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THE CONCEPT OF A STABLE POPULATION

APPLICATION TO THE STUDY OF POPULATIONS OF COUNTRIES WITH INCOMPLETE DEMOGRAPHIC STATISTICS

UNITED NATIONS

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United Nations

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CONTENTS

INTRODUCTION

If the contents of this work were to be judged by its title, there is little doubt that it would be classified among those theoretical studies which are useful for clarifying one's thoughts and setting them in order but have little prospect of practical application.

Those who are unfamiliar with demographic methods would first of all ask what is meant by a "stable population"; and when told that this is the name given to a limit population to which actual populations tend when their mortality and fertility remain constant, they would no doubt feel that this was a thoroughly abstract concept. For, to approach a limit is to come closer and closer to it without ever reaching it, like a hyperbola which approaches its asymptotes without ever touching them. If they were told that fertility and mortality would have to remain unchanged for about 100 years before an actual population came close enough to a stable population, they would be quite sure that this was a mathematical tool having only the most tenuous contacts with reality.

Demographers, and particularly those interested in theoretical demography, would take a somewhat more flexible attitude which would not differ fundamentally from that held by non-specialists.

They would point out that the limit stable state reached by a population is a function of the initial conditions. They would see it less as an abstract concept than as a consequence of those initial conditions, and hence as something providing information on the latter. Indeed, the concept of a stable population has for long been accepted as one of the tools of the demographer. Work of the kind carried out by Alfred J. Lotka has demonstrated the genuine value and usefulness of computing a stable population, given a set of initial conditions. The characteristics of a stable population computed in this way would reveal the properties of the corresponding set of demographic conditions. The net reproduction rate, the intrinsic rate of natural increase and its two components of mortality and fertility are now generally accepted as indices which are universally utilized.

Most demographers would agree, however, that the stable populations which they compute are very seldom found in reality. Any comparison of a stable population with the actual population in an attempt to estimate the demographic conditions of the population is therefore doomed to failure. If the actual demographic conditions are known, the corresponding stable population can be computed; and by comparing the stable state with the real population what may be regarded as transitional in the real population can be determined. The reverse is, however, fundamentally impossible. A stable state can be reached by starting from many actual populations, but it would be completely fortuitous if an actual population identical with a stable population were to be found. Nothing could be learnt of demographic conditions in the actual population from a mere knowledge of demographic conditions in the stable population thus identified. Nevertheless, we shall try to show, in the following pages, that the concept of a stable population has many practical applications.

In the first place, there is another theoretical concept, which relates to populations with unchanging age structures. Such populations possess some of the properties of a stable population, and there are many actual examples of them in the world, at least as a first approximation. It can be demonstrated that there are, at all times, in such populations the same relations among fertility, mortality and age structure as in a stable population. Consequently, so far as any phenomena involving age structure and its relations to mortality and fertility are concerned, populations with constant age structures can be assimilated to stable populations.[1] In order to express the fact that populations with unchanging age structures possess only some of the properties of stable populations, they have been given the name of *semi-stable populations*.

Many actual populations have age structures which, while not absolutely unchanging, change only slightly, and thus the concept of stable populations, which appeared at first to be a mathematical concept scarcely encountered in reality, becomes, through the theory of semi-stable populations, a familiar and common tool in population analysis. Most populations of developing countries, or almost three-quarters of the world population, may be considered semi-stable.

The constancy of the age structure of most populations of developing countries can be interpreted in terms of fertility and mortality, and this interpretation leads us to a further notion, that of the *quasi-stable population*. One of the fundamental characteristics of the present demographic evolution of the world is the decline in mortality. Starting from a level of mortality corresponding to an expectation of life at birth of about 25 years, the populations of the various areas of the world have been moving with varying degrees of rapidity towards a level corresponding to an expectation of life at birth of about 75 years. Observations show that this decline in mortality almost always follows the same pattern. It is, therefore, possible to construct a series of model life tables reflecting the various phases of mortality of the human species as experienced in the past.

As long as mortality remains within the "universe" defined above, variations in mortality have little effect on the age structure of a population, the latter being determined primarily by variations in fertility. The result is that, in populations where fertility remains unchanged while mortality varies within the universe of the life tables defined above, the age structure changes only slightly. Such populations have been given the name of *quasi-stable populations*. Roughly speaking, they may be

[1] This cannot be done where a phenomenon involves other factors, such as the past history of an individual. The average number of surviving children per family, the proportion of widowers and widows in the population, the proportion of orphans, and the absolute numbers of population, births, deaths and so forth, are examples of such phenomena.

considered to represent a particular series of semi-stable populations.

It can now be appreciated why little change occurs in the age structure of populations of developing countries. After a very long period during which the fertility and mortality of such populations remained, on the whole, relatively unchanged, mortality began to decline while fertility remained constant and the populations became quasi-stable.

It is worth noting that the concepts of a *stable population* and a *semi-stable population* are theoretical, and the study of their properties is a problem of pure mathematics. The concept of a *quasi-stable population*, on the other hand, is based on experience. The properties of quasi-stable populations are based on empirical data, which may be revised as a result of further experiments. There now follows a summary of the contents of the various chapters of this work.

Chapter I begins with a clarification of a point of terminology, since Lotka's definition of a stable population is not precisely what is stated above. Lotka defines a stable population as a particular case of a broader category of populations, called *Malthusian populations*. A Malthusian population is a population where the survivorship curve and the age structure are invariable.[2] There are an infinite number of such populations, which can be classified, initially, in two ways:

(1) Given a survivorship function, there corresponds a sub-set (H) of all the Malthusian populations based upon that survivorship function. By varying the survivorship function, an infinite number of sub-sets (H) embracing all Malthusian populations will then be obtained;

(2) Given an age structure, there corresponds a sub-set (F) of all the Malthusian populations based upon that age structure. By varying the age structure an infinite number of sub-sets (F) covering all Malthusian populations will then be obtained.

In a given sub-set (H_0) or (F_0) an additional condition is needed in order to determine a particular population. The stable population of a sub-set (H_0) is a particular population for which the fertility function is known. It can be demonstrated that for any given fertility function there exists one, and only one, stable population.

By varying the fertility function, on the other hand, all the populations of the sub-set (H) can be obtained. Thus, any Malthusian population can be considered a stable population, and indeed there are an infinite number of ways of doing this. That is why the two concepts are often confused, and such confusion has, in fact, become traditional. In this introduction, following the tradition, we have made references to a stable population, whereas in some cases the expression "Malthusian population" would have been more accurate. For the same reason, the expression "stable population" in the title of this work is not entirely in keeping with the contents. The expression "Malthusian population" can lead to confusion, since it may suggest a population practising birth control, whereas the subject of this manual is quite different. In order, therefore, to avoid using a title which might be misunderstood, we have preferred to be somewhat imprecise. In the body of this text, however, it is obviously necessary to distinguish clearly between the concept of a Malthusian population and that of a stable population, and this is the subject of the first chapter.

Lotka's definition of a stable population shows that such a population merely represents a particular case of a broader category, i.e., *the populations of a sub-set (H) or (F) which satisfy a given condition.* This general problem is posed in chapter I, but the study of the possible solutions to it is left to other chapters.

Lotka does not, however, simply define the concept of a stable population as a particular case of a Malthusian population. After asking the question of what happens to a population whose survivorship and fertility functions remain invariable, he demonstrates that such a population approaches the stable population defined by the sub-set of Malthusian populations based on the known survivorship function and the fertility function. A stable population thus appears as a limit state, and it was in this form that is was referred to at the beginning of this introduction.

This second definition of stable populations suggests a method of computing their characteristics. By simply computing far enough into the future a projection based on conditions of constant mortality and fertility, one gradually sees emerging from the projection the stable population corresponding to the given mortality and fertility. Chapter I ends with a description of such a method and establishes the properties of stable populations from actual examples of projections. These properties are established without the use of mathematical notation. It is somewhat like a person who, in attempting to demonstrate that the altitudes of a triangle meet at a point, begins by drawing triangles of different shapes and graphically verifies his thesis in each of the triangles he has drawn.

While this procedure is convenient for an understanding of the mechanism followed by a population in becoming stable, it is not suitable for practical computations,[3] which are based on the results of a mathematical analysis of "renewable resources". Chapter II describes and applies these methods. In this chapter we start from Lotka's definition of a stable population; that is to say, we take sub-sets (H) of Malthusian populations which have a given survivorship function. First, the formulae for the computation of the demographic characteristics of a particular population of the set satisfying a given condition are indicated. The chapter then goes on to actual applications of such formulae, beginning with the case where the given condition is knowledge of the fertility function, as in the case of a stable population. Various other conditions, such as knowledge of the crude death rate, crude birth rate and so forth are examined.

In chapter III, the same examples are taken up, but they are now considered as limit states of the process of demographic evolution. This chapter therefore deals with the theoretical aspect of the projections studied empirically in chapter I. By removing the condition of invariability

[2] It can be demonstrated that such a population has a constant rate of natural variation, and it is for this reason that such populations are known as Malthusian populations, Malthus having dealt in his works with populations increasing at a constant rate.

[3] The use of electronic computers may invalidate this statement, since such machines enable projections to be obtained in a few seconds, and the empirical method referred to in chapter I then becomes very easy to apply. See in this connexion the article by Nathan Keyfitz, "*L'utilisation des machines électroniques pour les calculs démographiques*", *Population* (Paris), No. 4, 1964, pp. 673-682.

of the age structure, a process of demographic evolution can be made to correspond to each of the examples given in chapter II. First, processes with constant survivorship and fertility functions leading at their limit to stable populations are studied. The chapter then proceeds to study other examples, such as processes with constant survivorship functions and crude birth rates, processes with constant survivorship functions and crude death rates, and so forth.

Chapter IV returns to the problem of determining a Malthusian population satisfying a given condition, this time by taking the sub-sets (F) defined by a given age structure rather than the sub-sets (H) defined by a given survivorship function.

Chapter V defines the concept of a "semi-Malthusian population" (or "semi-stable population", to use the not quite accurate terminology employed above). This, it should be recalled, is a population with an unchanging age structure. Whereas the earlier chapters described the well-known demographic works with a minimum of explanations because the full demonstration could always be found in the original works, this was not the case as regards chapter V. As the concept of a semi-Malthusian population is new in demography, it is necessary each time to justify the formulae used. Naturally, therefore, chapter V is highly theoretical.

The concept of a semi-Malthusian population has many practical applications. In the first place, it leads to a broadening of Lotka's definition of a Malthusian population. This is the subject of chapter VI, which takes up the problem of determining a Malthusian population satisfying a given condition within a new series of sub-sets (G) where the age structure of deaths is constant. However, the importance of the concept of a semi-Malthusian population lies in the fact that it opens the way to a solution of the problem from which this manual takes its title, namely, how to use the concept of a stable population for the purpose of estimating the fertility and mortality of actual populations.

In order to solve this problem, two lines of reasoning could legitimately be adopted according to whether the actual populations are assimilated to semi-Malthusian or to quasi-stable populations. The two methods are applied to populations whose age structure has changed little over time.

When a population is assimilated to a semi-Malthusian population, the little variation in its age structure is interpreted as the imperfect materialization of an invariable age distribution, while when a population is assimilated to a quasi-stable population, the little variation in its age structure is interpreted as the materialization of a population with constant fertility but with a mortality varying over the possible range of human mortality.

In the first case, when the actual population is assimilated to a semi-Malthusian population, it is convenient to treat the actual population as a Malthusian population for which certain demographic characteristics other than mortality and fertility are known, and the question is what can be said about the unknown mortality and fertility. This is the general problem studied in chapters II, IV and VI, and everything stated in those chapters is applicable to actual populations.

In the second case, when the actual population is assimilated to a quasi-stable population, the levels of mortality and fertility are sought along totally different lines. As the concept of a quasi-stable population is based on experience, it is by comparing actual populations with empirically constructed quasi-stable populations that the appropriate quasi-stable population to be assimilated to given actual population is determined. The mortality and fertility of this assimilated quasi-stable population are thus the estimated values sought. However, it is necessary first to construct schemes and framework of quasi-stable populations.

There are an infinite number of ways of calculating such schemes. In order to reduce the number, it may be noted that quasi-stable populations are always close to the stable populations of the moment. It was therefore assumed that stable populations, computed as having mortality levels within the range of human mortality, conveniently represented the possible range of quasi-stable populations. There remained the task of defining the possible range of human mortality. It was stated earlier that a series of model life tables representing the various phases of changes in human mortality could be conceived and constructed. In reality, and more exactly, however, human mortality vacillates around these successive phases, and it is therefore necessary to construct, in addition to the series of model life tables referred to, two other series reflecting tendencies deviating upwards and downwards. These two series mark the limits of the possible range of variation of actual human mortality. Details of the methods used to construct these three series of mortality tables will be found in annex II. It should be noted immediately that the intermediate model life table is practically indentical with the series of model tables computed by the United Nations in *Manual III*.[4]

With the aid of the three series of model life tables, three sets of stable populations, which were designated as the intermediate, the upward-deviating and the downward-deviating frameworks, were constructed. Methods of estimating mortality and fertility were developed by comparing actual populations with the populations of these three frameworks. This is the subject of chapter VII. These methods presuppose that the mortality sought conforms to the series of model life tables employed in the construction of the framework used in finding the quasi-stable population to be assimilated to the actual population. This is an assumption which cannot be verified, since the mortality is not known. It is therefore necessary to verify the fact that the assumption does not have any significant effect on the estimated values sought, and this is the subject of chapter VIII.

Finally, the manual has four annexes. Annex I deals with certain aspects, hitherto neglected by theoretical demographers, of the theory of a stable population considered as the limit of the process of evolution with constant levels of mortality and fertility. These aspects are not directly connected with the subject of this manual, and the results of annex I cannot be used to estimate the mortality and fertility of actual populations, but they are nevertheless the natural complement of the work referred to in chapter III. This annex involves the use of highly theoretical and mathematical notation and methods, and leads to the very concrete notion of the "growth poten-

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[4] *Methods of Estimating Population, Manual III: Methods for Population Projections by Sex and Age* (United Nations publication, Sales No.: 56.XIII.3).

tial" of a population — a most convenient concept, whose use is considered in chapter I and the computation of which presents no difficulty.

It has already been stated that annex II deals with the methods used in constructing the series of model life tables applicable for the range of possible variations of human mortality. This annex merely gives a summary of more extensive work on the subject published elsewhere.

Annex III gives the characteristics of the framework of intermediate model stable populations, while annex IV gives the characteristics of the upward-deviating and downward-deviating frameworks.

Chapter I

THE CONCEPT OF A STABLE POPULATION

A. Malthusian populations

The concept of a stable population was first introduced into demography by Alfred J. Lotka [1] as a particular case of a Malthusian population. Lotka went on to demonstrate that a stable population could also be considered as a limit state towards which populations with unchanging mortality and fertility tended, and it is primarily this second aspect which has received attention from demographers. The distinction between the two different approaches is not very clear in Lotka's work, however, and it seemed worth while to define it more precisely at the beginning of the present work.

A Malthusian population is a population whose mortality and sex-age structure are constant. It is important to note that these characteristics are not assumed to be known. The only assumption is that they are constant. Various properties of Malthusian populations follow from this assumption.

(1) As the age structure and mortality are constant, it follows that the age distribution of deaths is also constant.

(2) If a represents the age, $C(a)$ the age structure for both sexes together, and b the crude birth rate, we have:

$$C(0) = b$$

The crude birth rate is therefore constant.

(3) As both the mortality and the age structure are constant, it follows that the crude death rate d is also constant.

(4) The rate of natural variation $r = b - d$ is therefore also constant.[2] Thus for the total numbers of population, of births and of deaths at time t, we have the following expressions in which A is a constant equal to the total number of the population at the initial condition:

$$N(t) = Ae^{rt} = N(0)e^{rt}$$
$$B(t) = bN(0)e^{rt}$$
$$D(t) = dN(0)e^{rt}$$

(5) If $p(a)$ is the survivorship function for both sexes, the number of persons of age a at time t is:

$$B(t-a)p(a) = Abe^{rt}e^{-ra}p(a)$$

They also number:

$$N(t)C(a) = Ae^{rt}C(a)$$

so that we have:

$$C(a) = be^{-ra}p(a)$$

(6) If ω represents the upper limit age of life, as $C(a)$ is a distribution, it follows from the definition:

$$\int_0^\omega C(a)da = 1$$

which is written:

$$b\int_0^\omega e^{-ra}p(a)da = 1$$

or

$$b = \frac{1}{\int_0^\omega e^{-ra}p(a)da}$$

(7) It was stated above that in a Malthusian population the sex distribution is constant. In fact, however, it is sufficient to assume that the masculinity at birth is constant. If this masculinity—the ratio of male births to female births—is represented by m, the expressions for female and male births will be respectively:

$$B_f(t) = \frac{B(t)}{1+m} = \frac{bN(0)}{1+m}e^{rt}$$
$$B_m(t) = \frac{mB(t)}{1+m} = \frac{mbN(0)}{1+m}e^{rt}$$

If $p_f(a)$ is the female survivorship function, assumed to be invariable, then the number of females of age a is:

$$B_f(t-a)p_f(a) = \frac{bN(0)}{1+m}e^{rt}e^{-ra}p_f(a)$$

and the age distribution of the female population is written:

$$C_f(a) = \frac{\frac{bN(0)}{1+m}e^{rt}e^{-ra}p_f(a)}{\frac{bN(0)}{1+m}e^{rt}\int_0^\omega e^{-ra}p_f(a)da} = \frac{e^{-ra}p_f(a)}{\int_0^\omega e^{-ra}p_f(a)da}$$

Similarly, the age distribution of the male population will be:

$$C_m(a) = \frac{e^{-ra}p_m(a)}{\int_0^\omega e^{-ra}p_m(a)da}$$

where $p_m(a)$ represents the male survivorship function, assumed to be also invariable.

(8) In particular, the two expressions for the crude female and male birth rates will be as follows:

$$C_f(0) = b_f = \frac{1}{\int_0^\omega e^{-ra}p_f(a)da}$$

$$C_m(0) = b_m = \frac{1}{\int_0^\omega e^{-ra}p_m(a)da}$$

[1] Alfred J. Lotka, *Théorie analytique des associations biologiques, deuxième partie* (Paris, Hermann, 1939), 149 pages.

[2] This is why Lotka called these populations "Malthusian populations", since Malthus dealt with populations increasing by geometric progression.

There is a very simple approximate relationship between the three rates b_f, b_m and b. It may be noted that:

$$\frac{1}{b_f} + \frac{m}{b_m} = \int_0^\omega e^{-ra}[p_f(a) + mp_m(a)]da = \int_0^\omega e^{-ra}p(a)da = \frac{1}{b}$$

In practice, b_f and b_m are always close to one another, while m varies little from 1, so that we have:

$$b \# \frac{b_f + b_m}{2}$$

It also follows that:

$$d \# \frac{d_f + d_m}{2}$$

(9) If only the female sex is considered and $\varphi(a,t)$ is taken to represent the female fertility rate for women of age a, computed from the girls at time t, we obviously have:

$$b_f = \int_u^v C_f(a)\varphi(a,t)da = b_f \int_u^v e^{-ra}p_f(a)\varphi(a,t)da$$

where u and v represent the limits of the reproductive period. Finally, we can write:

$$\int_u^v e^{-ra}p_f(a)\varphi(a,t)da = 1 \quad (1)$$

Such are the main properties of a Malthusian population.[3]

B. Malthusian populations with known mortality (stable Malthusian populations)

Lotka goes on to consider the sub-sets H(r) obtained from the set of Malthusian populations thus defined by assuming certain fixed values for the mortality functions. Every survivorship function $p_0(a)$ has a corresponding sub-set $H_0(r)$, and all the populations of the sub-set can be obtained by successively associating with $p_0(a)$ all the possible values of r.[4]

It is within such a sub-set $H_0(r)$ that Lotka defines in the following manner what he means by a stable population. If, in addition to the mortality functions, values are also assumed for the fertility function—i.e., if it is assumed that $\varphi(a,t) = \varphi_0(a)$ is known and is independent of time, then expression (1) becomes an equation in r. It has only one real root: $r = \varrho$. The population of the sub-set H(r) which corresponds to this value ϱ is called a stable Malthusian population or, more simply, a stable population. To be more exact, it is *the* stable population corresponding to laws of mortality and fertility which are *assumed to be known and invariable*. In fact, since equation (1) in r has only one real root,[5] there is only one stable population corresponding to the given laws of mortality and fertility, and we can therefore speak of the stable population which corresponds to those laws.

Before studying the second definition of a stable population, i.e., the concept of a stable population as the limit state of a process of demographic evolution in which mortality and fertility remain unchanged, it is worth pausing to consider the principles on which the concept of the first type of stable population is based.

A stable population is a specific population from among a sub-set of Malthusian populations selected from the wider field of all possible Malthusian populations. The sub-set is selected on the basis of *knowledge* of the mortality function. It would have been possible to select another sub-set $F_0(r)$ if the sex-age distribution functions had been assumed to be known. The method adopted by Lotka therefore ignores a whole aspect of Malthusian populations, i.e., all the sub-sets F(r) of Malthusian populations with known age distribution. We shall revert later to the consideration of these sub-sets F(r), the study of which has many practical applications.

Let us now return to the sub-sets H(r) of Malthusian populations with known mortality function, as considered by Lotka. The stable population has been defined by the supplementary assumption that the fertility function is also known. Thus, the stable population has been defined as a Malthusian population with known mortality and fertility. It is obvious, however, that in a sub-set $H_0(r)$ there are other ways to determine a particular population.

Generally speaking, it is sufficient to assume as given any index or function leading to an equation in r with a finite number of real roots. Let us consider the following examples:[6]

(*a*) If we assume, for example, a birth rate $b = b_0$, we have the equation:

$$b_0 \int_0^\omega e^{-ra}p_0(a)da = 1$$

As equation (1), referred to above, has only one real root, there is therefore one, and only one, Malthusian population with a given mortality and a given crude birth rate.

(*b*) If we assume a death rate $d = d_0$, we arrive, since we have $b = d_0 + r$, at the following equation in r:

$$(d_0 + r)\int_0^\omega e^{-ra}p_0(a)da = 1$$

It can be shown that this equation has no solution, one solution or two solutions. Thus, there is not always a Malthusian population with the given mortality functions and the given crude death rate, and when one exists another generally exists also.

(*c*) It is also possible to assume a value for the age distribution at a given age h. This leads to the following equation in r:

$$C_0(h)\int_0^\omega e^{-ra}p_0(a)da = e^{-rh}p_0(h)$$

(*d*) We arrive at a similar equation if we assume the age distribution of deaths $d_0(h)$ for a given age h. This gives us the following equation in r:

$$d_0(h)\int_0^\omega e^{-ra}p_0(a)q_0(a)da = e^{-rh}p_0(h)q_0(h)$$

where $q(a)$ is the probability of dying at age a.

[3] These properties will be dealt with again in chapter II, one by one, and various methods of formulating them will be described.

[4] Mathematically speaking, r can vary from $-\infty$ to $+\infty$. In human populations, however, the actual variations of r are much narrower. We shall revert to this question later.

[5] We shall revert to the question of the existence of this one real root in later chapters.

[6] These examples will be dealt with again in detail in later chapters.

All these problems are similar in principle to that considered by Lotka in arriving at his definition of a stable population. However, the two conditions imposed by Lotka—knowledge of the mortality and the fertility—are obviously in a different category from the two conditions corresponding to each of the other problems. They are mathematically independent and express the fundamental characteristics of a population, namely, its mortality and its fertility. This is why the case of a stable population is of much greater interest than the other particular Malthusian populations described above. This remark is all the more important when the concept of a stable population is envisaged as the limit state of a process of demographic evolution in which the mortality and fertility remain unchanged. This is the case which will be discussed next.

C. Concept of a stable population considered as a limit

The two assumptions of constant age distribution and constant mortality will define all Malthusian populations.

The third assumption, that the mortality is known, enables us to define sub-set $H_0(r)$.

Finally, the fourth assumption that the female fertility function is known, enables us to define from sub-set $H_0(r)$ the stable population corresponding to the known laws of mortality and fertility.

What happens if we suppose that the first assumption is not made, i.e., if we do not assume that the age distribution is constant? In other words, what happens if the mortality and fertility of a population remain unchanged? Let us state immediately that such a population follows a path which brings it closer and closer to the structure of the stable population corresponding to the given laws of mortality and fertility.

The demonstration of this property is one of the classic problems of the theory of "renewable resources", the solution of which calls for difficult mathematical developments if the demonstration is to be completely accurate, although a rough proof of the results can be achieved by relatively simple methods, which are described in chapter III. It is also possible to examine empirically, on the basis of actual data, what happens in a population whose female mortality and fertility remain unchanged. The principle underlying such an examination is that of computing population projections and seeing how they evolve.

D. Study of the limit population on the basis of actual cases

Attempts are made here to provide examples which are the results of projections. The starting point is the population of Eastern Germany [7] in 1957.[8] This population was selected in order to have as a starting point a very distorted age distribution.

The life table, which is assumed to be invariable, is the level 80 table of the series of model life tables published in the *Manual on Methods for Population Projections by Sex and Age*. This table corresponds to an expectation of life at birth for both sexes of 60.4 years.

As stated in the introduction, it will be necessary in this work to use two other series of model life tables which are respectively above and below the series of model life tables given in the *Manual* on methods for population projections. The series of tables given in the *Manual* will be referred to as "intermediate model life tables".

The age distribution of the female fertility rates, which are assumed to be invariable, is as follows: [9]

TABLE I.1

Age group (years)	*Distribution*
15-19	100
20-24	273
25-29	263
30-34	188
35-39	121
40-44	55
ALL AGES	1 000

The total of these rates, which is the same as the gross reproduction rate, is assumed to equal 1.50.[10] Masculinity at birth is $m = 1.05$.

As in the case of mortality, we shall later need to use other patterns of fertility with an age distribution on either side of the distribution adopted for the computation of the projections. This latter distribution will be referred to as the "intermediate model fertility distribution".

Once these conditions have been set, a projection is computed by the traditional method described in the *Manual on Methods for Population Projections by Sex and Age*.[11]

Some explanations are called for regarding the selection of the model life table and the gross reproduction rate. When a projection is computed for a population with constant mortality and fertility, often troublesome discontinuity is being introduced if rates of mortality and fertility significantly differ from those actually observed in the initial population. It was, therefore, necessary to take a mortality and a fertility which were not too far removed from the mortality and fertility actually observed in Eastern Germany in recent years.

The same mortality and fertility will be used to compute another projection based on an estimate of the population of Thailand in 1955 and it will also be necessary that the mortality and fertility adopted should not be too far removed from the mortality and fertility observed in Thailand in recent years.

It should be stated immediately, as regards fertility, that it is impossible to select a gross reproduction rate which is close both to that of Eastern Germany and to that of Thailand, since the level of fertility in Thailand is almost three times as high as in Eastern Germany. If we adopt 1.50 as the gross reproduction rate, this conforms satisfactorily with the path along which fertility is developing in Eastern Germany, but it does not rid the

[7] The designations employed and the presentation of the material in this publication do not imply the expression of any opinion whatsoever on the part of the Secretariat of the United Nations concerning the legal status of any country or territory or of its authorities, or concerning the delimitation of its frontiers.

[8] *Demographic Yearbook, 1958* (United Nations publication, Sales No.: 58.XIII.1), table 5, p. 132.

[9] We shall revert to the reasons for selecting this particular age distribution of the fertility rates later.

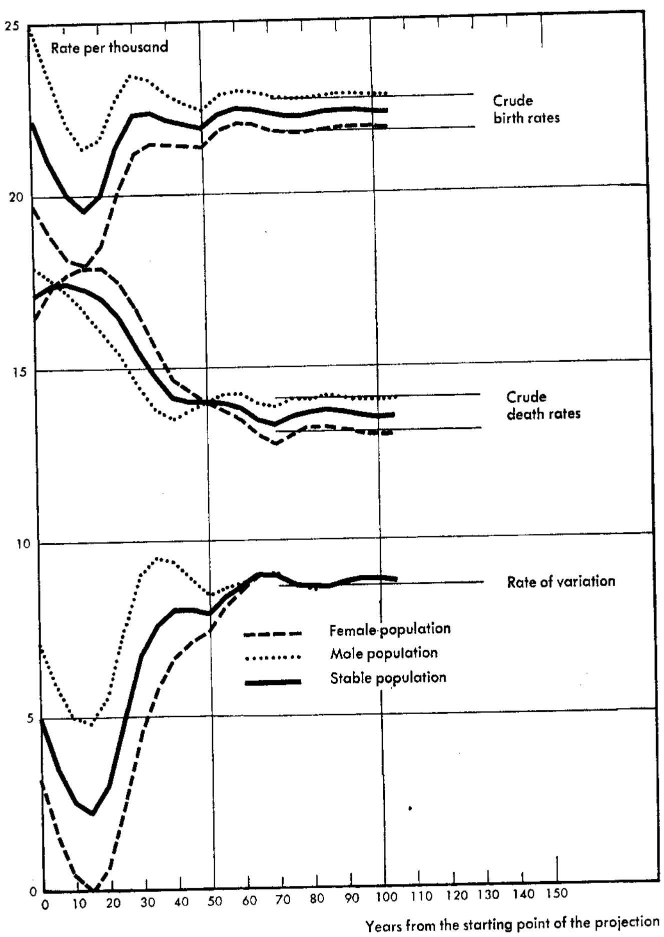
[10] Such a gross reproduction rate corresponds to a medium fertility which would lead, at the mortality rate assumed for the computations, to a stable population undergoing moderate increase.

[11] United Nations publication, Sales No.: 56.XIII.3.

projection based on the figures for Thailand of the discontinuity connected with the fertility assumption.[12] Where mortality is concerned, the level 80 of the intermediate model life table is approximately midway between the levels of mortality in Thailand and in Eastern Germany, and consequently it satisfies the requirements stated above.

These points having been made clear, we begin by commenting on the results obtained in the projection for the female population.

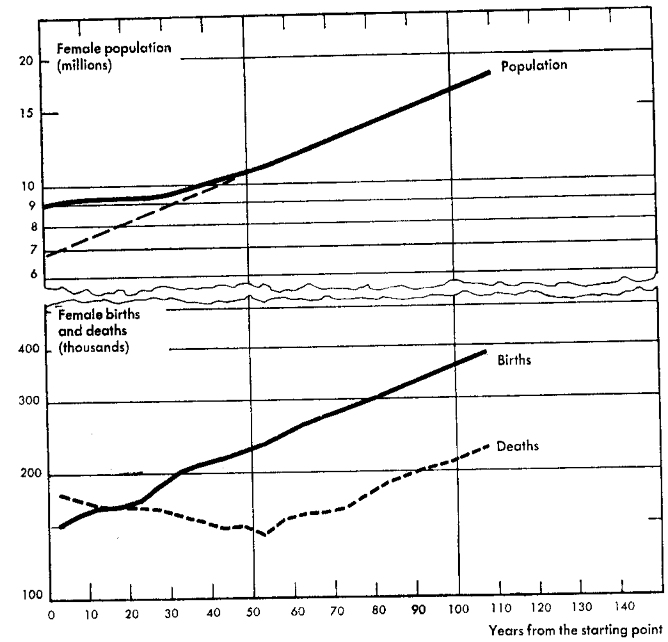
At the outset, the crude female birth and death rates show substantial fluctuations. These fluctuations diminish with time, however, and the rates finally stettle down at constant levels. The rate of natural increase of the female population, which is the difference between these two rates, follows a similar development (see graph I.1).



Graph I.1. Projections computed on the basis of the population of Eastern Germany in 1957. Variations in the crude birth rate, crude death rate and crude rate of natural increase

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50

The absolute figures for female deaths, female births and the total female population vary irregularly at the beginning but tend after a time to increase at a constant rate which is the same for births, deaths and total population. On a semi-logarithmic graph (with time indicated on the horizontal axis in a metric scale and the absolute numbers on the vertical axis in a logarithmic scale), the curves of the variation of births, deaths and total population will become, after a time, parallel straight lines whose slopes are equal to the stabilized rate of increase (see graph I.2). These straight lines will be referred to in this paper as stabilization lines.



Graph I.2. Projections computed on the basis of the population of Eastern Germany in 1957. Variations in the absolute numbers of persons in the female population, of female births and of female deaths

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50.

The projection for the male population leads to the same observations as the projection for the female population (see graph I.1). The crude male mortality and fertility rates stabilize, after a time, at a level slightly above that at which the female rates stabilize. The discrepancy between the birth rates is the same as that between the death rates, so that the male rate of variation stabilizes at the same level as the female rate of variation. In other words, at the limit the female population and the male population vary at the same rate.

Finally, the age structure of the two populations, which at first is very irregular, gradually becomes more regular and eventually acquires a very regular shape, which subsequently remains invariable (see graph I.3).

We therefore see that in this projection the population approaches a state in which the mortality and age distribution are invariable. This is an exact definition of a Malthusian population. In this Malthusian state, moreover, the mortality and fertility are known, and we have seen that such knowledge enables us to define the stable population of the sub-set $H_0(r)$ associated with the mortality used in the computation. We have therefore verified empirically the result given above: *a population whose mortality and fertility remain invariable tends to become the stable population corresponding to such levels of mortality and fertility.*

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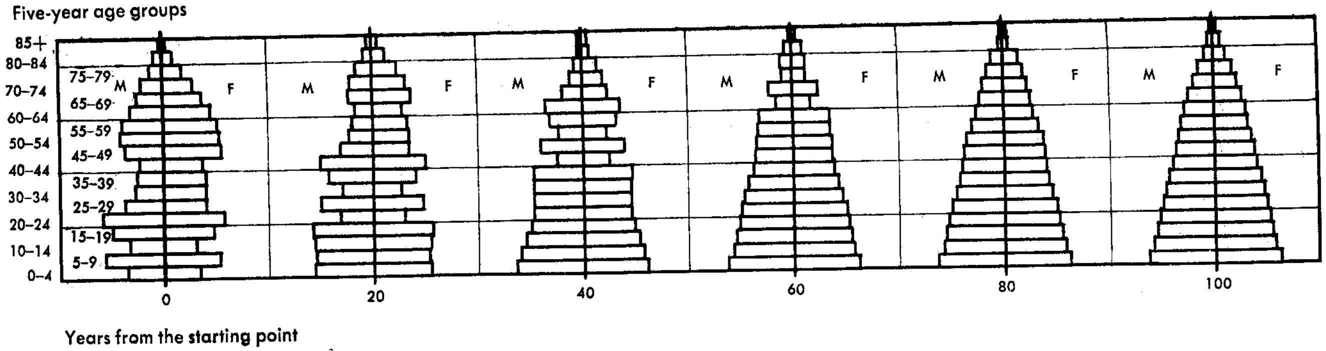
[12] This discontinuity is particularly marked in the population pyramids in graph I.8.



Graph I.3. Projections computed on the basis of the population of Eastern Germany in 1957. Variations in the age distribution of the population by five-year age groups

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50.

In the graphs, we find evidence of the following properties already given above as characteristic of Malthusian populations:

(i) The total number of population varies at a constant rate which is the same for both males and females;

(ii) Births and deaths vary at the same constant rate as the population, and consequently the crude death rates and birth rates are constant;

(iii) The crude death rates and birth rates computed on the basis of the female population, using female births and female deaths, are slightly different from the corresponding rates computed on the basis of the male population, using male births and male deaths;[13]

(iv) The age structure is invariable;

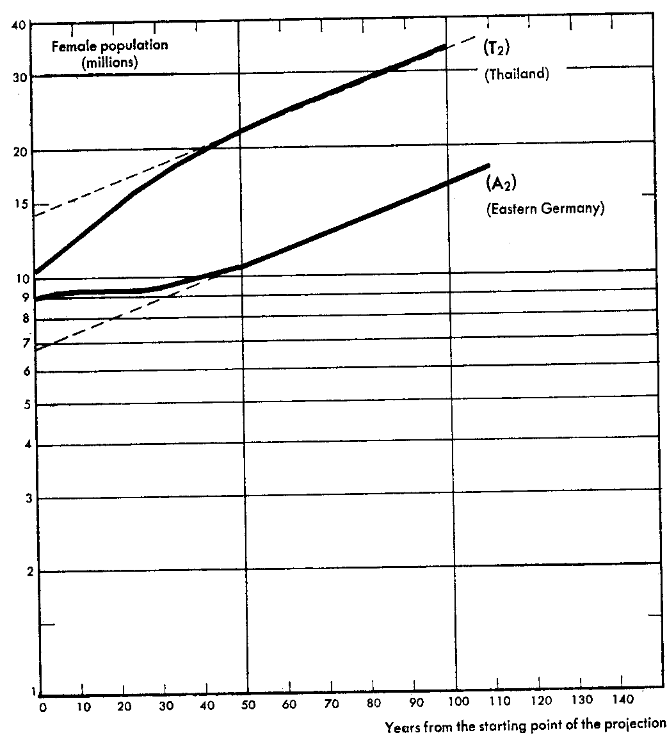
(v) There is a constant ratio between the total numbers of persons in the female and male populations.

As has been said, Lotka shows that the limit stable population is completely defined, once the laws of mortality and fertility are known. If, therefore, we compute a population projection on the basis of a population other than that of Eastern Germany in 1957, while keeping the same laws of mortality and fertility, we should arrive at a limit population identical with that described above.

Let us then repeat the preceding computations, retaining the same mortality and the same fertility, but starting from a population with an age structure different from that of the population of Eastern Germany. We have chosen for this purpose an estimate of the age structure of Thailand in 1955.[14] We find the same result as in the first computations; the demographic characteristics vary irregularly at the beginning, but become stable after a time. We note, however, that the crude birth rates, crude death rates and crude rates of increase settle down at the same levels in both projections (see graphs I.4 and I.5). The stabilized age structures are also identical in both computations (see graph I.6).

As expected, at the stable limit, the crude death rate, the crude birth rate and the crude rate of natural increase, as well as the age structure, are independent of the initial population and depend only on the laws of mortality and fertility.

In contrast, the irregular variations at the beginning and the level reached by the absolute numbers (of births, deaths, and number of persons in the population) when the rates become stabilized depend both on the initial conditions and on the laws of mortality and fertility.

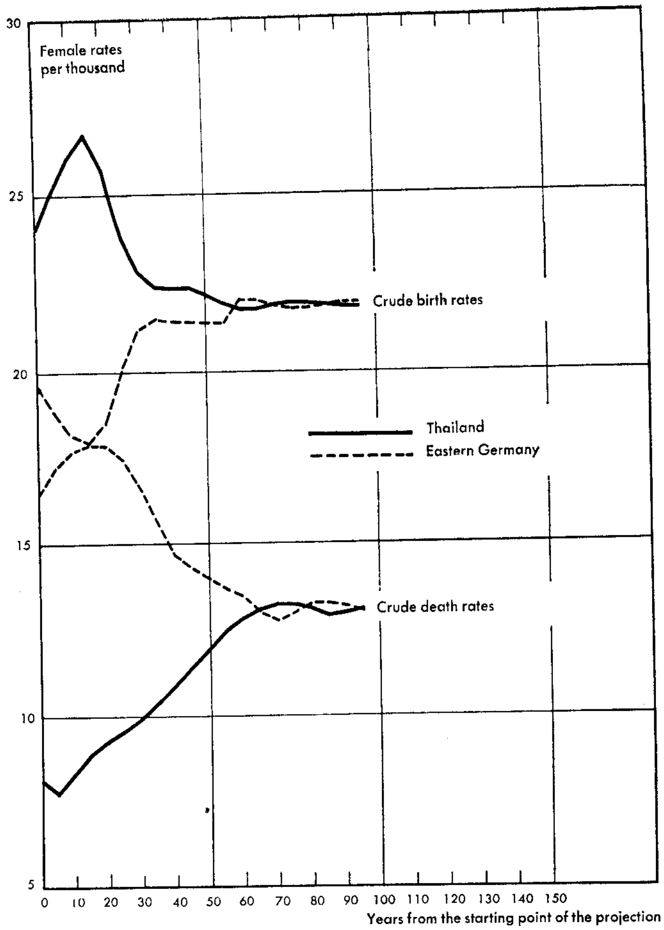


Graph I.4. Two sets of projections computed on the basis of the population of Eastern Germany in 1957 and of an estimate of the population of Thailand in 1955, respectively. Variations in the absolute number of persons in the female population

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50

[13] The male rates in the example chosen are higher than the female rates, and this characteristic is found in practice in most life tables. It is not, however, entirely impossible to perceive the contrary. This is an example of a property resulting from the fact that the mortality function is a function of human beings and as such, generally speaking, never varies greatly from the model mortality.

[14] *The Population of South-East Asia (including Ceylon and China (Taiwan))*, 1950-1980 (United Nations publication, Sales No.: 59.XIII.2).



Graph I.5. Two sets of projections computed on the basis of the population of Eastern Germany in 1955 and of an estimate of the population of Thailand in 1955, respectively. Variations in the crude female birth rate and the crude female death rate

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50

It is to be noted, first of all, that the levels reached in the stabilization phase vary considerably, according to the initial age structure.

One hundred years after the starting point, the female population is increasing each year by 8.7 per thousand, whether the age structure of Eastern Germany or that of Thailand is taken as the basis; when an initial population of one million women is assumed, however, the figure at the end of 100 years is 1,977,376 if the age structure of Eastern Germany is taken as a basis, but 3,275,000 if the age structure of Thailand is used.

The irregular oscillations observed at the beginning do not follow any general laws. The diversity of the initial age structures exerts its full effect in this phase of the computations. In contrast, certain regular patterns are to be observed in the levels reached by the absolute numbers at the period when stabilization is taking place.

In order to establish what these regular patterns are, let us repeat the preceding computations, retaining the same law of mortality, but taking different fertility rates in succession.

Two other projections (A_1) and (A_3) have been computed on the basis of the population of Eastern Germany by associating the same mortality as in projection (A_2) with two other levels of fertility, one corresponding to a gross reproduction rate of 0.75 [15] (projection A_1), the other corresponding to a gross reproduction rate of 1.17 (projection A_3). This latter level of fertility is such that the stable level corresponds to an unchanging number of persons in the population; the population is then said to be stationary. In both cases, intermediate model distributions of the age-specific fertility rates have been adopted.

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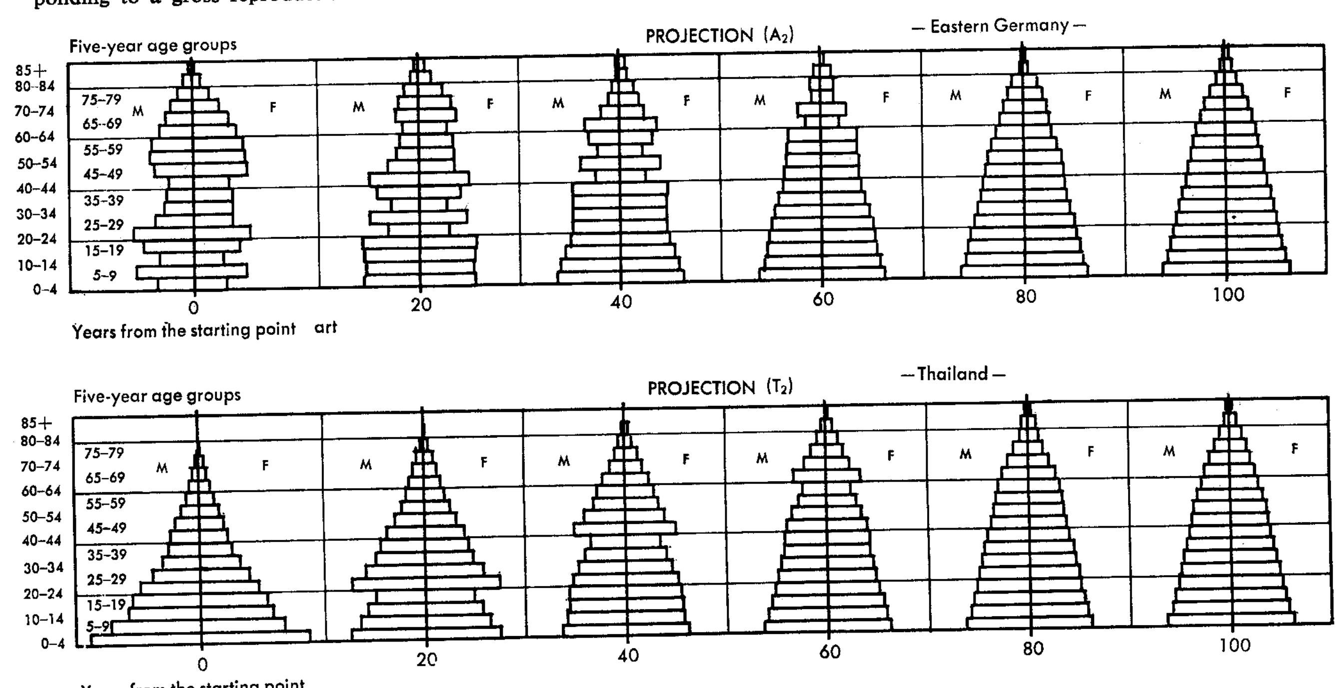
[15] Such a rate corresponds to a very low fertility leading to a declining stable population.



Graph I.6. Two sets of projections computed on the basis of the population of Eastern Germany in 1957 and of an estimate of the population of Thailand in 1955, respectively. Variations in the age distribution of the population by five-year age groups

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50.

Finally, similar projections (T_1) and (T_3) have been computed on the basis of the population of Thailand in 1955. Thus, we now have, for each of the two initial populations, two sets of three projections (A) and (T), which correspond to each other two by two in accordance with the assumptions regarding mortality and fertility.

Graphs I.7 and I.8 show how the age structure varies in these several projections.

At year zero, the population pyramids of each set are the same in all three projections. They correspond to the initial populations, which are the same in all three projections. The pyramid for set (A) is very different from that for set (T), however, as the initial populations chosen are, of course, very different.

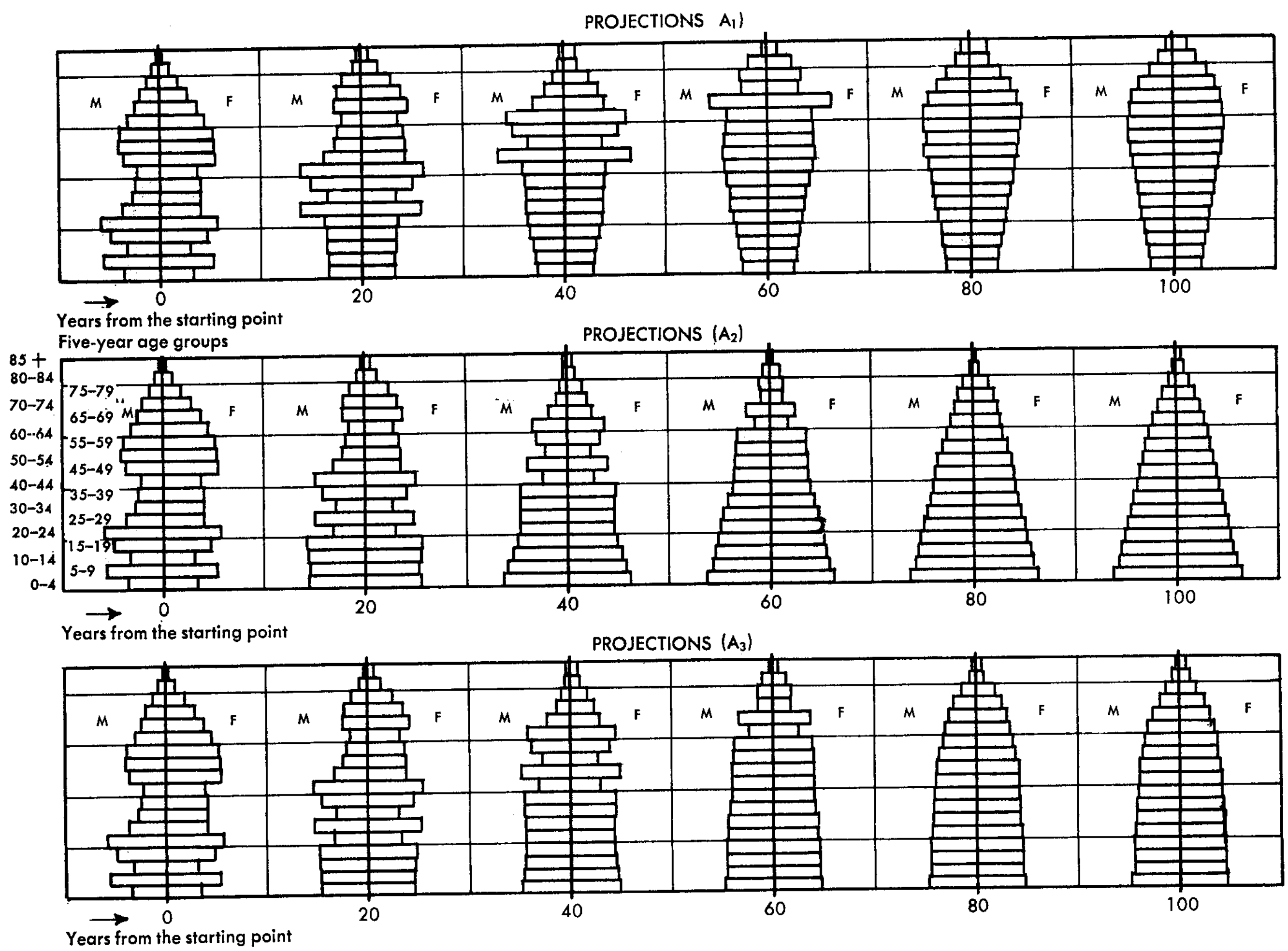
One hundred years after the starting point, the three pyramids of each of the two sets are very different from each other, because in the three projections very different fertilities were associated with the same mortality. However, the pyramids in sets (A) and (T) corresponding to the same fertility are identical, because the age structure of a stable population does not depend on the initial population.

On a semi-logarithmic graph for each projection, a stabilization straight line for total population, similar to those in graph I.4, can be drawn. Graph I.9 shows these straight lines for series (A), while graph I.10 shows them for series (T). Finally, graph I.11 shows the straight lines thus obtained for both series (A) and series (T).

When fertility varies in a continuous manner, we have a family of straight lines for each series, and it is to such a family that the three straight lines marked on graphs I.9, I.10 and I.11 belong. Each family has an envelope of which three tangents are known. The part of this envelope corresponding to the two series of projections [16] has been drawn in graph I.11. This envelope is a curve, concave to the origin, which passes through a maximum for the stationary population (points S and S′ in graph I.11).

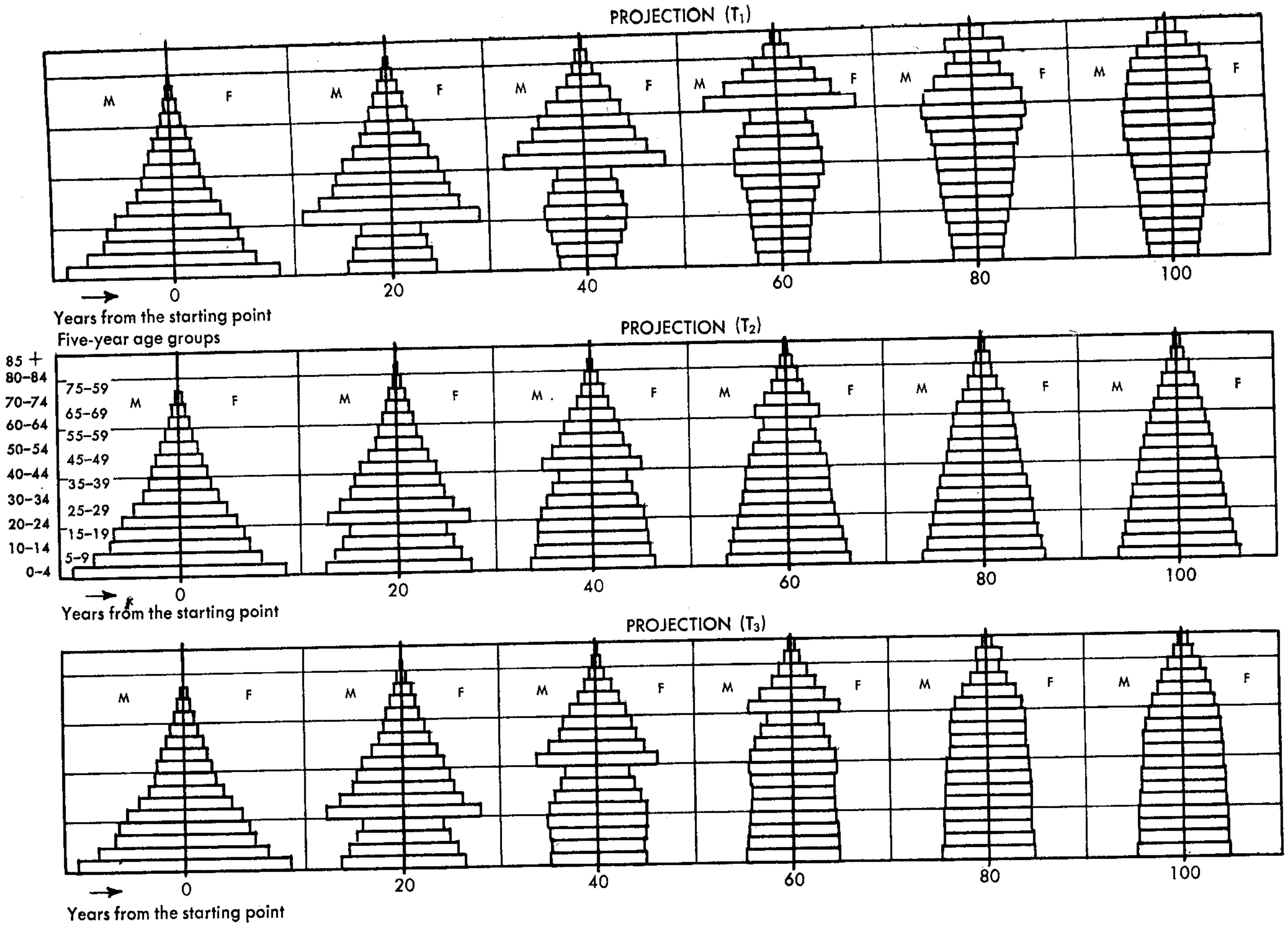
If the intrinsic rate of natural variation is small, as is often the case, the envelope is reduced in practice to its peak S. If, then, starting from different initial populations of equal numbers—for example, female populations of one million with different age distributions as may be found in different countries—we carry out the same computations as above, we obtain for each country a peak S, and the series of these peaks forms a curve the shape of which depends only to a minor degree on the fertility and mortality used for the computation. To all

[16] Formulae enabling the abscissae and ordinates of the points of contact of each straight line with the envelope to be calculated are to be found in annex I. See also the numerical applications in chapter III.



Graph I.7. Three sets of projections computed on the basis of the population of Eastern Germany in 1957. Variations in the age distribution of the population by five-year age groups

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 (projection A_1), 1.50 (projection A_2) and 1.17 (projection A_3).



Graph I.8. Three sets of projections computed on the basis of an estimate of the population of Thailand in 1955. Variations in the age distribution of the population by five-year age groups

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 projection (T_1), 1.50 (projection T_2) and 1.17 (projection T_3).

intents and purposes, the shape of this curve depends only on the initial age distributions of the populations, and it therefore enables those age distributions to be compared from the point of view of their growth potential.[17] See annex I for more details on how to construct and use these curves.

Such are the main properties of stable populations considered as the limit of a process of demographic evolution in which mortality and fertility remain unchanged. Obviously, the verification of these properties in some particular case is not sufficient to establish their validity, and a mathematical demonstration is necessary. This will be given in chapter III in a non-rigorous form used by Lotka. Already, one may note the usefulness of the concept of the limit stable population in demographic analysis.

If at a given moment the mortality and fertility of an actual population are stabilized at the levels attained at that moment, we set in motion a process of demographic evolution which approaches the stable population corresponding to the mortality and fertility of that moment.

This stable population is, in a way, the development of the conditions of mortality and fertility of the moment. It is determined entirely by the unique knowledge of the laws of fertility and mortality. It does not depend particularly on the age distribution of the moment. This age distribution makes itself felt at the beginning of the stabilization process, but with the passage of time its influence progressively diminishes, and at the limit it disappears completely. All the characteristics of this stable population are therefore actually characteristics of the moment. They describe what would happen if the mortality and fertility of the moment remained for ever unchanged from the level they had attained at that moment. They are by definition referred to as the intrinsic characteristics. We speak of the intrinsic crude birth rate, the intrinsic crude death rate and the intrinsic rate of natural variation. We must never forget, however, that these are characteristics of the moment, although they are defined as being part of a process of development.

E. A generalization of the concept of a limit stable population

What has been done for stable populations can obviously also be done for all the particular Malthusian populations of sub-set $H_0(r)$. We may ask, for example, what happens in a population where *the mortality and the crude birth rate* remain constant, or in a population

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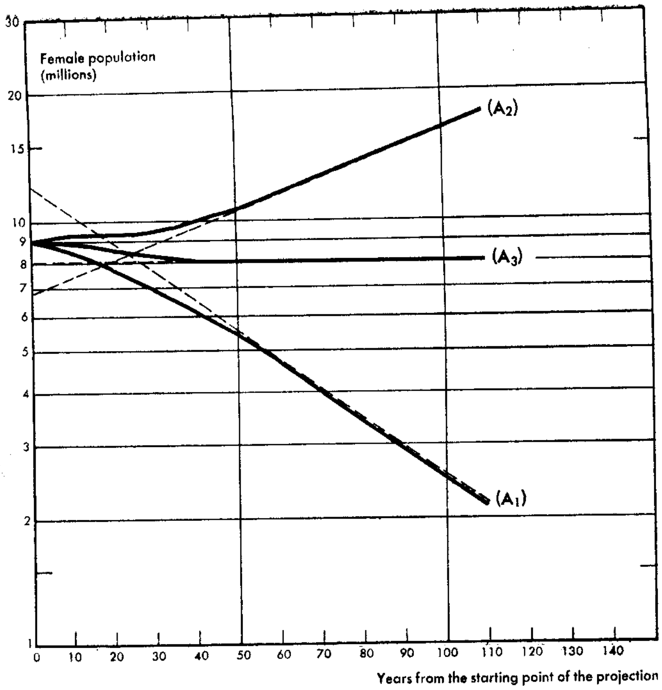
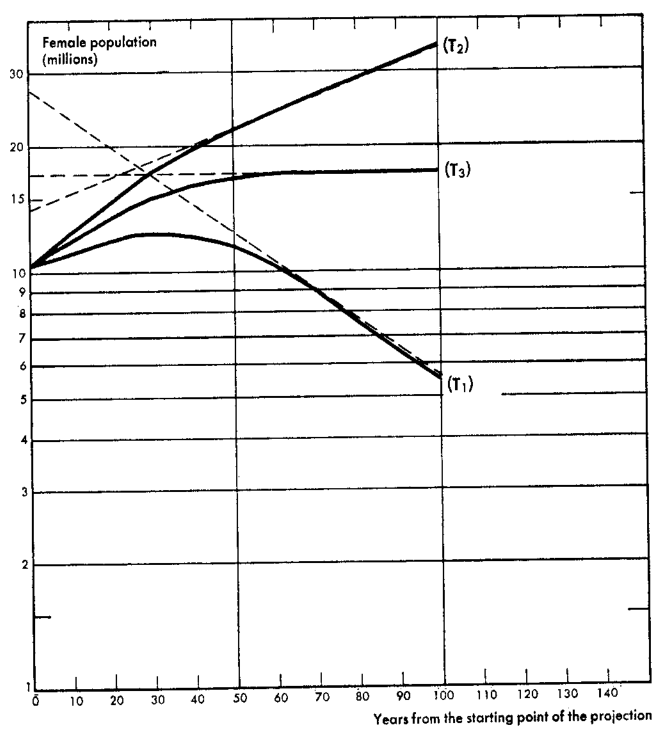
[17] This point was the subject of a paper submitted to the Société de statistique de Paris by Mr. P. Vincent. Cf. P. Vincent, "*Potentiel d'accroissement d'une population*", *Journal de la Société de statistique de Paris* (January-February 1945), pp. 16 *et seq.* Some comments on the method developed by Mr. Vincent will be found in annex I.



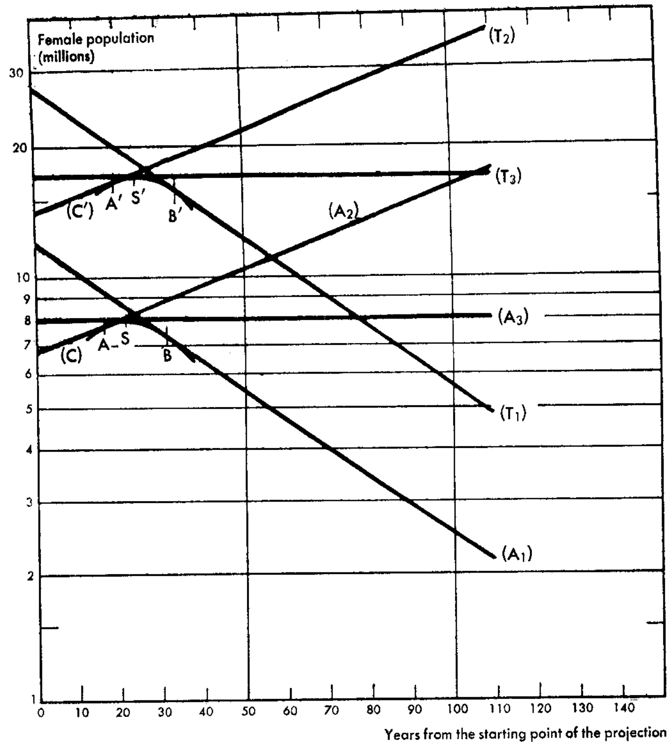
Graph I.9. Three sets of projections computed on the basis of the population of Eastern germany in 1957. Variations in the absolute number of persons in the female population

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 (projection A_1), 1.50 (projection A_2) and 1.17 (projection A_3).



Graph I.10. Three sets of projections computed on the basis of an estimate of the population of Thailand in 1955. Variations in the absolute number of persons in the female population

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 (projection T_1), 1.50 (projection T_2) and 1.17 (projection T_3)



Graph I.11. Two series of three sets of projections computed on the basis of the population of Eastern Germany in 1957 (series A) and of an estimate of the population of Thailand in 1955 (series T). Stabilization straight lines for the female population

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 (projections A_1 and T_1), 1.50 (projections A_2 and T_2) and 1.17 (projections A_3 and T_3)

where *the mortality and the crude death rate* remain constant, and so forth. In each of the cases considered above we show that, in general, the population approaches the corresponding Malthusian population of the sub-set $H_0(r)$ or, when the given conditions are applicable to several Malthusian populations[18] of sub-set $H_0(r)$, towards one of the corresponding Malthusian populations.

F. Priority of the concept of a stable population

It is easy to see why, as mentioned earlier, the concept of a stable population has become quite popular in demographic analysis. The laws of mortality and fertility of the moment can be considered, quite satisfactorily, as indices of health conditions and of the reproductive behaviour of couples. The stable population thus enables an assessment of health conditions and reproductive behaviour to be made. In contrast, the laws of mortality and of the crude birth rate, for example, while still forming a good index of health conditions, give a picture of the reproductive behaviour of couples which is distorted by the age distribution of the moment. Part of this current age distribution is thus transferred into the limit Malthusian population, while the influence of the age distribution of the moment disappears from the stable

[18] This is so, for example, in the case of a population whose survivorship function and crude death rate remain invariable (see above).

population. What is stated above regarding the laws of mortality and of the crude birth rate taken together is equally true for all the other associations considered.

It is no doubt of interest from a theoretical point of view to see how the concept of a stable population fits into a wider set of Malthusian populations of different types and to show that, by the use of mathematics, it is possible to define processes similar to those leading to the limit of the notion of stable populations, but taking as the starting point conditions different from those which are at the basis of stable populations. It must be kept in mind, however, that none of these attempts can go very far, whereas the concept of stable populations has many useful applications.

G. Malthusian populations with known age distributions

In contrast, a study of the other sub-sets of Malthusian populations which were referred to at the beginning of this chapter (the sub-sets F(*r*)) leads to important results. These other sub-sets are in a sense symmetrical with the sub-sets H(*r*). Each sub-set H(*r*) is made up of all Malthusian populations for which the mortality function is known. The populations of each sub-set H(*r*) therefore satisfy the following conditions:

(i) *Constant* age distribution;

(ii) *Constant and known* mortality.

Each sub-set F(*r*) consists of Malthusian populations where age distribution is known. The populations of each sub-set F(*r*) therefore satisfy the following conditions:

(i) *Constant and known* age distribution;

(ii) *Constant* mortality.

