value added by industry, however, can be those for the entire country even though a projection for urban and rural areas is desired.

2. Preparation of the inputs

To apply the method, projections of value added by industry are required. These projections are part of many quantitative development planning exercises. Hence, procedures for making value added projections are not discussed in the present volume except for the procedure briefly outlined in chapter VI, box 17. Projections of the total labour force can be prepared as described in chapter V, while those of non-civilian employment can be obtained by considering likely future developments in the non-civilian sector of the economy. Estimates of the coefficients of employment-value added functions can be prepared as described below.

(a) Estimates of the coefficients of employment-value added functions

Estimates of the coefficients of employment-value added functions for each industry would be typically prepared using a standard method of regression analysis, such as ordinary least squares (OLS). Depending on the type of projection intended (national or urban-rural), the estimates of the functions would be for the entire country or for urban and rural areas. The estimation would make use of time series data on employment and value added, by industry, at the national level or for urban and rural areas.

(i) Time series data

Time series data on employment by industry at the national or urban-rural level can be obtained from annual surveys of establishments or from periodic labour force surveys of households. If those data refer only to modern establishment of various industries, which is sometimes the case in developing countries, and data on traditional establishments are available for a few years, the latter data may be used as a basis for inflating the employment in modern establishments with a view to estimating total employment levels by industry over time.

Time series data on value added by industry would normally be obtained from the national accounts. Unfortunately, the national accounts or other data sources will rarely include value added information for industries classified by urban-rural location. Where projections of employment are desired by location, it may be necessary to use data on employment by industry and location along with those on value added by industry without the locational breakdown in order to estimate requisite employment-value added functions.

(ii) Estimation procedures

Estimating employment-value added functions at the national or urban-rural level would normally entail preparing estimates of functions having different forms, which exclude and include the time variable. This would be necessary in view of the fact that it would never be possible to determine *a priori* which functional form may be more suitable than the other as well as whether including the time variable improves the fit or not. As a result, the user of the method would typically be required to estimate functions of different forms, compare the results, and select for use those estimates that appear most robust in terms of goodness of fit, significance of coefficients and so on.

This section will first describe a method to estimate coefficients of employment-value added functions of different forms at the national level. It will then describe procedures to estimate those functions at the urban-rural level. Subsequently, estimation of those functions will be illustrated using the time series data on employment and value added.

a. National level

Procedures used to estimate employment-value added functions will differ depending on whether or not those functions include time as a variable.

i. Functions without the time variable

In order to obtain estimates of linear employment value-added functions that do not include the time variable, it would be necessary to add to the functions shown in equation (1) random disturbance terms to obtain:

$$EM(i,t') = a(i) + b(i) \cdot VA(i,t') + u(i,t'); \quad i = 1,\ldots,I, \tag{57}$$

where:

$u(i,t')$ is the random disturbance term for industry i in year t'.

The functions shown in equation (57) would be then estimated with a regression technique, such as OLS, using time series data on employment and value added.

The following standard approach can be used for estimating coefficients of non-linear functions having the multiplicative form, which do not include time as a variable. First, it would be necessary to take logarithms of the functions shown in equation (2) and add a disturbance term to each:

$$\ln EM(i,t') = \ln a(i) + b(i) \cdot \ln VA(i,t') + u(i,t'); \quad i = 1,\ldots,I. \tag{58}$$

The resultant log-linear functions could then be estimated using a regression technique, such as OLS.

The result would be estimates of the logarithms of the intercept coefficients of the non-linear functions, $[\ln a(i)]^*$'s, and estimates of the partial coefficients, $b^*(i)$'s. The estimate of the partial coefficients can be used as obtained by OLS, while those of the logarithms of the intercept coefficients need to be transformed into estimates of the intercept coefficients. This can be done by taking antilogarithms of the estimates of the logarithms of intercept coefficients:

$$a^*(i) = \text{antiln}[\ln a(i)]^*; \qquad (59)$$

$$i = 1,\ldots,I.$$

To estimate the log-linear employment-value added functions, it would be initially necessary to add random disturbance terms to those functions (equation (3)) and to obtain functions indicated in equation (58). Those functions could be estimated by a regression technique such as OLS. The estimates of the logarithms of the intercept coefficients and of partial coefficients can be used directly from this estimation.

ii. <u>Functions with the time variable</u>

To derive estimates of linear employment value-added functions which include the time variable, it would initially be necessary to add random disturbance terms to those functions (equation (4)) to obtain:

$$EM(i,t') = a(i) + b(i) \cdot VA(i,t') + c(i) \cdot t' + u(i,t'); \qquad (60)$$

$$i = 1,\ldots,I.$$

These functions would then be estimated with a regression technique such as OLS, using appropriate time series data.

To estimate non-linear functions with the time variable, it would initially be necessary to take logarithms of those functions (equation (5)) and add a disturbance term to each:

$$\ln EM(i,t') = \ln a(i) + b(i) \cdot \ln VA(i,t') + c(i) \cdot t' + u(i,t'); \qquad (61)$$

$$i = 1,\ldots,I.$$

The log-linear functions can be estimated from time series data using a regression technique such as OLS. This should be followed by the derivation of estimates of the intercept coefficients, as indicated by equation (59).

To derive estimates of log-linear functions with time as a variable, it would first be necessary to add disturbance terms to those functions (equation (6)) and in the process derive a set of functions shown in equation (61). Such functions could be estimated by a regression technique, such as OLS. The estimates of the logarithms of the intercept coefficients and of partial coefficients can be used in the form obtained by this estimation.

b. Urban-rural level

The methods for estimation of employment-value added functions at the urban-rural level will also differ, depending on whether or not the functions include the time variable.

i. Functions without the time variable

To obtain estimates of linear functions without the time variable for urban and rural areas, it would be initially necessary to add random disturbance terms to the functions shown in equation (36) to obtain:

$$EM(i,k,t') = a(i,k) + b(i,k) \cdot VA(i,t') + u(i,k,t'); \qquad (62)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

$u(i,k,t')$ is the random disturbance term for industry i in location k in year t'.

The functions shown in equation (62) would be estimated with OLS, using time series data on employment and value added.

If estimates of non-linear functions without time as a variable are desired, it would be initially necessary to take logarithms of the functions shown in equation (37) and add a disturbance term to each:

$$\ln EM(i,k,t') = \ln a(i,k) + b(i,k) \cdot \ln VA(i,t') + u(i,k,t'); \qquad (63)$$

$$i = 1,\ldots,I;$$

$$k = 1,2.$$

The resultant log-linear functions can be estimated using a regression technique, such as OLS.

The result would include estimates of the logarithms of the intercept coefficients of the non-linear functions, $[\ln a(i,k)]^*$'s, and estimates of the partial coefficients, $b^*(i,k)$'s. While the estimate of the partial coefficients can be used as obtained by OLS, those of the logarithms of the intercept coefficients must be transformed into estimates of the intercept coefficients themselves. This can be accomplished by taking antilogarithms of the estimates of the logarithms of intercept coefficients:

$$a^*(i,k) = \text{antiln}[\ln a(i,k)]^*; \quad i = 1,\ldots,I; \quad k = 1,2. \tag{64}$$

To estimate the log-linear employment-value added functions, it would be initially necessary to add random disturbance terms to those functions (equation (38)) and to obtain functions indicated in equation (63). Those functions can be estimated using regression techniques, such as OLS. The estimates of the logarithms of the intercept coefficients and of partial coefficients can be used in the form obtained by this estimation.

ii. Functions with the time variable

To derive estimates of linear employment value-added functions with the time at the urban-rural level, it is necessary to add disturbance terms to those functions (equation (39)) to obtain:

$$EM(i,k,t') = a(i,k) + b(i,k) \cdot VA(i,t') + c(i,k) \cdot t' + u(i,k,t'); \quad i = 1,\ldots,I; \quad k = 1,2. \tag{65}$$

These functions would then be estimated with a regression technique, such as OLS.

Non-linear functions with time as a variable can be estimated by taking logarithms of those functions (equation (40)) and adding a disturbance term to each:

$$\ln EM(i,k,t') = \ln a(i,k) + b(i,k) \cdot \ln VA(i,t') + c(i,k) \cdot t' + u(i,k,t'); \quad i = 1,\ldots,I; \quad k = 1,2. \tag{66}$$

These log-linear functions can be estimated from time series data using a regression technique, such as OLS. Estimates of the intercept coefficients can be derived as indicated by equation (64).

To derive estimates of log-linear functions with time as a variable, it is necessary to add disturbance terms to those functions (equation (41)) and in the process derive a set of functions shown in equation (66). Such functions would be estimated by a regression technique, such as OLS. The estimates of the logarithms of the intercept coefficients and of partial coefficients can be used in the form obtained by this estimation.

(iii) Illustrative estimation

This section will use the time series data shown in tables 86 through 88 to illustrate a technique for estimating employment-value added functions of various specifications, first at the national level and then at the urban-rural level. Reasons will be presented why estimates of functions of a given form (for example those with time as a variable) may be preferable to those of other forms (such as those without the time variable).

a. National level

The estimation of the national-level functions without time as a variable will be illustrated, and then the estimation of the functions with time as a variable will be discussed.

i. Functions without the time variable

To project employment using estimates of national-level linear employment-value added functions without time as a variable, it would be necessary to estimate the functions indicated in equation (57). If OLS regression techniques are employed along with the data presented in tables 86 and 87 to estimate those functions, the results will be those shown in table 89.

For the most part, those results will be satisfactory as a basis for making projections of employment. All of the estimated partial coefficients (column 3) are positive, as expected, and most are statistically significant at the 0.05 level. However, forecast errors could be fairly high in the case of functions with coefficients of determination, R^2's (column 4), under 0.90, as is the case with several of the estimated functions. Column 5 of table 89 also shows Durbin-Watson statistics (box 23), which indicate whether or not the disturbance terms are serially correlated (box 24).

To obtain estimates of non-linear employment-value added functions, it is necessary to estimate the log-linear functions indicated in equation (58). The estimates of the coefficients of those functions based on the time series data shown in tables 86 and 87 are those presented in table 90.

Table 86. Employment for the entire country, by industry: 1968–1978

(Thousands of employed persons)

Year	Agriculture	Mining	Manufacturing	Utilities	Construction	Trade	Transport	Services
1968	1656.5	5.0	124.5	9.6	57.6	79.3	68.5	312.8
1969	1688.5	5.4	127.7	9.2	53.8	79.7	73.9	330.1
1970	1739.8	6.3	137.8	8.9	55.8	77.4	84.4	336.8
1971	1766.1	7.2	155.4	9.0	59.8	85.4	82.1	365.9
1972	1764.8	7.1	145.5	11.3	64.3	83.9	86.2	375.7
1973	1843.5	7.4	159.8	9.6	69.4	81.1	82.5	391.4
1974	1846.3	9.4	171.8	10.0	76.0	100.5	89.2	442.1
1975	1887.0	8.3	170.3	13.7	71.0	94.1	83.7	468.5
1976	1917.8	10.3	184.0	14.4	81.8	104.9	88.4	483.3
1977	1955.6	9.5	198.2	15.6	85.4	108.5	89.5	503.3
1978	1994.0	6.7	217.3	15.3	93.8	109.4	95.1	516.2

Table 87. Value added for the entire country, by industry: 1968-1978

(Millions of local currency units; constant 1968 prices)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
1968	155.2	2.2	44.6	8.2	18.4	41.2	38.1	119.2
1969	165.4	2.0	48.6	9.1	18.6	44.0	38.6	128.1
1970	172.4	2.6	52.5	9.8	19.0	44.6	41.2	139.0
1971	175.9	2.7	59.3	10.6	20.2	47.1	43.1	152.9
1972	189.3	2.5	63.6	11.6	23.1	42.6	42.4	172.2
1973	199.4	3.6	70.8	11.9	23.8	45.5	45.2	185.7
1974	202.6	3.9	74.9	12.7	22.2	46.4	44.7	207.1
1975	237.1	3.5	75.5	13.7	21.2	49.5	42.0	223.0
1976	235.2	3.8	89.6	15.4	20.7	51.8	46.4	237.5
1977	256.5	4.0	103.9	16.5	22.3	50.5	47.7	252.6
1978	260.3	2.6	118.9	18.0	23.5	50.9	50.3	266.9

Table 88. Employment for urban and rural areas, by industry: 1968-78

(Thousands of employed persons)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
				Urban				
1968	21.0	4.4	113.6	8.3	54.3	77.9	67.2	214.1
1969	19.3	4.8	115.9	8.6	52.5	75.1	72.9	218.1
1970	18.2	5.4	117.2	8.6	52.7	73.7	83.4	221.6
1971	18.1	5.9	132.2	8.1	52.8	81.7	79.3	240.3
1972	19.7	5.8	128.2	8.9	56.3	76.5	84.6	267.0
1973	16.8	5.9	137.9	8.9	59.4	72.9	81.2	274.9
1974	18.5	7.5	148.7	9.1	66.6	91.9	86.5	286.3
1975	20.4	6.5	146.9	12.5	64.4	85.2	80.7	299.2
1976	18.8	8.1	158.7	12.3	73.2	94.5	85.9	306.2
1977	21.2	7.8	169.4	12.5	77.0	96.9	86.6	319.4
1978	22.2	5.9	184.2	12.7	81.4	98.8	92.2	334.3
				Rural				
1968	1635.5	0.5	10.9	1.3	3.3	1.4	1.3	98.7
1969	1668.8	0.6	11.7	0.6	1.3	4.7	1.1	112.0
1970	1721.6	0.9	20.6	0.3	3.1	3.7	1.0	115.2
1971	1748.0	1.3	23.2	0.9	7.0	3.7	2.8	125.6
1972	1745.1	1.4	17.4	2.5	8.0	7.4	1.7	108.8
1973	1826.7	1.5	21.9	0.7	10.0	8.2	1.4	116.5
1974	1827.7	1.8	23.1	0.9	9.4	8.7	2.7	155.8
1975	1866.6	1.8	23.4	1.2	6.6	8.8	3.0	169.3
1976	1898.9	2.1	25.3	2.1	8.6	10.4	2.5	177.2
1977	1934.3	1.7	28.8	3.1	8.4	11.6	2.9	184.0
1978	1971.8	0.8	33.1	2.6	12.4	10.6	2.9	181.9

Table 89. Estimates of the coefficients of linear employment-value added functions without the time variable, by industry: entire country a/

Industry	Coefficients		R-square	Durbin-Watson
	Intercept	Value added b/		
(1)	(2)	(3)	(4)	(5)
Agriculture	1235.7727	2.87485 (13.810)	0.955	2.10
Mining	0.9902	2.14693 (7.059)	0.847	2.46
Manufacturing	73.1422	1.23128 (17.817)	0.972	1.94
Utilities	1.6971	0.78495 (6.805)	0.837	1.66
Construction	-27.7123	4.60744 (2.836)	0.472	0.54
Trade	-50.1969	3.02735 (5.053)	0.739	2.36
Transport	6.4904	1.77633 (5.829)	0.791	2.25
Services	141.9969	1.42219 (28.608)	0.989	1.73

a/ Estimated by ordinary least squares (OLS) .
b/ t values are shown in parentheses.

Table 90. Estimates of the coefficients of log-linear employment-value added functions without the time variable, by industry: entire country a/

Industry	Coefficients		R-square	Durbin-Watson
	Intercept	Value added b/		
(1)	(2)	(3)	(4)	(5)
Agriculture	5.7760	0.32625 (15.226)	0.963	2.35
Mining	1.0547	0.86530 (7.280)	0.855	2.40
Manufacturing	2.6863	0.56365 (18.948)	0.976	2.47
Utilities	0.4165	0.80202 (5.802)	0.789	1.49
Construction	-0.1260	1.42891 (3.134)	0.522	0.57
Trade	-1.3045	1.51233 (4.937)	0.730	2.34
Transport	0.8048	0.96016 (5.798)	0.789	2.18
Services	2.6878	0.63680 (24.260)	0.985	1.44

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

Box 23

Glossary

Durbin-Watson statistic
The statistic, developed by J. Durbin and G. S. Watson as a weighted ratio of the sum of squared differences in successive residuals, which is used to test for autocorrelation of residuals.

Serially correlated
Disturbance terms of a regression equation fitted to time series data are said to be serially correlated if there is a degree of stochastic dependence between those terms. Serial correlation occurs when effects due to particular chance disturbances or omitted variables tend to persist, through several periods or years. It could also be occasioned by methods of data collecting or reporting that incorporate elements of smoothing and interpolation which average the "true" disturbances over adjacent periods.

These results are equally satisfactory and sometimes better than those shown in table 89. The only exception are the results for utilities, for which both R^2 and the Durbin-Watson statistics are lower than those obtained by estimating the linear employment-value added function for this industry.

The estimates of partial coefficients such as those presented in table 90 can be used as derived by OLS, while the estimates of the logarithms of the intercepts must be transformed into estimates of the intercept coefficients themselves. As illustrated in table 91, the latter estimates (column 3) can be obtained by taking antilogarithms of the estimates of the logarithms of the intercept coefficients (column 2). Thus, the estimate of the intercept coefficient for agriculture, 322.467, can be calculated as:

$$322.467 = \text{antiln}(5.77600), \qquad (59)$$

where 5.77600 is the estimate of the logarithm of the intercept coefficient of the non-linear function for agriculture.

Estimates of the log-linear functions can be obtained directly by estimating the functions shown in equation (58). Estimates of such functions based on the time series data presented in tables 86 and 87 are those shown in table 90 and discussed above.

Table 91. Computing estimates of intercept coefficients of non-linear employment-value added functions without the time variable, by industry: entire country

Industry (1)	Intercepts of log-linear functions a/ (2)	Intercepts of non-linear functions b/ (3)
Agriculture	5.77600	322.467
Mining	1.05470	2.871
Manufacturing	2.68634	14.678
Utilities	0.41649	1.517
Construction	-0.12600	0.882
Trade	-1.30452	0.271
Transport	0.80483	2.236
Services	2.68782	14.700

a/ From table 90, col. 2.
b/ Antiln(Col. 2).

Box 24

Serial correlation

Serial correlation occurs frequently with time-series data and suggests either that important variables have been omitted from the equation estimated or that an incorrect functional form has been employed. Although levels of significance for the Durbin-Watson statistic depend on the number of independent variables and the number of observations in the estimated regression, values of less then 1 are highly suggestive of serial correlation. In table 89, for example, such a low value of the Durbin-Watson statistics is observed in the construction industry.

When serially correlated disturbances are suspected, one of several remedial steps may be tried. One such step would be to use an alternative form for the functions being estimated. This step could involve the use, for example, of a non-linear or log-linear rather than a linear form. Alternatively, other independent variables can be added to the function to see whether their inclusion reduces the extent of serial correlation. In many contexts, an additional variable that could be tried is the time variable. Other approaches to the problem of serial correlation can be also tried.a/

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a/ See, for example, Jan Kmenta, Elements of Econometrics (New York, Macmillan, 1971).

ii. Functions with the time variable

Estimates of national-level linear employment-value added functions with the time variable can be obtained by estimating the functions shown in equation (60). Estimates of such functions based on the data shown in tables 86 and 87 are given in table 92. 5/

When judged on the basis of R^2's and Durbin-Watson statistics, the results for all the industries are better than those for the linear functions without the time variable (table 89). With the exception of the construction industry, however, the results are only marginally better. Adding the time variable has greatly increased the values of the R^2 and the Durbin-Watson

Table 92. Estimates of the coefficients of linear employment-value added functions with the time variable, by industry: entire country a/

Industry	Coefficients b/			R-square	Durbin-Watson
	Intercept	Value added	Time variable		
(1)	(2)	(3)	(4)	(5)	(6)
Agriculture	-60421.5146	0.11880 (0.190)	31.53616 (4.499)	0.987	3.01
Mining	-192.5399	1.84087 (4.454)	0.09856 (1.082)	0.867	2.53
Manufacturing	-5234.8860	0.86031 (3.478)	2.70405 (1.553)	0.979	2.62
Utilities	1413.1940	1.53768 (2.080)	-0.72018 (-1.031)	0.856	1.86
Construction	-7440.9921	0.01652 (0.018)	3.80665 (6.815)	0.922	1.62
Trade	-5477.2335	0.70268 (0.685)	2.80572 (2.550)	0.856	2.46
Transport	-1384.2503	1.19043 (1.539)	0.71784 (0.827)	0.807	2.11
Services	25170.6595	2.24062 (3.236)	-12.76418 (-1.185)	0.991	1.97

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

statistics for the construction industry. On the other hand, the value added coefficients for a number of industries are no longer statistically significant (agriculture, trade and transport). Furthermore, the value added coefficient for construction continues to be non-significant.

For five of the industries, the time variable coefficients are not statistically significant. Also, for all industries except two (utilities and services), the signs of the time variable coefficients are positive. The positive signs are contrary to a priori expectations which would call for a negative value for the coefficients in an employment-value added function, since both technical progress and capital deepening tend to reduce the amount of labour required to produce a given level of value added.

On purely statistical grounds, the results for the construction industry are clearly to be preferred to those reported for that industry in table 89. (This is not the case for the results for other industries). However, the sign of the coefficient of the time variable for the construction industry is also inconsistent with prior expectations suggested by the economic theory.

Such inconsistent results, however, are fairly common occurrences, suggesting that important variables may have been omitted from a function. Such results should not automatically be discarded, however. Instead, an effort should be made to identify the factors that have accounted for the irregular past performance, such as a steadily increasing employment in the construction industry in the presence of fairly stable levels of production (i.e., declining labour productivity). If there is reason to believe that such factors are likely to continue to exert the same impact over the planning horizon, the estimated function can be used as a basis for making future projections. If not, ad hoc adjustments will have to be made to the function for projection purposes.

Alternatively, employment-value added functions that reflect more typical conditions may be produced by deleting the observations referring to the typical periods, which reflect the effects of unusual circumstances in the past. Special statistical approaches, (such as introducing dummy variables) can be used to achieve essentially the same result without sacrificing all of the information contained in the atypical observations (Kmenta, 1971).

To obtain estimates of non-linear functions that include the time variable, it would be necessary to estimate the log-linear functions indicated in equation (61). If they were estimated on the basis of the data shown in tables 86 and 87, using ordinary least squares techniques, the results would be those presented in table 93.

Most of these results are just marginally better than the comparable results obtained without using the time variable (table 90). The only exceptions are the results for the construction industry, but as in the case of the results for this industry that were obtained by estimating a linear function with the time variable, the coefficient of this variable is

Table 93. Estimates of the coefficients of log-linear employment-value added functions with the time variable, by industry: entire country a/

Industry	Coefficients b/			R-square	Durbin-Watson
	Intercept	Value added	Time variable		
(1)	(2)	(3)	(4)	(5)	(6)
Agriculture	-28.7872	-0.00693 (-0.078)	0.01841 (3.787)	0.987	3.02
Mining	-27.5627	0.72835 (4.499)	0.01458 (1.211)	0.877	2.61
Manufacturing	-10.5030	0.49130 (2.218)	0.00684 (0.330)	0.976	2.58
Utilities	75.0737	1.31027 (0.747)	-0.03848 (-0.291)	0.791	1.47
Construction	-99.4136	0.07935 (0.305)	0.05240 (7.131)	0.935	1.98
Trade	-58.2461	0.31108 (0.609)	0.03120 (2.656)	0.857	2.54
Transport	-12.6445	0.69933 (1.632)	0.00731 (0.664)	0.800	2.02
Services	-28.8026	0.44341 (1.472)	0.01647 (0.645)	0.986	1.60

a/ Estimated by ordinary least squares (OLS) .
b/ t values are shown in parentheses.

positive. This is contrary to what would be expected of the sign of this coefficient on theoretical grounds.

To estimate the intercept coefficients of the non-linear functions, it would be necessary to take antilogarithm of the estimated logarithms of the intercepts. This can be done as described in the case of non-linear functions, which did not include the time variable.

If estimates of log-linear functions with time as a variable are required, they can be obtained by estimating the functions as indicated in equation (61). Estimates of those functions based on data given in tables 86 and 87 are presented in table 93. Such estimates can be used in the form as they are obtained by OLS.

b. Urban-rural level

This section will initially consider the estimation of functions that do not include the time variable, and then it will discuss estimation of the functions that include this variable. The discussion will be confined to functions using the national level of value added for each industry as an independent variable.

i. Functions without the time variable

To project employment for urban and rural areas using linear functions that do not include the time variable, it is necessary to estimate functions such as those indicated in equation (62). Estimates of such functions for urban and rural areas, which were obtained by OLS from the data of tables 87 and 88, are shown respectively in tables 94 and 95.

These results are largely satisfactory--all of the estimated partial coefficients are positive and most are statistically significant. The exception is the coefficient for urban agriculture (t-statistics is 1.55). However, for a few industries in either location, coefficients of determination, R^2, and Durbin-Watson statistics are lower than desired. The most obvious example is the equation for the construction industry in the urban areas, which has an R^2 of 0.330 and a Durbin-Watson statistic of 0.41. In addition, judged on the basis of the latter statistic, estimates for urban agriculture, utilities and services, as well as those for rural services are borderline in nature.

Better estimates for these industries might be obtained by using a different specification of the functions. Estimates of the non-linear functions without the time variable would be obtained by estimating the log-linear functions shown in equation (63), followed by taking antilogarithms of the estimates of the logarithms of the intercept coefficient.

Table 94. Estimates of the coefficients of linear employment-value added functions without the time variable, by industry: urban areas a/

Industry	Coefficients		R-square	Durbin-Watson
	Intercept	Value added b/		
(1)	(2)	(3)	(4)	(5)
Agriculture	15.4221	0.01980 (1.555)	0.212	1.38
Mining	1.7300	1.46615 (6.160)	0.808	2.29
Manufacturing	70.1197	0.97430 (21.114)	0.980	2.10
Utilities	2.9915	0.56431 (6.163)	0.808	1.51
Construction	-2.3773	3.07618 (2.104)	0.330	0.41
Trade	-22.4318	2.27942 (4.365)	0.679	2.42
Transport	10.8482	1.62845 (5.406)	0.765	2.36
Services	116.0514	0.81798 (25.124)	0.986	1.16

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

Table 95. Estimates of the coefficients of linear employment-value added functions without the time variable, by industry: rural areas a/

Industry	Coefficients		R-square	Durbin-Watson
	Intercept	Value added b/		
(1)	(2)	(3)	(4)	(5)
Agriculture	1220.1759	2.85558 (13.113)	0.950	2.06
Mining	-0.6826	0.65595 (5.928)	0.796	1.67
Manufacturing	3.0025	0.25725 (6.532)	0.826	1.59
Utilities	-1.2738	0.21972 (3.220)	0.535	1.97
Construction	-25.3350	1.53126 (6.304)	0.815	2.35
Trade	-27.5401	0.74332 (3.887)	0.627	1.68
Transport	-4.1849	0.14453 (2.653)	0.439	2.15
Services	25.9060	0.60456 (8.246)	0.883	1.34

a/ Estimated by ordinary least squares (OLS) .
b/ t values are shown in parentheses.

Estimates of such functions made by OLS and based on the data of tables 87 and 88 are given in tables 96 and 97, respectively. When compared to the results for linear functions, which do not include the time variable (tables 94 and 95), these estimates are, on average, less satisfactory. This indicates that for the data used here, the linear specification without the time variable appears to be more suitable than the non-linear specification excluding that variable.

To complete the estimation of the non-linear functions without the time variable, it would be necessary to take antilogarithms of the estimates of the logarithms of the intercepts as indicated in equation (64). In the case of the estimate of the functions shown in tables 96 and 97, the estimates of the intercepts can be obtained as indicated in table 98, where the estimate of an intercept for a given industry in a particular location (column 3) is obtained as the antilogarithm of the estimate of the logarithm of the intercept for the same industry and location (column 2). For example, the estimate of the intercept for urban agriculture, 7.447, is obtained as:

$$7.447 = \text{antiln}(2.00785), \qquad (64)$$

where 2.00785 is the estimate of the logarithm of the intercept coefficient for urban agriculture.

If log-linear functions without time as a variable for urban and rural areas are required, it is necessary to estimate the functions indicated in equation (63). Estimates of those functions using OLS and based on data given in tables 87 and 88 are presented in tables 96 and 97. Such estimates can be used as they are obtained by OLS.

ii. Functions with the time variable

If linear functions with the time variable are to be used to project urban and rural employment, it will be necessary to estimate the functions shown in equation (65).

The estimates of such functions, derived by OLS from the data of tables 87 and 88 for the urban areas, are presented in table 99, while those for the rural areas are shown in table 100. Some of these results represent a significant improvement over those obtained earlier by estimating linear functions without time as a variable (e.g. the results for agriculture in the urban areas), while others are only slightly better (for example, utilities in the urban areas). However, most of the coefficients of the time variable are not statistically significant and have an unexpected positive sign.

Time series data can be used to estimate non-linear employment-value added functions which include the time variable. This can be done by estimating the functions indicated in equation (66) and then taking the antilogarithms of the estimates of the logarithms of the intercepts.

Table 96. Estimates of the coefficients of log-linear employment-value added functions without the time variable, by industry: urban areas a/

Industry	Coefficients		R-square	Durbin-Watson
	Intercept	Value added b/		
(1)	(2)	(3)	(4)	(5)
Agriculture	2.0079	0.18058 (1.297)	0.157	1.40
Mining	1.0409	0.70495 (6.221)	0.811	2.30
Manufacturing	2.7744	0.50974 (21.261)	0.980	2.48
Utilities	0.6165	0.67028 (5.726)	0.785	1.31
Construction	0.9619	1.03814 (2.245)	0.359	0.42
Trade	-0.3037	1.23106 (4.186)	0.661	2.41
Transport	0.9953	0.90305 (5.397)	0.764	2.28
Services	2.6496	0.56462 (26.861)	0.988	1.48

a/ Estimated by ordinary least squares (OLS) .
b/ t values are shown in parentheses.

Table 97. Estimates of the coefficients of log-linear employment-value added functions without the time variable, by industry: rural areas a/

Industry	Coefficients		R-square	Durbin-Watson
	Intercept	Value added b/		
(1)	(2)	(3)	(4)	(5)
Agriculture	5.7557	0.32803 (14.471)	0.959	2.32
Mining	-1.7077	1.73724 (5.734)	0.785	1.40
Manufacturing	-1.1202	0.97796 (5.667)	0.781	1.51
Utilities	-4.6021	1.91228 (2.655)	0.439	1.83
Construction	-16.8638	6.12216 (5.001)	0.735	2.32
Trade	-22.3092	6.28241 (3.371)	0.558	1.63
Transport	-12.2902	3.43645 (2.771)	0.460	2.16
Services	0.8769	0.77605 (7.176)	0.851	1.30

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

Table 98. Computing estimates of intercept coefficients of non-linear employment-value added functions without the time variable, by industry: urban and rural areas

Industry (1)	Intercepts of log-linear functions a/ (2)	Intercepts of corresponding non-linear functions b/ (3)
	Urban	
Agriculture	2.00785	7.447
Mining	1.04094	2.832
Manufacturing	2.77442	16.029
Utilities	0.61652	1.852
Construction	0.96192	2.617
Trade	-0.30370	0.738
Transport	0.99535	2.706
Services	2.64962	14.149
	Rural	
Agriculture	5.75573	315.996
Mining	-1.70771	0.181
Manufacturing	-1.12016	0.326
Utilities	-4.60211	0.010
Construction	-16.86382	0.000
Trade	-22.30920	0.000
Transport	-12.29016	0.000
Services	0.87692	2.403

a/ From tables 96 and 97, col. 2.
b/ Antiln(Col. 2).

Table 99. Estimates of the coefficients of linear employment-value added functions with the time variable, by industry: urban areas a/

Industry	Coefficients b/			R-square	Durbin-Watson
	Intercept	Value added	Time variable		
(1)	(2)	(3)	(4)	(5)	(6)
Agriculture	2643.5283	0.13728 (2.370)	-1.34421 (-2.065)	0.486	2.04
Mining	-205.2709	1.13878 (3.767)	0.10542 (1.582)	0.854	2.71
Manufacturing	-3345.9050	0.73556 (4.406)	1.74021 (1.481)	0.984	2.81
Utilities	191.3496	0.66477 (1.066)	-0.09610 (-0.163)	0.809	1.54
Construction	-6700.6656	-1.07196 (-1.291)	3.43951 (6.999)	0.906	1.24
Trade	-3170.7871	0.93082 (0.863)	1.62767 (1.408)	0.743	2.34
Transport	-845.8819	1.26753 (1.616)	0.44220 (0.502)	0.772	2.25
Services	-2506.0234	0.73224 (1.492)	1.33721 (0.175)	0.986	1.20

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

Table 100. Estimates of the coefficients of linear employment-value added functions with the time variable, by industry: rural areas a/

Industry	Coefficients b/			R-square	Durbin-Watson
	Intercept	Value added	Time Variable		
(1)	(2)	(3)	(4)	(5)	(6)
Agriculture	-63135.5310	-0.02109 (-0.032)	32.91633 (4.464)	0.986	3.05
Mining	-1.0586	0.65536 (4.070)	0.00019 (0.005)	0.796	1.67
Manufacturing	-1913.9193	0.12329 (0.805)	0.97652 (0.906)	0.842	1.75
Utilities	1226.2377	0.87433 (2.174)	-0.62630 (-1.647)	0.653	2.23
Construction	-740.3265	1.08848 (3.683)	0.36714 (2.099)	0.881	3.04
Trade	-2304.8547	-0.23217 (-1.110)	1.17734 (5.251)	0.916	2.16
Transport	-524.7971	-0.07479 (-0.641)	0.26871 (2.053)	0.632	2.65
Services	27683.7275	1.50896 (1.424)	-14.10500 (-0.855)	0.893	1.51

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

Such functions, which were estimated by the OLS using tables 87 and 88, are shown in table 101 for urban areas and in table 102 for the rural areas. The results are almost uniformly less satisfactory than those obtained by estimating the linear function with the time variable.

To complete the estimation of the coefficients of non-linear functions with the time variable, it is necessary to take the antilogarithms of the estimated logarithms of the intercept coefficients. This can be done using a procedure illustrated with reference to table 98.

If log-linear functions with the time variable are to be used in projections, it is necessary to estimate the functions indicated in equation (66). Estimates of those functions, derived by OLS and based on data given in tables 87 and 88, were presented in tables 101 and 102. Such estimates can be used in the same form as they are obtained.

(b) Calibration of the empirically estimated functions

After obtaining satisfactory estimates of employment-value added functions, the planner will sometimes desire to make special adjustments in the estimated coefficients. Although the adjustments may apply to the estimates of the intercepts as well as to those of the partial coefficients, they will often be restricted to the intercept estimates. These adjustments, which are typically referred to as "calibration", ensure that once adjusted, the functions are capable of precisely predicting the levels of employment by industry for a particular year or a group of years of the time period to which the data used pertain, given the level of value added by industry for that year or group of years. (If left unadjusted, the functions will be capable of predicting mean levels of employment over the entire time period to which data refer, using the average levels of value added for the period.) The calibration procedures for employment-value added functions of different forms are described in annex I.

D. Illustrative examples of projections

The examples presented below will illustrate the use of employment-value added functions to prepare a national projection and an urban-rural projection, respectively. These examples, using log-linear and linear employment-value added functions, will indicate how the relevant calculations are made for the projection interval 0-5. In addition, they will provide complete projection results for a 20-year period.

This section will not, however, illustrate how to make projections of employment using estimates of the coefficients of the non-linear functions. This alternative approach would give the same results as that using the estimates of the coefficients of the log-linear functions.

Table 101. Estimates of the coefficients of log-linear employment-value added function with the time variable, by industry: urban areas a/

Industry	Coefficients b/			R-square	Durbin-Watson
	Intercept	Value added	Time variable		
(1)	(2)	(3)	(4)	(5)	(6)
Agriculture	108.0827	1.20312 (1.345)	-0.05651 (-1.156)	0.278	1.55
Mining	-35.5475	0.52984 (3.715)	0.01864 (1.758)	0.864	2.68
Manufacturing	-4.2995	0.47094 (2.629)	0.00367 (0.219)	0.981	2.55
Utilities	-106.5143	-0.05904 (-0.040)	0.05522 (0.498)	0.791	1.31
Construction	-100.0441	-0.33477 (-1.296)	0.05332 (7.305)	0.916	1.37
Trade	-35.8235	0.48173 (0.800)	0.01946 (1.405)	0.728	2.35
Transport	-6.7802	0.75226 (1.707)	0.00423 (0.373)	0.768	2.17
Services	33.9520	0.75686 (3.183)	-0.01637 (-0.812)	0.989	1.49

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

Table 102. Estimates of the coefficients of log-linear employment-value added functions with the time variable, by industry: rural areas a/

Industry	Coefficients b/			R-square	Durbin-Watson
	Intercept	Value added	Time variable		
(1)	(2)	(3)	(4)	(5)	(6)
Agriculture	-30.3726	-0.02023 (-0.210)	0.01925 (3.665)	0.985	2.97
Mining	-3.1059	1.73055 (3.855)	0.00071 (0.021)	0.785	1.39
Manufacturing	-94.2981	0.46685 (0.365)	0.04833 (0.403)	0.785	1.54
Utilities	991.4696	8.69332 (0.981)	-0.51343 (-0.768)	0.478	1.75
Construction	-138.9310	4.46296 (2.605)	0.06443 (1.331)	0.783	2.54
Trade	-380.3987	-1.27194 (-0.422)	0.19621 (2.833)	0.779	1.81
Transport	-256.2927	-1.29566 (-0.472)	0.13272 (1.881)	0.626	2.60
Services	-168.1228	-0.26181 (-0.215)	0.08840 (0.855)	0.864	1.42

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

1. National projection

The calculations presented in this example will be based on the inputs contained in table 103, which shows projected levels of value added by industry, along with selected estimates of the log-linear employment-value added functions presented in tables 90 and 93. For all industries except construction and trade, the estimates of the coefficients refer to the functions that do not include the time variable. For the construction and trade industries, the estimates of the coefficients refer to the functions which include the time variable. The functions used in this example were calibrated as explained in annex I. The value added levels are given for dates five years apart, starting with the initial year of the projection, which is denoted as year 0.

(a) Employment by industry

To derive the levels of employment by industry for a given date using log-linear functions, one could first obtain the logarithms of employment levels by evaluating the estimates of those functions for that date. This is done by using the logarithms of the levels of value added for the date, if necessary, along with appropriate value of the time variable.

In the case of log-linear functions without the time variable, the logarithms of the levels of employment are obtained as indicated by equation (9), while in the case of log-linear functions with time included, they are derived as shown by equation (13). Table 104 illustrates how the logarithms of the levels of employment by industry for the end of the projection interval 0-5 are calculated using log-linear functions, first without and then with the time variable.6/

In particular, the log of the employment level for each industry in year 5 (column 7), except that for the construction or trade industry, is obtained by adding the adjusted intercept coefficient (column 2) to the product of the estimate of the value added coefficient (column 3) and the logarithm of the projected level of value added in year 5 (column 5). For example, the logarithm of the level of employment in agriculture in year 5, 7.65296, is calculated as follows:

$$7.65296 = 5.78334 + 0.32625 \cdot \ln(308.2), \qquad (9)$$

where 5.78334 is the adjusted intercept coefficient for agriculture, 0.32625 is the estimate of the value added coefficient for this industry, and 308.2 is the projected value added in agriculture for year 5.

The logarithm of the level of employment for the construction or trade industry in year 5 (column 7) is obtained by adding the adjusted intercept coefficient for the industry (column 2) to a sum of two products. The first product is that of the estimate of the value added coefficient for the

Table 103. Inputs for projecting employment, by industry: entire country

Industry	Value added in year					Estimates of the coefficients of log-linear employment-value added functions		
	0	5	10	15	20	Adjusted intercept	Value added	Time variable
	(millions of LCUs a/)							
Agriculture	273.1	308.2	347.8	392.5	443.1	5.78333	0.33	
Mining	2.9	4.0	5.5	7.6	10.4	1.07529	0.87	
Manufacturing	140.4	212.7	322.5	489.3	742.7	2.68799	0.56	
Utilities	21.2	31.9	48.0	72.5	109.4	0.40971	0.80	
Construction	25.6	31.5	38.9	48.0	59.3	-99.37402	0.08	0.05240
Trade	59.0	85.1	122.9	177.6	256.7	-58.24014	0.31	0.03120
Transport	56.0	73.0	95.3	124.4	162.5	0.79301	0.96	
Services	312.5	464.3	691.2	1031.2	1541.4	2.68876	0.64	

a/ Local currency units.

Table 104. Deriving employment, by industry: entire country, year 5

Industry	Estimates of the coefficients of log-linear employment-value-added functions			In year 5			
	Adjusted intercept	Value added	Time variable	Value added a/ (millions of LCUs e/)	Value of time variable b/	Logarithm of projected employment c/	Projected employment d/ (thousands of persons)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Agriculture	5.78333	0.326		308.2		7.65	2106.9
Mining	1.07529	0.865		4.0		2.28	9.8
Manufacturing	2.68799	0.564		212.7		5.71	301.6
Utilities	0.40971	0.802		31.9		3.19	24.2
Construction	-99.37402	0.079	0.052	31.5	1985	4.93	138.6
Trade	-58.24014	0.311	0.031	85.1	1985	5.07	159.7
Transport	0.79301	0.960		73.0		4.91	136.0
Services	2.68876	0.637		464.3		6.60	734.4

a/ From table 103.
b/ Based on assumption that 1980 is the initial year of the projection period.
c/ (Col. 2) + (Col. 3) . (ln(Col. 5)) + (Col. 4) . (Col. 6).
d/ Antiln(Col. 7).
e/ Local currency units.

construction or trade industry (column 3) and the logarithm of the projected level of value added in year 5 for this industry (column 5). The second product is that of the estimate of the time variable coefficient for the industry (column 4) and the value of the time variable for year 5 (column 6). For example, the log of employment level for construction industry in year 5, 4.93134, is obtained as:

$$4.93134 = -99.37402 + [0.07935 \cdot \ln(31.5)] + [0.05241 \cdot (1980 + 5.0)], \quad (13)$$

where -99.37402 is the adjusted intercept coefficient for the industry, and 0.07935 and 31.5 are, respectively, the estimate of the value added coefficient and the projected level of value added for year 5. The estimate of the time variable coefficient is 0.05241, while 1980 is assumed to be the initial year of the projection period and 1985 is the last year of the projection interval, 0-5 (year 5).

Once the logarithms of the levels of employment by industry are obtained for a given date, it is then necessary to take the antilogarithms of those results in order to derive from them the levels of employment. Thus, for year 5, the level of employment in each industry (column 8) is obtained by taking the antilogarithm of the result obtained by evaluating the function for that date (column 4). For example, the level of employment in agriculture in year 5, 2,106.9, is obtained as:

$$2{,}106.9 = \text{antiln}\ (7.65296), \quad (14)$$

where 7.65296 is the logarithm of the level of employment in agriculture in year 5.

Performing the calculations illustrated for year 5 for dates five years apart over the entire projection period produces the projected levels of employment by industry for the entire period, which are shown in table 105.

(b) Other results 7/

Other results that are useful in planning can be obtained as part of a projection at the national level. These include various aggregates, indicators of the structure of and the rates of growth of employment.

(i) Employment aggregates

The employment aggregates that can be derived from the projections by industry include total employment along with employment in various sectors at dates five years apart. They also include increases in total employment and employment by sector over the intervening projection intervals.

Table 105. Projected employment, by industry: entire country

(Thousands of persons)

Industry	Year				
	0	5	10	15	20
Agriculture	2025.4	2106.9	2191.6	2279.9	2371.8
Mining	7.5	9.8	12.9	16.9	22.1
Manufacturing	238.6	301.6	381.4	482.4	610.3
Utilities	17.4	24.2	33.6	46.7	65.1
Construction	104.9	138.6	183.1	242.0	319.8
Trade	121.9	159.7	209.3	274.3	359.5
Transport	105.3	136.0	175.7	226.9	293.2
Services	570.7	734.4	946.2	1220.8	1576.8

a. Total employment

Total employment at the end of a given projection interval is obtained by aggregating the projected levels of employment by industry. Total employment in year 5, 3.611.2, is computed by adding the projected levels of employment by industry. This number is shown in table 106 (in the column corresponding to year 5) along with other results derived for the entire 20-year projection period. The increase in total employment over this period is indicated in figure XXII.

b. Employment by sector

Employment in the primary, secondary and the tertiary sector can be obtained by aggregating employment projected for various industries, ranging from agriculture to services, using appropriate aggregation rules.

Employment in the primary sector in year 5, 2,116.7, is obtained as:

$$2{,}116.7 = 2{,}106.9 + 9.8, \tag{15}$$

where 2,106.9 and 9.8 are, respectively, projected levels of employment in agriculture and mining.

Employment in the secondary sector in year 5, 464.4, is obtained as:

$$464.4 = 301.6 + 24.2 + 138.6, \tag{16}$$

where 301.6, 24.2 and 138.6 are, respectively, projected levels of employment in manufacturing, utilities and construction.

Employment in the tertiary sector in year 5, 1,030.1, is obtained as:

$$1{,}030.1 = 159.7 + 136.0 + 734.4, \tag{17}$$

where 159.7, 136.0 and 734.4 are, respectively, projected levels of employment in trade, transportation and services.

Employment by sector obtained for different dates over the projection period is presented in figure XXIII.

c. Growth in total employment

The growth in total employment over a given projection interval equals the difference between total employment at the end of the interval and total

Table 106. Employment aggregates, structure and rates of growth: entire country

Indicators	Year 0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	3191.8	3611.2	4133.8	4789.9	5618.6
Primary	2032.9	2116.7	2204.5	2296.8	2394.0
Secondary	360.9	464.4	598.1	771.1	995.1
Tertiary	798.0	1030.1	1331.2	1722.0	2229.5
Growth in employment					
Total		419.4	522.7	656.0	828.7
Primary		83.8	87.8	92.3	97.2
Secondary		103.5	133.7	173.0	224.0
Tertiary		232.2	301.1	390.8	507.5
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.64	0.59	0.53	0.48	0.43
Secondary	0.11	0.13	0.14	0.16	0.18
Tertiary	0.25	0.29	0.32	0.36	0.40
Rates of growth of employment (percentage)					
Total		2.50	2.74	2.99	3.24
Primary		0.81	0.82	0.82	0.83
Secondary		5.17	5.19	5.21	5.23
Tertiary		5.24	5.26	5.28	5.30

Figure XXII. Total employment

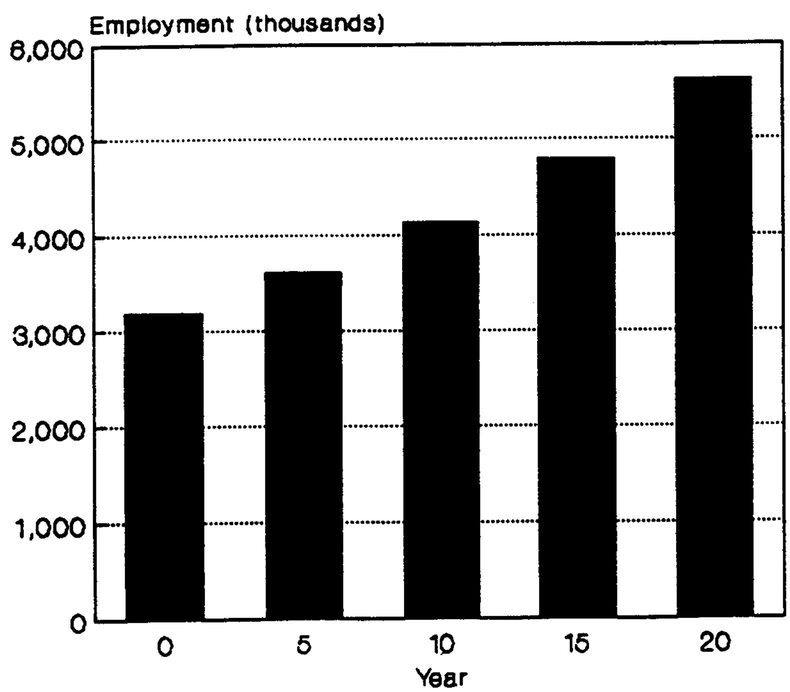
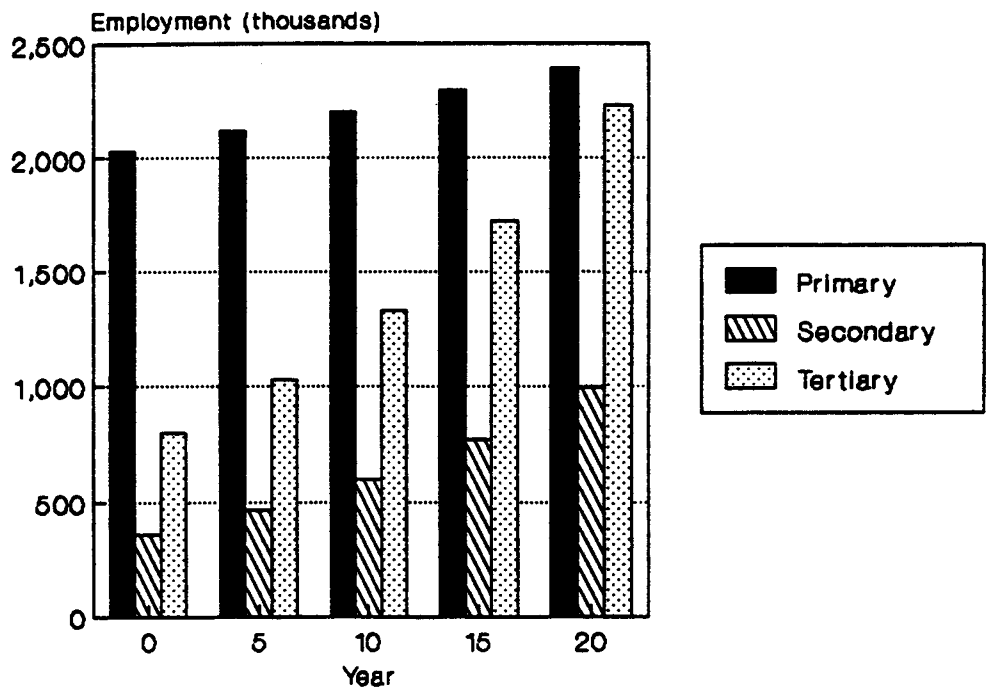


Figure XXIII. Employment: primary, secondary and tertiary sectors



employment at its beginning. For the interval 0-5, the growth in total employment 419.4, is obtained as:

$$419.4 = 3{,}611.2 - 3{,}191.8, \qquad (18)$$

where 3,191.8 and 3,611.2 are, respectively, total employment at the beginning and the end of the interval (shown in columns corresponding to years 0 and 5 respectively).

d. Growth in employment by sector

The increase in employment over the interval 0-5 in various sectors is obtained as follows:

Growth of employment in the primary sector, 83.8, is:

$$83.8 = 2{,}116.7 - 2{,}032.9, \qquad (19)$$

where 2,032.9 and 2,116.7 are, respectively, the levels of employment in the primary sector in years 0 and 5;

Growth of employment in the secondary sector, 103.5, is:

$$103.5 = 464.4 - 360.9, \qquad (20)$$

where 360.9 and 464.4 are the levels of employment in the secondary sector in years 0 and 5; and

Growth of employment in the tertiary sector, 232.2, is:

$$232.2 = 1{,}030.1 - 798.0, \qquad (21)$$

where 798.0 and 1,030.1 are the levels of employment in the tertiary sector in years 0 and 5.

(ii) Indicators of the structure of employment

Indicators of the structure of employment that can be calculated as part of an employment projection include proportions of total employment found in each sector.

a. Proportions by sector

For the end of the interval 0-5, these proportions are obtained as follows:

The proportion of employment in the primary sector, 0.59, is:

$$0.59 = 2{,}116.7 \;/\; 3{,}611.2, \qquad (22)$$

where 2,116.7 and 3,611.2 are, respectively, employment in the primary sector and the total employment;

The proportion of employment in the secondary sector, 0.13, is:

$$0.13 = 464.4 \;/\; 3{,}611.2, \qquad (23)$$

where 464.4 is employment in the secondary sector;

The proportion of employment in the tertiary sector, 0.29, is:

$$0.29 = 1{,}030.1 \;/\; 3{,}611.2, \qquad (24)$$

where 1,030.1 is employment in the tertiary sector.

(iii) Rates of growth of employment

The rates of growth of employment can be calculated for total employment and for employment in each sector.

a. Rate of growth of total employment

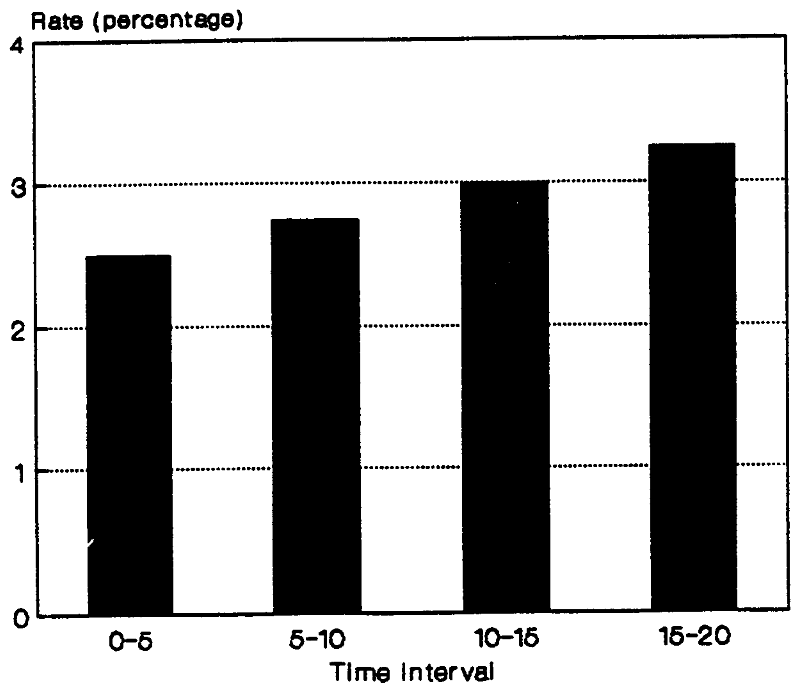
If growth in employment is assumed to occur over discrete intervals, the average annual growth rate of total employment for a given interval is obtained using the geometric growth rate formula. For the projection interval 0-5, this annual growth rate, 2.50 per cent (table 106), is obtained as follows:

$$2.50 = [\, (3{,}611.2 \;/\; 3{,}191.8)^{1/5} - 1 \,] \cdot 100, \qquad (25)$$

where 3,191.8 and 3,611.2 are the levels of total employment in years 0 and 5, respectively, and 5 is the length of the interval.