The study of the sub-sets F(*r*) lends itself to developments similar to those described above in connexion with the sub-sets H(*r*).

Here we are breaking new ground, since Lotka and his successors did not study the sub-sets F(*r*). The main properties of the sub-sets F(*r*) are indicated in chapter IV. For the moment, we shall confine ourselves, in chapters II and III, to a further study of the properties of the populations of sub-sets H(*r*) with known mortality functions and, in particular, to giving practical ways of calculating the various characteristics of these populations.

Chapter II

MALTHUSIAN POPULATIONS WITH KNOWN MORTALITY FUNCTIONS (sub-sets H(r))

The way in which a sub-set of Malthusian populations $H_0(r)$ could be defined for each survivorship function $p_0(a)$ was described in the preceding chapter. It was also stated that the knowledge of an additional condition made it possible to determine from such a sub-set a particular Malthusian population, or at least a limited number of Malthusian populations which satisfied that condition. In particular, knowledge of the fertility function makes it possible to single out a particular Malthusian population called "stable population".

Chapter II resumes the study of the sub-sets H(r) with reference to actual cases, or, more precisely, the determination of particular populations subject to a knowledge of an additional condition.

A. Fundamental formulae

Notation used

The study of the sub-sets H(r) involves most of the mathematical functions used in pure demography. The notation employed has been more or less fixed by usage, but as the available notation has been built up over the years in a somewhat unco-ordinated manner, it sometimes happens that there are too many symbols or, conversely, that different functions are expressed by almost identical symbols. We must therefore make it clear what notation is to be used in this study, and this notation is set out in table II.1. Generally speaking, each function can refer to the female population, the male population or both together. Symbols are given the subscript f when used for the female population and the subscript m when used for the male population. For example, the survivorship function at age a is written $p(a)$ for both sexes together, $p_f(a)$ for the female population, and $p_m(a)$ for the male population.

In table II.1, the only variable is the age (or age group). It frequently happens, however, that the functions under consideration depend also on other variables, such as time. In such a case, the same symbols are used, but the additional variables are written next to the age. For example, a female survivorship depending on time is written $p_f(a, t)$.

Examination of table II.1 shows that for any given concept there are generally two functions: one valid continuously (instantaneous indices) and the other valid discontinuously (age-specific indices). Generally speaking, the functional notation of the form $f(x)$ is reserved for continuous notation, while the notation using indices of the form i_m is used for discontinuous notation.

Table II.1. The main symbols used to represent the functions encountered in this study

Survivorship function at age a	$p(a)$
Instantaneous death rate at age a	$q(a)$
Probability of death within n years from age a	${}_nq_a$
Death rate for a specific age group * (e.g., the age group 20-24)	$m_{20\text{-}24}$
Stationary population * (e.g., population within an age group 20-24)	$L_{20\text{-}24}$
Life expectancy at age a	$\overset{a}{e}_0$
Instantaneous probability o survival *	$s(a)$
Probability of survival from age a to age $a + n$	${}_ns_a$
Survival ratio from one age group to the next, * $\frac{L_{25\text{-}29}}{L_{20\text{-}24}}$	$S_{20\text{-}24}$
Instantaneous fertility rate at age a	$\varphi(a)$
Fertility rate for a specific age group * (e.g., 20-24)	$f_{20\text{-}24}$
Age distribution of fertility rates	$F(a)$
Distribution of fertility rates by age groups * (e.g., for the age group 20-24)	$F_{20\text{-}24}$
Total number of persons of all ages in the population	K
Number of persons aged between a and $a + da$ years	$K(a)$
Number of persons in a given age group * (e.g., 20-24)	$K_{20\text{-}24}$
Age distribution of the population at age a	$C(a)$
Distribution of the population by age groups * (e.g., 20-24)	$C_{20\text{-}24}$
Logarithmic derivative of C(a) with changed sign:	
(1) In continuous form: $\frac{-C'(a)}{C(a)} = Q(a)$	

(2) In discontinuous form: $-\dfrac{C(a+5)-C(a)}{C(a)} = {}_5Q_a$	
Number of deaths between a and $a + da$ years of age	$D(a)$
Number of deaths of a given age group * (e.g., 20-24)	$D_{20\text{-}24}$
Age distribution of deaths	$d(a)$
Distribution of deaths by age groups * (e. g., the age group 20-24 whose median age is 22.5 years)	$d_{20\text{-}24}$
Number of children born to mothers aged between a and $a + da$ years	$B(a)$
Number of children born to mothers of a given age group * (e.g., 20-24)	$B_{20\text{-}24}$
Number of births at a given point in time	$B(t)$
Crude birth rate	b
Number of deaths at a given point in time	$D(t)$
Crude death rate	d
Age at the beginning of the female reproductive period	u
Age at the end of the female reproductive period	v
Age limit of human life	ω
Masculinity at birth (number of male births per female birth)	m

* When the interval of the age group in question is known, only the lower age is given in the notation, e.g., m_{20}, L_{20}, S_{20}, f_{20}, F_{20}, K_{20}, C_{20}, D_{20}, d_{20}, B_{20}, instead of m_{20-24}, L_{20-24}, S_{20-24}, f_{20-24}, F_{20-24}, K_{20-24}, C_{20-24}, D_{20-24}, d_{20-24}, B_{20-24}.

DEFINITIONAL RELATIONS AMONG VARIOUS FUNCTIONS

The various functions of table II.1 are not independent. By definition, they are linked by certain relations, of which the main ones are the following:

(*a*) For mortality functions, we have the following well-known equations:

$$q(a) = -\frac{p'(a)}{p(a)} = \frac{D(a)}{K(a)}$$

$$_nq_a = \frac{p(a) - p(a+n)}{(pa)}$$

($_nq_a$ tends to $q(a)$ when n approaches zero)

$$e_0^o = \int_0^\omega p(a)da$$

$$e_0^a = \frac{1}{p(a)}\int_a^\omega p(a)da$$

$$m_{20\text{-}24} = \frac{D_{20\text{-}24}}{K_{20\text{-}24}}$$

(*b*) For fertility functions we have the following equations:

$$\varphi(a) = \frac{B(a)}{K(a)}$$

$$f_{20\text{-}24} = \frac{B_{20\text{-}24}}{K_{20\text{-}24}}$$

(*c*) The values relating to a given age group are the integrals of the corresponding instantaneous functions in that age group.

If we set aside the two extremities of life and if the interval k between the age groups is not too wide, we find that the value of a given attribute corresponding to a given age group is almost equal to k times the instantaneous value corresponding to the median age of that age group.

Thus, for five-year age groups ($k = 5$) we have:

$$L_{20\text{-}24} = \int_{20}^{25} p(a)da \,\#\, 5p(22.5)$$

$$K_{20\text{-}24} = \int_{20}^{25} K(a)da \,\#\, 5K(22.5)$$

$$C_{20\text{-}24} = \int_{20}^{25} C(a)da \,\#\, 5C(22.5)$$

$$D_{20\text{-}24} = \int_{20}^{25} D(a)da \,\#\, 5D(22.5)$$

$$d_{20\text{-}24} = \int_{20}^{25} d(a)da \,\#\, 5d(22.5)$$

$$B_{20\text{-}24} = \int_{20}^{25} B(a)da \,\#\, 5B(22.5)$$

It may be noted in passing that it follows from the first equation that:

$$\overset{0}{e_0} = \sum_0^\omega L_a$$

Morevoer (again disregarding the extremities of life), we have the approximate relationship:

$$\frac{p(22.5) + p(27.5)}{2} \,\#\, p(25) \,\#\, \frac{1}{10}[L_{20\text{-}24} + L_{25\text{-}29}]$$

It follows that:

$$\overset{25}{e_0} \,\#\, \frac{10\sum_{25}^\omega L_a}{L_{20\text{-}24} + L_{25\text{-}29}}$$

(*d*) As the rates are the quotient of two magnitudes, in the process the age-group interval k vanishes. The age-group rates are therefore equal to the instantaneous ones which correspond to the median age of the age group, and we can write:

$$m_{20\text{-}24} = \frac{D_{20\text{-}24}}{K_{20\text{-}24}} \# \frac{5D(22.5)}{5K(22.5)} = \frac{D(22.5)}{K(22.5)} = q(22.5)$$

$$f_{20\text{-}24} = \frac{B_{20\text{-}24}}{K_{20\text{-}24}} \# \frac{5B(22.5)}{5K(22.5)} = \frac{B(22.5)}{K(22.5)} = \varphi(22.5)$$

Some confusions to be avoided

The use of the letter "m" may cause confusion, and its meaning needs to be made quite clear:

(*a*) Used alone, this letter designates masculinity at birth;

(*b*) Used as a subscript, it indicates that the function in question refers to the male population;

(*c*) Used as a notation with subscripts, it designates the death rate for a certain age group. For example, when it is used to designate the death rate for the 20-24 age group it is written: $m_{20\text{-}24}$.

The letter f or F may also cause confusion:

(*a*) When f is used as subscript, it indicates that the function in question refers to the female population;

(*b*) When f is used as a notation with subscripts, it designates the fertility rate for a certain age group. For example, the fertility rate for the 20-24 age group is written: $f_{20\text{-}24}$;

(*c*) When F is used as a functional symbol, it designates the age distribution of fertility rates: $F(a)$.

(*d*) When F is used as a symbol with subscripts, it designates the distribution of fertility rates by age groups. For example, the distribution of birth rates for the 20-24 age group is written: $F_{20\text{-}24}$.

Fundamental formulae for the female population

The main formulae have already been given in chapter I. They are given again in this chapter in various equivalent forms, so that the form best adapted to the problem under consideration can be selected. Some additional formulae not included in chapter I are also given.

The formulae are given in continuous notation. They can easily be written in discontinuous notation by using the approximate relationships between the two systems of notation given above.

Finally, while most formulae are valid only for Malthusian populations, some are valid for all populations, and these are indicated in the course of the text.

(1) The crude female birth rate is written:

$$b_f = \frac{1}{\int_0^\omega e^{-ra} p_f(a)da} \qquad \text{(II.1)}$$

(2) The crude female death rate is:

$$d_f = b_f - r \qquad \text{(II.2)}$$

(3) The age distribution of the female population is written:

$$C_f(a) = b_f e^{-ra} p_f(a) \qquad \text{(II.3}a\text{)}$$

This formula can be written in the following equivalent forms:

$$C_f(a) = \frac{e^{-ra} p_f(a)}{\int_0^\omega e^{-ra} p_f(a)da} \qquad \text{(II.3}b\text{)}$$

$$C_f(a) = C_f(0) e^{-ra} p_f(a) \qquad \text{(II.3}c\text{)}$$

$$\frac{C_f(a)e^{ra}}{C_f(0)} = p_f(a) \qquad \text{(II.3}d\text{)}$$

Finally, for $a = 0$ we have:

$$C_f(0) = b_f \qquad \text{(II.3}e\text{)}$$

This last formula is valid for all populations.

(4) If we differentiate formula II.3*d* we can write: and consequently:

$$p_f'(a) = \frac{C_f'(a)e^{ra} + rC_f(a)e^{ra}}{C_f(0)}$$

and consequently:

$$q_f(a) = -\frac{p_f'(a)}{p_f(a)} = -\frac{C_f'(a) + rC_f(a)}{C_f(a)}$$

whence finally:

$$q_f(a) = -\frac{C_f'(a)}{C_f(a)} - r = Q_f(a) - r \qquad \text{(II.4)}$$

(5) At time t we have the following formulae for the absolute number of female births $B_f(t)$ and the absolute number of persons in the female population $N_f(t)$:

$$B_f(t) = Ae^{rt} \qquad \text{(II.5}a\text{)}$$

$$N_f(t) = \frac{B_f(t)}{b_f} = Ae^{rt}\int_0^\omega e^{-ra} p_f(a)da \qquad \text{(II.5}b\text{)}$$

where A is a constant equal to the absolute number of female births at time zero (i.e., $B_f(0) = A$).

(6) The age distribution of female deaths is given by the formula:

$$d_f(a) = \frac{C_f(a)q_f(a)}{\int_0^\omega C_f(a)q_f(a)da} = \frac{C_f(a)q_f(a)}{d_f} \qquad \text{(II.6}a\text{)}$$

This formula can be written in the following equivalent forms:

$$q_f(a) = \frac{d_f(a)}{C_f(a)}\int_0^\omega C_f(a)q_f(a)da = \frac{d_f(a)}{C_f(a)} d_f \qquad \text{(II.6}b\text{)}$$

or:

$$q_f(a) = \frac{d_f(a)}{C_f(a)} [C_f(0) - r] \qquad \text{(II.6}c\text{)}$$

Formulae II.6a, II.6b *and* II.6c *are valid for all populations.*

(7) From formula II.6*a* we deduce that:

$$C_f(a) = \frac{d_f(a)}{q_f(a)} d_f$$

whence, as $C_f(a)$ is a distribution:

$$1 = \int_0^\omega C_f(a)da = d_f \int_0^\omega \frac{d_f(a)}{q_f(a)} da$$

whence:

$$d_f = \frac{1}{\int_0^\omega \frac{d_f(a)}{q_f(a)} da}$$

If we transfer this value of d_f into II.6*b* we obtain:

$$C_f(a) = \frac{\frac{d_f(a)}{q_f(a)}}{\int_0^\omega \frac{d_f(a)}{q_f(a)} da} \qquad \text{(II.7)}$$

This last formula is valid for all populations.

(8) In a Malthusian population, the age distribution of deaths is also written:

$$d_f(a) = \frac{e^{-ra} p_f(a) q_f(a)}{\int_0^\omega e^{-ra} p_f(a) q_f(a) da} \qquad \text{(II.8)}$$

(9) As $p_f(a)q_f(a) = -p'_f(a)$, the formula II.8 is written:

$$\frac{d_f}{b_f} d_f(a)e^{ra} = -p'_f(a) \qquad \text{(II.9)}$$

(10) This last formula allows us to write, by integrating:

$$\frac{d_f}{b_f} \int_0^a d_f(a)e^{ra}da = -\int_0^a p'_f(a)da = 1 - p_f(a)$$

If we make $a = \omega$ in this formula, we have:

$$\frac{d_f}{b_f} \int_0^\omega d_f(a)e^{ra}da = 1$$

Whence, by dividing the last two formulae member by member, we have:

$$\frac{\int_0^a d_f(a)e^{ra}da}{\int_0^\omega d_f(a)e^{ra}da} = 1 - p_f(a)$$

whence finally:

$$p_f(a) = 1 - \frac{\int_0^a d_f(a)e^{ra}da}{\int_0^\omega d_f(a)e^{ra}da} \qquad \text{(II.10)}$$

From this value of $p_f(a)$ it is possible to obtain a useful formula for $q_f(a)$.

We have:

$$p'_f(a) = \frac{d_f(a)e^{ra}}{\int_0^\omega d_f(a)e^{ra}da}$$

whence:

$$q_f(a) = -\frac{p'_f(a)}{p_f(a)} = -\frac{d_f(a)e^{ra}}{\int_0^\omega d_f(a)e^{ra}da - \int_0^a d_f(a)e^{ra}da}$$

$$= -\frac{d_f(a)e^{ra}}{\int_a^\omega d_f(a)e^{ra}da} \qquad \text{(II.10}a\text{)}$$

(11) If we write formulae II.4 and II.6*c* side by side, whe have:

$$q_f(a) = -\frac{C'_f(a)}{C_f(a)} - r$$

$$q_f(a) = \frac{d_f(a)}{C_f(a)} [C_f(0) - r]$$

If we eliminate r from these two equations we obtain:

$$q_f(a) = \frac{d_f(a)}{C_f(a)} \times \frac{C'_f(a) + C_f(a)C_f(0)}{C_f(a) - d_f(a)} \qquad \text{(II.11)}$$

(12) If instead of eliminating r we eliminate $q_f(a)$, we obtain:

$$r = \frac{C_f(0)d_f(a) + C'_f(a)}{-C_f(a) + d_f(a)} \qquad \text{(II.12)}$$

(13) At a given time, we have the equation:

$$\int_u^v e^{-ra} p_f(a) \varphi_f(a, t) da = 1 \qquad \text{(II.13)}$$

In the particular case of the stable population, we assume a value of $\varphi_f(a)$ for the function $\varphi_f(a, t)$ which no longer depends on the time t. The intrinsic rate of natural variation is the real root of the r equation:

$$\int_u^v e^{-ra} p_f(a) \varphi_f(a) da = 1 \qquad \text{(II.14)}$$

This equation differs from equation II.13 only in that the fertility function no longer depends on time ($\varphi_f(a)$ instead of $\varphi_f(a, t)$).

Formulae for the male population

All these formulae are valid both for the male population and for populations of both sexes together, provided that the female mortality functions are replaced by the male mortality functions or by the functions for both sexes together, as the case may be.

For equation II.13, $\varphi_f(a, t)$ must be replaced by $\varphi_m(a, t)$, which represents the number of boys fathered by men of age a, while the limits u and v of the reproductive period must be modified. When both sexes are taken together, such an equation obviously becomes meaningless.

Formula II.14 can also be written for male populations. Values cannot be assumed for the functions $\varphi_f(a)$ and $\varphi_m(a)$ simultaneously, however.

If we assume a value for $\varphi_f(a)$, then equation (II.14) determines the intrinsic rate of natural variation ϱ_1 and, consequently, the corresponding stable population. $\varphi_m(a)$ must then satisfy the condition:

$$\int_0^{\omega} e^{-\varrho_1 a} p_m(a)\varphi_m(a)da = 1$$

Thus, we can no longer assume an arbitrary value for $\varphi_m(a)$.

Likewise, *if we assume a value for* $\varphi_m(a)$, then the equation:

$$\int_0^{\omega} e^{-ra} p_m(a)\varphi_m(a)da = 1$$

determines an intrinsic rate of natural variation ϱ_2. The function $\varphi_f(a)$ must then satisfy the condition:

$$\int_0^{\omega} e^{-\varrho_2 a} p_f(a)\varphi_f(a)da = 1$$

Thus, we can no longer assume an arbitrary value for $\varphi_f(a)$.[1]

[1] In an actual population, if $p_f(a)$, $p_m(a)$, $\varphi_f(a)$ and $\varphi_m(a)$ are the mortality and fertility functions *observed* respectively at a given time, the two integral equations:

$$\int_0^{\omega} e^{-ra} p_m(a)\varphi_m(a)da = 1 \quad \text{and} \quad \int_0^{\omega} e^{-ra} p_f(a)\varphi_f(a)da = 1$$

are generally incompatible.

Hereafter, any mention of a stable population will be a reference to the case where the female fertility function is given.

Table II.2 sets out the various formulae considered above and classifies them according to the parameter which they make it possible to calculate. In order to make them simpler to write, they have been given with neither female nor male subscripts, except in the case of the gross and net reproduction rates, which generally speaking are always for the female sex.

At the end of the list, we have added the traditional formulae giving the gross and net reproduction rates and the approximate formula described in chapter I linking the crude male and female death and birth rates with the same rates for both sexes together.

The formulae in table II.2 are given in continuous notation. They can easily be written in discontinuous notation, but their use then raises new problems which will be considered in connexion with each particular application.

TABLE II.2. BASIC FORMULAE LINKING THE VARIOUS CHARACTERISTICS OF MALTHUSIAN POPULATIONS

Crude birth rate

$$b = \frac{1}{\int_0^{\omega} e^{-ra} p(a)da} \qquad \text{(II.1)}$$

$$b = C(0) \qquad \text{(II.3}e\text{)*}$$

Crude death rate

$$d = b - r \qquad \text{(II.2)}$$

Age structure of the population

$$C(a) = be^{-ra}p(a) \qquad \text{(II.3}a\text{)}$$

$$C(a) = C(0)e^{-ra}p(a) \qquad \text{(II.3}c\text{)}$$

$$C(a) = \frac{e^{-ra}p(a)}{\int_0^{\omega} e^{-ra} p(a)da} \qquad \text{(II.3}b\text{)}$$

$$C(a) = \frac{\dfrac{d(a)}{q(a)}}{\int_0^{\omega} \dfrac{d(a)}{q(a)} da} \qquad \text{(II.7)}$$

$$C(a) = \frac{d(a)}{q(a)} \times d \qquad \text{(II.6}a\text{)}$$

Instantaneous death rate

$$q(a) = -\frac{C'(a)}{C(a)} - r = Q(a) - r \qquad \text{(II.4)}$$

$$q(a) = \frac{d(a)}{C(a)} [C(0) - r] \qquad \text{(II.6}c\text{)*}$$

$$q(a) = \frac{d(a)}{C(a)} \times d \qquad \text{(II.6}b\text{)*}$$

$$q(a) = -\frac{d(a)e^{ra}}{\int_a^{\omega} d(a)e^{ra}da} \qquad \text{(II.10}a\text{)}$$

$$q(a) = \frac{d(a)}{C(a)} \times \frac{C'(a) + C(a)C(0)}{C(a) - d(a)} \qquad \text{(II.11)}$$

Survivorship function

$$p(a) = \frac{C(a)}{C(0)} e^{ra} \qquad \text{(II.3}d\text{)}$$

$$p(a) = 1 - \frac{\int_0^a d(a)e^{ra}da}{\int_0^\infty d(a)e^{ra}da} \qquad \text{(II.10)}$$

Age structure of deaths

$$d(a) = \frac{C(a)q(a)}{\int_0^\infty C(a)q(a)da} \qquad \text{(II.6}a\text{)}$$

$$d(a) = -\frac{b}{d} p'(a)e^{-ra} \qquad \text{(II.9)}$$

$$d(a) = \frac{e^{-ra}p(a)q(a)}{\int_0^\infty e^{-ra}p(a)q(a)da} \qquad \text{(II.8)}$$

Approximate relation between the crude female and crude male birth rates

$$b \# \frac{b_m + b_f}{2}$$

Approximate relation between the crude female and crude male birth rates

$$d \# \frac{d_m + d_f}{2}$$

Relation among mortality, fertility and rate of natural increase

$$\int_u^v e^{-ra}p(a)\varphi(a, t)da = 1 \qquad \text{(II.13)}$$

In a stable population, $\varphi(a, t)$ is not dependent on time, and equation (II.13) is written:

$$\int_u^v e^{-ra}p(a)\varphi(a)da = 1 \qquad \text{(II.14)}$$

In addition, we have:

$$\int_u^v \varphi(a, t)C(a)da = b \qquad \text{(II.15)}$$

Gross reproduction rate

$$R' = \int_u^v \varphi_f(a, t)da$$

Net reproduction rate

$$R_0 = \int_u^v p_f(a)\varphi_f(a, t)da$$

Rate of natural variation

$$r = \frac{C(0)d(a) + C'(a)}{-C(a) + d(a)} \qquad \text{(II.12)}$$

Births at time t

$$B(t) = Ae^{rt} = B(0)e^{rt} \qquad \text{(II.5}a\text{)}$$

Population at time t

$$N(t) = B(0)e^{rt}\int_0^\infty e^{-ra}p(a)da \qquad \text{(II.5}b\text{)}$$

Increase at time t

$$rN(t) = rB(0)e^{rt}\int_0^\infty e^{-ra}p(a)da$$

Deaths at time t

$$D(t) = B(0)e^{rt}\left[1 - r\int_0^\infty e^{-ra}p(a)da\right]$$

(*) Formulae valid for all populations.

B. Determination of a population in a sub-set $H_0(r)$ (a Malthusian population whose mortality function is known and which satisfies a given condition)

It has already been stated on several occasions that in a sub-set $H_0(r)$ the knowledge of an additional condition makes it possible to determine a population [2] which satisfies that condition. The principle of this determination is that the additional condition makes it possible to write an equation in r, generally with only one real solution. The population of the sub-set corresponding to that solution [3] is the population satisfying the given condition. We now propose to show, in actual cases, how populations of a sub-set $H_0(r)$ which satisfy a condition are determined.

FIRST EXAMPLE: THE RATE OF NATURAL VARIATION IS KNOWN

The simplest way of determining a population of a sub-set $H_0(r)$ is obviously to take the rate of natural variation, $r = r_0$. Here is an example of the computations:

Let us consider the sub-set $H_0(r)$ associated with the model life tables given in *Manual III*, which correspond to an expectation of life at birth for both sexes of 50 years (level 60). Let us assume that $r = r_0 = 0.03$ and let us calculate the characteristics of the *female population* which has this rate of increase.

[2] Or at least a small number of populations.

[3] It may happen that the r equation which enables the additional condition to be written has several real roots, but there are never many of them. A population of the set $H(r)$ corresponds to each of these real roots.

Table II.3 gives details of the computation of the age distribution of the population and the age distribution of deaths.

Age distribution of the population

Let us suppose that we know the survivorship function, in discontinuous notation, for the following age groups: under 1 year, 1-4 years, 5-9 years, and so on by five-year age groups up to 80-84 years, and the 85 and over age group. The corresponding quantities L_a are given in the third column of table II.3.[4] The age distribution of the population is given in continuous notation by formula II.3*a* in table II.2:

$$C_f(a) = b_f e^{-ra} p_f(a) \qquad \text{(II.3}a\text{)}$$

For an age group a, $a + 5$ we assume that:

$$\int_a^{a+5} C_f(a)da = b_f \int_a^{a+5} e^{-ra} p_f(a)da = b_f e^{-r\alpha} L_a$$

where α is the median age of the age group (first column of the table).

The quantities $e^{-r\alpha}$ are given in the fourth column. By multiplying together the third and fourth columns we obtain the quantities $e^{-r\alpha}L_a$ (column 5). The distribution of these quantities is the distribution sought C_a (column 6).

Crude birth rate

The crude birth rate is given in continuous notation by formula II.1 in table II.2:

[4] These quantities are taken from table IV in the appendix to *Manual III*. See *Manual III: Methods for Population Projections by Sex and Age* (United Nations publication, Sales No.: 56.XIII.3).

TABLE II.3. COMPUTATION OF THE AGE DISTRIBUTION OF THE FEMALE POPULATION AND OF FEMALE DEATHS IN A STABLE POPULATION CORRESPONDING TO AN INTERMEDIATE MODEL LIFE TABLE WITH AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 50 YEARS AND AN INTRINSIC RATE OF NATURAL VARIATION OF 0.03

Median age α (1)	*Age group* (a) *(years)* a (2)	*Stationary population* L_a (3)	$e^{-r\alpha}$ *for* $r = 0.03$ (4)	*Product of the two preceding columns* $e^{-r\alpha}L_a$ (5)	*Age distribution* C_a (6)	*Death rate (per thousand)* m_a (7)	*Product of the two preceding columns (deaths)* (8)	*Age distribution of deaths* d_a (9)
0.5	0	90 719	0.98511	89 368	40 282	136.41	5 495	366 651
3.0	1-4	338 974	0.91393	309 799	139 640	16.19	2 261	150 864
7.5	5-9	406 628	0.79852	324 701	146 358	3.99	584	38 967
12.5	10-14	399 620	0.68729	274 655	123 799	2.96	366	24 421
17.5	15-19	392 370	0.59156	232 110	104 623	4.38	458	30 560
22.5	20-24	382 368	0.50916	194 686	87 754	5.97	524	34 964
27.5	25-29	370 680	0.43824	162 447	73 222	6.45	472	31 494
32.5	30-34	358 600	0.37719	135 260	60 968	6.80	415	27 691
37.5	35-39	346 202	0.32465	112 394	50 661	7.28	369	24 621
42.5	40-44	333 118	0.27943	93 083	41 957	8.15	342	22 820
47.5	45-49	318 325	0.24051	76 560	34 509	10.06	347	23 153
52.5	50-54	300 392	0.20701	62 184	28 029	13.22	371	24 755
57.5	55-59	277 922	0.17817	49 517	22 320	18.05	403	26 890
62.5	60-64	248 722	0.15336	38 144	17 193	26.79	461	30 760
67.5	65-69	210 400	0.13199	27 771	12 518	41.19	516	34 430
72.5	70-74	162 220	0.11361	18 430	8 307	65.38	543	36 231
77.5	75-79	108 068	0.09778	10 567	4 763	102.30	487	32 495
82.5	80-84	58 022	0.08416	4 883	2 201	154.48	340	22 686
87.5	85 +	27 443	0.07244	1 988	896	259.56	233	15 547
TOTAL . . .		5 130 792		2 218 547	1 000 000		14 987	1 000 000

(a) a represents the starting age of each age group.

$$b_f = \frac{1}{\int_0^{\infty} e^{-ra} p_f(a) da} \qquad \text{(II.1)}$$

In discontinuous notation, this formula becomes:

$$b_f = \frac{1}{\sum e^{-ra} L_a}$$

In the last line of table II.3 we have, for every 100,000 persons at birth:

$$\sum e^{-ra} L_a = 2\,218\,547$$

whence

$$b_f = \frac{100\,000}{2\,218\,547} = 0.045\,075$$

Crude death rate

$$d_f = b_f - r = 0.045\,075 - 0.030\,000 = 0.015\,075$$

Age distribution of deaths

In continuous notation, the deaths of persons aged between a and $a + da$ years are given, for a population equal to unity, by:

$$D(a) = C(a)\, q(a)\, da$$

In discontinuous notation, for an age group a, $a + 5$ we have:

$$D_a = \sum_a^{a+5} C_a . q_a$$

Let us assume that this expression is equal to $C_a . q_\alpha$, where α is the median age of the age group, and also that the probability of death at the median age q_α is equal to the death rate for the age group m_a. Finally,

$$D_a = C_a . m_a$$

In the particular case considered here, the rates m_a are given in column 7 of table II.3.[5] The products $D_a = C_a m_a$ are given in the following column (column 8). The distribution of the quantities D_a is the age distribution of deaths sought (column 9).

We have confined ourselves to the computation of the characteristics of the female population. The details of the computation of the characteristics of the male population and of the population of both sexes together will be seen in the next example.

Second example: the female fertility function is known (stable population)

This additional condition makes it possible to write the r equation (formula II.14 in table II.2):

$$\int_u^v e^{-ra} p_f(a) \varphi_f(a) da = 1 \qquad \text{(II.14)}$$

[5] These rates are taken from table I in the appendix to *Manual III* (op. cit.).

As will be seen, for a given sub-set $H_0(r)$ this equation has a single real root ρ. When this root is found, we are brought back to the problem dealt with above, i.e., the problem of computing the characteristics of the population of the sub-set $H_0(r)$, whose rate of natural variation is equal to ρ. *This population is called the stable population corresponding to the laws $p_f(a)$ and $\varphi_f(a)$.*

In order to solve equation II.14, we shall first use a graphic method. Such a method has the advantage that it can be applied in all cases, i.e., whatever the functions $p_f(a)$ and $\varphi_f(a)$. We shall then give numerical methods utilizing approximate formulae, which are valid only because the mortality and fertility functions are those applicable to the human species and thus obey certain rules which ensure that their form is not arbitrary. Finally, we shall describe methods using successive approximations which are also valid for human populations.

Uniqueness of the real root ϱ of the fundamental equation

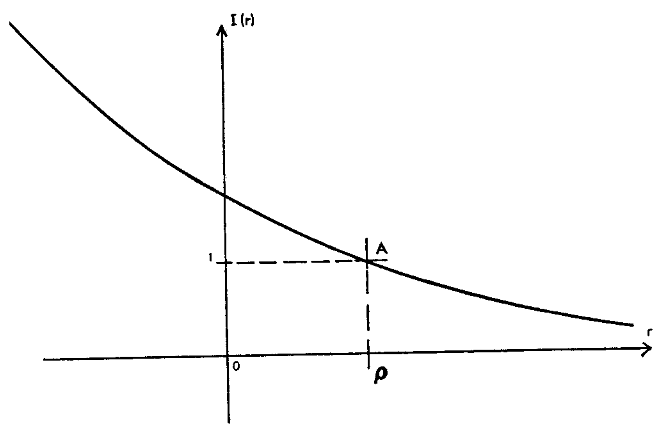
Let us consider the integral

$$I(r) = \int_u^v e^{-ra} p_f(a) \varphi_f(a) da$$

When r approaches $+\infty$, e^{-ra} tends towards zero and $I(r)$ therefore also tends towards zero. When r approaches $-\infty$, e^{-ra} tends towards $+\infty$ and $I(r)$ also tends towards $+\infty$. In addition, we can easily see that $I(r)$ is a decreasing function [6] of r. The curve representing $I(r)$ is thus of the form shown in graph II.1, where r is represented by the horizontal axis and $I(r)$ by the vertical axis.

The straight line of the ordinate equal to unity intersects the curve at one, and only one, point A, whose abscissa ρ is the real root of the equation.

Equation II.14 has therefore only one real root.



Graph II.1. Diagram illustrating the calculation of ϱ by the graphic method

The graphical calculation of ϱ

In order to determine ρ, we must plot the curve representing the integral $I(r)$ and then read off the abscissa of the point of the curve $I(r)$ whose ordinate is equal to unity. By adopting a sufficiently large scale, we can determine ρ with adequate accuracy.

[6] The derivative $\frac{I(r)}{dr} = - \int_u^v e^{-ra} . a p_f(a) \varphi_f(a) da$ is a negative quantity.

By way of example, we shall calculate the rate ρ corresponding to the mortality and fertility conditions used in chapter I and compute the population projections which, at the limits, approach stable populations. We shall thus be able to verify experimentally that these projections do indeed approach the stable populations which can be computed by solving equation II.14.

The survivorship functions will be those corresponding to the intermediate model life table which provides an expectation of life at birth for both sexes of 60.4 years. We are thus taking a new sub-set, $H_1(r)$, rather than that used in the first example.

The age distribution of female fertility rates will be that described in chapter I. (For reasons which will be given later, this distribution is being called the "intermediate model fertility distribution".) The gross reproduction rates R′ will be equal to 1.5, 0.75 and 1.74675 respectively.[7]

Equation (II.14) is written in continuous notation, but in practice we must write it in discontinuous notation. If we have the data by five-year age groups, then we have:

$$\sum_{15}^{40} e^{-r\alpha} L_a f_a = 1$$

where α is the median age of the successive age groups, i.e. 17.5, 22.5 ... 42.5, and a is the starting point of the age groups.

[7] It will be remembered that the last-mentioned value is such that the corresponding stable population is identical with the stationary population (i.e., a population whose rate of natural increase is zero).

As we have:

$$R' = 5\sum_{15}^{40} f_a$$

and

$$F_a = \frac{f_a}{\sum_{15}^{40} f_a} = \frac{5 f_a}{R'}$$

equation (II.14) is written, in discontinuous notation:

$$\sum_{15}^{40} e^{-r\alpha} L_a F_a = \frac{5}{R'} \qquad \text{(II.14}bis\text{)}$$

Table II.14 gives the computation of the sum:

$$\sum_{15}^{40} e^{-r\alpha} L_a F_a$$

for various values of r and for 100,000 girls at birth. Graph II.2 gives the curve representing this sum as a function of r.

For a gross reproduction rate $R' = 1.5$, we have $5/R' = 3.333$ ..., and the point on the curve having an ordinate of 3.333 ... corresponds to an abscissa $\rho_1 = 0.009$. For a reproduction rate $R' = 1.174675$, we have $5/R' = 4.26$ and the point on the curve corresponding to an ordinate of 4.26 for abscissa[7] $\rho_2 = 0$.

Finally, for a gross reproduction rate $R' = 0.75$, we have $5/R' = 6.666$ ..., and the point on the curve having an ordinate of 6.666 ... corresponds to an abscissa $\rho_3 = 0.017$. For computations of this nature, we are constantly in need of the values of the function $e^{-r\alpha}$

TABLE II.4. COMPUTATION OF THE SUM $\sum_{15}^{40} e^{-r\alpha} L_a F_a$ FOR VARIOUS VALUES OF r

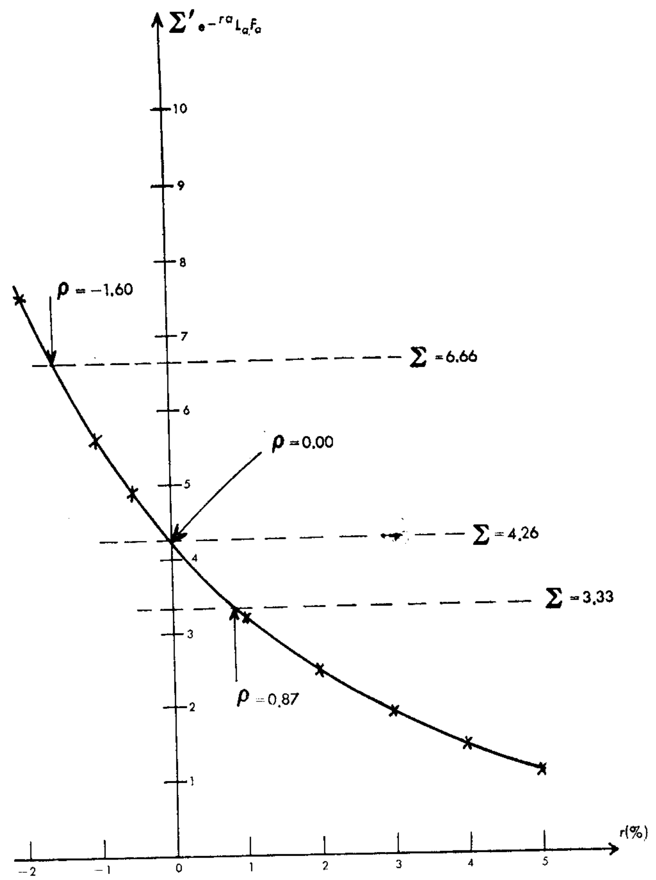
Conditions: intermediate model life table corresponding to an expectation of life at birth for both sexes of 60.4 years, and intermediate model fertility distribution

Age group (years) (1)	*Median age α (2)*	*Female stationary population (L_a) (per 100,000 births) (3)*	*Age distribution of female fertility rates (F_a) (4)*	*Product of the two preceding columns (5)*
15-19	17.5	439 970	0.100	43 997
20-24	22.5	434 040	0.273	118 493
25-29	27.5	427 035	0.263	112 310
30-34	32.5	419 610	0.188	78 887
35-39	37.5	411 672	0.121	49 812
40-44	42.5	402 742	0.055	22 151

Product of the multiplication of column (5) by $e^{-r\alpha}$ for the following values of r (percentages)

Age group (years)	*5*	*4*	*3*	*2*	*1*	*0.5*	*0.0*	*−0.5*	*−1.0*	*−2.0*
15-19	18 341	21 848	26 027	31 005	36 934	40 311	43 997	48 064	52 412	62 484
20-24	38 469	48 176	60 332	75 555	94 619	105 885	118 493	132 614	148 391	185 834
25-29	28 396	37 385	49 219	64 797	85 307	97 882	112 310	128 864	147 860	194 673
30-34	15 534	21 499	29 755	41 183	56 998	67 055	78 887	92 807	109 182	151 111
35-39	7 639	11 115	16 171	23 530	34 236	41 295	49 812	60 085	72 476	105 453
40-44	2 645	4 047	6 190	9 469	14 482	17 910	22 151	27 865	33 882	51 812
TOTAL	111 024	144 070	187 694	245 539	322 575	370 339	425 650	490 299	564 203	751 367

for various values of r and for the traditional five-year age groups. These can be calculated once for all, as has been done in tables II.5 and II.6.



Graph II.2. Curve showing variation of the sum $\sum_{15}^{40} e^{-ra}L_aF_a$ as a function of r (the figures in the last line of table II.4, divided by 100,000)

At the scale used in graph II.2, the accuracy achieved is not very great. It could obviously be improved by enlarging the scale, but approximate formulae can be established; these will be dealt with next.

Practical limits of the variation of the intrinsic rate

In practice, the intrinsic rate of natural variation varies in the human species only within quite narrow limits. If, in the preceding formulae, we replace the various functions of r by their expansions in an increasing-power series in r, we can, because of the smallness of r, disregard the terms of these expansions beyond a certain order and establish approximate numerical formulae which enable us to compute the various characteristics of a stable population. It is therefore essential to determine exactly the practical limits of variation in the intrinsic rate of natural variation.

If we assume that no women are sterile between the ages of 15 and 45 and that each woman gives birth to one child every year, the female fertility rate is constant from the age of 15 to the age of 45 and is approximately [8] equal to 0.5

If we also assume that mortality is nil up to the age of 45, we have $p_f(a) = 1$, and equation (II.14) is written:

$$\int_u^v e^{-ra}\, da = 2$$

The real root of this equation is very close to $\rho = 0.10$. Thus, a rate of increase of 10 per cent appears to be the upper limit of the intrinsic rate of natural variation. In fact, however, there are women who have no children, either because they are sterile or because they are unmarried. Moreover, even fertile women do not conceive

[8] We say "approximately" because more boys than girls are born; consequently, one child born every year would mean slightly less than 0.5 girls per year.

TABLE II.5. VALUES OF e^{-ra} FOR VARIOUS VALUES OF a AND r

Age group (years)	Median age a	Intrinsic rate of natural variation r (percentage)											
		−2.0	−1.5	−1.0	−0.5	+0.5	+1.0	+1.5	+2.0	+2.5	+3.0	+3.5	+4.0
0 . . .	0.5	1.01000	1.00750	1.00500	1.00250	0.99005	0.99181	0.99501	0.99750	0.98758	0.98511	0.98236	0.98020
1-4 . .	3.0	1.06180	1.04600	1.03050	1.01510	0.98511	0.97044	0.95600	0.94176	0.92774	0.91393	0.90033	0.88691
0-4 . .	2.5	1.05137	1.03821	1.02532	1.01268	0.98758	0.97531	0.96319	0.95123	0.93941	0.92774	0.91622	0.90482
5-9 . .	7.5	1.16283	1.11917	1.07788	1.03821	0.96319	0.92774	0.89360	0.86071	0.82901	0.79852	0.76913	0.74082
10-14 .	12.5	1.28402	1.20623	1.13315	1.06559	0.93941	0.88250	0.82903	0.77880	0.73162	0.68729	0.64565	0.60653
15-19 .	17.5	1.42017	1.30128	1.19125	1.09244	0.91622	0.83946	0.76913	0.70469	0.64565	0.59156	0.54199	0.49659
20-24 .	22.5	1.56831	1.40144	1.25232	1.11917	0.89360	0.79852	0.71355	0.63763	0.56977	0.50916	0.45498	0.40657
25-29 .	27.5	1.73335	1.51059	1.31653	1.14740	0.87153	0.75957	0.66199	0.57695	0.50283	0.43824	0.38194	0.33287
30-34 .	32.5	1.91554	1.62829	1.38403	1.17645	0.85002	0.72253	0.61416	0.52205	0.44375	0.37719	0.32061	0.27253
35-39 .	37.5	2.11700	1.75515	1.45499	1.20623	0.82903	0.68729	0.56978	0.47237	0.39160	0.32465	0.26915	0.22313
40-44 .	42.5	2.33905	1.89174	1.52959	1.25796	0.80856	0.65377	0.52861	0.42742	0.34559	0.27943	0.22594	0.18268
45-49 .	47.5	2.58571	2.03918	1.60801	1.26818	0.78860	0.62189	0.49042	0.38674	0.30498	0.24501	0.18966	0.14957
50-54 .	52.5	2.85805	2.19899	1.69046	1.30128	0.76913	0.59156	0.45498	0.34994	0.26913	0.20701	0.15922	0.12246
55-59 .	57.5	3.15829	2.36918	1.77713	1.33319	0.75014	0.56271	0.42211	0.31664	0.23752	0.17817	0.13365	0.10026
60-64 .	62.5	3.49034	2.55369	1.86824	1.36684	0.73162	0.53526	0.39161	0.28651	0.20961	0.15336	0.11220	0.08209
65-69 .	67.5	3.85743	2.75257	1.96403	1.40144	0.71355	0.50916	0.36331	0.25924	0.18508	0.13199	0.09418	0.06721
70-74 .	72.5	4.26311	2.96685	2.06473	1.43092	0.69593	0.48432	0.33706	0.23457	0.16235	0.11361	0.07906	0.05502
75-79 .	77.5	4.71157	3.19192	2.17059	1.47339	0.67875	0.46070	0.31270	0.21225	0.14406	0.09778	0.06637	0.04505
80-84 .	82.5	5.20708	3.44709	2.28188	1.51169	0.66199	0.43824	0.29011	0.19205	0.12714	0.08416	0.05572	0.03688
85 + .	87.5	5.75560	3.71665	2.39887	1.51705	0.64565	0.41686	0.26915	0.17377	0.11220	0.07244	0.04677	0.03020

Note: $e = 2.7182818$; Log e = 4342945.

at the rate of one child per year; the interval between births is generally about 2.5 years. Finally, the mortality rate up to the age of 45 will never be nil. Thus, the limit of 10 per cent is very liberally calculated; indeed, rates even as high as 4 per cent are very seldom encountered in practice.

Where negative intrinsic rates of natural variation are concerned, there may well be cases of very rapid population decline. Examples could be found of rapid population decline in certain cities[9] or certain sub-groups of the population. However, an intrinsic rate of natural variation of −10 per cent would mean an extremely rapid natural decrease, for in twenty years the population would decline by 93 per cent.

In the final analysis, then, it seems that if we assume that intrinsic rates of natural variation always remain between −10 per cent and +10 per cent we are sure to encompass nearly all actual populations.

It is frequently necessary, in this study, to seek to determine the form of the curves representing various functions of r, and perhaps to examine how they behave for values of r far removed from the range of −0.10 to +0.10 or even to consider what happens to the functions in the case of infinitely great values of r, as in the case of the integral $I(r)$ above. This obviously has no practical significance; the only object of such computations is to permit a more accurate determination of the form of the curves, and only that part of the curves corresponding to values of r between −0.10 and +0.10 is of any significance.

[9] This refers to natural variation, and not to total variation (i.e., it does not include the effects of migratory movement). A city with very low fertility may have a very high negative intrinsic rate of natural variation, and yet its population may not decrease, owing to extensive immigration.

TABLE II.6. VALUES OF e^{-ra} FOR VARIOUS VALUES OF a AND TWO VALUES OF r ($r = 0.01$ and $r = -0.01$)

Age group (years)	Median age a	Intrinsic rate of natural variation r (percentage) −10	+10
0	0.5	1.0513	0.95123
1-4	3.0	1.3191	0.74330
0-4	2.5	1.2840	0.77880
5-9	7.5	2.1170	0.47237
10-14	12.5	3.4903	0.28650
15-19	17.5	5.7546	0.17377
20-24	22.5	9.4877	0.10540
25-29	27.5	15.643	0.06393
30-34	32.5	25.790	0.03877
35-39	37.5	42.521	0.02352
40-44	42.5	70.105	0.01426
45-49	47.5	115.58	0.00865
50-54	52.5	190.57	0.00525
55-59	57.5	314.19	0.00318
60-64	62.5	518.01	0.00193
65-69	67.5	854.06	0.00117
70-74	72.5	1 408.1	0.00071
75-79	77.5	2,321.6	0.00043
80-84	82.5	3 827.6	0.00026
85 and over	87.5	6 310.7	0.00016

Approximate numerical formulae for the computation of the intrinsic rate of natural variation

There are, of course, various ways of establishing approximate formulae. The method proposed by Lotka involves the use of expansion in series of the moments R_n and the cumulants μ_n of a function $f(x)$. The moment R_n is defined by the integral

$$R_n = \int_0^{+\infty} x^n f(x)dx$$

while the cumulants $\mu_1, \mu_2, \ldots, \mu_n$ are deduced from the moments R by the solution of the following equations:

$$\begin{aligned} R_1 &= \mu_1 R_0 \\ R_2 &= \mu_1 R_1 + \mu_2 R_0 \\ R_3 &= \mu_1 R_2 + 2\mu_2 R_1 + \mu_3 R_0 \\ R_4 &= \mu_1 R_3 + 3\mu_2 R_1 + 3\mu_3 R_0 + \mu_4 R_0 \end{aligned}$$

etc.

The coefficients of the successive equations are those of Newton's binomial formula.

First approximation

This is obtained by disregarding all the terms above the first order. It is written:[10]

$$\varrho = \frac{\text{Log } R_0}{\mu_1 \times 0.4342945} = \text{Log } R_0 \times \frac{2.302584}{\mu_1} \qquad \text{(II.16)}$$

where R_0 is the zero-order moment of the function $p_f(a)m_f(a)$ and μ_1 is the first-order cumulant of the same function. The expression:

$$R_0 = \int_u^v p_f(a)\varphi_f(a)da$$

gives us the net reproduction rate, and

$$\mu_1 = \frac{R_1}{R_0} = \frac{\int_u^v a p_f(a)\varphi_f(a)da}{\int_u^v p_f(a)\varphi_f(a)da}$$

is therefore the average age of mothers at the birth of their children *in the stationary population.*

Second approximation

This is obtained by disregarding all the terms above the second order:

$$\varrho = \frac{\mu_1 \pm \sqrt{\mu_1^2 - 2\mu_2 \text{ Log } R_0 \times 2.302584}}{\mu_2} \qquad \text{(II.17)}$$

where R_0 is always the zero-order moment of the function $p_f(a)\varphi_f(a)$ and μ_1 and μ_2 are the first-order and second-order cumulants of the same function.[11]

[10] The numerical coefficient in the denominator of this formula is simply: $\text{Log } e = 0.4342945$.

[11] We select for placing before the root symbol the symbol which gives a value close to that given by the first approximate formula.

Tables II.7 and II.8 give details of the application of these two formulae in the three cases so far considered.

The last line of table II.8 gives the values of r obtained by the geometric method. It will be seen that, for the two values of the gross reproduction rate used here (1.50 and 0.75), the three methods give very similar results. In order to obtain at least an idea of the maximum value of the gross reproduction rate for which the approximation formula can be used, we have added to the preceding table the results obtained by the three methods using a gross reproduction rate of 4.0. Even at such a high value, the first formula still gives a good approximation. The second formula and the geometric method give practically identical results. There is very little likelihood of ever encountering a human fertility rate which gives a gross reproduction rate of over 4.0. Thus, we see that the approximation formulae are generally quite adequate in practice for the computation of the intrinsic rate of natural variation of stable populations.

Computation of the intrinsic rate of natural variation by the method of successive approximations

Many ways of computing the intrinsic rate of natural variation by the method of successive approximations can be envisaged, but we shall confine ourselves to two of them.

First method (the mean interval between two generations)

The first method involves a notion frequently utilized in demography, namely, that of the mean interval between two generations.

The word "generation" is used in two senses in demography. First, it may mean a group of persons born at the same time—for example, all the persons born during a calendar year. It is also used to mean a group of children in relation to their parents, or *vice versa*. The group of parents is called the first generation, the group of children is called the second generation, and it is easy to see what is meant by third generation, fourth generation, and so on. What, then, does the expression "interval between two generations" mean?

Let us consider a set of girls born at the same time, and let us represent the mean age of their mothers at that time by A. If the risk of death was nil, the mean age of the set of girls under consideration throughout their lives would be A years less than the mean age of their mothers. In fact, however, death exercises an effect, and the difference between the two mean ages declines as the "generation" of girls grows older, although the decline is quite slow.[12] It seems logical to call this mean age A the "interval between two generations". In a stable population it is written:

[12] Assuming mortality to be nil, if in a census every woman is asked the age of her mother and the replies are classified according to the age of the daughters, we shall have for each age of the daughters a mean age of the mothers which is greater than the age of the daughters by A years. Mortality affects the results in two ways. In the first place, as the group of daughters grows older, the older mothers die and the difference between the mean age of the mothers and that of the daughters becomes less. This effect can be corrected by asking the daughters the year of birth of their mothers, whether or not they are still alive. However, mortality can also affect the difference A if there is a correlation between the fertility of a mother and the age at death of her daughters. In fact, it is recognized that such a correlation exists. The difference between the mean age of daughters and the mean age of mothers is greater in the lower than in the higher social classes, because of a difference in the fertility of the two classes. Women of the lower social classes continue to have children at relatively advanced ages, while in the higher social classes childbearing ceases quite early. However, mortality is also higher in the lower social classes. The result is that daughters of the lower social classes are represented to an increasingly small extent in the group of daughters questioned as the age of the daughters increases, thus tending to reduce A, although this latter factor exercises its effect only at advanced ages, when mortality becomes high.

TABLE II.7. COMPUTATION OF THE FIRST-ORDER AND SECOND-ORDER MOMENTS OF THE FUNCTION $p_f(a)m_f(a)$ FOR A GROSS REPRODUCTION RATE OF 5.0 ([a]) (LEVEL-80 MODEL LIFE TABLE AND INTERMEDIATE MODEL FERTILITY DISTRIBUTION)

Age group (years)	*Median age* α	*Zero-order* ([b]) *moment:* R_0	*First-order* ([c]) *moment:* R_1	*Second-order* ([c]) *moment:* R_2
15-19	17.5	43 997	769 948	13 474 090
20-24	22.5	118 493	2 666 093	59 987 092
25-29	27.5	112 310	3 088 575	84 934 438
30-34	32.5	78 887	2 563 828	83 324 410
35-39	37.5	49 812	1 867 950	70 048 125
40-44	42.5	22 151	941 418	40 010 265
TOTAL		425 650	11 897 812	351 778 420
Moments . . .		4.2565	118.9781	3 517.7842

([a]) The gross reproduction rate is equal to five times the total of the female fertility rates by five-year age groups. In table II.4, as F_a is a distribution, we have:

$$\sum_{15}^{40} F_a = 1$$

and consequently:

$$5 \sum_{15}^{40} F_a = 5$$

We can therefore consider F_a as the fertility rate by age groups where the gross reproduction rate is equal to 5.

([b]) These are the figures from column 5 of table II.4.

([c]) These are the figures from the previous column, multiplied by the median age.

$$A(r) = \frac{\int_u^v ae^{-ra}p_f(a)\varphi_f(a)da}{\int_u^v e^{-ra}p_f(a)\varphi_f(a)da}$$

The problem can also be reasoned in the following manner. Let us consider a group of girls all born during the same year. In the course of their reproductive life, they give birth to daughters at various ages. When they reach the end of the reproductive period, we can calculate the mean age B at which they gave birth to their daughters. If the risk of death is nil, there will in the future always be the same difference B between the mean age of the group of mothers under consideration and the mean age of their daughters. In fact, however, mortality occurs, and the difference increases slowly as the group of mothers grows older.[13] It therefore appears to be quite as logical as in the preceding case to call the mean age B the "mean interval between two generations". Its value is:

$$B = \frac{\int_u^v ap_f(a)\varphi_f(a)da}{\int_u^v p_f(a)\varphi_f(a)da}$$

It will be seen that B is different from A_r. More precisely, B is equal to the value assumed by A_r when the population is stationary. We can write: $B = A_0$.

[13] If a population census is taken, the two aspects referred to in the preceding foot-note will be noted.

In his theory of stable populations, Lotka gives the name of "interval between two generations" to yet another formula. He considers the integral:

$$I(r) = \int_u^v e^{-ra}p_f(a)\varphi_f(a)da$$

and he writes

$$\frac{dI(r)}{dr} = -\int_u^v ae^{-ra}p_f(a)\varphi_f(a)da$$

If we put:

$$A(r) = \frac{\int_u^v ae^{-ra}p_f(a)\varphi_f(a)da}{\int_u^v e^{-ra}p_f(a)\varphi_f(a)da}$$

we have

$$\frac{dI(r)}{dr} = -(Ar)I(r)$$

whence

$$I(r) = Ke^{-\int_0^r A(r)dr}$$

where K is a constant.

When $r = 0$ we have $I(0) = K = R_0$ (the net reproduction rate). We therefore have:

$$I(r) = R_0e^{-\int_0^r A(r)dr}$$

TABLE II.8. COMPUTATION OF THE INTRINSIC RATE OF NATURAL VARIATION BY APPROXIMATION

Stage of calculation	*Gross reproduction rate:* R′ 1.50	0.75	4.00
Zero-order moment (a) (R_0)	1.27695	0.638475	3.4052
Log R_0	0.1061739	−0.1948561	0.5321426
Cumulant (b) $\mu_1 = \frac{R_1}{R_0}$	27.9251	27.9521	27.9521
Cumulant (b) $\mu_2 = \frac{R_2}{R_1}$	29.5666	29.5666	29.5666
		First approximation	
μ_1 Log $e = \mu_1 \times 0.4342945$	12.13944	12.13944	12.13944
$r = \frac{\text{Log } R_0}{\mu_1 \text{ Log } e}$	+0.00874	−0.01605	+0.04384
		Second approximation	
$\frac{2\mu_2}{\text{Log } e} = 2\mu_2 \times 2.302584$	136.1504	136.1504	136.1504
$\frac{\text{Log } R_0 \times 2\mu_2}{\text{Log } e}$	14.4563	−26.4309	72.4546
$(\mu_1)^2$	781.3199	781.3199	781.3199
Difference between the two preceding lines	766.8636	807.8508	708.8653
Square root of the preceding line	27.6923	28.4228	26.6245
μ_1 less the preceding line	0.2598	−0.4707	1.3276
The preceding line divided by μ_2	0.00879	−0.01592	+0.04490
Resultats obtained by the geometric method	0.0090	−0.0160	+0.0450

(a) The zero-order moment of table II.7 multiplied by R′/5.
(b) As the cumulants are equal to the ratios between two successive moments, they become independent of the gross reproduction rate as soon as a value is assumed for the age distribution of the fertility rates.

Lotka then puts:

$$\int_0^r A(r)dr = rT_r$$

and finally we write:

$$I(r) = R_0 e^{-rT_r}$$

In the stable population, $I(r) = 1$. Thus, we have:

$$R_0 = e^{rT_r}$$

T_r is therefore the time needed for the stable population to be multiplied by the net reproduction rate. Lotka calls this time T_r the "mean interval between two generations".

Thus, we have three definitions, all of them apparently valid, for the interval between two generations. Fortunately, when r is small there is a very simple approximate relationship between these three formulae.

Let us revert to the equation:

$$A(r) = \frac{\int_u^v a e^{-ra} p_f(a) \varphi_f(a) da}{\int_u^v e^{-ra} p_f(a) \varphi_f(a) da}$$

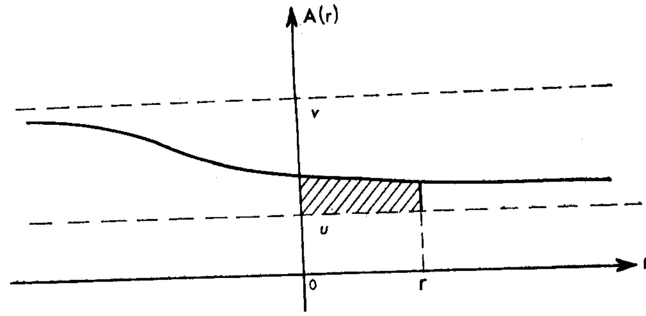
If we introduce the age distribution of fertility rates F(a), this equation is then written:

$$A(r) = \frac{\int_u^v a e^{-ra} p_f(a) F(a) da}{\int_u^v e^{-ra} p_f(a) F(a) da}$$

In order to clarify our ideas, let us now take the case where F(a) does not vary, i.e., where the age distribution of fertility rates remains constant when fertility varies. In such conditions, A(r) is a function only of r, of which it is a decreasing function. In order to obtain the values of A(r) for infinitely great values of r, it is convenient to imagine the formula above written in discontinuous notation. If r tends towards $+\infty$, then the term close to u becomes preponderant over all the others and A(r) tends towards:

$$\frac{u e^{-ru} p_f(u) F(a)}{e^{-ru} p_f(u) F(a)} = u$$

Likewise, if r tends towards $-\infty$ we see that A(r) tends towards V. The curve representing A(r) as a function of r therefore has the form shown in graph II.3.



Graph II.3. Curve representing the variation of A(r) as a function of r

If $r = 0$, then we obviously have $Ar = A_0 = B$. Around $r = 0$, the variations of A(r) as a function of r are almost linear. This can be seen from table II.9, where we have calculated, for seven values of r ranging from 0 to 7 per cent, the values of A(r) computed by associating the level—80 intermediate model life table (giving an expectation of life at birth for both sexes of 60.4 years) with a fertility function of the intermediate model age distribution. The slight variation in the difference computed in the third line of table II.9 clearly shows that A(r) is, practically speaking, a decreasing linear function of r. For other age distributions of the fertility rate and for other model life tables we should have other values of A(r), but we should still find a linear relation between A(r) and r. It was seen above that for $r = 0$ we have $A_0 = B$. We therefore have, as a good approximation:

$$T_r = \frac{1}{r} \int_0^r A_r dr \ \# \ \frac{A_r + A_0}{2}.$$

We see that T_r is also, practically speaking, a linear function of r. In particular, when r tends towards 0, T_r tends towards A_0. The last two lines of the table enable the results of the computation of T_r by approximation to be compared with the results of the exact computation.

Naturally, all these formulae are valid only for values of r close to $r = 0$. As soon as we take very large values of r they no longer hold good.[14]

[14] The hatched area on graph II.3 represents the integral

$$\int_0^r A(r)\, dr$$

As r approaches $+\infty$, the integral tends towards ur, and consequently T_r tends towards u. Similarly, when r tends towards $-\infty$, T_r tends towards v. The curve representing T_r has the same asymptotes as the curve representing A_r, and for $r = 0$ we have $T_r = A_r = B_0$. The two curves representing A_r and T_r are very close to each other.

TABLE II.9. VARIATIONS IN THE MEAN AGE A_r AS A FUNCTION OF r (UNITED NATIONS INTERMEDIATE MODEL LIFE TABLE GIVING AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS). INTERMEDIATE MODEL FERTILITY DISTRIBUTION (TABLE I.1 IN CHAPTER I, SECTION D)

Rate r (percentage)	0	1	2	3	4	5	6	7
Age A_r (years)	27.952	27.506	27.074	26.655	26.251	25.861	25.486	25.126
Difference between successive values of A_r	0.446	0.432	0.419	0.404	0.390	0.375	0.360	
Calculation of T_r (1) Approximate formula .		27.729	27.509	27.294	27.084	26.878	26.878	26.481
(2) Exact formula		27.729	27.513	27.303	27.102	26.906	26.719	26.539

Let us return to the equation:

$$R_0 = e^{rT_r}$$

whence we obtain:

$$r = \frac{\text{Log } R_0}{T_r \text{ Log } e}$$

This is the equation which will be used to calculate r by the method of successive approximations. The value sought, r, is the abscissa of the point of intersection of the two curves:

(I) $$T_r = \frac{\text{Log } R_0}{\text{Log } e} \cdot \frac{1}{r}$$

and (II) $$T_r = \frac{1}{r}\int_0^r A_r dr$$

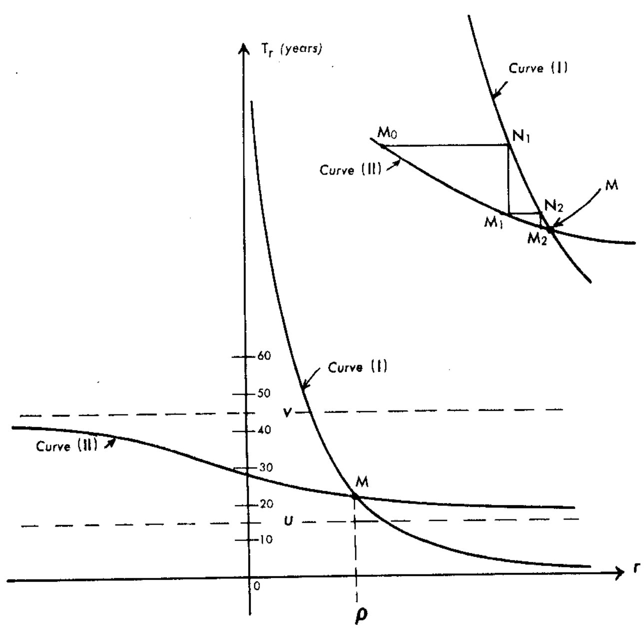
plotted on a graph where r is represented by the horizontal axis and T_r by the vertical. Graph II.4 shows these curves for a gross reproduction rate of 4.00 (the age distribution of fertility being those of the intermediate model) and for an intermediate model life table corresponding to an expectation of life at birth for both sexes of 60.4 years. Curve I is a branch of an equilateral hyperbola, while curve II is of the form shown in graph II.3. The two curves intersect at a point M whose abscissa is the intrinsic rate of natural variation ρ. Let r_0 be an approximate value of r, and let us consider the series:

$$r_1 = \frac{\text{Log } R_0}{T_{r_0} \text{ Log } e} \qquad r_2 = \frac{\text{Log } R_0}{T_{r_1} \text{ Log } e}$$

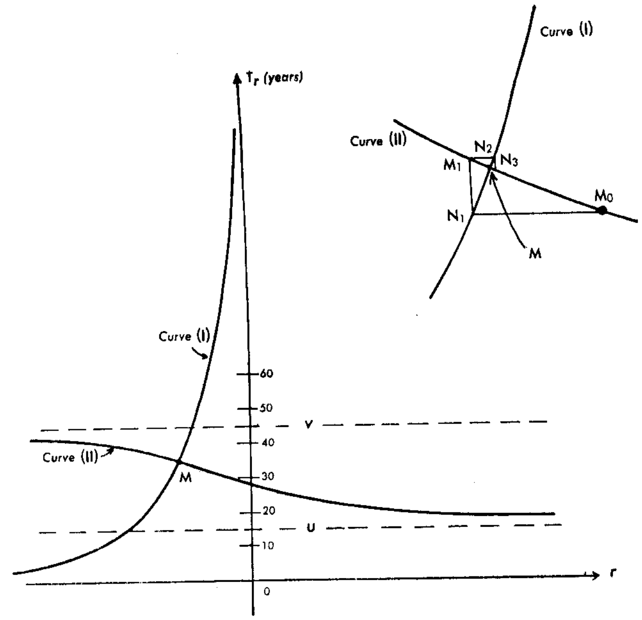
$$r_3 = \frac{\text{Log } R_0}{T_{r_2} \text{ Log } e} \text{ etc.}$$

and T_{r_0}, T_{r_1}, T_{r_2}, etc., being calculated by the formula

$$T_r = \frac{1}{r}\int_0^r A_r dr$$



Graph II.4. Graphic illustration of the first method of determining the intrinsic rate of natural variation ϱ by successive approximations (where ϱ is positive)



Graph II.5. Illustration of the first method of determining the intrinsic rate of natural variation ϱ by successive approximations (where ϱ is negative)

Let us begin from the point M_0 of curve II which has as its abscissa r_0, and let us plot the path $M_0N_1M_1N_2M_2$. As the inset in the graph indicates, the points M_0, M_1, M_2, etc., have as their abscissae terms of the series r_0, r_1, r_2, etc. The form of the curves is such that this series tends toward r. If we start from an approximate value which is too low we tend towards r but remain always below it, while if we start from an approximate value which is too high we tend towards r but remain always above it. In this way, we can close in on the true value of r by successive approximations. Curve II is very flat, and consequently the series converges very rapidly on r.

If r is negative (gross reproduction rate of 0.75), the results are slightly different. Graph II.5 shows that we converge successively on the true value of r. If r_0 is smaller than ρ, r_1 will be larger and r_2 will be smaller, and so on. Let us see how this method is applied to an actual example. The model life table is of the intermediate model, giving an expectation of life at birth for both sexes of 60.4 years, while the gross reproduction rate is 4.0. Let us take $r_0 = 0.04$ in our first calculation and $r_0 = 0.05$ in our second. Here are the successive phases of the computation:

(*a*) For a gross reproduction rate $R' = 5.0$, we have $R_0 = 4.25650$ (figures taken from table II.7).

(*b*) For a gross reproduction rate $R' = 4.0$, we have

$$R_0 = \frac{4.25650 \times 4}{5} = 3.4052$$

(figures taken from table II.8).

(*c*) We deduce from this that:

$$\frac{\text{Log } R_0}{\text{Log } e} = 1.225303$$

(*d*) For $r = 0.04$, we have

$$T_r = \frac{A_0 + A_r}{2} = \frac{26.251 + 27.952}{2} = 27.102$$

(e) For $r = 0.05$ we have

$$T_r = \frac{A_0 + A_r}{2} = \frac{25.861 + 27.952}{2} = 26.906$$

(In these two computations of T_r, the values of A_0 and A_r are taken from table II.8.)

(f) If we start from $r_0 = 0.04$, we have

$$r_1 = \frac{1.225303}{27.102} = 0.0452$$

(g) If we start from $r_0 = 0.05$, we have

$$r_1 = \frac{1.225303}{26.906} = 0.0455$$

We ultimately find that the value of r is between 0.0452 and 0.0455. Although the approximate starting values may be very far from the true value, we obtain at the first approximation values very close to the value sought. It should be noted that the first approximate formula used earlier in this section is simply the first term of the above series when we start from $r_0 = 0$.

Second method of successive approximations

Let r_0 be an approximate value of ρ. Let us assume that $r_0 + \varepsilon = \rho$. The basic equation is then written:

$$\int_u^v e^{-(r_0+\varepsilon)a} p_f(a)\varphi_f(a)da = 1$$

or:

$$\int_u^v e^{-r_0 a} p_f(a)\varphi_f(a) e^{-\varepsilon a}\, da = 1$$

and, as ε is small, we can write:

$$\int_u^v e^{-r_0 a} p_f(a)\varphi_f(a)\,(1 - \varepsilon a)da = 1$$

whence we have:

$$\varepsilon = \frac{\int_u^v e^{-ra} p_f(a)\varphi_f(a)da - 1}{\int_u^v e^{-ra} a p_f(a)\varphi_f(a)da} \qquad \text{(II.18)}$$

$1 - \varepsilon a$ is smaller than $e^{-\varepsilon a}$. If we replace $e^{-\varepsilon a}$ by $1 - \varepsilon a$, we therefore find a value of ε which is too small. The quantity $r_1 = r_0 + \varepsilon$ is a better approximation than r_0. If we begin the computation over again with r_1 we therefore obtain a still better value r_2, and so forth. The graphic interpretation of the method is very simple. On a graph where r is represented on the horizontal axis, the value of r which is sought is the abscissa of the point of intersection of the two curves:

$$\text{(I)} \qquad y = \frac{\int_u^v e^{-ra} p_f(a)\varphi_f(a)da - 1}{\int_u^v a e^{-ra} p_f(a)\varphi_f(a)da} + r$$

$$\text{(II)} \qquad y = r$$

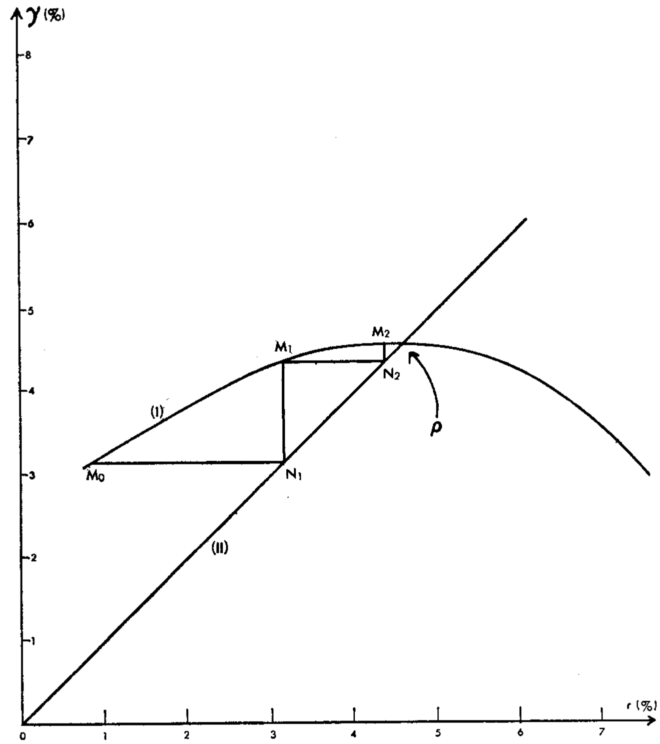
Graph II.6 shows the two curves (I) and (II) corresponding to a gross reproduction rate of 4.00. If, starting from a point on curve (I) with an abscissa r_0, we construct the path $M_0N_1M_1N_2M_2$ we see that the points M_0, M_1, M_2 etc., have as their abscissae r_0, r_1, r_2 etc. Graph II.6 shows that the series converges towards ρ. If we start from a value of r_0 which is too small, we remain consistently below the true value. If we start from a value which is too high, we shall be too low at the first approximation and shall remain so subsequently. Thus, in this method the series always converges towards r by giving values which are too small.

The correction for which a formula has been given above can be put in an approximate form which makes possible a very rapid method of computing ρ. We can write:

$$\varepsilon = \frac{1 - \dfrac{1}{\int_u^v e^{-r_0 a} p_f(a)\varphi_f(a)da}}{\dfrac{\int_u^v e^{-ra} a p_f(a)\varphi_f(a)da}{\int_u^v e^{-ra} p_f(a)\varphi_f(a)da}} = \frac{1 - \dfrac{1}{\int_u^v e^{-r_0 a} p_f(a)\varphi_f(a)da}}{A(r_0)}$$

In the case of the human species, the quantities $A(r_0)$ are always close to A_0, and we therefore have, approximately:

$$\varepsilon = \frac{\int_u^v e^{-r_0 a} p_f(a)\varphi_f(a)da - 1}{A_0 \int_u^v e^{-r_0 a} p_f(a)\varphi_f(a)da}$$



Graph II.6. Graphic illustration of the second method of determining ρ by successive approximations

and if we put:

$$\int_u^v e^{-r_0 a} p_f(a)\varphi_f(a)da = 1 + \delta$$

we have:

$$\varepsilon = \frac{\delta}{A_0(1+\delta)}$$

If r_0 is sufficiently close to the value of ρ, however, the quantity δ is small, and in these circumstances:[15]

$$\varepsilon = \frac{\delta}{A_0} \qquad \text{(II.19)}$$

A special case

There is one case in which the computation of the rate of variation ρ becomes very easy. This is where the fertility function $\varphi_f(a)$ amounts to a single value equal to the gross reproduction rate R′. Such a simplification of the fertility function would at first sight seem very artificial, since it is manifestly impossible for a woman to bring into the world at a given time R′ daughters. As will be seen later, however, stable populations computed with a simplified fertility function in this way are identical with stable populations computed with actually observed fertility functions, provided that the age to which the fertility function is reduced is close to the average age of mothers at the birth of their children. In such circumstances, the simplification of the fertility function is no more than a straightforward mathematical device.

The median age 27.5 years of the 25-29 five-year age group is close to the average age of mothers at the birth of their children. It is therefore particularly convenient to take this age and to consider the fertility function as being reduced to $\varphi_f(27.5) = R'$.

Equation (1) is written:

$$e^{-27.5r} p_f(27.5)\varphi_f(27.5) = 1$$

whence we have:

$$\varrho = \frac{\text{Log } p_f(27.5) + \text{Log } \varphi_f(27.5)}{27.5 \text{ Log } e}$$

or

$$\varrho = \frac{\text{Log } p_f(27.5) + \text{Log } \varphi_f(27.5)}{11.9431}$$

If, for example, we take a gross reproduction rate of 3.0

$$R' = \varphi_f(27.5) = 3.0$$

and use the intermediate model life table corresponding to an expectation of life at birth for both sexes of 40 years, we have the following calculation:

$p_f(27.5) = 0.62222$ (taken from the model life table)[16]

$\text{Log } 0.62222 = \bar{1}.793{,}9440$

$\text{Log } 3.0 = 0.477{,}1213$

$\text{Log } p_f(27.5) + \text{Log } \varphi_f(27.5) = 0.2710653$

$$\varrho = \frac{0.2710653}{11.9431} = 0.022705$$

The age structure in the stable state

When we know the rate of variation in the stable state, it is easy to calculate the age structure by applying the method given in the first example.

Tables II.10 to II.13 give details of the calculations for the case considered here, where the gross reproduction rate R′ is 1.50. In the calculations, a value of $\rho = 0.00087$ has been used. Table II.13 gives details of the calculation of the age composition of the total population of both sexes together. We had omitted this calculation in the first example.

[15] We see here the formula proposed by Coale in his article "A new method for calculating Lotka's *r*, the intrinsic rate of growth in a stable population", *Population Studies*, vol. XI, No. 1, July 1957.

[16] *Methods of Estimating Population, Manual III: Methods for Population Projections by Sex and Age*, appendix, table III.

TABLE. II.10. COMPUTATION OF e^{-ra} AND e^{-ra} FOR $r = 0.0087$

Median age a	*Age group (years)*	ra	$ra \times \log e = \log e^{+ra}$	e^{+ra}	colog e^{ra}	e^{-ra}
2.5 . . .	0-4	0.0218	0.009468	1.0220	$\bar{1}$.990532	0.97844
7.5 . . .	5-9	0.0652	0.028316	1.0674	$\bar{1}$.971684	0.93688
12.5 . . .	10-14	0.1088	0.047251	1.1159	$\bar{1}$.952749	0.89691
17.5 . . .	15-19	0.1522	0.066100	1.1654	$\bar{1}$.933890	0.85880
22.5 . . .	20-24	0.1958	0.085035	1.2173	$\bar{1}$.914965	0.82218
27.5 . . .	25-29	0.2392	0.103883	1.2702	$\bar{1}$.896117	0.78726
32.5 . . .	30-34	0.2828	0.122818	1.3268	$\bar{1}$.877182	0.75367
37.5 . . .	35-39	0.3262	0.141667	1.3857	$\bar{1}$.858333	0.72166
42.5 . . .	40-44	0.3698	0.160602	1.4474	$\bar{1}$.839398	0.69087
47.5 . . .	45-49	0.4132	0.179450	1.5116	$\bar{1}$.820540	0.66152
52.5 . . .	50-54	0.4568	0.198386	1.5790	$\bar{1}$.801614	0.63331
57.5 . . .	55-59	0.5002	0.217234	1.6491	$\bar{1}$.782766	0.60641
62.5 . . .	60-64	0.5438	0.236169	1.7225	$\bar{1}$.763831	0.58084
67.5 . . .	65-69	0.5872	0.255018	1.7999	$\bar{1}$.744982	0.55588
72.5 . . .	70-74	0.6308	0.273953	1.8791	$\bar{1}$.726047	0.53227
77.5 . . .	75-79	0.6742	0.292801	1.9625	$\bar{1}$.707199	0.50956
82.5 . . .	80-84	0.7178	0.311737	2.0499	$\bar{1}$.688263	0.48782
87.5 . . .	85 +	0.7830	0.340053	2.1880	$\bar{1}$.659947	0.45703

TABLE II.11. COMPUTATION OF THE AGE DISTRIBUTION OF THE FEMALE STABLE POPULATION CORRESPONDING TO A RATE OF VARIATION OF $r = +0.0087$. INTERMEDIATE MODEL LIFE TABLE GIVING AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS. INTERMEDIATE MODEL FEMALE FERTILITY GIVING A GROSS REPRODUCTION RATE OF 1.50

Median age a	Age group (years)	Female stationary population (per 100,000 births) L_a	e^{-ra}	Female stable population (product of the two preceding columns)	Age distribution of the female stable population C_a
2.5	0-4	460 386	0.97844	450 460	98 137
7.5	5-9	448 010	0.93688	419 732	91 444
12.5	10-14	444 150	0.89691	398 363	86 787
17.5	15-19	439 970	0.85880	377 846	82 318
22.5	20-24	434 040	0.82218	356 859	77 745
27.5	25-29	427 035	0.78726	336 188	73 242
32.5	30-34	419 610	0.75367	316 247	68 898
37.5	35-39	411 672	0.72166	297 087	64 723
42.5	40-44	402 742	0.69087	278 242	60 706
47.5	45-49	391 728	0.66152	259 136	56 455
52.5	50-54	377 275	0.63331	238 932	52 054
57.5	55-59	357 718	0.60641	216 924	47 259
62.5	60-64	330 472	0.58084	191 852	41 797
67.5	65-69	291 642	0.55588	162 118	35 319
72.5	70-74	238 028	0.53227	126 895	27 602
77.5	75-79	171 308	0.50956	87 292	19 017
82.5	80-84	102 020	0.48782	49 767	10 842
87.5	85 +	56 792	0.45703	25 956	5 655
ALL AGES		6 204 598		4 589 696	1 000 000

The stable female birth rate is written:

$$b_f = \frac{100,000}{4,589,696} = 21.79 \text{ per thousand.}$$

The stable female death rate is written:

$$d_f = 21.79 - 8.70 = 13.09 \text{ per thousand.}$$

TABLE II.12. COMPUTATION OF THE AGE DISTRIBUTION OF THE MALE STABLE POPULATION CORRESPONDING TO A RATE OF VARIATION OF $r = +0.0087$. INTERMEDIATE MODEL LIFE TABLE GIVING AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS. INTERMEDIATE MODEL FEMALE FERTILITY GIVING A GROSS REPRODUCTION RATE OF 1.50

Median age a	Age group (years)	Male stationary population (per 100,000 births) L_a	e^{-ra}	Male stable population (product of the two preceding columns)	Age distribution of the male stable population C_a
2.5	0-4	453 493	0.97844	443 716	101 037
7.5	5-9	440 230	0.93688	412 443	93 916
12.5	10-14	436 230	0.89691	391 259	89 092
17.5	15-19	431 860	0.85880	370 881	84 452
22.5	20-24	425 305	0.82218	349 677	79 624
27.5	25-29	417 588	0.78726	328 750	74 624
32.5	30-34	409 715	0.75367	308 790	70 314
37.5	35-39	401 195	0.72166	289 526	65 928
42.5	40-44	391 022	0.69087	270 536	61 603
47.5	45-49	377 658	0.66152	249 828	56 888
52.5	50-54	359 368	0.63331	227 591	51 824
57.5	55-59	334 412	0.60641	202 791	46 177
62.5	60-64	300 470	0.58054	174 435	39 720
67.5	65-69	255 450	0.55588	142 000	32 334
72.5	70-74	199 095	0.53227	105 972	24 131
77.5	75-79	135 710	0.50956	69 152	15 746
82.5	80-84	75 538	0.48782	36 849	8 391
87.5	85 +	38 093	0.45703	17 410	3 964
ALL AGES		5 882 432		4 391 606	1 000 000

The stable male birth rate is written:

$$b_m = \frac{100,000}{4,391,600} = 22.77 \text{ per thousand.}$$

The stable male death rate is written:

$$d_m = 22.77 - 8.70 = 14.07 \text{ per thousand.}$$

TABLE II.13. COMPUTATION OF THE AGE DISTRIBUTION OF THE STABLE POPULATION (BOTH SEXES TOGETHER) CORRESPONDING TO A RATE OF VARIATION OF $r = +0.0087$. INTERMEDIATE MODEL LIFE TABLE GIVING AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS. INTERMEDIATE MODEL FEMALE FERTILITY GIVING A GROSS REPRODUCTION RATE OF 1.50

Age group (years)	*Male stable population multiplied by 1.05*	*Female stable population*	*Stable population (both sexes)*	*Age distribution of the stable population (both sexes)*
0-4	465 902	450 460	916 362	99 595
5-9	433 065	419 732	852 797	92 686
10-14	410 822	398 363	809 185	87 946
15-19	389 425	377 946	767 271	83 391
20-24	367 161	356 859	724 020	78 690
25-29	345 188	336 188	681 376	74 055
30-34	324 230	316 247	640 477	69 610
35-39	304 002	297 087	601 089	65 329
40-44	284 063	278 242	562 305	61 157
45-49	262 319	259 136	521 455	56 674
50-54	238 971	238 932	477 903	51 941
55-59	212 931	216 924	429 855	46 719
60-64	183 157	191 852	375 009	40 758
65-69	149 100	162 118	311 218	33 825
70-74	111 271	126 695	237 966	25 863
75-79	72 610	87 292	159 902	17 379
80-84	38 691	49 767	88 458	9 614
85 +	18 280	25 956	44 236	4 808
ALL AGES	4 611 188	4 589 696	9 200 884	1 000 000

The stable birth rate for both sexes together is written:

$$b = \frac{205{,}000}{9{,}200{,}884} = 22.28 \text{ per thousand.}$$

The arithmetic mean of the stable female and male birth rates is:

$$\frac{21.77 + 22.77}{2} = 22.28 \text{ per thousand.}$$

The stable death rate for both sexes togegher is written: $d = 22.28 - 8.70 = 13.58$.
The arithmetic mean of the stable female and male death rates is:

$$\frac{13.09 + 14.07}{2} = 13.58 \text{ per thousand.}$$

Computation of the stable birth rate

The stable birth rate of the female population can be computed as in the first example

$$b_f = \frac{1}{\int_0^\infty e^{-\varrho a} p_f(a) da}$$

This is in fact a by-product of the preceding computation. The rates will be found at the foot of the tables.

We can establish approximate formulae for the computation of the stable birth rate, just as we did for the computation of the intrinsic rate of natural variation ρ. These formulae involve not only the moments and cumulants of the function $p_f(a)\varphi_f(a)$, but also the moments and cumulants of the survivorship function $p_f(a)$.

First approximation

$$b_f = \frac{R_0}{E_0} \qquad \text{(II.20)}$$

where R_0 is the net reproduction rate and E_0 is the first-order moment of the function $p_f(a)$, i.e., the expectation of life at birth E_0.

Second approximation

$$b_f = \frac{R_0}{E_0} \cdot \frac{1 - \varrho\mu_1}{1 - \varrho\lambda_1} \qquad \text{(II.21)}$$

where μ_1 is the first-order cumulant of the function $p_f(a)\varphi_f(a)$, λ_1 is the first-order cumulant of the function $p_f(a)$, and ρ is the intrinsic rate of variation.

The same formulae can be used to compute the stable male birth rate, provided that the male survivorship function is used instead of the female survivorship function in calculating the expectation of life and the cumulant λ_1.

Tables II.14 and II.15 give details of the application of these formulae. We know that:

$$\lambda_1 = \frac{E_1}{E_0}$$

where

$$E_0 = \int_0^\infty p(a) da$$

and

$$E_1 = \int_0^\infty a p(a) da$$

Consequently, λ_1 is simply the mean age of the stationary population (table II.14). μ_1 has already been determined (table II.8). Thus, we have the calculations given in table II.15.

When the intrinsic rate of variation is *positive*, it can be demonstrated that the first and second approximations encircle the true value. We thus have an idea of the error resulting from use of the approximate formula. If the rate of variation is negative, however, we have no idea what this error will be.

TABLE II.14. COMPUTATION OF THE MEAN AGE OF THE FEMALE STATIONARY POPULATION. INTERMEDIATE MODEL LIFE TABLE GIVING AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS. INTERMEDIATE MODEL FEMALE FERTILITY GIVING A GROSS REPRODUCTION RATE OF 1.50

Median age a	*Age group (years)*	*Female stationary population per 100 000 births*	*Product of the preceding column and the median age*
2.5	0-4	460 386	1 150 960
7.5	5-9	448 010	3 360 080
12.5	10-14	444 150	5 551 880
17.5	15-19	439 970	7 699 480
22.5	20-24	434 040	9 765 900
27.5	25-29	427 035	11 743 460
32.5	30-34	419 610	13 637 320
37.5	35-39	411 672	15 437 700
42.5	40-44	402 742	17 116 540
47.5	45-49	391 728	18 607 080
52.5	50-54	377 275	19 806 940
57.5	55-59	357 718	20 568 780
62.5	60-64	330 472	20 654 500
67.5	65-69	291 642	19 685 840
72.5	70-74	238 028	17 257 030
77.5	75-79	171 308	13 276 370
82.5	80-84	102 020	8 416 650
87.5	85 +	56 792	5 111 280
	ALL AGES	6 204 598	228 847 790

$$\lambda_1 = \frac{228,847,790}{6,204,598} = 36.884 \text{ years.}$$

Age composition of deaths

The method given in the first example is used. We have not reproduced the calculation here.

THIRD EXAMPLE: THE CRUDE BIRTH RATE b_0 IS KNOWN

In this case, we can write the r equation:

$$\int_0^{\omega} e^{-ra} p(a)da = \frac{1}{b} \qquad \text{(II.1)}$$

In order to solve this equation, let us consider the integral:

$$J(r) = \int_0^{\omega} e^{-ra} p(a)da$$

J (r) a decreasing function of r. For infinitely great values of r it takes the following values:[17]

When r tends towards $+\infty$, J(r) tends towards zero; and when r tends towards $-\infty$, J(r) tends towards $+\infty$.

The curve representing b_r is thus of the form shown in graph II.7. The straight line of the ordinate b_0 cuts this curve at a single point M. Equation (II.1) has thus only

[17] As already pointed out, all that is needed in order to see how J(r) behaves for infinitely great values of r is to imagine J(r) written in discontinuous notation.

(*a*) When r tends towards $+\infty$, the first term (that corresponding to the lowest age) becomes preponderant over all the others, and consequently J(r) behaves like $e^{-r\varepsilon}L_{\varepsilon}$, where ε is a small but fixed quantity. When r tends towards $+e^{-re}$, L_ε tends towards zero.

(*b*) When r tends towards $-\infty$, the last term becomes preponderant over all the others, and J(r) behaves like $e^{-r(w-\varepsilon)}L_{w-\varepsilon}$: a quantity which tends towards $+\infty$.

TABLE II.15. COMPUTATION OF THE STABLE FEMALE BIRTH RATE BY APPROXIMATION

A. First formula: $b_f = \frac{R_0}{E_0}$

Gross reproduction rate	*Zero-order moment of the function:* $p_f(a)\varphi_f(a)$	*Female expectation of life at birth:* E_0	*Stable female birth rate,* b_f *(per thousand)*
1.50	1.276950	62.04598	20.58
0.75	0.638475	62.04598	10.29

B. Second formula: $b_f = \frac{R_0}{E_0} \cdot \frac{1 - \rho\,\mu_1}{1 - \rho\,\lambda_1}$

Gross reproduction rate	ϱ	μ_1	λ_1	$\varrho\mu_1$	$\varrho\lambda_1$	$1 - \varrho\mu_1$	$1 - \varrho\lambda_1$	$\frac{1-\varrho\mu_1}{1-\varrho\lambda_1}$	b_f
1.50	+0.0087	27.9521	36.884	0.24318	0.32089	0.75682	0.67911	1.114	22.93
0.75	−0.0157	27.9521	36.884	−0.43885	−0.57908	1.43885	1.57908	0.911	9.37

C. Comparison of various values of the stable female birth rate (per 1,000)

Gross reproduction rate	*First approximate formula*	*Second approximate formula*	*Rate computed by numerical integration*
1.50	20.58	22.93	21.79
0.75	10.29	9.37	8.46

one real root r_0, whose value is the abscissa of the point M. In order to calculate r_0 in practice, we can use a graphic method, of which graph II.7 gives the principles and graph II.8 gives a specific illustration.

In graph II.8 we have taken the same sub-set $H_0(r)$ as in the second example, i.e., we have taken the sub-set of all the Malthusian populations corresponding to a level-80 of intermediate model life table, with an expectation of life at birth for both sexes of 60.4 years. We have seen that the stable population of $H_0(r)$ corresponding to a gross reproduction rate [18] of $R' = 1.50$ has as its intrinsic rate of natural variation $\rho = 0.0087$ and as its stable birth rate for both sexes together $b = 0.02228$ (table II.13).

Let us now determine which population of the sub-set $H_0(r)$ has a birth rate (both sexes) of $b_0 = 0.02228$, i.e., exactly equal to the birth rate (both sexes) of the stable population whose characteristics we have just given.

In graph II.8 we have drawn, for both sexes together, the curve showing the variation of

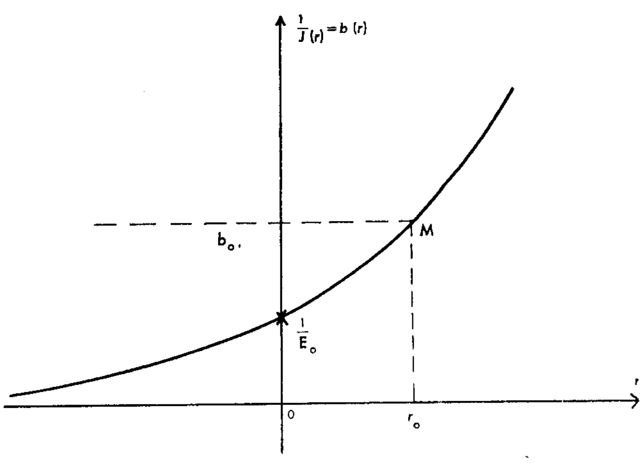
$$\frac{1}{J(r)}$$

in the sub-set $H_0(r)$ under consideration.[19] The straight line of $b = 22.28$ per thousand cuts this curve at a point M whose abscissa r must equal the value of the intrinsic rate of natural variation of the population which we have just given (i.e., $\rho = 0.0087$).

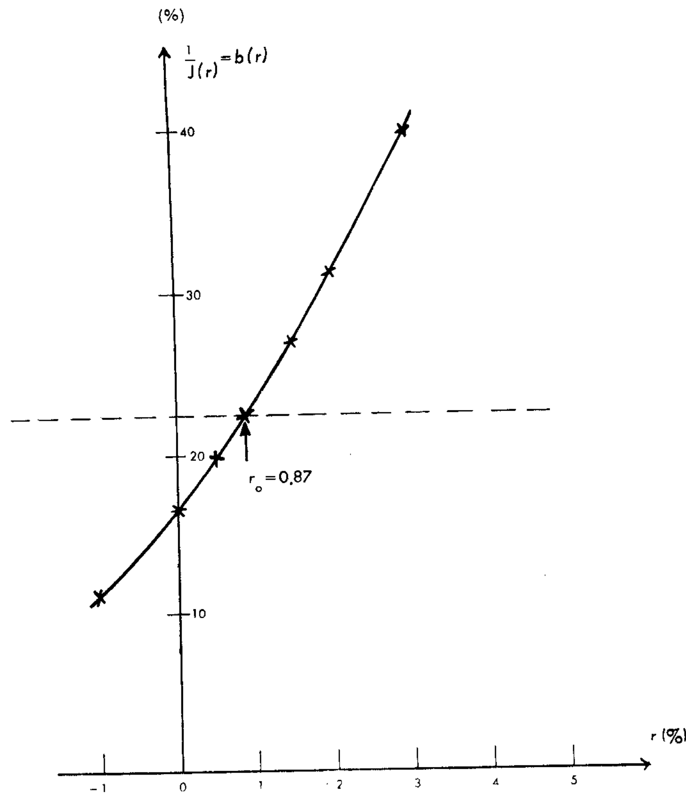
On the scale of the graph, we can indeed see that r_0 is between 0.008 and 0.009. All that is necessary in order to determine r_0 more precisely is to draw a graph on a larger scale. We can also, however, use a method by successive approximations similar to that described in the second example.

If r_1 is an approximate value of r_0, the true value of which is $r_0 = r_1 + \varepsilon$, we can write:

$$\int_0^{\omega} e^{-(r_1+\varepsilon)A} p(a)da = \frac{1}{b}$$



Graphic II.7. Form of the curve representing $\frac{1}{J(r)} = b\ (r)$ as a function of r in a sub-set $H_0(r)$ of Malthusian populations, i.e., a sub-set of all Malthusian populations belonging to the same life table



Graph II.8. Graphic determination of the rate of increase r_0 of the Malthusian population of the sub-set $H_0(r)$ associated with the intermediate model life table giving an expectation of life at birth for both sexes of 60.4 years and based on a crude birth rate (boys and girls) of 22.28 per thousand

and as ε is small, if we assimilate $e^{-\varepsilon a}$ to $1 - \varepsilon a$ we find that:

$$\varepsilon = \frac{\int_0^{\omega} e^{-r_1 a} p(a)da - \frac{1}{b}}{\int_0^{\omega} a e^{-r_1 a} p(a)da} \qquad \text{(II.22)}$$

In the particular case considered here, if we take $r_1 = 0.008$ we obtain $\varepsilon = 0.0007$, which does indeed give $r_0 = 0.0087$. Table II.16 gives details of the computation as it is performed for the female sex.

FOURTH EXAMPLE: THE CRUDE DEATH RATE d_0 IS KNOWN

In this case we can write the r equation:

$$b - r = \frac{1}{\int_0^{\omega} e^{-ra} p(a)da} - r = d_0 \qquad \text{(II.2)}$$

which is simply equation (II.2) in table II.2. In order to solve this equation, let us consider the equation:

$$K(r) = \frac{1}{\int_0^{\omega} e^{-ra} p(a)da} - r$$

When r tends towards $\pm\infty$, this equation tends towards $+\infty$.[20]

---

[18] The age distribution of female fertility rates being those of the intermediate model.

[19] We can plot the variation curves in this way for either the female or the male sex. The same applies to all the examples which follow.

[20] When r approaches $+\infty$, K(r) behaves like $e^{-r\varepsilon}/L_\varepsilon - r$ where ε is small but fixed. In these conditions, $e^{r\varepsilon}$ preponderates over r and consequently K(r) approaches $+\infty$. When r approaches $-\infty$, K(r) behaves like $(e^{r(\omega-\varepsilon)}/L_{\omega-\varepsilon}) - r$. As r is negative and infinitely great, $e^{r(\omega-\varepsilon)}/L_\varepsilon$ approaches zero and $-r$ approaches $+\infty$. K(r) thus approaches $+\infty$.

TABLE II.16. APPLICATION OF THE METHOD OF SUCCESSIVE APPROXIMATIONS IN ORDER TO COMPUTE THE RATE OF A MALTHUSIAN POPULATION WHOSE MORTALITY AND CRUDE BIRTH RATE ARE KNOWN (FORMULA II.22 IN THE TEXT). LEVEL-80 FEMALE MODEL MORTALITY RATE (EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS). FEMALE CRUDE BIRTH RATE $b_0 = 0.02179$ (VALUE TAKEN FROM TABLE II.11). APPROXIMATE VALUE OF r_0, $r_1 = 0.008$

Median age of age group a *(1)*	*Age group (years) (2)*	ra *for* $r = 0.008$ *(3)*	ra Log e *(4)*	e^{ra} *(5)*	*Stationary population* L_a *(6)*	*Quotient of (6) divided by (5)* $e^{-ra}L_a$ *(7)*	*Preceding column multiplied by* a *(8)*
2.5	0-4	0.02	0.00869	1.020	460 386	451 390	1.1285
7.5	5-9	0.06	0.02606	1.062	448 010	421 900	3.1695
12.5	10-14	0.10	0.04343	1.105	444 150	401 950	5.0250
17.5	15-19	0.14	0.06080	1.150	439 970	382 610	6.6970
22.5	20-24	0.18	0.07817	1.197	434 040	362 610	8.1600
27.5	25-29	0.24	0.09554	1.246	427 035	342 770	9.4260
32.5	30-34	0.26	0.11292	1.297	419 610	323 600	10.5850
37.5	35-39	0.30	0.13029	1.350	411 672	304 950	11.4370
42.5	40-44	0.34	0.14766	1.405	402 742	286 690	12.1850
47.5	45-49	0.38	0.16503	1.462	391 728	267 970	12.7360
52.5	50-54	0.42	0.18240	1.522	377 275	247 870	13.0170
57.5	55-59	0.46	0.19878	1.584	357 718	225 860	12.989
62.5	60-64	0.50	0.21715	1.649	330 472	200 160	12.511
67.5	65-69	0.54	0.23452	1.716	291 642	169 950	11.472
72.5	70-74	0.58	0.25189	1.786	238 028	133 220	9.658
77.5	75-79	0.62	0.26926	1.859	171 308	92 220	7.1420
82.5	80-84	0.66	0.28663	1.935	102 020	52 720	4.3490
87.5	85 +	0.70	0.30401	2.014	56 792	28 200	2.4680
ALL AGES						4 696 580	154.4155

$b_0 = 0.02179$ (value taken from table II.11).

$\frac{1}{b_0} = 45.89. \qquad \varepsilon = \frac{46.97 - 45.89}{1\ 544.155} = 0.0006978.$

$r_0 = r_1 + \varepsilon = 0.008 + 0.0006978 = 0.0086978.$

Moreover:

$$\frac{dK(r)}{dr} = \frac{\int_0^\infty ae^{-ra}p(a)da}{\left[\int_0^\infty e^{-ra}p(a)da\right]^2} - 1$$

We can easily see that the expression:

$$\frac{\int_0^\infty ae^{-ra}p(a)da}{\left[\int_0^\infty e^{-ra}p(a)da\right]^2}$$

increases from zero to $+\infty$ when r varies from $-\infty$ to $+\infty$. It thus takes once the value 1 and the derivative is set at zero for a value r_m of r. The curve representing K(r) is thus of the form shown in graph II.9. It passes through a minimum for $r = r_m$.

The straight line of ordinate d_0 cuts this curve at zero point or at one point or at two points. Let us take the case where there are two points of intersection M and M′; the abscissae r_0 and r'_0 of these two points are the real solutions of equation II.2. Consequently, if we assume a value for the crude death rate, there will be zero, one or two populations in the set $H_0(r)$ with that death rate. The result depends on the value of d_0.

When there is only one solution, we have $r_0 = r'_0 = r_m$. The corresponding Malthusian population possesses a remarkable property, for we have for this population:

$$\frac{dK(r)}{dr} = 0$$

which is written:

$$\frac{\int_0^\infty ae^{-ra}p(a)da}{\int_0^\infty e^{-ra}p(a)da} \times \frac{1}{\int_0^\infty e^{-ra}p(a)da} = 1$$

or

$$A(r_m)b(r_m) = 1$$

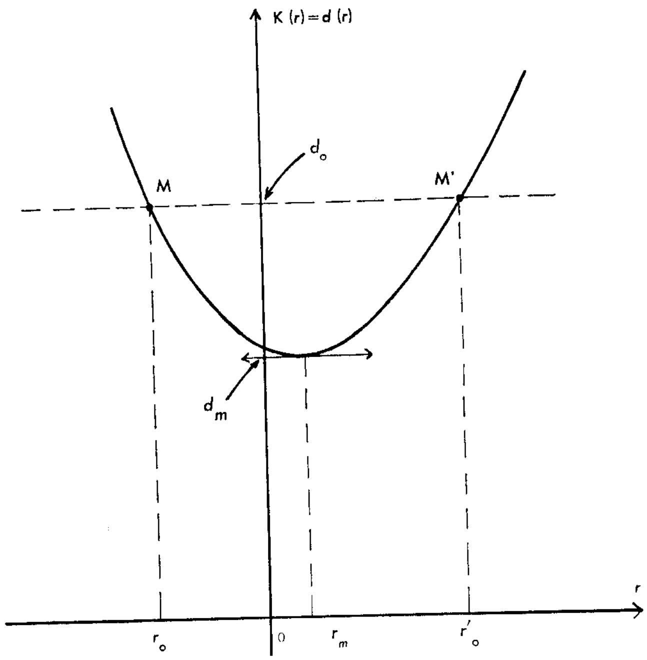
where $A(r_m)$ is the mean age of the population. In other words, in the Malthusian population corresponding to the minimum of K(r), the mean age equals the inverse of the crude birth rate.

Finally, for a sub-set $H_0(r)$, K(r) represents all the death rates of the Malthusian populations of that sub-set. We see that these death rates pass through a minimum d_m.

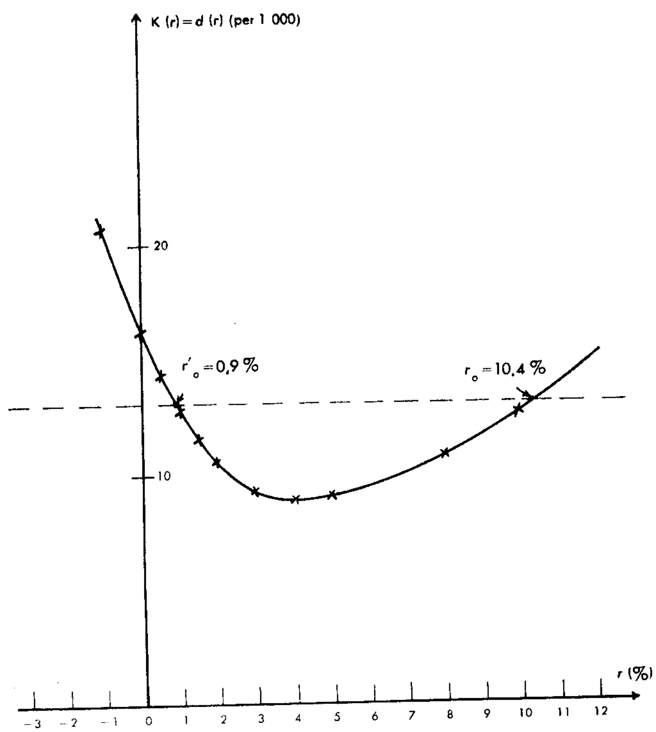
Graph II.9 gives the principle of the graphic solution of equation II.2, while graph II.10 gives a specific application of it. We take the same sub-set $H_0(r)$ as in the two preceding examples (i.e., intermediate model life table with an expectation of life at birth for both sexes of 60.4 years). *For the female population*, the curve of graph II.10 is the curve of variation of K(r) in this sub-set.

In the second example considered above, the stable population corresponding to the gross reproduction rate $R' = 1.50$ had as its crude female death rate $d_0 = 0.01309$, while the intrinsic rate of natural variation was $r_0 = 0.0087$.

Let us determine the Malthusian populations of the sub-set $H_0(r)$ which have in fact crude female death rates of exactly the value obtained for the stable populations whose characteristics were referred to above. The straight line of the ordinate 0.01309 cuts the curve of graph II.10 at two points, M and M′, whose abscissae are the values of r which are sought. One of these values must be equal to the intrinsic rate of variation of the stable population which we have already computed, and this is in fact exactly what we obtain.



Graph II.9. Form of the curve representing the variation of K(r) = d(r) as a function of r in a sub-set H₀(r) of Malthusian populations, i.e., the sub-set of all Malthusian populations having the same life table



Graph II.10. Graphic determination of the rates of increase r_0 and r'_0 of the two Malthusian populations of the sub-set $H_0(r)$, associated with the intermediate model life table giving an expectation of life at birth for both sexes of 60.4 years, for which the crude female death rate is 13.09 per thousand

At the scale of the graph we have:

$r_0 = 0.0090$

$r'_0 = 0.104$

If greater precision is desired, a graph on a larger scale must be traced. A method by successive approximation can also be used, as in the preceding examples. If r_1 is an approximate value of r_0 and if we assume that $r_0 = r_1 + \varepsilon$, we easily find by transferring this value of r_0 into II.2 and assimilating $e^{-\varepsilon a}$ to $1 - \varepsilon a$ that:

$$\varepsilon = \frac{1 - (d_0 + r_1)\int_0^\infty e^{-r_1 a}\, p(a)da}{\int_0^\infty e^{-r_1 a}\, p(a)da - (d_0 + r_1)\int_0^\infty a e^{-r_1 a} p(a)da}$$

This can also be written:

$$\varepsilon = \frac{b(r_1) - (d_0 + r_1)}{1 - (d_0 + r_1)\mathrm{A}(r_1)} = \frac{d(r_1) - d_0}{1 - (d_0 + r_1)\mathrm{A}(r_1)} \qquad \text{(II.23)}$$

where $d(r_1)$ is the death rate of the Malthusian population corresponding to r_1 and $\mathrm{A}(r_1)$ is the mean age of the same population.

Let us apply this formula for $r_1 = 0.008$. According to table II.16, we have:

$$b(r_1) = \frac{100\,000}{4\,696\,580} = 0.021292$$

and consequently $d(r_1) = 0.021292 - 0.008 = 0.013292.$

We also have, again according to table II.16:

$$\mathrm{A}(r_1) = \frac{154\,415\,500}{4\,696\,580} = 32.882$$

We therefore have:

$$\varepsilon = \frac{0.013292 - 0.013090}{1 - (0.013090 + 0.008000)32.882} = \frac{0.000202}{1 - 0.69355}$$

and finally:

$\varepsilon = 0.000659$, whence $r_0 = 0.008 + 0.000659 = 0.008659$,

i.e., roughly 0.0087. We thus arrive, once again, at the intrinsic rate of variation of the stable population.

As regards $r'_1 = 0.103$, there would obviously be another correction.

Fifth example: the age distribution $c(a_0)$ at a given age a_0 is known

In this case we must write the r equation:

$$\frac{e^{-ra_0}p(a_0)}{\int_0^\infty e^{-ra}p(a)da} = c(a_0) \qquad \text{(II.3}b\text{)}$$

This is formula (II.3b) in table II.2 Let us consider the equation:

$$\mathrm{U}(r) = \frac{e^{-ra_0}p(a_0)}{\int_0^\infty e^{-ra}p(a)da}$$

When r tends towards $\pm\infty$, U(r) tends towards zero.[21] In addition, its derivative may be written:

$$\frac{dU(r)}{dr} = \frac{-ae^{-ra_0}p(a_0)\int_0^{\omega} e^{-ra}p(a)da + e^{-ra_0}p(a_0)\int_0^{\omega} ae^{-ra}p(a)da}{\left[\int_0^{\omega} e^{-ra}p(a)da\right]^2}$$

which becomes:

$$\frac{dU(r)}{dr} = b(r)e^{-ra_0}p(a_0)\,[A(r) - a_0]$$

where $b(r)$ and A(r) are, respectively, the crude birth rate and the mean age of the Malthusian population having a rate of increase r. As r increases from $-\infty$ to $+\infty$, the mean age of the population decreases from ω to zero. It assumes once the value of a_0, and for the corresponding value of r, i.e., r_m, the derivative is cancelled out.

Thus, the curve representing the variation of U(r) is of the form shown in graph II.11 and passes through a maximum.[22]

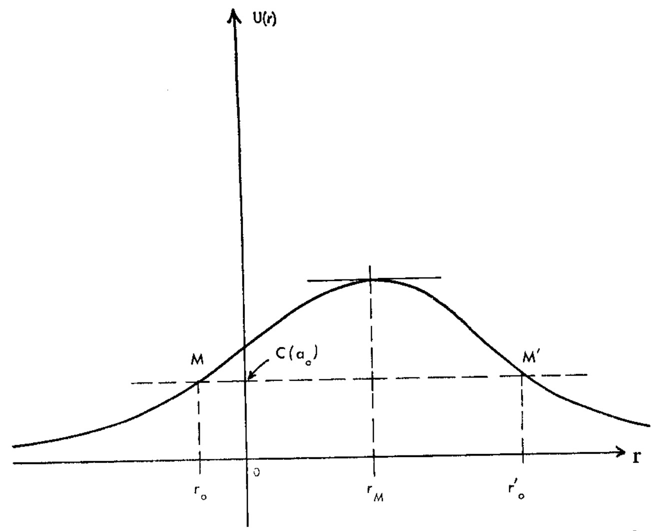
The straight line of ordinate $c(a_0)$ cuts this curve at zero point or at one point or at two points, according to the value of $c(a_0)$. If we take the case where there are two points of intersection, M and M′, then the abscissae r_0 and r'_0, of these two points are the real roots of equation II.3*b*. We thus have a result similar to that obtained in the fourth example.

Graph II.11 shows the principle of the graphic solution of equation II.3*b*, while graph II.12 gives a specific illustration of this. We continue to take the same sub-set $H_0(r)$ (intermediate model life table giving an expectation of life at birth for both sexes of 60.4 years). The curve of the graph for $U_0(r)$ is the curve representing the variation, in the sub-set in question, of the proportion C_{25-29} *of the female population*. We propose to determine, for the sub-set $H_0(r)$, the populations for which $C_{25-29}=0.073242$. This value of C_{25-29} is that of the stable population corresponding in the second example of $R' = 1.50$. In these conditions, one of the two values r_0 or r'_0 must be equal to the intrinsic rate of natural variation of this stable population, $\rho = 0.0087$. This is in fact what we observe in graph II.12. At the scale of this graph, we have the following approximate values:

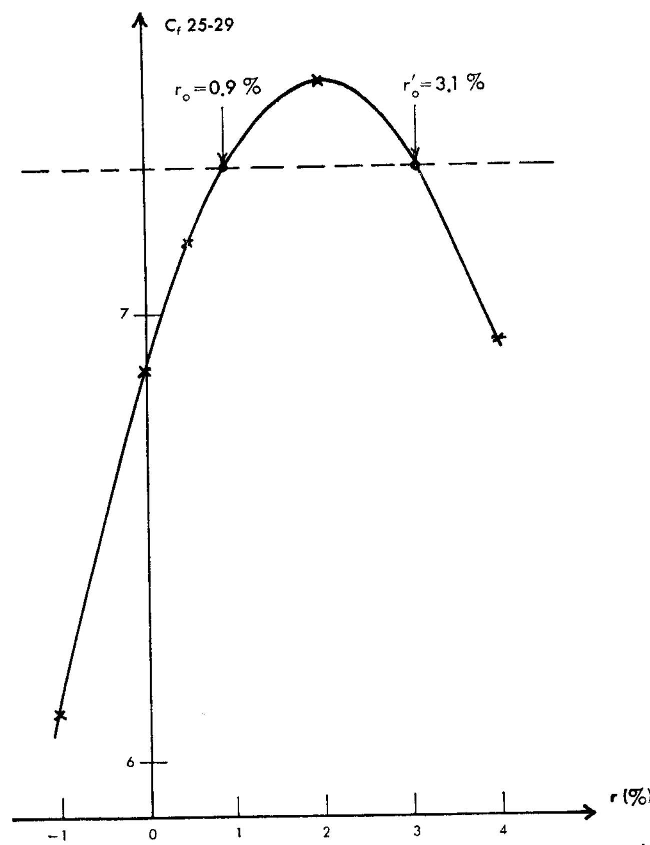
$r_0 = 0.0090$

$r'_0 = 0.0310$

As in the preceding examples, it is easy to apply a more accurate method of computation by successive approximations. No details are given here of these computations, since they are similar in all respects to the computations described in preceding examples.



Graph II.11. Form of the curve representing the variation of the integral U(r) as a function of r



Graph II.12. Graphic illustration of the rates of increase r_0 and r'_0 of the two Malthusian populations of the sub-set $H_0(r)$, associated with the intermediate model life table corresponding to an expectation of life at birth for both sexes of 60.4 years, in which women aged 25 to 29 represent 7.3 per cent of the total

Sixth example: the age distribution of deaths $d(a_0)$ at a given age a_0 is known

In this case, we have to write the r equation:

$$d(a_0) = \frac{e^{-ra_0}p(a_0)q(a_0)}{\int_0^{\omega} e^{-ra}p(a)q(a)da} \qquad \text{(II.8)}$$

[21] When r approaches $+\infty$, U(r) behaves like

$$\frac{e^{-ra_0}p(a_0)}{e^{-r\varepsilon}}$$

The value ε can always be selected smaller than a_0, so that e^{-ra_0} will predominate over $e^{-r\varepsilon}$ and U(r) will approach zero. When r tends towards $-\infty$, U(r) behaves like

$$\frac{e^{-ra_0}p(a_0)}{e^{-r(\omega-\varepsilon)}L_{\omega-\varepsilon}}$$

If the quantity ε is selected small enough, $e^{-r(\omega-\varepsilon)}$ predominates over e^{-ra_0}, since r is negative, and consequently U(r) tends towards zero.

[22] At the maximum, the derivative is cancelled out and we therefore have $A(r) = a_0$. The mean age of the population is equal to the age a_0, for which we know the age structure $C(a_0)$.

This is formula II.8 in table II.2, and it is of the same form as formula II.3*b* in the fifth example. Let us consider the equation:

$$M(r) = \frac{e^{-ra_0}p(a_0)q(a_0)}{\int_0^{\omega} e^{-ra}p(a)q(a)da}$$

The curve representing M(r) as a function of r is of the same form as the curve representing U(r) (graph II.11). M(r) passes through a maximum, and the straight line of the ordinate $d(a_0)$ cuts it at zero point or at one point or at two points. The r equation (II.8) thus has no solution, one solution or two solutions, according to the value of $d(a_0)$.

Graph II.13 gives an example of the graphic solution of equation II.8. In this case, we have taken a sub-set $H_0(r)$ which is different from the sub-set considered in examples 2 to 5, being the one used in the first example,[23] associated with the level-60 of the intermediate model life table giving an expectation of life at birth for both sexes of 50 years. It will be recalled that in the first example we calculated the age distribution of female deaths in the Malthusian population of the sub-set $H_0(r)$ which had a rate of natural variation of 0.03, and we found that 22.82 per cent of female deaths occurred in the 40-44 age group.

Let us now determine the Malthusian populations of the sub-set $H_0(r)$ in which the proportion of female deaths in the 40-44 age group is exactly 22.82 per cent. We must find the Malthusian population whose rate of variation is 0.03, but there is also another one, as can be seen from graph II.13, on which we have plotted the curve representing the variation of M(r) as a function of r for the 40-44 age group. The straight line of the ordinate of 0.2282 cuts this curve at two points M and M′ whose abscissae are:

$$r_0 = -0.0125$$

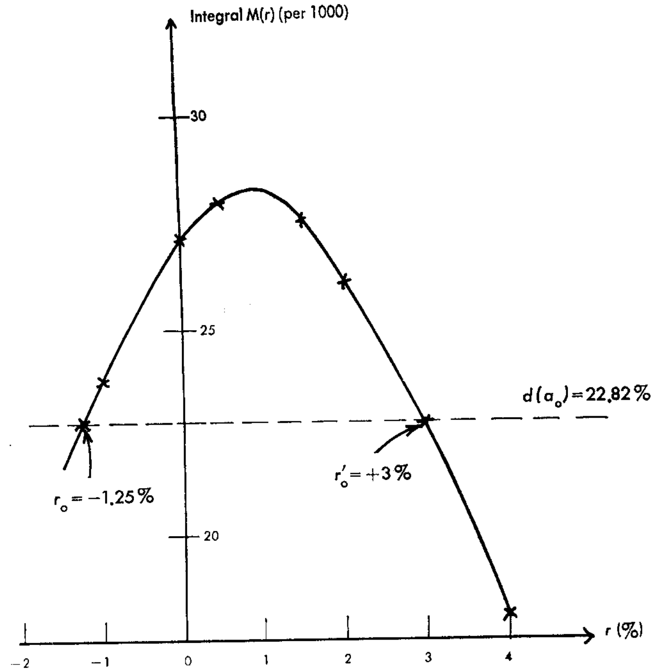
and

$$r'_0 = +0.030$$

Thus we find, once again, the exponential population with a rate of variation of 0.03, and another with a rate of variation of −0.0125.

Obviously, the graphic solution of equation II.8 is an imprecise method. Successive approximations enable us to calculate more accurate values similar to that worked out in the previous examples.

We shall confine ourselves to these six examples, but many others can readily be imagined. Instead of assuming the age distribution at a given age, for example, we can assume the ratio between two age groups, such as the ratio of the number of old people to the number of persons of working age, or any other combination of ages. All these problems are treated in the same way. From the mathematical point of view, they all lead to an equation in r, and all that is necessary is to find the real roots of that equation.



Graph II.13. Graphic determination of the rates of increase r_0 and r'_0 of the two Malthusian populations of the sub-set H(r) associated with the intermediate model life table giving an expectation of life at birth for both sexes of 50 years, in which the proportion of deaths of women aged 40 to 44 represents 22.82 per cent of total female deaths

In all these examples, the value or values of r used in the equation are exact values. If we use approximate values and content ourselves with solving equations whose solutions are "statistical" variables diverging to a greater or less extent from an average value, we begin to encounter other problems, of which we shall now give two examples.

COMPATIBILITY OF A GIVEN AGE DISTRIBUTION OF THE POPULATION WITH A GIVEN MORTALITY

For a given sub-set $H_0(r)$, we cannot make an arbitrary assumption of the whole of the age distribution $C_0(a)$. Indeed, we showed in the fifth example that we need only know the age distribution at a single age $C_0(a_0)$ in order to know the age distribution for all ages.[24]

Let us, then, consider an actual population, and let $p_0(a)$ and $c_0(a)$ be the survivorship function and the age distribution observed at a given time. Let us consider the sub-set $H_0(r)$ corresponding to $p_0(a)$, and let us pose the following question: does the age distribution $C_0(a)$ belong to the sub-set $H_0(r)$?

If we want $C_0(a)$ to coincide exactly with the age distribution of a population of the sub-set $H_0(r)$, we shall generally have to answer this question in the negative. It will be recalled from what was stated above in connexion with the fifth example that each term of the sequence $C_0(a)$ determines a particular population of the sub-set $H_0(r)$, and it is unlikely that all the particular populations thus determined will be *absolutely* identical. It may happen that all the populations are *almost* identical, and if we are willing to accept an approximate coincidence, the question which we have posed takes on a totally different meaning.

[23] The first example is the only one in which we calculated the age distribution of deaths in a Malthusian population. That is why we have reverted to this example to illustrate the case where the age distribution of deaths at a given age a_0 is known.

[24] Obviously, we are assuming the case where the straight line of ordinate $C_0(a_0)$ intersects the curve of graph II.11.

If the populations coincide exactly, then there are two numbers b_0 and r_0 which are such that:

$$b_0 e^{-r_0 a} p_0(a) = C_0(a)$$

If we take the logarithms, this is written:

$$\text{Log } b_0 - r_0 a \text{ Log } e = \text{Log } C_0(a) - \text{Log } p_0(a) \quad (II.23)$$

Let us now assume that:

$$y = \text{Log } p_0(a) - \text{Log } C_0(a)$$

and

$$x = a,$$

we shall then have:

$$y = x r_0 \text{ Log } e - \text{Log } b_0 \quad (II.24)$$

To each value of a there corresponds a pair of values (x, y), and if we plot a graph with x on the horizontal axis and y on the vertical axis the points obtained will be on a straight line given by the equation II.24.

If there is only an approximate coincidence, we shall obtain a cluster of points which can be more or less adjusted by the straight line of equation II.24.

Let us see how the problem occurs in a specific example. Table II.17 gives the age distribution of the female population of Mexico according to the 1940 census and the life table computed for the same population for 1940. In order to apply the foregoing formulae in discontinuous form, we must use the approximate formulae described earlier:

$$5C_0(22.5) = C_{20\text{-}24}$$

and

$$5p_0(22.5) = L_{20\text{-}24}$$

We then see that:

$$y = \text{Log } p_0(22.5) - \text{Log } C_0(22.5) = \\ = \text{Log } L_{20\text{-}24} - \text{Log } C_{20\text{-}24}$$

and we shall assume that $x = 22.5$

Table II.17 gives details of the calculation of y for successive age groups, and graph II.14 shows that the set of points (x, y) can be adjusted by a straight line. It is true that there are some points which diverge from the adjustment line fitted, particularly at the two extremities, but it is well known that young children (0 and 1-4 years of age) and old people (80 and over) are often incorrectly recorded in censuses, and it is therefore to be expected that the points corresponding to very young and very old people should show some divergences.

The ordinate at the origin of the adjustment line is written according to formula II.23: $-\text{Log } b_0 = 1.29$, whence $\text{Log } b_0 = 2.71$, which gives us: $b_0 = 51.29$ per thousand.

TABLE II.17. COMPUTATION OF THE QUANTITIES y OF FORMULA (II.24) IN THE TEXT, WITH A VIEW TO STUDYING THE COMPATIBILITY OF THE AGE DISTRIBUTION OF THE FEMALE POPULATION OF MEXICO AS DETERMINED IN THE 1940 CENSUS, C_a, WITH THE LIFE TABLE COMPUTED FOR THE SAME POPULATION AT THE SAME DATE, L_a

Median age, $a = x$	*Age group (years)*	*Female population according to 1940 census,* C_a	*Life table for Mexico in 1940,* L_a	Log C_a	Log L_a	*Difference* Log. L_a — Log C_a $= y$
0.5	0	2 628	88 740 (b)	3.41963	4.94812	1.52849
2.5	1-4	11 599	311 000 (c)	4.06446	5.49276	1.42830
7.5	5-9	13 936	349 092 (d)	4.14457	5.54294	1.39837
12.5	10-14	11 611	338 925	4.06483	5.53011	1.46528
17.5	15-19	10 314	330 632	4.01326	5.51934	1.50608
22.5	20-24	8 114 (a)	318 650	3.90924	5.50331	1.59407
27.5	25-29	8 432 (a)	304 600	3.92593	5.48373	1.55780
32.5	30-34	6 874	290 035	3.83721	5.46244	1.62523
37.5	35-39	7 041	274 487	3.84763	5.43853	1.59090
42.5	40-44	4 897	257 710	3.68993	5.41113	1.72120
47.5	45-49	3 970	239 580	3.59879	5.37945	1.78066
52.5	50-54	3 182	219 947	3.50270	5.34232	1.83962
57.5	55-59	2 205	197 375	3.34341	5.29530	1.95189
62.5	60-64	2 157	168 947	3.33385	5.22776	1.89391
67.5	65-69	1 158	134 345	3.06371	5.12824	2.06453
72.5	70-74	845	96 272	2.92686	4.98350	2.05664
77.5	75-79	450	59 575	2.65321	4.77506	2.12185
82.5	80-84	336	30 702	2.52634	4.48717	1.96083
87.5	85 +	251	20 180 (e)	2.39967	4.30492	1.90535
ALL AGES . . .		100 000				

SOURCE: For the age distribution C_a, see *Demographic Yearbook, 1940* (United Nations publication, Sales No.: 49.XIII.1). For the life table, see *Demographic Yearbook, 1961* (United Nations publication, Sales No.: 62.XIII.1).

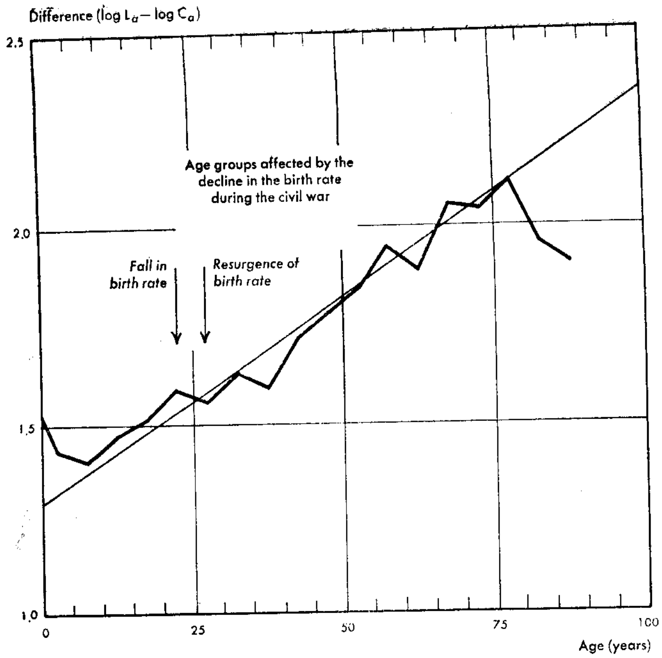
(a) Decline in birth rate during the revolution of 1911-1921, followed by a resurgence of the birth rate after the civil war. If these events were ignored it would be necessary to correct the age distribution. A graphic adjustment would involve taking 8,814 instead of 8,114 for the 20-24 age group and 7,732 instead of 8,432 for the 25-29 age group.

(b) $L_0 = 0.25p(0) + 0.75p(1)$.

(c) $L_1 = 1.9p(1) + 2.1p(5)$.

(d) $L_{5\text{-}9} = \frac{p(5) + p(10)}{2} \times 5$. This formula is valid for all five-year age groups.

(e) $L_{85+} = p(85) \times \text{Log } p(85)$, $p(85)$ being expressed per 100,000 births.



Graph II.14. Compatibility of the age distribution of the female population of Mexico, as determined in the 1940 census, with the life table computed for the same population at the same date

The ordinate of the point corresponding to 100 years on the horizontal axis equals:

$$\text{Log } b_0 + 100\, r_0 \text{ Log } e = 2.37$$

We therefore have: $100\, r_0 \text{ Log } e = 1.08$, whence [25]

$$r_0 = \frac{1.08}{43.429} = 0.02487$$

Finally, the crude death rate is:

$$d_0 = b_0 - r_0 = 26.42 \text{ per thousand}$$

Thus, the female population of Mexico, as recorded in the 1940 census, coincides quite well with a Malthusian population whose death rate is given in the life table in table 11.17 and whose crude birth and death rates and rate of increase are, respectively:

$b_0 = 51.29$ (per thousand)

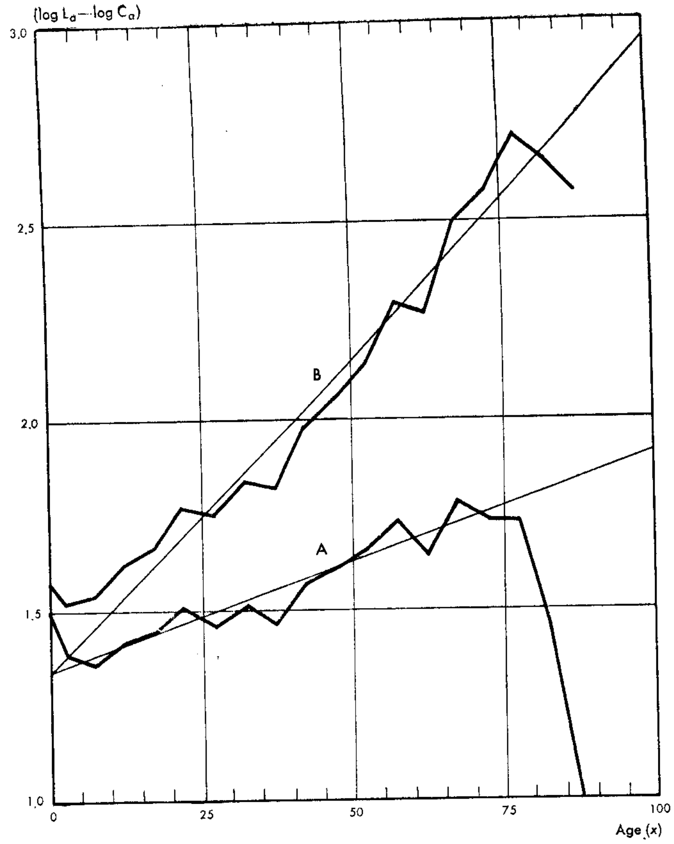
$d = 26.42$ (per thousand)

$r_0 = 24.87$ (per thousand)

The fact that we have obtained a good adjustment with the life table for Mexico in 1940 must not delude us, however. We have contented ourselves with an approximate coincidence, and there are a number of other Malthusian populations which coincide quite as well as that described above.