Rates of growth of total employment over the 20-year projection period, which were computed using the geometric rate formula, are shown in figure XXIV.

Figure XXIV. Rate of growth in total employment



If the planner assumes that growth in employment is continuous, the average annual growth rate of total employment for a given interval is obtained by substituting the same data as above in the exponential growth rate formula. For the projection interval 0-5, this annual growth rate, 2.47 per cent, is obtained as follows:

$$2.47 = [\ \ln (3{,}611.2 \ / \ 3{,}191.8) \ / \ 5\ ] \cdot 100. \tag{26}$$

b. <u>Rates of growth of employment by sector</u>

Assuming discrete growth, the rates of increase in employment by sector for the interval 0-5 are calculated as follows:

The annual rate of growth of employment in the primary sector, 0.81 per cent, is calculated as follows:

$$0.81 = [\ (2{,}116.7 \ / \ 2.032.9)^{1/5} - 1\ ] \cdot 100, \tag{27}$$

where 2.032.9 and 2,116.7 are the levels of employment in the primary sector in years 0 and 5, respectively;

The rate of growth of employment in the secondary sector, 5.17 per cent, is calculated as:

$$5.17 = [\ (464.4 \ / \ 360.9)^{1/5} - 1\ ] \cdot 100, \tag{28}$$

where 360.9 and 464.4 are the levels of employment in the secondary sector in years 0 and 5;

The rate of growth of employment in the tertiary sector, 5.24, is obtained as:

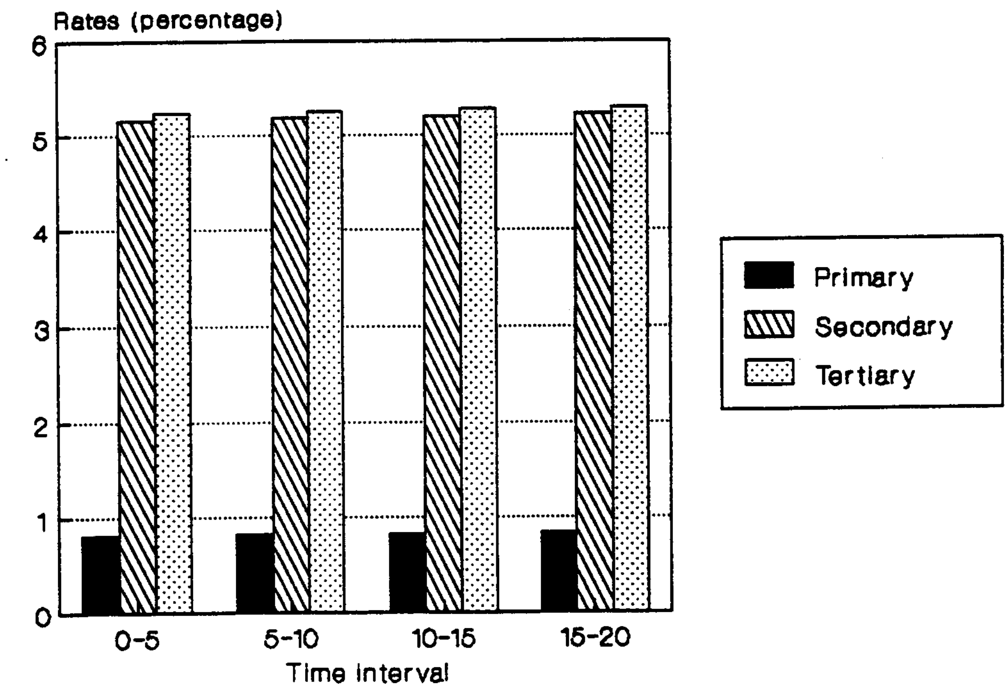
$$5.24 = [\ (1{,}030.1 \ / \ 798.0)^{1/5} - 1\ ] \cdot 100, \tag{29}$$

where 798.0 and 1,030.1 are the levels of employment in the tertiary sector in years 0 and 5.

Rates of growth of employment in primary, secondary and tertiary sectors over the 20-year projection interval are shown in figure XXV.

If continuous growth is assumed, rates of employment by sector would be calculated using the exponential growth rate formula. The calculations would be performed by steps indicated by equations (30) through (32).

Figure XXV. Rates of growth of employment: primary, secondary and tertiary sectors



(iv) Labour market balances

If projections of the labour force are available, it is possible to calculate the level of excess demand for or excess supply of labour. Also, it is possible to calculate the excess demand for or excess supply of labour as a percentage of the level of labour supply. The calculations are based on the projected total labour force, diminished where necessary by the size of non-civilian employment and the projected total employment as indicators of the labour supply and the labour demand. In order to illustrate these calculations, we shall use projections of the total labour force and of total employment (shown, respectively, in tables 107 and 108) along with the illustrative projections of non-civilian employment, which are shown in table 107. The calculations are illustrated for the end of the projection interval 0-5 using tables 107-108.

The civilian labour force for the end of the 0-5 interval, 3,615.4, can be calculated as follows:

$$3{,}615.4 = 3{,}651.9 - 36.5, \qquad (33)$$

where 3,651.9 and 36.5 are the projected total labour force and the projected non-civilian employment for year 5, shown in columns 2 and 3. The calculated civilian labour force for year 5 is shown in column 4.

The excess supply of labour for the same date, 4.2, is calculated as follows:

$$4.2 = 3{,}615.4 - 3{,}611.2, \qquad (34)$$

where 3,615.4 is the civilian labour force and 3,611.2 is the total employment shown in columns 4 and 5, respectively.

The excess supply expressed as a percentage of the civilian labour force at the end of the 0-5 interval, 0.12 is:

$$0.12 = (4.2 / 3{,}615.4) \cdot 100. \qquad (35)$$

This ratio is shown in column 7.

2. Urban-rural projection

The procedure for projecting employment for urban and rural areas using employment-value added functions is similar to that for the country as a whole. This example will illustrate a method by which such a projection can be prepared using the inputs shown in tables 109 and 110, which indicate for

Table 107. Projected non-civilian employment: entire country

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	32.5
5	36.5
10	41.5
15	47.4
20	53.8

Table 108. Labour market balances: entire country

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
		(thousands of persons)				(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	3251.5	32.5	3219.0	3191.8	27.2	0.85
5	3651.9	36.5	3615.4	3611.2	4.2	0.12
10	4147.8	41.5	4106.3	4133.8	-27.5	-0.67
15	4735.4	47.4	4688.0	4789.9	-101.9	-2.17
20	5377.4	53.8	5323.6	5618.6	-295.0	-5.54

a/ From table 37, "Labour force (Total)".
b/ From table 107.
c/ (Col. 2) - (Col. 3).
d/ From table 106, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Table 109. Inputs for projecting employment, by industry: urban areas

	National value added in year (millions of LCUs a/)					Estimates of the coefficients of linear employment-value added functions		
	0	5	10	15	20	Adjusted intercept	Value added	Time variable
Agriculture	273.1	308.2	347.8	392.5	443.1	2645.31363	0.14	-1.34421
Mining	2.9	4.0	5.5	7.6	10.4	2.08799	1.47	
Manufacturing	140.4	212.7	322.5	489.3	742.7	68.35567	0.97	
Utilities	21.2	31.9	48.0	72.5	109.4	190.82840	0.66	-0.09610
Construction	25.6	31.5	38.9	48.0	59.3	-6696.75995	-1.07	3.43951
Trade	59.0	85.1	122.9	177.6	256.7	-17.22250	2.28	
Transport	56.0	73.0	95.3	124.4	162.5	10.28867	1.63	
Services	312.5	464.3	691.2	1031.2	1541.4	115.98101	0.82	

a/ Local currency units.

Table 110. Inputs for projecting employment, by industry: rural areas

	National value added in year					Estimates of the coefficients of linear employment-value added functions		
	0	5	10	15	20	Adjusted intercept	Value added	Time variable
	(millions of LCUs a/)							
Agriculture	273.1	308.2	347.8	392.5	443.1	1228.49142	2.86	
Mining	2.9	4.0	5.5	7.6	10.4	-0.90547	0.66	
Manufacturing	140.4	212.7	322.5	489.3	742.7	2.51193	0.26	
Utilities	21.2	31.9	48.0	72.5	109.4	1225.68768	0.87	-0.62630
Construction	25.6	31.5	38.9	48.0	59.3	-23.58472	1.53	
Trade	59.0	85.1	122.9	177.6	256.7	-27.23500	0.74	
Transport	56.0	73.0	95.3	124.4	162.5	-524.85848	-0.07	0.26871
Services	312.5	464.3	691.2	1031.2	1541.4	20.54159	0.60	

a/ Local currency units.

urban and rural areas projected levels of value added along with selected estimates of linear employment-value added functions. The selected functions come from tables 94 and 95, along with tables 99 and 100. The intercepts have been adjusted as described in annex I.

The example will focus on the calculations that are unique to linear functions and those related to an urban-rural projection of employment. As in the previous example, it will be assumed that growth in employment occurs over discrete time intervals.

(a) Employment by industry

For any given date, such as the end of the projection interval 0-5, the levels of employment in urban and rural areas, by industry, are obtained from calculations that are similar to those employed in making the national projection. In the urban-rural projection, however, these calculations are performed for either area. If linear employment-value added functions are used, the levels of employment are obtained directly by evaluating the relevant functions. Thus, the levels of employment obtained using the linear functions for the end of the interval 0-5 for the urban areas are calculated as illustrated in table 111.

For example, the levels of employment in urban manufacturing in year 5, 275.6, is:

$$275.6 = 68.35568 + 0.97430 \cdot 212.7, \qquad (36)$$

where 68.35568 is the adjusted intercept coefficient of the employment-value added function for manufacturing in urban areas respectively. The coefficient of the value added variable of this function is 0.97430 and the projected level of the national value added for this industry for year 5 is 212.7.

The level of employment in urban agriculture in year 5, 19.4, is calculated as follows:

$$19.4 = 2{,}645.31363 + [0.13728 \cdot 308.2] + [(-1.34421) \cdot (1980 + 5)], \qquad (39)$$

where 2,645.31363 is the adjusted intercept coefficient of the function for urban agriculture. The coefficient of the value added variable in this function is 0.13728 and the projected level of value added for urban agriculture for year 5 for the entire country is 308.2. The coefficient of the time variable is -1.34421, while 1985 (= 1980 + 5) is year 5 of the projection period, the initial year of which is assumed to be 1980.

Table 111. Deriving employment, by industry: urban areas, year 5

Industry	Estimates of coefficients of linear employment-value added functions			In year 5		
	Adjusted intercept	Value added	Time variable	National value added a/ (LCUs d/)	Value of time variable b/	Projected employment c/ (thousands of persons)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Agriculture	2645.31363	0.137	-1.344	308.2	1985	19.4
Mining	2.08799	1.466		4.0		8.0
Manufacturing	68.35567	0.974		212.7		275.6
Utilities	190.82840	0.665	-0.096	31.9	1985	21.3
Construction	-6696.75995	-1.072	3.440	31.5	1985	96.9
Trade	-17.22250	2.279		85.1		176.8
Transport	10.28867	1.628		73.0		129.2
Services	115.98101	0.818		464.3		495.7

a/ From table 109.
b/ Based on assumption that 1980 is the initial year of the projection period.
c/ (Col. 2) + (Col. 3) . (Col. 5) + (Col. 4) . (Col. 6).
d/ Local currency units.

Projected levels of employment by industry for urban and rural areas at dates five years apart can be found by performing these calculations for the relevant dates over the projection period, starting with the initial year of the projection. Those levels can be further aggregated across the two locations to obtain the levels of employment, by industry, for the entire country. Tables 112 through 114 display urban, rural and national projected levels of employment, by industry, respectively.

(b) Other results 8/

An urban-rural projection of employment involves calculations of all those additional results that can be obtained as part of the national projection. Those results, which refer to urban and rural areas as well as the entire country, include various employment aggregates and indicators of employment structure and growth, as well as labour market balances. They can be derived by means of steps illustrated above in connection with the national projection. The results also include proportions of employment which are urban and rural.

Figure XXVI indicates projected levels of total employment for urban and rural areas and for the entire country, which are obtained in this illustrative projection.

(i) Proportions urban and rural

These proportions can be obtained for total employment and for employment by sector.

a. Proportions of total employment

The proportion of total employment that is urban for the end of the projection interval is calculated as a ratio of total employment in the urban areas to the total employment in the entire country for the date. For the end of the interval 0-5, the proportion of total employment that is urban, 0.32, is obtained as:

$$0.32 = 1{,}222.9 - 3{,}765.7, \qquad (49)$$

where 1,222.9 is total employment in the urban areas and 3,765.7 is total employment for the entire country.

The proportion of total employment that is rural, 0.68, is calculated as a complement of the proportion urban:

$$0.68 = 1 - 0.32, \qquad (50)$$

Table 112. Projected employment, by industry: urban areas

(Thousands of persons)

Industry	Year				
	0	5	10	15	20
Agriculture	21.3	19.4	18.1	17.5	17.7
Mining	6.4	8.0	10.2	13.2	17.3
Manufacturing	205.1	275.6	382.6	545.1	792.0
Utilities	14.6	21.3	31.5	47.3	71.3
Construction	86.1	96.9	106.2	113.6	118.7
Trade	117.2	176.8	263.0	387.7	567.9
Transport	101.4	129.2	165.5	212.9	274.9
Services	371.6	495.7	681.4	959.5	1376.8

Table 113. Projected employment, by industry: rural areas

(Thousands of persons)

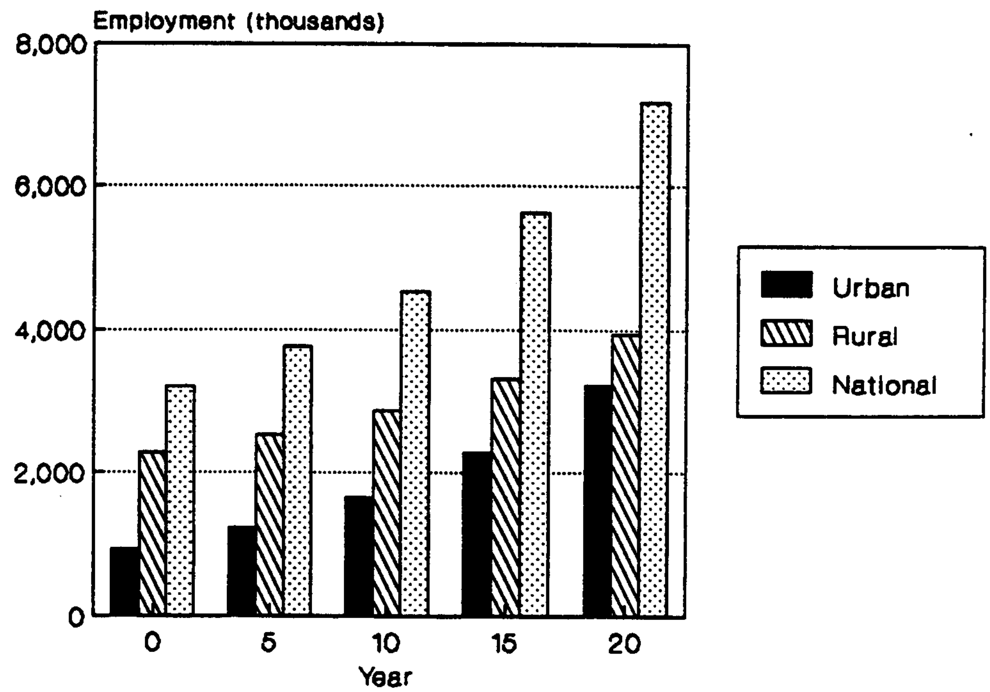
Industry	Year 0	5	10	15	20
Agriculture	2008.3	2108.4	2221.6	2349.3	2493.7
Mining	1.0	1.7	2.7	4.1	5.9
Manufacturing	38.6	57.2	85.5	128.4	193.6
Utilities	4.1	10.4	21.3	39.6	68.7
Construction	15.5	24.7	36.0	49.9	67.2
Trade	16.6	36.0	64.2	104.8	163.6
Transport	3.0	3.1	2.8	1.9	0.4
Services	209.5	301.2	438.4	644.0	952.4

Table 114. Projected employment, by industry: entire country

(Thousands of persons)

Industry	Year 0	5	10	15	20
Agriculture	2029.6	2127.8	2239.6	2366.8	2511.4
Mining	7.4	9.7	12.9	17.2	23.2
Manufacturing	243.8	332.9	468.1	673.5	985.6
Utilities	18.8	31.6	52.9	86.8	140.1
Construction	101.6	121.6	142.1	163.5	185.9
Trade	133.7	212.8	327.2	492.5	731.5
Transport	104.4	132.3	168.3	214.9	275.3
Services	581.1	797.0	1119.8	1603.5	2329.2

Figure XXVI. Total employment: urban, rural and national



where 0.32 is the proportion urban.

These proportions, along with all other results for the entire country excepting the labour market balances, are shown in table 117. Similar results for urban and rural areas are shown in tables 115 and 116. Labour market balances for urban and rural areas and the entire country were calculated using, among other things, illustrative projected levels of non-civilian employment, which are shown in tables 118 through 120. The results relating to the labour market balances are shown, respectively, in tables 121 through 123.

Proportions of total employment for the 20-year projection period that are urban and rural are shown in figure XXVII.

b. Proportions of employment by sector

Proportions of employment for the end of the interval 0-5, that are urban, can be calculated by sector as follows:

The proportion of employment in the primary sector that is urban, 0.01, is obtained as:

$$0.01 = 27.4 / 2{,}137.5, \tag{51}$$

where 27.4 is employment in the primary sector in the urban areas and 2,137.5 is employment in the primary sector in the entire country;

The proportion of employment in the secondary sector that is urban, 0.81, is:

$$0.81 = 393.7 / 486.0, \tag{52}$$

where 393.7 is employment in the secondary sector in the urban areas and 486.0 is employment in the secondary sector in the entire country;

The proportion of employment in the tertiary sector that is urban, 0.70, is calculated as:

$$0.70 = 801.8 / 1{,}142.1, \tag{53}$$

where 801.8 is employment in the tertiary sector in the urban areas and 1,142.1 is employment in the tertiary sector in the entire country.

Proportions of employment, by sector, that are rural can be obtained as complements of proportions of employment, by sector, that are urban:

Table 115. Employment aggregates, structure and rates of growth: urban areas

Indicators	Year 0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	923.7	1222.9	1658.5	2296.8	3236.6
Primary	27.7	27.4	28.3	30.7	35.0
Secondary	305.8	393.7	520.3	706.0	982.1
Tertiary	590.2	801.8	1109.9	1560.1	2219.6
Growth in employment					
Total		299.2	435.6	638.3	939.9
Primary		-0.3	0.9	2.4	4.3
Secondary		87.9	126.6	185.7	276.1
Tertiary		211.6	308.2	450.2	659.4
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.03	0.02	0.02	0.01	0.01
Secondary	0.33	0.32	0.31	0.31	0.30
Tertiary	0.64	0.66	0.67	0.68	0.69
Rates of growth of employment (percentage)					
Total		5.77	6.28	6.73	7.10
Primary		-0.23	0.65	1.65	2.67
Secondary		5.18	5.73	6.29	6.82
Tertiary		6.32	6.72	7.05	7.31

Table 116. Employment aggregates, structure and rates of growth: rural areas

Indicators	Year 0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	2296.7	2542.8	2872.4	3322.0	3945.5
Primary	2009.3	2110.2	2224.3	2353.4	2499.5
Secondary	58.3	92.3	142.8	217.9	329.5
Tertiary	229.1	340.3	505.3	750.7	1116.4
Growth in employment					
Total		246.1	329.6	449.5	623.5
Primary		100.9	114.1	129.1	146.2
Secondary		34.0	50.5	75.1	111.6
Tertiary		111.3	165.0	245.4	365.7
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.87	0.83	0.77	0.71	0.63
Secondary	0.03	0.04	0.05	0.07	0.08
Tertiary	0.10	0.13	0.18	0.23	0.28
Rates of growth of employment (percentage)					
Total		2.06	2.47	2.95	3.50
Primary		0.98	1.06	1.13	1.21
Secondary		9.62	9.13	8.82	8.62
Tertiary		8.24	8.23	8.24	8.26

Table 117. Employment aggregates, structure and rates of growth: entire country

	Year				
	0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	3220.4	3765.7	4530.9	5618.8	7182.1
Primary	2037.0	2137.5	2252.5	2384.0	2534.5
Secondary	364.1	486.0	663.1	923.9	1311.6
Tertiary	819.2	1142.1	1615.3	2310.8	3336.0
Growth in employment					
Total		545.3	765.3	1087.8	1563.3
Primary		100.6	115.0	131.5	150.5
Secondary		121.9	177.1	260.8	387.7
Tertiary		322.9	473.2	695.5	1025.1
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.63	0.57	0.50	0.42	0.35
Secondary	0.11	0.13	0.15	0.16	0.18
Tertiary	0.25	0.30	0.36	0.41	0.46
Indicators of employment distribution					
Proportions urban					
Total	0.29	0.32	0.37	0.41	0.45
Primary	0.01	0.01	0.01	0.01	0.01
Secondary	0.84	0.81	0.78	0.76	0.75
Tertiary	0.72	0.70	0.69	0.68	0.67
Proportions rural					
Total	0.71	0.68	0.63	0.59	0.55
Primary	0.99	0.99	0.99	0.99	0.99
Secondary	0.16	0.19	0.22	0.24	0.25
Tertiary	0.28	0.30	0.31	0.32	0.33
Rates of growth of employment (percentage)					
Total		3.18	3.77	4.40	5.03
Primary		0.97	1.05	1.14	1.23
Secondary		5.94	6.41	6.86	7.26
Tertiary		6.87	7.18	7.42	7.62

Table 118. Projected non-civilian employment: urban areas

(Thousands of persons)

Year	Non-civilian employment
(1)	(2)
0	29.3
5	32.9
10	37.4
15	42.6
20	48.4

Table 119. Projected non-civilian employment: rural areas

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	3.3
5	3.7
10	4.2
15	4.7
20	5.4

Table 120. Projected non-civilian employment: entire country

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	32.5
5	36.5
10	41.5
15	47.4
20	53.8

Table 121. Labour market balances: urban areas

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	968.3	29.3	939.0	923.7	15.3	1.63
5	1383.8	32.9	1350.9	1222.9	128.0	9.48
10	1881.1	37.4	1843.7	1658.5	185.2	10.05
15	2438.7	42.6	2396.1	2296.8	99.3	4.15
20	3032.4	48.4	2984.0	3236.6	-252.6	-8.47

a/ From table 46, "Labour force (Total)".
b/ From table 118.
c/ (Col. 2) - (Col. 3).
d/ From table 115, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Table 122. Labour market balances: rural areas

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	2282.6	3.3	2279.3	2296.7	-17.4	-0.76
5	2269.1	3.7	2265.4	2542.8	-277.4	-12.25
10	2268.5	4.2	2264.3	2872.4	-608.1	-26.86
15	2298.6	4.7	2293.9	3322.0	-1028.1	-44.82
20	2344.1	5.4	2338.7	3945.5	-1606.8	-68.70

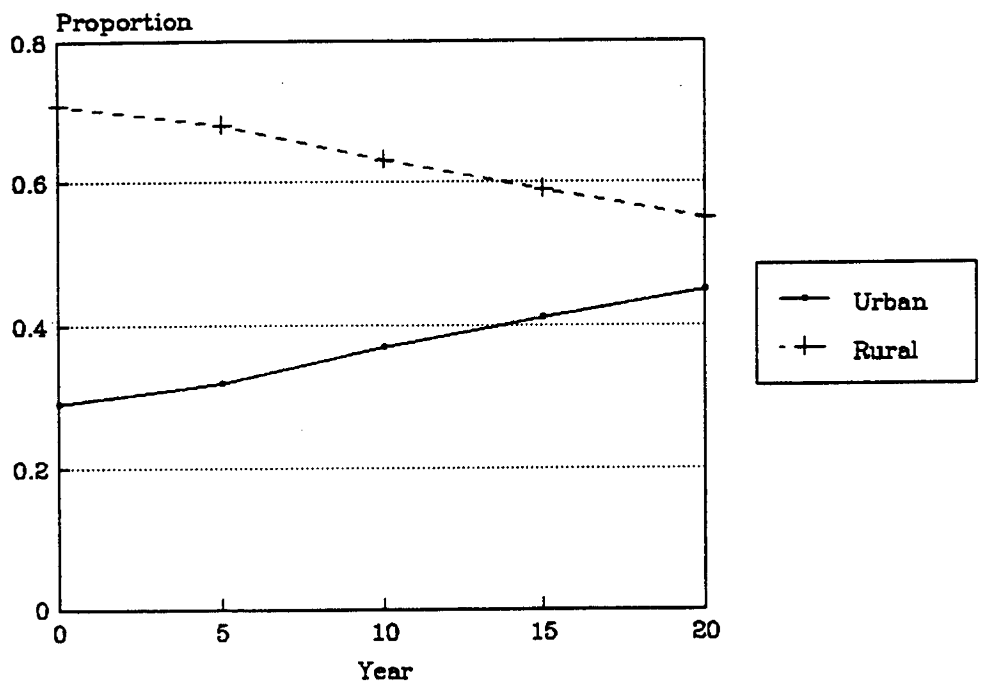
a/ From table 47, "Labour force (Total)".
b/ From table 119.
c/ (Col. 2) - (Col. 3).
d/ From table 116, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Table 123. Labour market balances: entire country

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	3250.9	32.5	3218.4	3220.4	-2.0	-0.06
5	3652.9	36.5	3616.4	3765.7	-149.3	-4.13
10	4150.2	41.5	4108.7	4530.9	-422.2	-10.28
15	4737.3	47.4	4689.9	5618.8	-928.9	-19.81
20	5376.5	53.8	5322.7	7182.1	-1859.4	-34.93

a/ From table 48, "Labour force (Total)".
b/ From table 120.
c/ (Col. 2) - (Col. 3).
d/ From table 117, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Figure XXVII. Proportions of total employment: urban and rural



The proportion of employment in the primary sector that is rural, 0.99, is:

$$0.99 = 1 - 0.01, \quad (54)$$

where 0.01 is the proportion of employment in the primary sector that is urban;

The proportion of employment in the secondary sector that is rural, 0.19, is:

$$0.19 = 1 - 0.81, \quad (55)$$

where 0.81 is the proportion of employment in the secondary sector that is urban;

The proportion of employment in the tertiary sector that is rural, 0.30, is:

$$0.30 = 1 - 0.70, \quad (56)$$

where 0.70 is the proportion of employment in the tertiary sector that is urban.

The proportions of employment, by sector, that are urban and rural for the entire projection interval are shown in table 117.