By way of example, we have plotted in graph II.15 the cluster of points (similar to those in graph II.14) for two other life tables in the United Nations series: one for level 20 (expectation of life at birth for both sexes of 30 years) and one for level 100 (expectation of life at birth for both sexes of 70.2 years). In both cases, we have adjustment straight lines quite as good as those in graph II.14. Indeed, we should have good adjustment straight lines for all the intermediate model life tables between the two levels considered in graph II.15.

[25] It may be recalled that Log e = 0.43429.



Graph II.15. Compatibility of the age distribution of the female population of Mexico, according to the 1940 census— with the two intermediate model life tables corresponding to expectations of life at birth for both sexes of 30 years (curve A) and 70.2 years (curve B)

Finally, for the three Malthusian populations considered in graphs II.14 and II.15 we have the following crude rates:

Crude rate (per thousand)	*Life table for Mexico, 1940*	*Intermediate model life table*	
		Level 20 (a)	*Level 100* (b)
b_0	51.29	45.71	45.71
d_0	26.42	32.59	8.41
r_0	24.87	13.12	37.30

(a) Expectation of life at birth for both sexes of 30 years.
(b) Expectation of life at birth for both sexes of 70.2 yea

The variations in the crude birth rate are not very great, and it can no doubt be accepted that the crude female birth rate in Mexico in 1940 was between 45 and 50 per thousand. However, there is considerable uncertainty regarding the crude death rate and rate of increase. Here we encounter for the first time a difficulty resulting from the fact that variations in mortality of the kind which occur in the human species [26] have little effect on the age structure of populations. Conversely,

[26] It would perhaps be better to say "of the kind which have occurred in the human species", since there is no absolute certainty that mortality in countries where it is still high will decline in the same way as it did in the past in the developed countries.

therefore, very diverse assumptions regarding the unknown level of mortality can be compatible with the age structures observed.

The crude birth rate determined with the aid of graph II.15 (51.29 per thousand) seems a little high, because the crude birth rate recorded in 1940 was 44.3 per thousand, and if the true rate was 51.29 per thousand it would be necessary to assume under-registration of births of the order of 15 per cent. Although such a percentage is not unusual in developing countries, it is generally agreed that it would be unusual in the case of Mexico, where registration of births is considered to be almost total.

Compatibility of a given distribution of deaths with a given death rate

We can pose a problem similar to that dealt with above, using the age distribution of deaths instead of the age distribution of the population.

We can ask to what extent the observed age distribution of deaths $d(a)$ in Mexico in 1940 belongs to the sub-set $H_0(r)$ associated with the life table computed for Mexico for 1940.

If there were a perfect coincidence between the observed age distribution of deaths $d(a)$ and the age distribution of deaths of a particular Malthusian population of the sub-set $H_0(r)$, we should have:

$$d(a) = -\frac{b}{d}p'(a)e^{-ra}$$

and if we take the logarithms:

$$\text{Log}\,\frac{d(a)}{-p'(a)} = -ra\,\text{Log}\,e - \text{Log}\,\frac{d}{b}$$

It may be noted in passing that $-p'(a)$ is simply the age distribution of deaths in the stationary population corresponding to the life table for 1940.

If we assume that:

$$y = \text{Log}\,\frac{d(a)}{-p'(a)} \quad \text{and} \quad x = a$$

then the points (x, y) will be on the straight line:

$$y = -rx\,\text{Log}\,e - \text{Log}\,\frac{d}{b} \qquad \text{(II.26)}$$

Table II.18 gives details of the computation of y for the female population and graph II.16 shows that if we are satisfied with an approximate coincidence the pattern of points (x, y) can be satisfactorily adjusted by a straight line. As in graph II.14, the points which deviate from the straight line are those corresponding to young children (0 and 1-4 years of age) and old people (80 and over).

The ordinate at the origin of the straight line gives:

$$-\text{Log}\,\frac{d}{b} = 0.330$$

whence

$$\text{Log}\,\frac{d}{b} = 1.670$$

and finally

$$\frac{d}{b} = 0.46774$$

or

$$\frac{b-r}{b} = 1 - \frac{r}{b} = 0.46774$$

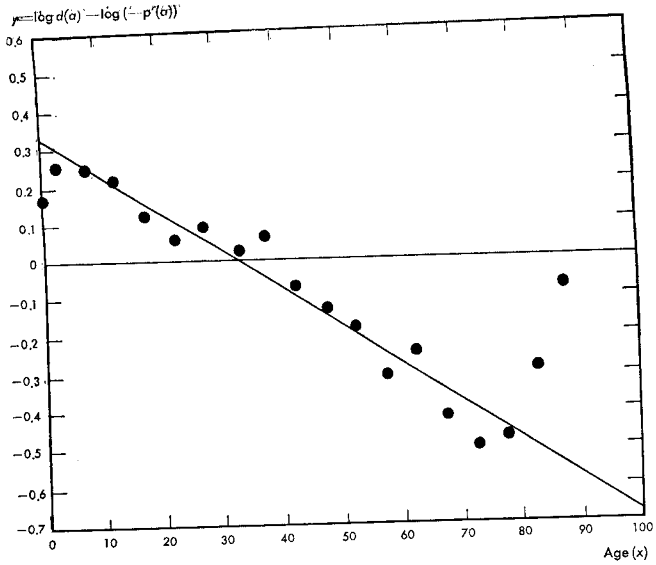
whence

$$\frac{r}{b} = 0.53226$$

Table II.18. Calculation of the quantities y of formula (II.25) in the text with a view to studying the compatibility of the age distribution of female deaths recorded in Mexico in 1940 ($d(a)$) with the female life table computed for Mexico for the same year ($p(a)$)

Age group (years)	*Distribution of female deaths by age groups observed in 1940* $d(a)$	*Distribution of deaths in the stationary population* $-p'(a)$	Log $d(a)$	Log $[-p'(a)]$	*Difference* Log $d(a)$ — Log $[-p'(a)]$
Under 1 year	22 604	15 072	4.35419	4.17644	0.17775
1-4	25 297	13 794	4.40307	4.13969	0.26338
5-9	4 922	2 751	3.69214	3.43949	0.25265
10-14	2 020	1 216	3.30535	3.08493	0.22042
15-19	2 699	2 001	3.43120	3.30125	0.12995
20-24	3 184	2 792	3.50297	3.44592	0.05705
25-29	3 506	2 828	3.54481	3.45148	0.09333
30-34	3 184	2 998	3.50297	3.47683	0.02614
35-39	3 699	3 221	3.56808	3.50799	0.06009
40-44	2 966	3 490	3.47217	3.54283	—0.07006
45-49	2 789	3 762	3.44545	3.57542	—0.12997
50-54	2 714	4 191	3.43361	3.62232	—0.18871
55-59	2 367	4 738	3.37420	3.67560	—0.30140
60-64	3 781	6 633	3.57761	3.82171	—0.24410
65-69	2 740	7 128	3.43775	3.85297	—0.41522
70-74	3 214	8 181	3.50705	3.91281	—0.49576
75-79	2 210	6 498	3.34439	3.81278	—0.46839
80-84	2 551	5 051	3.40671	3.70338	—0.29667
85 +	3 553	3 615	3.55059	3.55811	—0.00752
	100 000	100 000			

Sources: For the age distribution of deaths, see *Demographic Yearbook, 1951* (United Nations publication, Sales No.: 52.XIII.1). For the life table, see *Demographic Yearbook, 1961* (United Nations publication, Sales No.: 62.XIII.1).



Graph II.16. Compatibility of the age distribution of female deaths observed in Mexico in 1940, $d(a)$, with the female life table computed for Mexico in 1940, $p(a)$

The abscissa of the point corresponding to 100 on the adjustment line is:

$$-0.680 = -100r \text{ Log } e + 0.330, \text{ whence } r = 0.02326.$$

When we know r we can calculate b and d by the formulae given earlier, so that we finally have:

$b = 0.04370$
$d = 0.02044$
$r = 0.02326$

Compared with the result of graph II.14, we have approximately the same value for the rate of increase, r, but the crude birth and death rates are different, the crude birth rate being this time a little too low.[27]

Effects of census and registration errors

We have used crude data just as they were collected, but we know that these data suffer from certain errors, including, in particular:

(*a*) Under-estimation of the number of very young persons (particularly children under 1 year of age and, to a less extent, children between the ages of 1 and 4);

(*b*) Exaggeration of the ages of elderly persons (80 or over). These errors affect the data both for the number of persons in the population and for the number of deaths, so that between the ages of 5 and 80 the values $C(a)$ and $d(a)$ used to construct graphs II.14 and II.16 are lower than they should be.

We have also used for these graphs the life table calculated for 1940 (function $p(a)$ for graph II.14 and function $-p'(a)$ for graph II.16). This table has, however, been at least partially corrected for the errors mentioned.[28] Correction of the data would thus have the result, between the ages of 5 and 80, of raising the ordinates of the points in graph II.14 and lowering the ordinates of the points in graph II.16, and the two characteristics of the Malthusian populations determined from the age structure and the age distribution of deaths respectively would consequently be closer to each other. In other words, the two values obtained for the crude birth rates would probably straddle the true rates. The mean figure:

$$\frac{51.3 + 43.4}{2} = 47.5 \text{ (per thousand)}$$

is in fact in conformity with everything we know about the birth rate in Mexico in 1940.

What can be assumed regarding female fertility when there are not data for it?

In all the preceding examples, except that leading to the concept of a stable population, the fertility of the Malthusian populations has remained unknown throughout.[29] We know, however, that it must satisfy the equation:

$$\int_u^v e^{-ra} p_f(a) \varphi_f(a, t) da = 1 \qquad \text{(II.13)}$$

In discontinuous notation, this equation is written, for 100,000 girls at birth:

$$\sum_u^v e^{-ra} L_a f_a(t) = 100\,000 \qquad \text{(II.13}b\text{)}$$

The fertility will be determined only if we assume an additional condition.

The variation in the female fertility rate as a function of the age of the woman always follows the same pattern, beginning from zero at about 15 years of age, rising to a maximum between 20 and 30, and then declining until it returns to zero at about the age of 50. The rising phase is dependent primarily on marital status, since at the ages below 50 few women are sterile and under conditions of natural fertility, i.e., when there is no birth control, a woman's reproductive activity is determined almost entirely by whether or not she is married. Even where there is birth control, it has little influence at the time when a family is being started, and marital status therefore remains the preponderant factor. The earlier marriage takes place, the earlier the age at which maximum fertility is reached.

The declining phase of the age-specific fertility rate depends mainly, under natural conditions, on the fact that more and more women become sterile as they grow older.[30] Where birth control is practised, the situation is affected by voluntary sterility through the use of contraceptive methods in addition to the natural phenomenon of increasing female sterility with increasing age.

Table II.19 gives the age distribution of fertility rates observed recently in various countries of the world. We see that this distribution varies relatively little, and the distributions observed in Jamaica in 1951 and in Spain in 1940 may be considered extremes. It might be thought surprising that the mean distributions in countries with high fertility are very close to those of countries with

[27] It may be recalled that the recorded crude rate was 44.3 per thousand and that, as we stated, there was a relatively small, but nevertheless not negligible, under-registration of births.

[28] For example, according to the recorded deaths the female infant mortality rate in 1940 was 118.1 per 1 000 live births, while in the life table it was 150.1 per 1 000 live births.

[29] In the example for a stable population, the data included the fertility.

[30] This does not necessarily mean biological fertility, for social factors also enter into the question. The adjective "natural" simply signifies that we are not concerned with sterility consciously connected with family size, as in the case of voluntary sterility through the use of contraceptive methods.

TABLE II.19. DISTRIBUTION OF AGE-SPECIFIC FERTILITY RATES ([a]) IN SELECTED AREAS

Age group (years)	*Average ([b]) for 52 countries*	*Average for 15 countries with high fertility*	*Average for 37 countries with low fertility*	*Jamaica (1951)*	*Spain (1940)*	*Intermediate distribution*
15-19 ([c]) . . .	63	93	51	136	14	100
20-29	253	251	254	292	147	273
25-29	276	235	285	249	303	263
30-34	211	196	216	166	272	188
35-39	134	137	132	110	179	121
40-44 ([c])	63	69	60	47	85	55
15-44 years . .	1 000	1 000	1 000	1 000	1 000	1 000

([a]) The distribution covers both male and female fertility, but as the rate of masculinity of births may be considered constant the distribution indicated is valid for female fertility rates.

([b]) This average is calculated on the basis of all the data given in the *Demographic Yearbook, 1954* (United Nations publication, Sales No.: 54.XIII.1), table 11, pp. 283-294.

([c]) Births to women under the age of 15 have been included in the 15-19 group, while births to women over the age of 45 have been included in the 40-44 group.

low fertility. However, what we have said regarding the factors governing observed distributions enables us to understand this apparent contradiction. The rate at which "natural" sterility increases with the age of women is by no means the same everywhere. In populations of European origin, for instance, the increase is much slower than in Asian, African or Latin American countries. Thus, where there is no birth control, we observe that the decline in the fertility rate with age, once the maximum has been passed, is much slower in Europe than elsewhere. The use of birth control practices compensates for this slowness, however, and this is why the declining phase of the fertility rates is similar in both groups of countries. The rising phase depends on marital status, and when we consider the average figures [31] we obtain variations which are more or less identical.

As a first approximation, we can define a model fertility for the human species by taking an invariable age distribution of fertility rates. If this is done, a given fertility is then determined by a single parameter, namely the total of the fertility rates or (which amounts to the same thing) the gross reproduction rate.

For reasons which will be explained later, it is convenient to adopt, when defining this model fertility, the distribution given in the last column of table II.19, which has been termed the "intermediate distribution". In fact, this distribution has already been used several times in the study of stable populations.

If F_a represents this intermediate distribution and $R'(t)$ represents the gross reproduction rate, we have:

$$R'(t)F_a = 5f_a(t)$$

and equation II.13 is written

$$R'(t)\sum_u^v e^{-ra}L_aF_a = 500\ 000$$

whence

$$R'(t) = \frac{500\ 000}{\sum_u^v e^{-ra}L_aF_a}$$

which is not dependent on time. Thus, the assumption that fertility follows a well-defined pattern of variation enables us to determine the model fertility function, once the other characteristics of the Malthusian population are known.

Here, by way of illustration, are details of how the fertility function is calculated in the first example of the determination of a Malthusian population, where the rate of increase was assumed to be known. Table II.20 gives details of the computations. Once we know the gross reproduction rate R′, we obviously obtain the age-specific fertility rates through the formula

$$f_a = \frac{R'\ F_a}{5}$$

TABLE II.20. CALCULATION OF THE FERTILITY OF EXAMPLE ([a]) No. 1 (INTERMEDIATE MODEL FERTILITY)

Median age a	*Age group (years)*	$e^{-ra}L_a$ *(figures taken from table II.3)*	*Distribution of fertility rates* F_a	*Product of the two preceding columns* $e^{-ra}L_aF_a$
17.5 . .	15-19	232 110	0.100	23 211
22.5 . .	20-24	194 686	0.273	53 169
27.5 . .	25-29	162 447	0.263	42 724
32.5 . .	30-34	135 620	0.188	25 497
37.5 . .	35-39	112 394	0.121	13 600
42.5 . .	40-44	93 083	0.055	5 120
All ages			1.000	163 321

([a]) Malthusian population with a rate of increase $r = 0.03$ in the sub-set $H_0(r)$ associated with the intermediate model life table giving an expectation of life at birth for both sexes of 50 years.

From the table we know that :

$$R' = \frac{500\ 000}{163\ 321} = 3.068$$

However, the model fertility described above, which is based on the assumption of an invariable age distribution of fertility rates, is of course, simply one of many methods. It merely represents a first approximation based on the consideration of averages computed for a large number of countries. It is quite certain that, for a country with a given nuptiality and "natural" sterility, the ever-wider

[31] This does not apply, of course, if we consider individual countries.

TABLE II.21. MODEL FERTILITY TABLES (FERTILITY RATES CALCULATED PER 1,000 WOMEN ON THE BASIS OF BOYS AND GIRLS TOGETHER)

Age group (years)	Observed fertility rate in Chile (a) in 1952	Gross reproduction rate 2.50	2.75	3.00	3.25	3.50
		I. Late fertility with slow decline (rate per 1 000)				
15-19 . . .	75.1	78.1	82.6	86.9	90.8	94.6
20-24 . . .	223.7	235.4	252.8	269.8	286.2	302.0
25-29 . . .	232.2	247.0	269.6	292.1	314.2	335.9
30-34 . . .	192.3	206.8	229.6	252.3	275.2	298.1
35-39 . . .	145.3	158.0	178.2	198.8	219.9	241.3
40-44 . . .	71.1	78.3	89.7	101.5	113.8	126.5
45-49 . . .	19.2	21.4	24.9	28.6	32.5	36.6
	Puerto Rico (b) in 1950	*II. Early fertility with rapid decline (rate per 1 000)*				
15-19 . . .	99.2	96.3	103.8	109.4	114.7	120.0
20-24 . . .	279.7	272.6	295.5	317.4	337.8	357.3
25-29 . . .	260.3	254.7	279.5	303.7	327.8	351.8
30-34 . . .	200.0	196.3	217.5	240.0	262.6	285.5
35-39 . . .	143.1	141.0	158.7	176.8	196.2	216.0
40-44 . . .	53.1	52.5	59.3	67.4	75.9	84.7
45-49 . . .	11.7	11.6	13.2	15.3	17.5	19.7

(a) Gross reproduction rate observed in 1952 in Chile: R′ = 2.339.
(b) Gross reproduction rate observed in 1950 in Puerto Rico: R′ = 2.554.

use of contraceptive practices cannot be reconciled with an invariable age distribution of fertility rates.

Observation shows that, in such circumstances, it is the rates of fertility at the higher ages which begin to decline, and this diminution also increasingly affects the rates at earlier ages.

The Regional Centre for Demographic Training and Research in Latin America at Santiago, Chile, which operates under United Nations auspices within the framework of the technical assistance programme, attempted on this basis to define a number of fertility models where the age distribution of fertility rates varied with the level of fertility.

A series of fertility rates for relatively late fertility with a slow decline at the higher ages was determined on the basis of observed fertility in Chile in 1952, while a series of fertility rates for relatively early fertility with a rapid decline at the higher ages was prepared on the basis of observed fertility in Puerto Rico in 1950. These rates are given in table II.21. The rates in question are calculated for girls and boys together. In order to obtain the female rates, the rates given must be divided by 2.05. The distributions at extreme ages are given in table II.22, together with some other distributions already encountered.

Let us take up the preceding problem, using the model fertility determined on the basis of the Chilean fertility. We have the equation:

$$\sum_u^v e^{-ra} L_a f_a(t) = 100\,000.$$

Let us calculate the results of this equation for various values of R′ and find out by interpolation the value of R′ for which the integral equals 100,000. Details of the calculation are given in table II.23. We find that R′ equals 3.28. From R′ we calculate by interpolation the

TABLE II.22. AGE DISTRIBUTION OF FERTILITY RATES FOR MODEL TABLES OF EXTREME FERTILITY FROM TABLE II.21 AND COMPARISON OF THOSE RATES WITH THE INTERMEDIATE DISTRIBUTION, THE DISTRIBUTIONS OBSERVED IN JAMAICA IN 1951 AND SPAIN IN 1940, AND THE AVERAGE DISTRIBUTION FOR 52 COUNTRIES

Age group (years)	Gross reproduction rate 2.50 (Late fertility with slow decline)	3.50 (Late fertility with slow decline)	Intermediate distribution (a)	Spain 1940 (a)
15-19 . . .	76	66	100	14
20-24 . . .	230	210	273	147
25-29 . . .	241	234	263	303
30-34 . . .	202	208	188	272
35-39 . . .	154	168	121	179
40-44 . . .	76	88	55 (40-49)	85 (40-49)
45-49 . . .	21	26		
ALL AGES .	1 000	1 000		

Age group (years)	Gross reproduction rate 2.50 (Early fertility with rapid decline)	3.50 (Early fertility with rapid decline)	Average for 52 countries (a)	Jamaica 1951 (a)
15-19 . . .	94	84	63	136
20-24 . . .	266	249	253	292
25-29 . . .	249	245	276	249
30-34 . . .	191	199	211	166
35-39 . . .	138	150	134	110
40-44 . . .	51	59	63 (40-49)	47 (40-49)
45-49 . . .	11	14		
ALL AGES .	1 000	1 000	1 000	1 000

(a) Figures taken from table II.19.

TABLE II.23. CALCULATION OF THE FERTILITY OF THE POPULATION OF EXAMPLE No. 1. LATE FERTILITY WITH SLOW DECREASE, BASED ON OBSERVED FERTILITY IN CHILE IN 1952 (SEE TABLE II.21)

Median age a (1)	Age group (years) (2)	$e^{-ra}L_a$ (3)	Product of column 3 by the female fertility rates corresponding, in table II.21, to the following gross reproduction rates: [a] 3.00	3.25	3.50
17.5	15-19	232 110	9 839	10 280	10 170
22.5	20-24	194 686	25 623	27 180	28 680
27.5	25-29	162 447	23 150	24 900	26 620
32.5	30-34	135 260	16 691	18 206	19 720
37.5	35-39	112 394	10 900	12 056	13 229
42.5	40-44	93 083	4 608	5 167	5 743
47.5	45-49	76 500	1 068	1 251	1 367
ALL AGE .			91 879	99 040	106 069

By interpolation, we see that the amount in the last line is equal to 100 000 for R′ = 3.28.

(a) In table II.21, the fertility rate in question is the one computed on the basis of all male and female births together. In order to obtain the female fertility rates, the rates given in table II.21 should be divided by 2.05 (this assumes a masculinity at birth of 105).

values of the corresponding fertility rates in table II.21. We find the following rates (per 1,000 women):

Age group	Fertility rate
15-19	91.3
20-24	288.4
25-29	317.2
30-34	279.1
35-39	222.8
40-45	115.5
45-49	33.1

Here again, the rates are not time-dependent. The fertility can be defined in accordance with the assumption that it is of the same form as the model fertility defined at the top of table II.21.

We should not, however, lose sight of the fact that the result obtained depends on the model fertility used in the calculation. Table II.24 gives some indication of the divergences which may be encountered in the determination of the gross reproduction rates (R′) if we take one model fertility rather than another. In these comparative calculations we have taken the sub-set $H_0(r)$, the intermediate model life table giving an expectation of life at birth for both sexes of 60.4 years, and a rate of natural increase of $r = 3.5$ per cent. It will be noted that what we called the "special case" gives a gross reproduction rate R′ which lies between the extreme values corresponding to the distributions observed in Jamaica and Spain. We can obviously determine the distributions which give exactly the same result as the "special case". The values will coincide only for a single value of the gross reproduction rate, but threre are grounds for hoping that the divergences will not be very great for other levels of fertility. It was in this way that we determined what we termed the "intermediate distribution". This method gives *approximately* the same result as the "special case". Thus, in table II.24, the "special case" gives R′ = 3.066, and the intermediate distribution R′ = 3.042. Other examples of approximate coincidences for other fertility

TABLE II.24. SIX SERIES OF FEMALE FERTILITY RATES [a] (PER 1 000 WOMEN) LEADING TO A STABLE POPULATION WITH AN INTRINSIC RATE OF NATURAL VARIATION OF 3.5 PER CENT WHEN ASSOCIATED WITH AN INTERMEDIATE MODEL LIFE TABLE CORRESPONDING TO AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS

Age group (years)	Nature of the model fertility table (births of both sexes): Constant distribution (Jamaica, 1951)	Intermediate constant distribution	Special case	Relatively early fertility	Relatively late fertility	Constant distribution (Spain, 1940)
15-19	164.5	124.7		112.0	91.2	19.6
20-24	353.2	340.5		327.4	287.8	206.5
25-29	301.1	328.0	1 256,9	315.6	316.4	425.6
30-34	200.8	234.5		251.1	277.5	382.1
35-39	133.0	150.9		185.3	221.9	251.4
40-44	56.8	68.6		71.6	115.1	119.4
45-49				16.4	32.9	
Gross reproduction rate	2.950	3.042	3.066	3.120	3.275	3.424
Net reproduction rate	2.518	2.590	2.619	2.651	2.767	2.888
Intrinsic rate of natural variation (percentage)	3.5	3.5	3.5	3.5	3.5	3.5
Expectation of life at birth for both sexes (years)	60.4	60.4	60.4	60.4	60.4	60.4

(a) These rates are computed on the basis of all births (girls and boys).

levels will be given in chapter VII. This intermediate distribution was determined by trial and error, and it depends to some extent on how the method of trial and error has been carried out. Thus, other distributions could give results which are at least as good, and perhaps better.

Reconsideration of the special case of stable populations

As was pointed out above, the calculation of the characteristics of a stable population was greatly simplified when the fertility function $\varphi(a)$ was reduced to a single value $\varphi(27.5)$, which was then equal to the gross reproduction rate R′. We stressed the artificial nature of such a fertility function.

Let us disregard this artificiality for a moment, however, and consider only the result, i.e., the population calculated by these means. This is a population in which:

(*a*) The age distribution is constant;

(*b*) The mortality is constant and known;

(*c*) The rate of increase is known.

Thus, it is the Malthusian population of the sub-set $H_0(r)$ associated with a known mortality function whose rate of increase r_0 is also known. We have seen that the fertility of such a population is determined if we assume knowledge of the age distribution of fertility rates, which we assume to be constant. This fertility can be that of the special case used in the calculation, but it can also be very different, and its level then depends on the age distribution of fertility rates adopted, although if we adopt the intermediate distribution, the level of fertility is very close to that corresponding to the special case.

In other words, if we assume the mortality function and the value of the gross reproduction rate R′, and if we calculate two stable populations by applying the simplified method of the special case on the one hand and the general method, using the intermediate age distribution of fertility, on the other, the two populations thus arrived at are very close to each other. The method of the special case is then revealed as a device for the easy calculation of stable populations having an intermediate age distribution of fertility rates.

Chapter III

MALTHUSIAN POPULATIONS WITH KNOWN MORTALITY FUNCTIONS, CONSIDERED AS THE LIMIT IN A DEMOGRAPHIC EVOLUTION

A. Introduction

Throughout chapter II, the populations considered were Malthusian populations, i.e., populations with constant mortality and age distribution, and in addition the mortality was assumed to be known for each sub-set H(r). Finally, knowledge of one additional condition made it possible to identify one population, or at least a small finite number of populations in each sub-set H(r).

Let us take, for example, the case of a stable population corresponding to a constant and known fertility function while belonging to a sub-set H(r). This stable population has been defined by the following properties:

(a) *Constant* age distribution;

(b) *Constant* and *known* mortality;

(c) *Constant* and *known* fertility.

In another example (the first example in chapter II) we arrived at a population with the following properties:

(a) *Constant* age distribution;

(b) *Constant* and *known* mortality;

(c) *Constant* and *known* rate of increase.

We can easily spell out the conditions corresponding to the various cases envisaged in chapter II, and in each case we shall find a constant age distribution associated with a known and constant mortality, plus one other condition.

Let us imagine that in each of these cases we dispense with the first condition, i.e., the constant age distribution. We are then left with a set of demographic conditions which can be imposed on a population from a given arbitrary initial state. We thus set in motion a process of demographic evolution, but is it possible to bring out the simple laws of such a process? That is the problem which we propose to examine next.

It may be recalled that we have already dealt experimentally, in chapter I, with the case where the given demographic conditions are those of constant mortality and constant fertility. We have shown in several actual cases that in a process of demographic evolution under these conditions the population approaches the stable Malthusian population corresponding to the given laws of mortality and fertility. This is the case dealt with by Lotka in the work already referred to.[1]

Various problems involved

We can obviously imagine a wide range of problems of the type which we have defined above and of which the concept of a limit stable population is an example. All the problems in this range are not equally important, however, and they can be classified in various categories:

(a) *Determinate problems.* These are problems leading to population evolution which is not impossible. The way to find out whether a problem is determinate or not is to try to compute a population projection, as was done in chapter I with Lotka's theorem. This theorem is the very model of the determinate problem. Whatever the functions $p_f(a)$ and $\varphi_f(a)$ and whatever the initial age structure may be, it is always possible to compute a projection. Other cases can be envisaged, however, where the problem is determinate only *in general terms.* Let us suppose, for example, that we assume the age structure $C_f(a)$ and the rate of increase r to be independent of time. The projection will then be calculated as follows: beginning with a female population $N_f(t)$ having an age distribution $C_f(a)$ at time t, we calculate the population at time $t+1$ by the formula: $N_f(t+1)=(1+r)N_f(t)$ and we distribute $N_f(t+1)$ according to the distribution $C_f(a)$. We thus obtain the age distribution of the population at time $t+1$. By repeating the same operation, we can therefore compute a projection of the age distribution of the population. If, however, we wish to follow persons of an initial age group throughout the projection computed in this way, we must find a number of persons which will consistently decrease, for if r is too great we may very well see the number increasing rather than decreasing. The problem which we have posed is, therefore, not always determinate. It may, in some cases, become impossible.

We see at the same time what is meant by "impossible". What we are referring to is a logical "impossibility"—in the case in question, a survivorship function which does not decrease. Provided that there is no logical impossibility, however, we classify a case as "possible", even if this logical possibility leads to a type of population development which is very unlikely to occur.

(b) *Indeterminate problems.* Let us suppose that we assume a crude birth rate b and a crude death rate d. Starting from an initial population, we can obviously compute the total number of persons in the population at any time in the future, but there are an infinite number of mortality and fertility functions through which such results can be attained, and we can hardly make any definite statement concerning these functions. Thus, the problem can be considered indeterminate.

(c) *Indeterminate problems which become determinate when it is also assumed that the mortality and fertility functions form part of the "universe" of model mortalities and fertilities.* When this assumption is made, the preceding problem, for example, becomes determinate in most cases. For a given age structure there is, generally speaking, only one model mortality function giving a crude death rate equal to d and only one model fertility

[1] Alfred J. Lotka, *Théorie analytique des associations biologiques ; Deuxième partie* (Paris, Hermann, 1939), 149 pp.

function giving a crude birth rate equal to b. The reservation "generally speaking" is necessary because it may happen, depending on the given age structure, that either there is no model mortality or model fertility compatible with the values b and d or there are several such functions.

(d) *Impossible problems.* We cannot, for example, assume as given the mortality and fertility functions as well as the rate of variation. The first two functions are sufficient to determine the problem; the third is superflous and may make the problem impossible.

Whe shall proceed to consider only problems of the first type, i.e., determinate problems. It will therefore be necessary to verify, in each case, that the conditions which we set are not contradictory or indeterminate at any time during the evolutionary process.

These "determinate" problems correspond to the various examples of the determination of Malthusian populations studied in chapter II. The examples are the same, in fact, except that the condition of an invariable age structure has been dispensed with. We shall take them up successively, after first defining the general conditions of possibility.

Conditions of possibility

Starting from a given initial population, we assume that the mortality of both sexes remains constant and that one other demographic characteristic also remains constant. In order to see whether the process of demographic evolution defined in this way leads to impossibilities or not, we need only show that it is in fact possible in such circumstances to compute, on the basis of the initial state (period zero), the population at the next period (period 1), then at period 2, 3 and so forth, or, in other words, to show that a "population projection" can be computed. For an understanding of how certain contradictions may arise, it may be best to take a specific example.

Let us take as our initial state (state zero) the female population of Eastern Germany according to the 1957 census, which we have already used in chapter I. This population is given in five-year age groups in the second column of table III.1. Starting from this initial state, let us keep the mortality constant at level 80 of the intermediate model life table (expectation of life at birth for both sexes of 60.4 years). The survival ratios from one age group to the next over a five-year period are given in the last column of table III.1. By multiplying each term of the second column by the corresponding survival ratio, we obtain the number of survivors five years later. We can thus determine for each group of five years the female population aged 5 and over. We thus find that: $N_{5\ \&\ \text{over}} = 8{,}351{,}319$ for an initial population of $N_{0\ \&\ \text{over}} = 9{,}031{,}093$ (these two figures are given in the last line of table III.1).

Let us represent by B the average annual female births during the five years under consideration, and let b, d and r represent the average annual female rates of birth, death and natural increase, respectively, during those five years.

The survival ratio to the year 5 of mean annual births B is 0.9208 (the first figure in the last column of table III.2). In other words, girls aged 0-4 will number, in year 5:

$$0.9208\ \text{B} \times 5 = 4.604\ \text{B}$$

TABLE III.1. FIVE-YEAR PROJECTIONS BASED ON THE FEMALE POPULATION OF EASTERN GERMANY ACCORDING TO THE 1957 CENSUS

Age group (years)	*Female population in year* 0 (a)	5	*Survival ratio* (c) *from one age group to the next* 0.9208 (d)
0-4	440 981		0.9731
5-9	706 869	429 119	0.9914
10-14	410 485	700 790	0.9906
15-19	617 750	406 626	0.9865
20-24	748 500	609 410	0.9839
25-29	536 107	736 449	0.9826
30-34	543 514	526 779	0.9811
35-39	545 370	533 243	0.9783
40-44	490 705	533 535	0.9727
45-49	715 494	477 309	0.9631
50-54	712 084	689 092	0.9482
55-59	680 800	675 198	0.9238
60-64	595 017	628 923	0.8825
65-69	495 159	525 103	0.8162
70-74	372 892	404 149	0.7197
75-79	240 681	268 370	0.5955
80-84	127 718	143 326	0.3576 (e)
85 and over .	50 967	63 898	
ALL AGES . .	9 031 093	8 351 319 (b)	

(a) Female population according to 1957 census.
(b) Total population aged 5 and over.
(c) Survival ratios corresponding to the level-80 model life table (expectation of life at birth for both sexes of 60.4 years).
(d) Survival ratio at the end of the fifth year from births spread evenly over the five years.
(e) Survival ratio over the five-year period of persons aged 80 and over.

while the total population in year 5 will be:

$$N_{5\ \&\ \text{over}} + 4.604\ \text{B} = 8{,}351{,}319 + 4.604\ \text{B}$$

The mean population during the five years is:

$$\frac{9{,}031{,}093 + 8{,}351{,}319 + 4.604\ \text{B}}{2} = 8{,}691{,}206 + 2.252\ \text{B}$$

and the mean annual increase in population is:

$$\frac{(8{,}351{,}319 + 4.604\ \text{B} - 9{,}031{,}093)}{5} = \\ = -\ 135{,}954.8 + 0.9208\ \text{B}.$$

We then have the following formulae for the rates b, d and r:

$$b = \frac{\text{B}}{8{,}691{,}206 + 2.252\ \text{B}}$$

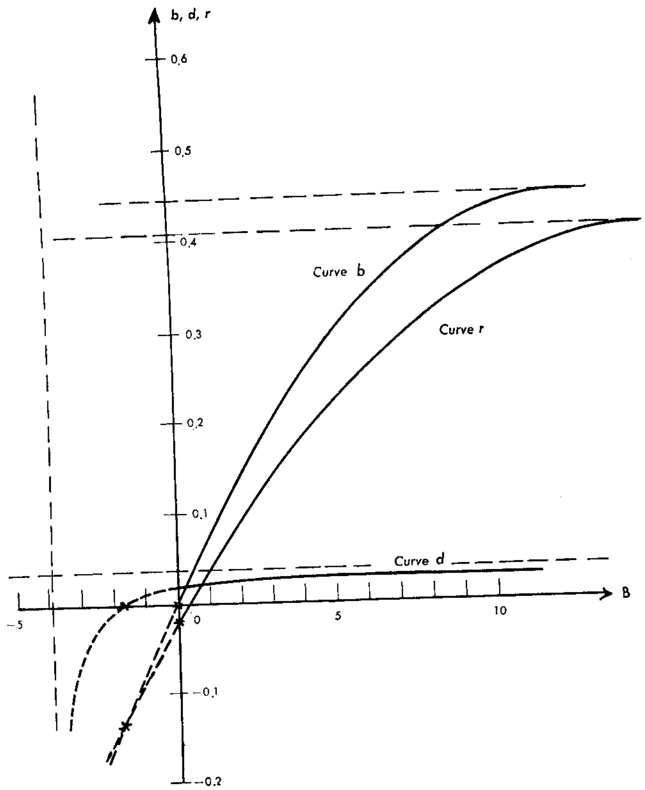
$$d = \frac{135{,}954.8 + 0.0792\ \text{B}}{8{,}691{,}206 + 2.252\ \text{B}}$$

$$r = \frac{-135{,}954.8 + 0.9208\ \text{B}}{8{,}691{,}206 + 2.252\ \text{B}}$$

Let us examine the variations of b, d and r as a function of B. These quantities are homographic functions of B, and thus the corresponding curves are branches of equilateral hyperbolae. In graph III.1, the horizontal axis represents B and the vertical axis represents b, d, and r.

The three curves have in common as asymptote a straight line whose abscissa is given by:

$$\text{B} = -\frac{8{,}691{,}206}{2.252} = -\ 3{,}859{,}000$$



Graph III.1. Determination of the conditions of compatibility of system III.A in the text

Each of the curves also has as a horizontal asymptote whose ordinate is given by a straight line:

$$b_a = 0.440$$
$$d_a = 0.0352$$
$$r_a = 0.4088$$

We also, of course, have: $b_a - d_a = r_a$.

Finally, the smallest value that B can have is $B = 0$, and for this value of B we have:

$$b_0 = 0$$
$$d_0 = 0.0156$$
$$r_0 = -0.0156$$

In addition, of course, we have: $b_0 - d_0 = r_0$.

Finally, b, d, and r can vary over that part of their curves which is shown as a solid line in graph III.1. We therefore see that these three quantities cannot assume arbitrary values during the five years under consideration. We must have:

$$\left.\begin{array}{c} 0 < b < 0.4440 \\ 0.0156 < d < 0.0352 \\ -0.0156 < r < 0.4088 \end{array}\right\} \qquad \text{(III.A)}$$

If the conditions defining the process of demographic evolution involve values of b, d *or* r *which do not satisfy system (III.A), then it will not be possible to compute the projection.*[2]

[2] The fact that human fertility rates cannot exceed certain values causes further limitation as to the possible variation of b, d and r. This is a different problem, however. We are concerned only with *logical* contradictions.

It should be noted, in passing, that knowledge of the age-specific female fertility rates determines the mean annual number of births without ever introducing any contradiction. In order to obtain B, all we have to do is to apply the fertility rates to the 15-49 age groups at year zero and year 5. The average of the results obtained is the number B which is sought.

Let us suppose that we have taken demographic characteristics satisfying the conditions under III.A. We are thus certain of being able to calculate the population at year 5 on the basis of the initial state. If we wish to continue beyond year 5 and calculate the population in year 10, however, we are confronted with a problem similar to that described above, and we shall, in fact, encounter such a problem at each new five-year period. The horizontal asymptotes of the curves representing the variation of b, d and r are always the same at each period, because their ordinates depend only on the mortality, but the vertical asymptote common to the three curves changes at each period, although its abscissa is always negative. The ordinates at the origin of the curves representing d and r also change at each period. Hoewever, we always have: $d_0 = -r_0$, and d_0 is always positive.

Finally, if d_m represents the highest value of d_0 in the process and d_m the lowest value of d_0, then b and r must satisfy the conditions:

$$\left.\begin{array}{c} 0 < b < 0.4440 \\ 0 < d_m < d < 0.0352 \\ -d_m < 0 < r < 0.4088 \end{array}\right\} \qquad \text{(III.A}bis\text{)}$$

In all that follows we shall assume that we are dealing with a case where these conditions are satisfied.

B. Limit stable population

Let us consider a given initial population in which female mortality and female fertility remain invariable. We have seen above that, in such a process, the conditions under III.A*bis* are always satisfied.

Let us consider only the female population. The females of age a at time t, who are the survivors of the $B(t-a)$ girls born at time $t-a$, number $B_f(t-a)p_f(a)$. Consequently, the total female births at time t are written:

$$B_f(t) = \int_0^{\omega} B_f(t-a)p_f(a)\varphi_f(a)da \qquad \text{(III.1)}$$

This is the basic equation of the process of demographic evolution with constant mortality and fertility.

In order to solve this equation, we seek a solution of the form:

$$B_f(t) = A_1e^{r_1t} + A_2e^{r_2t} + A_3e^{r_3t} + \dots \qquad \text{(III.2)}$$

We can thus write:

$$A_1e^{r_1t} + A_2e^{r_2t} + A_3e^{r_3t} + \dots$$
$$= A_1e^{r_1t}\int_0^{\omega} e^{-r_1a}p_f(a)\varphi_f(a)da$$
$$+ A_2e^{r_2t}\int_0^{\omega} e^{-r_2a}p_f(a)\varphi_f(a)da + \dots$$

We see that if r_1, r_2, r_3 etc., are the roots of the equation:

$$\int_0^\infty e^{-ra} p_f(a)\varphi_f(a)da = 1 \qquad \text{(III.3)}$$

formula III.2 is the solution of III.1.

We recognize in equation III.3 formula II.14 linking the rate of natural increase, the mortality and the fertility in a stable population. We have seen that this equation has a single real root ρ, which we termed the intrinsic rate of natural variation.

This root is associated with an infinity of complex roots, which possess the following two properties:

(1) Let $x + iy$ be one of these roots. We then have:

$$\int_0^\infty e^{-(x+iy)a} p_f(a)\varphi_f(a)da = 1$$

which is written:

$$\int_0^\infty e^{-xa} [\cos y\, a - i \sin y\, a]\, p_f(a)\varphi_f(a)da = 1$$

Where "i" represents $\sqrt{-1.}$

In separating the real part from the imaginary part, we obtain:

$$S \begin{cases} \int_0^\infty e^{-xa} \cos yap_f(a)\varphi_f(a)da = 1 \\ \int_0^\infty e^{-xa} \sin yap_f(a)\varphi_f(a)da = 0 \end{cases}$$

For a given value of y, we have:

$$1 = \int_0^\infty e^{-xa} \cos yap_f(a)\varphi_f(a)da < \int_0^\infty e^{-xa} p_f(a)\varphi_f(a)da$$

and as:

$$\int_0^\infty e^{-\varrho a} p_f(a)\varphi_f(a)da = 1$$

we therefore have:

$$\int_0^\infty e^{-\varrho a} p_f(a)\varphi_f(a)da < \int_0^\infty e^{-xa} p_f(a)\varphi_f(a)da$$

from which it follows that:

$$\varrho > x.$$

The intrinsic rate ρ is therefore superior to all the quantities x.

(2) If $(x + iy)$ is a root, then the conjugate complex number $(x - iy)$ is also a root, since system (S)is satisfied by both quantities. As the number of births $B_f(t)$ is necessarily a real number, the coefficients A_n corresponding to the two conjugate roots are equal, and we finally have for $B_f(t)$ the formula:

$$B_f(t) = A_1 e^{\rho t} + A_2 e^{x_2 t} \cos y_2 t + A_3 e^{x_3 t} \cos y_3 t + \ldots$$

in which ρ is higher than x_2, x_3 etc.

We see that the imaginary roots introduce oscillations in the number of births. Since ρ is greater than all the quantities x, as the time t increases indefinitely the term $A_1 e^{\varrho t}$ becomes preponderant over all the others, the latters giving rise to damped oscillations.

In other words, the number of births at time t asymptotically approaches:

$$B_f(t) \rightarrow A_1 e^{\varrho t}$$

i.e., it tends to follow the law of births in the stable population corresponding to the laws $p_f(a)$ and $\varphi_f(a)$. The population itself approaches this stable state. It is this result, arrived at empirically in chapter I, which constitutes what may be termed Lotka's theorem.

We showed in the first part of this chapter how to calculate the intrinsic rate, the death rate and the age distribution of the stable state. Generally speaking, we confine ourselves to a consideration of these characteristics. For completeness, however, it is also necessary to calculate the constants A_1, A_2, A_3 ... Annex I gives details of a method which makes it possible to determine the constant A_1 corresponding to the intrinsic rate ρ and thus permits the complete calculation of the stable state.

$$B_f(t) = A_1 e^{\varrho t} \qquad \text{(III.4)}$$

The formula at which we arrive is the following:

$$A_1 = \int_0^v \frac{K_f(a, 0)}{p_f(a)} G(a) e^{\varrho a} da = \int_0^v S(a)da$$

Where

$$S(a) = \frac{K_f(a, 0)\, G(a)}{p_f(a)}$$

$K_f(a, o)$ represents the number of women of age a at the starting point and $G(a)$ is a function of a which does not depend on the initial conditions and is practically the same, whatever the human fertility and mortality functions may be.[3]

In the stable state, the total population $N(t)$ is obtained by dividing the births by the birth rate:

$$b = \frac{1}{\int_0^\infty e^{-\varrho a} p_f(a)da}$$

Thus we have:

$$N(t) = e^{\varrho t} \int_0^\infty e^{-\varrho a} p_f(a)da \int_0^\infty S(a)da \qquad \text{(III.5)}$$

Reconsideration of the special case of chapter II

We pointed out earlier that the computation of the intrinsic rate ρ becomes very easy when the fertility function $\varphi_f(a)$ is reduced to a single value $\varphi_f(27.5)$. It was stated that in such circumstances we were dealing with the "special case". We shall see that in this case the oscillations in the number of births corresponding to the complex roots of the fundamental equation do not disappear with the passage of time.

The basic equation is written:

$$e^{-27.5r} R_0 = 1$$

If we assume that $r = x + iy$, the equation becomes:

$$[\cos 27.5y + i \sin 27.5y] e^{-27.5x} = \frac{1}{R_0}$$

---

[3] The values of $G(a)$ are to be found in table III.2 below.

which breaks down into two equations:

$$\begin{cases} e^{-27.5x} \cos 27.5y = \frac{1}{R_0} \\ e^{-27.5x} \sin 27.5y = 0 \end{cases}$$

whence we have:

$$\sin 27.5y = 0$$

$$e^{-27.5x} = \frac{1}{R_0}$$

or, finally:[4]

$$27.5y = 2\lambda\pi$$

$$x = \varrho$$

The number of births at time t is therefore:

$$B_f(t) = A_1 e^{\varrho t} + A_2 e^{\varrho t} \frac{\cos 2\pi t}{27.5} + A_3 e^{\varrho t} \frac{\cos 6\pi t}{27.5} + \ldots$$

or:

$$B_f(t) = e^{\varrho t} A_1 + \Sigma A_n \cos \frac{2nt}{27.5}$$

Thus, births oscillate continuously a round the exponential term:

$$A_1 e^{\varrho t}$$

There will therefore not be any damping out of the oscillations. In this case, the stable state no longer appears as a limit state, but rather as a mean value. It should be noted that *the special case is the only case* where the oscillations due to the complex roots of the fundamental equation are not damped out.

Numerical application

Tables III.2 and III.3 give an example of the application of the formula III.4 (for determining the absolute number of births, $B_f(t)$ in the process of projection making based on the population of Thailand in 1955.

The annual number of births in the stationary population is:

$$B_f(t) = 275,634$$

The stationary population is obtained by dividing the births by the crude female birth rate, which is the same thing as multiplying the births by the female expectation of life at birth (see table II.11): $275,634 \times 62.05 = 17,103,000$.

In the second stable population the annual number of female births at time $t = 0$ is $B_f(^0) = 308,380$.

If we divide this number by the crude female birth rate $b_f = 0.02179$ (see table II.11), we obtain for the total number of persons in the population at time $t = 0$:

$$N_f(0) = \frac{B_f(0)}{b_f} = \frac{308,380 \times 1,000}{21.79} = 14,152,000$$

In the case of the third stable population calculated in chapter I on the basis of the population of Thailand and the three stable populations calculated in the same chapter on the basis of the population of Eastern Germany in 1957, we shall confine ourselves to giving the results of the computations (table III.4). It was with the aid of these results that we plotted the straight lines on graphs I.2, I.4, I.9, I.10 and I.11 in chapter I.

C. A limit Malthusian population with constant mortality and a constant and given crude birth rate

In this second problem, we shall suppose that, starting off with a given initial condition, we keep the mortality $p_f(a)$ and the crude birth rate b_f constant. This case corresponds to the third example in chapter II.

We assume, of course, that the condition imposed on b_f in the system of inequalities III.A *bis* above is satisfied; in fact, the relevant in equality is almost always satisfied for b_f, since the uppser limit of b_f is always much higher than the crude birth rates encountered in the human species.

---

[1] The solution $27.5k = \lambda\pi$ would give a negative value for $\cos 27.5y$, which is impossible, since we have:

$$\cos 27.5y = \frac{e^{27.5x}}{R_0}$$

Table III.2. Computation of the annual number of female births in the stationary population on the basis of the population of Thailand in 1955

Median age a	Age group (years)	Initial number of women $K_a(0)$	Stationary female population ([a]) L([b]) (per 100 000 births)	Ratio of the two preceding columns	G(a) ([b])	Product of the two preceding columns (stationary births per 100 000)	Product of preceding column and median age ([c])
2.5	0-4	1 809 000	460 386	3.930	0.18	0.70740	1.7685
7.5	5-9	1 397 000	448 010	3.118	0.18	0.56124	4.2093
12.5	10-14	1 196 000	444 150	2.692	0.18	0.48456	6.0570
17.5	15-19	1 110 000	439 970	2.522	0.17	0.42874	7.5030
22.5	20-24	987 000	434 040	2.274	0.13	0.29562	6.0515
27.5	25-29	824 000	427 035	1.930	0.09	0.17370	4.7768
32.5	30-34	658 000	419 610	1.568	0.05	0.07840	2.5480
37.5	35-39	549 000	411 672	1.334	0.02	0.02668	1.0005
	All ages					2.75634	34.5146

([a]) Intermediate model life table corresponding to an expectation of life at birth for both sexes of 60.4 years.
([b]) The function G(a) is the same for all populations. See annex I for more details.
([c]) The computation shown in this last colum is used in annex I.

TABLE III.3. COMPUTATION OF THE ANNUAL NUMBER OF FEMALE BIRTHS IN THE STABLE POPULATION ON THE BASIS OF THE POPULATION OF THAILAND IN 1955 (a)

Median age a	*Age group (years)* (a)	$e^{\rho a}$	*Stationary births* (b) *(per 100 000)*	*Product of the two preceding columns (initial* (c) *stable births)*	*Product of the preceding column and the "median age" column* (d)
2.5	0-4	1.0220	0.70740	0.7230	1.8075
7.5	5-9	1.0674	0.56124	0.5990	4.4925
12.5	10-14	1.1159	0.48456	0.5408	6.7600
17.5	15-19	1.1654	0.42874	0.4996	8.7430
22.5	20-24	1.2173	0.29562	0.3598	8.0955
27.5	25-29	1.2702	0.17370	0.2206	6.0665
32.5	30-34	1.3268	0.07840	0.1040	3.3800
37.5	35-39	1.3857	0.02668	0.0370	1.3875
	ALL AGES			3.0838	40.7325

(a) Intermediate fertility model with constant distribution corresponding to a gross reproduction rate of 1.50 and model life table corresponding to an expectation of life at birth for both sexes of 60.4 years.
(b) Figures taken from table II.11.
(c) Figures related to time $t = 0$.
(d) The computation shown in this last column is used in annex I.

TABLE III.4. ABSOLUTE ANNUAL NUMBER OF FEMALE BIRTHS AND ABSOLUTE NUMBER OF WOMEN IN THREE STABLE POPULATIONS COMPUTED ON THE BASIS OF (*a*) POPULATION OF EASTERN GERMANY IN 1957; (*b*) POPULATION OF THAILAND IN 1955

Gross reproduction rate	*Births* $B_f(t)$	*Population (millions)* $N_f(t)$
(a) *Based on East Germany, 1957*		
1.50	$148\,980\ e^{0.0087t}$	$6\,802\ e^{0.0087t}$
1.17	128 980	8 003
0.75	$102\,060\ e^{-0.0157t}$	$12\,064\ e^{-0.0157t}$
(b) *Based on Thailand, 1955*		
1.50	$308\,380\ e^{0.0087t}$	$14\,152\ e^{0.0087t}$
1.17	275 634	17 103
0.75	$228\,530\ e^{-0.0157t}$	$27\,013\ e^{-0.0157t}$

At time t, we can write for the total female population $N_f(t)$:

$$N_f(t) = \int_0^\omega B_f(t-a)p_f(a)da$$

and the births at time t are written:

$$B_f(t) = b_f \int_0^\omega B_f(t-a)p_f(a)da$$

Thus, we have an equation of the same type as equation III.3, and the solution sought is:

$$B_f(t) = A_1e^{\rho t} + A_2e^{r_2t} + A_3e^{r_3t} + \ldots$$

where ρ is the real root and r_2, r_3 ... are the complex roots of the equation:

$$b_f \int_0^\omega e^{-ra}p_f(a)da = 1$$

or

$$\int_0^\omega e^{-ra}p_f(a)da = \frac{1}{b}$$

The population approaches the Malthusian population corresponding to the function $p_f(a)$ and the crude birth rate b_f.

D. A limit Malthusian population with constant mortality and constant rate of natural variation

Let us now suppose that, starting off with a given initial state, we keep the mortality $p_f(a)$ and the rate of natural variation r_0 constant. We assume, of course, that the value r_0 selected for the rate of variation satisfies the relevent inequality in system III.A *bis*. It should be noted that, if r_0 is positive, this inequality is always satisfied, since the upper limit of r in system III.A *bis* is always much higher than the values encountered in the human species.

We can write for the female population:

$$N_f(t) = N_f(0)e^{r_0t}$$

However, we also have:

$$N_f(t) = \int_0^\omega B_f(t-a)p_f(a)da$$

whence we have the equation:

$$N_f(0)e^{r_0t} = \int_0^\omega B_f(t-a)p_f(a)da$$

If we seek a solution of the form:

$$B_f(t) = A_1e^{r_1t} + A_2e^{r_2t} + A_3e^{r_3t} + \ldots$$

we have:

$$N_f(0)e^{r_0t} = A_1e^{r_1t}\int_0^\omega e^{-r_1a}p_f(a)da + A_2e^{r_2t}\int_0^\omega e^{r_2a}p_f(a)da + \ldots$$

If we assume that:

$$r_1 = r_0$$

and

$$A_1 \int_0^{\omega} e^{-r_0 a} p_f(a) da = N(0)$$

and if r_2, r_3, ... are the complex roots of the equation:

$$\int_0^{\omega} e^{-ra} p_f(a) da = 0 \qquad \text{(III.6)}$$

then the expression:

$$B_f(t) = \frac{N(0)}{\int_0^{\omega} e^{-r_0 a} \; p_f(a) da} e^{r_0 t} + A_2 e^{r_2 t} + A_3 e^{r_3 t} + \dots$$

is the solution sought.

The crude birth rate is written:

$$b_f(t) = \frac{1}{\int_0^{\omega} e^{-r_0 a} p_f(a) da} + A_2 \frac{e^{(r_2 - r_0)t}}{N(0)} + A_3 \frac{e^{(r_3 - r_0)t}}{N(0)} + \dots$$

There is an upper limit to the crude birth rate, so that we have in reality:

$$b_f = r_0 - d_f, \text{ and thus } b_f < r_0$$

It must therefore be true that: $r_2 - r_0 < 0$, $r_3 - r_0 < 0$, and so on. r_0 is therefore greater than all the r_2, r_3, r_4 and so on, and the population tends towards the Malthusian population of the set $H_0(r)$ connected with $p_f(a)$, whose rate of natural variation is equal to r_0.

E. A limit Malthusian population with constant mortality and constant crude death rate

Let us now suppose that, starting from any initial state, we keep the mortality $p_f(a)$ and the crude death d_0 constant. This case corresponds to the fourth example in chapter II.

Here again, we suppose that the value d_0 chosen for the crude death rate satisfies the corresponding inequality of system III. A *bis*. We have already seen that the inequality corresponding to b_f in system III.A *bis* is practically always satisfied and that the inequality corresponding to r is very frequently satisfied (in practice, it is sufficient that r should be positive in order for the inequality to be satisfied). The same does not apply in the case of d, however.

In the case given as an example in table III.1 (where the initial population is the female population of Eastern Germany according to the census of 1957 and the mortality is that of level-80 of the intermediate model life table), d had to be under 35.2 per thousand and over 15.6 per thousand, although the latter limit, which was calculated only on the basis of the first years of the projection, might prove inadequate in the computations. Thus, the problem posed is impossible unless d_0 falls within quite narrow limits.

The study of the conditions governing the possibility or impossibility of the problem is thus of the greatest importance here, and it can be carried out only when the initial state, the mortality and the crude death rate are known. In succeeding pages it will be assumed that the compatibility of the conditions imposed has been verified.

The number of females aged a at time t is:

$$K_f(a,t) = B_f(t-a) \, p_f(a)$$

The number of female deaths at time t is:

$$\int_0^{\omega} B_f(t-a) p_f(a) q_f(a) da$$

and consequently the crude death rate is:

$$d_0 = \frac{\int_0^{\omega} B_f(t-a) p_f(a) q_f(a) da}{\int_0^{\omega} B_f(t-a) p_f(a) da}$$

We thus have the equation:

$$\int_0^{\omega} B_f(t-a) p_f(a) \, [d_0 - q_f(a)] da = 0$$

The number of births is of the form:

$$B_f(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} + A_3 e^{r_3 t} + \dots$$

where r_1, r_2, r_3 etc., are the roots of the equation:

$$\int_0^{\omega} e^{-ra} p_f(a) [d_0 - q_f(a)] da = 0$$

Taking account of the formula:

$$p_f(a) q_f(a) = - p'_f(a)$$

and integrating by parts, we finally have the equation:

$$(d_0 + r) \int_0^{\omega} e^{-ra} p_f(a) da = 1 \qquad \text{(III.7)}$$

This is an r equation which can be written:

$$d_0 = \frac{1}{\int_0^{\omega} e^{-ra} p_f(a) da} - r = F(r)$$

We have already encountered the expression $F(r)$ in connexion with the fourth example in chapter II. Graph II.9 gave the form of the curve representing the variation of $F(r)$ as a function of r. The abscissae of the points of intersection of this curve with the straight line of the ordinate d_0 are the real roots of equation (III.7). There are zero, one or two real points of intersection, depending on the value of d_0. Equation III.7 thus has zero, one or two real roots. It also has an infinite number of complex roots conjugate in pairs. Let ρ_1 and ρ_2 be the real roots and $x_3 + iy_3$, $x_4 + iy_4$ etc., the sequence of the complex roots. We have the following formula for the births at time t:

$$B_f(t) = A_1 e^{\rho_1 t} + A_2 e^{\rho_2 t} + A_3 e^{x_3 t} \cos y_3 t + A_4 e^{x_4 t} \cos y_4 t + \dots$$

When the time increases indefinitely, the term corresponding to the largest of the quantities ρ_1, ρ_2, x_3, x_4 etc., becomes preponderant over all the others. In the preceding examples, the term which thus became preponderant over all the others was always a real one, but this is not the case here. This is obvious when equation III.7 does not have any real roots, but it can happen even when real roots exist. If it is one of the quantities in x which is the greatest, the births are expressed in the form:

$$B = Ae^{xt} \cos yt$$

The births continually oscillate around a mean value, and the limit population is no longer a Malthusian population.

We shall confine ourselves to these few cases of limit populations, but as in the examples in chapter II it would be easy to imagine many other cases.

In the next chapter, we shall leave the sub-sets H(r) associated with a given mortality and revert to the general subject of all Malthusian populations, considering it this time as an infinity of sub-sets associated with a given age structure or, in other words, the sub-sets F(r) to which we have already referred briefly in chapter I.

Chapter IV

MALTHUSIAN POPULATIONS WITH KNOWN AGE DISTRIBUTION: SUB-SETS F(r)

We propose to study in this chapter the sub-sets $F(r)$ which were defined in chapter I as a kind of counterpart of the sub-sets $H(r)$. If we assume that the age distribution $C_0(a)$ is *constant and given*, while the survivorship function $p(a)$ is constant but not known, then all the Malthusian populations satisfying these conditions form the sub-set $F_0(r)$ corresponding to the age distribution $C_0(a)$, and by varying $C_0(a)$ we obtain the series of sub-sets $F(r)$.

A. The fundamental formulae

The basic formulae for the sub-sets $F(r)$ are the same as for the sub-sets $H(r)$. We shall set them out once again, using a particular sub-set $F_0(r)$, in a form expressing the unknown functions in terms of the age distribution $C_0(a)$.

(1) Let us note, first of all, that $C_0(0) = b_0$. Consequently, when we assume a given age distribution we also assume a corresponding crude birth rate;

(2) The survivorship function is written:

$$p(a) = \frac{C_0(a)}{C_0(0)} e^{ra} \qquad \text{(II.3}d\text{)}^1$$

and thus we obtain all the survivorship functions of the sub-set $F_0(r)$ by successively giving r all its possible values;

(3) The rate r must be such that the function $p(a)$ thus calculated is a decreasing function. We therefore must have:

$$\frac{dp(a)}{da} < 0$$

or, in other terms,

$$r < -\frac{C_0'(a)}{C_0(a)}$$

Let

$$-\frac{C'(a)}{C(a)} = Q(a)$$

r must therefore satisfy the inequality:

$$r < Q_0(a) \qquad \text{(IV.1)}$$

For a given function $C_0(a)$, $Q_0(a)$ passes through a minimum for a certain age a_m. This minimum thus represents a maximum for r. Finally, we can arrive at all the populations of the sub-set $F(r)$ by varying r from $-\infty$ to $Q_0(a_m)$.[2]

(4) The probabilities of death given in the various model life tables calculated with the aid of formula II.3*d* have an important property, since we have:

$$q(a) = Q_0(a) - r \qquad \text{(II.4)}$$

Each series of probabilities is obtained by subtracting r from the series of probabilities $Q_0(a)$.

(5) The fertility function $\varphi(a, t)$ satisfies the condition:

$$\int_0^{\infty} \varphi(a,t) C_0(a)\, da = b_0 = C_0(0) \qquad \text{(II.15)}$$

It may be recalled that in a sub-set $H(r)$ we could arbitrarily assume the function $\varphi(a, t)$. This value then determined a particular Malthusian population which we called a stable population. This no longer applies in the case of a sub-set $F(r)$, however. Here, we cannot arbitrarily assume the fertility function, which must satisfy II.15.

B. Populations of a sub-set F(r) satisfying certain conditions

In order to define a particular population of the sub-set $F(r)$, we must have an additional condition. We shall now discuss some of these conditions, which are of particular interest in practice.

First example: Malthusian population with age distribution $C_0(a)$ and given rate of variation r_0

The simplest way to define a particular population of the sub-set $F(r)$ is obviously to assume the rate of variation r_0 as given. We then apply formula II.3*d* to calculate the corresponding survivorship function.

$$p_0(a) = \frac{C_0(a)}{C_0(0)} e^{r_0 a} \qquad \text{(II.3}d\text{ bis)}$$

The numerical calculations raise various practical problems which we shall now examine.

Discontinuous data. The age distribution $C_0(a)$ is generally not known in continuous form but is, rather, a function which is known for various age groups. We shall therefore take the following conventional age groups : 0 (i.e., less than 1 year of age),[3] 1-4, 5-9, 10-14, and so on by successive five-year age groups. The last age group usually consists of several five-year groups together. For clarity, we shall make this last age group cover all persons aged 85 and over.

[1] This is formula II.3*d* in table II.2.

[2] This is obviously only a theoretical possibility. We have already pointed out that in actual populations other limits might apply.

[3] We shall consider later the case where the 0 and 1-4 age groups are combined into a single age group 0-4.

One of the first consequences of the fact that we know the age distribution in discontinuous notation is the following: in continuous notation, we had $C_0(0) = b_0$, thus knowledge of the age distribution automatically meant that we also knew the crude birth rate; in discontinuous notation, however, with the age groups considered here, this no longer applies and the crude birth rate is unknown.

In order to apply formula II.3*d bis*, we use approximate relationships based on the following assumptions:

(*a*) For a five-year age group a, $a+5$ we assume that:[4]

$$b_0 L_a = C_a e^{r(a+2.5)}$$

For the first two age groups, we assume that:

$$b_0 L_0 = C_0 e^{0.5r}$$

and

$$b_0 L_{1\text{-}4} = C_{1\text{-}4} e^{3r}$$

Finally, for the last age group, we assume that:

$$b_0 L_{85+} = C_{85+} e^{87.5r}$$

These formulae make it possible to calculate the series:

$$b_0 L_0,\ b_0 L_{1\text{-}4},\ b_0 L_{5\text{-}9},\ \dots\ b_0 L_{85+}$$

(*b*) On the basis of this series, we can calculate the series of values of $b_0\ p_0(a)$ by applying the following formulae:

(1) For each five-year age group—e.g., L_{10-14}—we assume that:

$$L_{10\text{-}14} = \frac{5}{2}\,[p(10) + p(15)]$$

We also have the approximate formula:

$$p(10) + p(15) = 2p(12.5)$$

so that:

$$L_{10\text{-}14} = 5p(12.5)$$

Similarly, we can write:

$$L_{5\text{-}9} = 5p(7.5)$$

and finally:

$$L_{10\text{-}14} + L_{5\text{-}9} = 5p(12.5) + 5p(7.5) = 10p(10)$$

We can thus very easily calculate the series:

$$b_0 p(7.5),\ b_0 p(10),\ b_0 p(12.5),\ \dots\ b_0 p(82.5)$$

Finally, the formula:

$$L_{5\text{-}9} = \frac{5}{2}\,[p(5) + p(10)]$$

enables $b_0 p(5)$ to be calculated.

(2) For the last age group, we apply the formula:[5]

$$L_{85+} = p(85)\ \text{Log}\ 100{,}000p(85)$$

We determine $b_0 p(85)$ by a process of trial and error.

(3) It now remains for us to find the values of the survivorship function under the age of 5. For the 1-4 age group, we write:

$$L_{1\text{-}4} = 1.9p(1) + 2.1p(5)$$

As we already know $b_0 p(5)$, we can calculate $b_0 p(1)$ with the aid of this formula.

The problem of the 0 age group. In the case of the 0 age group the problem is more complicated; for, if λ represents the proportion of deaths of children under 1 year of age which occur in the calendar year of their birth, we have the following approximate formula:

$$L_0 = 1 - \lambda + \lambda p(1)$$

which can be written:

$$b_0 L_0 = C_0 e^{0.5r} = b_0(1 - \lambda) + b_0 \lambda p(1)$$

from which we have:

$$b_0 = \frac{C_0 e^{0.5r} - \lambda b_0 p(1)]}{1 - \lambda} \qquad \text{(IV.2)}$$

We already know $b_0 p(1)$, but we do not know λ to complete the calculation. We again come up against the fact mentioned above, namely, that knowledge of the age distribution in discontinuous terms does not mean that we know the crude birth rate.

The proportion λ is not the same for all populations. In practice[6] it varies from 0.6 to 0.9. By varying λ between these limits, we can therefore determine with the use of formula IV.2 a whole series of crude birth rates.

As long as r remains small—as it always does in the case of the human species—the results scarcely depend on the value of r adopted for the computation. We can therefore speak of crude birth rates compatible with the age distribution, and it is useful to determine this series of compatible crude birth rates. If b_0 is given as well as the age distribution, we can check that it is indeed within the series; when it is not given, we merely have to select of value of b_0 which is within the series in order to complete the computation.

Use of the infant mortality. Determination of the compatible crude birth rates is tantamount to choosing the infant mortality measured for the probability of death between the ages of 0 and 1. In fact, choosing b_0 when $b_0 p(1)$ is known is tantamount to choosing the infant probability of death q_0.

[4] It may be recalled that:

$$L_a = \int_a^{a+5} p(a)da$$

[5] This formula is given in the manual on the computation of population projections, *Manual III. Methods for Population Projections by Sex and Age* (United Nations publication, Sales No.: 56.XIII.3). It is an experimental formula which is found to be satisfied within the range of variations of mortality of the human species.

[6] It is convenient to draw a distinction between the two main categories of infant deaths: endogenous deaths, which are due to factors present when the infant is born (genetic factors arising at conception, factors transmitted by the mother during pregnancy, or factors arising at confinement), and exogenous deaths, which are due to the environment—in the broad sense of the term—in which the infant lives after birth. Endogenous deaths occur for the most part soon after birth, and they can be considered in practice to occur entirely within the calendar year of birth, but exogenous deaths occur throughout the calendar year of birth and the following year. Observation reveals that this distribution is remarkably stable in time and space. If e is the endogenous infant mortality (per 1,000 births) and E is the exogenous infant mortality, distributed between kE, the calendar year of birth, and $(1 - k)$E, the following year, we can write:

$$\lambda = \frac{e + kE}{e + E}$$

where k is a coefficient which is the same for all populations. λ is then a function of e and E. On the basis of observed values of e and E, we find λ to be between 0.6 and 0.9.

As we are assuming that the mortality is that of the human species, we know that in practice this probability can only vary between 10 and 500 per thousand and that the value of q_0 selected must come within these limits. By using the formulae given above, we can easily see that we have:

$$\lambda = \frac{b_0 p(1) - C_0 e^{0.5}(1 - q_0)]}{q_0 b_0 p(1)} \qquad \text{(IV.3)}$$

Once we have chosen q_0, therefore, we can calculate λ, which must be between 0.6 and 0.9. If λ is outside these limits, the value originally chosen for q_0 must be rejected. We can thus determine a range of variation for infant mortality compatible with the age distribution, just as we did for the crude birth rate. The range thus determined is generally smaller than the range of 10 to 500 per thousand which was quoted above as representing the practical limits of infant mortality in the human species. We can, in fact, write formula (IV.3) as follows:

$$q_0 = \frac{1 - \dfrac{C_0 e^{0.5r}}{b_0 p(1)}}{\lambda - \dfrac{C_0 e^{0.5r}}{b_0 p(1)}}$$

In this form, we can easily calculate the limits of variation of q_0 by successively taking $\lambda = 0.6$ and $\lambda = 0.9$.

When the first age group is 0-4 years. It may happen that the age distribution is not known separately for age groups 0 and 1-4, but only for the combined 0-4 age group. As in the previous case, we first of all determine the series:

$$b_0 L_{0\text{-}4},\ b_0 L_{5\text{-}9},\ b_0 L_{10\text{-}14} \text{ etc.}$$

by the formulae:

$$C_{0\text{-}4} e^{2.5r},\ C_{5\text{-}9} e^{7.5r},\ C_{10\text{-}14} e^{12.5r} \text{ etc}$$

From this, we have no difficulty in progressing, as described above, to the series: $b_0 p(5)$, $b_0 p(10)$, $b_0 p(15)$ etc. and in order to complete the calculation we must, as before, assume a value for b_0, or for the infant mortality q_0. Can we also calculate for these two quantities what we called above a "range compatible with the age distribution"? We can write: $b_0 p(1) = b_0(1 - q_0)$ and in accordance with the foregoing formulae:

$$b_0 L_0 = b_0(1 - \lambda) + \lambda b_0 p(1) = b_0(1 - \lambda) + \lambda b_0(1 - q_0)$$

$$\begin{aligned} b_0 L_{1\text{-}4} &= 1.9 b_0 p(1) + 2.1 b_0 p(5) \\ &= 1.9 b_0(1 - q_0) + 2.1 b_0 p(5) \end{aligned}$$

Whence we finally have:

$$\begin{aligned} b_0 L_{0\text{-}4} &= b_0 L_0 + b_0 L_{1\text{-}4} \\ &= b_0(1 - \lambda) + \lambda b_0(1 - q_0) + 1.9 b_0(1 - q_0) + 2.1 b_0 p(5) \end{aligned}$$

We know that: $b_0 L_{0\text{-}4} = C_{0\text{-}4} e^{2.5r}$

We therefore have the equation:

$$\begin{aligned} C_{0\text{-}4} e^{2.5r} = {} & b_0(1 - \lambda) + \lambda b_0(1 - q_0) \\ & + 1.9 b_0(1 - q_0) + 2.1 b_0 p(5) \qquad \text{(IV.4)} \end{aligned}$$

Let us suppose that q_0 is given. We then have a relationship between b_0 and λ, in which to each value for λ corresponds one value for b_0. Experience shows that, if λ varies from 0.6 to 0.9, the corresponding values for b_0 will not be very different. If q_0 varies from 10 to 500 per thousand and λ from 0.6 to 0.9, we obtain a series of rates b_0 which vary within two limits. These limits define the range of variation of b_0 compatible with the age distribution. The result is practically independent of the value of r used in the calculations.

In the preceding case, where we knew the age distribution of the 0-1 and 1-4 age groups, we completed the calculation by assuming a value for the infant mortality q_0, and λ was then determined. This determination of λ amounted, in fact, to the determination of the distribution of q_0 between endogenous and exogenous causes of death.

In the present case, where we know only the age distribution for the 0-4 age group, assumption of a value for the infant mortality q_0 will only enable us to complete the calculation with a certain margin of error which is fortunately quite small, corresponding to the possibility of variation of λ between 0.6 and 0.9. In order to eliminate this possible error, we must also assume a value for λ, i.e., we must select a value for the distribution of the infant mortality between endogenous and exogenous causes of death.

In the first case knowledge of λ was enough to rule out any question of a choice, but in the second case knowledge of λ does not eliminate the need to choose a value for q_0 and only removes a secondary uncertainty. These explanations will be made clearer and more specific if we give some numerical applications.

A numerical application. In this first example, we shall use the age distribution of the female Malthusian population having a rate of natural variation $r = 0.03$ and a mortality identical with that of the intermediate model life table giving an expectation of life at birth for both sexes of 50 years. This age distribution has been computed in table II.3 and is given again in table IV.2.

Leaving aside the way in which this age distribution has been computed and taking it as a known quantity, we shall consider the sub-set $F_0(r)$ which corresponds to it.

We shall first of all determine the series of crude birth rates compatible with this age distribution, by applying formula IV.2. As already stated, the result does not in practice depend on the value adopted for r. In order to verify this fact, we made two calculations in which we successively took the value of r as 0 and 0.015. Table IV.1 shows that we do indeed obtain two almost identical series of compatible rates. In practice, of course, we should only made a single calculation, and the simplest procedure would be to assume that $r = 0$.

Let us now examine the computation for this case in detail:[7]

$$\begin{aligned} b_0 p(10) = C(10) &= \frac{C_{5\text{-}9} + C_{10\text{-}14}}{10} \\ &= \frac{146{,}358 + 123{,}799}{10} = 27{,}015.7 \end{aligned}$$

$$b_0 p(7.5) = C(7.5) = \frac{C_{5\text{-}9}}{5} = \frac{146{,}358}{5} = 29{,}271.6$$

As C(5) is symmetrical with C(10) in relation to C(7.5), we have:

$$(C\ 5) = 29{,}271.6 + (29{,}271.6 - 27{,}015.7) = 31{,}527.5$$

[7] The numerical values used in this computation are given in table IV.2.

Let us now proceed to the computation of $b_0C(1)$. We have:

$$C_{1\text{-}4} = b_0L_{1-4} = 1.9b_0p(1) + 2.1b_0p(5) = 139{,}640$$

whence we have:

$$b_0p\,(1) = 38{,}648.6$$

We can then apply formula IV.2 for various values of r. The results are given in the column headed $r = 0$ in table IV.1. The same calculation for $r = 0.015$ gives the series of crude birth rates appearing in the column headed $r = 0.015$. As already stated, the two series are almost identical.

TABLE IV.1. CRUDE BIRTH RATE PER 1,000 COMPATIBLE WITH THE AGE DISTRIBUTION OF THE POPULATION IN TABLE IV.2 (COLUMN 3)

a	$r = 0$	$r = 0.015$
0.6	42.7	42.9
0.7	44.1	44.1
0.8	46.8	46.6
0.9	55.0	54.2

We now propose to find the population of sub-set $F_0(r)$ corresponding to $r = 0.015$. Table IV.2 gives the details of the computation.

Column 1 of the table gives the median age a of the age groups whose limits are indicated in column 2. The age distribution of the population, C_a, which is the same as the age distribution of the population computed in table II.3, is given in column 3. Column 4 gives the sequence of the coefficients $e^{r\alpha}$ for $r = 0.015$. These are the data of the problem. The computations are shown from column 5 onwards.

We begin by calculating the sequence of the quantities b_0L_a. These are given in column 5 and are obtained by multiplying columns 3 and 4.

Column 6 gives the sequence of the quantities b_0, $b_0p(1)$, $b_0p(5)$, $b_0p(10)$, $b_0p(15)$, and so forth. These are obtained in the following manner:

(*a*) The figures below the horizontal line are obtained by successively adding together in pairs the figures in column 5 and dividing the result by 10. Thus, the first figure below the line comes to:

$$\frac{(163{,}799 + 149{,}330)}{10} = 31{,}312.9 = b_0p(10)$$

b) For $b_0p(5)$, we have:

$$b_0p(5) = 32{,}759.3 + (32{,}759.3 - 31{,}312.9) = 34{,}206.8$$

(*c*) For $b_0p(1)$, we have:

$$1.9b_0p(1) + 2.1b_0p(5) = 146{,}063$$

TABLE IV.2. COMPUTATION OF THE SURVIVORSHIP FUNCTIONS CORRESPONDING IN A MALTHUSIAN POPULATION TO A GIVEN AGE DISTRIBUTION C_a AND AN INTRINSIC RATE OF NATURAL VARIATION OF 0.015

Median age a (1)	Age group (years) (2)	Age distribution (a) C_a (3)	e^{ra} for $r = 0.015$ (4)	Product of the two preceding columns: $C_ae^{ra} = b_0L_a$ (5)	$\frac{b_0L_a + b_0L_{a+5}}{10} = b_0p(a+5)$ (6)	Differences between successive figures in the preceding column (7)	Death rate m_a (per thousand); column (7) divided by column (5) (8)	Survivors to the initial age of the age group; column (6) divided by 45,074.6 and multiplied by 100,000 (9)
0.5	0	40 282	1.00750	40 584	45 074.6	6 006.8	147.53	100 000
3.0	1-4	139 640	1.04600	146 063	39 067.7	4 860.9	33.53	86 717
7.5	5-9	146 358	1.11917	163 799	34 206.8	2 893.9	17.56	75 851
12.5	10-14	123 799	1.20623	149 330	31 312.9	2 765.5	18.52	69 469
17.5	15-19	104 623	1.30128	136 144	28 547.4	2 634.8	19.36	63 334
22.5	20-24	87 754	1.40144	122 982	25 912.6	2 553.6	20.76	57 488
27.5	25-29	73 222	1.51059	110 608	23 359.0	2 370.8	21.43	51 823
32.5	30-34	60 968	1.62829	99 274	20 988.2	2 169.0	21.86	46 563
37.5	35-39	50 661	1.75515	88 918	18 819.2	1 990.2	22.39	41 751
42.5	40-44	41 957	1.89174	79 372	16 829.0	1 854.8	23.37	37 336
47.5	45-49	34 509	2.03918	70 370	14 974.2	1 773.7	25.19	33 221
52.5	50-54	28 029	2.19899	61 635	13 200.5	1 749.0	28.38	29 286
57.5	55-59	22 320	2.36918	52 880	11 451.5	1 772.9	33.52	25 406
62.5	60-64	17 193	2.55369	43 906	9 678.6	1 842.3	41.98	21 472
67.5	65-69	12 518	2.75257	34 457	7 836.3	1 926.0	55.90	17 385
72.5	70-74	8 307	2.96685	24 646	5 910.3	1 922.5	78.00	13 112
77.5	75-79	4 763	3.19792	15 232	3 987.8	1 705.9	111.97	8 847
82.5	80-84	2 201	3.44709	7 587	2 281.9	1 287.1	169.68	5 063
87.5	85-89	896	3.71655	3 330	994.8	994.8	298.97	2 207
TOTAL . . .				1 451 088				

(a) Figures taken from table II.3 (chap. II).

As we already know $b_0p(5)$, we obtain from this equation: $b_0p(1) = 39{,}067.7$.

(*d*) Finally, in order to complete the computation, we must select a value of b_0 from the series of compatible crude birth rates. We have selected $b_0 = 0.0450746$, which is the crude birth rate of the Malthusian population constructed as a first step in table II.3 in order to obtain the age distribution C_a. We need not have made this particular choice, but could have adopted any value compatible with the age distribution.

Column 7 gives the differences between successive figures in column 6.

Column 8 gives the death rates by age groups m_a. These are obtained by dividing column 7 by column 5.

Finally, column 9 gives the survivors to the initial ages of the age groups: $p(0)$, $p(1)$, $p(5)$, $p(10)$, $p(15)$, and so forth.

It should be noted that it is not necessary to know b_0 in order to calculate the survivorship function from 1 year of age onwards. All that is necessary is to divide the terms of the series $b_0p(1)$, $b_0p(5)$, $b_0p(10)$, $b_0p(15)$, and so forth, by $b_0p(1)$.

Similarly, all the death rates m_a except the first can be calculated without reference to b_0.

Let us now suppose that we know only the 0-4 age group. Table IV.3 is then written as follows:

TABLE IV.4. CRUDE BIRTH RATES COMPATIBLE WITH THE AGE DISTRIBUTION IN TABLE IV.2, FOR VARIOUS LEVELS OF INFANT MORTALITY, TWO VALUES OF λ AND TWO RATES OF NATURAL VARIATION (APPLICATION OF FORMULA IV.4)

q_0 (rate per thousand)	$\lambda = 0.6$		$\lambda = 0.9$	
	$r = 0.015$	$r = 0$	$r = 0.015$	$r = 0$
10	40.0	39.6	40.0	39.6
50	41.4	41.0	41.7	41.3
100	43.4	43.0	43.9	43.4
200	47.9	47.4	49.1	48.6
300	53.5	52.9	55.8	55.2
400	60.5	59.8	64.6	63.9
500	69.7	68.9	76.6	75.8

It can be seen very clearly that the variation of λ has little effect on b_0. Finally, the crude birth rates compatible with the age distribution vary from 40.0 to 76.6. We may recall that for the same age distribution (but on the assumption that we knew both the 0 and the 1-4 age groups) we found compatible crude birth rates varying from 42.7 to 55.0.

Direct computation of probabilities of death over a finite age range. We can obviously determine the probabilities of death from the survivorship table in table IV.2.

TABLE IV.3

Median age a (1)	Age group (years) (2)	Age distribution of the population, C_a (3)	e^{ra} (4)	Product of the two proceding columns, $C_ae^{ra} = b_0L_a$ (5)	$\frac{b_0L_a + b_0L_{a+5}}{10} = b_0p(a+5)$ (6)
2.5	0-4	179 922	1.03821	186 797	
7.5	5-9	146 358	1.11397	163 799	34 206.8
12.5	10-14	123 799	1.20623	149 330	31 312.9
17.5	15-19	104 623	1.30128	136 144	28 547.4
. . . .	...	...	...	...	...
. . . .	...	...	...	...	...
. . . .	...	...	...	...	...

Over 4 years of age, nothing needs to be changed. Only the first line, which is a combination of the first two lines of table IV.2, is changed, so that we have:

$$C_{0\text{-}4}e^{2.5r} = 186{,}797$$

and $2.1b_0p(5) = 2.1 \times 34{,}206.8 = 71{,}834.3$.

Formula IV.4 is written:

$$\frac{(186{,}797 - 71{,}834)}{1 - \lambda + \lambda(1 - q_0) + 1.9(1 - q_0)} = b_0$$

or

$$b_0 = \frac{114{,}963}{2.9 - (\lambda + 1.9)q_0}$$

This formula enables b_0 to be computed, once λ and q_0 are known. Table IV.4 gives the results of this computation. It was stated above that the result depends scarcely at all on the value of r used. This can easily be verified in the following table, which shows two series of compatible crude birth rates corresponding to $r = 0$ and $r = 0.015$. The two series are almost identical.

These probabilities can also be obtained directly, however, without making use of the survivorship table.

If we divide each five-year age group by the group which precedes it, we can write:

$$e^{5r}\frac{C_{a+5}}{C_a} = \frac{L_{a+5}}{L_a} = \frac{p(a+7.5)}{p(a+2.5)}$$

We thus obtain the survivorship rates for:

from 7.5 to 12.5 years of age
from 12.5 to 17.5 years of age
from 17.5 to 22.5 years of age
... ...
from 77.5 to 82.5 years of age
from 82.5 to 85 and over

For earlier ages, we can write:

$$e^{4.5r}\frac{C_{5\text{-}9}}{C_{1\text{-}4}}\frac{4}{5} = \frac{p(7.5)}{p(3)}$$

i.e., the survivorship rate from 3 to 7.5 years of age, and likewise:

$$e^r \frac{C_{1-4}}{C_0} \frac{1}{4} = \frac{p(3)}{p(0.5)}$$

i.e., the survivorship rate from 0.5 to 3 years of age, and finally:

$$e^r \frac{C_0}{b_0} = \frac{p(0.5)}{p(0)}$$

i.e., the survivorship rate from birth to 0.5 years of age.

The complements to 1 of all these survivorship rates are the probabilities of death calculated for finite age ranges: 0.5 years in the case of the first age group, 2.5 years for the second, 4.5 years for the third and 5 years for all the others. All this clearly shows that we must know b_0 in order to obtain the complete life table: otherwise, it is possible to determine the probabilities only from the age of 0.5 years.

An important property of the survival ratios. We see that the survival ratios for five-year periods have a remarkable property: over the age of 7.5 years, they can be deduced from the series of coefficients

$$\frac{C_a + 5}{C_a}$$

by multiplying by e^{5r}.

This remarkable property corresponds to that already pointed out above in the case of the instantaneous probabilities. It will be recalled that these probabilities were deduced from $Q(a)$ (the logarithmic derivative bearing the sign of $C(a)$) by taking away a constant, r.

$$q(a) = Q(a) - r$$

In the case of the finite probabilities of death, this property is only approximate. Thus, we have:

$$_5q_a = \frac{p(a) - p(a+5)}{p(a)} = \frac{C(a) - C(a+5)e^{5r}}{C(a)} = 1 - \frac{C(a+5)}{C(a)} e^{5r}$$

We also have:

$$_5Q_a = \frac{C(a) - C(a+5)}{C(a)} = 1 - \frac{C(a+5)}{C(a)}$$

whence we have the difference:

$$_5Q_a - {_5q_a} = \frac{C(a+5)}{C(a)} (e^{5r} - 1)$$

For a given value of r, this difference varies with age, since $C(a+5)/C(a)$ is not constant. If we set aside the extreme ages of life, however, $C(a+5)/C(a)$ remains in reality close to unity, and as r is relatively small, we have approximately:

$$_5q_a \# {_5Q_a} - 5r$$

i.e., an approximate formula similar to the exact formula regarding the instantaneous probabilities.

The condition of compatibility of r. As was seen above, not all values of r are compatible with a given age distribution. r must therefore satisfy the condition:[8]

$$r < -\frac{C'(a)}{C(a)}$$

This condition is easily expressed in discontinuous notation when considering the survivorship rates. It is tantamount to saying that survivorship rates computed in the manner described above must all be less than unity.

Numerical example of the direct computation of survival ratios. Let us again take the age distribution in table II.3 and compute the life table corresponding to $r = 0.03$. It will be noted that this is exactly the same value as was used for the original construction of the age distribution in table II.3. If, therefore, we take $b_0 = 0.0450751$, we should obtain the same life table as was used with $r = 0.03$ for that construction. In fact, however, as we are using approximate formulae, we cannot hope to obtain exactly the same life table as in the first case, and indeed the differences enable us to evaluate the usefulness of the approximate formulae.

Table IV.5 gives the details of the computation, which does not call for any special comments, since it is the direct application of the formulae already given. A comparison of columns 10 and 11 and of columns 12 and 14 shows that the approximate formulae give very good results.

Determination of the values of r *compatible with the age distribution.* We shall, however, describe at this point how the computations in table IV.5 enable the values of r compatible with the age distribution to be determined. We have already seen that the condition of compatibility is tantamount to saying that the survivorship rates must all be less than unity. These survivorship rates, for $r = 0.03$, are given in column 6. They pass through a maximum equal to $0.84586 \times e^{5 \times 0.03}$ for the 5-9 age group, whatever the value of r. We should therefore have: $0.84586e^{5r} < 1$, which gives: $r < 0.033480$.

Thus, all the values of r below this limit of $r_m = 0.033480$ are compatible with the age distribution computed in table IV.5.

Verification of the important property of the instantaneous death rates. If we leave aside the extreme ages, the death rate for an age group m_a is close to the probability of death at the median age of the age group. Consequently, the two series of death rates in tables IV.2 and IV.5 should illustrate in an approximate manner the remarkable property of the instantaneous death rates mentioned above. There should be a difference between the two series, at all ages, of about 0.015,[9] and this is in fact exactly what we observe in table IV.6.

Numerical application on the basis of an actually observed age distribution. We now propose to repeat the same calculations, by using the age distributions of actual populations. We shall begin by using an age distribution

[8] In the example in table III.2, we did not take the trouble to verify that this condition was fulfilled because we were quite sure that it was, since the age distribution used had been constructed as a first step, by using the value $r = 0.03$. When using this age distribution in the opposite sense, we were sure that at least all the values of r equal to or less than 0.03 would be compatible with this age distribution.

[9] 0.015 is the difference between the rate of $r = 0.015$ used in table IV.2 and the rate of $r = 0.030$ used in table IV.5.

TABLE IV.5. COMPUTATION OF THE MORTALITY RATES CORRESPONDING, IN A MALTHUSIAN POPULATION, TO A GIVEN AGE DISTRIBUTION C_a AND AN INTRINSIC RATE OF NATURAL VARIATION OF $r = 0.03$ (DIRECT CALCULATION OF THE PROBABILITIES OF DEATH)

Median age *a* (1)	Age group (years) "*a*" (2)	Age distribution of the population Ca (3)	$\frac{C_a + 5}{Ca}$ (4)	e^{ra} for $r = 0.03$ (c) (5)	Survivorship rate $\frac{L_a + 5}{L_a}$ (6)	Probability of death between one age and the following age (per 1,000) (7)	Survivors to age "*a*" *p*(*a*) (8)	Deaths between age "*a*" and the following age (9)	Survivors in each age group: L_a reconstitued (10)	Survivors in each age group: L_a initial (11)	Death rate (per 1 000): m_a reconstitued (12)	Death rate (per 1 000): m_a initial (13
0 . . .	Births	45 075 (a)	0.89366 (b)	1.0151 (c)	0.90715	92.85	100 000	9 285				
0.5 . . .	Under 1 year	40 282	0.86664	1.0779	0.93415	65.85	90 715	5 974	90 715	90 719	136.41	136.41
3.0 . . .	1-4	139 640	0.83849	1.1445	0.95965	40.35	84 741	3 419	338 964	338 974	16.19	16.19
7.5 . . .	5-9	146 358	0.84586	1.1618	0.98272	17.28	81 322	1 405	406 610	406 628	3.72	3.99
12.5 . . .	10-14	123 799	0.84510	1.1618	0.98184	18.16	79 917	1 451	399 585	399 620	3.57	2.96
17.5 . . .	15-19	104 623	0.83876	1.1618	0.97447	25.53	78 466	2 003	392 330	392 370	4.40	4.38
22.5 . . .	20-24	87 754	0.83440	1.1618	0.96941	30.59	76 463	2 339	382 315	382 368	5.67	5.97
27.5 . . .	25-29	73 222	0.83265	1.1618	0.96737	32.63	74 124	2 419	370 620	370 680	6.41	6.45
32.5 . . .	30-34	60 968	0.83094	1.1618	0.96539	34.61	71 705	2 482	358 525	358 600	6.83	6.80
37.5 . . .	35-39	50 661	0.82819	1.1618	0.96219	37.81	69 223	2 617	346 115	346 202	7.36	7.28
42.5 . . .	40-44	41 957	0.82248	1.1618	0.95556	44.44	66 606	2 960	333 030	333 118	8.37	8.15
47.5 . . .	45-49	34 509	0.81222	1.1618	0.94364	56.36	63 646	3 587	318 230	318 325	10.28	10.06
52.5 . . .	50-54	28 029	0.79632	1.1618	0.92516	74.84	60 059	4 495	300 295	300 392	13.45	13.22
57.5 . . .	55-59	22 320	0.77030	1.1618	0.89493	105.07	55 564	5 838	277 820	277 922	18.59	18.05
62.5 . . .	60-64	17 193	0.72809	1.1618	0.84589	154.11	49 726	7 663	248 630	248 722	27.15	26.79
67.5 . . .	65-69	12 518	0.66360	1.1618	0.77097	229.03	42 063	9 634	210 315	210 400	41.12	41.19
72.5 . . .	70-74	8 307	0.57337	1.1618	0.66614	333.86	32 429	10 827	162 145	162 220	63.08	65.38
77.5 . . .	75-79	4 763	0.46210	1.1618	0.53687	463.13	21 602	10 005	108 010	108 068	96.42	102.30
82·5 . . .	80-84	2 201	0.40709	1.1618	0.47296	527.04	11 597	6 112	57 985	58 022	163.49	154.48
87.5 . . .	85 and over	896					5 485		27 425	27 443	259.56	259.56

(a) This refers to the crude female birth rate $b_f = 45.075$ per thousand.
(b) The first figure in the column represents the ratio C_0/b, the second $C_{1\text{-}4}/4C_0$ and the third $4C_{5\text{-}9}/5C_{1\text{-}4}$.
(c) The first figure in the column corresponds to $k = 0.5$, the second to $k = 2.5$, the third to $k = 4.5$ and all the others to $k = 5$.

TABLE IV.6. COMPARISON OF DEATH RATES BY AGE GROUPS m_a IN TWO MALTHUSIAN POPULATIONS COMPUTED ON THE BASIS OF AGE DISTRIBUTION IN TABLE II.3 (ASSUMED TO BE CONSTANT) BY TAKING SUCCESSIVELY TWO VALUES, 0.030 AND 0.015, AS THE RATE OF NATURAL VARIATION r

Age group (a)	Death rate m_a (per thousand) $r = 0{,}015$ (a)	$r = 0{,}030$ (b)	Difference
0	147.53	136.41	11.12
1-4	33.53	16.19	17.34
5-9	17.56	3.72	13.84
10-14	18.52	3.57	14.95
15-19	19.36	4.40	14.96
20-24	20.76	5.67	15.09
25-29	21.43	6.41	15.02
30-34	21.86	6.83	15.03
35-39	22.39	7.36	15.03
40-44	23.37	8.37	15.00
45-49	25.19	10.28	14.91
50-54	28.38	13.45	14.93
55-59	33.52	18.59	14.93
60-64	41.98	27.15	14.83
65-69	55.90	41.12	14.78
70-74	78.00	63.08	14.92
75-79	111.97	96.42	15.55
80-84	169.68	163.49	6.19
85 and over .	298.97	259.56	39.41

(a) Rate taken from column 8 of table IV.2.
(b) Rate taken from column 12 of table IV.5.

TABLE IV.7. AGE DISTRIBUTION OF THE FEMALE POPULATION OF BRAZIL AT THE 1900, 1940 AND 1950 CENSUSES, AND ADJUSTED DISTRIBUTION BASED ON THE RESULTS OF THE CENSUSES

Age group (years)	Year of census 1900	1940	1950	Adjusted distribution
0-4	4 443 (0-14)	1 546	1 590	1 764
5-9		1 376	1 329	1 367
10-14		1 284	1 209	1 203
15-19	1 134	1 110	1 099	1 047
20-24	927	960	1 003	902
25-29	864	829	808	776
30-34	532	622	624	650
35-39	624	560	583	544
40-44	356	459	447	456
45-49	387	343	368	378
50-54	195	294	298	301
55-59	223	187	198	223
60-64	101	171	178	155
65-69	94	97	100	107
70-74	40	76	76	58
75-79	41	38	39	39
80-84	15	27	28	20
85 and over .	24	21	23	10
ALL AGES . .	10 000	10 000	10 000	10 000
0-14	4 443	4 206	4 128	4 334

adjusted to the female population of Brazil, as recorded in the censuses of 1900, 1940 and 1950. The age distributions of these three censuses are close to one another and the adjustment, made graphically by hand, simply smoothed out some irregularities in the age pyramid. Table IV.7 gives the three age distributions provided by the censuses as will as the smooth and adjusted age distribution. It should be noted in passing, however, that there is still a good deal of uncertainty about the first age group. Moreover, it should be noted that this first age group covers the first five years of life, whereas in the preceding example those five years were divided into the age groups under 1 year and 1-4.

Let us now consider the sub-set F(r) attached to the adjusted age distribution, and let us determine which of the populations of that sub-set corresponds to $r = 0.025$. We can use either of the two methods already mentioned:

(1) We can determine the sequence of the quantities: $b_0L_{0\text{-}4}$, $b_0L_{5\text{-}9}$, $b_0L_{10\text{-}14}$ etc., by multiplying the sequence of the quantities, $C_{0\text{-}4}$, $C_{5\text{-}9}$, $C_{10\text{-}14}$, etc. by $e^{2.5r}$, $e^{7.5r}$, $e^{12.5r}$ etc., respectively. We can then determine without difficulty the series of quantities: $b_0p(5)$, $b_0p(10)$,$b_0p(15)$ etc.

(2) We can also calculate the survivorship rates directly by multiplying the quantities:

$$\frac{C_{5\text{-}9}}{C_{0\text{-}4}}, \frac{C_{10\text{-}14}}{C_{5\text{-}9}}, \frac{C_{15\text{-}19}}{C_{10\text{-}14}}, \text{etc.,}$$

by e^{5r}. We thus obtain the survivorship rates from 2.5 7.5 years of age, from 7.5 to 12.5, from 12.5 to 17.5. and so on.

In both methods it is necessary, in order to complete the computation, to make an assumption regarding b_0, and we must therefore begin by determining the series of birth rates compatible with the age structure. Table IV.8 gives the essentials of the computation.

In order to complete the computation, we have arbitrarily assumed that $b_0 = 45$ per thousand. As can be seen from table IV.8, this is tantamount to assuming an infant mortality of between 96 and 106 per thousand, depending on the value of λ (96 per thousand corresponds to $\lambda - 0.6$, while 106 per thousand corresponds to $\lambda = 0.9$). Once a value has been adopted for b_0, the computation presents no difficulty. Table IV.9 gives details of the computation by the second method, which involves first computing the survivorship rates.

We have dealt with this first example at some length because, as will be seen later, the other examples can always be referred back to it.

SECOND EXAMPLE: MALTHUSIAN POPULATION WITH GIVEN AGE DISTRIBUTION $C_0(a)$ AND KNOWN SURVIVAL RATIO AT A GIVEN AGE a_0

If r_0 represents the rate of natural variation of the population which is sought, we have:

$$p(a_0) = \frac{C_0(a_0)}{C_0(0)} e^{r_0 a}$$

This formula enables r_0 to be calculated, and this brings us back to the preceding case. In discontinuous notation, we apply the preceding approximate formulae, the best method being to begin by calculating $C_0(a_0)$.

If, for example, $a_0 = 21$ years, we write:

$$C_{15-19} = 5C(17.5)$$

$$C_{20-24} = 5C(22.5)$$

and we determine C(21) by interpolation. We then have the formula:

$$p(21) = \frac{C(21)}{C(0)} e^{21 r_0}$$

TABLE IV.8. COMPUTATION OF THE CRUDE BIRTH RATES COMPATIBLE WITH THE AGE STRUCTURE OF THE FEMALE POPULATION OF BRAZIL, AS ADJUSTED TO FIT THE RESULTS OF THE CENSUSES OF 1900, 1940 AND 1950

Median age, a	*Age group*	C_a	e^{-ra}	$C_a e^{ra} = b_0 L_a$	$\frac{b_0 L_a + b_0 L_{a+5}}{10} = b_0 p(a)$	
(A) $r = 0.025$						
2.5	0-4	1 764	0.93941	1 878		
7.5	5-9	1 367	0.82901	1 649	330.3 (a)	$b_0p(5)$
12.5	10-14	1 203	0.73162	1 644	329.3	$b_0p(10)$
17.5	15-19	1 047	0.64565	1 622	326.6	$b_0p(15)$
(B) $r = 0$ (stationary population)						
2.5	0-4	1 764		1 764		
7.5	5-9	1 367		1 367	289.8 (b)	$b_0p(5)$
12.5	10-14	1 203		1 203	257.0	$b_0p(10)$
17.5	15-19	1 047		1 047	225.0	$b_0p(15)$

(C) Compatible crude birth rates (per thousand), application of formula IV.4

Infant mortality (q_0 per thousand)	*Value of λ:* 0.6		0.9	
	$r = 0.025$	$r = 0$	$r = 0.025$	$r = 0$
10	41.2	40.2	41.2	40.2
50	42.7	41.6	42.9	41.9
100	44.7	43.6	45.2	44.1
200	49.3	48.1	50.6	49.4
300	55.1	53.7	57.5	56.1
400	62.3	60.8	66.5	64.9
500	71.8	70.0	78.9	77.0

(a) We assume that $b_0p(7.5) = 1\,649/5 = 329.8$ and we selected $b_0p(5)$ symmetric of $b_0p(10)$ with respect to $b_0p(7.5)$.

(b) We assume that $b_0p(7.5) = 1\,367/5 = 273.4$ and we selected $b_0p(5)$ symmetric of $b_0p(10)$ with respect to $b_0p(7.5)$.

which makes it possible to calculate r_0, provided that $C(0)$—in other words, b_0—is known. If it is not known, it can be chosen from the series of birth rates compatible with the age distribution. There is thus an infinity of possible values of r_0 and consequently an infinity of populations satisfying the given conditions. Thus, the problem, which was a determinate one in continuous notation, becomes indeterminate in discontinuous notation.

THIRD EXAMPLE: MALTHUSIAN POPULATION WITH GIVEN AGE DISTRIBUTION $C_0(a)$ AND KNOWN CRUDE DEATH RATE d_0

In continuous notation, the problem is determinate, since knowledge of d_0 is tantamount to knowledge of $r_0 = b_0 - d_0 = C(0) - d_0$, thus bringing us back to the first example. In discontinuous notation, however, the problem is not determinate. All values of r_0, such as $r_0 = b_0 - d_0$, are valid, provided that b_0 is within the range of values compatible with the age distribution.

FOURTH EXAMPLE: MALTHUSIAN POPULATION WITH GIVEN AGE DISTRIBUTION $C_0(a)$ AND KNOWN AGE DISTRIBUTION OF DEATHS AT A GIVEN AGE a_0

In continuous notation, we calculate r by the formula:

$$r = \frac{C(0)d(a_0) + C'(a_0)}{-C(a_0) + d(a_0)} \qquad \text{(II.12)}$$

In discontinuous notation—if, for example, we know the proportion $d_{20\text{-}24}$ of deaths in the 20-24 age group—we shall assume that we can write:

$$5d(22.5) = d_{20\text{-}24}$$

and

$$5C(22.5) = C_{20\text{-}24}$$

In order to apply the preceding formula, we must calculate $C'(22.5)$. We assume that we can write:

$$5C'(22,5) = 5\,\frac{C(17.5) - C(27.5)}{10} =$$

$$= \frac{1}{2}\,\frac{C_{15\text{-}19}}{5} - \frac{C_{25\text{-}29}}{5} = \frac{1}{10}(C_{15\text{-}19} - C_{25\text{-}29})$$

TABLE IV.9. COMPUTATION OF THE FEMALE MORTALITY RATES IN A MALTHUSIAN FEMALE POPULATION WITH A DISTRIBUTION BY AGE GROUPS ADJUSTED TO FIT THE FEMALE POPULATION OF BRAZIL, AS RECORDED IN THE CENSUSES OF 1900, 1940 AND 1950, AND WITH A RATE OF NATURAL VARIATION OF $r = 0.025$

Median age, a	Age group (years) a	Distribution by age groups C_a	$\frac{C_a + 5}{C_a}$	Multiplier (a) $k = e^{5r}$	$k \frac{C_a + 5}{C_a}$	Probability of death from one age to the next (per 1 000)	Survivors to age a	Deaths between age a and the next age	Survivors in each age group	Survivors to initial age of age group	Deaths between one age and the next	Death rate m_a (per 1 000)
0	Births (5b)	2 250	0.7840	1.0645 (a)	0.8346	165.4	10 000	1 654				
2.5 . . .	0-4	1 764	0.7749	1.1332	0.8781	121.9	8 346	1 017	41 730	10 000	2 660	63.7
7.5 . . .	5-9	1 367	0.8800	1.1332	0.9972	2.8	7 329	21	36 645	7 340	22	0.6
12.5 . . .	10-14	1 203	0.8703	1.1332	0.9862	13.8	7 308	101	36 540	7 318	60	1.7
17.5 . . .	15-19	1 047	0.8615	1.1332	0.9762	23.8	7 207	171	36 035	7 258	137	3.8
22.5 . . .	20-24	902	0.8603	1.1332	0.9748	25.2	7 036	177	35 180	7 121	173	4.9
27.5 . . .	25-29	776	0.8376	1.1332	0.9791	50.9	6 859	349	34 295	6 948	264	7.7
32.5 . . .	30-34	650	0.8369	1.1332	0.9483	51.7	6 510	336	32 550	6 684	342	10.5
37.5 . . .	35-39	544	0.8382	1.1332	0.9498	50.2	6 174	310	30 870	6 342	323	10.5
42.5 . . .	40-44	456	0.8289	1.1332	0.9393	60.7	5 864	356	29 320	6 019	333	11.3
47.5 . . .	45-49	378	0.7963	1.1332	0.9023	97.7	5 508	537	27 540	5 686	446	16.2
52.5 . . .	50-54	301	0.7409	1.1332	0.8396	160.4	4 971	798	24 855	5 240	668	26.9
57.5 . . .	55-59	223	0.6951	1.1332	0.7877	212.3	4 173	886	20 865	4 572	842	40.4
62.5 . . .	60-64	155	0.6903	1.1332	0.7822	217.8	3 287	718	16 435	3 730	802	48.8
67.5 . . .	65-69	107	0.5421	1.1332	0.6143	305.7	2 569	981	12 845	2 928	850	66.1
72.5 . . .	70-74	58	0.6724	1.1332	0.7619	238.1	1 588	385	7 940	2 078	682	86.0
77.5 . . .	75-79	39	0.5128	1.1332	0.5811	418.9	1 203	504	6 015	1 396	445	73.9
82.5 . . .	80-84	20	0.5000	1.1332	0.5666	433.4	699	303	3 495	951	404	115.5
87.5 . . .	85 +	10					396		1 980	547	547	276.5

(a) Except for the first line, where $k = e^{2.5r}$

We shall finally obtain:

$$r = \frac{b_0 d_{20\text{-}24} + \frac{1}{10}(C_{15\text{-}19} - C_{25\text{-}29})}{-C_{20\text{-}24} + d_{20\text{-}24}} \qquad \text{(IV.5)}$$

As in the previous examples, to each value of r corresponds some value of b_0 which is compatible with the age distribution. There will thus be an infinite number of populations satisfying the condition.

Finally, in discontinuous notation, formula IV.5, which gives the value of r by approximation, is not valid at the beginning and the end of life.

Fifth example: Malthusian population with given age distribution $C_0(a)$ and known probability of dying at a given age $q(a_0)$

In continuous notation we write:

$$q(a_0) + \frac{C'(a_0)}{C(a_0)} = -r$$

which brings us back to the first example.

In discontinuous notation, the formulae which we have written for the fourth example make it easy for us to calculate such quantities as:

$$\frac{C'(22.5)}{C(22.5)} = \frac{C_{15\text{-}19} - C_{25\text{-}29}}{10C_{20\text{-}24}}$$

Furthermore, if $m_{20\text{-}24}$ is the death rate between the ages of 20 and 24, we have, approximately, $q(22.5) = m_{20\text{-}24}$, whence we have the formula;

$$m_{20\text{-}24} + \frac{C_{15\text{-}19} - C_{25\text{-}29}}{10C_{20\text{-}24}} + r = 0$$

Sixth example: Malthusian population with given age distribution $C_0(a)$ and known mean death rate for five-year groups between the ages of 5 and 34

In this example, the quantity which we assume to be known in addition to the age distribution is written:

$$A = \frac{(m_{5\text{-}9} + m_{10\text{-}14} + m_{15\text{-}19} + m_{20\text{-}24} + m_{25\text{-}29} + m_{30\text{-}34})}{6}$$

In accordance with the formula in the fifth example, we can write:

$$A = -r - \frac{1}{60}\left[\frac{C_{0\text{-}4} - C_{10\text{-}14}}{C_{5\text{-}9}} + \frac{C_{5\text{-}9} - C_{15\text{-}19}}{C_{10\text{-}14}} + \dots + \frac{C_{25\text{-}29} - C_{35\text{-}39}}{C_{30\text{-}34}}\right],$$

This formula enables us to calculate r, thus bringing us back to the first example.

Introduction of statistical variables

Although it would be easy to think of many other examples, we shall confine ourselves to the six described above. All these examples lead us to an r equation the solution of which gives us an *exact and determinate* value of r or, at most, a small number of exact and determinate values. If we are satisfied with approximate values, then we must resolve equations whose solutions are "statistical variables" diverging to a greater or lesser degree from the mean values, and new problems then arise. We have already encountered problems of this kind when dealing with the sub-sets $H(r)$, and we have already studied the compatibility between:

(*a*) An age distribution of the population and a given death rate;

(*b*) An age distribution of deaths and a given death rate.

In the study of the sub-set $F_0(r)$ a third problem is encountered: that of the *compatibility of a given age distribution of deaths and a given age distribution of the population.*

In a sub-set $F_0(r)$ we cannot arbitrarily assume knowledge of the entire age distribution of deaths. As was seen in the fourth example above, knowledge of this age distribution of deaths at a single age $d(a_0)$ was sufficient to determine a population from $F_0(r)$ and was therefore sufficient to define $d(a)$ at all ages.

Let us now consider an actual population in which $C_0(a)$ and $d_0(a)$ are the observed age distributions of the population and the distribution of the observed deaths. Let us take the sub-set $F_0(r)$ corresponding to $C_0(a)$ and pose the following question: is the age distribution $d_0(a)$ consistent with the sub-set $F_0(r)$? If we want $d_0(a)$ to coincide *exactly* with the age distribution of deaths of a population from $F_0(r)$, then obviously the answer will in most cases be "no", for it is unlikely that we should find in $F_0(r)$ an age distribution of deaths which coincides exactly with the observed age distribution $d_0(a)$. If we are satisfied with an approximate coincidence, however, the question takes on a different aspect.

If there is exact coincidence, we can write in accordance with the formulae in table II.2 (see chapter II):

$$-\frac{C'(a)}{C(a)} - r = \frac{d(a)}{C(a)}(b_0 - r)$$

which is written:

$$C'(a) + rC(a) + dd(a) = 0$$

If we put:

$$\frac{C'(a)}{d(a)} = y \qquad \text{and} \qquad \frac{C(a)}{d(a)} = x$$

we therefore have:

$$y + rx + d = 0 \qquad \text{(IV.6)}$$

For each value of a we have a pair of values (x, y), and if we plot a graph with x on the horizontal axis and y on the vertical axis, the points obtained will be on a straight line defined by the equation IV.6.

If there is no exact coincidence, we shall obtain a cluster of points to which a straight line can be fitted and whose equation enables us to compute r_0. The population from $F_0(r)$ corresponding to r_0, which is the population sought, is one whose age distribution of deaths *coincides approximately* with the age distribution of deaths $d_0(a)$ actually observed.

In discontinuous notation, the formulae giving x and y are easily written. In accordance with the previous example, we have:

$$5C'(22.5) = \frac{1}{10}(C_{15\text{-}19} - C_{25\text{-}29})$$

$$5\,C(22.5) = C_{20\text{-}24}$$

and $$5d(22.5) = d_{20\text{-}24}$$

We therefore have:

$$y = \frac{C_{15\text{-}19} - C_{25-29}}{10d_{20\text{-}24}}\,y$$

$$x = \frac{C_{20\text{-}24}}{d_{20\text{-}24}}\,x$$

Tables IV.11 and IV.12 give an example of the application of this formula, using for C_a the age distribution adjusted to fit the results of the population censuses carried out in Mexico in 1930, 1940 and 1950 and for d_a the age distribution of deaths actually observed in Mexico in 1950.

Table IV.10 shows that the age distribution of the female population of Mexico varied only slightly in the three censuses mentioned. Graph IV.1 shows how the adjustment was made. It was drawn by hand so as to eliminate the fluctuations in age distribution observed in the three censuses. As a result, the fluctuations caused by variations in the birth rate during the civil war of 1911-1921 and the period which followed it were eliminated.

In the case of the female deaths observed in 1950, we have taken the crude figures without making any adjustment. This does not mean that we considered the crude figures to be more accurate than the population figures given by the censuses; on the contrary, the age distribution

TABLE IV.10. AGE DISTRIBUTION OF THE FEMALE POPULATION OF MEXICO, AS RECORDED IN THE 1930, 1940 AND 1950 CENSUSES, AND AGE DISTRIBUTION ADJUSTED ON THE BASIS OF THE RESULTS OF THE THREE CENSUSES

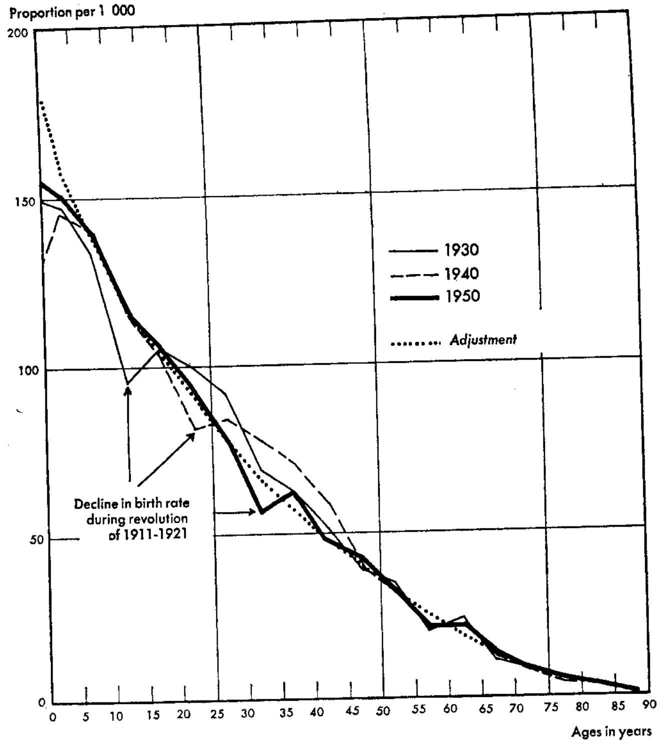
Age group (years)	*Year of census* 1930	1940	1950	*Adjusted distribution*
Under 1 year	29 772	26 278	30 830	35 796
1-4	117 442	115 992	119 929	125 921
5-9	133 338	139 361	138 465	135 521
10-14	95 335 (a)	116 107	115 558	117 193
15-19	105 799 (b)	103 136	105 875	103 624
20-24	99 863	81 141 (a)	94 320	90 714
25-29	91 695	84 317 (b)	79 433	77 745
30-34	68 903	68 743	56 082 (a)	65 707
35-39	62 710	70 407	61 097 (b)	55 900
40-44	50 661	48 971	47 634	46 621
45-49	38 067	39 699	41 235	40 358
50-54	34 326	31 818	32 359	31 672
55-59	19 261	22 054	20 411	24 027
60-64	23 547	21 571	22 106	18 633
65-69	10 175	11 583	12 967	12 691
70-74	8 866	8 446	9 735	9 528
75-79	4 014	4 500	5 049	4 942
80-84	3 831	3 363	3 941	2 603
85-89	1 139	2 510	2 974	804
90-94	734			
95-99	336			
100 and over .	187			
ALL AGES . .	1 000 000	1 000 000	1 000 000	1 000 000

(a) Decline in birth rate during revolution of 1911-1921.
(b) Resurgence of birth rate after the civil war.

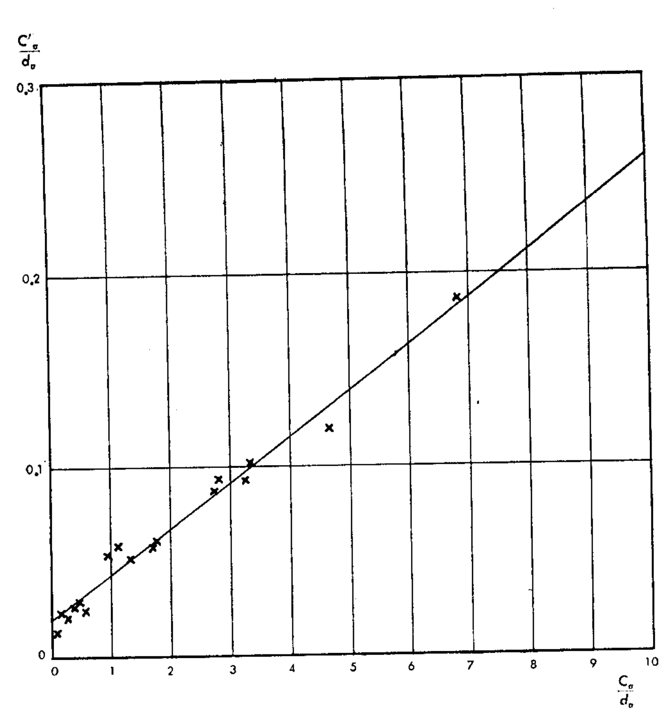
TABLE IV.11. COMPUTATION OF THE DERIVATIVE $C'(a)$ CORRESPONDING TO THE AGE DISTRIBUTION OF THE FEMALE POPULATION OF MEXICO, AS ADJUSTED TO FIT THE RESULTS OF THE CENSUSES OF 1930, 1940 AND 1950 (SEE TABLE IV.10)

Median age a	*Age group (years)* a	*Distribution by age groups* (a) C_a	*Distribution by years of age for median years* $C(a) = C_a/5$	*Successive differences* $C(a) - C(a+5)$	*Average of two successive differences* $C'(a)$
0.5	0	35 796	35 796 (b)	1 726.4 (d)	
3.0	1-4	125 921	31 480 (c)	972.6 (e)	1 349.5 (f)
7.5	5-9	135 521	27 104	733.0	852.8 (f)
12.5	10-14	117 193	23 439	542.8	637.9
17.5	15-19	103 624	20 725	516.4	529.6
22.5	20-24	90 714	18 143	518.8	517.6
27.5	25-29	77 745	15 549	481.6	500.2
32.5	30-34	65 707	13 141	392.2	436.9
37.5	35-39	55 900	11 180	371.2	381.7
42.5	40-44	46 621	9 324	250.4	310.8
47.5	45-49	40 358	8 072	347.6	299.0
52.5	50-54	31 672	6 334	305.8	326.7
57.5	55-59	24 027	4 805	215.6	260.7
62.5	60-64	18 633	3 727	237.8	226.7
67.5	65-69	12 691	2 538	126.4	182.1
72.5	70-74	9 528	1 906	183.6	155.0
77.5	75-79	4 942	988	93.4	138.5
82.5	80-84	2 603	521	72.0	82.7 (f)
87.5	85 and over	804	161		
	ALL AGES	1 000 000			

(a) Adjusted age distribution of table IV.10.
(b) $C(0.5) = C_0$.
(c) $C(3.0) = (1/4)C_{1-4}$.
(d) $C(0.5) - C(3.0)/2.5$.
(e) $C(3.0) - C(7.5)/4.5$.
(f) Approximate figures.



Graph IV.1. Distribution by five-year age groups of the female population of Mexico, as recorded in the 1930, 1940 and 1950 censuses— and an adjusted distribution on the basis of the results of those three censuses



Graph IV.2. Compatibility of the age distribution of the female population of Mexico, as adjusted to fit the age distributions recorded in the 1930, 1940 and 1950 censuses—with the age distribution of female deaths recorded in Mexico in 1950

TABLE IV.12. COMPATIBILITY OF THE AGE DISTRIBUTION OF A POPULATION AND ITS AGE DISTRIBUTION OF DEATHS

Computation of the quantities x *and* y *of equation IV.6 in the text, using an age distribution adjusted to that of the female population of Mexico, as recorded in the censuses of 1930, 1940 and 1950 (see table IV.10), and the age distribution of female deaths recorded in Mexico in 1950*

Median age a	*Age group (years)* a	C(a) *Figures from column 4 of table IV.11*	C'(a) *Figures from the last column of table IV.11*	*Distribution by age group of female deaths* ([a]) d_a	*Distribution by years of age of female deaths, for the median age* $d(a) = d_a/5$	$\frac{C(a)}{d(a)} = x$	$\frac{C'(a)}{d(a)} = y$
0.5	0	35 796		256 964	256 964 ([b])		
3.0	1-4	31 480	1 349.5 ([c])	221 823	55 431 ([d])	0.568	0.0243 ([c])
7.5	5-9	27 104	852.8 ([c])	40 466	8 093	3.349	0.1053 ([c])
12.5	10-14	23 439	637.9	17 141	3 428	6.837	0.1861
17.5	15-19	20 725	529.6	22 245	4 449	4.658	0.1195
22.5	20-24	18 143	517.6	27 877	5 575	3.254	0.0928
27.5	25-29	15 549	500.2	28 168	5 634	2.760	0.0888
32.5	30-34	13 141	436.9	23 396	4 679	2.809	0.0934
37.5	35-39	11 180	381.7	31 107	6 221	1.797	0.0614
42.5	40-44	9 324	310.8	26 139	5 228	1.784	0.0595
47.5	45-49	8 072	299.0	28 861	5 772	1.330	0.0518
52.5	50-54	6 334	326.7	27 922	5 584	1.134	0.0585
57.5	55-59	4 805	260.7	24 320	4 864	0.988	0.0537
62.5	60-64	3 727	226.7	38 411	7 682	0.485	0.0295
67.5	65-69	2 538	182.1	35 025	7 005	0.362	0.0260
72.5	70-74	1 906	155.0	38 391	7 678	0.248	0.0202
77.5	75-79	988	138.5	29 329	5 866	0.168	0.0236
82.5	80-84	521	82.7 ([c])	31 524	6 305	0.083	0.0131 ([c])
87.5	85 and over	161		50 890	10 178		
	ALL AGES			1 000 000			

SOURCE: United Nations *Demographic Yearbooks.*

([a]) Female deaths recorded in Mexico in 1950.

([b]) $d(0.5) = d_0$.

([c]) Approximate figures.

([d]) $d(3.0) = (1/4)d_{1-4}$.

of deaths is no doubt less accurate than the age distribution of the population—but the errors involved are such as to be very difficult to correct and, since any adjustment would be arbitrary, it was decided to keep the crude figures as they were.

Graph IV.2 was drawn by using the values x and y of table IV.12. As may be seen, to the points obtained can easily be fitted a straight line with the equation:

$$y + rx + d = 0$$

The ordinate-intercept of this straight line is equal to d. Graph IV.2 gives: $d = 0.020$.

The slope of the straight line is equal to r and we have: $r = 0.024$, whence we obtain the crude birth rate $b = d + r = 0.044$.

Thus, the sub-set $F_0(r)$ of Malthusian populations associated with the adjusted age distribution of the female population of Mexico, as recorded in the 1930, 1940 and 1950 censuses and having the vital rates:

$$\begin{cases} r = 0.024 \\ d = 0.020 \\ b = 0.044 \end{cases}$$

has an age distribution of female deaths which coincides *approximately* with the age distribution of female deaths observed in Mexico in 1950.

We shall now leave the sub-sets $F(r)$ of Malthusian populations and turn to an examination of the "processes of demographic evolution where the age distribution is invariable".

Chapter V

PROCESSES OF DEMOGRAPHIC EVOLUTION WHERE THE AGE DISTRIBUTION IS INVARIABLE: SEMI-MALTHUSIAN POPULATIONS

A. Introduction

The populations considered in chapters II and IV were Malthusian populations, i.e., populations with constant mortality and age distributions.

In chapter II it was further assumed that mortality was known, and this additional condition defined what we termed the sub-sets H(r). The age distribution remained invariable but was not assumed to be known.

In chapter IV it was assumed that the age distribution was known, and this additional condition defined what we termed the sub-sets F(r). The mortality in these sub-sets remained invariable but was not assumed to be known.

In each of those sub-sets, knowledge of an additional condition generally determined a particular Malthusian population (or a small number of particular Malthusian populations). Ultimately, each particular population was determined by three conditions which were, for the two types of sub-sets, as follows:

SUB-SETS H(r)

(a) Mortality invariable and known;

(b) Age distribution invariable but not known;

(c) Knowledge of one other characteristic of the population, e.g.:

(i) Rate of natural variation;

(ii) Crude birth rate;

(iii) Crude death rate;

(iv) Age distribution at a given age;

(v) Age-specific fertility ("stable" Malthusian population).

SUB-SETS F(r)

(a) Mortality invariable but not known;

(b) Age distribution invariable and known;

(c) Knowledge of one other characteristic of the population, e.g.:

(i) Rate of natural variation;

(ii) Crude death rate;

(iii) Age distribution of deaths at a given age;

(iv) Survivorship function at a given age.

In chapter III we defined some processes of demographic evolution where the condition that the *age distribution was invariable but not known* was eliminated from the conditions of sub-set H(r).

We first considered the possibility of the existence of such processes and then went on to study their properties.

We concluded that in such processes the population generally developed towards the Malthusian population (or one of the Malthusian populations) defined by restoring the condition of invariability of the age distribution.

In the case of the sub-sets F(r) we may pose the same question as in the case of the sub-sets H(r), and we can study the processes of demographic evolution defined by eliminating from the conditions of the sub-set F(r) the condition that *the mortality is invariable but not known.*

This is the subject of the present chapter.

B. Processes where the age structure is invariable

DEFINITION OF SEMI-MALTHUSIAN POPULATIONS

Let us consider a population at time t, defined by N(t) as the total population, $r(t)$ the rate of natural variation, $p(a, t)$ the survivorship function, and C(a) the age distribution assumed to be independent of time.

Let us now see what happens to this population at time $t + dt$. The number of persons aged a at time t is equal to N(t)C(a). At time $t + dt$, the survivors are aged $a + dt$ and their number is equal to:

$$N(t + dt)C(a + dt) = N(t)C(a + dt)[1 + r(t)dt]$$

We can also estimate the number of survivors by applying the survivorship function. The number of survivors is:

$$N(t)C(a)\frac{p(a + dt, t)}{p(a, t)}$$

The two expressions obtained for the number of persons surviving to time $t + dt$, give:

$$N(t)C(a + dt)[1 + r(t)dt] = N(t)C(a)\frac{p(a + dt, t)}{p(a, t)}$$

which is written:

$$\frac{C(a + dt)}{C(a)}[1 + r(t)dt] = \frac{p(a + dt, t)}{p(a, t)}$$

or:

$$\left[1 + \frac{C'(a)}{C(a)}dt\right][1 + r(t)dt] = 1 + \frac{p'_a(a, t)}{p(a, t)}dt$$

and if we disregard the second-order terms, we obtain:

$$1 + \frac{C'(a)}{C(a)}dt + r(t)dt = 1 + \frac{p'_a(a, t)}{p(a, t)}dt$$

and finally,

$$\frac{C'(a)}{C(a)} = \frac{p'_a(a,t)}{p(a,t)} - r(t)$$

By integration, we obtain:

$$C(a) = b_0 e^{-r(t)a} p(a,t) \qquad (V.1)$$

where b_0 is a constant equal to C(0), the crude birth rate. Formula V.1 is the fundamental formula for Malthusian populations. We can therefore state the following result:

In a process of demographic evolution where the age structure of the population is constant, there are, at all times, the same relations between the demographic characteristics of the population as in a Malthusian population.

We shall term populations with a constant age distribution "*semi-Malthusian populations*".

Another definition of semi-Malthusian populations

Let us assume that, from a given time onwards, the mortality and fertility of a population remain invariable at the level they have reached at that moment. It was seen in chapter III that this led to a process of population evolution in which the population approached the stable Malthusian population corresponding to the mortality and fertility of the moment. Generally, the limit age distribution is different from the age distribution at the time when the process began. It should be noted that we say "generally", because, as has been seen, in a population which is assumed to maintain an invariable age distribution, the age distribution of the limit stable population coincides exactly at all times with the current age distribution.

In other words, in order for the current age distribution to coincide at all times with the age distribution of the limit stable population, it is sufficient that the population should have an invariable age structure.