In addition to proportions of employment by sector, table 117 shows other projection results for the entire country. The results in this table were obtained from those for the urban and rural areas, shown respectively in tables 115 and 116, derived in the course of preparing the urban-rural projection of employment. The results for the entire country (table 117) differ from those obtained as part of the national projection (table 106). This is due to differences in inputs used in the national and urban-rural projections.

E. Summary

This chapter has described the method for preparing employment projections using employment-value added functions by industry to make national or urban-rural projections. As part of the description of the method, procedures used in making both national and urban-rural projections were presented. In addition, types of inputs required by the method were described, and preparation of the inputs was discussed. Lastly, examples were given of a national projection using a log-linear employment-value added function and an urban-rural projection using a linear employment-value added function. A complete listing of the outputs that can be generated by the method is shown in box 25.

Box 25

Outputs of the method for making employment projections using employment-value added functions

1. Employment by industry (national or urban, rural and national)

2. Employment aggregates (national or urban, rural and national)

Levels of employment:

Total

Primary sector
Secondary sector
Tertiary sector

Growth in employment:

Total

Primary sector
Secondary sector
Tertiary sector

3. Indicators of the structure of employment (national or urban, rural and national)

Proportions of employment, by sector:

Primary sector
Secondary sector
Tertiary sector

4. Indicators of the urban-rural distribution of employment (national only; if urban and rural employment is being projected)

Proportions of employment urban:

Total

Primary sector
Secondary sector
Tertiary sector

(continued)

Box 25 (continued)

Proportions of employment rural:

Total

Primary sector
Secondary sector
Tertiary sector

5. Rates of growth of employment (national or urban, rural and national)

Total

Primary sector
Secondary sector
Tertiary sector

6. Labour market balances (national or urban, rural and national)

Excess supply of or excess demand for labour

Percentage excess supply of or excess demand for labour

F. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

i = 1,...,I	are industries of the nation's economy
k = 1,2	are urban and rural locations
t	is the year of the projection period
t'	is the calendar year
$\bar{t}$'	is the calendar year designated as the initial year of the projection period

(b) List of variables

CLF(t+5)	is the civilian labour force at the end of the interval
EGREM	is the average annual exponential growth rate of total employment for the interval
EGREMP	is the average annual exponential growth rate of employment in the primary sector for the interval
EGREMS	is the average annual exponential growth rate of employment in the secondary sector for the interval
EGREMT	is the average annual exponential growth rate of employment in the tertiary sector for the interval
EM(i,k,t')	is the employment in industry i in location k in year t'
EM(i,k,t+5)	is the employment in industry i in location k at the end of the interval
EM(i,t')	is the employment in industry i in year t'
EM(i,t+5)	is the employment in industry i at the end of the interval
EM(k,t+5)	is the total employment in location k at the end of the interval

EM(t+5)	is the total employment at the end of the interval
EMGR	is the growth of total employment during the interval
EMP(k,t+5)	is the employment in the primary sector in location k at the end of the interval
EMP(t+5)	is the employment in the primary sector at the end of the interval
EMPGR	is the growth of employment in the primary sector during the interval
EMS(k,t+5)	is the employment in the secondary sector in location k at the end of the interval
EMS(t+5)	is the employment in the secondary sector at the end of the interval
EMSGR	is the growth of employment in the secondary sector during the interval
EMT(k,t+5)	is the employment in the tertiary sector in location k at the end of the interval
EMT(t+5)	is the employment in the tertiary sector at the end of the interval
EMTGR	is the growth of employment in the tertiary sector during the interval
EX(t+5)	is the excess supply of labour (if positive) or excess demand for labour (if negative) for the end of the interval
GGREM	is the average annual geometric growth rate of total employment for the interval
GGREMP	is the average annual geometric growth rate of employment in the primary sector for the interval
GGREMS	is the average annual geometric growth rate of employment in the secondary sector for the interval
GGREMT	is the average annual geometric growth rate of employment in the tertiary sector for the interval
LF(t+5)	is the total labour force at the end of the interval

NEM(t+5)	is the non-civilian employment at the end of the interval
PEMP(t+5)	is the proportion of employment in the primary sector at the end of the interval
PEMPRUR(t+5)	is the proportion of employment in the primary sector which is rural at the end of the interval
PEMPURB(t+5)	is the proportion of employment in the primary sector which is urban at the end of the interval
PEMRUR(t+5)	is the proportion of total employment which is rural at the end of the interval
PEMS(t+5)	is the proportion of employment in the secondary sector at the end of the interval
PEMSRUR(t+5)	is the proportion of employment in the secondary sector which is rural at the end of the interval
PEMSURB(t+5)	is the proportion of employment in the secondary sector which is urban at the end of the interval
PEMT(t+5)	is the proportion of employment in the tertiary sector at the end of the interval
PEMTRUR(t+5)	is the proportion of employment in the tertiary sector which is rural at the end of the interval
PEMTURB(t+5)	is the proportion of employment in the tertiary sector which is urban at the end of the interval
PEMURB(t+5)	is the proportion of total employment which is urban at the end of the interval
PEXL(t+5)	is the excess supply of labour or excess demand for labour as a percentage of the total labour force at the end of the interval
VA(i,t')	is the value added in industry i in year t'
VA(i,t+5)	is the value added in industry i at the end of the interval

(c) <u>List of special symbols</u>

a(i)	is the intercept coefficient of the linear or non-linear employment-value added function for industry i
a(i,k)	is the intercept coefficient of the linear or non-linear employment-value added function for industry i in location k
a*(i)	is the estimate of the intercept coefficient of the linear or non-linear employment-value added function for industry i
a*(i,k)	is the estimate of the intercept coefficient of the linear or non-linear employment-value added function for industry i in location k
antiln	is the antilogarithm of the natural logarithm
b(i)	is the partial coefficient of the value added variable in the linear or non-linear employment-value added function for industry i
b(i,k)	is the partial coefficient of the value added variable in the linear or non-linear employment-value added function for industry i in location k
b*(i)	is the estimate of the partial coefficient of the value added variable in the linear or non-linear employment-value added function for industry i
b*(i,k)	is the estimate of the partial coefficient of the value added variable in the linear or non-linear employment-value added function for industry i in location k
c(i)	is the partial coefficient of the time variable in the linear or non-linear employment-value added function for industry i
c(i,k)	is the partial coefficient of the time variable in the linear or non-linear employment-value added function for industry i in location k
c*(i)	is the estimate of the partial coefficient of the time variable in the linear or non-linear employment-value added function for industry i

$c^*(i,k)$	is the estimate of the partial coefficient of the time variable in the linear or non-linear employment-value added function for industry i in location k
e	is the base of the natural logarithm
I	is the number of industries
Ip	is the number of industries in the primary sector
Is	is the number of industries in the secondary sector
ln	is the natural logarithm
$[\ln a(i)]^*$	is the estimate of the logarithm of the intercept coefficient of the non-linear function for industry i
u(i,k,t')	is the random disturbance term for industry i in location k in year t'
u(i,t')	is the random disturbance term for industry i in year t'

2. Equations

B. The technique

1. Overview

(a) Employment-value added functions, by industry

(i) Functions without the time variable

$$EM(i,t') = a(i) + b(i) \cdot VA(i,t'); \quad i = 1,\ldots,I \qquad (1)$$

$$EM(i,t') = a(i) \cdot VA(i,t')^{b(i)}; \quad i = 1,\ldots,I \qquad (2)$$

$$\ln EM(i,t') = \ln a(i) + b(i) \cdot \ln VA(i,t'); \quad i = 1,\ldots,I \qquad (3)$$

(ii) Functions with the time variable

$$EM(i,t') = a(i) + b(i) \cdot VA(i,t') + c(i) \cdot t'; \quad i = 1,\ldots,I \tag{4}$$

$$EM(i,t') = a(i) \cdot VA(i,t') \cdot e^{[c(i)\ .\ t']}; \quad i = 1,\ldots,I \tag{5}$$

$$\ln EM(i,t') = \ln a(i) + b(i) \cdot \ln VA(i,t') + c(i) \cdot t'; \quad i = 1,\ldots,I \tag{6}$$

(b) Employment by industry

(i) Functions without the time variable

$$EM(i,t+5) = a^*(i) + b^*(i) \cdot VA(i,t+5); \quad i = 1,\ldots,I \tag{7}$$

$$EM(i,t+5) = a^*(i) \cdot VA(i,t+5)^{b^*(i)}; \quad i = 1,\ldots,I \tag{8}$$

$$\ln EM(i,t+5) = [\ln a(i)]^* + b^*(i) \cdot \ln VA(i,t+5); \quad i = 1,\ldots,I \tag{9}$$

$$EM(i,t+5) = \text{antiln}[\ \ln EM(i,t+5)\ ]; \quad i = 1,\ldots,I \tag{10}$$

(ii) Functions with the time variable

$$EM(i,t+5) = a^*(i) + b^*(i) \cdot VA(i,t+5) + c^*(i) \cdot (\bar{t}'+t+5); \quad i = 1,\ldots,I \tag{11}$$

$$EM(i,t+5) = a^*(i) \cdot VA(i,t+5)^{b^*(i)} \cdot e^{[c^*(i).(\bar{t}'+t+5)]}; \quad i = 1,\ldots,I \tag{12}$$

$$\ln EM(i,t+5) = [\ln a(i)]^* + b^*(i) \cdot \ln VA(i,t+5) + c^*(i) \cdot (\bar{t}'+t+5); \quad i = 1,\ldots,I \tag{13}$$

(c) Other results

(i) Employment aggregates

a. Total employment

$$EM(t+5) = \sum_{i=1}^{I} EM(i,t+5) \tag{14}$$

b. Employment by sector

i. Employment in the primary sector

$$EMP(t+5) = \sum_{i=1}^{Ip} EM(i,t+5) \tag{15}$$

ii. Employment in the secondary sector

$$EMS(t+5) = \sum_{i=Ip+1}^{Ip+Is} EM(i,t+5) \tag{16}$$

iii. Employment in the tertiary sector

$$EMT(t+5) = \sum_{i=Ip+Is+1}^{I} EM(i,t+5) \tag{17}$$

c. Growth in total employment

$$EMGR = EM(t+5) - EM(t) \quad (18)$$

d. Growth of employment by sector

$$EMPGR = EMP(t+5) - EMP(t) \quad (19)$$

$$EMSGR = EMS(t+5) - EMS(t) \quad (20)$$

$$EMTGR = EMT(t+5) - EMT(t) \quad (21)$$

(ii) Indicators of the structure of employment

a. Proportions by sector

$$PEMP(t+5) = EMP(t+5) / EM(t+5) \quad (22)$$

$$PEMS(t+5) = EMS(t+5) / EM(t+5) \quad (23)$$

$$PEMT(t+5) = EMT(t+5) / EM(t+5) \quad (24)$$

(iii) Rates of growth of employment

a. Rate of growth of total employment

$$GGREM = [(EM(t+5) / EM(t))^{1/5} - 1] \cdot 100 \quad (25)$$

$$EGREM = [\ln (EM(t+5) / EM(t)) / 5] \cdot 100 \quad (26)$$

b. Rates of growth of employment by sector

$$GGREMP = [(EMP(t+5) / EMP(t))^{1/5} - 1] \cdot 100 \quad (27)$$

$$GGREMS = [(EMS(t+5) / EMS(t))^{1/5} - 1] \cdot 100 \qquad (28)$$

$$GGREMT = [(EMT(t+5) / EMT(t))^{1/5} - 1] \cdot 100 \qquad (29)$$

$$EGREMP = [\ln (EMP(t+5) / EMP(t)) / 5] \cdot 100 \qquad (30)$$

$$EGREMS = [\ln (EMS(t+5) / EMS(t)) / 5] \cdot 100 \qquad (31)$$

$$EGREMT = [\ln (EMT(t+5) / EMT(t)) / 5] \cdot 100 \qquad (32)$$

(iv) Labour market balances

$$CLF(t+5) = LF(t+5) - NEM(t+5) \qquad (33)$$

$$EXL(t+5) = CLF(t+5) - EM(t+5) \qquad (34)$$

$$PEXL(t+5) = [EXL(t+5) / CLF(t+5)] \cdot 100 \qquad (35)$$

3. Urban-rural level

(a) Employment-value added functions by industry

(i) Functions without the time variable

$$EM(i,k,t') = a(i,k) + b(i,k) \cdot VA(i,t'); \qquad (36)$$

$i = 1,\ldots,I;$

$k = 1,2$

$$EM(i,k,t') = a(i,k) \cdot VA(i,t')^{b(i,k)}; \qquad (37)$$

$i = 1,\ldots,I;$

$k = 1,2$

$$\ln EM(i,k,t') = \ln a(i,k) + b(i,k) \cdot \ln VA(i,t'); \qquad (38)$$

$i = 1,\ldots,I;$

$k = 1,2$

(ii) Functions with the time variable

$$EM(i,k,t') = a(i,k) + b(i,k) \cdot VA(i,t') + c(i,k) \cdot t'; \qquad (39)$$

$i = 1,\ldots,I;$

$k = 1,2$

$$EM(i,k,t') = a(i,k) \cdot VA(i,t')^{b(i,k)}\, e^{[c(i,k) \cdot t']}; \qquad (40)$$

$i = 1,\ldots,I$

$$\ln EM(i,k,t') = \ln a(i,k) + b(i,k) \cdot \ln VA(i,t') + c(i,k) \cdot t'; \qquad (41)$$

$i = 1,\ldots,I;$

$k = 1,2$

(b) Employment by sector

(i) Functions without the time variable

$$EM(i,k,t+5) = a^*(i,k) + b^*(i,k) \cdot VA(i,t+5); \qquad (42)$$

$i = 1,\ldots,I;$

$k = 1,2$

$$EM(i,k,t+5) = a^*(i,k) \cdot VA(i,t+5)^{b^*(i,k)}; \qquad (43)$$

$i = 1,\ldots,I;$

$k = 1,2$

$$\ln EM(i,k,t+5) = [\ln a(i,k)]^* + b^*(i,k) \cdot \ln VA(i,t+5); \qquad (44)$$

$$i = 1,\ldots,I;$$

$$k = 1,2$$

$$EM(i,k,t+5) = \text{antiln}[\ \ln EM(i,k,t+5)\ ]; \qquad (45)$$

$$i = 1,\ldots,I;$$

$$k = 1,2$$

(ii) Functions with the time variable

$$EM(i,k,t+5) = a^*(i,k) + b^*(i,k) \cdot VA(i,t+5) + c^*(i,k) \cdot (\bar{t}'+t+5); \qquad (46)$$

$$i = 1,\ldots,I;$$

$$k = 1,2$$

$$EM(i,k,t+5) = a^*(i,k) \cdot VA(i,t+5)^{b^*(i,k)} \cdot e^{[c^*(i,k)\ \cdot\ (\bar{t}'+t+5)]}; \qquad (47)$$

$$i = 1,\ldots,I;$$

$$k = 1,2$$

$$\ln EM(i,k,t+5) = [\ln a(i,k)]^* + b^*(i,k) \cdot \ln VA(i,t+5) \qquad (48)$$

$$+ c^*(i,k) \cdot (\bar{t}'+t+5);$$

$$i = 1,\ldots,I;$$

$$k = 1,2$$

(c) Other results

(i) Proportions of employment, urban and rural

a. Proportions of total employment

$$PEMURB(t+5) = EM(1,t+5)/EM(t+5) \qquad (49)$$

PEMRUR(t+5) = 1 - PEMURB(t+5) (50)

b. Proportions of employment by sector

PEMPURB(t+5) = EMP(1,t+5)/EMP(t+5) (51)

PEMSURB(t+5) = EMS(1,t+5)/EMS(t+5) (52)

PEMTURB(t+5) = EMT(1,t+5)/EMT(t+5) (53)

PEMPRUR(t+5) = 1 - PEMPURB(t+5) (54)

PEMSRUR(t+5) = 1 - PEMSURB(t+5) (55)

PEMTRUR(t+5) = 1 - PEMTURB(t+5) (56)

C. The inputs

1. Types of inputs required

2. Preparation of the inputs

(a) Estimates of the co-efficients of employment-value added functions

(i) Time series data

(ii) Estimation procedures

a. National level

i. Functions without the time variable

EM(i,t') = a(i) + b(i) • VA(i,t') + u(i,t'); (57)

i = 1,...,I

$$\ln EM(i,t') = \ln a(i) + b(i) \cdot \ln VA(i,t') + u(i,t'); \qquad i = 1,\dots,I \tag{58}$$

$$a^*(i) = \text{antiln}[\ln a(i)]^*; \qquad i = 1,\dots,I \tag{59}$$

ii. <u>Functions with the time variable</u>

$$EM(i,t') = a(i) + b(i) \cdot VA(i,t') + c(i) \cdot t' + u(i,t'); \qquad i = 1,\dots,I \tag{60}$$

$$\ln EM(i,t') = \ln a(i) + b(i) \cdot \ln VA(i,t') + c(i) \cdot t' + u(i,t'); \qquad i = 1,\dots,I \tag{61}$$

(b) <u>Urban-rural level</u>

i. <u>Functions without the time variable</u>

$$EM(i,k,t') = a(i,k) + b(i,k) \cdot VA(i,t') + u(i,k,t'); \qquad i = 1,\dots,I; \quad k = 1,2 \tag{62}$$

$$\ln EM(i,k,t') = \ln a(i,k) + b(i,k) \cdot \ln VA(i,t') + u(i,k,t'); \qquad i = 1,\dots,I; \quad k = 1,2 \tag{63}$$

$$a^*(i,k) = \text{antiln}[\ \ln a(i,k)\ ]^*; \qquad i = 1,\dots,I; \quad k = 1,2 \tag{64}$$

ii. Functions with the time variable

$$EM(i,k,t') = a(i,k) + b(i,k) \cdot VA(i,t') + c(i,k) \cdot t + u(i,k,t'); \quad i = 1,\ldots,I; \quad k = 1,2 \tag{65}$$

$$\ln EM(i,k,t') = \ln a(i,k) + b(i,k) \cdot \ln VA(i,t') + c(i,k) \cdot t + u(i,k,t'); \quad i = 1,\ldots,I; \quad k = 1,2 \tag{66}$$

Notes

1/ Throughout this chapter, "value added" will refer to value added measured in constant prices.

2/ This is equivalent to assuming that the average ratio of employment to value-added equals the marginal ratio of employment to value-added.

3/ Much of the material presented in this section is similar to that given in chapter VI, section B.2(c). The reader who is familiar with its content may wish to skip the present section.

4/ Much of the material presented in this section is similar to that given in chapter VI, section B.3(c). The reader who is familiar with it may wish to skip the present section.

5/ Values assigned to the time variable were 1968 through 1978.

6/ The employment-value added functions used in this and the following example were estimated, among other things, from the time series on employment shown in tables 86 and 88, which are expressed in units of 1,000 employed persons. Therefore, the levels of employment in these illustrative examples will be given in thousands of employed persons.

7/ Much of the material presented in this section is similar to that given in chapter VI, section D.1(c). The reader who is familiar with it may wish to skip the present section.

8/ Much of the material presented in this section is similar to that given in chapter VI, section D.2(c). The reader who is familiar with its content may wish to skip the present section.

Annex I

PROCEDURE TO CALIBRATE EMPLOYMENT-VALUE ADDED FUNCTIONS

A planner may wish to make adjustments in the estimated employment-value added functions by industry, so that they will accurately predict employment by industry for a given historical year or time period on the basis of the value added levels for that year or period. These adjustments, which are normally referred to as "calibration" may be employed, for example, where the functions are to be used to make projections of employment that originate in the given historical year or period to which the data used to estimate the functions refer, rather than in the later, initial year of the plan.

Calibrating employment-value added functions may involve adjustments in estimates of the intercept coefficients, or in estimates of the partial coefficients or both. Since adjustments in the intercept coefficients are more straightforward than those in the partial coefficients, calibration is often restricted to intercepts. Therefore, this annex will describe how the intercept coefficients of employment-value added functions of different forms can be calibrated by first describing the calibration procedure and then selectively illustrating its application.

A. The procedure

The principles of adjusting the intercept coefficients of employment-value added functions are the same, irrespective of the type of functions involved. The steps make use of the estimates of the partial coefficients of the functions, as well as the observed levels of employment and value added, for the selected year or time period. The actual steps involved in adjusting the intercepts vary, however, depending on the type of functions estimated and used in the projections, as well as whether those functions are for the entire country or for urban and rural areas.

1. National level

This section will describe the procedure as it applies to functions estimated for the entire country, first to those without the time variable and then to the functions with this variable. Subsequently, the procedure applicable to the urban-rural level will be explained.

(a) Functions without the time variable

The method used to obtain adjusted intercepts will vary depending on the whether the functions are linear, non-linear or log-linear.

(i) In the case of linear functions, they are calculated as follows:

$$[a^*(i)]' = EM(i,t') - b^*(i) \cdot VA(i,t'); \qquad (1)$$

$$i = 1,\dots,I,$$

where:

i = 1,...,I	are industries in the nation's economy,
I	is the number of industries,
t'	is the given calendar year,
EM(i,t')	is the observed employment in industry i in year t',
VA(i,t')	is the observed value added in industry i in year t',
[a*(i)]'	is the adjusted intercept coefficient of the linear employment-value added functions for industry i, and
b*(i)	is the estimate of the partial coefficient of the value added variable in the linear employment-value added function for industry i.

(ii) In the case of non-linear functions having the multiplicative form, adjusted intercepts can be obtained as follows:

$$[a^*(i)]' = EM(i,t') \;/\; [\; VA(i,t')^{b^*(i)} \;]; \qquad (2)$$

$$i = 1,\dots,I,$$

where:

[a*(i)]'	is the adjusted intercept coefficient of the non-linear employment-value added function for industry i, and
b*(i)	is the estimate of the partial coefficient of the value added variable in the non-linear employment-value added function for industry i.

(iii) If adjusted intercepts of log-linear employment-value added functions are needed, they can be obtained as follows:

$$[[\ln a(i)]^*]' = \ln EM(i,t') - b^*(i) \cdot \ln VA(i,t'); \qquad (3)$$

$$i = 1,\ldots,I,$$

where:

ln	is the natural logarithm, and
$[[\ln a(i)]^*]'$	is the adjusted logarithm of the intercept coefficient of the non-linear employment-value added function for industry i.

If the planner wishes to perform adjustments in the intercept coefficients using data for a few or several years rather then a single year, the adjustments can be also made using expressions shown in equation (1) through (3). In that instance, the observed levels of employment and value added used would be mean levels of employment and value added for several years centred on one particular year, t'.

(b) <u>Functions with the time variable</u>

If the functions include a time variable, the calculation of adjusted intercepts would also involve the use of a suitable value for the time variable.

(i) In the case of linear functions with the time variable, the adjusted intercepts would be:

$$[a^*(i)]' = EM(i,t') - [\ b^*(i) \cdot VA(i,t') + c^*(i) \cdot t'\ ]; \qquad (4)$$

$$i = 1,\ldots,I,$$

where:

$c^*(i)$	is the estimate of the partial coefficient of the time variable in the linear employment-value added function for industry i.

(ii) If adjusted intercepts of non-linear functions with the time variable which have the multiplicative form are sought, they would be obtained as:

$$[a^*(i)]' = EM(i,t') \;/\; [\; VA(i,t')^{b^*(i)} \cdot e^{(c^*(i) \cdot t')} \;]; \qquad (5)$$

$$i = 1,\ldots,I,$$

where:

$c^*(i)$ is the estimate of the partial coefficient of the time variable in the non-linear employment-value added function for industry i.

(iii) In the case of log-linear employment-value added functions, adjusted intercepts can be obtained as follows:

$$[[\ln a(i)]^*]' = \ln EM(i,t') - [\; b^*(i) \cdot \ln VA(i,t') + c^*(i) \cdot t' \;]; \qquad (6)$$

$$i = 1,\ldots,I.$$

1. Urban-rural level

If time series data on employment and value added are available for urban and rural areas by industry, such data could be used to estimate employment-value added functions for the two locations by industry. After estimating the functions, the intercept coefficients could be adjusted using the estimates of the partial coefficients and the observed levels of employment and value added for a given year or time period. Depending on the type of functions estimated, those adjustments could be performed using urban-rural equivalents of steps described in equations (1) through (6).

In some countries, time series on value added by industry will not be available for urban and rural areas. To estimate and use employment-value added functions for urban and rural areas, it may be necessary to estimate functions that relate employment in urban and rural areas by industry to the value added at the national level by industry. The procedure for adjusting the intercept coefficients of such functions is described below.

(a) Functions without the time variable

(i) In the case of linear employment-value added functions that do not include the time variable, adjusted intercepts are obtained as follows:

$$[a^*(i,k)]' = EM(i,k,t') - b^*(i,k) \cdot VA(i,t'); \qquad (7)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

k = 1,2	are urban and rural locations,
EM(i,k,t')	is the observed employment in industry i in location k in year t',
[a*(i,k)]'	is the adjusted intercept coefficient of the linear employment-value added function for industry i in location k, and
b*(i,k)	is the estimate of the partial coefficient of the value added variable in the linear employment-value added function for industry i in location k.

(ii) Adjusted intercept coefficients of the non-linear employment-value added functions that do not include the time variable would be obtained as:

$$[a^*(i,k)]' = EM(i,k,t') \;/\; [\; VA(i,t')^{b^*(i,k)} \;]; \qquad (8)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

[a*(i,k)]'	is the adjusted intercept coefficient of the non-linear employment-value added functions for industry i in location k, and
b*(i,k)	is the estimate of the partial coefficient of the value added variable in the non-linear employment-value added function for industry i in location k.

(iii) Adjusted intercept coefficients of the log-linear employment-value added functions that do not include the time variable can be derived as:

$$[[\ln a(i,k)]^*]' = \ln EM(i,k,t') - b^*(i,k) \cdot \ln VA(i,t'); \quad (9)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

$[[\ln a(i,k)]^*]'$ is the adjusted logarithm of the intercept coefficient of the log-linear employment-value added function for industry i in location k.

(b) <u>Functions with the time variable</u>

(i) For employment-value added functions which include the time variable, adjusted intercepts in the linear functions would be obtained as follows:

$$[a^*(i,k)]' = EM(i,k,t') - [\, b^*(i,k) \cdot VA(i,t') + c^*(i,k) \cdot t' \,]; \quad (10)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

$c^*(i,k)$ is the estimate of the partial coefficient of the time variable in the linear employment-value added function for industry i in location k.

(ii) In the case of non-linear employment-value added functions, adjusted constants can be obtained as:

$$[a^*(i,k)]' = EM(i,k,t') \,/\, [\, VA(i,t')^{b^*(i,k)} \cdot e^{(c^*(i,k) \cdot t')} \,]; \quad (11)$$

$$i = 1,\ldots,I;$$

$$k = 1,2,$$

where:

$c^*(i,k)$	is the estimate of the partial coefficient of the time variable in the non-linear employment-value added function for industry i in location k.

(iii) In the case of log-linear functions, adjusted constants can be obtained as:

$$[[\ln a(i,k)]^*]' = \ln EM(i,k,t') - [\ b^*(i,k) \cdot \ln VA(i,t') + c^*(i,k) \cdot t'\ ]; \quad i = 1,\ldots,I; \quad k = 1,2. \tag{12}$$

B. Illustrative examples of calibration

The examples presented below will not attempt to exhaustively illustrate the procedure to calibrate all employment-value added functions, which include linear, non-linear and log-linear functions, with and without the time variable, for the entire country and urban and rural areas. Rather they will show how to calibrate the functions that were employed in chapter VII in order to illustrate their use in preparing employment projections. In particular, the first example will show how to derive adjusted intercept coefficients for log-linear employment-value added functions for the entire country by industry, two of which include the time variable. The second example will indicate the way to obtain adjusted intercepts for linear functions for urban and rural areas, by industry, several of which include the time variable. Both examples will use as inputs, the observations on employment and value added for 1978, which are shown in tables 86 through 88.