It is easy to show that this condition is *not only sufficient, but also necessary*. The fact that at time t the age distribution $C_f(a, t)$ coincides with the age distribution according to the laws of $p_f(a, t)$ and $\varphi_f(a, t)$ means that, if we assume that from time t onwards the mortality and fertility functions retain the values they had reached at time t, the population will approach a stable state whose age distribution will indeed be $C_f(a, t)$. Thus we hall have:

$$p(a,t) = \frac{C(a,t)}{C(0,t)} e^{r(t)a}$$

In such a life table we have:

$$\frac{p(a+da,t)}{p(a,t)} = \frac{C(a+da,t)}{C(a,t)} e^{r(t)da}$$

At time $t + dt$ the survivors of the persons aged a at time t will be aged $a + dt$ and will number:

$$N(t)C(a,t)\frac{p(a+dt,t)}{p(a,t)} =$$

$$= N(t)C(a,t)\frac{C(a+dt,t)}{C(a,t)} e^{r(t)dt} = N(t)C(a+dt,t)e^{r(t)dt}$$

However, they will also number:

$$N(t)e^{r(t)dt}C(a+dt, t+dt)$$

whence we obtain by equating the two expressions:

$$C(a+dt,t) = C(a+dt, t+dt)$$

In other words, at time $t + dt$ the age distribution is the same as at time t. The age distribution is therefore invariable. *Thus, semi-Malthusian populations are identical with populations for which the current age structure coincides with the stable age structure.*

Relationship between semi-Malthusian populations and quasi-stable populations

In a previous study,[1] quasi-stable populations were defined as populations with constant fertility where the mortality varied within the universe of the intermediate series of model life tables. It was noted that such populations had current age structures which almost coincided with their stable structures. *Quasi-stable populations thus offer an example of the near-achievement of semi-stable populations. We shall revert to this point in chapters VII and VIII.*

Fertility in semi-Malthusian populations

The fertility function must verify the relation:

$$\int_u^v C(a)\varphi(a,t)da = C(0) = b_0 \qquad (V.2)$$

There is no strictly mathematical reason why $\varphi(a, t)$ should not vary over time. The variations must be such, however, that formula (V.2) is at all times verified. This means that the variations in the fertility function at a certain age must be exactly compensated by the variations at other ages. This condition is of purely theoretical significance, however, since such compensation never takes place in reality. In an actual development with constant age distribution,[2] formula (V.2) involves invariability of the fertility function over time.

C. Semi-Malthusian populations satisfying a given condition

The fundamental property of semi-Malthusian populations makes it very simple to study the properties of processes of demographic evolution with constant age distribution associated with another condition. We shall now revert, one by one, to the examples given in chapter IV.

First example

We consider the two conditions:

(i) Constant age distribution;

(ii) Constant rate of natural variation.

In such conditions, r is no longer dependent on time and formula (V.1) shows that the survivorship function is also no longer dependent on time. In such a development, the population is Malthusian from the very beginning.

We have, however, the formula:

$$q(a) = -r - \frac{C'(a)}{C(a)}$$

[1] *The Future Growth of World Population* (United Nations publication, Sales No.: 58.XIII.2).

[2] In fact, the age distribution is almost invariable in actual developments.

and we must therefore have:

$$-r - \frac{C'(a)}{C(a)} > 0$$

or, alternatively:

$$r < -\frac{C'(a)}{C(a)} \qquad (V.3)$$

We cannot therefore select at the outset an arbitrary value for the rate of natural variation which is assumed to be constant. Formula (V.3) must be complied with.

Second example

Here we consider the two conditions:

(i) Constant age distribution;

(ii) Survivorship function constant at a given age a_0. For this age a_0, formula (V.1) is written:

$$C(a_0) = b_0 e^{-r(t)a_0} p(a_0) \qquad (V.4)$$

which shows that the rate of natural increase is not independent of time. This therefore brings us back to the first example. The value of r taken from formula (V.4) must of course satisfy (V.3).

Third example

Here we consider the two conditions:

(i) Constant age distribution;

(ii) Constant crude death rate.

Assuming the crude death rate as given is tantamount to assuming the rate of natural increase as given, since the crude birth rate $b = C(0)$ is already known. We therefore have $r = C(0) - d$, and if we know r this brings us back to the first example. The value of r thus determined must also satisfy formula (V.3).

Fourth example

Here we consider the two conditions:

(i) Constant age distribution;

(ii) Constant age distribution of deaths for a given age a_0.

For age a_0 we can write:

$$r(t) = \frac{b_0 d(a_0) + C'(a_0)}{-C(a_0) + d(a_0)} \quad \text{(formula II.12)}$$

which shows that $r(t)$ is not time-dependent, and thus we are once more brought back to the first example. As in the previous cases, r must, of course, satisfy formula (V.3).

Examples using actual age structure

In the numerical applications in previous chapters we have sometimes used actually observed age distributions.

These examples take on a new aspect when interpreted in the light of the properties of semi-Malthusian populations.

Let us revert to sub-set F(r) corresponding to the age distribution adjusted to fit the female population of Brazil, as recorded in the censuses of 1900, 1940 and 1950. It may be recalled that we proposed, in chapter IV, to determine the population of this sub-set corresponding to a given value r_0 of the rate of natural variation. We showed that the survivorship curve of such a population is given by the formula:

$$p(a) = \frac{C(a)}{C(0)} e^{r_0 a}$$

where r_0 must satisfy the condition:

$$r_0 < -\frac{C'(a)}{C(a)}$$

By varying r_0, we obtain all the survivorship functions of the sub-set F(r). It must be made clear that, in doing so we do not pose the question whether or not the population of Brazil formed part of the sub-set F(r). We considered the totality of the Malthusian populations having the same age distribution as the age distribution of the population of Brazil adjusted to fit the three censuses of 1900, 1940 and 1950. This is what we termed the sub-set F(r) corresponding to that age distribution. Before the population of Brazil could be included among these Malthusian populations, it would have been necessary to establish that it was a Malthusian population. We did not pose this question, and consequently it would have been imprudent to conclude that the mortality in Brazil was among the mortalities of the sub-set F(r) obtained by varying r_0 within the limits indicated above. Indeed, such a conclusion was not given in chapter IV.

We now know, however, that it would have been sufficient to establish that the population of Brazil was "semi-Malthusian" in order to be in a position to give such a conclusion. According to table IV.7, there was little variation in the age distribution of the female population of Brazil between 1900 and 1950. Over that period, therefore, the Brazilian population was semi-Malthusian. In the circumstances, then, the mortalities in the sub-set F(r) did in fact define the universe of possible variations in the mortality in Brazil between 1900 and 1960.

Determination of the universe to which the mortality belongs does not, of course, fully determine the mortality itself, since we have a choice between all the mortalities belonging to that universe. Often, however, we have at our disposal additional information which enables us to make the choice. In the present case, for example, we can assume that the mean annual rate of variation calculated from the census results of 1940 and 1950 gives an estimate of the annual rate of natural variation in the middle of the period, i.e., at the beginning of 1945. We thus find a rate $r = 0.0239$. The survivorship function in Brazil in 1945 is then determined by the formula:

$$p_f(a) = \frac{C_f(a)}{C_f(0)} e^{0{,}0239a}$$

The same reasoning holds good in the case of Mexico. Table IV.10 shows that from 1940 to 1950 the population of Mexico was semi-Malthusian. From 1940 to 1950, the mean rate of annual variation was equal to $r = 0.0269$. The survivorship function in Mexico in 1945 was thus equal to:

$$p_f(a) = \frac{C_f(a)}{C_f(0)} e^{0{,}0269a}$$

It must be borne in mind, in these two examples, that all the difficulties of computation resulting from the fact that the age distributions are known in discontinuous terms continue to exist.

Let us now turn to the example in chapter IV, where we seek to determine a Malthusian population *almost compatible* with the age distribution of the population of Mexico and the age distribution of deaths recorded in 1950.

In this case also, the result obtained was as follows: there is a Malthusian population whose age distribution is almost identical with that of the female population of Mexico adjusted to fit the 1930, 1940 and 1950 census results and which has an age distribution of deaths almost identical with the age distribution of deaths recorded in Mexico in 1950. This Malthusian population has a rate of natural variation $r = 0.024$ and a crude death rate $d = 0.020$.

We had, however, no grounds for concluding that those were the values for the rate of natural variation and the crude death rate in Mexico in 1950. In order that such a conclusion be true, we should show that the population of Mexico is Malthusian. We now know that it is enough to show that it is semi-Malthusian, and this is indeed the case, according to table IV.10 in chapter IV.

Since most developing countries have semi-Malthusian populations, the potential usefulness of the methods studied in the foregoing chapters is immediately apparent.

D. Reconsideration of the definition of Malthusian populations

It will be recalled that, at the beginning of this work, a Malthusian population was defined as a population in which the age distribution of the population and the mortality remained constant. It was deduced from this that many other demographic characteristics must also remain constant in such a population, particularly the age distribution of deaths. We were thus led to consider three functions of age:

The age distribution of the population $C(a)$;

The survivorship function $p(a)$;

The age distribution of deaths $d(a)$.

If we combine these three functions two by two, we obtain the following three pairs:

(1) $p(a)$, $C(a)$

(2) $p(a)$, $d(a)$

(3) $C(a)$, $d(a)$

The definition given for a Malthusian population corresponds to constancy of the first pair. Let us now consider what corresponds to constancy of the other pairs.

If $p(a)$ and $d(a)$ are invariable, we can write at time t:

$$C(a, t) = \frac{\frac{d(a)}{q(a)}}{\int_0^{\omega} \frac{d(a)}{q(a)} da} \tag{II.7}$$

This is valid for all populations. This formula shows that, if $p(a)$ and $d(a)$ are invariable, $C(a)$ is invariable too. The population is therefore Malthusian.

If $C(a)$ and $d(a)$ are invariable, we are dealing with a semi-Malthusian population, and we have seen that there exist at all times between the demographic characteristics of such a population the same relationships as in a Malthusian population. We can therefore write:

$$r = \frac{C(0)d(a) + C'(a)}{- C(a) + d(a)}$$

This means that r is also invariable, and as:

$$p(a) = \frac{C(a)}{C(0)} e^{ra}$$

the survivorship function is also invariable. The population is therefore not merely semi-Malthusian, but Malthusian.

To sum up, if in a given population two of the three functions $p(a)$, $C(a)$ and $d(a)$ are invariable, that population is Malthusian. We can therefore broaden the definition of a Malthusian population given at the beginning of chapter I by stating that:

A Malthusian population is a population in which two of the three functions $p(a)$, $C(a)$ and $d(a)$ are invariable.

Let it be quite clear that only the *invariability* of the functions is required. *It is not assumed that the functions are known.*

If we assume that $p(a)$ is known, we can define particular sub-sets $H(r)$; similarly, if we assume that $C(a)$ is known, we can define a series of other particular sub-sets $F(r)$. Thus, there emerges the possibility of defining a third series of particular sub-sets if we assume $d(a)$ to be known. We shall term this third series $G(r)$, and it will be studied in chapter VI below. One last remark: the three series $H(r)$ and $G(r)$ are not really distinct from each other. When the functions $p(a)$, $C(a)$ and $d(a)$ vary as well as r, we obtain all possible Malthusian populations. The three series differ only in the classification of all these possible Malthusian populations.

Chapter VI

MALTHUSIAN POPULATIONS WITH KNOWN AGE DISTRIBUTION OF DEATHS: THE SUB-SETS G(r) AND PROCESSES OF DEMOGRAPHIC EVOLUTION WITH CONSTANT AGE DISTRIBUTION OF DEATHS

In this chapter, we propose to study the sub-sets G(r) described above. We assume that the age distribution of deaths, $d_0(a)$ is constant and given, while the mortality function and the age distribution of the population are constant but are not given. All Malthusian populations corresponding to these conditions constitute the sub-sets $G_0(r)$ linked to the age distribution of deaths $d_0(a)$. By varying $d_0(a)$ we obtain the series of sub-sets G(r).

The basic relations used for studying the sub-sets G(r) are the same as those used for the study of the preceding sub-sets H(r) and F(r). We shall present them once again below, in connexion with the determination in a given set $G_0(r)$ of a population satisfying a given condition.

A. Population of a sub-set $G_0(r)$ satisfying certain conditions

FIRST EXAMPLE: THE RATE OF NATURAL VARIATION r IS GIVEN

The survivorship function is written:

$$p(a) = 1 - \frac{\int_0^a d(a)e^{ra}da}{\int_0^\omega d(a)e^{ra}da} \qquad \text{(II.10)}$$

(This is formula II.10 in chapter II.)

For any given value of r the quantity:

$$\frac{\int_0^a d(a)e^{ra}da}{\int_0^\omega d(a)e^{ra}da}$$

increases from 0 to 1 when a increases from 0 to ω.

Consequently, formula (II.10) is valid for all values of r.

Table VI.1 gives details of the calculation for applying this formula in the following case: the function $d(a)$ is the age distribution of deaths calculated in chapter II (table II.3). It corresponds to a Malthusian population with a rate of natural variation of $r = 0.03$, whose mortality is that of the intermediate model life table with an expectation of life at birth for both sexes of fifty years. Here we disregard the way in which this age distribution of deaths was calculated, taking it as an established fact, and we proceed to consider the sub-set $G_0(r)$ linked to this age distribution.

We propose to determine the population of this set $G_0(r)$, which has a rate of natural increase of $r = 0.03$. By applying formula (II.10), we should again find the function $p(a)$ which was used in the calculation of $d(a)$, and this is in fact what table VI.1 shows.

Once we know the survivorship function $p(a)$, it is easy to calculate all the other characteristics of the population.

The fact that formula II.10 is written in continuous notation and that we have the age distribution of deaths only in discontinuous terms causes no difficulty here, as it did in the case of the study of the sub-sets F(r). It was simply assumed, for the purpose of calculating the integrals of formula II-10, that for the 20-24 age group, for example, we had:

$$\int_{20}^{25} d(a)e^{ra}da \# e^{22.5r}d_{20\text{-}24}$$

Tables VI.2 and VI.3 give computations similar to those in table VI.1 for the other two values of the rate of natural increase: $r = 0.015$ and $r = 0$.

Table VI.5 gives the age distributions of the populations.

We shall see in a moment how these computations can be used for the three different rates of variation.

SECOND EXAMPLE: THE CRUDE BIRTH RATE b_0 IS GIVEN

As was seen above, for each value of r there is a survivorship function $p(a)$ and, consequently, a crude birth rate:

$$b_0 = \frac{1}{\int_0^\omega e^{-ra}p(a)da}$$

We can therefore regard the crude birth rate b as a function of r. The three computations in tables VI.1, VI.2 and VI.3 give the values of this function for $r = 0.03$, $r = 0.015$ and $r = 0$. If we prepare a graph with r on the horizontal axis and b on the vertical axis, we have three points on the curve representing $b(r)$. Graph VI.1 was drawn in this way. We see that, when r varies from $-\infty$ to $+\infty$, b passes through a minimum b_m.

If $b_0 > b_m$, the straight line of the ordinate b_0 intersects the curve G(r) at two points M_1 and M_2, having as their abscissae r_1 and r_2. There are therefore two Malthusian populations corresponding to the rates of natural variation r_1 and r_2. Thus, for the purpose of calculating the characteristics of these populations, we are brought back to the first example, since we know r_1 and r_2.

If $b_0 < b_m$, there is no solution to the problem, and if b_0 equals b_m there is one double solution.

Table VI.1. Computation of the female survivorship function which, when associated with a rate of natural variation of 0.03, leads to a female Malthusian population having a distribution of deaths by age groups identical with the distribution d_a shown in the third column of the table

Median age a	Age group (years) a	Distribution of deaths by age groups d_a	e^{-ra} for $r = 0.03$	Quotient of the two preceding columns $e^{ra}d_a$	Cumulative totals of age groups $\int_0^a d(a)e^{ra}da$	Distribution by age groups of the cumulative totals	Survivors at the beginning of each age group	Survivors in each age group	Initial survivorship function
0.5	Under 1	366 651	0.98511	372 193	372 193	12 372	100 000	90 721	100 000
3.0	1-4	150 864	0.91393	165 072	537 265	17 859	87 628	339 008	87 625
7.5	5-9	38 967	0.79852	48 799	586 064	19 481	82 150	406 672	82 136
12.5	10-14	24 421	0.68729	35 532	621 596	20 663	80 519	399 640	80 515
17.5	15-19	30 560	0.59156	51 660	673 256	22 380	79 331	392 392	79 333
22.5	20-24	34 964	0.50916	68 670	741 926	24 663	77 620	382 392	77 615
27.5	25-29	31 494	0.43824	71 865	813 791	27 051	75 337	370 715	75 332
32.5	30-34	27 691	0.37719	73 414	887 205	29 492	72 949	358 642	72 940
37.5	35-39	24 621	0.32465	75 839	963 044	32 013	70 508	346 237	70 500
42.5	40-44	22 820	0.27943	81 666	1 044 710	34 727	67 987	333 150	67 981
47.5	45-49	23 153	0.24051	96 266	1 140 976	37 927	65 273	318 365	65 266
52.5	50-54	24 755	0.20701	119 584	1 260 560	41 903	62 073	300 425	62 064
57.5	55-59	26 890	0.17817	150 923	1 411 483	46 919	58 097	277 942	58 093
62.5	60-64	30 760	0.15336	200 574	1 612 057	53 587	53 080	248 732	53 076
67.5	65-69	34 430	0.13199	260 853	1 872 910	62 258	46 413	210 387	46 413
72.5 . . .	70-74	36 231	0.11361	318 907	2 191 817	72 859	37 742	162 207	37 747
77.5	75-79	32 495	0.9778	332 328	2 524 145	83 906	27 141	108 087	27 141
82.5	80-84	22 685	0.8416	269 546	2 793 691	72 866	16 094	58 070	16 086
87.5	85 +	15 547	0.7244	214 619	3 008 310	100 000	7 134	27 480	7 123

Table VI.2. Computation of the female mortality functions which, when associated with a rate of natural variation of 0.015, leads to a female Malthusian population having a distribution of deaths by age groups identical with the distribution d_a shown in the third column of the table

Median age a	Age group (years) a	Distribution of deaths by age groups d_a	$e^{-ra}da$ for $r = 0.015$	Quotient of the two preceding columns $e^{-ra}d_a$	Cumulative totals of age groups	Distribution by age groups of the cumulative totals	Survivors at the beginning of each age group	Deaths from one age to the next	Survivors in each age group L_a	Death rate (per 1 000) m_a
0.5 . .	Under 1	366 651	1.00750	369 401	369 401	23 769	100 000	23 769	82 713	287.37
3.0 . .	1-4	150 864	1.04600	157 804	527 205	33 922	76 231	10 153	283 603	35.80
7.5 . .	5-9	38 967	1.11917	43 611	570 816	36 728	66 078	2 806	323 375	8.68
12.5 . .	10-14	24 421	1.20623	29 457	600 273	38 624	63 272	1 896	311 620	6.08
17.5 . .	15-19	30 560	1.30128	39 767	640 040	41 182	61 376	2 558	300 485	8.51
22.5 . .	20-24	34 964	1.40144	49 000	689 040	44 335	58 818	3 153	286 208	11.02
27.5 . .	25-29	31 494	1.51059	47 575	736 615	47 396	55 665	3 153	270 673	11.31
32.5 . .	30-34	27 691	1.62829	45 089	781 704	50 298	52 604	2 902	255 765	11.35
37.5 . .	35-39	24 621	1.75515	43 214	824 918	53 078	49 702	2 780	241 560	11.51
42.5 . .	40-44	22 820	1.89174	43 170	868 088	55 856	46 922	2 778	227 665	12.20
47.5 . .	45-49	23 153	2.03918	47 213	915 301	58 894	44 144	3 038	213 125	14.25
52.5 . .	50-54	24 755	2.19899	54 436	969 737	62 396	41 106	3 502	196 775	17.80
57.5 . .	55-59	26 890	2.36918	63 707	1 033 444	66 495	37 604	4 099	177 773	23.06
62.5 . .	60-64	30 760	2.55369	78 552	1 111 996	71 550	33 505	5 055	154 888	32.64
67.5 . .	65-69	34 450	2.75257	94 771	1 206 767	77 648	28 450	6 098	127 005	48.01
72.5 . .	70-74	36 231	2.96685	107 492	1 314 259	84 564	22 352	6 916	94 470	73.21
77.5 . .	75-79	32 495	3.19792	103 916	1 418 175	91 250	15 436	6 686	60 465	110.58
82.5 . .	80-84	22 686	3.44709	78 201	1 496 376	96 282	8 750	5 032	31 170	161.44
87.5 . .	85 +	15 547	3.71655	57 781	1 554 157	100 000	3 718	3 718	13 274	280.10

TABLE VI.3. COMPUTATION OF FEMALE MORTALITY FUNCTIONS SUCH THAT IN THE CORRESPONDING STATIONARY POPULATION DISTRIBUTION OF DEATHS BY AGE GROUPS IS IDENTICAL WITH THE DISTRIBUTION d_a SHOWN IN THE THIRD COLUMN OF THE TABLE

Median age α	*Age group (years)* a	*Distribution of deaths by age groups* d_a	*Cumulative totals*	*Survivors at the beginning of each age group*	*Deaths from one age to the next*	*Survivors in each age group* L_a	*Death rate per 1 000* m_a
0,5	Under 1	366 651	366 651	100 000	36 665	72 501	505.71
3.0	1-4	150 864	517 515	63 335	15 337	221 658	69.19
7.5	5-9	38 967	556 482	48 248	3 896	231 500	16.83
12.5	10-14	24 421	580 903	44 352	2 442	215 655	11.32
17.5	15-19	30 560	611 463	41 910	3 056	201 910	15.14
22.5	20-24	34 964	646 427	38 854	3 497	185 528	18.85
27.5	25-29	31 494	677 921	35 357	3 149	168 913	18.64
32.5	30-34	27 691	705 612	32 208	2 769	154 118	17.97
37.5	35-39	24 621	730 233	29 439	2 462	141 040	17.56
42.5	40-44	22 820	753 053	26 977	2 282	129 180	17.66
47.5	45-49	23 153	776 206	24 695	2 316	117 685	19.68
52.5	50-54	24 755	800 961	22 379	2 475	105 708	23.41
57.5	55-59	26 890	827 851	19 904	2 689	92 798	28.98
62.5	60-64	30 760	858 611	17 215	3 076	78 385	39.24
67.5	65-69	34 430	893 041	14 139	3 443	62 088	55.45
72.5	70-74	36 231	929 272	10 696	3 623	44 423	81.56
77.5	75-79	32 495	961 767	7 073	3 250	27 240	119.31
82.5	80-84	22 686	984 453	3 823	2 268	13 445	168.69
87.5	85 +	15 547	1 000 000	1 555	1 555	4 963	313.32

THIRD EXAMPLE: THE CRUDE DEATH RATE d_0 IS GIVEN

The crude death rate is equal to the difference between the crude birth rate and the rate of natural increase. Thus, it is a function of r. On graph VI.1 the curve $d(r) = b(r) - r$ is easily traced.

The straight line of the ordinate d_0 cuts this curve at a point M_0 whose abscissa is the rate of natural increase r_0 of the population. There is one population, and only one which answers the question.

FOURTH EXAMPLE: THE AGE DISTRIBUTION OF THE POPULATION AT A GIVEN AGE $C_0(_0)$ IS KNOWN

The method in this case is exactly the same. For a given age, $C_0(a_0)$ is a function of r. Table VI.5 enables these functions to be plotted for various age groups. It may be seen in graph VI.2 that above the 25-29 age group the curve passes through a minimum, while below that age group it passes through a maximum. The abscissae of the points of intersection of the straight

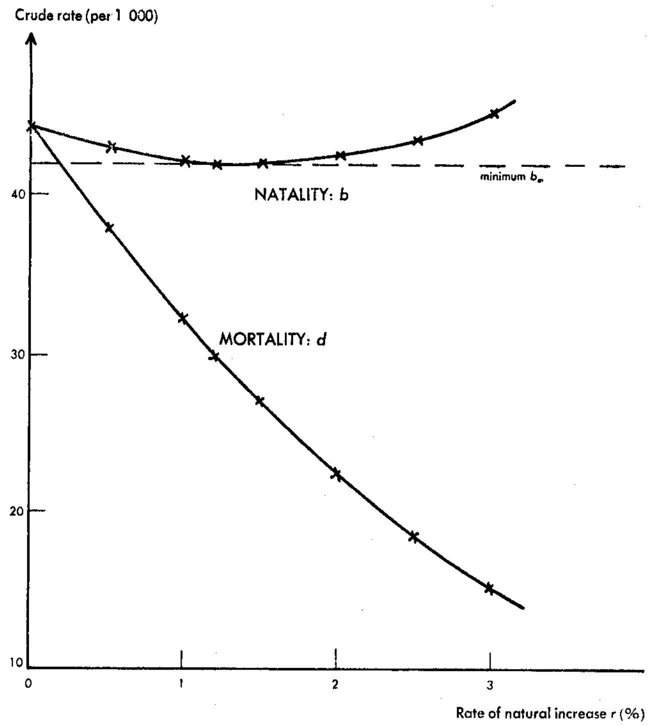
TABLE VI.4. FEMALE SURVIVORSHIP FUNCTION OF MALTHUSIAN POPULATIONS OF THE SUB-SET $G_0(r)$ LINKED TO THE AGE DISTRIBUTION OF FEMALE DEATHS GIVEN IN TABLES VI.1, VI.2 AND VI.3, FOR THE THREE RATES OF VARIATION: $r = 0.000$ (STATIONARY POPULATION), $r = 0.015$ AND $r = 0.030$

Age	$r = 0.000$ (a)	$r = 0.015$ (b)	$r = 0.030$ (c)
0	100 000	100 000	100 000
1	63 335	76 231	87 628
5	48 248	66 078	82 141
10	44 352	63 272	80 519
15	41 910	61 376	79 337
20	38 854	58 818	77 620
25	35 357	55 665	75 337
30	32 208	52 604	72 949
35	29 439	49 702	70 508
40	26 977	46 922	67 987
45	24 695	44 144	65 273
50	22 379	41 106	62 073
55	19 904	37 604	58 097
60	17 215	33 505	53 080
65	14 139	28 450	46 413
70	10 696	22 352	37 742
75	7 073	15 436	27 141
80	3 823	8 750	16 094
85	1 555	3 718	7 134

(a) Figure taken from the fifth column of table VI.3.
(b) Figure taken from the eighth column of table VI.2.
(c) Figure taken from the eighth column of table VI.1.

TABLE VI.5. DISTRIBUTION BY AGE GROUPS OF WOMEN IN MALTHUSIAN POPULATIONS OF THE SUB-SET $G_0(r)$, LINKED TO THE AGE DISTRIBUTION OF FEMALE DEATHS GIVEN IN TABLES VI.1, VI.2 and VI.3, FOR THE THREE RATES OF VARIATION: $r = 0.000$ (STATIONARY POPULATION), $r = 0.015$ and $r = 0.030$.

	$r = 0.000$	$r = 0.015$	$r = 0.030$
Crude birth rate per 100 000 . . .	44.070	42.165	45.070
Age group	*Age distribution*		
0	3 196	3 462	4 028
1-4	9 770	11 438	13 964
5-9	10 205	12 186	14 636
10-14	9 506	10 896	12 380
15-19	8 900	9 744	10 462
20-24	8 178	8 615	8 775
25-29	7 446	7 557	7 322
30-34	6 793	6 621	6 097
35-39	6 215	5 803	5 066
40-44	5 693	5 075	4 196
45-49	5 186	4 485	3 451
50-54	4 660	3 773	2 803
55-59	4 090	3 163	2 232
60-64	3 455	2 558	1 719
65-69	2 736	1 945	1 252
70-74	1 950	1 343	831
75-79	1 209	798	476
80-84	0 593	381	220
85 and over	0 219	151	90
ALL AGES	100 000	100 000	100 000



Graph VI.1. Variations, as a function of *r*, in the crude birth rate and crude death rate of Malthusian populations of the set G(*r*), linked to the age distribution of deaths d_a in tables VI.1, VI.2 and VI.3 (the distribution is the same in all three tables)

line of the ordinate $C_0(a_0)$ with the corresponding curve are the rates of natural increase of the populations sought. As in the first example, there is not always a solution, and when there is one solution there is generally a second one also.[1]

FIFTH EXAMPLE: THE FERTILITY FUNCTION (a) IS GIVEN

In the populations sought we have the relation:

$$\int_u^v e^{-ra} p_f(a) \varphi_f(a) da = 1$$

Let us consider the integral

$$I(r) = \int_u^v e^{-ra} p_f(a) \varphi_f(a) da$$

This can be calculated very easily with the aid of tables VI.1, VI.2 and VI.3 for the values of r assumed to be $r = 0$, $r = 0.015$ and $r = 0.03$.

Table VI.6 gives the computation for a fertility function corresponding to a gross reproduction rate of 2.9 according to the intermediate model fertility distribution. On graph VI.3 we have traced the curve representing $I(r)$ as a function of r. In the graph the straight line of the ordinate 1 cuts the curve at two points M_1 and M_2 whose abscissae r_1 and r_2 define two populations satisfying the given condition, i.e., having the specific fertility defined above.

As has been seen, there is not always a solution to the problem posed. Depending on the values of $\varphi(a)$, it may be that the curve of graph VI.3 is entirely above the straight line of the ordinate 1. We have also seen that, when there is one solution, there is generally a second one also.

This fifth example is similar to the second example of the sub-sets H(r), which was defined by knowledge of the survivorship function $p(a)$ and the additional knowledge of the fertility function $\varphi(a)$. This was shown to be a particular Malthusian population to which Lotka gave the name of a stable Malthusian population or, more simply, a stable population.

Here, in a sub-set G(r) defined by a constant age distribution of deaths, the additional knowledge of the fertility function defines, if certain conditions are satisfied, two particular Malthusian populations.

We shall confine ourselves to these few examples, although others could easily be imagined. We shall now proceed to a study of processes of demographic evolution where there is a constant age distribution of deaths.

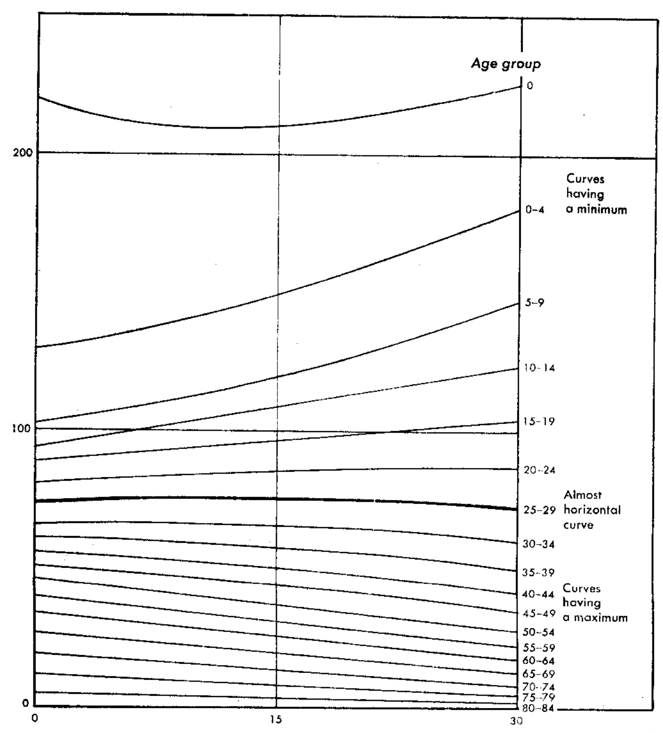
B. Processes of demographic evolution where there is constant age distribution of deaths

As in the case of the sub-sets H(r) and F(r), we can try to establish a correspondence between the sub-sets G(r) and processes of demographic evolution starting from a given initial state and assuming a constant age distribution of deaths associated with another condition implied in the series of examples considered above.

It is easy to see in a particular case that such processes of population evolution are not defined by these two conditions. Let us imagine, for example, that we assume the following two conditions:

(i) Constant age distribution of deaths $d_0(a)$; and

(ii) Constant crude death rate d_0.



Graph VI.2. Graphic illustration of variations, as a function of *r*, in the age distribution of women of Malthusian populations of the sub-set $G_0(r)$, linked to the age distribution of female deaths d_a in tables VI.1, VI.2 and VI.3 (the distribution is the same in all three tables)

[1] There is only one solution if the straight line of the ordinate $C_0(a_0)$ is tangent to the curve corresponding to age a_0.

TABLE VI.6. COMPUTATION OF THE INTEGRAL: $I(r) = \int_u^v e^{-ra} p_f(a) \varphi_f(a) da$ IN THREE MALTHUSIAN FEMALE POPULATIONS OF THE SUB-SET $G_0(r)$, LINKED TO AGE DISTRIBUTION OF FEMALE DEATHS IN TABLES VI.1, VI.2 AND VI.3, CORRESPONDING TO THREE RATES OF NATURAL VARIATION: $e = 0{,}030$, $r = 0.015$ AND $r = 0.000$

The gross reproduction rate is 2.9 and the distribution of the female fertility rates is that of the intermediate model

Median age α	Age group (years) a	Age distribution of female fertility rates	r = 0.000: $e^{-r\alpha}L_a$ (a)	r = 0.000: Product of the two preceding columns	r = 0.015: $e^{-r\alpha}L_a$ (b)	r = 0.015: Product of preceding column and third column	r = 0.030: $e^{-r\alpha}L_a$ (c)	r = 0.030: Product of preceding column and third column
17.5	15-19	0.100	201 910	20 191	230 920	23 092	232 110	23 211
22.5	20-24	0.273	185 528	50 650	204 270	55 770	194 686	53 150
27.5	25-29	0.263	168 913	44 440	179 200	47 130	162 447	42 730
32.5	30-34	0.188	154 118	28 980	157 080	29 520	135 260	25 430
37.5	35-39	0.121	141 040	17 060	137 620	16 650	112 394	13 600
42.5	40-44	0.055	129 080	7 105	120 350	6 620	93 083	5 120
	15-44 years	1.000		168 426		178 782		163 241
	I(r) (d)			0.977		1.037		0.9469

(a) Figures from the seventh column of table IV.3.
(b) Figures from the penultimate column of table IV.2, multiplied by $e^{-r\alpha}$
(c) Figures from the fifth column of table II.3.
(d) Figures from the preceding line, multiplied by (2.9/5.0) (1/100 000). The division by 100 000 is necessary because the life tables are expressed on the basis of an initial number at birth equal to 100 000.

These two conditions correspond to the third example given above.

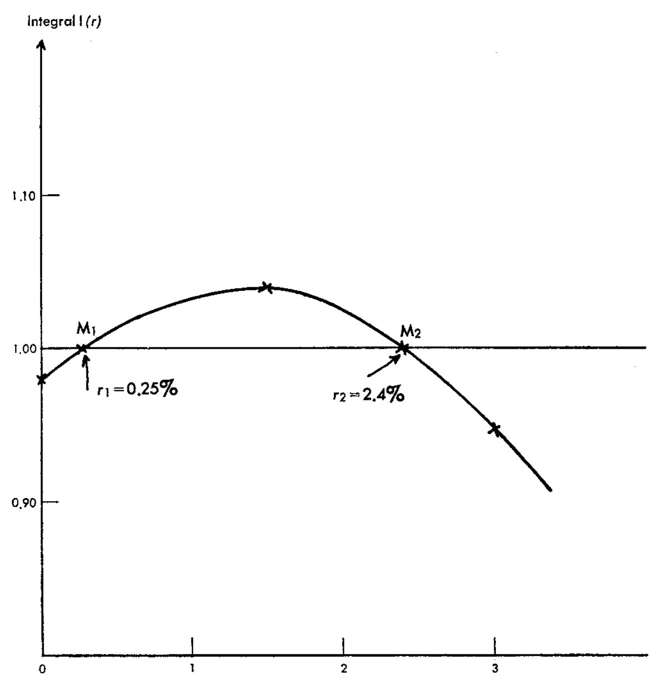
Starting from a given initial state at time t, the two conditions enable us to determine at each age the survivors at time $t + dt$ of the persons living at time t. They do not, however, enable us to determine the survivors at time $t + dt$ of persons born between times t and $t + dt$. In order to calculate these survivors we need a third condition, such as the fertility function. Let us then add this third condition. We can then compute a population projection, starting from the initial time. Let us consider what would happen to the population if we continued the computation of the projection indefinitely.

It is easy to see that, generally, the characteristics of the population computed in this way will not approach any limit, but will continue to fluctuate indefinitely without any diminution of the fluctuations with the passage of time.

In fact, if a limit existed—if, for example, the age distribution of the population tended to become invariable—this would mean that at the limit we should find a Malthusian population of the sub-set $G_0(r)$ linked to the age distribution of deaths $d_0(a)$. In such a sub-set, knowledge of the crude death rate defines a particular Malthusian population whose rate of natural increase is r, computed as described in the third example above. This rate r_0 would be that of the limit population.

In the sub-set $G_0(r)$, however, knowledge of the fertility function $\varphi(a)$ determines, subject to certain conditions, two particular Malthusian populations whose rate of increase r_1 and r_2 are computed as described in the fifth example above, and the rate of natural increase of the imagined limit population must be one of these two values r_1 and r_2.

Generally, r_0 will be different from r_1 and r_2, and therefore the limit population envisaged cannot exist. It is only when the fertility function $\varphi(a)$ is precisely so selected that one of these values r_1 or r_2 is equal to r_0 that a limit population can exist. It would still remain to be seen, however, whether the limit population really existed, since the fact that it can exist does not necessarily mean that it does exist.



Graph VI.3. Graphic presentation of the integral

$$\int_u^v e^{-ra} p_f(a) \varphi_f(a) da$$

of Malthusian female populations of the sub-set $G_0(r)$, linked to the age distribution of female deaths in tables VI.I, VI.2 and VI.3 (the distribution is the same in all three tables), the gross reproduction rate being 2.9 and the age distribution of the female fertility rates being the intermediate model fertility distribution

To sum up, processes of demographic evolution where the age distribution of deaths is constant do not generally lead to stable situations, as was the case with processes of demographic evolution where the survivorship function was constant.

We shall confine ourselves to these remarks and shall now proceed, in the next two chapters, to a study of quasi-stable populations, which, it may be recalled, represent an approximation to stable populations.

Chapter VII

THE NETWORK OF INTERMEDIATE MODEL STABLE POPULATIONS

A. Introduction

A solution to the problem posed by the very title of this work was given when, in chapter V, we established that a population having an age distribution which remains invariable with the passage of time can be assimilated at all times to a Malthusian population (we gave the name of "semi-Malthusian" to these populations with constant age distribution).

Once we verify that an actual population is semi-Malthusian, as most populations of developing countries are, all the methods studied in connexion with Malthusian populations, with the aim of determining the characteristics of a population when some of those characteristics are known, become applicable. As has already been mentioned more than once, however, the invariability of the age distribution of many actual populations could be interpreted in a different manner; instead of assimilating them to semi-Malthusian populations, one could assimilate them to quasi-stable populations.

The concept of quasi-stable populations was introduced into demography for the first time in the United Nations study on world population prospects,[1] where models of demographic evolution were constructed for the purpose of estimating the future population of the various countries of the world. These numerical calculations showed, among other things, that a population with invariable fertility but with mortality varying within the universe of the intermediate series of model life tables also had an almost invariable age distribution. That result was given more general application; *a quasi-stable population was defined as a population in which fertility had for long been constant and the age distribution also remained more or less constant.*

The concept of quasi-stable populations is therefore based on experience, and it was defined as a result of numerical calculations carried out on a necessarily limited scale. It is therefore not impossible that other calculations made with other universes of model life tables would result in a modification of the definition of this concept. For the time being, however, we shall take this concept exactly as it has been defined. It is easy to see how it can be used to solve the problem posed by the title of the present work. It is known that the fertility of most of the developing countries has remained constant, or at least has varied only slightly, for a long time past. Their populations are therefore quasi-stable populations, and it can be verified that their age distribution varies little with the passage of time.

Let us now imagine that we have available a network of quasi-stable populations, computed once for all by associating a series of model fertilities with a series of model life tables, and let us take an actual population of which we already know certain characteristics. With the aid of these known characteristics, we can seek out from the network the population which coincides with the actual population, and the unknwon characteristics of the actual population will be the same as the corresponding characteristics of the population thus chosen from the network. This is the principle of the method dealt with in chapter VII.

(*a*) The application of the principle is not as simple as its definition would seem to indicate, however. In the first place, there will never be in the network of quasi-stable populations a population which coincides exactly with the actual population; the coincidence will always be approximate.

(*b*) Moreover, a network of quasi-stable populations is, as we have just seen, linked to a universe of model life tables. We have already pointed out in previous chapters that, in order to describe all the possible life tables applicable to human species, we had to define three universes of life tables—intermediate model life tables, upward-deviating model life tables, and downward-deviating model life tables. In this chapter, we shall use the universe of intermediate model life tables, and in the following chapters we shall consider how the conclusions we reach must be modified when considered in the light of the other two universes.

(*c*) This having been assumed, a network of quasi-stable populations is made up of a number of population projections computed for constant fertility and for a mortality varying within the universe of model life tables adopted. There are, of course, an infinite number of possible variations in such a universe, and there are therefore an infinite number of possible networks.

(*d*) In order to eliminate this indeterminateness, we have used the following property of quasi-stable populations: in a quasi-stable population, the age distribution is at each moment very close to the age distribution of the stable population of that moment.[2] In other words, a network of stable populations constructed on the model life table adopted can be considered to represent the universe within which all the possible quasi-stable populations linked to the model life table in question vary.

In chapter VII, we shall consider a network of stable populations (or a network of Malthusian populations, which amounts to the same thing) constructed by associating a series of model fertilities with a series of intermediate model life tables. By assimilating actual populations to the populations in this network, we shall study the methods of evaluating the demographic characteristics of these actual populations.

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[1] *The Future Growth of World Population*, United Nations publication, Sales No.: 58.XIII.2 (see, in particular, para. (*b*) in chapter IV).

[2] This property was stated in chap. V.

B. The network of intermediate model stable populations

We have selected six intermediate model life tables corresponding to expectations of life at birth of 20, 30, 40, 50, 60.4 and 70.2 years. In our first series, we have associated these model life tables with six gross reproduction rates of 4.0, 3.0, 2.5, 2.0, 1.5 and 1.0, and we have thus calculated thirty-six stable populations *by applying the simplified method*, i.e., by assuming that the female fertility function is reduced to a single value $\varphi(27.5)$. In our second series, we have associated the same types of mortality with the following seven rates of natural variation (percentages): -1.0, $+0.5$, $+1.0$, $+1.5$, $+2.0$, $+3.0$ and $+4.0$. We thus obtain forty-two stable populations. The computation of the stable populations of this second series is very simple. Since the rate of natural variation is known, all we have to do is to multiply the survivorship functions $p_f(a)$, $p_m(a)$ and $p(a)$ by the coefficients e^{-ra}. The crude birth rates and death rates can be deduced immediately. There then remains to be calculated the gross reproduction rate, which was among the data for the first series. The difference between the two series is more apparent than real, however, for in the first series the gross reproduction rate is known only if we assume that the female fertility function is reduced to a single value $\varphi(27.5)$. If the fertility function has a different age distribution, it is necessary to compute the gross reproduction rate compatible with that distribution and the age structure of the population calculated by the simplified method. We are then confronted with the same problem as in the case of the forty-two populations of the second series, but this problem is easily solved and has been dealt with at the end of chapter II. We assume the distribution $M_f(a)$ of the fertility function and we write:

$$R'\int_u^v e^{-ra}p_f(a)M_f(a)da = 1$$

If we use five-year age groups for the calculation, and if we introduce into the calculation the age distribution of the total population, the formula is written:

$$R' \times 2{,}05\sum_u^v C_a M_a = b$$

where C_a represents the proportion *in the total population* of women in the five-year age group a, $a+5$; b is the crude female and male birth rate; 2.05 is a multiplication factor of the gross reproduction rate which serves the purpose of introducing into the formula the total of all the female and male births; and M_a is the rate of female fertility in the age group C_a.

Table VII.1 shows, by way of example, what results would be obtained from fertilities distributed in the same way as those of Jamaica in 1953 and of Spain in 1940 for the thirty-six stable populations. It will be recalled that the intermediate distribution was determined in such a way as to result, for a given value of the gross reproduction rate, in a stable population close to that arrived at by the "special case" method. Table VII.1 makes it possible to evaluate empirically the degree to which the intermediate distribution adopted achieves the desired goal.

If we adopt the same fertility distribution for the thirty-six and the forty-two stable populations, we obviously obtain the same network of stable populations in both cases. In the thirty-six populations, the mortality

TABLE VII.1. VALUE OF THE GROSS REPRODUCTION RATE IN THE NETWORK OF INTERMEDIATE MODEL STABLE POPULATIONS, FOR VARIOUS DISTRIBUTIONS OF FERTILITY BY AGE

Expectation of life at birth for both sexes (years)	*Function of fertility distribution:* Reduced to a single value φ(27.5) (special case)	Similar to that of Jamaica in 1953	Similar to that of Spain in 1940	Intermediate
20	4.00	3.94	4.17	4.05
30	4.00	3.86	4.13	3.99
40	4.00	3.82	4.11	3.96
50	4.00	3.80	4.10	3.94
60.4	4.00	3.77	4.09	3.92
70.2	4.00	3.76	4.09	3.91
20	3.00	3.00	3.12	3.06
30	3.00	2.95	3.10	3.02
40	3.00	2.92	3.09	3.00
50	3.00	2.91	3.08	2.99
60.4	3.00	2.89	3.08	2.98
70.2	3.00	2.88	3.08	2.98
20	2.50	2.52	2.59	2.56
30	2.50	2.48	2.58	2.53
40	2.50	2.46	2.57	2.51
50	2.50	2.45	2.57	2.50
60.4	2.50	2.44	2.57	2.50
70.2	2.50	2.43	2.57	2.49
20	2.00	2.03	2.06	2.05
30	2.00	2.00	2.05	2.03
40	2.00	1.99	2.05	2.02
50	2.00	1.98	2.05	2.01
60.4	2.00	1.97	2.05	2.01
70.2	2.00	1.97	2.05	2.00
20	1.50	1.53	1.53	1.53
30	1.50	1.51	1.52	1.52
40	1.50	1.50	1.52	1.51
50	1.50	1.50	1.52	1.51
60.4	1.50	1.49	1.52	1.51
70.2	1.50	1.49	1.53	1.51
20	1.00	1.02	0.99	1.01
30	1.00	1.01	0.99	1.00
40	1.00	1.01	0.99	1.00
50	1.00	1.00	1.00	1.00
60.4	1.00	1.00	1.00	1.00
70.2	1.00	1.00	1.00	1.00

and fertility are given and the rate of natural variation is a result. In the forty-two populations, the mortality and the rate of natural variation are given and the fertility is a result. For a given problem, we can use either of these two series and there is no theoretical reason for preferring one to the other, but one may be more convenient than the other, according to the calculations.

Annex III contains detailed results of the computation of the two series of stable populations. We shall confine ourselves in this chapter to giving, in tables VII.2 and VII.3, the main characteristics of the network of stable populations thus calculated. We shall refer to this network hereafter as *the network of intermediate model stable populations.*

C. Relationships among gross reproduction rate, expectation of life at birth, stable birth rate and stable death rate

In graph VII.1, the horizontal axis represents the stable death rate and the vertical axis the stable birth rate for the series of thirty-six stable populations. There is one point in the graph for each stable population, and we

TABLE VIII.2. CHARACTERISTICS OF THE NETWORK OF INTERMEDIATE MODEL STABLE POPULATIONS, AS A FUNCTION OF THE GROSS REPRODUCTION RATE AND OF THE EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES

Gross reproduction rate	Expectation of life at birth for both sexes (years)	Distribution of the population by age groups (%)			Stable rates per 1 000			Distribution by age groups of deaths over 5 years of age (%)		
		Under 15	15-19	60 and over	Births	Deaths	Natural variation	5-14	15-59	60 and over
4 . . .	20	45.2	52.4	2.4	63.8	53.0	10.8	19.6	68.3	12.1
3 . . .		38.5	57.6	3.9	50.5	50.2	0.3	14.7	68.4	16.9
2.5 . .		34.1	60.7	5.2	42.8	49.1	−6.3	11.8	66.5	21.7
2 . . .		28.9	64.0	7.1	34.2	48.6	−14.4	9.1	65.4	25.5
1.5 . .		22.6	66.9	10.5	24.8	49.7	−24.9	6.2	61.3	32.5
1 . . .		14.8	68.3	16.9	14.6	54.4	−39.8	3.3	53.6	43.1
4 . . .	30	48.2	49.2	2.6	59.8	35.3	24.5	19.5	64.7	15.8
3 . . .		41.3	54.5	4.1	47.7	33.7	14.0	14.5	63.4	22.1
2.5 . .		36.9	57.6	5.5	40.6	33.2	7.4	11.6	61.6	26.8
2 . . .		31.4	60.9	7.7	32.7	33.6	−0.9	8.7	58.4	32.9
1.5 . .		24.7	63.8	11.5	23.8	35.0	−11.2	5.7	53.0	41.3
1 . . .		16.3	65.0	18.7	14.0	39.9	−25.9	2.9	44.0	53.1
4 . . .	40	50.0	47.3	2.7	57.3	24.1	33.2	18.8	60.8	20.4
3 . . .		43.1	52.5	4.4	46.0	23.3	22.7	13.6	58.1	28.3
2.5 . .		38.5	55.6	5.9	39.3	23.2	16.1	10.8	55.4	33.8
2 . . .		32.9	58.8	8.3	31.7	23.7	8.0	7.9	51.1	41.0
1.5 . .		25.9	61.6	12.5	23.1	25.6	−2.5	5.0	44.6	50.4
1 . . .		17.0	62.6	20.4	13.6	30.9	−17.3	2.4	35.0	62.6
4 . . .	50	51.5	45.8	2.7	55.7	16.2	39.5	17.5	56.5	26.0
3 . . .		44.6	50.9	4.5	44.9	15.8	29.1	12.3	52.3	35.4
2.5 . .		40.0	53.9	6.1	38.4	16.0	22.4	9.5	48.8	41.7
2 . . .		34.2	57.2	8.6	31.1	16.8	14.3	6.7	43.7	49.6
1.5 . .		27.0	60.0	13.0	22.7	18.8	3.9	4.1	36.7	59.2
1 . . .		17.8	60.7	21.5	13.4	24.3	−10.9	1.9	27.3	70.8
4 . . .	60.4	52.9	44.4	2.7	54.1	9.4	44.7	14.8	51.1	34.1
3 . . .		46.0	49.6	4.4	43.8	9.6	34.2	9.9	45.5	44.6
2.5 . .		41.4	52.6	6.0	37.7	10.1	27.6	7.4	41.3	51.3
2 . . .		35.6	55.8	8.6	30.6	11.1	19.5	5.0	35.9	59.1
1.5 . .		28.2	58.7	13.1	22.5	13.5	9.0	2.9	29.0	68.1
1 . . .		18.7	59.4	21.9	13.3	19.0	−5.7	1.3	20.6	78.1
4 . . .	70.2	54.1	43.3	2.6	52.7	4.1	48.6	9.2	42.8	48.0
3 . . .		47.3	48.4	4.3	42.9	4.8	38.1	5.7	36.1	58.2
2.5 . .		42.7	51.4	5.9	37.0	5.5	31.5	4.0	31.9	64.1
2 . . .		36.8	54.7	8.5	30.1	6.8	23.3	2.6	26.8	70.6
1.5 . .		29.3	57.7	13.0	22.3	9.4	12.9	1.4	21.1	77.5
1 . . .		19.5	58.6	21.9	13.3	15.1	−1.8	0.6	14.5	84.9

thus have thirty-six points. We have joined together those points corresponding to the same expectation of life at birth and those corresponding to the same gross reproduction rate. Finally, we have plotted a network of straight lines inclined at an angle of 45 degrees, each corresponding to a constant rate of variation. In this way, we have obtained five networks of superimposed lines:

(*a*) A network of vertical straight lines for constant death rate;

(*b*) A network of horizontal straight lines for constant birth rate;

(*c*) A network of curves for constant expectation of life at birth;

(*d*) A network of curves for constant gross reproduction rate;

(*e*) A network of straight lines inclined at 45 degrees for a constant rate of variation.

This first diagram enables us to begin to grasp the properties of the intermediate model stable populations.

It will be observed that the network of curves for constant expectation of life, like the network of curves for constant gross reproduction rate, do not turn back on themselves. It was seen in chapter II that knowledge of the life table and of the gross reproduction rate (once the age distribution of the fertility rates was fixed) was sufficient to determine one, and only one, stable population. In other words, there corresponds to each pair of

TABLE VII.3. CHARACTERISTICS OF THE NETWORK OF INTERMEDIATE MODEL STABLE POPULATIONS, AS A FUNCTION OF THE INTRINSIC RATE OF NATURAL VARIATION AND OF THE EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES

Intrinsic rate of natural variation (%)	*Expectation of life at birth for both sexes (years)*	*Distribution of the population by age groups (%)*			*Stable rates per 1 000*		*Gross reproduction rate*
		Under 15	*15-59*	*60 and over*	*Births*	*Deaths*	
−1.0	20	31.7	62.3	6.0	38.8	48.8	2.26
0		38.2	57.8	4.0	50.1	50.1	2.97
0.5		41.5	55.3	3.2	56.3	51.3	3.10
1.0		44.8	52.7	2.5	62.8	52.8	3.91
1.5		47.9	50.1	2.0	69.5	54.5	4.48
2.0		51.0	47.4	1.6	76.5	56.5	5.15
3.0		56.9	42.1	1.0	91.0	61.0	6.77
4.0		62.3	37.1	0.6	106.2	66.2	8.92
−1.0	30	25.4	63.6	11.0	24.7	34.7	1.55
0		31.9	60.6	7.5	33.4	33.4	2.04
0.5		35.3	58.6	6.1	38.2	33.2	2.34
1.0		38.6	56.4	5.0	43.4	33.4	2.69
1.5		42.0	54.0	4.0	48.8	33.8	3.06
2.0		45.3	51.5	3.2	54.5	34.5	3.54
3.0		51.6	46.4	2.0	66.5	36.5	4.66
4.0		57.6	41.2	1.2	79.1	39.1	6.13
−1.0	40	21.2	62.6	16.2	17.9	27.9	1.22
0		27.5	61.1	11.4	25.0	25.0	1.61
0.5		30.9	59.7	9.4	29.1	24.1	1.84
1.0		34.3	58.0	7.7	33.5	23.5	2.11
1.5		37.8	56.0	6.2	38.2	23.2	2.43
2.0		41.2	53.8	5.0	43.2	23.2	2.79
3.0		48.0	48.9	3.1	53.8	23.8	3.67
4.0		54.3	43.8	1.9	65.1	25.1	4.83
−1.0	50	18.3	60.8	20.9	13.9	23.9	1.03
0		24.4	60.6	15.0	20.0	20.0	1.35
0.5		27.8	59.8	12.4	23.6	18.6	1.55
1.0		31.2	58.5	10.3	27.5	17.5	1.78
1.5		34.7	56.9	8.4	31.7	16.7	2.04
2.0		38.2	55.0	6.8	36.2	16.2	2.34
3.0		45.2	50.5	4.3	45.8	15.8	3.08
4.0		51.8	45.5	2.7	56.2	16.2	4.05
−1.0	60.4	16.2	58.8	25.0	11.2	21.2	0.89
0		22.2	59.7	18.1	16.6	16.6	1.17
0.5		25.5	59.3	15.2	19.7	14.7	1.34
1.0		28.9	58.5	12.6	23.2	13.2	1.54
1.5		32.5	57.2	10.3	27.0	12.0	1.77
2.0		36.0	55.6	8.4	31.1	11.1	2.03
3.0		43.2	51.5	5.3	39.9	9.9	2.67
4.0		49.9	46.7	3.4	49.5	9.5	3.52
−1.0	70.2	14.8	57.1	28.1	9.5	19.5	0.80
0		20.6	58.7	20.7	14.3	14.3	1.05
0.5		23.9	58.7	17.4	17.1	12.1	1.20
1.0		27.3	58.2	14.5	20.3	10.3	1.38
1.5		30.8	57.3	11.9	23.8	8.8	1.59
2.0		34.4	55.9	9.7	27.5	7.5	1.82
3.0		41.6	52.2	6.2	35.7	5.7	2.40
4.0		48.5	47.6	3.9	44.7	4.6	3.16

values—the expectation of life at birth e_0 and the gross reproduction rate R′—a unique other pair of values: the stable birth rate b and the stable death rate d. The fact that the two networks under consideration do not overlap shows that the converse of this property is equally true in the network of intermediate model stable populations: there corresponds to a pair of values for the stable birth rate and the stable death rate a unique pair of values for the expectation of life at birth and the gross reproduction rate. Here we have a particular case of a much more general property of the stable populations considered in this chapter, which can be stated as follows: if we consider any two independent characteristics (C_1 and C_2) of such a stable population—for example, the gross reproduction rate and the percentage of the population over 15 years of age, or the percentage of deaths at 60 and over and the stable birth rate—we can state that, *generally speaking, a pair (C_1 and C_2) tends to determine without ambiguity a stable population within the network.*

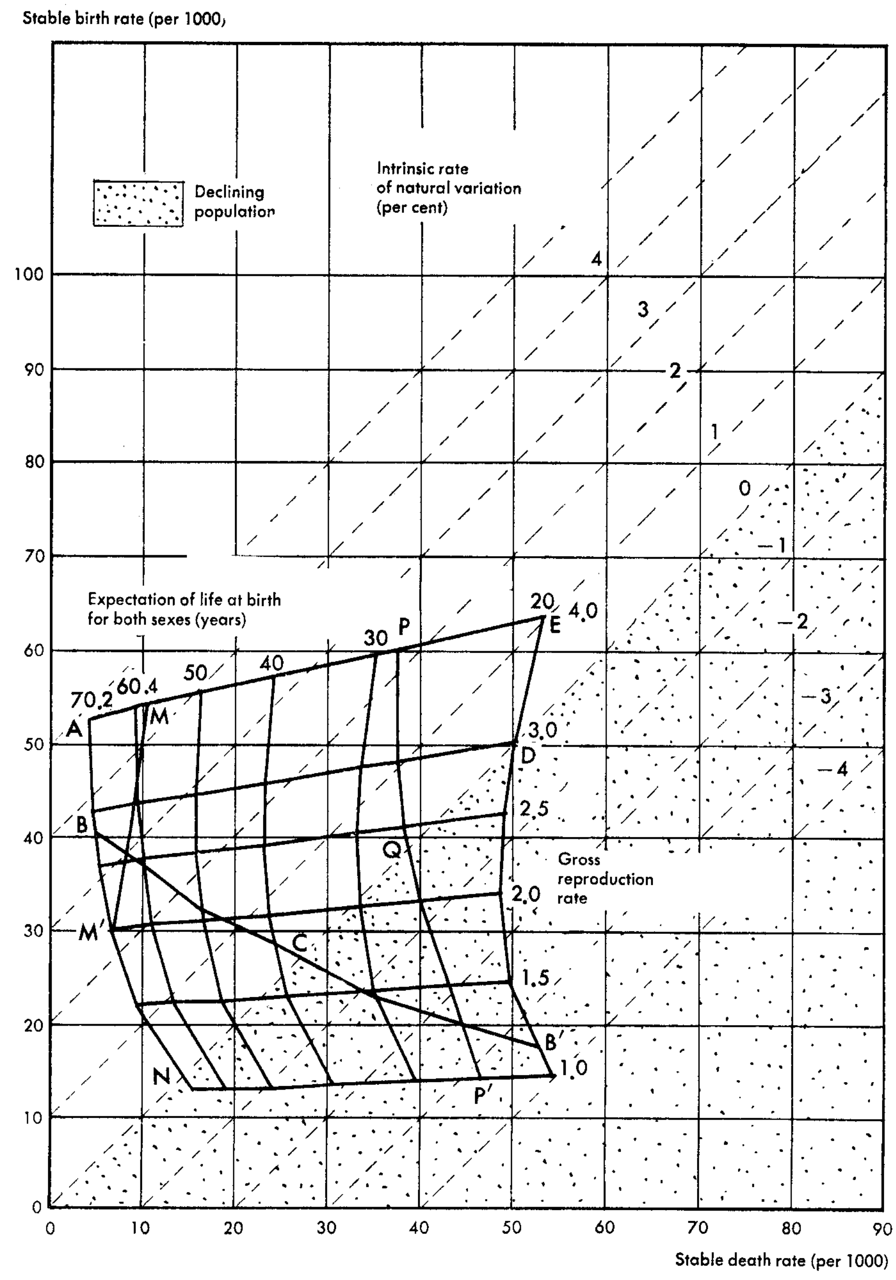
We are not dealing here with a theoretical property of stable populations. The precise nature of this property is due to the fact that the stable populations considered in this chapter have mortalities conforming to the model life tables. More precisely, it is due to the fact that the "universe" of intermediate model life tables is a stratified universe, in the sense that the curves for probabilities of death and the survivorship curves of the series of model tables do not cross, but are entirely one below the other.

However, the property under consideration *is true only in a general sense.* We shall give below, by way of example, a case where it is not true.

The curves for constant expectation of life in the case of medium and high fertilities (gross reproduction rate over 2.0) diverge relatively little from the vertical. In other words, at these levels of fertility the stable death rate is a fairly good index of the mortality of the intermediate model stable populations. If fertility is low, however, the stable death rate may give a very false idea of the mortality level.

The curves for constant gross reproduction rate diverge relatively little from the horizontal, particularly in the case of medium or low fertilities. The stable birth rate is therefore a fairly good index of the fertility of stable populations. However, this assessment of the value of the vital rates for measuring fertility and mortality can be made a little more precise.

On each curve for constant expectation of life we have plotted the points whose abscissa (the vital death rate)



Graph VII.1. Illustration of relations among stable death rate, stable birth rate, intrinsic rate of natural variation, gross reproduction rate and expectation of life at birth in the network of intermediate model stable populations

does not diverge more than 5 per cent from the abscissa of the point of intersection with the corresponding curve for the gross reproduction rate of 3.0. We thus trace the curve BB′.

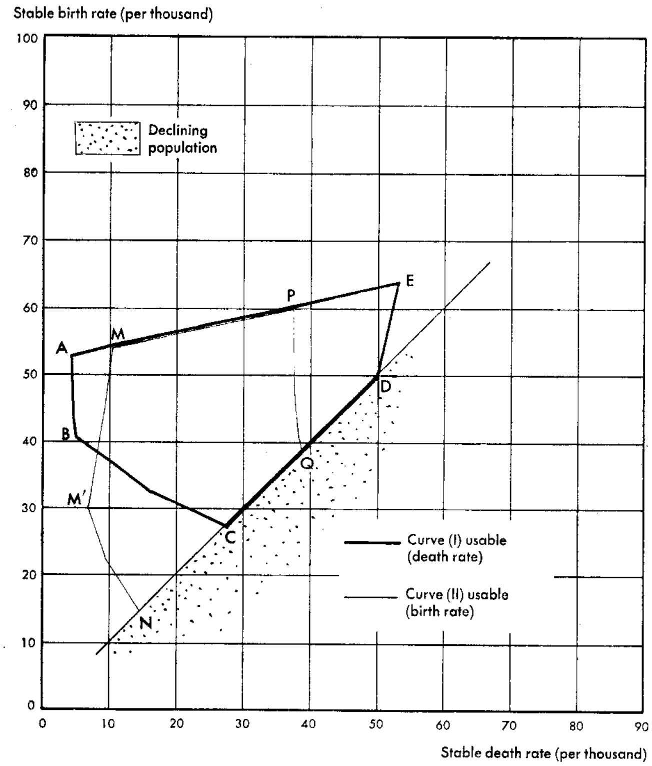
On each curve for constant gross reproduction rate we have marked the points whose ordinate (the birth rate) does not diverge more than 5 per cent from the abscissa of the point of intersection with the curve corresponding to an expectation of life at birth of 40 years. We thus obtain lines MM′ and PP′.

If we eliminate the declining stable populations (the hatched area of the diagram) we may say that the stable death rate is a good index for measuring mortality in the area ABCDE (graphs VII.1 and VII.1 *bis*).

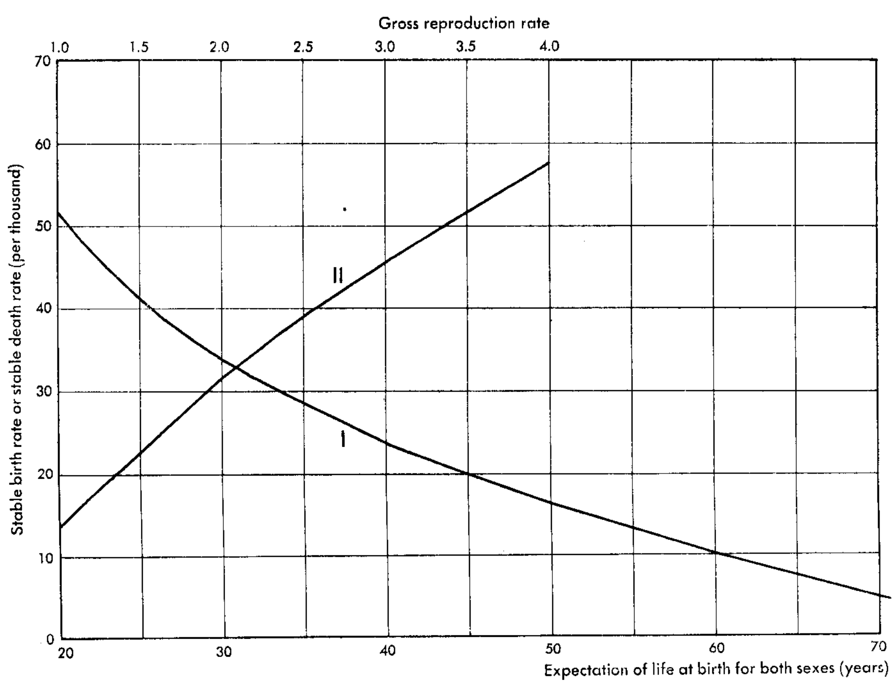
In table VII.4, we have reproduced the stable death rate corresponding to the thirty-six stable populations and have underlined those which are within or very close to the area ABCDE. The last line of the table gives, for each expectation of life, the average of the rates underlined. The relationship linking this average with the expectation of life enables us to pass very easily from the stable death rate to the expectation of life of populations conforming to the conditions of the network of intermediate model stable populations (graph VII.2). It must, however, be borne in mind that graph VII.2 cannot be applied outside the area ABCDE.