1. National level

This example will indicate how to calibrate estimates of country-level log-linear employment-value added functions without the time variable for all industries except construction and trade (the unadjusted coefficients are shown in table 90), and with the time variable for these two industries (the unadjusted coefficients are shown in table 92). The adjusted intercepts for these functions will be derived using estimates of the partial coefficients of those functions along with the levels of employment and value added by industry for 1978 which are shown in tables 86 and 87.

Table 124 illustrates the calculation of the adjusted logarithms of intercepts for the functions in question. The adjusted logarithm of intercept coefficient (column 7) for any industry (other than the construction or trade industry) is obtained as the difference between the logarithm of the observed level of employment for the industry in 1978 (column 6) and the product of the estimated value added coefficient (column 2) and the logarithm of the observed level of value added for the industry in 1978 (column 4).

The adjusted logarithm of the intercept coefficient (column 7) for the construction or trade industry is obtained as the difference between the logarithm of the observed level of employment for the industry in 1978 (column 6) and the sum of two products. The first product is obtained by multiplying the estimated value added coefficient for the construction or trade industry (column 2) by the logarithm of the observed level of value added for the industry in 1978 (column 4). The second product is the result of multiplying the time variable coefficient for the industry (column 3) by the value of the time variable for year 1978 (column 5).

Thus, the adjusted logarithm of the intercept in the function for agriculture, 5.78334, is obtained as follows:

$$5.78334 = \ln(1,994.0) - 0.32625 \cdot \ln(260.3), \qquad (3)$$

where 1,994.0 is the employment in agriculture in 1978, while 0.32625 and 260.3 are, respectively, the estimate of the value added coefficient for agriculture and the value added in agriculture in 1978.

The adjusted logarithm of the intercept in the function for construction, -99.37402, is calculated as:

$$-99.37402 = \ln(93.8) - [\ 0.07935 \cdot \ln(23.5) + 0.05241 \cdot 1978\ ], \qquad (6)$$

where 93.8 is the employment in construction in 1978, while 0.07935 and 23.5 are, respectively, the estimate of the value added coefficient for construction and the value added in this industry in 1978. 0.05241 is the estimate of the time variable coefficient for the construction industry and 1978 is the value of the time variable.

1. Urban-rural level

This example shows how to calibrate estimates of linear employment-value added functions for urban and rural areas by industry, some of which exclude the time variable while others include it. Among the functions for urban areas, the functions for mining, manufacturing, trade, transport and services do not include the time variable (table 93), while those for agriculture,

Table 124. Computing adjusted intercept coefficients of selected log-linear functions, by industry: entire country, year 1978

Industry	Estimates of partial coefficients		In year 1978			
	Value added a/	Time variable b/	Value added c/	Value of time variable	Employment d/	Adjusted intercept coefficient e/
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Agriculture	0.32625		260.3		1994.0	5.78333
Mining	0.86530		2.6		6.7	1.07529
Manufacturing	0.56365		118.9		217.3	2.68799
Utilities	0.80202		18.0		15.3	0.40971
Construction	0.07934	0.052	23.5	1978	93.8	-99.37402
Trade	0.31107	0.031	50.9	1978	109.4	-58.24014
Transport	0.96016		50.3		95.1	0.79301
Services	0.63680		266.9		516.2	2.68876

a/ All industries except construction and trade: from table 90, col. 3; construction and trade industry: from table 93, col. 3.
b/ From table 93, col. 4.
c/ From table 87, year 1978.
d/ From table 86, year 1978.
e/ All industries except construction and trade: (ln(Col. 6)) - ((Col. 2) . (ln(Col. 4))); construction and trade: (ln(Col. 6)) - ((Col. 2) . (ln(Col. 4)) + (Col. 3) . (Col. 5)).

utilities and construction include it (table 97). Among the functions for rural areas, those for agriculture, mining, manufacturing, construction, trade and services do not include the time variable (table 94), whereas those for utilities and transport include it (table 98). To derive adjusted intercepts for these functions, estimates of the partial coefficients of those functions will be used along with the levels of employment and value added by industry for 1978 (tables 87 and 88).

Tables 125 and 126 illustrate calculations of the adjusted intercepts for the functions in question. The adjusted intercept coefficient (column 7) for each industry having a function that excludes the time variable is obtained as the difference between the observed level of employment for the industry in urban or rural areas in 1978 (column 6) and the product of the estimated value added coefficient (column 2) and the observed level of national value added for the industry in 1978 (column 4).

The adjusted intercept coefficients (column 7) for each industry having a function which includes the time variable are obtained as the difference between the observed level of employment for the industry in urban or rural areas in 1978 (column 6) and the sum of two products. The first product is derived by multiplying the estimated value added coefficient for the industry (column 2) by the observed level of national value added for the industry in 1978 (column 4). The second product is obtained by multiplying the time variable coefficient for the industry (column 3) and the value of the time variable for year 1978 (column 5).

For example, the adjusted intercept coefficient in the function for urban mining, 2.08799, is obtained as follows:

$$2.08799 = 5.9 - 1.46616 \cdot 2.6, \qquad (7)$$

where 5.9 is the employment in urban mining in 1978, while 1.46616 and 2.6 are, respectively, the estimate of the value added coefficient for urban mining and the national value added in mining in 1978.

The adjusted intercept in the function for urban agriculture, 2,645.31363, is calculated as:

$$2{,}645.31363 = 22.2 - (\ 0.13728 \cdot 260.3 + (-1.34421) \cdot 1978\ ), \qquad (10)$$

where 22.2 is the employment in urban agriculture in 1978, while 0.13728 and 260.3 are, respectively, the estimate of the value added coefficient for urban agriculture and the national value added in agriculture in 1978. -1.34421 is the estimate of the time variable coefficient for the urban agriculture and 1978 is the value of the time variable.

Table 125. Computing adjusted intercept coefficients for selected linear functions, by industry: urban areas, year 1978

Industry	Estimates of partial coefficients		In year 1978			
	Value added a/	Time variable b/	National value added c/	Value of time variable	Employment d/	Adjusted intercept coefficient e/
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Agriculture	0.13728	-1.344	260.3	1978	22.2	2645.31363
Mining	1.46615		2.6		5.9	2.08799
Manufacturing	0.97430		118.9		184.2	68.35567
Utilities	0.66476	-0.096	18.0	1978	12.7	190.82840
Construction	-1.07195	3.440	23.5	1978	81.4	-6696.75995
Trade	2.27942		50.9		98.8	-17.22250
Transport	1.62845		50.3		92.2	10.28867
Services	0.81798		266.9		334.3	115.98101

a/ Mining, manufacturing, trade, transport and services: from table 94, col. 3; agriculture, utilities and construction: from table 99, col. 3.
b/ From table 99, col. 4.
c/ From table 87, year 1978.
d/ From table 88, urban, year 1978.
e/ Mining, manufacturing, trade, transport and services: (Col. 6) - ((Col. 2) . (Col. 4)); agriculture, utilities and construction: (Col. 6) - ((Col. 2) . (Col. 4) + (Col. 3) . (Col. 5)).

Table 126. Computing adjusted intercept coefficients for selected linear functions, by industry: rural areas, year 1978

Industry	Estimates of partial coefficients		In year 1978			Adjusted intercept coefficient e/
	Value added a/	Time variable b/	National value added c/	Value of time variable	Employment d/	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Agriculture	2.85558		260.3		1971.8	1228.49142
Mining	0.65595		2.6		0.8	-0.90547
Manufacturing	0.25725		118.9		33.1	2.51193
Utilities	0.87432	-0.626	18.0	1978	2.6	1225.68768
Construction	1.53126		23.5		12.4	-23.58472
Trade	0.74332		50.9		10.6	-27.23500
Transport	-0.07479	0.269	50.3	1978	2.9	-524.85848
Services	0.60456		266.9		181.9	20.54159

a/ Agriculture, mining, manufacturing, construction, trade and services: from table 95, col. 3; utilities and transport: from table 100, col. 3.
b/ From table 100, col. 4.
c/ From table 87, year 1978.
d/ From table 88, rural, year 1978.
e/ Agriculture, mining, manufacturing, construction, trade and services: (Col. 6) - ((Col. 2) . (Col. 4)); utilities and transport: (Col. 6) - ((Col. 2) . (Col. 4) + (Col. 3) . (Col. 5)).

C. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

i = 1,...,I	are industries of the nation's economy
k = 1,2	are urban and rural locations
t'	is the given calendar year

(b) List of variables

EM(i,k,t')	is the observed employment in industry i in location k in year t'
EM(i,t')	is the observed employment in industry i in year t'
VA(i,t')	is the observed value added in industry i in year t'

(c) List of special symbols

[a*(i)]'	is the adjusted estimate of the intercept coefficient of the linear or non-linear employment-value added function for industry i
[a*(i,k)]'	is the adjusted estimate of the intercept coefficient of the linear or non-linear employment-value added function for industry i in location k
b*(i)	is the estimate of the partial coefficient of the value added variable in the linear or non-linear employment-value added function for industry i
b*(i,k)	is the estimate of the partial coefficient of the value added variable in the linear or non-linear employment-value added function for industry i in location k
c*(i)	is the estimate of the partial coefficient of the time variable in the linear or non-linear employment-value added function for industry i
c*(i,k)	is the estimate of the partial coefficient of the time variable in the linear or non-linear employment-value added function for industry i in location k

I	is the number of industries
ln	is the natural logarithm
[[lna(i)]*]'	is the adjusted estimate of the logarithm of the intercept coefficient of the non-linear employment-value added function for industry i
[[lna(i,k)]*]'	is the adjusted estimate of the logarithm of the intercept coefficient of the non-linear employment-value added function for industry i in location k

2. Equations

A. The procedure

1. National level

(a) Functions without the time variable

$$[a^*(i)]' = EM(i,t') - b^*(i) \cdot VA(i,t'); \qquad i = 1,\ldots,I \tag{1}$$

$$[a^*(i)]' = EM(i,t') / [\ VA(i,t')^{b^*(i)}\ ]; \qquad i = 1,\ldots,I \tag{2}$$

$$[[\ln a(i)]^*]' = \ln EM(i,t') - b^*(i) \cdot \ln VA(i,t'); \qquad i = 1,\ldots,I \tag{3}$$

(b) Functions with the time variable

$$[a^*(i)]' = EM(i,t') - [\ b^*(i) \cdot VA(i,t') + c^*(i) \cdot t'\ ]; \qquad i = 1,\ldots,I \tag{4}$$

$$[a^*(i)]' = EM(i,t') / [\ VA(i,t')^{b^*(i)} \cdot e^{(c^*(i) \cdot t')}\ ]; \qquad i = 1,\ldots,I \tag{5}$$

$$[[\ln a(i)]^*]' = \ln EM(i,t') - [\, b^*(i) \cdot \ln VA(i,t') + c^*(i) \cdot t' \,]; \quad i = 1,\ldots,I \tag{6}$$

2. Urban-rural level

(a) Functions without the time variable

$$[a^*(i,k)]' = EM(i,k,t') - b^*(i,k) \cdot VA(i,t'); \quad i = 1,\ldots,I; \quad k = 1,2 \tag{7}$$

$$[a^*(i,k)]' = EM(i,k,t') \,/\, [\, VA(i,t')^{b^*(i,k)} \,]; \quad i = 1,\ldots,I; \quad k = 1,2 \tag{8}$$

$$[[\ln a(i,k)]^*]' = \ln EM(i,k,t') - b^*(i,k) \cdot \ln VA(i,t'); \quad i = 1,\ldots,I; \quad k = 1,2 \tag{9}$$

(b) Functions with the time variable

$$[a^*(i,k)]' = EM(i,k,t') - [\, b^*(i,k) \cdot VA(i,t') + c^*(i,k) \cdot t' \,]; \quad i = 1,\ldots,I; \quad k = 1,2 \tag{10}$$

$$[a^*(i,k)]' = EM(i,k,t') \,/\, [\, VA(i,t')^{b^*(i,k)} \cdot e^{(c^*(i,k)\,\cdot\, t')} \,]; \quad i = 1,\ldots,I; \quad k = 1,2 \tag{11}$$

$$[[\ln a(i,k)]^*]' = \ln EM(i,k,t') - [\, b^*(i,k) \cdot \ln VA(i,t') + c^*(i,k) \cdot t' \,]; \quad i = 1,\ldots,I; \quad k = 1,2 \tag{12}$$

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VIII. MAKING EMPLOYMENT PROJECTIONS USING INVERSE COBB-DOUGLAS PRODUCTION FUNCTIONS

A. Introduction

This chapter presents a method for projecting employment by industry, using employment functions (box 26) which are the inverse of Cobb-Douglas production functions (described in annex I), or their transformations.1/ These functions relate employment to value added, the capital stock and time, by industry.2/ Like the labour productivity technique described in chapter VI and the employment-value added function method described in chapter VII, this method can be used to project employment by industry as well as employment aggregates and indicators of the structure and growth of employment.

The method dispenses with a number of the explicit or implicit assumptions used by the simpler methods described in earlier chapters. Furthermore, unlike those techniques, this method explicitly allows for possible changes in the relative growth rates of various inputs (e.g., substitution of capital for labour).3/

Unlike the simpler techniques, the method is also capable of making separate estimates of the effects of input growth on employment and the effects of technical change. This makes it possible to make long-run employment projections which are more accurate than those prepared by the simpler methods, particularly if different inputs are expected to grow at different rates.

Deciding whether employment functions derived from Cobb-Douglas production functions are more suited than simpler methods for preparing long-term employment projection will involve a number of considerations. Among these considerations are: whether the Cobb-Douglas production function correctly specifies the technological relationships existing in various industries; whether adequate data are available to estimate parameters of the industry-specific Cobb-Douglas production functions (or their transformations); and whether the parameters of those functions can be correctly estimated from suitable data. In view of the importance of these considerations, they will be briefly reviewed below.

In deciding whether the Cobb-Douglas production function is the correct specification, it is important to recognize that it is only one, although the most popular, form of production function. Although this particular production function has a number of desirable properties, there may be no a priori reasons for choosing it over other types of production functions as a basis for making employment projections. This specific form may not be the most appropriate to use in any given situation. If the Cobb-Douglas function is a misspecification of the actual relationship between output and production inputs, the method may yield misleading projections of employment.

Box 26

Glossary

Average capital-output ratio
: The capital stock of a firm, industry or economy over a time period, divided by the output produced during that period.

Employment function
: A mathematical expression describing a relationship between employment and other variables, typically a measure of output and other inputs, in which employment is the dependent variable and the other variables are independent. The inverse of a Cobb-Douglas production function in which employment is the dependent variable would be an employment function.

Cobb-Douglas production function
: A mathematical expression describing a relationship between a measure of output and two or more inputs (such as employed labour and capital). The function is multiplicative in the natural numbers and linear when transformed into logarithms.

Human capital
: Productive investments embodied in human persons. These include skills, abilities, ideals, health etc., that result from expenditures on education, on-the-job-training and medical care.

Incremental capital-output ratio
: The increase in the capital stock of a firm, industry or economy over a time period, divided by the increase in output over that period.

Simultaneous equation bias
: A bias arising in statistical estimation when the dependent variable has a causal effect on the independent variables, rather than vice versa.

The method might not be readily applicable in many developing countries because reliable data needed to estimate parameters of the Cobb-Douglas production functions are not widely available. This is particularly true if the functions are to be estimated on the basis of time series data, which may be essential whenever estimates of the rate of technical change are needed.

At present, the data necessary for applying this method at the rural-urban level are not available in most developing countries.

In any given country, time series data on production and inputs may be unavailable, not sufficiently reliable or could refer to too short a time period. Cross sections including relevant information at several points in time could be even scarcer than sufficiently long time series. However, some time series data, or at least cross sections for a few different years, are essential in order to estimate the rate of technical change. Even if data of the right type are available, it may not be possible to obtain robust estimates of the relevant parameters owing to the lack of variability in the economic conditions from one year to the next or from one firm to another.

Although simple methods are available for estimating the Cobb-Douglas production function, there is an extensive econometric literature that casts doubt on the quality and utility of the estimates obtained by such methods. In particular, questions have been raised about whether the functions so estimated are really production functions at all, or merely some mixture of the production function and the profit maximizing conditions. In addition, the estimates of the parameters of the production functions obtained by ordinary least squares may be subject to a <u>simultaneous equations bias.</u> That is, changes in the independent variable may be caused by changes in the dependent variable.

There may be no such thing as an aggregate production function, that is, a technical production function at the level of industry or the entire economy. Most approaches to aggregation assume that the underlying production functions from which the aggregate production function of an industry or the economy is to be derived are additive. (An example of an additive production function is a linear production function.) Cobb-Douglas functions, however, are not additive. Thus if various segments of the industry, such as the modern or the traditional segments are growing at different rates, the estimated Cobb-Douglas production function for the industry will tend to yield poor results when used as a basis for projecting employment. That is, the coefficients of the estimated production function would take on different values depending on the relative rates of growth of the modern and traditional sectors. This is called aggregation bias.

If the <u>human capital</u> composition of the labour force or the age composition of the physical capital stock is expected to change significantly in the future (which may often be the case in developing countries), the traditional specification of the Cobb-Douglas production function can be expected to provide a poor basis for projecting employment. This is so since neither labour nor capital is homogeneous, though they are traditionally entered as homogeneous inputs in a Cobb-Douglas production function and, by implication, into any of its transformations.

This amounts to a misspecification since employees differ from one another in the amount of human capital they embody, or machines of different vintages differ from one another in the type of technology they embody.

Hence, to avoid misspecification, labour should be disaggregated by type, or a weighted index of labour inputs should be constructed and used as an input. Similarly, the capital stock need to be subdivided by vintage (or date of production), or a weighted index of capital of different ages should be prepared and used as the input.

Even if it is possible to specify, estimate and use a Cobb-Douglas production function correctly, little may be gained by using this method rather than simpler techniques to project employment, particularly if the objective is medium-term (i.e., five-year) projections. Since capital-output ratios are unlikely to change much in the medium-term, relatively simple projection techniques should serve almost as well as those involving estimated production functions. Hence production functions may perhaps be best used for longer-term projections.

B. The technique

1. Overview

This overview lists inputs required by the method based on the inverse Cobb-Douglas production functions and indicates the types of results it can generate. The overview also outlines the computational steps involved in making an employment projection with this method.

(a) Inputs

To project employment with this method the following inputs are required:

(i) Projected levels of value added, by industry;
(ii) Projected levels of the capital stock, by industry;
(iii) Estimates of the coefficients of employment functions which are the inverse of Cobb-Douglas production functions (or their transformations), by industry.4/

If, in addition to employment, shortages and/or surpluses in the labour market are to be projected, the inputs should also include:

(iv) Projected total labour force;
(v) Projected non-civilian employment.

The inputs are listed in box 27.

Since this method is described as a procedure for making quinquennial projections, the projected levels of value added and the capital stock would be for dates five years apart, starting with the initial year of the plan. Projected total labour force and projected non-civilian employment would be for the same dates. Given the appropriate annual inputs, however, the method could be used for making annual projections.

Box 27

Inputs for projecting employment using inverse Cobb-Douglas production functions or their transformations

1. Value added by industry

2. Capital stock by industry

3. Coefficients of the employment functions

   Coefficients of inverse Cobb-Douglas production functions or their transformations, by industry

4. Total labour force (if projection of labour market balances is desired)

5. Non-civilian employment (if projection of labour market balances is desired)

(b) Outputs

The outputs which this method can generate include:

(i) Levels of employment by industry;
(ii) Various employment aggregates, such as total employment and the growth in total employment;
(iii) Indicators of the structure of employment, such as proportions of total employment found in each sector, (e.g. primary, secondary and tertiary);
(iv) Rates of change in employment, including that of total employment or employment by sector.

If the inputs include projected total labour force and projected non-civilian employment, the outputs could also include:

(v) Absolute and relative levels of excess supply of labour and/or excess demand of labour.

The types of outputs that the technique can generate as part of the projection, are shown in box 28. They would be obtained for dates five years apart or for the intervening projection intervals.

(c) Computational steps

Levels of employment, by industry, for a given date can be projected by evaluating employment functions that are the inverses of Cobb-Douglas production functions or their transformations, using as inputs projected levels of value added and the capital stock. The procedure also permits

Box 28

Types of outputs of employment projections using inverse Cobb-Douglas production functions or their transformations

1. Employment by industry

2. Employment aggregates

   Total employment and employment by sector (e.g. primary, secondary and tertiary)

   Growth in total employment and employment by sector

3. Indicators of the structure of employment

   Proportions of employment, by sector

4. Rates of growth of employment

   Rates of growth in total employment and employment by sector

6. Labour market balances

   Absolute and relative levels of excess supply of and/or excess demand for labour

deriving for each projection date, total employment and employment in major sectors, such as primary, secondary and tertiary, along with other date-specific indicators. Also, the projection provides an estimate of the absolute growth and growth rates in employment for the intervening intervals. If a labour force projection is available, the projected total employment and the projected labour force size can be compared to calculate the surplus or shortage of labour.

2. Description

This section will first describe a particular type of Cobb-Douglas production function and then introduce employment functions that are the inverses of those Cobb-Douglas production functions and their transformations. Then it will show how to project levels of employment by industry and explain steps to derive other results. A summary of those steps is shown in box 29.

Box 29

Computational steps to project employment at the national level using inverse Cobb-Douglas production functions or their transformations

The steps used to project employment at the national level over a five-year projection interval are:

(1) Derive levels of employment or logarithms of those levels, by industry, at the end of the interval by evaluating empirically estimated industry-specific inverse Cobb-Douglas production functions or their log-linear transformations. In the process, use the assumed levels of value added, the capital stock and the suitable value of the time variable. If the logarithms of employment levels are computed, take antilogarithms of the results obtained in order to reach the levels of employment.

(2) Calculate various employment aggregates, such as total employment and the increase in total employment.

(3) Derive indicators of the employment structure, such as proportions of employment by sector.

(4) Obtain rates of growth of employment, such as the rate of growth of total employment.

(5) If the labour force projection is available, derive labour market balances by calculating the absolute and percentage levels of excess supply of or excess demand for labour.

(a) Cobb-Douglas production functions

Employment functions by industry can be obtained by inverting alternative specifications of industry-specific Cobb-Douglas production functions and, if so desired, by finding transformations of those inverse production functions. This section will describe industry-specific Cobb-Douglas production functions, which may be most suitable as the basis for deriving employment functions to be used in developing countries. These Cobb-Douglas production functions, in which the rate of technical change is considered independent of the growth of other inputs, are as follows:

$$VA(i,t') = z(i) \cdot CAP(i,t')^{a(i)} \cdot EM(i,t')^{b(i)} \cdot e^{[r(i) \cdot t']}; \qquad (1)$$

$$i = 1,\ldots,I,$$

where:

t'	is the calendar year,
$i = 1,\ldots,I$	are the industries of the nation's economy,
I	is the number of industries,
$VA(i,t')$	is the value added in industry i in year t',
$CAP(i,t')$	is the capital stock in industry i in year t',
$EM(i,t')$	is the labour employed in industry i in year t',
$z(i)$	is the intercept parameter for industry i,
$a(i)$	is the elasticity parameter relating value added to the capital stock for industry i,
$b(i)$	is the elasticity parameter relating value added to labour for industry i,
$r(i)$	is the constant rate of technical change for industry i, and
e	is the base of the natural logarithms.5/

The form of the Cobb-Douglas production functions indicated in equation (1) is one of several alternative forms (annex I). It has been selected here for the two basic reasons: first, this specification allows for technical change, which may be very important from the point of view of countries undergoing rapid economic growth based on the use of new technologies; and second, the data requirements for this specification are fewer than those of some alternative specifications.

(b) Inverse Cobb-Douglas production functions

The production functions indicated in equation (1) can be manipulated in a variety of ways. They can be inverted to obtain employment functions, which treat employment (or labour input) as the dependent variable and consider value added, the capital stock and time as the independent variable. These employment functions, which are sometimes referred to as inverse Cobb-Douglas production functions, would be as follows:

$$EM(i,t') = a'(i) \cdot VA(i,t')^{b'(i)} \cdot CAP(i,t')^{c'(i)} \cdot e^{[d'(i)\ .\ t']}; \qquad (2)$$

$$i = 1,\ldots,I,$$

where:

a'(i)	is the intercept parameter for industry i,
b'(i)	is the elasticity parameter relating labour to value added in industry i,
c'(c)	is the elasticity parameter relating labour to the capital stock in industry i, and
d'(i)	is the parameter relating labour to the time variable in industry i.6/

The employment functions shown in equation (2) can be in turn transformed by taking the logarithms of their left and right hand sides to obtain what could be referred to as log-linear transformations of inverse Cobb-Douglas production functions. The transformed functions are as follows:

$$\ln EM(i,t') = \ln a'(i) + b'(i) \cdot \ln VA(i,t') + c'(i) \cdot \ln CAP(i,t') \qquad (3)$$

$$+ d'(i) \cdot t';$$

$$i = 1,\ldots,I,$$

where:

ln	is the natural logarithm.

The advantage of this log-linear transformation is that it can be estimated easily using the ordinary least squares (OLS) method. Moreover, once the estimates of the coefficients of the log-linear transformation are available, they can be easily used to make projections of employment.

(c) Employment by industry

Employment by industry can be projected by evaluating empirically estimated employment functions or their transformations for various dates over the projection period, using projected levels of value added and the capital stock by industry. Procedures used to evaluate the functions will depend on their functional form. Thus, if untransformed inverse Cobb-Douglas production functions were used, employment levels, by industry, for the end of the projection interval (t to t+5) would be obtained as follows:

$$EM(i,t+5) = [a'(i)]^* \cdot VA(i,t+5)^{[b'(i)]^*} \cdot CAP(i,t+5)^{[c'(i)]^*} \cdot e^{[\,[d'(i)]^* \cdot (\bar{t}'+t+5)]}; \quad i = 1,\ldots,I, \qquad (4)$$

where:

t	is the year of the projection period,
$\bar{t}'$	is the calendar year designated as the initial year of the projection period,
EM(i,t+5)	is the labour employed in industry i at the end of the interval,
VA(i,t+5)	is the value added in industry i at the end of the interval,
CAP(i,t+5)	is the capital stock in industry i at the end of the interval,
[a'(i)]*	is the estimate of the intercept coefficient of the inverse Cobb-Douglas production function for industry i,
[b'(i)]*	is the estimate of the partial coefficient of the value added variable in the inverse Cobb-Douglas production function for industry i,
[c'(i)]*	is the estimate of the partial coefficient of the capital stock variable in the inverse Cobb-Douglas production function for industry i, and
[d'(i)]*	is the estimate of the partial coefficient of the time variable in the inverse Cobb-Douglas production function for industry i.