Similarly, the stable birth rate is a good index for measuring fertility in the area MM′NQP (graphs VII.1 and VII.1 *bis*).

In table VII.5, we have reproduced the stable birth rates of the thirty-six stable populations and have underlined those which are within or very close to the area MM′NQP. In the last column of the table we have calculated, for each gross reproduction rate, the average of the rates underlined. The relationship linking this average with the gross reproduction rate enables us to pass very easily from the stable birth rate to the gross reproduction rate (graph VII.2). Here, again, however, we should be sure, before using graph VII.2, that we are really within the area MM′NQP.



Graph VII.1 *bis*. Areas of graph VII.1 in which the curves of graph VII.2 are usable



Graph VII.2. Approximate relationships between: (I) stable death rate and expectation of life at birth; (II) stable birth rate and gross reproduction rate

TABLE VII.4. STABLE DEATH RATE IN INTERMEDIATE MODEL STABLE POPULATIONS
(Per thousand)

Gross reproduction rate	*Expectation of life at borth for both sexes (years)* 20	30	40	60	60.4	70.2
4.0	53.0	35.3	24.1	16.2	9.4	4.1
3.0	50.2	33.7	23.3	15.8	9.6	4.8
2.5	49.1	33.2	23.2	16.0	10.1	5.5
2.0	48.6	33.6	23.7	16.8	11.1	6.8
1.5	49.7	35.0	25.6	18.8	13.5	9.4
1.0	54.4	39.9	30.9	24.3	19.0	15.1
Average of rates underlined	51.6	33.9	23.6	16.2	9.7	4.5

TABLE VII.5. STABLE BIRTH RATE IN INTERMEDIATE MODEL STABLE POPULATIONS
(Per thousand)

Gross reproduction rate	*Expectation of life at birth for both sexes (years)* 20	30	40	50	60.4	70.2	*Average of rates underlined*
4.0 . . .	63.8	59.8	57.3	55.7	54.1	52.7	57.6
3.0 . . .	50.5	47.7	46.0	44.9	43.8	42.9	45.6
2.5 . . .	42.8	40.6	39.3	38.4	37.7	37.0	39.0
2.0 . . .	34.2	32.7	31.7	31.1	30.6	30.1	31.4
1.5 . . .	24.8	23.8	23.1	22.7	22.5	22.3	22.7
1.0 . . .	14.6	14.0	13.6	13.4	13.3	13.3	13.3

TABLE VII.6. TREND OF THE RECORDED DEATH RATE IN CEYLON

Year	*Recorded death rate (per thousand)*	*Expectation of life at birth for both sexes (in years) read off from graph VII.2 (curve I)*	*Expectation of life at birth for boths sexes (in years) calculated from the age-specific death rates*
1946 . .	19.8	44.7	42.8
1947 . .	14.0	53.6	51.9
1948 . .	12.9	55.6	54.1
1949 . .	12.3	56.2	55.5
1950 . .	12.4	56.1	55.6
1951 . .	12.7	55.7	55.1
1952 . .	11.8	57.0	56.6
1953 . .	10.7	58.8	58.2
1954 . .	10.2	59.6	59.8

PRACTICAL APPLICATIONS

Here now are two examples of the practical application of the two curves of graph VII.2. The first one concerns mortality in Ceylon.[3] Comparison of the last two columns of table VII.6 shows that, in the case of that country, the rapid determination of the expectation of life at birth from the death rate, using curve I of graph VII.2, gives approximate values which are sufficiently accurate for practical use. It should be clearly noted, however, that the concordance of the figures in the last two columns of table VII.6 does not mean that the level of mortality thus determined is the actual level. All the figures in table VII.6 refer only to the *recorded* mortality, which may be lower than the true rate.

The second example concerns fertility in Chile[4] for 1935-1939. During that period, an average crude birth rate of 32.9 per thousand was recorded in Chile. From graph VII.2 (curve II) we see that a gross reproduction rate of 2.08 corresponds to this value for the crude birth rate. The gross reproduction rate computed directly from the recorded rate of age-specific female fertility is equal to 2.077. The concordance between the results of the two computations is excellent. As in the previous example, however, these are merely the recorded figures, and the true level of fertility may be different.

A GUIDELINE

It will readily be understood that we can multiply diagrams similar to graph VII.1 almost to infinity. All we have to do is to indicate any characteristic (C_1) of a stable population of the network on the horizontal axis and any other characteristic (C_2) on the vertical axis. It is therefore necessary to make a choice among all the possible diagrams. In order to guide this choice, it is desirable to clarify somewhat the use that can be made of these diagrams and to consider the documentation at our disposal.

(1) Generally speaking, the populations that can be assimilated to stable populations are those of less developed countries. The mortality and fertility levels of such populations are usually unknown. The registration of births and deaths, if it exists at all, is almost always very incomplete, and it may therefore be considered that the expectation of life at birth and the gross reproduction rate are the values sought.

(2) The age distribution is better known. This is an item of data provided by censuses, and we often have a number of censuses at our disposal. If we have no census results, a sample survey can be used instead. We must, of course, take into account errors in the statement of ages which may affect the census or sample survey data in less developed countries, particularly in the case of children under 5 years of age.

(3) The annual rate of natural variation is linked with the annual rate of variation, which is known from the comparison of successive census results. We pass from one to the other by taking migratory movement into account. A sample survey can also provide a measurement of the rate of natural variation. Here, again, however, account must be taken of errors which may affect the data. In many less developed countries, for example, recent census counts or sample surveys are more complete than those made previously, with the result that they show a rate of increase higher than the true rate.

(4) The registration of deaths is often very incomplete. Except in the case of very young children, however, it is possible that under-registration does not vary greatly with age. We can, at all events, assume this as a working

[3] The age distribution of the population of Ceylon has varied little with the passage of time. We can therefore assimilate this population *at all times* to a quasi-stable population.

[4] The age distribution of the population of Chile has varied little with the passage of time, and we can therefore assimilate it *at all times* to a quasi-stable population.

hypothesis and can reconsider the validity of the assumption if the results obtained seem to be improbable. This being so, we know the age distribution of deaths, or rather of deaths excluding young children.

(5) Censuses sometimes give direct information about fertility by providing data from which the average number of children born to each woman can be calculated. In populations where fertility is constant, we can obtain the gross reproduction rate by multiplying this average number by the proportion of girls at birth (approximately 0.49). Unfortunately, the data assembled by censuses in this respect are often imprecise.

(6) We can obtain information on mortality by comparing the numbers of persons in corresponding age groups in two censuses. This procedure is a well-known one, but the crude results given by it are sometimes difficult to interpret because of irregularities in the age distribution due to erroneous statements of age or to incomplete coverage of certain age groups of the population. Comparison of the observed age distribution with stable distributions enables these irregularities to be reduced to some extent, but it is always necessary to act cautiously in these matters. If we replace the observed age distribution by an adjusted stable distribution, this is usually tantamount to constructing a life table by the methods described in chapters II or IV. A variant of the method of comparing the age groups at two successive censuses consists of comparing only the corresponding populations above a certain age in the two censuses. If, for example, we have two good censuses nine years apart and the population is a closed population (or if we can take migratory movement into account, which amounts to the same thing), the difference between the total population at the first census and the population aged 9 years and over at the second census is approximately equal to the number of deaths of persons aged 4.5 years and over during the interval between the two censuses ($4.5 = 9/2$). If we divide this approximate number of deaths by the number of persons aged 4.5 years and over [5] at the middle of the interval, we obtain the mean death rate for those aged 4.5 years and over during the interval between the two censuses.

Knowing what data we have and what we hope to deduce from them, we can now proceed to select the most useful diagrams. We shall consider in succession the following cases:

(*a*) Only the age distribution of the population is known. This would be the case, for example, if we had the results of only one census. We shall take as characteristics of the stable population C_1 and C_2 two characteristics of the age distribution, such as the percentage of the population aged 60 and over (on the horizontal axis) and the percentage of the population under 15 (on the vertical axis);

(*b*) The age distribution of the population and the annual rate of natural variation are known. In this case, we shall let the horizontal axis represent the proportion of the population under 15 and the vertical axis the rate of natural variation;

(*c*) The age distribution of deaths is known. Data on the age distribution of deaths are of a different nature from those mentioned above, which are known only for certain dates—the age distribution at the time of a census, and the rate of natural variation at the middle of the interval between two censuses— while the age distribution of deaths, which is obtained from the statistics of population movement, is generally known annually. We may consider three diagrams:

(i) Only the age distribution of deaths is known. We can, for example, indicate on the horizontal axis the percentage of deaths at 60 and over and on the vertical axis the percentage of deaths under 15. We can also disregard children under 5, whose deaths are usually stated less accurately than those of older persons, and we shall indicate on the horizontal axis the proportion of deaths at 60 and over in the total number of deaths over the age of 5 and on the vertical axis the proportion of deaths between the ages of 5 and 14 in the total number of deaths over the age of 5;

(ii) The age distribution of the population and the age distribution of deaths are known. We may plot proportion of deaths at 60 and over in the total number of deaths over the age of 5 on the horizontal axis and the proportion of the population aged 60 and over on the vertical axis;

(iii) The age distribution of deaths and the rate of natural variation are known. The horizontal axis may represent the proportion of deaths of 60 and over in the total number of deaths over the age of 5 and the vertical axis may represent the rate of natural variation.

Before considering the diagram mentioned in paragraph (*a*) above, we would comment on the accuracy with which the age distribution is known.

Age distribution of the population

As was stated above, we often know the age distribution of a population; there are even time series available for many countries. However, certain qualifications must be made regarding this statement.

In developing countries, as was just pointed out, statements of age are often inaccurate and the counting of certain age groups may be incomplete, resulting in irregularities in the population pyramid. It is essential to make a careful examination of these irregularities and to effect the necessary corrections before using the data. Manual III [6] describes the various methods which may be used for this purpose. We shall confine ourselves here to showing how the net work of intermediate model stable populations enables the under-enumeration of children aged 0 to 4 to be corrected.

We have calculated the following ratios for each of the thirty-six stable populations:

$$F_0 = \frac{\text{girls aged 0 to 4}}{\text{women aged 15 to 44}}$$

$$F_1 = \frac{\text{girls aged 5 to 9}}{\text{women aged 20 to 49}}$$

$$F_2 = \frac{\text{girls aged 10 to 14}}{\text{women aged 25 to 54}}$$

[5] This population is the half-sum of the total number of persons in the population at the first census and the total number of persons aged 9 and over at the second census.

[6] *Methods of estimating population. Manual III: Methods for Population Projections by Sex and Age* (United Nations publication, Sales No.: 56.XIII.3).

It will be observed from table VII.7 that in the case of a stable population the three ratios F_0, F_1 and F_2 are very close to each other, F_0 and F_2 in particular being almost always practically identical. If, therefore, in dealing with an *actual population which can be assimilated to a stable population*, we find ratios F_0, F_1 and F_2 which deviate from this pattern, we can correct the age distribution accordingly.

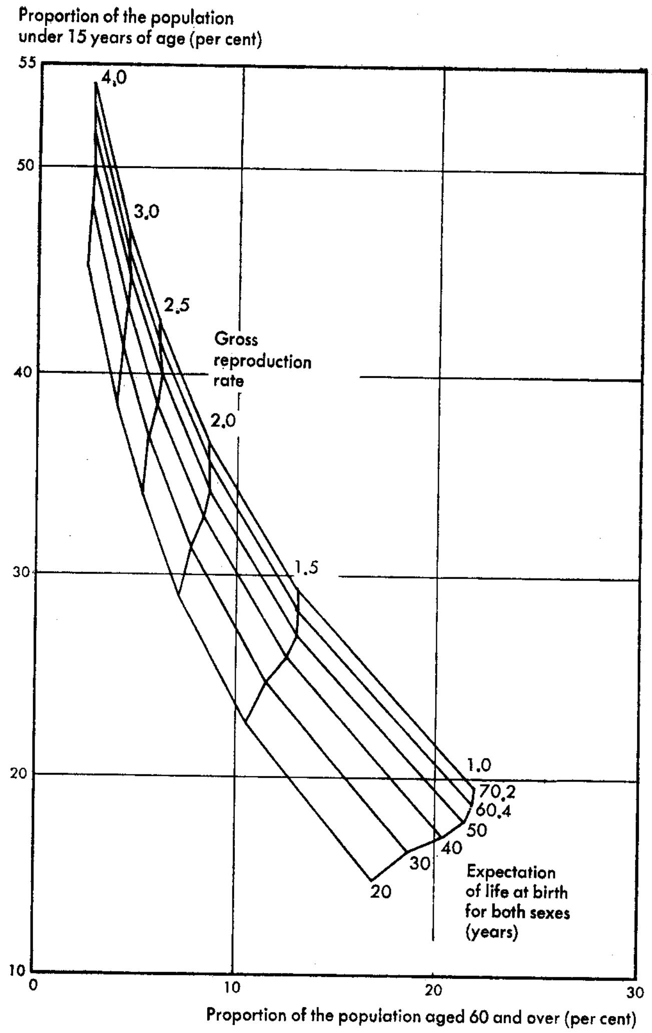
TABLE VII.7. VALUES OF THE RATIOS OF GIRLS TO WOMEN (F_0, F_1 AND F_2) IN THE NETWORK OF INTERMEDIATE MODEL STABLE POPULATIONS

	Expectation of life at birth for both sexes (years)					
	20	*30*	*40*	*50*	*60.4*	*70.2*
Gross reproduction rate: 4.0						
F_0 . . .	0.4598	0.4806	0.5129	0.5418	0.5712	0.5973
F_1 . . .	0.4041	0.4484	0.4931	0.5296	0.5658	0.5974
F_2 . . .	0.4092	0.4706	0.5090	0.5402	0.5726	0.6018
Gross reproduction rate: 3.0						
F_0 . . .	0.3368	0.3703	0.3960	0.4188	0.4122	0.4630
F_1 . . .	0.2940	0.3460	0.3810	0.4097	0.4382	0.4632
F_2 . . .	0.3162	0.3638	0.3936	0.4181	0.4438	0.4668
Gross reproduction rate: 2.5						
F_0 . . .	0.2837	0.3127	0.3348	0.3544	0.3746	0.3926
F_1 . . .	0.2482	0.2925	0.3222	0.3468	0.3712	0.3928
F_2 . . .	0.2675	0.3078	0.3332	0.3540	0.3761	0.3960
Gross reproduction rate: 2.0						
F_0 . . .	0.2291	0.2531	0.2714	0.2877	0.3044	0.3194
F_1 . . .	0.2009	0.2371	0.2615	0.2816	0.3018	0.3197
F_2 . . .	0.2170	0.2499	0.2705	0.2876	0.3059	0.3224
Gross reproduction rate: 1.5						
F_0 . . .	0.1727	0.1915	0.2057	0.2183	0.2314	0.2431
F_1 . . .	0.1519	0.1796	0.1983	0.2138	0.2295	0.2434
F_2 . . .	0.1647	0.1896	0.2054	0.2186	0.2327	0.2456
Gross reproduction rate: 1.0						
F_0 . . .	0.1144	0.1275	0.1373	0.1460	0.1551	0.1633
F_1 . . .	0.1011	0.1199	0.1325	0.1431	0.1539	0.1636
F_2 . . .	0.1102	0.1269	0.1375	0.1465	0.1562	0.1652

D. Age distribution and levels of mortality and fertility

Graph VII.3 enables us to study how the age distribution of intermediate model stable populations varies as a function of mortality and fertility. We have plotted on the horizontal axis the proportion of persons aged 60 and over and on the vertical axis the proportion of persons under 15. We have also plotted curves for constant mortality and constant fertility.

The two extreme curves for constant mortality enclose an elongated band which is quite narrow towards both the horizontal and the vertical axes, particularly if we disregard the very high mortalities (expectation of life at birth of less than 30 years) which, after all, represent quite exceptional situations. Moreover, the curves for constant fertility deviate only slightly from the vertical.



Graph VII.3. Relationships among age distribution, mortality level and fertility level in the network of intermediate model stable populations

It follows that *the age distribution of the population is not greatly influenced by variations in mortality.* At the higher ages, this unresponsiveness of the age distribution to variations in mortality is further accentuated, in the case of medium and high fertilities, by the fact that the curves for constant fertility are almost vertical. In other words, the proportion of persons aged 60 and over is almost independent of the mortality.[7]

Let us now consider, taking an actual example, how to use such a graph. We have chosen for this study the population of Venezuela, the distribution of which by five-year age groups, as obtained from the three censuses of 1930, 1941 and 1950, is given in table VII.8. At first sight, the distribution of this population by age groups seems to have varied little; however, two facts, one of a general nature and the other peculiar to Venezuela, should be taken into account:

(*a*) The enumeration of children is defective in almost every population census, even in countries having reliable statistics, and the magnitude of the errors in such enumeration varies from census to census;

[7] For more details, see ***The Aging of Populations and Its Economic and Social Implications*** (United Nations publication, Sales No.: 56.XIII.6).

TABLE VII.8. DISTRIBUTION OF THE POPULATION OF VENEZUELA BY FIVE-YEAR AGE GROUPS, AS RECORDED IN THE CENSUSES OF 1936, 1941 AND 1950 ([a])

Age group (years)	*1936*	*1941*	*1950*
ALL AGES . . .	100 000	100 000	100 000
0-4	14 078	15 010	16 838
5-9	14 015	13 573	13 652
10-14	12 249	12 288	11 415
15-19	10 378	10 453	9 864
20-24	9 900	9 694	9 409
25-29	8 471	8 353	7 957
30-34	6 472	6 530	6 507
35-39	5 951	5 591	5 989
40-44	5 014	4 946	4 759
45-49	3 721	3 949	3 670
50-54	3 304	3 072	3 272
55-59	1 933	1 989	2 023
60-64	1 983	1 886	1 837
65-69	878	915	951
70-74	691	699	760
75-79	325	349	373
80-84	303	299	329
85-89	104	110	107
90-94	67	67	77
95-99	31	32	39
100 and over .	32	16	12
Age not stated	100	179	160

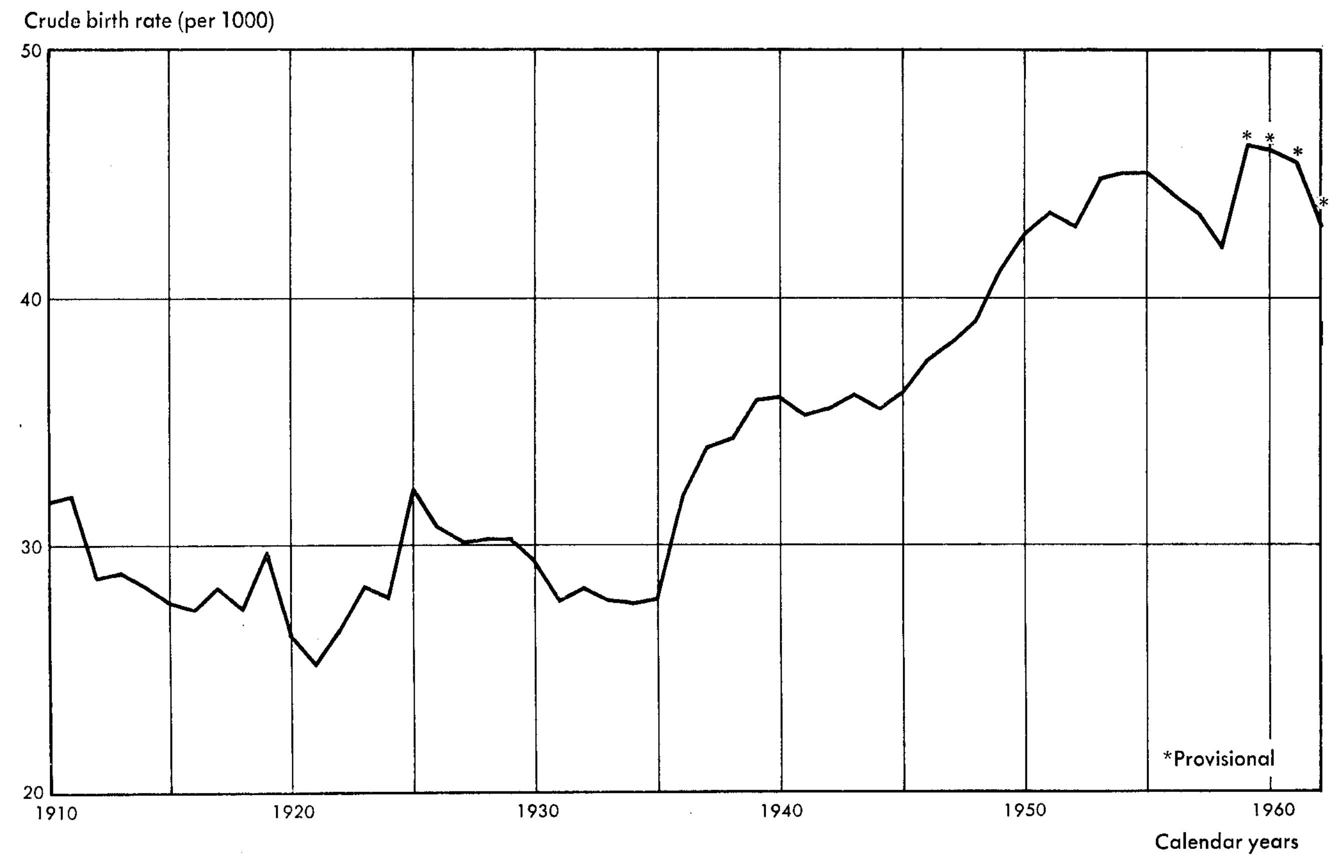
SOURCE: *Octavo Censo General de Poblacion. Edad y estado civil por entidades y distritos, y resumen nacional,* Caracas, 1954. See, in particular, table 2 on page 4.

([a]) Excluding Indians in the jungle.

(*b*) The crude birth rate recorded in Venezuela has increased considerably since 1936 (graph VII.4). Before 1936 it fluctuated around an average of 28 per thousand, and since 1955 it has apparently been stabilized around 45 per thousand. Thus, it increased by almost 70 per cent between 1936 and 1955. A part of this increase is no doubt due to improvements in the registration of births, but an increase in fertility due to improved economic conditions cannot be excluded.

This possible increase in fertility combined with a change in the number of cases of under-enumeration of children aged 0 to 4 might give the impression of an invariable age distribution, while in actual fact there was a rejuvenation at the base of the population pyramid. If we confine ourselves to the population aged 5 and over, however, we almost completely eliminate the effects of this possible rejuvenation, because children over the age of 5 are generally much more accurately enumerated than children from 0 to 4 and also because, if there was an increase in fertility after 1936, the population aged 5 and over enumerated in the 1950 census was scarcely affected by the increase. Table VII.9 gives the distribution by five-year age groups of the population aged 5 and over, as enumerated in the 1936, 1941 and 1950 censuses. We can assume in this case that the age distribution of the population aged 5 and over varied very little from census to census.

We shall select from the network of intermediate model stable populations the population whose age distribution above the age of 5 coincides, at least by broad age groups, with the age distribution observed in Venezuela in the 1936, 1941 and 1950 censuses. We have drawn graph VII.5 along the same lines as graph VII.3, but this time only



Graph VII.4. Variations in the crude birth rate recorded in Venezuela from 1910 to 1962

TABLE VII.9. DISTRIBUTION OF THE POPULATION OF VENEZUELA AGED 5 AND OVER, BY FIVE-YEAR AGE GROUPS, AS RECORDED IN THE CENSUSES OF 1936, 1941 AND 1950 ([a])

(Figures taken from table VII.8)

Age group (years)	*1936*	*1941*	*1950*
5 and over . .	100 000	100 000	100 000
5-9	16 330	16 004	16 448
10-14	14 273	14 489	13 753
15-19	12 092	12 325	11 884
20-24	11 535	11 430	11 336
25-29	9 870	9 849	9 587
30-34	7 541	7 699	7 840
35-39	6 934	6 592	7 215
40-44	5 842	5 832	5 734
45-49	4 336	4 656	4 422
50-54	3 850	3 622	3 942
55-59	2 252	2 345	2 437
60-64	2 311	2 224	2 213
65-69	1 023	1 079	1 146
70-74	805	824	916
75-79	379	412	449
80-84	353	353	396
85-89	121	130	129
90-94	78	79	93
95-99	36	38	47
100 and over .	37	19	14

([a]) Excluding Indians in the jungle.

for the population aged 5 and over (whereas graph VII.3 related to the total population). Moreover, we have reproduced only that portion of the graph within which the age distribution of Venezuela occurs.

The distribution by broad age groups of the population aged 5 and over at the three censuses in question was as follows (figures taken from table VII.9):

Age group (years)	*1936*	*1941*	*1950*
5-14	30 603	30 493	30 201
15-59	64 254	64 349	64 396
60 and over . .	5 143	5 158	5 403
All ages . . .	100 000	100 000	100 000

Between 1941 and 1950 there was a resurgence of immigration into Venezuela, and it is preferable to exclude aliens when comparing age structures. This can be achieved roughly by taking the non-alien population for 1941 and the population born in Venezuela for 1950. For 1936, we do not know the age distribution of aliens, but only their total number (45,184) which is close to the number of aliens enumerated in the 1941 census (47,704). Assuming that the distribution of aliens by age group was the same in 1936 as in 1941, we obtain the following three distributions by age groups (table VII.10):

TABLE VII.10. DISTRIBUTION OF THE NON-ALIEN POPULATION OF VENEZUELA AT VARIOUS CENSUSES BY BROAD AGE GROUPS

Age group (years)	*Non-alien population*		*Population born in Venezuela*	*Average of the three censuses*
	1936	*1941*	*1950*	
5-14 . . .	30 931	30 820	31 201	30 984
15-59 . . .	64 007	64 102	63 350	63 820
60 and over	5 062	5 078	5 449	5 196
All ages. . .	100 000	100 000	100 000	100 000

The average distribution for the three censuses is given in the last column of table VII.10. In graph VII.5 a point M, with ordinate 30.984 and abscissa 5.196, corresponds to this average distribution. The curves corresponding to a gross reproduction rate of 3.00 and an expectation of life at birth $^0e_0 = 37.5$ years respectively pass through this point M. Let us now consider the meaning of these two values of fertility and mortality in terms of the population of Venezuela.

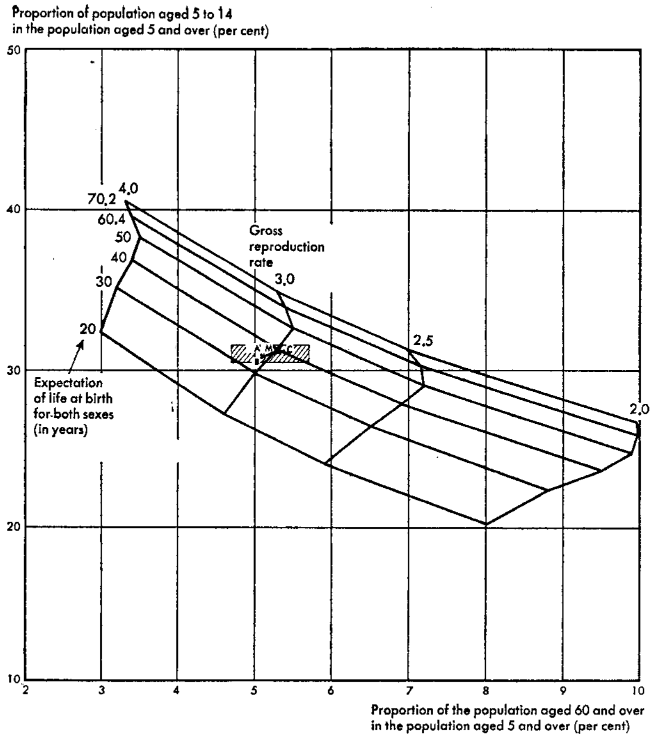
We showed in the introduction to this chapter how the network of stable populations employed here could be considered to be a network of quasi-stable populations. It will be recalled that quasi-stable populations are, by definition, populations with constant fertility but with a mortality varying within a universe of model mortalities. Experience has also shown that, when these conditions are realized, the age structure of quasi-stable populations does not vary greatly.

In a graph such as graph VII.5, a quasi-stable population is represented by a point which will vary within a small area (owing to the slight variability of the age structure) with only small deviations from the constant fertility curve [8] corresponding to the fertility of the quasi-stable population in question.

Let us revert to the three age distributions in table VII.10. They are represented in graph VII.5 by the line ABMC, which remains within a small area and hardly deviates from the constant fertility curve corresponding to a gross reproduction rate of 3.0. This line corresponds to the evolution of quasi-stable populations, and it is reasonable to suppose that a quasi-stable population which followed the pattern ABMC would provide a good picture of the population trends of Venezuela from 1936 to 1950.[9] However, this is not the only development of a quasi-stable population which is compatible with the results of the three censuses. There are some errors in the census results, and the three age structures are known only approximately. In fact, all quasi-stable populations capable of being represented by lines remaining within a small area around the point M could be considered compatible with the results of the censuses. It is difficult to

[8] If we had a true network of quasi-stable populations, the representative point would remain on the constant fertility curve. Here we are taking a network of stable populations as an approximation of a network of quasi-stable populations, and the representative point therefore deviates from the constant fertility curve but the deviations are small.

[9] Bearing in mind the inexactitude of censuses, one would be quite as justified in assimilating the population trends of Venezuela from 1936 to 1950 to that of a quasi-Malthusian population and applying the methods set out in chapters II and IV.



Graph VII.5. Relationships between age distribution above the age of 5, mortality level and fertility level in the network of intermediate model stable populations

define the term "a small area" precisely; its meaning depends on the degree of precision which may reasonably be attributed to the knowledge of the age structure acquired at the various censuses. For example, at opposite extremes, the following two structures may both be considered compatible with the results of the three censuses:

5-14 years	30.5	31.5
15-59 years	63.8	63.8
60 and over	5.7	4.7
	100.0	100.0

The two extreme structures define the shaded area in graph VII.5. Consequently, any quasi-stable population falling within that area gives a picture of the population trend of Venezuela from 1936 to 1950.

In all these populations, fertility will remain near the level corresponding to a gross reproduction rate of 3.00.[10] We may therefore conclude that from 1936 to 1950 fertility of Venezuela remained at about this level, varying only slightly. However, as we have taken into account only the population aged 5 and over, children bornafter 1945 have not been included in the computation and the value obtained for the gross reproduction rate is applicable only to the years before 1945. It is easy to calculate the crude birth rate from the gross reproduction rate. Here, use may be made of graph VII.2, from which we obtain $b = 45$ per thousand for a gross reproduction rate of 3.00. We may therefore say that before 1945 the crude birth rate of the population of Venezuela remained with little variation around an average level of 45 per thousand.

[10] In the shaded rectangle, the gross reproduction rate remains within the limits 2.84 and 3.20.

A comparison of this level with the recorded level brings us back to the possibility, which we have thus far left aside, of an increase in fertility after 1936. A crude birth rate of 45 per thousand represents a very high rate in the gamut of human natality. Consequently, if there has been any increase in fertility since 1936, it cannot have been very great. According to the Venezuelan Government Statistical Office, the registration of births, which was very incomplete in the past, has improved considerably in recent years and may now be regarded as complete; yet the recorded crude birth rate has for some years remained at about 45 per thousand, i.e., at the pre-1945 level. This is a further reason for supposing that the increase in fertility can only have been very small. As a first approximation we can without doubt ignore it and assume that up to 1960, the latest year for which birth statistics are available, fertility in Venezuela remained more or less constant, at a level corresponding to a gross reproduction rate of 3.00, which in turn corresponds to a crude birth rate of 45 per thousand.

In the case of mortality the results are less satisfactory. In all quasi-stable populations falling inside the shaded area of graph VII.5, the expectation of life at birth may vary between 28 and 46 years. It can therefore be said that from 1936 to 1950 mortality in Venezuela remained between those two limits. This does not tell us very much.

The following important conclusion therefore emerges: *knowledge of the age distribution of a population which can be assimilated to a quasi-stable population provides a reliable estimate of fertility, but gives little information about mortality.*

Graphs VII.3 and VII.5 are but two examples among many of the relationships which exist between the age structure and the levels of fertility and mortality, and all graphs of this type lead to the same conclusion. In practice, one chooses the graph best suited to each individual case. If, for example, there is reason to believe that one age group is more accurately enumerated than another, the proportion of persons within that age group will preferably be chosen for the horizontal or vertical axes of the graph.

We now give a further example which could be of considerable practical use. Table VII.11 gives the ratio f of the number of women aged 15 to 44 to the number of women aged 45 and over in the thirty-six female stable populations. It is immediately apparent that the ratio is very close to the gross reproduction rate, and it is more or less independent of the mortality.

For Venezuela in 1950, the ratio f in respect of the native population is:

$$f = \frac{1\,064\,368}{335\,626} = 3.171$$

According to table VII.11, the gross reproduction rate corresponding to this Venezuelan age structure[11] is R = 3.00.

[11] The estimate of the gross reproduction rate, made on the basis of the ratio f, relates, of course, to a period ending fifteen years before the census, since any variation in fertility during the fifteen years preceding the census does not affect the ratio f. Consequently, the gross reproduction rate of 3.00 which we obtain here for Venezuela relates to the years before 1936.

TABLE VII.11. RATIO *f* OF THE NUMBER OF WOMEN AGED 15 TO 44 TO THE NUMBER OF WOMEN AGED 45 AND OVER IN THE INTERMEDIATE MODEL STABLE POPULATIONS

Expectation of life at birth (years)	*Gross reproduction rate* 4.0	3.0	2.5	2.0	1.5	1.0
20	4.296	3.199	2.654	2.112	1.569	1.031
30	4.278	3.158	2.604	2.055	1.513	0.978
40	4.243	3.110	2.553	2.002	1.460	0.931
50	4.276	3.117	2.548	1.987	1.439	0.906
60.4	4.410	3.201	2.608	2.026	1.458	0.909
70.2	4.571	3.317	2.624	2.083	1.492	0.924

Ratio *f* can be put to valuable use in sample surveys. It is close to the ratio between the number of women who have passed the age of puberty and have not yet reached the menopause and the number of women who have passed the menopause.

In countries where the people are uncertain of their age, it is difficult to establish an exact age structure. It is, however, relatively easy to ascertain the number of women who have reached the age of puberty and the number of women who have passed the menopause. This makes it possible to compute the ratio *f*, or at least to allocate an approximate value to it. For example, the 1950 census of the Sudan yielded a figure of 2,272,000 women who had passed the age of puberty but had not yet reached the menopause and a figure of 724,000 who had passed the menopause. An approximate value for the ratio *f* is therefore:

$$f = \frac{2\,272}{724} = 3.1381$$

A gross reproduction rate of 3.0 appears compatible with this value of the ratio *f*.

If we could know by means of supplementary sampling, for instance, the average ages at which women stated that they had reached puberty and menopause, a method could easily be devised by preparing, with the aid of the data given in annex III, a table similar to table VII.11, using for limit age group the average ages at which puberty and menopause were said to have occurred.

E. Correction of the age structure

We are now in a position to correct the errors in the enumeration of children aged 0 to 4 which occurred in the three censuses we are considering. We give below the ratio of the number of children aged 0 to 4 to the number of persons aged 5 and over in some intermediate model stable populations.

Gross reproduction rate	*Expectation of life at birth for both sexes (in years)*	*Ratio of the number of children aged 0 to 4 to the number of persons aged 5 and over*
	20	0.185
	30	0.197
3.00	40	0.206
	50	0.216
	60.4	0.225
	70.2	0.233

Within the shaded area of graph VII.5, it may be assumed that this ratio does not deviate greatly from a figure of 0.21. In the three Venezuelan censuses we find the following ratios :

Year of census	*Ratio of the number of children aged 0 to 4 to the number of persons aged 5 and over*	*Percentage of children aged 0 to 4 enumerated*
1936	0.164	78.0
1941	0.177	84.1
1950	0.203	96.4

By comparing these ratios with the expected value of 0.21, we can estimate the percentage of children aged 0 to 4 who were enumerated. Dividing the number of children enumerated by the percentages, we obtain the following corrected figures :

Year of census	*Children aged 0 to 4 enumerated*	*Percentage of children aged 0 to 4 enumerated*	*Corrected figures for children aged 0 to 4*
1936	473 628	78.0	607 215
1941	577 993	84.1	687 268
1950	847 748	96.4	879 406

TABLE VII. 12. VENEZUELA — FEMALE POPULATION

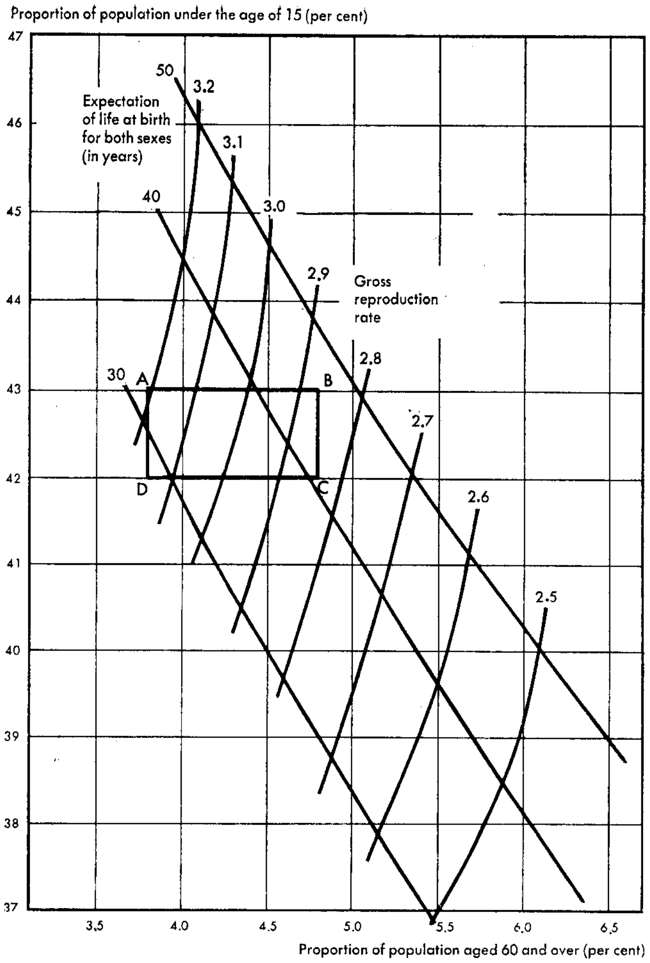
Age group (years)	*1936*	*1941*	*1950*
0-4	299 369 (a)	339 501 (b)	430 567 (c)
5-9	230 133	255 893	334 560
10-14	197 209	226 819	276 204
15-44	812 239	902 401	1 105 480
20-49	687 340	763 031	940 041
25-54	572 025	633 165	784 543
F_0 = 0-4/15-44 . .	0.3685	0.3762	0.3895
F_1 = 5-9/20-49 . .	0.3348	0.3354	0.3559
F_2 = 10-14/25-54 .	0.3448	0.3582	0.3521

(a) Corrected figures 233 508/0.780 = 299 369.
(b) Corrected figures 285 520/0.841 = 339 501.
(c) Corrected figures 415 067/0.964 = 430 567.

It is possible, using the corrected figures, to calculate the ratios F_0, F_1, F_2 and to compare them with the same ratios in table VII.7. For this purpose, we shall assume that the percentage of children aged 0 to 4 who were enumerated is the same for boys and girls. Table VII.12 provides the data for the computation of F_0, F_1 and F_2. The improvement in the census enumeration of children aged 0 to 4 may be seen in the increase over the years in the percentage of children enumerated. In 1950 the degree of precision appears comparable to that achieved in countries with complete demographic statistics. The distribution of the population of Venezuela by broad age groups at the three censuses is given, with the corrected figures, in table VII.13.

TABLE VII.13. DISTRIBUTION OF THE POPULATION OF VENEZUELA BY BROAD AGE GROUPS AT THE 1936, 1941 AND 1950 CENSUSES, CORRECTED TO ALLOW FOR UNDER-ENUMERATION OF CHILDREN AGED 0 TO 14

Age group (years)	*Year of census* 1936	1941	1950	*Average of the three censuses*
0-14	42 661	42 576	42 334	42 524
15-59	53 090	53 164	53 200	53 151
60 and over .	4 249	4 260	4 466	4 325
All ages . . .	100 000	100 000	100 000	100 000



Graph VII.6. Relationships between age distribution, mortality level and fertility level in the area of the network of intermediate model stable populations containing the population of Venezuela from 1936 to 1950 (enlargement of graph VII.3)

Keeping the same margin of error as in our previous calculation, we shall find that the age distribution of the population of Venezuela from 1936 to 1950 remains within the limits set by the following two extreme distributions:

0-14	42.0	43.0
15-59	53.2	53.2
60 and over	4.8	3.8
All ages	100.0	100.0

We can now use graph VII.3, or rather graph VII.6, which is an enlargement of the area of graph VII.3 within which the two extreme distributions are situated. All quasi-stable populations situated in the rectangle ABCD of graph VII.6 are compatible with the census results. The graph yields the same readings as graph VII.5, i.e., a gross reproduction rate within the limits of 3.20 and 2.84 and an expectation of life at birth for both sexes within the limits of 28 and 46 years.

Before making further use of the networks of intermediate model stable populations, we must remember that everything said above assumes a mortality conforming to the intermediate model life tables. Until the effect of the deviations in relation to these model life tables has been studied,[12] the results obtained must be considered provisional.

[12] This will be done in chapter VIII.

F. Age structure and rate of natural variation

We shall now assume that, in addition to the age structure, the rate of natural variation is known. This further information makes it possible to achieve greatly improved estimates of mortality and fertility, in a way that may be readily seen. Turning once more to graph VII.6, we are able to plot on it a third network of lines: those of the constant intrinsic rate of natural variation. This has been done on graph VII.7. We saw how all the points in the rectangle ABCD could represent the population of Venezuela at any time between 1936 and 1950. If we assume it to be known that, at a given time, the rate of natural variation was between 22 and 23 per thousand, the corresponding representative points must lie within the part of the rectangle enclosed by the curves marked 22 and 23, i.e., within the small shaded area. As will be seen, this greatly reduces the indeterminateness of the levels of fertility and mortality. Let us apply this method to Venezuela.[13]

Table VII.14 shows how it is possible to estimate the rate of natural variation of the population of Venezuela between the 1941 and 1950 censuses. The total corrected population, less aliens and naturalized Venezuelans, as well as the Indians in the jungle, has been regarded as a closed population and the rate of variation computed on the basis of these population figures has been considered approximately as the rate of natural variation. The value thus obtained for the rate of variation has been assigned to the median year between the two censuses. Lastly, we find a rate of natural variation equivalent to 22.6 per thousand. We have assumed that this was only an approximation and that the true rate lay between 22 and 23 per thousand. This brings us into the small shaded area of graph VII.6. Within this area, the gross reproduction rate lies between 2.84 and 3.00 and the expectation of life at birth for both sexes lies between 39 and 43 years. These values provide an estimate of the fertility and mortality of the population of Venezuela midway between the two censuses, i.e., about mid-1945.[14]

Graph VII.7 is an example of a graph containing three elements of stable populations:

(*a*) The proportion of persons under the age of 15;

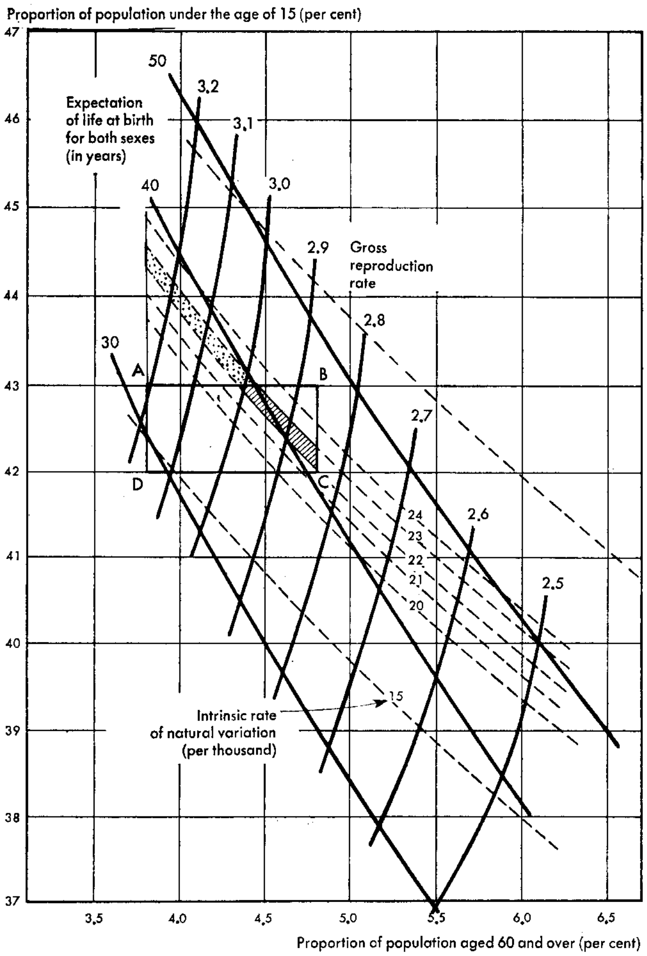
(*b*) The proportion of persons aged 60 and over;

(*c*) The intrinsic rate of natural variation.

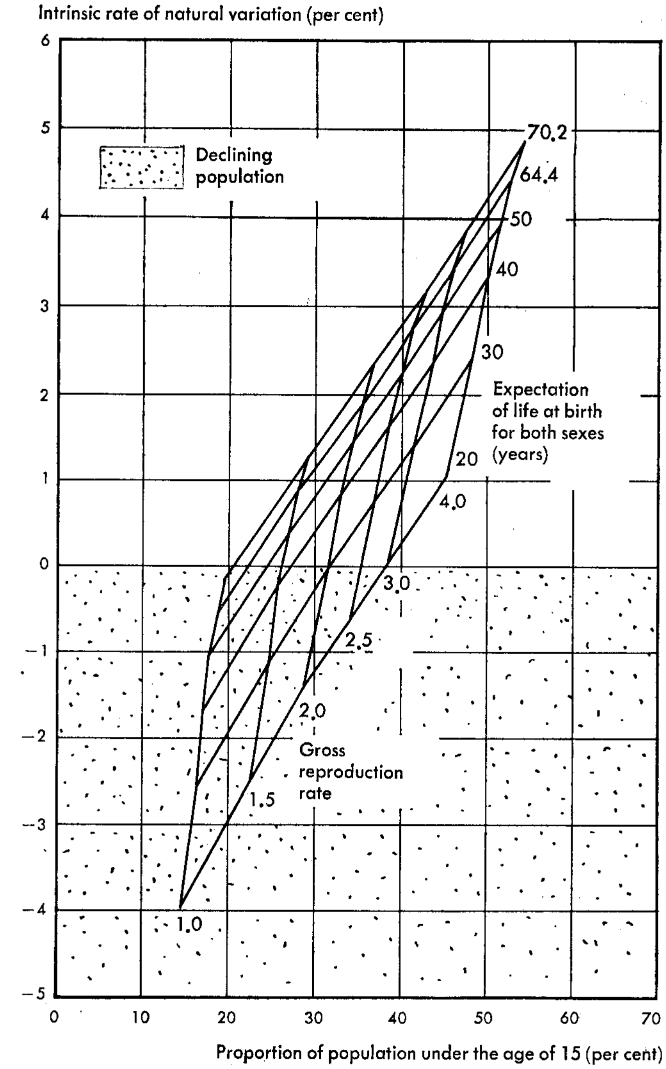
Such graphs clearly provide more reliable estimates than graphs involving only two factors. They have, however, the disadvantage of being quite difficult to draw, and in a specific case it may be sufficient to only one characteristic of the age structure—for example, the proportion of persons under the age of 15—and the intrinsic rate of natural variation. Graph VII.8, an enlargement of which is given in graph VII.9, is of this

[13] If we assume that the population of Venezuela was quasi-Malthusian from 1936 to 1950—and it has already been shown that this assumption is perfectly compatible with the results of the three censuses—then knowledge of the age structure and of the rate of natural variation completely determines the mortality and fertility of the population (see the first example in chapter IV). However, the methods which must then be employed involve relatively lengthy calculations, and it might be desirable to obtain rapid—though less precise—estimates of fertility and mortality. That is the purpose of graphs such as graph VII.7.

[14] The censuses relate to about the end in the respective years: 7 December 1941 and 26 November 1950.



Graph VII.7. Relationships between age distribution, mortality level, fertility level and intrinsic rate of natural variation in the area of the network of intermediate model stable populations containing the population of Venezuela from 1936 to 1950 (graph similar to graph VII.6, plus curves of constant intrinsic rates of natural variation)

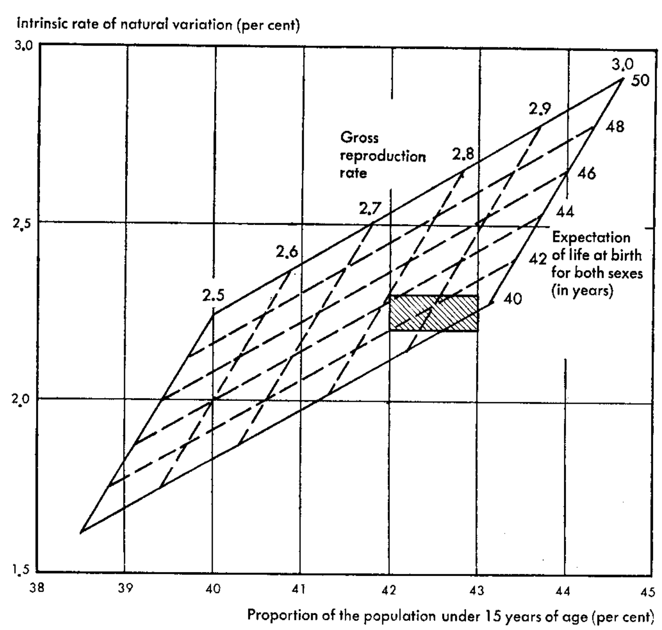


Graph VII.8. Relationships between the proportion of the population under the age of 15 and the intrinsic rate of natural variation in the network of intermediate model stable populations

TABLE VII.14. ESTIMATE OF THE ANNUAL RATE OF NATURAL VARIATION OF THE POPULATION OF VENEZUELA FROM 1941 TO 1950

	1941	*1950*
Total population enumerated	3 850 771	5 034 838
Correction for under-enumeration of children aged 0-4	109 275	31 658
Corrected total population (A)	3 960 046	5 066 496
Naturalized Venezuelans	2 224	12 622
Aliens	47 704	194 145
TOTAL, number of persons of foreign origin (B)	49 928	206 767
Indians in the jungle (C)	100 000	56 705
(A) + (C) − (B)	4 010 118	4 916 434
Increase between the censuses.		905 316
Average annual (a) increase between the censuses		100 591
Average population between the censuses.		4 463 576
Average annual rate of variation (per thousand)		22.56

(a) The 1941 census relates to 7 December 1941 and the 1950 census to 26 November 1950. An interval of nine years has been assumed.



Graph VII.9. Relationships between the proportion of the population under the age of 15 and the intrinsic rate of natural variation in the area of the network of intermediate model stable populations containing the population of Venezuela from 1936 to 1950 (enlargement of graph VII.8)

type. In the shaded area of graph VII.9 the gross reproduction rate lies between 2.80 and 3.02 and the expectation of life lies between 39 and 44.3 years. The accuracy of the estimates obtained is almost as good as in the case of graph VII.6.[15]

Graphs such as graph VII.7 must be used with care. We have used it above to delimit values for the gross reproduction rate. We might be tempted to perform the operation in reverse, i.e., to delimit values for the rate of natural variation from the age structure and the gross reproduction rate. The precision of an estimate obtained in this way would be very poor. If we retain, for example, the rectangle already drawn as the area within which the age structure varies and add that the gross reproduction rate lies between 3.0 and 3.1, graph VII.7 shows that the rate of natural variation then lies between 15 and 22 per thousand.

Graphs like VII.7 somewhat resemble those chemical reactions which occur in only one direction. Here, the gross reproduction rate R′ appears as a function of the age structure C and of the rate of natural variation *r*. If we write $R' = f(C, r)$, there is nothing, in theory, to prevent us from expressing *r* as a function of R′ and C, but as the estimate obtained for *r* is insufficiently precise, we shall write, as with chemical reactions:

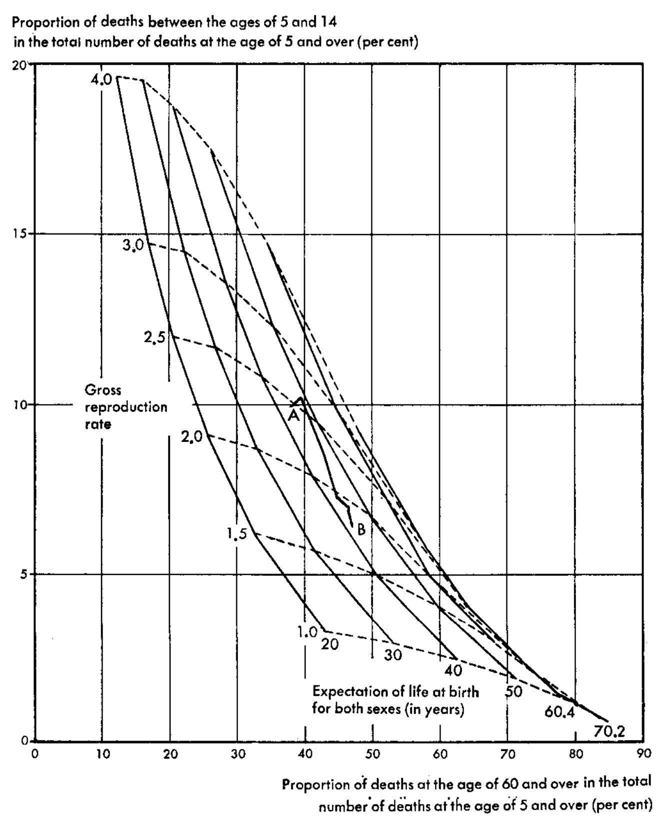
$$R' \rightarrow f(C, r).$$

G. Age structure of deaths

In graph VII.10, the horizontal axis represents the proportion of deaths at the age of 60 and over in the total number of deaths at the age of 5 and over and the vertical axis represents the proportion of deaths between the ages of 5 and 14 in the total number of deaths at the age of 5 and over. In theory, these two characteristics are sufficient to determine a stable population within the network. However, it will be noticed that in the case of low mortalities the curves of constant mortality are very close together. We are almost in a situation in which the network turns back on itself and in which the theory stated above, namely, that any two characteristics are sufficient to define a stable population within the network, no longer holds true. In the case of medium and high mortalities, however, the curves are spaced well apart, and for such mortality values the use of graph VII.10 involves no risk of ambiguity.

We have plotted in graph VII.10 the line AB, using the age-specific death statistics of China (Taiwan)[16] for 1950-1956. We are dealing here with a population whose fertility has remained practically invariable over a long period of time, the gross reproduction rate having been constant at 3.0. The curve for 1950-1956 ought therefore to lie in the neighbourhood of the curve of constant fertility marked 3.0, but in fact there is a considerable discrepancy between the two. According to the line AB, China (Taiwan) should have had, during 1950-1956, an almost constant mortality rate combined with declining fertility, causing the gross reproduction rate to fall from 2.5 to 1.7. Yet births and deaths are properly registered in China (Taiwan), and there has been no decline in fertility; on the contrary, mortality has declined sharply. We must therefore conclude that the age distribution of deaths is incorrect. The case of China (Taiwan) is not an isolated one. Estimates of mortality and fertility based on age distribution of deaths invariably differ from those obtained by other indirect methods and, worse still, from those provided by statistics based on direct observation in cases where such statistics are reliable. We shall therefore disregard the other graphs in which the age distribution of deaths is combined with either the age distribution of the population or the rate of natural variation, since they are, for this reason, of limited value.

Does this mean that we must altogether abandon the use of variations in the age distribution of recorded deaths for measuring variations in mortality? Even if the line AB in graph VII.10 deviates from the constant fertility curve corresponding to a gross reproduction rate of 3.0, its direction is the same as that of the curve. What is mainly at fault, therefore, is the line's position in the graph. This being so, while we cannot hope to measure the absolute level of mortality, have we at least some hope of measuring its movement?



Graph VII. 10. Relationships between the age distribution of deaths above the age of 5, the mortality level and the fertility level in the network of intermediate model stable populations

[15] In some cases, the loss of precision caused by leaving out one variable may be much greater. If, for example, we used a graph similar to graph VII.9 but showing on the horizontal axis the proportion of the population over the age of 60, this would have the effect of widening our limits to embrace the area represented on graph VII.7 by the shaded and dotted areas combined.

[16] The population of Venezuela has not been used as an example here for two reasons: (*a*) Venezuelan death statistics are published by age groups, from which it is not possible to calculate the proportion of deaths between the ages of 5 and 14 in the total number of deaths; and (*b*) as the use of graph VII.10 almost always results in estimates different from those obtained by using other graphs, it appeared preferable to take a country from whose statistics the levels of mortality and fertility could be definitely ascertained, so that it might be possible to evaluate the usefulness of the graph. China (Taiwan) partly fulfils these requirements. However, we shall shortly revert to Venezuela.

In table VII.15 we have computed for the thirty-six populations of the network of intermediate model stable populations the ratio of deaths of 60 and over to the total number of deaths of 5 and over, and we have plotted in graph VII.11 the variations in this ratio as a function of mortality and fertility.

TABLE VII.15. PROPORTION OF DEATHS OF 60 AND OVER IN THE TOTAL NUMBER OF DEATHS OF 5 AND OVER IN THE NETWORK OF INTERMEDIATE MODEL STABLE POPULATIONS

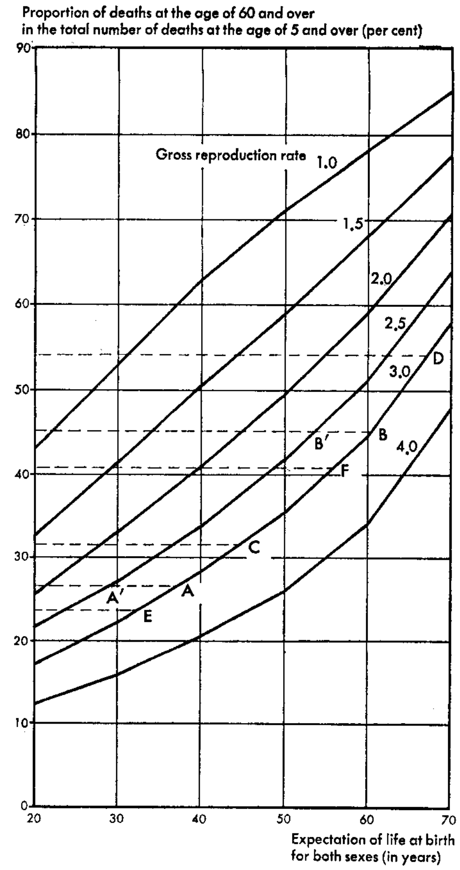
Expectation of life at birth for both sexes (in years)	*Gross reproduction rate* 1.0	1.5	2.0	2.5	3.0	4.0
20	43.1	32.5	25.5	21.7	16.9	12.1
30	53.1	41.3	32.9	26.8	22.1	15.8
40	62.6	50.4	41.0	33.8	28.3	20.4
50	70.8	59.2	49.6	41.7	35.4	26.0
60.4	78.1	68.1	59.1	51.3	44.6	34.1
70.2	84.9	77.5	70.6	64.1	58.2	48.0

Let us take a quasi-stable population whose gross reproduction rate has remained constant at a value of 3.0. If the age distribution of deaths is known exactly, the ratio of deaths of 60 and over to the total number of deaths of 5 and over will trace the curve marked 3.0 on graph VII.11.

Table VII.16 contains the values for Venezuela from 1939 to 1960 of the ratio of deaths at the age of 60 and over to the total number of deaths at the age of 5 and over.

Let us assume that the age distribution of deaths is known exactly and that fertility is such that the gross reproduction rate equals 3.0.[17] Reading from the curve marked 3.0 on graph VII.11, it is possible to find for each ratio in table VII.16 a corresponding value for the expectation of life at birth. Thus, we see that between 1939 and 1960 the expectation of life rose from 37.0 years to 60.2 years, an increase of 23.2 years (points A and B).

[17] This is the value which our study of the age structure yielded.



Graph VII.11. Variations in age distribution of deaths in the network of intermediate model stable populations

We have assumed a gross reproduction rate of 3.0, but this is only an estimate. Let us suppose that the gross reproduction rate is really 2.5, in which case, instead of reading from the curve marked 3.0, we should read from the curve marked 2.5. We should then find that from 1939 to 1960 the expectation of life at birth had risen from

TABLE VII.16. VARIATIONS IN THE PROPORTION OF RECORDED DEATHS AT THE AGE OF 60 AND OVER IN THE TOTAL NUMBER OF REGISTERED DEATHS AT THE AGE OF 5 AND OVER IN VENEZUELA, 1939-1960

Age (in years)	*Year of registration* 1939	1940	1941	1942	1943	1944	1945	1946	1947	1948	1949
60 and over	10 320	9 753	9 785	10 233	10 979	12 008	11 613	11 056	11 160	10 845	10 780
5 and over	39 227	36 224	36 495	37 167	37 998	40 577	38 266	36 664	34 946	33 398	31 570
Ratio of the two preceding lines	0.2631	0.2692	0.2681	0.2753	0.2889	0.2959	0.3035	0.3015	0.3193	0.3247	0.3415

Age (in years)	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
60 and over	10 465	11 506	11 639	11 870	12 375	13 022	13 549	14 124	13 858	14 558	13 940
5 and over	29 752	30 564	30 195	29 309	30 460	31 106	30 693	33 560	32 603	33 349	30 854
Ratio of the two preceding lines	0.3517	0.3765	0.3855	0.4050	0.4063	0.4186	0.4414	0.4208	0.4251	0.4365	0.4518

NOTE: From 1958 onwards, the figures are not strictly comparable with those for the years before 1957.

29.7 to 53.8. This would be an increase of 24.1 years (points A′ and B′), i.e., almost the same increase as in the case of our last reading. On the other hand, the mortality levels are not the same in the two readings. Thus, ignorance of the true level of fertility affects the measurement of the absolute level of mortality but has practically no effect on the measurement of variations in mortality.

It has thus far been assumed that the age distribution of deaths was adequately known. This is notoriously not the case. There are two main causes of error which cause the ratio of deaths of 60 and over to the total number of deaths of 5 and over to vary:

(*a*) Misreporting of age;

(*b*) Differences in under-registration at different ages.

Misreporting of age is of various kinds. First, there are those who have forgotten their age or whose age is stated by persons other than themselves. In these cases, there is a tendency to round off ages and to express them in multiples of 10 or, somewhat less often, in ages ending with 5. The age of 60, in particular, is noteworthy, and this tends to increase the ratio in question.