Estimates of the log-linear transformation of the inverse Cobb-Douglas production functions could be used to project employment. Those functions would be first used to obtain the logarithms of employment levels by industry as follows:

$$\ln EM(i,t+5) = [\ln a'(i)]^* + [b'(i)]^* \cdot \ln VA(i,t+5) + [c'(i)]^* \cdot \ln CAP(i,t+5) + [d'(i)]^* \cdot (\bar{t}'+t+5); \quad i = 1,\ldots,I, \qquad (5)$$

where:

[lna'(i)]* is the estimate of the logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for industry i.

Once the logarithms of the labour input or employment by industry are obtained as indicated by equation (5), employment levels by industry can be obtained by taking the antilogarithms of those results:

$$EM(i,t+5) = \text{antiln}[\ \ln EM(i,t+5)\ ]; \quad i = 1,\ldots,I, \tag{6}$$

where:

antiln is the antilogarithm of the natural logarithm.

This step completes the description of this technique for projecting employment by industry.

(d) <u>Other results</u>

Once the levels of employment by industry are derived for the end of a given projection interval, several useful indicators can be calculated. These indicators include employment aggregates and indicators of the employment structure as well as the rate of change in employment. (Since much of this section is very similar to sections B.2(c) of chapters VI and VII, the reader who is familiar with this material may wish to move directly to the next section.)

(i) <u>Employment aggregates</u>

A key aggregate that can be calculated from the projected levels of employment by industry is the level of total employment. It is also possible to obtain the levels of employment in sectors, such as the primary, secondary and tertiary sectors. Once these levels are obtained for different dates five years apart, one can further calculate increases in total and sectoral employment over the intervening five-year projection intervals.

a. <u>Total employment</u>

Total employment can be obtained by aggregating the levels of employment across industries. For the end of a projection interval (t to t+5) this number is:

$$EM(t+5) = \sum_{i=1}^{I} EM(i,t+5), \quad (7)$$

where:

EM(t+5) is the total employment at the end of the interval.

b. Employment by sector

A variety of criteria can be used to aggregate industries into sectors. Thus, industries could be aggregated into primary, secondary and tertiary sectors or into agricultural, industrial and service sectors. For illustrative purposes, the primary-secondary-tertiary-sector mode of aggregation will be used. In addition, it will be assumed that the numbering of industries for which the levels of employment are being projected lists industries of the primary, secondary and tertiary sectors one after another.

i. Employment in the primary sector

Using these classifications and aggregation rules, employment in the primary sector for the end of the projection interval (t to t+5) can be obtained as:

$$EMP(t+5) = \sum_{i=1}^{Ip} EM(i,t+5), \quad (8)$$

where:

Ip is the number of industries in the primary sector, and

EMP(t+5) is the employment in the primary sector at the end of the interval.

ii. Employment in the secondary sector

Employment in the secondary sector can be obtained as:

$$EMS(t+5) = \sum_{i=Ip+1}^{Ip+Is} EM(i,t+5), \quad (9)$$

where:

Is is the number of industries in the secondary sector, and

EMS(t+5) is the employment in the secondary sector at the end of the interval.

iii. Employment in the tertiary sector

Employment in the tertiary sector can be calculated as:

$$EMT(t+5) = \sum_{i=Ip+Is+1}^{I} EM(i,t+5), \qquad (10)$$

where:

EMT(t+5) is the employment in the tertiary sector at the end of the interval.

c. Growth in total employment

The growth in total employment over the projection interval (t to t+5) equals the difference between total employment at the end and total employment at the beginning of the interval:

$$EMGR = EM(t+5) - EM(t), \qquad (11)$$

where:

EMGR is the growth in total employment during the interval.

d. Growth in employment, by sector

The increase in employment in the primary, secondary and tertiary sectors over the projection interval is respectively obtained as follows:

Growth of employment in the primary sector is calculated as:

$$EMPGR = EMP(t+5) - EMP(t), \qquad (12)$$

Growth of employment in the secondary sector is calculated as:

$$EMSGR = EMS(t+5) - EMS(t), \quad (13)$$

Growth of employment in the tertiary sector is calculated as:

$$EMTGR = EMT(t+5) - EMT(t), \quad (14)$$

where:

EMPGR is the growth of employment in the primary sector during the interval,

EMSGR is the growth of employment in the secondary sector during the interval, and

EMTGR is the growth of employment in the tertiary sector during the interval.

(ii) Indicators of the structure of employment

Once the various employment aggregates are obtained, it is possible to derive the proportions of employment accounted for by each sector (primary, secondary and tertiary).

a. Proportions by sector

The proportions of total employment accounted for by each sector can be obtained as follows:

The proportion of employment in the primary sector is calculated as:

$$PEMP(t+5) = EMP(t+5) / EM(t+5), \quad (15)$$

The proportion of employment in the secondary sector is calculated as:

$$PEMS(t+5) = EMS(t+5) / EM(t+5), \quad (16)$$

The proportion of employment in the tertiary sector is calculated as:

$$PEMT(t+5) = EMT(t+5) / EM(t+5), \quad (17)$$

where:

PEMP(t+5)	is the proportion of employment accounted for by the primary sector at the end of the interval,
PEMS(t+5)	is the proportion of employment accounted for by the secondary sector at the end of the interval, and
PEMT(t+5)	is the proportion of employment accounted for by the tertiary sector at the end of the interval.

(iii) <u>Rates of growth of employment</u>

As part of an employment projection, it is also possible to compute average annual rates of growth in employment, for the total employment and employment in major sectors.

a. <u>Rate of growth in total employment</u>

The average annual rate of growth of total employment for a given projection interval can be computed from the total employment at the beginning and the end of the interval. If, as part of the projection process, the planner makes the assumption that growth occurs over discrete intervals, then the percentage growth rate can be obtained using the formula for calculating a geometric growth rate:

$$GGREM = [(EM(t+5) / EM(t))^{1/5} - 1] \cdot 100, \quad (18)$$

where:

GGREM	is the average annual geometric growth rate of total employment for the interval.

Alternatively, if the planner assumes that growth is continuous, then the percentage growth rate of total employment can be calculated using the formula for calculating an exponential growth rate:

$$EGREM = [(\ln (EM(t+5) / EM(t))) / 5] \cdot 100, \quad (19)$$

where:

EGREM	is the average annual exponential growth rate of employment for the interval.

b. Rates of growth in employment, by sector

Assuming discrete growth, the percentage rates of growth of employment for major sectors can be obtained as follows:

Geometric growth rate for the primary sector is calculated as:

$$GGREMP = [(EMP(t+5) / EMP(t))^{1/5} - 1] \cdot 100, \quad (20)$$

Geometric growth rate for the secondary sector is calculated as:

$$GGREMS = [(EMS(t+5) / EMS(t))^{1/5} - 1] \cdot 100, \quad (21)$$

Geometric growth rate for the tertiary sector is calculated as:

$$GGREMT = [(EMT(t+5) / EMT(t))^{1/5} - 1] \cdot 100, \quad (22)$$

where:

GGREMP is the average annual geometric growth rate of employment in the primary sector for the interval,

GGREMS is the average annual geometric growth rate of employment in the secondary sector for the interval, and

GGREMT is the average annual geometric growth rate of employment in the tertiary sector for the interval.

If the projections were based on the assumption of continuous growth, then the percentage rates of growth of employment by major sector would be calculated using the formula for obtaining the exponential growth rate. The calculations would be as follows:

Exponential growth rate for the primary sector is calculated as:

$$EGREMP = [(\ln (EMP(t+5) / EMP(t))) / 5] \cdot 100, \quad (23)$$

Exponential growth rate for the secondary sector is calculated as:

$$EGREMS = [(\ln (EMS(t+5) / EMS(t))) / 5] \cdot 100, \quad (24)$$

Exponential growth rate for the tertiary sector is calculated as:

$$EGREMT = [(\ln (EMT(t+5) / EMT(t))) / 5] \cdot 100, \tag{25}$$

where:

EGREMP	is the average annual exponential growth rate of employment in the primary sector for the interval,
EGREMS	is the average annual exponential growth rate of employment in the secondary sector for the interval, and
EGREMT	is the average annual exponential growth rate of employment in the tertiary sector for the interval.

(iv) Labour market balances

Once various projection results are obtained, it is possible to calculate the excess demand for labour or excess supply of labour using projections of labour force and employment as indicators of the future supply of, and demand for labour, respectively. Also, it is possible to calculate the excess demand or excess supply as a percentage of the total labour force.

In countries where there is sizeable non-civilian employment, which may include military or internal security personnel, the projected labour force to be used in these calculations should not be the projected total labour force, which can be obtained as described in chapter V. The projected labour force that should be used is the projected civilian labour force obtained as the difference between the projected total labour force and the projected non-civilian employment, where the latter projection is an additional input.

The reason for this is related to the fact that in the projections relating to the labour market, projections of the demand for labour (or employment) will normally be those for the civilian segment of the labour market. Therefore, projections of the supply of labour (or labour force) used to compute excess supply or demand must also be those for this segment of the market.

To calculate excess supply or excess demand, therefore, the civilian labour force may first have to be calculated; this, for the end of the time interval (t to t+5), can be obtained as:

$$CLF(t+5) = LF(t+5) - NEM(t+5), \tag{26}$$

where:

CLF(t+5) is the civilian labour force at the end of the interval,

LF(t+5) is the total labour force at the end of the interval, and

NEM(t+5) is the non-civilian employment at the end of the interval.

The excess supply of (or demand for) labour for the end of the interval can be obtained as the difference between the projected civilian labour force and the projected employment for that date:

$$EXL(t+5) = CLF(t+5) - EM(t+5), \quad (27)$$

where:

EXL(t+5) is the excess supply of labour (if positive) or excess demand for labour (if negative) for the end of the interval.

The excess demand or excess supply as a percentage of the civilian labour supply (civilian labour force) can be calculated as:

$$PEXL(t+5) = [\ EXL(t+5) \ / \ CLF(t+5)\ ] \cdot 100, \quad (28)$$

where:

PEXL(t+5) is the excess supply of labour or excess demand for labour as a percentage of the civilian labour force at the end of the interval.

C. The inputs

This section will first list the inputs used to project employment using inverse Cobb-Douglas production functions or their log-linear transformations and then describe how those inputs can be prepared.

1. Types of inputs required

The following inputs are needed to project employment using inverse Cobb-Douglas production functions or their transformations:

(i) Projected levels of value added by industry;

(ii) Projected levels of the capital stock by industry;

(iii) Estimates of the coefficients of inverse Cobb-Douglas production functions or log-linear transformations of inverse Cobb-Douglas production functions, by industry.

If projections of labour surpluses and/or shortages are also to be prepared, the inputs should include:

(iv) Projected total labour force;

(v) Projected non-civilian employment.

2. Preparation of the inputs

In order to employ the method, the user will need to have projections of value added and the capital stock, by industry. These projections, which are part of many planning exercises can be prepared by means of suitable procedures. A simple procedure for preparing value added projections is outlined in box 17 (chapter VI). A procedure for deriving historical time series of the capital stock from time series of gross investment, which can be used to make projections of the capital stock using projections of investment,is described below (box 31). Projections of the total labour force can be prepared as described in chapter V, while those of non-civilian employment can be obtained by considering likely future developments in the non-civilian sector of the economy. The user of the method must also estimate coefficients of inverse or log-linear inverse Cobb-Douglas production functions by industry. Methods for estimating these coefficients are described below.

(a) Estimates of inverse Cobb-Douglas production functions and their transformations

Estimates of the coefficients of inverse or log-linear inverse Cobb-Douglas production functions, by industry, can be obtained by a standard method of regression analysis such as OLS. Depending on the form of the function, the coefficients can be estimated from time series data, cross section data, or from a combination of the two. No matter what kind of data is used, it should include information on value added and on production inputs, labour and capital.7/

If time series data are to be used exclusively, there should be a minimum of 10 annual observations, but preferably 20 or more. Where time series are in short supply, however, they may have to be used along with cross sectional data. If cross sectional data are to be used alone, they must refer to several points in time in order to produce a suitable estimate of the rate of technical progress.

The following section describes a method which can be used to estimate parameters of the functions using time series data, by industry. In connection with this, we shall first discuss time series data required to estimate the functions. Cross section data, although not demonstrated here, are discussed in box 30.

(i) Time series data

Time series data on value added by industry can usually be obtained from the national accounts. Time series data on labour inputs are typically available in the form of employment data (i.e., number of employees), collected either through periodic labour force surveys of households or through periodic surveys of establishments. In the case of data obtained from establishments, coverage is often insufficient, since such surveys often exclude traditional (or informal) and very small establishments. This raises the possibility that the data on value added, which generally include the traditional sectors, and the data on labour inputs will not refer to the same units. Where the traditional sector has been omitted from the available reports on employment, it would be useful to acquire data on traditional establishments from some other source and use it as a basis for adjusting, so that total (i.e., traditional plus modern) employment can be estimated.

Box 30

Cross section data on employment, output and the capital stock

Cross section data normally come from periodic surveys of establishments, such as manufacturing surveys or farm management surveys. Such data usually include reported levels of output and reported levels of production inputs.

For establishments in non-agricultural industries the data on output normally refer to value added, while for agricultural establishments they typically refer to gross output. Given the information on gross output, levels of value added can be obtained as a difference between gross production and the value of intermediate inputs.

Establishment survey data on inputs typically include the number of employees, as a measure of labour inputs, and frequently also some estimate of the number of days or hours actually worked. Although data on actual labour inputs are conceptually more appealing for the estimation of production functions, use of the former may provide a better basis for projecting employment.

Capital stock data are typically found in one of two forms in enterprise surveys: in the form of reported book value of the firm's plant, equipment and inventory; or, particularly in the case of farm data, in the form of reported numbers and characteristics of various physical capital items (e.g. livestock structures, tractors, electric pumps). Data on physical assets can be combined into an estimate of the current value of the capital stock by assigning each item of physical capital an estimate of its market price and then adding the various items together.

Another possible problem with the available data on labour inputs is that they typically refer to the number of persons employed rather than to labour inputs per se (e.g., man-hours of actual work). The two variables differ to the extent that there is variation in the number of hours worked per employee, owing either to seasonal variation or to the presence of part-time workers. Although such measurement errors would result in inaccurate estimates of the true parameters of the production function, it is less important in the present context since the objective in making employment projections is typically to project the number of workers rather than the number of hours worked.

Time series data on the capital stock by industry are generally not available in the national accounts. Therefore, it is customary to use annual data on gross investment in order to estimate the value of the capital stock over time, by successively depreciating it in each year. To initiate this process, it is necessary to estimate the value of the capital stock in each industry for the initial year of the time period for which the estimation is done. Generally, this value can be estimated by assuming that the average capital-output ratio of the industry during the period is equal to the incremental capital-output ratio. Box 31 describes the principles of the procedure.

Examples of the requisite time series data on employment, value added and the capital stock, by industry, are shown in tables 127-129. These data will be used below to illustrate the estimation of the coefficients of the inverse or log-linear inverse Cobb-Douglas production functions shown in equation (2) with OLS.

(ii) Estimation procedure

If employment is to be projected using inverse Cobb-Douglas production functions, the estimates of the coefficients of those functions can be obtained as follows: take the logarithms of the inverse Cobb-Douglas production functions, shown in equation (2), and obtain the log-linear inverse Cobb-Douglas production functions indicated in equation (3). Then, to each log-linear function, add a random disturbance term to obtain:

$$\ln EM(i,t') = \ln a'(i) + b'(i) \cdot \ln VA(i,t') + c'(i) \cdot \ln CAP(i,t') + d'(i) \cdot t' + u(i,t'); \quad i = 1,\ldots,I. \qquad (29)$$

where:

$u(i,t')$ is the random disturbance term for industry i in year t'.

Box 31

The procedure to derive time series of the capital stock from time series on gross investment

The value of the capital stock over time, by industry, can be estimated by successively depreciating it in each year using a constant rate of depreciation along with annual levels of gross investment by industry. The formula most often used is the following one:

$$CAP(i,t') = CAP(i,t'-1) \cdot [\ 1 - DR(i)\ ] + INV(i,t'); \qquad (1)$$

$$i = 1,\ldots,I,$$

where:

CAP(i,t') is the capital stock in industry i in year t'.

CAP(i,t'-1) is the capital stock in industry i in year t'-1,

DR(i) is the constant annual rate of depreciation of the capital stock in industry i, and

INV(i,t') is gross investment in industry i in year t'.

However, to initiate the process of estimating the capital stock using this formula, it is necessary to have an estimate of the value of the capital stock in each industry for the initial year of the time period for which the estimation is being performed. Most of the time this value itself would have to be estimated, using the assumption that for each industry the incremental capital-output ratio equals the average capital-output ratio during this year. On the basis of this assumption, the capital stock in the initial year of the period in question, by industry, can be obtained as follows:

$$CAP(i,0) = [\ INV(i,1) \cdot VA(i,0)\ ] \ /$$

$$[\ VA(i,1) - VA(i,0) \cdot (\ 1 - DR(i))\ ], \qquad (2)$$

$$i = 1,\ldots,I,$$

(continued)

Box 31 (continued)

where:

CAP(i,0) is the capital stock in the first (initial) year of the time period for which the time series of the capital stock by industry is being estimated,

INV(i,1) is the gross investment in the second (next to the initial) year of the time period for which the time series of the capital stock by industry is being estimated,

VA(i,0) is the value added in the first (initial) year of the time period for which the time series of the capital stock by industry is being estimated, and

VA(i,1) is the value added in the second (next to the initial) year of the time period for which the time series of the capital stock by industry is being estimated.

The above formula for calculating the initial-year capital stock by industry, which is shown in equation (2), can be derived using the formula shown in equation (1) at the beginning of the box and the assumption on the equality of the average and incremental capital-output ratios.

In order to obtain estimates of the capital stock for the initial year, as indicated in equation (2), information on value added, by industry, in that year along with the information on value added and gross investment, by industry, in the subsequent year is needed. In addition, information on depreciation rates, by industry, is also needed.

Moreover, in order to use the formula indicated in equation (2), the assumption on equality of average and incremental capital-output ratios must be approximated. Unfortunately, the data used in the course of estimating the initial-year levels of the capital stock would not frequently enable one to check as to whether or not the assumption is satisfied. Under certain conditions, the data can, however, point to the fact that the assumption is violated. Thus, whenever the value of the term [VA(i,1) - VA(i,0) · (1 - DR(i))] is smaller than or equal to zero for any given industry, one can be sure that the assumption is violated for this industry.

The functions indicated in equation (29) can be estimated by OLS using time series data on employment, value added and the capital stock, such as those shown in tables 127-129. The results would include estimates of the logarithms of the intercept coefficients of the inverse Cobb-Douglas production functions, [lna'(i)]*'s, and estimates of the partial coefficients of the value added, capital stock and the time variable--[b'(i)]*'s, [c'(i)]*'s and [d'(i)]*'s.

While the estimates of the partial coefficients can be used as they are, those of the logarithms of the intercept coefficients must be transformed into estimates of the intercept coefficients themselves. This can be accomplished by taking antilogarithms of the estimates of the logarithms of the intercept coefficients:

$$[a'(i)]^* = \text{antiln} \, [\, [\ln a'(i)]^* \,]; \qquad (30)$$

$$i = 1,\ldots,I.$$

If employment is to be projected using log-linear transformations of inverse Cobb-Douglas production functions, the estimates of the coefficients of those transformations can be directly obtained by estimating functions indicated in equation (29) by OLS using data such as those shown in tables 127-129. The result of this estimation would be estimates of the logarithms of the intercept coefficients of the inverse Cobb-Douglas production functions, [lna'(i)]*'s, and estimates of the partial coefficients of those functions -- [b'(i)]*'s [c'(i)]*'s and [d'(i)]*'s.

(iii) Illustrative estimation

This section will illustrate estimation of the coefficients of the inverse Cobb-Douglas production functions using the time series data presented in tables 127-129. To obtain the estimates of the coefficients of those functions one would use those data with OLS, the result of which would be the estimates presented in table 130. 8/

The results shown in table 130 appear satisfactory inasmuch as the R^2's are relatively high and the estimated coefficients have expected signs. The exceptions are a few positive coefficients of the time variable, most of which are, however, statistically insignificant. In addition, the Durbin-Watson statistics do not suggest autocorrelation.9/

If the estimates of the coefficients are not statistically significant or do not have the expected signs and alternative data are not available, it may be preferable to use the simpler projection techniques discussed in chapters VI and VII.

Table 127. Employment for the entire country, by industry: 1968-1978

(Thousands of employed persons)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
1968	1656.5	5.0	124.5	9.6	57.6	79.3	68.5	312.8
1969	1688.5	5.4	127.7	9.2	53.8	79.7	73.9	330.1
1970	1739.8	6.3	137.8	8.9	55.8	77.4	84.4	336.8
1971	1766.1	7.2	155.4	9.0	59.8	85.4	82.1	365.9
1972	1764.8	7.1	145.5	11.3	64.3	83.9	86.2	375.7
1973	1843.5	7.4	159.8	9.6	69.4	81.1	82.5	391.4
1974	1846.3	9.4	171.8	10.0	76.0	100.5	89.2	442.1
1975	1887.0	8.3	170.3	13.7	71.0	94.1	83.7	468.5
1976	1917.8	10.3	184.0	14.4	81.8	104.9	88.4	483.3
1977	1955.6	9.5	198.2	15.6	85.4	108.5	89.5	503.3
1978	1994.0	6.7	217.3	15.3	93.8	109.4	95.1	516.2

Table 128. Value added for the entire country, by industry: 1968-1978

(Millions of local currency units; constant 1968 prices)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
1968	155.2	2.2	44.6	8.2	18.4	41.2	38.1	119.2
1969	165.4	2.0	48.6	9.1	18.6	44.0	38.6	128.1
1970	172.4	2.6	52.5	9.8	19.0	44.6	41.2	139.0
1971	175.9	2.7	59.3	10.6	20.2	47.1	43.1	152.9
1972	189.3	2.5	63.6	11.6	23.1	42.6	42.4	172.2
1973	199.4	3.6	70.8	11.9	23.8	45.5	45.2	185.7
1974	202.6	3.9	74.9	12.7	22.2	46.4	44.7	207.1
1975	237.1	3.5	75.5	13.7	21.2	49.5	42.0	223.0
1976	235.2	3.8	89.6	15.4	20.7	51.8	46.4	237.5
1977	256.5	4.0	103.9	16.5	22.3	50.5	47.7	252.6
1978	260.3	2.6	118.9	18.0	23.5	50.9	50.3	266.9

Table 129. Capital stock for the entire country, by industry: 1968-1978

(Millions of local currency units; constant 1968 prices)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
1968	15.1	17.4	27.2	7.8	19.1	39.4	102.6	8.2
1969	16.1	14.1	28.8	8.5	18.3	42.1	93.5	8.6
1970	16.2	18.2	33.5	9.0	17.8	42.7	88.0	9.4
1971	16.7	17.7	34.5	9.6	17.1	40.6	85.5	9.8
1972	18.6	15.3	39.1	10.2	17.9	35.7	78.4	10.8
1973	19.2	23.5	39.2	11.1	17.0	35.3	74.4	11.4
1974	19.8	22.9	39.3	11.3	15.7	30.2	65.5	11.9
1975	25.4	21.5	42.1	11.2	13.4	32.6	58.4	12.1
1976	24.8	20.3	45.5	12.0	12.8	31.2	55.8	12.4
1977	28.2	20.8	52.5	12.9	10.9	29.3	53.1	12.8
1978	28.0	21.0	58.7	14.1	9.4	27.3	50.3	13.1

Table 130. Estimates of the coefficients of log-linear inverse Cobb-Douglas production functions, by industry: entire country a/

Industry	Intercept	Coefficients b/ Value added	Capital stock	Time variable	R-square	Durbin-Watson
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Agriculture	-14.21215	0.45694 (2.180)	-0.24581 (-2.353)	0.010 (1.939)	0.992	2.77
Mining	-37.48326	1.07969 (7.602)	-0.73550 (-3.585)	0.021 (2.624)	0.956	2.74
Manufacturing	-11.05266	0.63665 (2.133)	-0.20063 (-0.753)	0.007 (0.337)	0.977	2.24
Utilities	-112.70138	2.12628 (1.938)	-2.96373 (-3.775)	0.059 (0.695)	0.931	1.79
Construction	-66.71019	0.24704 (0.732)	-0.19959 (-0.807)	0.036 (1.640)	0.940	2.22
Trade	62.82720	1.24755 (3.128)	-0.96450 (-3.784)	-0.030 (-1.703)	0.952	2.45
Transport	124.92383	1.24437 (1.405)	-0.76079 (-0.710)	-0.062 (-0.631)	0.813	1.82
Services	57.02152	1.47648 (4.730)	-0.87672 (-4.019)	-0.029 (-1.530)	0.995	1.92

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

The estimates of partial coefficients (such as those presented in table 130) can be used as they are to project employment using inverse Cobb-Douglas production functions. The estimates of the logarithms of the intercept coefficients need, however, to be transformed into estimates of the intercept coefficients themselves. As illustrated in table 131, the latter estimates (column 3) can be obtained by taking antilogarithms of the estimates of the logarithms of the intercept coefficients (column 2). Thus, the estimate of the intercept coefficient for agriculture, 0.000000673, can be calculated as follows:

$$0.000000673 = \text{antiln}(-14.21214) \tag{30}$$

where -14.21214 is the estimate of the logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for agriculture.

If estimates of the log-linear transformations of inverse Cobb-Douglas production functions are to be used to prepare employment projections, those estimates can be obtained directly by estimating functions shown in equation (29). Estimates of such functions based on the time series data presented in tables 127-129 are those shown in table 130 and discussed above.

(b) Calibration of the empirically estimated functions

After obtaining satisfactory estimates of the inverse production functions, the planner will sometimes desire to make special adjustments in the estimated coefficients. These adjustments, which are normally referred to as "calibration", are designed to make the functions better predict the levels of employment for a particular year, or group of years, of the time period to which the data used pertain, given the levels of value added and the capital stock for that year or group of years. (If left unadjusted, the functions predict the mean level of employment over the entire time period to which data refer, using the average levels of value added and the capital stock for the period).

Although the adjustments may apply to the estimates of the intercepts as well as to those of the partial coefficients, they will be most often restricted to the intercept estimates. A calibration procedure applying to intercepts of inverse and log-linear inverse Cobb-Douglas production functions is described in annex II.

D. Illustrative example of projections

The example presented below will illustrate the use of the method based on employment functions which are log-linear transformations of inverse Cobb-Douglas production functions to prepare a national projection of employment. The example will show how the relevant calculations are made for the projection interval 0-5 and will also provide complete projection results for a 20-year projection period.

Table 131. Computing estimates of intercept coefficients of inverse Cobb-Douglas production functions, by industry

Industry (1)	Intercept of log-linear inverse Cobb-Douglas production function a/ (2)	Intercept of non-linear inverse Cobb-Douglas production function b/ (3)
Agriculture	-14.21214	6.7260×10^{-7}
Mining	-37.48326	5.2629×10^{-17}
Manufacturing	-11.05265	1.5845×10^{-5}
Utilities	-112.70137	1.1335×10^{-49}
Construction	-66.71018	1.0669×10^{-29}
Trade	62.82719	1.9298×10^{27}
Transport	124.92382	1.7936×10^{54}
Services	57.02152	5.8094×10^{24}

a/ From table 130, col. 2.
b/ Antiln(Col. 2).

The calculations presented in the example will be based on the inputs contained in table 132, which shows projected levels of value added and the capital stock, by industry, for dates five years apart, starting with the initial year of the plan, which is denoted as year 0. Table 132 also shows estimates of the partial coefficients of the log-linear transformations of inverse Cobb-Douglas production functions, presented above in table 130, along with adjusted logarithms of the intercept coefficients shown in table 138 in annex II.

(a) Employment by sector

To obtain the levels of employment by industry for a given date, it is necessary to evaluate estimated log-linear transformations of inverse Cobb-Douglas production functions, by industry, using the levels of value added and the capital stock along with the appropriate value of the time variable. This would result in the logarithms of the projected levels of employment. To obtain the levels of employment themselves, it would be necessary to take antilogarithms of those results. Table 133 illustrates how the levels of employment by industry for the end of the projection interval 0-5 are calculated.10/

In particular, the logarithm of the level of employment for each industry in year 5 (column 9), is obtained as the adjusted intercept coefficient (column 2) plus the sum of three products. The first product is that of the estimate of the value added coefficient (column 3) and the logarithm of the projected level of value added (column 6). The second product is that of the estimate of the capital stock coefficient and the logarithm of the projected level of the capital stock (column 7). The third product is that of the time variable coefficient (column 5) and the value of the time variable (column 8). The level of employment in any industry (column 10) is then calculated as the antilogarithm of the logarithm of the employment level.

For example, the logarithm of the level of employment in agriculture in year 5, 7.65723, is obtained as follows:

$$7.65723 = -14.20909 + 0.45694 \cdot \ln 308.2 + (-0.24582) \cdot \ln 40.2 + 0.01015 \cdot (1980 + 5), \qquad (5)$$

where -14.20909 is the adjusted intercept coefficient for agriculture and where 0.45694 and 308.2 are, respectively, the estimate of the coefficient of the value added variable and the projected level of value added in agriculture in year 5. The estimate of the coefficient of the capital stock variable is -0.24582, and the projected level of the capital stock in this industry is 40.2. Assuming that 1980 is the initial year of the projection, the estimate of the time variable coefficient, 0.01015, is multiplied by 1985 (= 1980 + 5), the value of the time variable.

Table 132. Inputs for projecting employment, by industry: entire country

Industry	Year 0	5	10	15	20
	(Millions of local currency units)				
	Value added				
Agriculture	273.1	308.2	347.8	392.5	443.1
Mining	2.9	4.0	5.5	7.6	10.4
Manufacturing	140.4	212.7	322.5	489.3	742.7
Utilities	21.2	31.9	48.0	72.5	109.4
Construction	25.6	31.5	38.9	48.0	59.3
Trade	59.0	85.1	122.9	177.6	256.7
Transport	56.0	73.0	95.3	124.4	162.5
Services	312.5	464.3	691.2	1031.2	1541.4
	Capital stock				
Agriculture	31.0	40.2	52.0	67.4	87.2
Mining	22.9	28.4	35.2	43.7	54.2
Manufacturing	70.9	113.7	182.2	292.1	468.4
Utilities	16.1	22.6	31.8	44.5	62.5
Construction	10.4	13.5	17.5	22.6	29.3
Trade	29.9	37.7	47.4	59.6	75.0
Transport	60.8	97.4	156.1	250.3	401.3
Services	14.3	17.7	22.0	27.2	33.8

Estimates of the coefficients of log-linear inverse Cobb-Douglas production functions

	Adjusted intercept	Value added	Capital stock	Time variable
Agriculture	-14.20909	0.45694	-0.246	0.010
Mining	-37.46840	1.07969	-0.736	0.021
Manufacturing	-11.04406	0.63665	-0.201	0.007
Utilities	-112.66345	2.12628	-2.964	0.059
Construction	-66.69979	0.24704	-0.200	0.036
Trade	62.82550	1.24755	-0.965	-0.030
Transport	124.91470	1.24437	-0.761	-0.062
Services	57.01587	1.47648	-0.877	-0.029

Table 133. Deriving employment, by industry: entire country, year 5

Industry	Estimates of coefficients of log-linear inverse Cobb-Douglas production functions a/				In year 5				
	Adjusted intercept	Value added	Capital stock	Time variable	Value added a/	Capital stock a/	Value of time variable b/	Logarithm of projected employment c/	Projected employment d/ (thousands of persons)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Agriculture	-14.20909	0.45694	-0.24581	0.010	308.2	40.2	1985	7.657	2115.9
Mining	-37.46840	1.07969	-0.73550	0.021	4.0	28.4	1985	2.298	10.0
Manufacturing	-11.04406	0.63665	-0.20063	0.007	212.7	113.7	1985	5.669	289.8
Utilities	-112.66345	2.12628	-2.96373	0.059	31.9	22.6	1985	2.954	19.2
Construction	-66.69979	0.24704	-0.19959	0.036	31.5	13.5	1985	4.793	120.6
Trade	62.82550	1.24755	-0.96450	-0.030	85.1	37.7	1985	4.815	123.3
Transport	124.91470	1.24437	-0.76079	-0.062	73.0	97.4	1985	4.084	59.4
Services	57.01587	1.47648	-0.87672	-0.029	464.3	17.7	1985	6.599	734.2

a/ From table 132.
b/ Based on assumption that 1980 is the initial year of the projection period.
c/ (Col. 2) + (Col. 3) . (ln(Col. 6)) + (Col. 4) . (ln(Col. 7) + (Col. 5) . (Col. 8).
d/ Antiln(Col. 9).

Given the logarithm of the level of employment in agriculture in year 5, the level of employment itself, 2,115.9, is obtained by taking the antilogarithm of this result:

$$2,115.9 = \text{antiln}\ (7.65723). \qquad (6)$$

Performing the calculations illustrated for the end of the interval 0-5 for each five-year interval of the entire projection period produces the projected levels of employment by industry for the entire period. The projected levels for the 20-year projection interval are shown in table 134.

(b) Other results

Other results that are useful in planning can be obtained as part of a projection at the national level. These include various employment aggregates, indicators of the structure of employment and the rates of growth of employment.11/

(i) Employment aggregates

The employment aggregates, which can be derived from the projection by industry, include total employment and employment in various sectors at dates five years apart. They also include increases in total employment and employment by sector over the intervening projection intervals.

a. Total employment

Total employment at the end of a given projection interval is obtained by aggregating the projected levels of employment by industry. Total employment in year 5, 3,472.3, is computed by adding the projected levels of employment by industry. Total employment is shown in table 135 for the entire 20-year projection period. The increase in total employment over this period is indicated in figure XXVIII.

b. Employment by sector

Employment in the primary, secondary and the tertiary sectors can be obtained by aggregating employment projected for various industries, using appropriate aggregation rules. For illustrative purposes, it will be assumed that the primary sector consists of agriculture and mining, the secondary sector of manufacturing, utilities and construction, and the tertiary sector of trade, transportation and services.

Table 134. Projected employment, by industry: entire country

(Thousands of persons)

Industry	Year 0	5	10	15	20
Agriculture	2027.8	2115.9	2207.9	2303.9	2404.3
Mining	7.5	10.0	13.2	17.5	23.3
Manufacturing	235.9	289.8	356.2	437.9	538.6
Utilities	16.3	19.2	22.6	26.7	31.7
Construction	100.8	120.6	144.3	172.8	206.8
Trade	113.2	123.3	134.4	146.5	159.7
Transport	83.1	59.4	42.4	30.3	21.6
Services	570.5	734.2	947.8	1227.3	1593.6

Table 135. Employment aggregates, structure and rates of growth: entire country

	Year								
	0		5		10		15		20
Employment aggregates (thousands of persons)									
Levels of employment									
Total	3155.1		3472.3		3868.8		4362.9		4979.6
Primary	2035.3		2125.8		2221.1		2321.5		2427.6
Secondary	353.0		429.6		523.2		637.4		777.0
Tertiary	766.8		916.8		1124.6		1404.0		1774.9
Growth in employment									
Total		317.2		396.5		494.1		616.6	
Primary		90.5		95.2		100.4		106.1	
Secondary		76.6		93.5		114.3		139.6	
Tertiary		150.0		207.7		279.5		370.9	
Indicators of employment structure									
Proportions of total employment by sector									
Primary	0.65		0.61		0.57		0.53		0.49
Secondary	0.11		0.12		0.14		0.15		0.16
Tertiary	0.24		0.26		0.29		0.32		0.36
Rates of growth of employment (percentage)									
Total		1.93		2.19		2.43		2.68	
Primary		0.87		0.88		0.89		0.90	
Secondary		4.01		4.02		4.03		4.04	
Tertiary		3.64		4.17		4.54		4.80	

Figure XXVIII. Total employment

i. <u>Employment in the primary sector</u>

Employment in the primary sector in year 5, 2,125.8, is obtained as:

2,125.8 = 2,115.9 + 10.0, (8)

where 2,115.9 and 10.0 are, respectively, projected levels of employment in agriculture and mining.

ii. <u>Employment in the secondary sector</u>

Employment in the secondary sector in year 5, 429.6, is obtained as:

429.6 = 289.8 + 19.2 + 120.6, (9)

where 289.8, 19.2 and 120.6 are, respectively, projected levels of employment in manufacturing, utilities and construction.

iii. <u>Employment in the tertiary sector</u>

Employment in the tertiary sector in year 5, 916.8, is obtained as:

916.8 = 123.3 + 59.4 + 734.2, (10)

where 123.3, 59.4 and 734.2 are, respectively, projected levels of employment in trade, transportation and services.

Employment by sector obtained for different dates over the projection period is presented in figure XXIX.

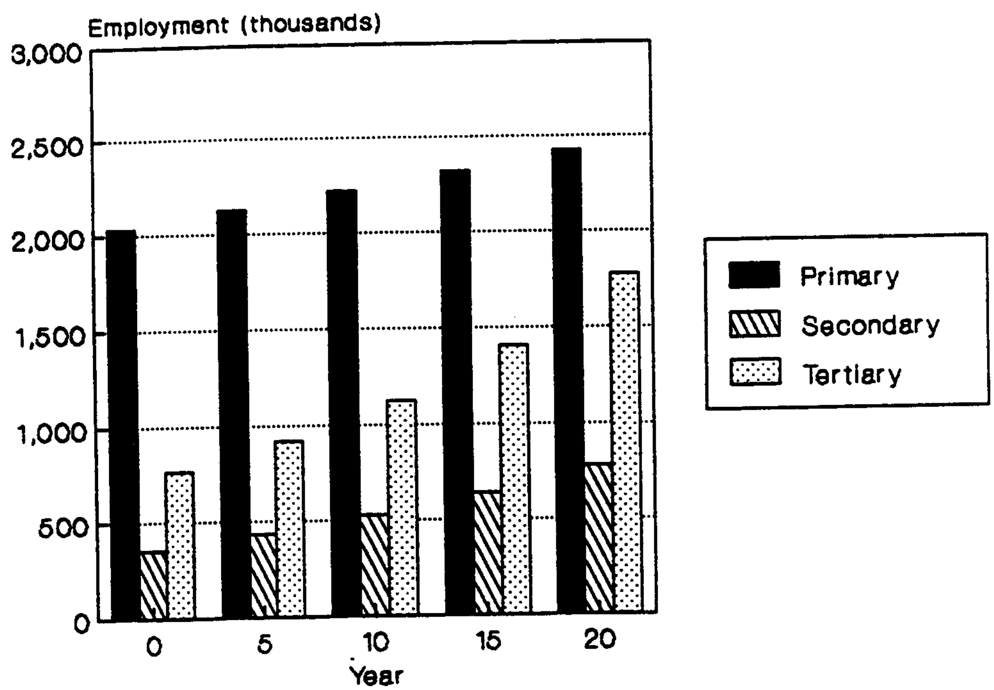
c. <u>Growth in total employment</u>

The growth in total employment over a given projection interval equals the difference between total employment at the end of the interval and total employment at its beginning. For the interval 0-5, the growth in total employment, 317.2, is obtained as:

317.2 = 3,472.3 - 3,155.1, (11)

where 3,155.1 and 3,472.3 are, respectively, total employment at the beginning and the end of the interval (shown in columns corresponding to years 0 and 5 respectively).

Figure XXIX. Employment: primary, secondary and tertiary sectors



d. Growth in employment, by sector

The increase in employment over the interval 0-5 in various sectors is obtained as follows:

Growth of employment in the primary sector, 90.5, is:

$$90.5 = 2{,}125.8 - 2{,}035.3, \qquad (12)$$

where 2,035.3 and 2,125.8 are, respectively, total employment in the primary sector in years 0 and 5;

Growth of employment in the secondary sector, 76.6, is:

$$76.6 = 429.6 - 353.0, \qquad (13)$$

where 353.0 and 429.6 are the levels of employment in the secondary sector in years 0 and 5; and

Growth of employment in the tertiary sector, 150.0, is:

$$150.0 = 916.8 - 766.8, \qquad (14)$$

where 766.8 and 916.8 are total employment in the tertiary sector in years 0 and 5.

(ii) Indicators of the structure of employment

Indicators of the structure of employment that can be calculated as part of an employment projection include proportions of total employment found in each sector.

a. Proportions by sector

For the end of the interval 0-5, these proportions are obtained as follows:

The proportion of employment in the primary sector, 0.61, is:

$$0.61 = 2{,}125.8 \;/\; 3{,}472.3, \qquad (15)$$

where 2,125.8 and 3,472.3 are, respectively, employment in the primary sector and the total employment;

The proportion of employment in the secondary sector, 0.12, is:

$$0.12 = 429.6 / 3{,}472.3, \tag{16}$$

where 429.6 is employment in the secondary sector;

The proportion of employment in the tertiary sector, 0.26, is:

$$0.26 = 916.8 / 3{,}472.3, \tag{17}$$

where 916.8 is employment in the tertiary sector.

(iii) Rates of growth of employment

The rates of growth of employment can be calculated for total employment and for employment in each sector.

a. Rate of growth in total employment

If growth in employment is assumed to occur over discrete intervals, the average annual growth rate of total employment for a given interval is obtained using the geometric growth rate formula. For the projection interval 0-5, this annual growth rate, 1.93 per cent (table 134), is obtained as follows:

$$1.93 = [\ (3{,}472.3 / 3{,}155.1)^{1/5} - 1\ ] \cdot 100, \tag{18}$$

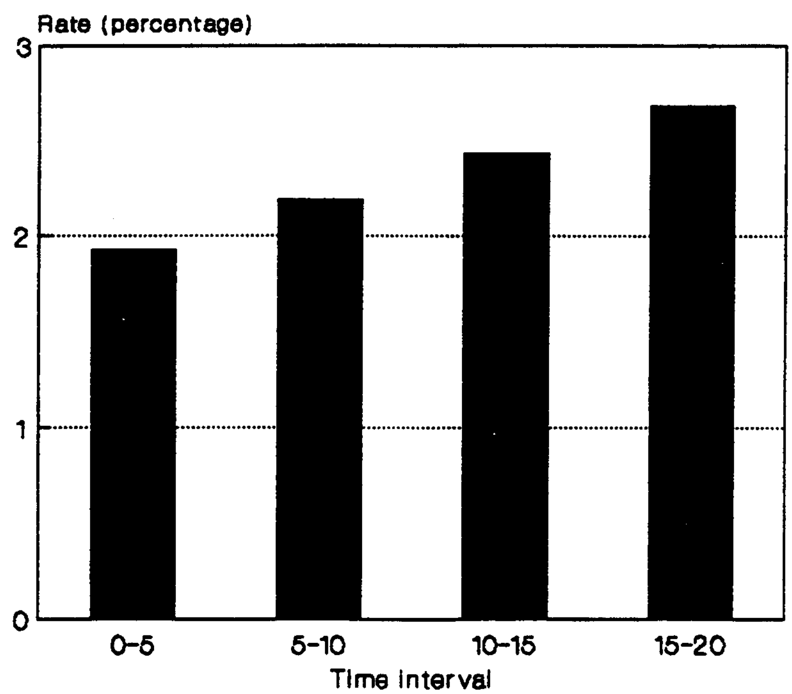
where 3,155.1 and 3,472.3 are the levels of total employment in years 0 and 5, respectively, and 5 is the length of the interval.

Rates of growth of total employment over the 20-year projection period which were computed using the geometric rate formula are shown in figure XXX.

If it is assumed that growth in employment is continuous, the average annual growth rate of total employment for a given interval is obtained by substituting the same data as above in the exponential growth rate formula. For the projection interval 0-5, this annual growth rate, 1.92 per cent, is obtained as follows:

$$1.92 = [\ \ln (3{,}472.3 / 3{,}155.1) / 5\ ] \cdot 100, \tag{19}$$

Figure XXX. Rate of growth in total employment



b. Rates of growth in employment, by sector

Assuming discrete growth, the rates of increase in employment, by sector, for the interval 0-5 are calculated as:

The annual rate of growth of employment in the primary sector, 0.87 per cent, is obtained as:

$$0.87 = [\ (2{,}125.8\ /\ 2{,}035.3)^{1/5} - 1\ ] \cdot 100, \tag{20}$$

where 2,035.3 and 2,125.8 are the levels of employment in the primary sector in years 0 and 5, respectively;

The annual rate of growth of employment in the secondary sector, 4.01 per cent, is obtained as follows:

$$4.01 = [\ (429.6\ /\ 353.0)^{1/5} - 1\ ] \cdot 100, \tag{21}$$

where 353.0 and 429.6 are the levels of employment in the secondary sector in years 0 and 5, respectively;

The annual rate of growth of employment in the tertiary sector, 3.64 per cent, is obtained as follows:

$$3.64 = [\ (916.8\ /\ 766.8)^{1/5} - 1\ ] \cdot 100, \tag{22}$$

where 766.8 and 916.8 are the levels of employment in the tertiary sector in years 0 and 5, respectively.

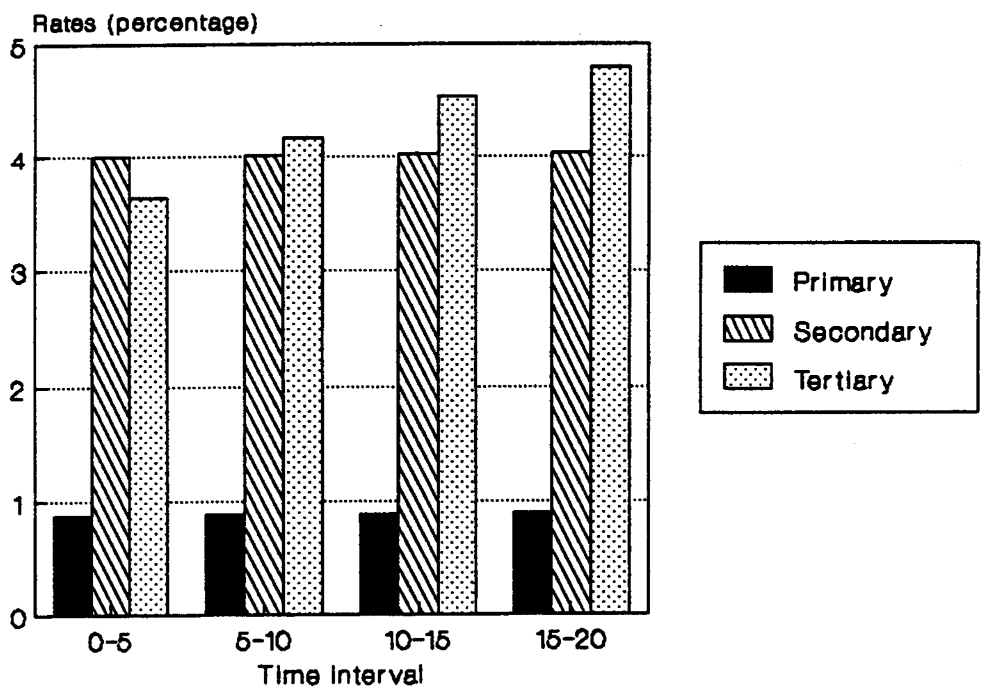
Rates of growth of employment in primary, secondary and tertiary sectors over the 20-year projection interval are shown in figure XXXI.

If continuous growth is assumed, rates of employment by sector would be calculated using the exponential growth rate formula. The calculations would be analogous to that indicated by equation (19) for total employment.

(iv) Labour market balances

If labour force projections are available for the same years as the employment projection, it is possible to calculate the levels of excess demand for or excess supply of labour. Also, it is possible to calculate the excess demand for or excess supply of labour as a percentage of the level of labour supply.

Figure XXXI. Rates of growth of employment: primary, secondary and tertiary sectors



The calculations are based on the projected total labour force, diminished where necessary by the size of non-civilian employment, and the projected total employment as indicators of the labour supply and the labour demand.

In order to illustrate these calculations, we shall use projections of the total labour force and the total employment (shown, respectively, in tables 37 and 135) along with the illustrative projections of non-civilian employment, which are shown in table 136. These calculations are illustrated for the end of the projection interval 0-5 in table 137.

The civilian labour force in year 5, 3,615.4, can be calculated as follows:

$$3{,}615.4 = 3{,}651.9 - 36.5, \tag{26}$$

where 3,651.9 and 36.5 are the projected total labour force and the projected non-civilian employment for year 5, shown in columns 2 and 3. The calculated level of civilian labour force for year 5 is shown in column 4.

The excess supply of labour for the same date, 143.1, is calculated as follows:

$$143.1 = 3{,}615.4 - 3{,}472.3, \tag{27}$$

where 3,615.4 is the civilian labour force and 3,472.3 is the total employment shown in columns 4 and 5, respectively.

The excess supply expressed as a percentage of the civilian labour force in year 5, 3.96 per cent, is calculated as follows:

$$3.96 = (143.1 \ / \ 3{,}615.4) \cdot 100. \tag{28}$$

This percentage is shown in column 7.

E. Summary

This chapter has described the method which uses empirically estimated inverse Cobb-Douglas production functions or their log-linear transformations, by industry, to make employment projections for the entire country. As part of the description of the method, the procedure utilising these functions to make projections by industry was presented. In addition, the types of inputs required by the method were described and the preparation of the inputs was discussed. Lastly, an example of a projection was described, including the various outputs that can be generated. A complete listing of the outputs that the method is capable of producing is shown in box 32.

Table 136. Projected non-civilian employment: entire country

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	32.5
5	36.5
10	41.5
15	47.4
20	53.8

Table 137. Labour market balances: entire country

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	3251.5	32.5	3219.0	3155.1	63.9	1.98
5	3651.9	36.5	3615.4	3472.3	143.1	3.96
10	4147.8	41.5	4106.3	3868.8	237.5	5.78
15	4735.4	47.4	4688.0	4362.9	325.1	6.93
20	5377.4	53.8	5323.6	4979.6	344.0	6.46

a/ From table 37, "Labour force (Total)".
b/ From table 136.
c/ (Col. 2) - (Col. 3).
d/ From table 135, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Box 32

Outputs of the method for making employment projections using inverse Cobb-Douglas production functions or their transformations

1. Employment by industry

2. Employment aggregates

Levels of employment:

Total

Primary sector
Secondary sector
Tertiary sector

Growth in employment:

Total

Primary sector
Secondary sector
Tertiary sector

3. Indicators of the structure of employment

Proportions of employment by sector:

Primary sector
Secondary sector
Tertiary sector

4. Rates of growth of employment

Total

Primary sector
Secondary sector
Tertiary sector

5. Labour market balances

Excess supply of or excess demand for labour

Percentage excess supply of or excess demand for labour

F. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

$i = 1,\ldots,I$	are the industries of the nation's economy
t	is the year of the projection period
t'	is the calendar year
$\bar{t}'$	is the calendar year designated as the initial year of the projection

(b) List of variables

CAP(i,t')	is the capital stock in industry i in year t'
CAP(i,t+5)	is the capital stock in industry i at the end of the interval
CLF(t+5)	is the civilian labour force at the end of the interval
DR(i)	is the constant annual rate of depreciation of the capital stock in industry i
EGREM	is the average annual exponential growth rate of employment for the interval
EGREMP	is the average annual exponential growth rate of employment in the primary sector for the interval
EGREMS	is the average annual exponential growth rate of employment in the secondary sector for the interval
EGREMT	is the average annual exponential growth rate of employment in the tertiary sector for the interval
EM(i,t')	is the labour employed in industry i in year t'
EM(i,t+5)	is the labour employed in industry i at the end of the interval
EM(t+5)	is the total employment at the end of the interval
EMGR	is the growth in total employment during the interval

EMP(t+5)	is the employment in the primary sector at the end of the interval
EMPGR	is the growth of employment in the primary sector during the interval
EMS(t+5)	is the employment in the secondary sector at the end of the interval
EMSGR	is the growth of employment in the secondary sector during the interval
EMT(t+5)	is the employment in the tertiary sector at the end of the interval
EMTGR	is the growth of employment in the tertiary sector during the interval
EXL(t+5)	is the excess supply of labour (if positive) or excess demand for labour (if negative) for the end of the interval
GGREM	is the average annual geometric growth rate of total employment for the interval
GGREMP	is the average annual geometric growth rate of employment in the primary sector for the interval
GGREMS	is the average annual geometric growth rate of employment in the secondary sector for the interval
GGREMT	is the average annual geometric growth rate of employment in the tertiary sector for the interval
INV(i,t')	is gross investment in industry i in year t'
LF(t+5)	is the labour force size at the end of the interval
NEM(t+5)	is the non-civilian employment at the end of the interval
PEMP(t+5)	is the proportion of employment accounted for by the primary sector at the end of the interval
PEMS(t+5)	is the proportion of employment accounted for by the secondary sector at the end of the interval
PEMT(t+5)	is the proportion of employment accounted for by the tertiary sector at the end of the interval

PEXL(t+5)	is the excess supply of labour or excess demand for labour as a percentage of the total labour force at the end of the interval
VA(i,t')	is the value added in industry i in year t'
VA(i,t+5)	is the value added in industry i at the end of the interval

(c) List of special symbols

a'(i)	is the intercept parameter for industry i
a(i)	is the elasticity parameter relating value added to the capital stock for industry i
antiln	is the antilogarithm of the natural logarithm
b'(i)	is the elasticity parameter relating labour to value added for industry i
b(i)	is the elasticity parameter relating value added to labour for industry i
c'(i)	is the elasticity parameter relating labour to the capital stock for industry i
d'(i)	is the parameter relating labour to the time variable for industry i
e	is the base of the natural logarithm
I	is the number of industries
Ip	is the number of industries in the primary sector
Is	is the number of industries in the secondary sector
ln	is the natural logarithm
r(i)	is the constant rate of technical change for industry i
u(i,t')	is the random disturbance term for industry i in year t'
z(i)	is the intercept parameter for industry i

[a'(i)]*	is the estimate of the intercept coefficient of the inverse Cobb-Douglas production function for industry i
[b'(i)]*	is the estimate of the partial coefficient of the value added variable in the inverse Cobb-Douglas production function for industry i
[c'(i)]*	is the estimate of the partial coefficient of the capital stock variable in the inverse Cobb-Douglas production function for industry i
[d'(i)]*	is the estimate of the partial coefficient of the time variable in the inverse Cobb-Douglas production function for industry i
[lna'(i)]*	is the estimate of the logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for industry i

2. Equations

A. Description

(a) Cobb-Douglas production functions

$$VA(i,t') = z(i) \cdot CAP(i,t')^{a(i)} \cdot EM(i,t')^{b(i)} \cdot e^{[r(i) \cdot t']}; \qquad (1)$$

$$i = 1,\ldots,I$$

(b) Inverse Cobb-Douglas production functions

$$EM(i,t') = a'(i) \cdot VA(i,t')^{b'(i)} \cdot CAP(i,t')^{c'(i)} \cdot e^{[d'(i) \cdot t']}; \qquad (2)$$

$$i = 1,\ldots,I,$$

$$\ln EM(i,t') = \ln a'(i) + b'(i) \cdot \ln VA(i,t') + c'(i) \cdot \ln CAP(i,t') \qquad (3)$$

$$+ d'(i) \cdot t';$$

$$i = 1,\ldots,I,$$

(c) Employment by industry

$$EM(i,t+5) = [a'(i)]^{*} \cdot VA(i,t+5)^{[b'(i)]^{*}} \cdot CAP(i,t+5)^{[c'(i)]^{*}} \cdot e[\,[d'(i)]^{*} \cdot (\bar{t}'+t+5)];\quad i = 1,\ldots,I, \tag{4}$$

$$\ln EM(i,t+5) = [\ln a'(i)]^{*} + [b'(i)]^{*} \cdot \ln VA(i,t+5) + [c'(i)]^{*} \cdot \ln CAP(i,t+5) + [d'(i)]^{*} \cdot (\bar{t}'+t+5);\quad i = 1,\ldots,I, \tag{5}$$

$$EM(i,t+5) = \text{antiln}[\,\ln EM(i,t+5)\,];\quad i = 1,\ldots,I, \tag{6}$$

(d) Other results

(i) Employment aggregates

a. Total employment

$$EM(t+5) = \sum_{i=1}^{I} EM(i,t+5) \tag{7}$$

b. Employment by sector

i. Employment in the primary sector

$$EMP(t+5) = \sum_{i=1}^{Ip} EM(i,t+5) \tag{8}$$

ii. Employment in the secondary sector

$$EMS(t+5) = \sum_{i=Ip+1}^{Ip+Is} EM(i,t+5) \tag{9}$$

iii. Employment in the tertiary sector

$$EMT(t+5) = \sum_{i=Ip+Is+1}^{I} EM(i,t+5) \qquad (10)$$

c. Growth in total employment

$$EMGR = EM(t+5) - EM(t) \qquad (11)$$

d. Growth in employment, by sector

$$EMPGR = EMP(t+5) - EMP(t) \qquad (12)$$

$$EMSGR = EMS(t+5) - EMS(t) \qquad (13)$$

$$EMTGR = EMT(t+5) - EMT(t) \qquad (14)$$

(ii) Indicators of the structure of employment

a. Proportions by sector

$$PEMP(t+5) = EMP(t+5) / EM(t+5) \qquad (15)$$

$$PEMS(t+5) = EMS(t+5) / EM(t+5) \qquad (16)$$

$$PEMT(t+5) = EMT(t+5) / EM(t+5) \qquad (17)$$

(iii) Rates of growth of employment

a. Rate of growth in total employment

$$GGREM = [(EM(t+5) / EM(t))^{1/5} - 1] \cdot 100, \qquad (18)$$

$$EGREM = [(\ln (EM(t+5) / EM(t))) / 5] \cdot 100, \qquad (19)$$

b. Rates of growth in employment by sector

$$GGREMP = [(EMP(t+5) / EMP(t))^{1/5} - 1] \cdot 100, \quad (20)$$

$$GGREMS = [(EMS(t+5) / EMS(t))^{1/5} - 1] \cdot 100, \quad (21)$$

$$GGREMT = [(EMT(t+5) / EMT(t))^{1/5} - 1] \cdot 100, \quad (22)$$

$$EGREMP = [(\ln (EMP(t+5) / EMP(t))) / 5] \cdot 100, \quad (23)$$

$$EGREMS = [(\ln (EMS(t+5) / EMS(t))) / 5] \cdot 100, \quad (24)$$

$$EGREMT = [(\ln (EMT(t+5) / EMT(t))) / 5] \cdot 100, \quad (25)$$

(iv) Labour market balances

$$CLF(t+5) = LF(t+5) - NEM(t+5), \quad (26)$$

$$EXL(t+5) = CLF(t+5) - EM(t+5), \quad (27)$$

$$PEXL(t+5) = [EXL(t+5) / CLF(t+5)] \cdot 100 \quad (28)$$

B. The inputs

1. Types of inputs required

2. Preparation of the inputs

(a) Estimates of inverse Cobb-Douglas production functions and their transformations

(i) Time series data

(ii) Estimation procedure

$$\ln EM(i,t') = \ln a'(i) + b'(i) \cdot \ln VA(i,t') + c'(i) \cdot \ln CAP(i,t') + d'(i) \cdot t' + u(i,t'); \quad i = 1,\ldots,I, \tag{29}$$

$$[a'(i)]^* = \text{antiln}\, [\, [\ln a'(i)]^* \,]; \quad i = 1,\ldots,I \tag{30}$$

$$a'(i) = z(i)^{-1/b(i)},$$

$$b'(i) = 1/b(i),$$

$$c'(i) = -a(i)/b(i),$$

and

$$d'(i) = -r(i)/b(i).$$

Hence, the parameters of the employment function indicated in equation (2) could be derived directly from the parameters of the Cobb-Douglas production function shown in equation (1).

Notes

1/ Annex I describes various forms of the Cobb-Douglas production function and their properties.

2/ Throughout the chapter, "value added" and "capital stock" will refer, respectively, to value added and the capital stock measured in constant prices.

3/ However, it assumes that the elasticity of substitution between production factors is equal to 1, which may not necessarily be the case in any given industry.

4/ This description of the method assumes that the only two relevant inputs of production are labour and capital.

5/ The functions shown in equation (1) were obtained by adding the industry dimension to the Cobb-Douglas production function shown in equation (2) in annex I.

6/ The parameters of employment functions indicated in equation (2) are related to the parameters of the Cobb-Douglas production functions shown in equation (1) as follows:

7/ Value added is most often used as the measure of output in production function analysis, although it is fairly common to use gross output as the dependent variable in agricultural production functions (in which case, purchased inputs, such as seed and fertilizer, are treated as inputs).

8/ To obtain the results shown in table 130, time series data presented in table 127 through 129 were subject to a logarithmic transformation. Values assigned to the time variable were 1968 through 1978.

9/ For the discussion of autocorrelation and the use of Durbin Watson statistics, see chapter VII, section C.

10/ The log-linear transformations of inverse Cobb-Douglas production functions used in this example were estimated, among other things, from the time series on employment shown in table 127, which are expressed in units of 1,000 employed persons. Therefore, the levels of employment in this illustrative example will be given in thousands of employed persons.

11/ Much of this section is similar to section D.1(c) in chapters VI or chapter VII. The reader who is familiar with that material may wish to move directly to the next section.

Annex I

THE COBB-DOUGLAS PRODUCTION FUNCTION

The Cobb-Douglas production function, which is by far the most widely used form of production function, has been applied in various planning exercises, especially in a wide range of planning models.a/ The function, which was formulated by Cobb and Douglas in an attempt to explain the relative constancy of the shares of capital and labour in national income, is widely used in theories of income distribution, production and economic growth.

The Cobb-Douglas function may assume a number of different specifications, several of which will be introduced and discussed below. Irrespective of the specification, however, the function has a number of properties. First, it embodies the assumption that inputs can be fairly freely substituted for one another, although not as freely as with some other functions. Secondly, it has the property that if one or more inputs is increased, the productivity of the other inputs always increases. Third, the function is homogeneous, in that when all inputs are simultaneously increased, output always increases by a constant proportion--not necessarily equal to 1 --of the increase in inputs.

Simpler specifications of the Cobb-Douglas function assume that production factors (labour and capital) are homogeneous or they do not allow for change in technology. More complex specifications treat some or all factors of production as composite production inputs or allow for technical change. This annex will introduce various specifications of the Cobb-Douglas production function, starting with the simplest. The calendar year will be used as unit observation.

A. Function without technical change

The simplest Cobb-Douglas production function, which assumes homogeneous inputs and no technical change, can be represented in the following general form:

$$VA(t') = z \cdot CAP(t')^{a} \cdot EM(t')^{b}, \qquad (1)$$

where:

t'	is the calendar year,
VA(t')	is the value added in year t',b/
CAP(t')	is the measure of the capital stock in year t',

EM(t')	is the measure of labour employed in year t',
z	is the intercept parameter,
a	is the elasticity parameter (box 33) relating to the capital stock, and
b	is the elasticity parameter relating to labour.

If the elasticity parameter of an input is less than one, there will be diminishing returns to that input. The sum of the two elasticity parameters, a + b, provides a measure of returns to scale. The returns to scale can be either increasing, constant, or decreasing, depending on whether the sum, a + b, is greater than, equal to, or less than unity. In this connection, if one is willing to assume constant returns to scale (an assumption often made in practice), the number of parameters to be estimated is reduced by 1, since in this case, b = 1 - a.

B. Functions with technical change

The Cobb-Douglas production function indicated in equation (1) is suitable for estimating production relationships on the basis of cross-section data. However, over time, the production function should reflect the process of technical change. This can be done in one of several ways, the most common of which is to assume that the technical change is "disembodied"; that is, it affects the function's intercept alone (Solow, 1957). The following specification is typical of this approach:

$$VA(t') = z \cdot CAP(t')^{a} \cdot EM(t')^{b} \cdot e^{(r\ t')}, \quad (2)$$

where:

r	is the constant rate of disembodied technical change, and
e	is the base of the natural logarithm.

Another approach to specifying technical change in the production function is to assume that it affects the elasticity parameters, a and b. The following specification provides an example of this approach:

$$VA(t') = z \cdot CAP(t')^{a(t')} \cdot EM(t')^{b(t')}, \quad (3)$$

Box 33

Glossary

<u>Diminishing returns</u>
A situation where if one factor of production is increased by small, constant amounts, all other factor quantities being held constant, then after some point the resulting increases in output become smaller and smaller.

<u>Disembodied technical change</u>
A type of technical change that influences output through shifts in the level of the production function rather than via factors of production.

<u>Elasticity parameter</u>
A parameter indicating the extent to which value added changes during a specified period of time (usually a year) in response to a given change in the amount of labour of capital used.

<u>Vintage capital models</u>
A class of economic models in which the aggregate capital stock consists of capital of different years of production (vintages).

where:

a(t')	is the elasticity parameter relating to the capital stock in year t', and
b(t')	is the elasticity parameter relating to labour in year t'.

As indicated by equation (3), this type of approach assumes that the elasticity parameters are related to time (Brown, 1966).

An alternative treatment of technical changes is to assume that it is embodied within one or more of the inputs. Thus, for example, it has been argued that technical change cannot occur independently of investment, and that it is in fact embodied in the physical characteristics of new machines. These ideas underlie the <u>vintage capital models</u> (Solow, 1959). Another approach has been to assume that technical change is embodied in the labour force in the form of "knowledge" (Griliches, 1967; Christensen and Jorgensan, 1970). The Cobb-Douglas production function with embodied technical change is written as follows:

$$VA(t') = z \cdot CAP^*(t')^a \cdot EM^*(t')^b, \quad (4)$$

where:

CAP*(t')	is the index of capital inputs in year t', which reflects quality improvements in this input over time, and
EM*(t')	is the index of labour inputs in year t', which reflects quality improvements in this input over time.

C. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

t'	is the calendar year

(b) List of variables

CAP(t')	is the measure of the capital stock in year t'
CAP*(t')	is the index of capital inputs in year t', which reflects quality improvements in this input over time
EM(t')	is the measure of labour employed in year t'
EM*(t')	is the index of labour inputs in year t', which reflects quality improvements in this input over time
VA(t')	is the value added in year t'

(c) List of special symbols

a	is the elasticity parameter relating to the capital stock
a(t')	is the elasticity parameter relating to the capital stock in year t'
b	is the elasticity parameter relating to labour

b(t')	is the elasticity parameter relating to labour in year t'
e	is the base of the natural logarithm
r	is the constant rate of disembodied technical change
z	is the intercept parameter

2. Equations

(a) Function without technical change

$$VA(t') = z \cdot CAP(t')^{a} \cdot EM(t')^{b} \quad (1)$$

(b) Functions with technical change

$$VA(t') = z \cdot CAP(t')^{a} \cdot EM(t')^{b} \cdot e^{(r\ t')} \quad (2)$$

$$VA(t') = z \cdot CAP(t')^{a(t')} \cdot EM(t')^{b(t')} \quad (3)$$

$$VA(t') = z \cdot CAP^{*}(t')^{a} \cdot EM^{*}(t')^{b} \quad (4)$$

Notes

a/ The Cobb-Douglas production function has been widely estimated during the past 50 years for a variety of sectors in many different countries, and with a variety of types of data including both aggregate and firm-level data, and both time series and cross-section data. See, for example: Walters (1970); Murti and Sastry (1957); Hildebrand and Liu (1965).

b/ In a Cobb-Douglas production function, one can use other measures of output instead of value added, such as gross output.

Annex II

PROCEDURE TO CALIBRATE INVERSE COBB-DOUGLAS PRODUCTION FUNCTIONS

The planner may wish to make adjustments in the estimated transformed Cobb-Douglas production functions by industry in order to make it possible to accurately predict employment, by industry, for a given historical year or time period, using the levels of value added and the capital stock for that year or period. These adjustments, which are usually referred to as "calibration", may be necessary, for example, where the planner wishes to use them to make projections of employment originating in the year that immediately follows the time period to which the data used to estimate the functions refer, rather than in the later, initial year of the plan.

Calibrating inverse Cobb-Douglas production functions may involve adjustments in the estimates of the intercept coefficients or in the estimates of the partial coefficients or both. Adjustments in the intercepts are more straightforward than those in the partial coefficients and, therefore, calibration is often restricted to the former coefficients. This annex describes a procedure that can be used to adjust intercepts of inverse Cobb-Douglas production functions as well as intercepts of log-linear inverse Cogg-Douglas production functions. It will also illustrate the use of the procedure to adjust intercepts of the latter form of functions.

A. The procedure

The procedure to calculate adjusted intercept coefficients of the inverse Cobb-Douglas production functions makes use of estimates of the partial coefficients of those functions as well as observed levels of employment, value added and the capital stock for the selected year or time period. Also, it uses an appropriate value of the time variable which refers to the chosen year or the mid-year of the selected time period. For a selected year, adjusted intercept coefficients are obtained as follows:

$$[\ [a'(i)]^*\ ]' = EM(i,t+5)\ /\ [\ VA(i,t+5)^{[b'(i)]^*} \cdot CAP(i,t+5)^{[c'(i)]^*} \cdot e^{[\ [d'(i)]^* \cdot t'\ ]}\ ]; \quad i = 1,\dots,I, \qquad (1)$$

where:

$i = 1,\dots,I$ are the industries of the nation's economy, I is the number of industries,

t' is the given calendar year,

EM(i,t') is the observed labour or employment in industry i in year t',

VA(i,t') is the observed value added in industry i in year t',

CAP(i,t') is the observed capital stock in industry i in year t',

[[a'(i)]*]' is the adjusted intercept coefficient of the inverse Cobb-Douglas production function for industry i,

[b'(i)]* is the estimate of the partial coefficient of the value added variable in the inverse Cobb-Douglas production function for industry i,

[c'(i)]* is the estimate of the partial coefficient of the capital stock variable in the inverse Cobb-Douglas production function for industry i, and

[d'(i)]* is the estimate of the partial coefficient of the time variable in the inverse Cobb-Douglas production function for industry i.

To adjust intercepts of log-linear inverse Cobb-Douglas production functions, one would use estimates of the partial coefficients as well as observed levels of employment, value added and the capital stock for the selected year or time period. Also, one would need to use an appropriate value of the time variable. For a selected year adjusted intercept coefficients would be obtained as follows:

$$[\,[\ln a'(i)]^*\,]' = \ln EM(i,t') - [\,[b'(i)]^* \cdot \ln VA(i,t') + [c'(i)]^* \cdot \ln CAP(i,t') + [d'(i)]^* \cdot t'\,]; \quad i = 1,\ldots,I, \qquad (2)$$

where:

[[lna'(i)]*]' is the adjusted logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for industry i.

In equations (1) and (2), t' stands for a selected year of the time period to which the data used to estimate inverse Cobb-Douglas production functions refer. In instances where the planner wishes to perform adjustments in the intercept coefficients using data for a few or several years rather then a single year, the adjustments can also be made using expressions shown in equation (1) or equation (2). In that instance, the observed levels of employment, value added and the capital stock used would be the mean levels of those variables for, say, three or five years centred on that particular year, t'.

B. Illustrative example of calibration

This example will illustrate the application of the calibration procedure by calculating adjusted intercepts of the log-linear inverse Cobb-Douglas production functions, by industry, estimated in chapter VIII (table 130). The example will use observations on employment, value added and the capital stock for 1978, which are respectively shown in tables 127-129.

Table 138 illustrates the calculation of the adjusted intercepts for the functions in question. The adjusted intercept coefficient (column 9) for any industry is obtained as the difference between the logarithm of the observed level of employment for the industry in 1978 (column 8) and the sum of three products. The first product is obtained by multiplying the estimated value added coefficient for the industry (column 2) by the logarithm of the observed level of value added for the industry in 1978 (column 5). The second product is found by multiplying the estimated capital stock coefficient (column 3) by the logarithm of the observed capital stock for the industry in 1978 (column 5). The third product is the result of multiplying the time variable coefficient for the industry (column 4) by the value of the time variable for year 1978 (column 6).

For example, the adjusted intercept in the function for agriculture, -14.20909, is obtained as follows:

$$-14.20909 = \ln(1{,}994.0) - [0.45694 \cdot \ln(260.3) + -0.24582 \cdot \ln(28.0) + 0.01015 \cdot 1978\,], \quad (1)$$

where 1,994.0 is the employment in agriculture in 1978, while 0.45694 and 260.3 are, respectively, the estimate of the value added coefficient in the function for agriculture and the value added in this industry in 1978 0-0.24582 and 28.0. The estimate of the capital stock coefficient in the function for agriculture and the capital stock in this industry in 1978 are -0.24582, 28.0 and 0.01015. The estimate of the time variable coefficient for agriculture in 1978 is the value of the time variable which is 0.01015.

Table 138. Computing adjusted intercept coefficients for log-linear inverse Cobb-Douglas production functions: entire country, year 1978

	Estimates of partial coefficients a/			In year 1978				
Industry	Value added	Capital stock	Time variable	Value added b/	Capital stock c/	Value of time variable	Employment d/	Adjusted intercept coefficient e/
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Agriculture	0.45695	-0.24581	0.01015	260.3	28.0	1978	1994.0	-14.20909
Mining	1.07969	-0.73550	0.02051	2.6	21.0	1978	6.7	-37.46840
Manufacturing	0.63666	-0.20063	0.00717	118.9	58.7	1978	217.3	-11.04406
Utilities	2.12628	-2.96373	0.05919	18.0	14.1	1978	15.3	-112.66345
Construction	0.24705	-0.19959	0.03584	23.5	9.4	1978	93.8	-66.69978
Trade	1.24756	-0.96450	-0.03025	50.9	27.3	1978	109.4	62.82550
Transport	1.24438	-0.76079	-0.06180	50.3	50.3	1978	95.1	124.91469
Services	1.47648	-0.87672	-0.02869	266.9	13.1	1978	516.2	57.01586

a/ From table 130.
b/ From table 128, year 1978.
c/ From table 129, year 1978.
d/ From table 127, year 1978.
e/ (ln(Col. 8)) - ((Col. 2) . (ln(Col. 5)) + (Col. 3) . (ln(Col. 6)) + (Col. 4) . (Col. 7)).

C. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

i = 1,...,I	are industries of the nation's economy
t'	is the given calendar year

(b) List of variables

CAP(i,t')	is the observed capital stock in industry i in year t'
EM(i,t')	is the observed labour or employment in industry i in year t'
VA(i,t')	is the observed value added in industry i in year t'

(c) List of special symbols

[[a'(i)]*]'	is the adjusted intercept coefficient of the inverse Cobb-Douglas production function for industry i
[b'(i)]*	is the estimate of the partial coefficient of the value added variable in the inverse Cobb-Douglas production function for industry i
[c'(i)]*	is the estimate of the partial coefficient of the capital stock variable in the inverse Cobb-Douglas production function for industry i
[d'(i)]*	is the estimate of the partial coefficient of the time variable in the inverse Cobb-Douglas production function for industry i
I	is the number of industries
[[lna'(i)]*]'	is the adjusted logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for industry i

2. Equations

A. The procedure

$$[\,[a'(i)]^*\,]' = EM(i,t+5) \,/\, [\, VA(i,t+5)^{[b'(i)]^*} \cdot CAP(i,t+5)^{[c'(i)]^*} \cdot e^{[\,[d'(i)]^* \cdot t'\,]}\,]; \quad i = 1,\ldots,I \tag{1}$$

$$[\,[\ln a'(i)]^*\,]' = \ln EM(i,t') - [\,[b'(i)]^* \cdot \ln VA(i,t') + [c'(i)]^* \cdot \ln CAP(i,t') + [d'(i)]^* \cdot t'\,]; \quad i = 1,\ldots,I \tag{2}$$

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Glossary

Asymptotically unbiased
An estimator, such as the coefficient in a regression equation, is said to be asymptotically unbiased if the probability that the estimator is different from the true value of the parameter it purports to assess approaches zero as the size of the sample approaches infinity.

Average capital-output ratio
The capital stock of a firm, industry or economy over a time period, divided by the output produced during that period.

Average employment-value added ratio
For a given time period, the quantity of labour employed, divided by the valued added produced.

Average labour productivity
The level of output per unit of labour input, usually measured as value added per man-hour or man-year.

Cobb-Douglas production function
A mathematical expression describing a relationship between a measure of output and two or more inputs (such as employed labour and capital). The function is multiplicative in the natural numbers and linear when transformed into logarithms.

Coefficient of determination, R^2
The measure of the goodness of fit of a regression equation, which denotes the proportion of the variance in the dependent variable associated with independent variable(s) included in the regression. The coefficient may lie between 0 and 1: when it is close to 0, it suggests a weak relationship; when it is close to 1, a strong one.

De jure population
A population enumerated on the basis of normal residence, excluding temporary visitors and including residents temporarily absent.

Demand for labour
The quantity of labour that users of labour services desire to purchase at prevailing wages and salaries.

Diminishing returns
A situation where if one factor of production is increased by small, constant amounts, all other factor quantities being held constant, then after some point the resulting increases in output become smaller and smaller.

Disembodied technical change

A type of technical change that influences output through shifts in the level of the production function rather than via factors of production.

Dropout rate

The proportion of students entering a given grade who withdraw from school before completing the grade.

Durbin-Watson statistic

The statistic, developed by J. Durbin and G. S. Watson as a weighted ratio of the sum of squared differences in successive residuals, which is used to test for autocorrelation of residuals.

Elasticity of employment with respect to value added

For a given time period, the proportionate change in the quantity of labour employed, divided by the proportionate change in the value added.

Elasticity parameter

A parameter indicating the extent to which value added changes during a specified period of time (usually a year) in response to a given change in the amount of labour of capital used.

Employment function

A mathematical expression describing a relationship between employment and other variables, typically a measure of output and other inputs, in which employment is the dependent variable and the other variables are independent. The inverse of a Cobb-Douglas production function in which employment is the dependent variable would be an employment function.

Enrolment ratio

The number of students attending school, divided by the number of persons of appropriate years of age. The ratio may refer to the entire educational system or to a given school level.

Excess demand for labour

The amount by which the quantity of labour demanded exceeds the quantity of labour available at the prevailing level of wages and salaries.

Excess supply of labour

The amount by which the quantity of labour available exceeds the quantity of labour demanded at the prevailing level of wages and salaries.

Factor substitution
The process by which one factor of production (e.g. labour) is replaced in production by some other factor of production (e.g. capital).

Human capital
Productive investments embodied in human persons. These include skills, abilities, ideals, health etc., that result from expenditures on education, on-the-job-training and medical care.

Incremental capital-output ratio
The increase in the capital stock of a firm, industry or economy over a time period, divided by the increase in output over that period.

Labour force participation rate
The number of persons in the labour force of a given age, sex and/or level of education, divided by the corresponding total number of persons with the same characteristics.

Labour market
The market in which labour services are bought and sold through a process that determines the number of persons employed as well as wages and salaries.

Level-specific enrolment ratio
The number of students attending schools at a given school level, divided by the number of persons of the years of age assumed to correspond to that school level.

Marginal employment-value added ratio
For a given time period, the change in the quantity of labour employed, divided by the change in the value added.

Primary sector
The part of the economy that specializes in the production of agricultural products and the extraction of raw materials. Major industries in the sector generally include: agriculture, forestry, fishing and mining.

Promotion rate
The proportion of students entering a given grade who upon completing it, enter the subsequent grade.

Random disturbance term
The term added to a regression equation as an average relationship between dependent and independent variables, which ensures equality between the left and the right hand side of the equation for each

observation. The disturbance or error term may represent random disturbances in an observation or it may reflect errors of measurement.

Repetition rate
The proportion of students entering a given grade who fail to complete it and then repeat the grade.

School-age population
The population of the school-age period, conventionally defined as ages 5 through 24 years, inclusive.

Secondary sector
The part of the economy that uses raw materials and intermediate products to produce final goods and other intermediate products. Major industries comprising the sector generally include: manufacturing, construction and utilities.

Serially correlated
Disturbance terms of a regression equation fitted to time series data are said to be serially correlated if there is a degree of stochastic dependence between those terms. Serial correlation occurs when effects due to particular chance disturbances or omitted variables tend to persist through several periods or years. It could also be occasioned by methods of data collecting or reporting that incorporate elements of smoothing and interpolation which average the "true" disturbances over adjacent periods.

Sex ratio of labour force
The number of males in the labour force, divided by the corresponding number of females and conventionally multiplied by a hundred.

Sex ratio of students
The number of male students for each female student, conventionally multiplied by 100.

Simultaneous equation bias
A bias arising in statistical estimation when the dependent variable has a causal effect on the independent variables, rather than vice versa.

Sprague interpolation
A type of non-linear interpolation.

Statistically significant
An estimate of a particular statistic, such as a partial regression coefficient, is said to be statistically significant if the probability that it could have occurred by chance is less than, say, 5 per cent.

Supply of labour
The quantity of labour that owners of labour services desire to sell at prevailing wages and salaries.

t-statistic
In regression analysis, a statistic calculated for each partial coefficient which makes it possible for the analyst to determine whether or not the coefficient is statistically significant.

Technical progress
The application of new scientific knowledge in the form of inventions and innovations to capital, both physical and human, usually leading to lower costs or increased output.

Tertiary sector
The part of the economy that provides various services to businesses and households. Major industries of the sector generally include: banking and insurance, public administration, health and education.

Value added
For a firm or farm, the difference between its total revenue and the cost of raw materials, services and components used in production over a specified time period; for the economy as a whole or any of its industries, the aggregate of value added of different firms or farms of which the economy or industry is composed.

Vintage capital models
A class of economic models in which the aggregate capital stock consists of capital of different years of production (vintages).

Working-age population
The population in the working ages, conventionally defined as 15 to 59 years or 15 to 64 years.

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