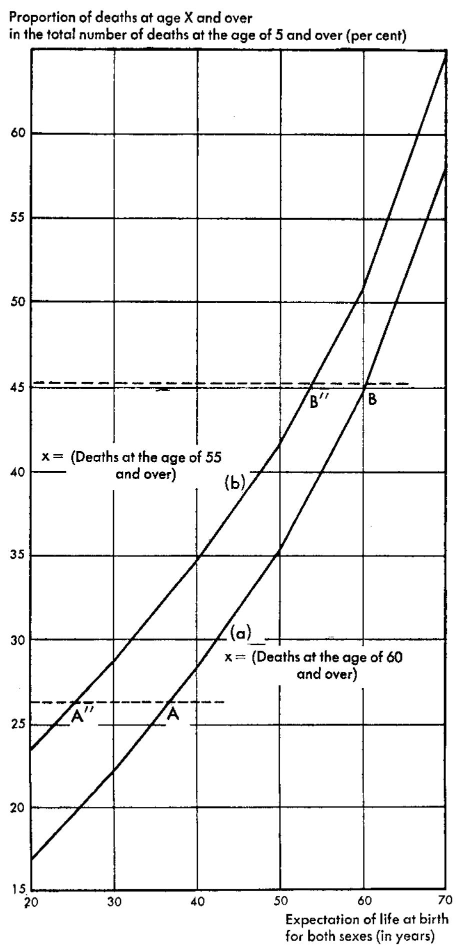
There are also persons who know their age but, for various reasons, state that they are younger or older. Old people who do this usually state a higher age, and this also tends to increase the proportion of deaths of 60 and over in the total number of deaths of 5 and over.

Ultimately, it is as though the group of deaths at 60 and over were really a group of deaths at some earlier age—for example, at 55 and over. In graph VII.12 we have plotted, for a gross reproduction rate of 3.0, the curve of variation in the proportion of deaths at 60 and over in the total number of deaths at 5 and over (curve (*a*), similar to that in graph VII.11), and the curve of variation in the proportion of deaths at 55 and over in the total number of deaths at 5 and over (curve (*b*)).

We assumed above, when estimating the variations in the Venezuelan mortality rate, that the age distribution of deaths was well known and we read the expectation of life at birth from curve (*a*). If we assume that the age distribution of deaths is not well known and that we are really dealing with a group of deaths at 55 and over, then we must read from curve (*b*). We then see from 1939 to 1960 the expectation of life at birth rose from 25.8 to 53.9 years, an increase of 28.1 (points A″ and B″). Here, again, the increase is of the same order of magnitude as in the other readings, but the level is very different.

Differences in under-registration at different ages are more difficult to evaluate. We can expect the proportion of deaths registered to vary in different socio-economic groups—urban and rural populations, highly educated and comparatively uneducated people, migrants and non-migrants, persons living in households and those living in institutions, persons related to heads of households and lodgers who are not part of the family and so forth. These various groups do not have a common age structure, and the differences in the completeness of registration of deaths from group to group are reflected in the total death statistics by variations in the proportion of deaths registered at different ages.

Nevertheless, wherever a high proportion of the total number of deaths is registered, such variations in registration at different ages are unlikely to be of much significance. For example, where 80 per cent of deaths between the ages of 5 and 59 are registered, it can no doubt be assumed, with little likelihood of error, that the proportion of deaths above the age of 59 which are registered is between 60 and 90 per cent. A differential variation of that order would have the following effects on the proportion of deaths at 60 and over in the total number of deaths at 5 and over (assuming a gross reproduction rate of 3.00):



Graph VII.12. Variations in the age distribution of deaths in two intermediate model stable populations corresponding to a gross reproduction rate of 3.00 combined with various mortality levels

	Differential effect of under-registration at different ages on the proportion of deaths at the age of 60 and over in the total number of deaths at the age of 5 and over (per cent)	
	80 per cent of deaths registered between the ages of 5 and 59	
Expectation of life at birth for both sexes (in years)	*60 per cent of deaths registered over the age of 59*	*90 per cent of deaths registered over the age of 59*
20	−21.9	+10.1
30	−20.4	+ 9.5
40	−19.4	+ 8.5
50	−17.8	+ 7.3
60.4	−15.5	+ 6.5
70.2	−12.3	+ 4.8

For expectations of life at birth ranging from 30 to 60 years, such differential under-registration has the effect, in one case, of reducing by about 20 per cent the proportion of deaths at 60 and over in the total number of deaths at 5 and over and, in the other case, of increasing it by about 9 per cent.

If Venezuela were in the first category, we should have to increase the ratios given in table VII.16 by 20 per cent and, using the increased ratios, read from the curve marked 3.0 in graph VII.11. In that case, between 1939 and 1960 the expectation of life at birth would rise from 44.4 to 67.0 years, an increase of 22.6 years (points C and D). If, on the other hand, Venezuela fell into the second category, we should have to reduce the ratios given in table VII.16 by 9 per cent and, using these adjusted ratios, read from the same curve. It would then be found that the expectation of life had increased between 1939 and 1960 from 32.5 to 55.9 years, an increase of 23.4 years (points E and F).

In every case that we have considered, the increase has always been of the same order of magnitude and the only variation has been in the level at which the increase occurred. The results obtained in the various cases are assembled in table VII.17.

Consequently, *if the degree of error in the registration of deaths remains more or less constant over the years*, the ratio of deaths at the age of 60 and over to the total number of deaths at the age of 5 and over appears to function as an index with the aid of which we can make fairly reliable estimates of variations in the mortality, at least where the proportion of deaths registered is relatively high. *It does not, however, enable us to estimate the level at which these variations occur.* In our last table we found that increases of the same order of magnitude occurred at levels varying by as much as 50 per cent.

The following is a summary of the method to be followed in estimating variations in mortality over a given period. We compute, for the beginning and the end of the period, the ratio of deaths at 60 and over, or at any other more suitable age, to the total number of registered deaths at 5 and over. We then read the expectation of life at birth which corresponds to these two ratios on the curve in graph VII.11 corresponding to an approximate value of the gross reproduction rate. The variation in expectation of life which is ascertained in this way provides an estimate of the variation in mortality over the period in question. Our last calculation showed that we need not have a very exact estimate of the gross

TABLE VII.17. ESTIMATED VARIATIONS IN THE MORTALITY OF THE POPULATION OF VENEZUELA BETWEEN 1939 AND 1960 BASED ON VARIOUS ASSUMPTIONS REGARDING THE DEGREE OF ERROR IN THE REGISTRATION OF DEATHS

Expectation of life at birth for both sexes (in years)		*Increase in expectation of life (difference between the two preceding columns)*	*Gross reproduction rate on which reading was based*	*Ratio on which reading was based*
1939	*1960*			
37.0	60.2	23.2	3.00	60 and over, as observed
44.4	67.0	22.6	3.00	60 and over, as observed, increased by 20 per cent
32.5	55.9	23.4	3.00	60 and over, as observed, reduced by 9 per cent
25.8	53.9	28.1	3.00	55 and over, as observed
29.7	53.8	24.1	2.50	60 and over, as observed

TABLE VII.18. INCREASE IN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES (IN YEARS) BETWEEN 1940-1942 AND 1958-1960 IN SEVENTEEN COUNTRIES

(Estimate based on age distribution of deaths)

Venezuela	21	Trinidad and Tobago	12
Panama	19	Nicaragua	11
Ceylon	18	Paraguay	10
		Honduras	10
El Salvador	15	Guatemala	10
China (Taiwan)	15	Colombia	9
Dominican Republic	14	Thailand	8
Mexico	13	India	8
Chile	13	Philippines	7

TABLE VII.19. ESTIMATED AVERAGE ANNUAL DEATH RATES AT THE AGE OF 9.5 AND OVER IN VENEZUELA BETWEEN THE 1941 AND THE 1950 CENSUSES

I. Census of 1941

Age group (years)	*Total population*	*Aliens*	*Non-alien population*	*Indians in the jungle*	*Combined total of Indians in the jungle and non-alien population*
0-4	579 032	1 290	577 742	15 283	593 025
5 and over	3 271 739	46 414	3 225 325	85 317	3 310 642
All ages	3 850 771	47 704	3 803 067	100 000	3 903 667

II. Census of 1950

Age group (years)	*Total population*	*Non-alien population born in Venezuela*	*Indians in the jungle*	*Combined total of Indians in the jungle and non-alien population*
0-13	2 013 055	1 985 328	23 327	2 008 655
14 and over	3 021 783	2 840 779	33 378	2 874 157
All ages	5 034 838	4 826 107	56 705	4 882 812

reproduction rate in order to choose the curve on which the reading is to be made. We have applied this method to the seventeen countries in table VII.18. In order to be sure that we were justified in assimilating the populations of all these countries during the period in question to the developments of quasi-stable populations, it would have been necessary, strictly speaking, to study the variations in their age structures. No such study has yet been made, and it may be debatable whether, in some cases, these populations can be assimilated to quasi-stable populations. However, this can only appyl in particular cases, and what we know in other respects of the demographic situation of the countries in table VII.18 justifies our assimilating their populations to quasi-stable populations. We have estimated the gain in expectation of life between the two periods 1940-1942 and 1958-1960, classifying the countries in diminishing order of gain. There was an interval of eighteen years between the two periods. If the mortality rate of these countries had fallen at the same speed as that recorded in Europe between the two world wars, we could have expected to find an increase in expectation of life of about 9 years. This is roughly true in the case of the last nine countries in the table. In the case of the first three countries in the table, the decline in mortality appears to have been twice as rapid as that which occurred in Europe between the two wars. The decline in the five remaining countries was about one and a half times as rapid.

TABLE VII.20. DEATH RATES (PER THOUSAND) ABOVE THE AGES INDICATED FOR VARIOUS FERTILITY AND MORTALITY LEVELS IN THE NETWORK OF INTERMEDIATE MODEL STABLE POPULATIONS

Age (in years)	*Expectation of life at birth (in years)* 20	30	40	50	60.4	70.2
Gross reproduction rate: 4.0						
0 and over. . . .	53.0	35.3	24.1	16.2	9.4	4.1
1 and over. . . .	34.3	21.8	14.3	9.3	5.5	3.8
5 and over. . . .	26.3	17.2	11.5	7.7	4.9	3.0
10 and over. . . .	27.8	18.6	12.7	8.7	5.7	3.6
15 and over. . . .	31.2	21.4	14.8	10.3	6.9	4.5
20 and over. . . .	35.2	24.3	17.0	12.0	8.2	5.6
25 and over. . . .	39.6	27.3	19.2	13.7	9.7	7.0
Gross reproduction rate: 3.0						
0 and over. . . .	50.2	33.7	23.3	15.8	9.6	4.8
1 and over. . . .	35.3	22.9	15.4	10.3	6.5	2.8
5 and over. . . .	29.3	19.6	13.4	9.2	6.2	4.1
10 and over. . . .	30.9	21.1	14.8	10.3	7.1	4.9
15 and over. . . .	34.3	23.8	16.9	12.0	8.4	5.9
20 and over. . . .	38.2	26.7	19.2	13.8	9.9	7.2
25 and over. . . .	42.5	29.8	21.4	15.6	11.5	8.7
Gross reproduction rate: 2.5						
0 and over. . . .	49.1	33.2	23.2	16.0	10.1	5.5
1 and over. . . .	36.5	24.1	16.5	11.3	7.5	4.7
5 and over. . . .	31.5	21.4	15.0	10.5	7.3	5.1
10 and over. . . .	33.3	23.0	16.4	11.7	8.4	6.0
15 and over. . . .	36.5	25.6	18.5	13.4	9.3	7.1
20 and over. . . .	40.3	28.5	20.8	15.3	11.3	8.4
25 and over. . . .	44.5	31.5	23.1	17.2	13.0	10.0
Gross reproduction rate: 2.0						
0 and over. . . .	48.6	33.6	23.7	16.8	11.1	6.8
1 and over. . . .	38.5	26.2	18.3	13.0	9.0	6.2
5 and over. . . .	34.8	24.3	17.3	12.6	9.1	6.7
10 and over. . . .	36.6	25.9	18.8	13.9	10.2	7.7
15 and over. . . .	39.7	28.5	20.9	15.6	11.7	8.9
20 and over. . . .	43.2	31.3	23.2	17.5	13.3	10.3
25 and over. . . .	47.3	34.4	35.5	19.6	15.1	12.0
Gross reproduction rate: 1.5						
0 and over. . . .	49.7	35.0	25.6	18.8	13.5	9.4
1 and over. . . .	42.7	29.7	21.7	16.1	12.0	9.0
5 and over. . . .	40.0	28.6	21.3	16.1	12.4	9.6
10 and over. . . .	41.7	30.2	22.8	17.4	13.5	10.7
15 and over. . . .	44.5	32.6	24.8	19.1	15.0	12.0
20 and over. . . .	47.9	35.3	27.0	21.0	16.7	13.5
25 and over. . . .	51.7	38.2	29.4	23.1	18.6	15.3
Gross reproduction rate: 1.0						
0 and over. . . .	54.4	39.0	30.9	24.3	19.0	15.1
1 and over. . . .	50.2	36.0	28.7	22.8	18.2	14.9
5 and over. . . .	49.0	36.5	28.7	23.1	18.7	15.6
10 and over. . . .	50.5	37.9	30.1	24.4	19.9	16.8
15 and over. . . .	52.9	40.0	31.9	25.9	21.3	18.1
20 and over. . . .	55.7	42.4	33.9	27.7	22.9	19.6
25 and over. . . .	59.1	45.1	36.2	29.7	24.8	21.4

TABLE VII.21. DEATH RATES (PER THOUSAND) AT THE AGE OF 9.5 AND OVER IN THE NETWORK OF INTERMEDIATE MODEL STABLE POPULATIONS

Gross reproduction rate	*Expectation of life at birth for both sexes (in years)* 20	30	40	50	60.4	70.2
4.0	27.6	18.5	12.6	8.6	5.6	3.5
3.0	30.7	20.9	14.7	10.2	7.0	4.8
2.5	33.1	22.8	16.3	11.6	8.3	5.9
2.0	36.4	25.7	18.6	13.8	10.1	7.6
1.5	41.5	30.0	22.6	17.3	13.4	10.6
1.0	50.3	37.8	30.0	24.3	19.8	16.7

H. Direct estimate of mortality by comparison of two censuses

We referred above to the method of determining the level of mortality by comparing the total numbers of persons in corresponding age groups, sometimes called generations, at different censuses. We shall attempt to show here how the network of intermediate model stable populations can be used when applying this method of comparison between generations by comparing total numbers above certain ages. We shall demonstrate the method, using Venezuela as an example. The data needed for the calculation are given in table VII.19.

In 1941, the non-alien population aged 5 and over numbered 3,310,642.[18] Nine years later, in 1950, the survivors of that population were aged 14 and over. In 1950, the total number of persons aged 14 and over who had been born in Venezuela was 2,874,157.[18] The

TABLE VII.22. ESTIMATED EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES ACCORDING TO THE AVERAGE CRUDE DEATH RATE AT THE AGE OF 9.5 AND OVER IN VENEZUELA BETWEEN 1941 AND 1950

Gross reproduction rate	4.0	3.0	2.5	2.0
Expectation of life (in years)	35.3	38.9	41.9	46.7

[18] Figure corrected to take account of Indians in the jungle.

difference of 436,485 represents approximately the number of deaths at the age of 9.5 years and over $\left(9.5 = \frac{5 + 14}{2}\right)$ which occurred between the two censuses among the population born in Venezuela. Between 1941 and 1950 the total number of persons aged 9.5 years and over averaged 3,092,400.

Between 1941 and 1950, therefore, the average crude annual death rate at 9.5 years and over was 15.7 per thousand.

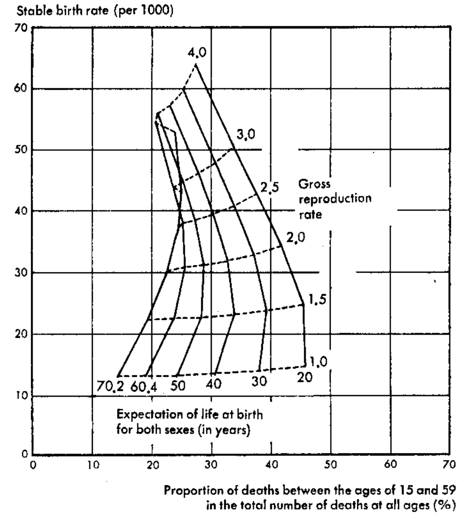
Table VII.20 gives the crude death rate above various ages in the network of intermediate model stable populations. By interpolation, it is possible to determine the crude death rate at 9.5 years and over in that network (table VII.21). A knowledge of the crude death rate at 9.5 years and over is clearly not sufficient to determine the level of mortality. For each gross reproduction rate there is a value of the expectation of life corresponding to a given value of the crude death rate at 9.5 years and over, and these expectations of life are all different. In the case of Venezuela, for example, we have the estimates given in table VII.22. It is therefore necessary to know the fertility rate in order to establish the level of mortality. However, this difficulty should not be exaggerated. Table VII.22 shows that an approximate value for the fertility rate is enough to establish the expectation of life to within a few years. In the case of Venezuela, for example, if the gross reproduction rate is known to lie between 2.8 and 3.2, the expectation of life will lie between 38.2 and 40.1 years.

I. Conclusion

The above examples have been given solely by way of illustration. It would not be difficult to imagine other applications of the network of intermediate model stable populations. Let us now summarize the principal findings.

1. Generally speaking, any two characteristics will determine without any ambiguity a population within the network of intermediate model stable populations. This property, on which all the practical applications are based, is true only as a general rule and must be verified on each occasion. We have already encountered one case where it was not easy to apply (graph VII.10). Here is one case in which the networks of curves turn back on themselves (graph VII.3). The property in question no longer holds good in the area where overlapping occurs.

2. Within a fairly wide field of variations in the fertility and mortality of the intermediate model stable populations, there is an almost fixed relation between the crude birth rate and the gross reproduction rate and a fixed relation between the crude death rate and the expectation of life at birth. With the use of these relations it is possible, by simply reading from a table or graph, to compute the gross reproduction rate from the crude birth rate and the expectation of life at birth from the crude death rate.



Graph VII.13. Relationships between age distribution of deaths, stable birth rate, expectation of life at birth and gross reproduction rate in the network of intermediate model stable populations

3. Variations in mortality have little influence on the age structure of the intermediate model stable populations, whereas variations in fertility have a great deal of influence. It follows that the inexactitude of observed age structures has little effect on estimates of fertility based on the age structure, but makes estimates of the level of mortality extremely uncertain.

4. In order to estimate mortality, other data in addition to the age structure are needed. Knowledge of the rate of natural variation appears most useful in this connexion.

5. Age distribution of deaths is, in general, too imperfectly known to be of use in determining *the level* of mortality and fertility. It can, however, be used to estimate variations in mortality wherever a relatively high proportion of deaths are registered.

6. Finally, it must be constantly borne in mind that the applications given in chapter IV rest on the assumption that the mortality of the populations in question conforms to the pattern of the intermediate model life tables,[19] or, at least, scarcely deviates from it. Since that assumption is difficult to verify, owing to the defective demographic statistics of the countries to which the methods in question are applied, it is essential to consider what effect it had on our estimates of mortality and fertility. This will be done in chapter VIII.

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[19] It must also be borne in mind that, in addition, the methods indicated in chapters II and IV can be used in many cases and that those methods do not generally assume that the mortality of the populations in question conforms to the pattern of any model life tables.

Chapter VIII

DEVIATING NETWORKS OF MODEL STABLE POPULATIONS

As was stated in the preceding chapter, the assumption that mortality varies in conformity with the intermediate model life tables is constantly present in all the developments covered by chapter VII. However, when trying to estimate the mortality of a population, as was done in the case of the population of Venezuela, one does not know whether or not that mortality lies within the universe of the intermediate model life tables. It therefore becomes necessary to examine the effects of the above assumption on the estimates made in chapter VII. For that purpose, the questions dealt with in chapter VII will be reconsidered, using the networks of stable populations which deviate in relation to the intermediate network.

A. Definition of deviating networks

Chapter I and annex II explain how a study of variations of mortality in space and time makes it possible to associate each intermediate model mortality level with upward-deviating and downward-deviating model mortalities between which mortalities of the same level are situated.[1] These deviating model mortalities can therefore be used to trace a network of upward-deviating model stable populations and a network of downward-deviating model stable populations. Practically all observable stable populations will come between these two extreme networks. To construct the network of intermediate model stable populations, six levels of mortality were used, numbered respectively 0, 20, 40, 60, 80 and 100, corresponding to the expectations of life at birth for the two sexes indicated in the second column of table VIII.1.

Level 0, which relates to very exceptional circumstances, has been omitted in the case of the upward-deviating and downward-deviating networks; since mortalities at this level are very high, the very notion of deviation loses its meaning. On the other hand, it has been considered necessary to add level 115, which is now being reached by more and more countries. Six mortality levels—20, 40, 60, 80, 100 and 115—have thus been used for the two deviating networks and the corresponding expectations of life at birth for both sexes are given in the third and fourth columns of table VIII.1. These six deviating levels of mortality have been associated with three simplified model levels of fertility corresponding to the three gross reproduction rates of 2.00, 3.00 and 4.00. In this way, we have computed a network of eighteen upward-deviating model stable populations and a network of eighteen downward-deviating model stable populations. The detailed results of the computations are given in appendix IV. Here we shall limit ourselves to indicating,

[1] As explained in annex II, all the life tables having the same female mortality between the ages of 5 and 34 have been defined as belonging to the same mortality level.

TABLE VIII.1. LEVELS OF MORTALITY AND EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES (IN YEARS) USED IN THE THREE NETWORKS OF STABLE POPULATIONS

	Expectation of life at birth		
Level of mortality ([a])	*Intermediate network*	*Upward-deviating network*	*Downward-deviating network*
0	20	Not computed	Not computed
20	30	17.0	41.9
40	40	28.9	50.8
60	50	41.0	60.4
80	60.4	52.6	66.9
100	70.2	63.9	73.6
115	73.0	70.0	76.1

([a]) The mortality levels are numbered as in the series of model life tables published in the appendix to *Manual III; Methods for Population Projections by Sex and Age* (United Nations publication, Sales No.: 56.XIII.3).

in tables VIII.2 and VIII.3, the principal characteristics of the two networks.

Only five levels—20, 40, 60, 80 and 100—are common to the three networks. However, what we are really trying to estimate here is not mortality level, but expectation of life at birth, and it becomes an easy matter to determine in the two deviating networks the characteristics of populations corresponding to the life expectancy values of the intermediate network. The results of this interpolation are shown in tables VIII.4 and VIII.5. With the help of these tables it is possible to compare the two deviating networks directly with the intermediate network and to construct graphs directly comparable with the graphs in chapter VII.

B. Summary comparison of the three networks

The three networks can be quickly compared with the help of tables VIII.6, VIII.7 and VIII.8.

Table VIII.6 relates to population structure by broad age groups. It will be seen at once that a given pair of values for expectation of life at birth and gross reproduction rate produces very similar age structures in all three networks. This is particularly true of the proportion of children (aged 0-14). More distinct differences between the networks are seen in the proportion of old people (60 and over), especially in the case of high mortalities (expectation of life at birth of 40 years and under).

Table VIII.7 compares the stable birth rates, stable death rates and intrinsic rates of natural variation. Here again, the rates for corresponding populations are close to each other in the three networks.

Table VIII.8 deals with the structure of deaths by broad age groups. This time, the structure is no longer the same in the three networks, as might have been

TABLE VIII.2. PRINCIPAL CHARACTERISTICS OF THE UPWARD-DEVIATING NETWORK OF STABLE POPULATIONS

Gross reproduction rate	Mortality level	Expectation of life at birth for both sexes (in years)	Distribution of population by age groups (in percentages)			Stable rates per 1 000			Distribution of deaths at 5 years and over by age groups (in percentages)		
			Under 15 years	15-59 years	60 years and over	Birth rate	Death rate	Intrinsic rate of natural variation	5-14 years	15-59 years	60 years and over
4	20	16.97	38.4	56.6	5.0	66.3	61.6	4.7	10.4	64.8	24.8
3			31.3	61.0	7.7	50.2	56.1	− 5.9	7.2	60.5	32.3
2			21.8	64.9	13.3	31.6	52.2	−20.6	3.9	52.5	43.6
4	40	28.85	46.3	50.7	3.0	62.6	38.8	23.8	13.6	65.2	21.2
3			39.3	55.9	4.8	49.5	36.1	14.4	9.6	62.0	28.4
2			29.3	61.9	8.8	33.3	34.7	−1.4	5.4	54.9	39.7
4	60	41.03	50.3	47.3	2.4	59.1	23.6	35.5	13.6	63.1	23.3
3			43.5	52.6	3.9	47.5	22.5	25.0	9.5	59.7	30.8
2			33.4	59.2	7.3	33.0	22.7	10.3	5.3	52.1	42.6
4	80	52.60	52.5	45.3	2.2	56.1	13.6	42.5	11.7	58.4	29.9
3			45.7	50.6	3.7	45.5	13.5	32.0	7.9	53.6	38.5
2			35.6	57.3	7.1	32.0	14.7	17.3	4.1	45.0	50.9
4	100	63.86	53.7	44.0	2.3	54.0	6.8	47.2	7.3	48.5	44.2
3			46.9	49.3	3.8	44.0	7.2	36.8	4.6	41.2	53.3
2			36.7	55.9	7.4	31.0	9.0	22.0	2.2	32.9	64.9
4	115	69.95	54.1	43.4	2.5	53.1	4.2	48.9	5.1	40.7	54.2
3			47.2	48.6	4.2	43.2	4.8	38.4	3.0	33.9	63.1
2			36.9	54.9	8.2	30.4	6.8	23.6	1.3	25.1	73.6

TABLE VIII.3. PRINCIPAL CHARACTERISTICS OF THE DOWNWARD-DEVIATING NETWORK OF STABLE POPULATIONS

Gross reproduction rate	Mortality level	Expectation of life at birth for both sexes (in years)	Distribution of population by age groups (in percentages)			Stable rates per 1 000			Distribution of deaths at 5 years and over by age groups (in percentages)		
			Under 15 years	15-59 years	60 years and over	Birth rate	Death rate	Intrinsic rate of natural variation	5-14 years	15-59 years	60 years and over
4	20	41.89	51.2	45.7	3.1	53.9	21.2	32.7	25.0	58.6	16.4
3			44.4	50.6	5.1	43.2	20.9	22.3	19.0	56.7	24.3
2			33.8	56.6	9.6	29.9	22.4	7.5	11.6	50.1	38.3
4	40	50.53	52.7	44.6	2.7	53.7	14.5	39.2	23.4	57.6	19.0
3			45.8	49.7	4.5	43.5	14.7	28.8	17.5	54.8	27.7
2			35.4	55.7	8.9	30.3	16.3	14.0	10.3	47.2	42.5
4	60	60.41	53.2	44.0	2.8	52.9	9.1	43.8	21.2	52.7	26.1
3			46.2	49.1	4.7	42.8	9.5	33.3	15.0	48.3	36.7
2			35.7	35.1	9.2	29.9	11.3	18.6	8.2	38.8	53.0
4	80	66.90	53.9	43.4	2.7	52.6	5.7	46.9	16.4	49.0	34.6
3			47.0	48.5	4.5	42.8	6.3	36.5	11.0	43.1	45.9
2			36.5	54.7	8.8	30.0	8.2	21.8	5.5	33.1	61.4
4	100	73.61	54.2	43.1	2.7	52.2	3.1	49.1	9.6	39.1	51.3
3			47.2	48.1	4.6	42.4	3.8	38.6	5.8	32.0	62.2
2			36.7	54.2	9.1	29.7	5.8	23.9	2.6	22.5	74.9
4	115	76.07	54.2	43.0	2.8	52.0	2.2	49.8	5.4	32.3	62.3
3			47.3	48.0	4.7	42.3	3.0	39.3	3.1	25.4	71.5
2			36.8	54.0	9.2	29.6	5.0	24.6	1.3	17.4	81.3

TABLE VIII.4. PRINCIPAL CHARACTERISTICS ([a]) OF THE UPWARD-DEVIATING NETWORK OF STABLE POPULATIONS

Gross reproduction rate	Expectation of life at birth for both sexes (in years)	Distribution of population by broad age groups (in percentages)			Stable rates per 1 000			Distribution of deaths at 5 years and over by broad age groups (in percentages)		
		Under 15 years	15-59 years	60 years and over	Birth rate	Death rate	Intrinsic rate of natural variation	5-14 years	15-59 years	60 years and over
4	30	46.8	50.3	2.9	62.3	37.4	24.9	13.7	65.1	21.2
3		39.9	55.4	4.7	49.0	34.7	14.3	9.8	61.8	28.4
2		29.8	61.6	8.6	33.4	33.3	0.1	5.5	54.7	39.8
4	40	50.0	47.6	2.4	59.3	24.5	34.8	13.8	63.3	22.9
3		43.3	52.8	3.9	47.7	23.5	24.2	9.5	60.0	30.5
2		33.2	59.5	7.3	33.1	23.5	9.6	5.4	52.5	42.1
4	50	52.3	45.5	2.2	56.7	16.0	40.7	12.2	59.7	28.1
3		45.2	51.1	3.7	45.9	15.6	30.3	8.3	55.1	36.6
2		35.2	57.7	7.1	32.4	16.4	16.0	4.4	46.8	48.8
4	60.4	53.5	44.3	2.2	54.7	9.7	46.0	8.9	52.3	38.8
3		46.7	49.5	3.8	44.4	9.0	35.4	5.8	45.7	48.5
2		36.8	56.0	7.2	31.3	10.6	20.7	2.8	37.0	60.2
4	70.2	54.2	43.2	2.6	53.0	4.0	49.0	5.0	40.4	54.6
3		47.4	48.4	4.2	44.2	4.6	39.6	2.9	33.9	63.2
2		37.0	54.7	8.3	30.3	6.8	23.5	1.3	24.9	73.8

([a]) Characteristics obtained by interpolation of figures in table VIII.2 for expectations of life at birth of the intermediate network.

TABLE VIII.5. PRINCIPAL CHARACTERISTICS ([a]) OF THE DOWNWARD-DEVIATING NETWORK OF STABLE POPULATIONS

Gross reproduction rate	Expectation of life at birth for both sexes (in years)	Distribution of population by broad age groups (in percentages)			Stable rates per 1 000			Distribution of deaths at 5 years and over by broad age groups (in percentages)		
		Under 15 years	15-59 years	60 years and over	Birth rate	Death rate	Intrinsic rate of natural variation	5-14 years	15-59 years	60 years and over
4	30	47.5 ([b])	48.5 ([b])	4.0 ([b])	55.0 ([b])	34.0 ([b])	21.0	26.0 ([b])	58.8 ([b])	15.2 ([b])
3		40.2 ([b])	53.4 ([b])	6.4 ([b])	43.6 ([b])	35.0 ([b])	8.6	20.0 ([b])	59.2 ([b])	20.8 ([b])
2		31.2 ([b])	57.2 ([b])	11.6 ([b])	29.9 ([b])	35.5 ([b])	−5.6	12.0 ([b])	53.0 ([b])	35.0 ([b])
4	40	50.4	46.4	3.2	53.9	23.1	30.8	25.2	58.7	16.1
3		43.6	51.2	5.2	43.2	23.4	19.8	19.0	57.3	23.7
2		33.4	56.7	9.9	29.9	24.6	5.3	11.9	50.4	37.7
4	50	52.6	44.6	2.8	53.6	14.7	38.9	23.5	57.7	28.8
3		45.8	49.6	4.6	43.4	15.0	28.4	17.6	55.0	27.4
2		35.4	55.7	8.9	30.2	16.6	13.6	10.4	47.5	42.1
4	60.4	53.2	44.0	2.8	52.9	9.1	43.8	21.2	52.7	26.1
3		46.2	49.1	4.7	42.8	9.5	33.3	15.0	48.3	36.7
2		35.7	55.2	9.1	29.9	11.3	18.6	8.2	38.8	53.0
4	70.2	54.2	43.1	2.7	52.4	4.3	48.1	13.4	45.8	40.8
3		47.4	48.1	4.5	42.6	4.8	37.8	8.6	39.2	52.2
2		36.6	54.5	8.9	29.8	6.7	23.1	4.3	27.9	67.8

([a]) Characteristics obtained by interpolation of figures in table VIII.3 for expectations of life at birth of the intermediate network.
([b]) Estimates obtained by extrapolation.

TABLE VIII.6. COMPARISON OF STRUCTURES BY BROAD AGE GROUPS IN THE THREE NETWORKS

Expectation of life at birth for both sexes (in years)	Gross reproduction rate	Distribution of population by broad age groups (in percentages): 0-14 years: Intermediate network	0-14 years: Upward-deviating network	0-14 years: Downward-deviating network	15-59 years: Intermediate network	15-59 years: Upward-deviating network	15-59 years: Downward-deviating network	60 years and over: Intermediate network	60 years and over: Upward-deviating network	60 years and over: Downward-deviating network
30	4.00	48.2	46.8	47.5 (a)	49.2	50.3	48.5 (a)	2.6	2.9	4.0 (a)
40		50.0	50.0	50.4	47.3	47.6	46.4	2.7	2.4	3.2
50		51.5	52.3	52.6	45.8	45.5	44.6	2.7	2.2	2.8
60.4		52.8	53.5	53.2	44.4	44.3	44.0	2.7	2.2	2.8
70.2		54.1	54.2	54.2	43.3	43.2	43.1	2.6	2.6	2.7
30	3.00	41.3	39.9	40.2 (a)	54.5	55.4	53.4 (a)	4.1	4.7	6.4 (a)
40		43.1	43.3	43.6	52.5	52.8	51.2	4.4	3.9	5.2
50		44.6	45.2	45.8	50.9	51.1	49.6	4.5	3.7	4.6
60.4		46.0	46.7	46.2	49.6	49.5	49.1	4.4	3.8	4.7
70.2		47.3	47.4	47.4	48.4	48.4	48.1	4.3	4.2	4.5
30	2.00	31.4	29.8	31.2 (a)	60.9	61.6	57.2 (a)	7.7	8.6	11.6 (a)
40		32.9	33.2	33.4	58.8	59.5	56.7	8.3	7.3	9.9
50		34.2	35.2	35.4	57.2	57.7	55.8	8.6	7.1	8.8
60.4		35.6	36.8	35.7	55.8	56.0	55.2	8.6	7.2	9.1
70.2		36.8	37.0	36.0	54.7	54.7	54.5	8.5	8.3	8.9

(a) Estimate obtained by extrapolation.

TABLE VIII.7. COMPARISON OF STABLE BIRTH RATES, STABLE DEATH RATES AND INTRINSIC RATES OF NATURAL VARIATION IN THE THREE NETWORKS

Expectation of life at birth for both sexes (in years)	Gross reproduction rate	Stable birth rate (per thousand): Intermediate network	Stable birth rate: Upward-deviating network	Stable birth rate: Downward-deviating network	Stable death rate (per thousand): Intermediate network	Stable death rate: Upward-deviating network	Stable death rate: Downward-deviating network	Intrinsic rate of natural variation (per thousand): Intermediate network	Intrinsic rate: Upward deviating network	Intrinsic rate: Downward-deviating network
30	4.00	59.8	62.3	55.0 (a)	35.3	37.4	34.0 (a)	24.5	24.9	21.0
40		57.3	59.3	53.9	24.1	24.5	23.1	33.2	34.8	30.8
50		55.7	56.7	53.6	16.2	16.0	14.7	39.5	40.7	38.9
60.4		54.1	54.7	52.9	9.4	9.7	9.1	44.7	46.0	43.8
70.2		52.7	53.0	52.4	4.1	4.0	4.3	48.6	49.0	48.1
30	3.00	47.7	49.0	43.6 (a)	33.7	34.7	35.0 (a)	14.0	14.3	8.6
40		46.0	47.7	43.2	23.3	23.5	23.4	22.7	24.2	19.8
50		44.9	45.9	43.4	15.8	15.6	15.0	29.1	30.3	28.4
60.4		43.8	44.4	42.8	9.6	9.0	9·5	34.2	35.4	33.3
70.2		42.9	44.2	42.6	4.8	4.6	4.8	38.1	39.6	37.8
30	2.00	32.7	33.4	29.9 (a)	36.6	33.3	35.5 (a)	−3.9	0.1	−5.6
40		31.7	33.1	29.9	23.7	23.5	24.6	8.0	9.6	5.3
50		31.1	32.4	30.2	16.8	16.4	16.6	14.3	16.0	13.6
60.4		30.6	31.3	29.9	11.1	10.6	11.3	19.5	20.7	18.6
70.2		30.1	30.3	29.2	6.8	6.8	6.7	23.3	23.5	23.1

(a) Estimate obtained by extrapolation.

Table VIII.8. Comparison of structures, by broad age groups, of deaths at 5 years and over in the three networks

Expectation of life at birth for both sexes (in years)	*Gross reproduction rate*	*Deaths between 5 and 14 years as percentages of all deaths at 5 years and over*			*Deaths at 60 years and over as percentages of all deaths at 5 years and over*		
		Intermediate network	*Upward-deviating network*	*Downward-deviating network*	*Intermediate network*	*Upward-deviating network*	*Downward-deviating network*
30	4.00	19.5	13.7	26.0 (a)	25.8	21.2	15.2 (a)
40		18.8	13.8	25.2	20.4	22.9	16.1
50		17.5	12.2	23.5	26.0	28.1	18.8
60.4		14.8	8.9	21.2	34.1	38.8	26.1
70.2		9.2	5.0	13.4	48.0	54.6	40.8
30	3.00	14.5	9.8	20.0 (a)	22.1	28.4	20.8 (a)
40		13.6	9.5	19.0	28.3	30.5	23.7
50		12.3	8.3	17.6	35.4	36.6	27.4
60.4		9.9	5.8	15.0	44.6	48.5	36.7
70.2		5.7	2.9	8.6	58.2	63.2	52.2
30	2.00	8.7	5.5	12.0 (a)	32.9	39.8	35.0 (a)
40		7.9	5.4	11.9	41.0	42.1	37.7
50		6.7	4.4	10.4	49.6	48.8	42.1
60.4		5.0	2.8	8.2	59.1	60.2	53.0
70.2		2.6	1.3	4.3	70.6	73.8	67.8

(a) Estimate obtained by extrapolation.

expected. At a given age, intermediate, upward-deviating and downward-deviating mortalities do not, by definition, have the same effects. These different effects upon the age structures necessarily produce different age structures of deaths.

The similarity of age structures and the similarity of stable death rates, stable birth rates and intrinsic rates of natural variation for corresponding populations in the three networks, indicates that the results obtained in the preceding chapter with the use of these demographic characteristics will not be radically affected. On the other hand, we may expect to have to make more extensive adjustments in the results obtained with the use of age structures of deaths.

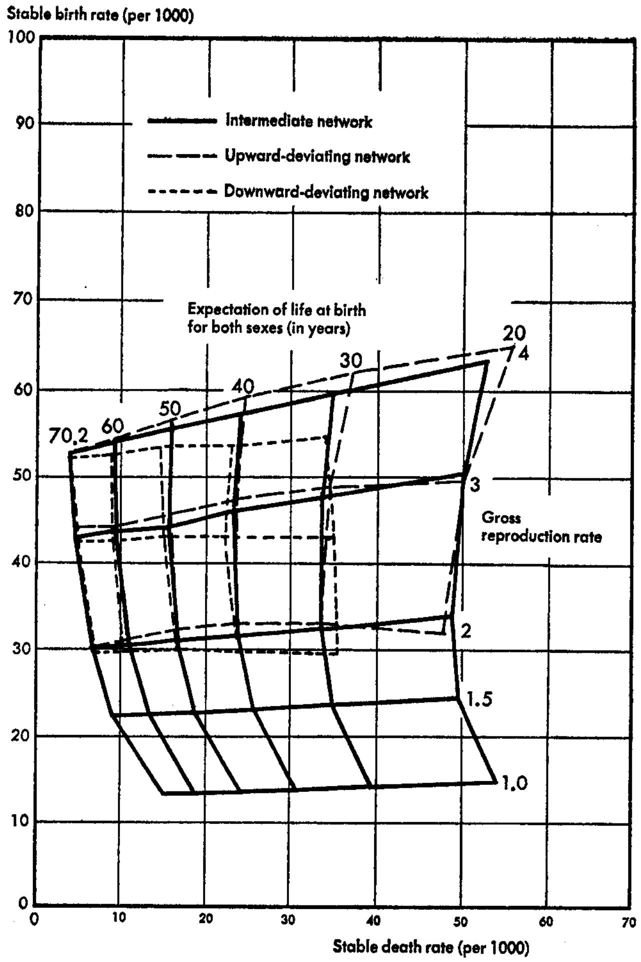
C. Relations among gross reproduction rate, expectation of life at birth, stable birth rate and stable death rate

Graph VIII.1 is similar to graph VII.1. The stable death rates for the series of stable populations of the three networks are represented by the horizontal axis and the stable birth rates by the vertical axis. The graph thus contains thirty-six points for the intermediate network and eighteen points for each of the two deviating networks. Constant life expectancy curves and constant gross reproduction rate curves were drawn, producing very similar graphs for the three networks.

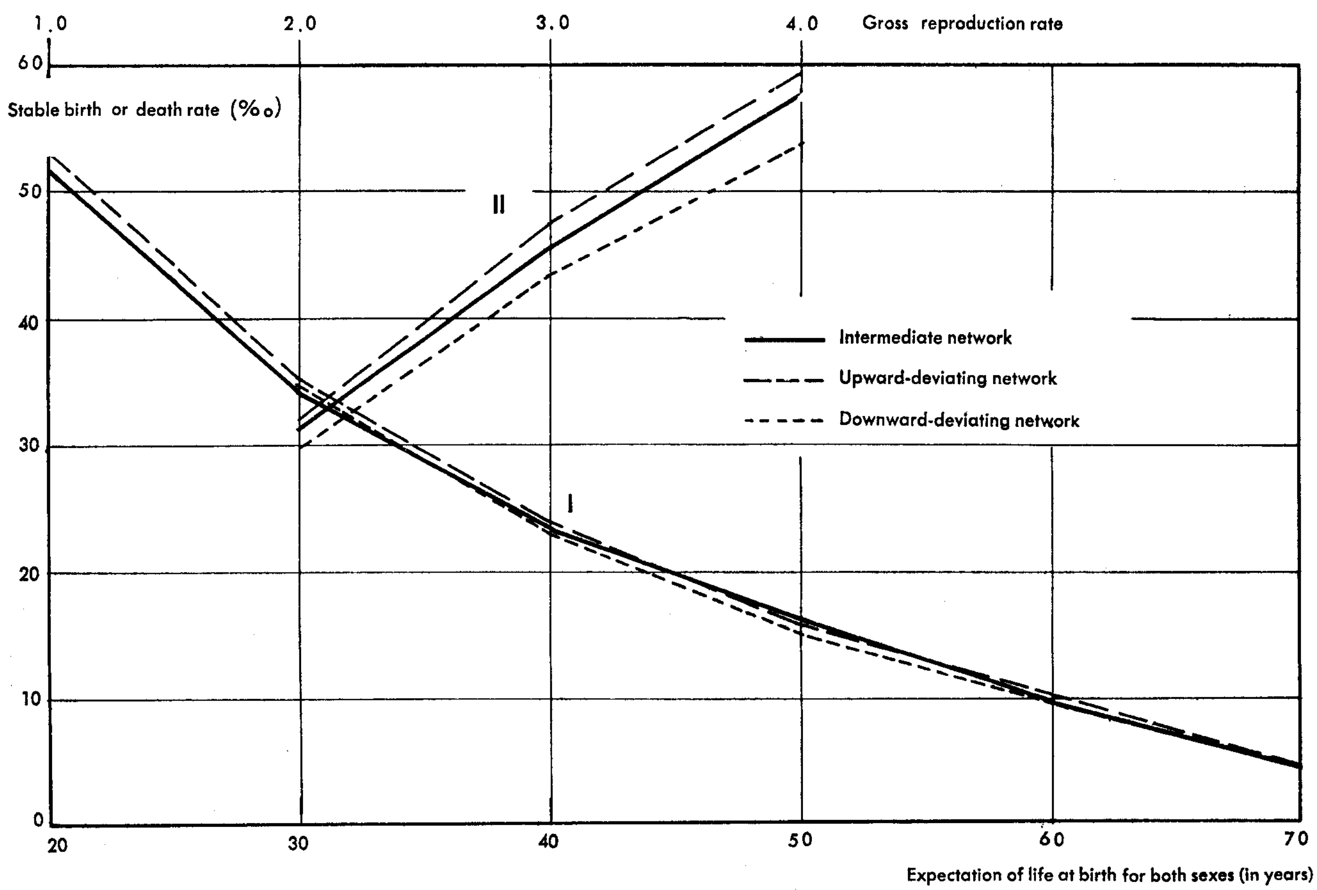
1. As in the case of the intermediate model stable populations, the networks of constant fertility and constant mortality curves in the upward-deviating and the downward-deviating model stable populations represented in graph VIII.1 do not overlap. Each pair of values for stable birth rate and stable death rate is associated with a single pair of values for expectation of life at birth and gross reproduction rate. As in the case of the intermediate networks, this is a particular property of general characteristics of deviating networks. *In general,* any two characteristics of an upward-deviating or) downward-deviating) stable population determine, without any ambiguity, one stable population in the upward-deviating (or downward-deviating) network. Here again, it must be emphasized that this is true only *in general,* for there are cases in which the networks of curves overlap.

2. In graph VIII.1, the curves of constant expectation of life are almost vertical for the average and high fertilities both in the two deviating networks and in the intermediate network. In each of the networks, therefore, the stable death rate is a good index of mortality. What is more, for a given expectation of life, the curves of constant expectation of life are close together in the three networks and therefore the stable death rate is a good index of mortality not only in each network, but also in all three networks. Table VIII.9 and graph VIII.2 illustrate this. Table VIII.9 reproduces, for intermediate model stable populations, the stable death rates in the area where there is practically a direct relationship between expectation of life at birth and stable death rate (the figures underlined in table VII.4). For the two deviating networks, the corresponding death rates have been given and the same averages have been calculated as for the intermediate network. The three curves I in graph VIII.2 correspond to these averages. It will be seen that these three curves are very close to one another and one may feel that one need simply use the general average, which is practically identical with the average of the intermediate network. In other words, curve I in graph VIII.2, which belongs to the intermediate model stable populations, is found to be equally applicable to the other two stable populations.

3. Graph VIII.1 shows that in all three networks the curves of constant gross reproduction rate do not deviate much from the horizontal, and in each network, at least in a certain area, there is practically a direct relationship between the gross reproduction rate and the stable birth rate. It will be noted, however, that for a given gross reproduction rate the constant gross reproduction rate



Graph VIII.1. Relations among stable death rate, stable birth rate, intrinsic rate of natural variation, gross reproduction rate, and expectation of life at birth in the three networks of model stable populations



Graph VIII.2. Approximate relations in the three networks of model stable populations between: (I) stable death rate and expectation of life at birth; (II) stable birth rate and gross reproduction rate

TABLE VIII.9. STABLE DEATH RATE IN MODEL STABLE POPULATIONS
(Per thousand)

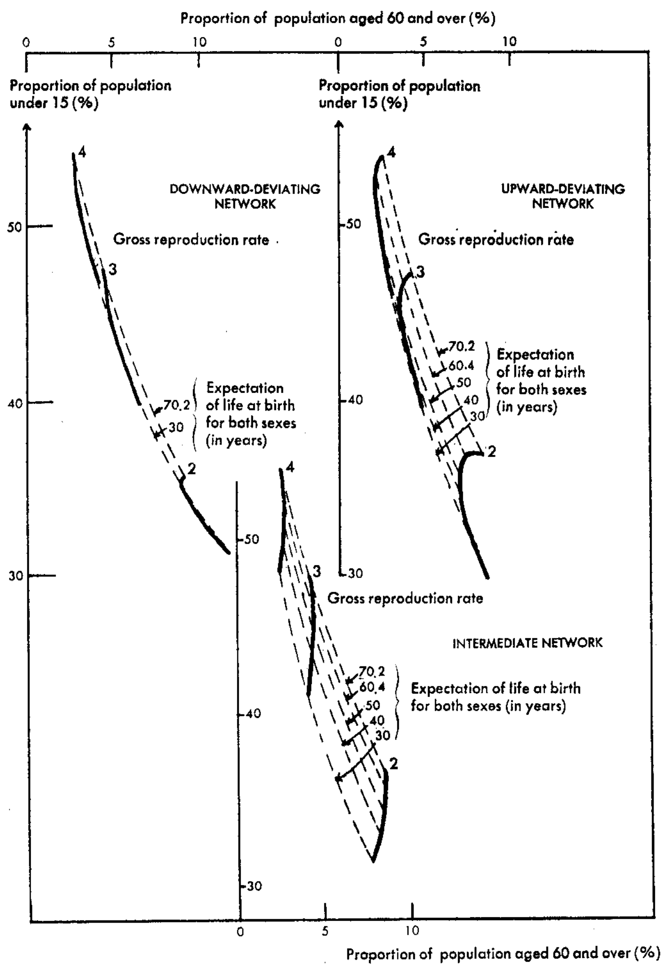
Gross reproduction rate	Expectation of life at birth for both sexes (in years)					
	20	30	40	50	60.4	70.2
I. Intermediate model stable populations						
4.0	53.0	35.3	24.1	16.2	9.4	4.1
3.0	50.2	33.7	23.3	15.8	9.6	4.8
2.0	48.6	33.6	23.7	16.8	11.1	6.8
Average of underlined rates . .	51.6	34.2	23.7	16.3	9.7	4.5
II. Upward-deviating model stable populations						
4.0	55.6	37.4	24.8	16.0	9.9	4.0
3.0	50.4	34.9	23.6	15.6	9.2	4.6
2.0	47.1	33.5	23.7	16.4	10.8	6.8
Average of underlined rates . .	53.0	35.3	24.0	16.0	10.0	4.3
III. Downward-deviating model stable populations						
4.0	Rates	34.0	23.2	14.8	9.1	4.4
3.0	not	35.0	22.4	15.1	9.5	5.0
2.0	computed	35.5	24.0	16.6	10.3	7.1
Average of underlined rates . .		34.8	23.2	15.5	9.6	4.7
IV. Three networks as a whole						
Average of underlined rates . .	52.3	34.7	23.6	15.9	9.7	4.5

curves show greater differences from network to network than do the curves for constant expectation of life corresponding to a given expectation of life. Table VIII.10 illustrates this clearly. It is the counterpart of table VIII.9 for the birth rate. The three averages computed for each network are represented by the curves II in graph VIII.2. In each network, the stable birth rate reveals itself as a fairly good index of fertility in the area of the rates underlined in table VIII.9. However, there are some divergences between the three curves which are not insignificant. Replacing them by their averages, in order to obtain the gross reproduction rate from the stable birth rate, will therefore produce estimates that are less precise than those for expectation of life at birth mentioned above.

D. Age structure and levels of mortality and fertility

Graph VIII.3 is identical with graph VII.3. For each of the three networks, the proportion of persons aged 60 and over has been indicated on the horizontal axis and the proportion of persons aged 0-14 on the vertical ax.is Curves have been plotted for constant mortality and constant fertility. In all three networks, the extreme curves of constant mortality enclose a rather narrow band elongated in the direction of both axes. Moreover, the curves of constant expectation of life are almost vertical, at least for expectations of life at birth of more than 30 years. It follows that, for expectations of life of more than 30 years, the age structure of populations in all three networks is not greatly affected by variations in mortality. This characteristic had already been noted in the case of the intermediate network. It is also a characteristic of the two deviating networks. What is more, for any given pair of values of life expectancy at birth and gross reproduction rate, the age structure is almost the same in all three networks, as can be seen from table VIII.6. One exception must be pointed out, however; in the case of high mortality there are proportions of persons aged 60 and over which vary considerably from network to network.

Once the "universes" of upward-deviating and downward-deviating model life tables have been defined, it is obviously possible to speak of upward-deviating and downward-deviating quasi-stable populations. An upward-deviating quasi-stable population, for example, will be one in which fertility is constant and mortality varies while remaining within the universe of the upward-deviating model life tables. It will be remembered that the basic characteristic of intermediate quasi-stable



Graph VIII.3. Relations between age structure, mortality level and fertility level in the three networks

TABLE VIII.10. STABLE BIRTH RATE IN MODEL STABLE POPULATIONS
(Per thousand)

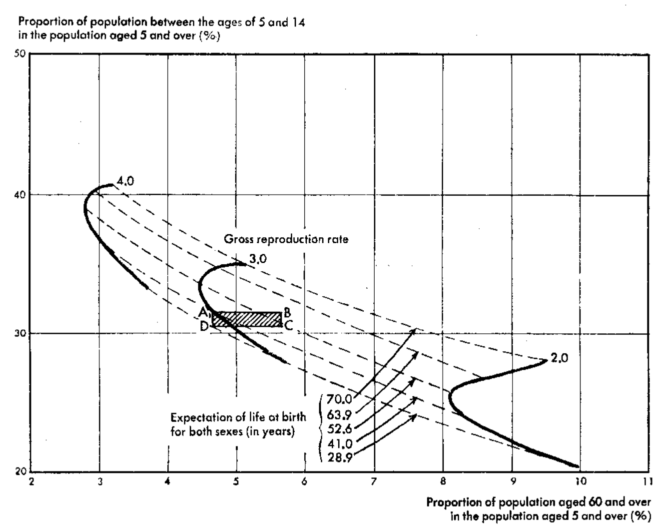
Gross reproduction rate	Expectation of life at birth for both sexes (in years): 20	30	40	50	60.4	70.2	Average of underlined rates
	I. Intermediate model stable populations						
4.0	63.8	59.8	57.3	55.7	54.1	52.7	57.6
3.0	50.5	47.7	46.0	44.9	43.8	42.9	45.6
2.0	34.2	32.7	31.7	31.1	30.6	30.1	31.4
	II. Upward-deviating model stable populations						
4.0	65.4	62.3	59.3	56.7	54.7	53.0	59.4
3.0	50.0	49.4	47.6	45.9	44.4	44.2	47.6
2.0	32.2	33.3	33.0	32.4	31.3	30.3	32.1
	III. Downward-deviating model stable populations						
4.0	Rates	55.0	53.9	53.6	52.9	52.4	53.6
3.0	not	43.6	43.2	43.4	42.8	42.6	43.1
2.0	computed	29.9	29.9	30.2	29.9	29.8	29.9
	IV. Over-all average of the three networks						
4.0							56.7
3.0							45.4
2.0							31.1

populations is that they are close to the stable populations at a given time. This characteristic is due to the fact that the age structure of intermediate stable populations is more or less independent of variations in mortality. As this quasi-independence also exists in the case of deviating stable populations, it was assumed that deviating quasi-stable populations also had the characteristic of being close to current stable populations.[2] By assimilating the trends of the population of Venezuela between 1936 and 1950 to a deviating quasi-stable population, one could repeat the study described in the preceding chapter with the help of graph VIII.3 in respect of the two deviating networks.

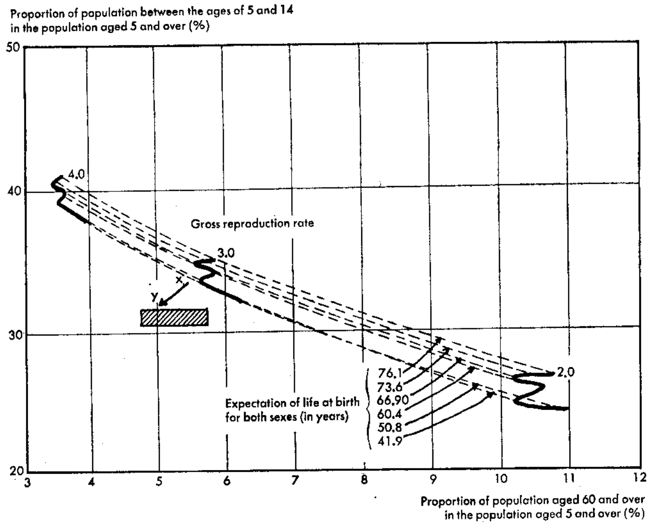
In order to avoid having to decide, at least temporarily, whether or not there was an increase in fertility in Venezuela after 1936, use will be made of graphs VIII.4 and VIII.5, which are limited to the population aged 5 and over and which are similar to graph VII.5. The shaded rectangles in these two graphs represent the area comprising all the deviating quasi-stable populations which are consistent with the census results in so far as the age structure is concerned.

[2] Strictly, in order to be sure of the validity of this characteristic, one would have to compute the quasi-stable populations with the use of deviating model mortality rates. No such computations have yet been made.

Graph VIII.4 relates to the upward-deviating network. In the shaded rectangle, the gross reproduction rate is close to 3.0 and the expectation of life at birth is between



Graph VIII.4. Relations among the age structure above the age of 5, the mortality level and the fertility level in the upward-deviating network of model stable populations



Graph VIII.5. Relations among the age structure above the age of 5, the mortality level and the fertility level in the downward deviating network of model stable populations

29 and 57 years. With the intermediate network, we had also found a gross reproduction rate close to 3.0 and an expectation of life of between 28 and 46 years. The estimated gross reproduction rate is thus almost the same for both networks. The estimate of expectation of life was not very good with the intermediate network, but it is even worse with the deviating network.

Graph VIII.5 relates to the downward-deviating network. Here, the shaded rectangle is outside the curves of the graph. This means that it is not possible to reconstruct the development of the population of Venezuela between 1936 and 1950 by using downward-deviating model life tables. However, the downward-deviating model life tables used here reflect extreme deviations. Smaller deviations would produce graphs similar to graph VIII.5, situated somewhere between that graph and graph VII.5, which relates to the intermediate network. If there was no deviation at all, the result would be graph VII.5 of the intermediate network. A series of such graphs reflecting increasingly smaller mortality deviations may be regarded as the result of a progressive deformation of graph VIII.5 towards graph VII.5. This deformation is tantamount to a dilation of graph VIII.5 in the direction of the arrow drawn on graph VIII.5, the upper curve of that graph remaining fixed.

When, in the course of this dilation, the curves of the graph again cross the shaded rectangle, the curve of constant gross reproduction rate marked "3.0" will be near the centre of the rectangle. Here again, therefore, one is led to conclude that the fertility of the population of Venezuela remained constant at a level corresponding to a gross reproduction rate of about 3.0.

Where mortality is concerned, estimates are even less precise than in the other two networks. In the dilation referred to above, it is the constant mortality curves corresponding to high mortality levels which first cross the rectangle. The lower limit of expectation of life at birth is therefore the lowest value for expectation of life in the series of downward-deviating life tables. The upper limit is obtained when there is no longer any deviation, and it is therefore identical with that found in the intermediate network.

In short, the conclusions reached in chapter VII by using graph VII.5 are the same as those we arrive at by using graphs VIII.4 and VIII.5. *On this particular point*, therfore, the assumption that the mortality of a given population varies within a certain universe of model life tables has no appreciable effect on our estimates of fertility and mortality. Thus, it may now be considered certain that up to 1945 the fertility of the population of Venezuela remained more or less constant at a level corresponding to a gross reproduction rate of 3.00.[3]

As was seen in chapter VII, a study of the variations in recorded crude birth rates makes it possible to extend this conclusion up to 1960. As to mortality, it cannot be conveniently estimated in any of the graphs under consideration.

A comparison in the three networks of the ratio *f* of the number of women aged 15 to 44 to the number of women aged 45 and over confirms these results. Table VIII.11 shows that for medium and low mortalities (expectation of life at birth of 40 years and over) this ratio shows little variation from network to network. In 1950, the ratio *f* observed in Venezuela was 3.1381. This value gives a gross reproduction rate of close to 3.00 in all three networks.

[3] This conclusion assumes, however, that the age distribution of the fertility in Venezuela is the intermediate distribution. With a different distribution, the conclusion would still be that fertility remained constant, but the corresponding value for the gross reproduction rate would not be 3.00.

TABLE VIII.11. RATIO *f* OF THE NUMBER OF WOMEN AGED 15 TO 44 TO THE NUMBER OF WOMEN AGED 45 AND OVER IN THE THREE NETWORKS OF STABLE POPULATIONS

	4.0			*3.0*			*2.0*		
Expectation of life at birth (in years)	*Intermediate network*	*Upward-deviating network*	*Downward-deviating network*	*Intermediate network*	*Upward-deviating network*	*Downward-deviating network*	*Intermediate network*	*Upward-deviating network*	*Downward-deviating network*
20	4.30	2.72	(a)	3.20	2.01	(a)	2.11	1.31	(a)
30	4.28	3.58	(a)	3.16	2.63	(a)	2.06	1.71	(a)
40	4.27	4.16	4.21	3.11	2.98	3.06	2.00	1.97	1.92
50	4.28	4.47	4.54	3.12	3.25	3.30	1.99	2.10	2.08
60.4	4.41	4.58	4.52	3.20	3.33	3.27	2.03	2.14	2.06
70.2	4.57	4.48	4.57	3.32	3.24	3.30	2.08	2.03	2.07

(a) Ratio not computed.

E. Correction of the age structure

Table VIII.12 permits comparison, in the three networks, of variations in the ratio of the number of children aged 0-4 to the number of persons aged 5 and over, in stable populations corresponding to a gross reproduction rate of 3.00. In chapter VII it was assumed that a ratio of 0.21 would have been observed in the Venezuelan censuses if children aged 0-4 had been properly enumerated. It can be seen from table VIII.12 that the use of one of the deviating networks would lead to substantially the same conclusion. Consequently, all that was said in chapter VII regarding correction of the age structure of the population of Venezuela remains valid. More particularly, the corrected structure by broad age groups can be used to estimate mortality and fertility with the help of graph VIII.3 or, better, with the help of graphs VIII.6 and VIII.7, which are enlargements of graph VIII.3 for the area in which the population of Venezuela is situated. These two graphs are, in fact, similar to graphs VIII.4 and VIII.5 (one set relating to the population aged 5 and over and the other to the total population), and both sets give identical estimates. They are given here merely as an introduction to the use of the next graph where it is assumed that the rate of natural variation, as well as the age structure, is known.

TABLE VIII.12. RATIO OF THE NUMBER OF CHILDREN AGED 0-4 TO THE NUMBER OF PERSONS AGED 5 AND OVER IN THE THREE NETWORKS OF STABLE POPULATIONS CORRESPONDING TO GROSS REPRODUCTION RATE OF 3.0

Expectation of life at birth for both sexes (in years)	*Intermediate network*	*Upward-deviating network*	*Downward-deviating network*
20	0.185	0.161	(a)
30	0.197	0.190	(a)
40	0.206	0.209	0.218
50	0.216	0.221	0.221
60.4	0.225	0.230	0.226
70.2	0.233	0.234	0.232

(a) Ratio not computed.

F. Age structure and rate of natural variation

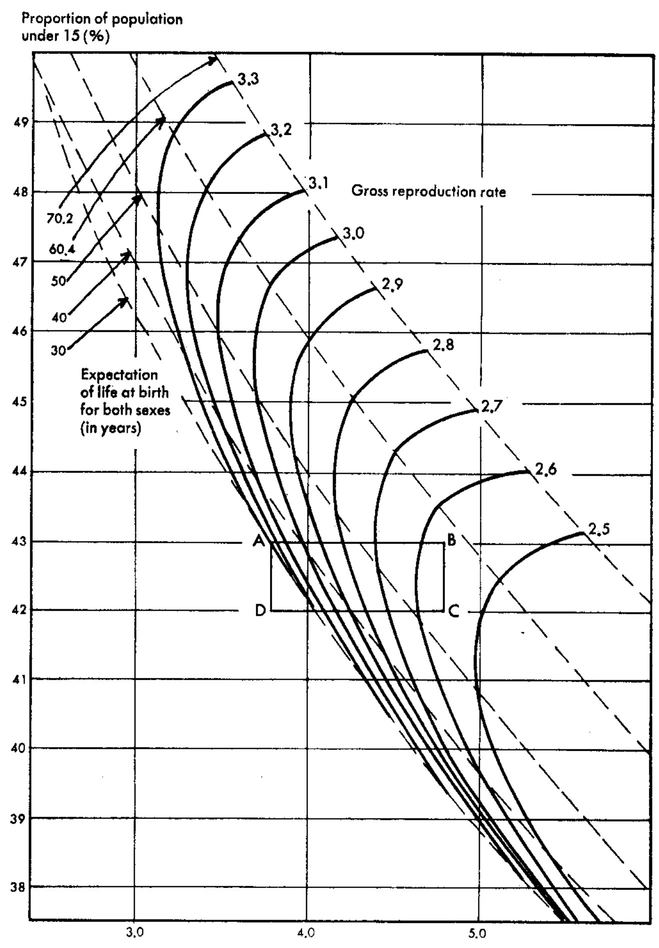
It was discussed in chapter VII how a third network of lines—lines of constant intrinsic rate of natural variations—could be plotted in such graphs as VIII.6 and VIII.7. This has been done in graphs VIII.8 and VIII.9. Chapter VII also indicated how it could be estimated that in mid-1946 the annual rate of natural variation of the population of Venezuela was between 22 and 23 per thousand.

Let us examine graph VIII.8, representing the upward-deviating network. The point representing the population of Venezuela in mid-1946 should be located somewhere in the rectangle ABCD between the curves of constant intrinsic rate of natural variation 22 and 23. This is the shaded area in graph VIII.8. In this area, the gross reproduction rate is between 2.80 and 3.10 and the expectation of life at birth for both sexes is between 36 and 42.

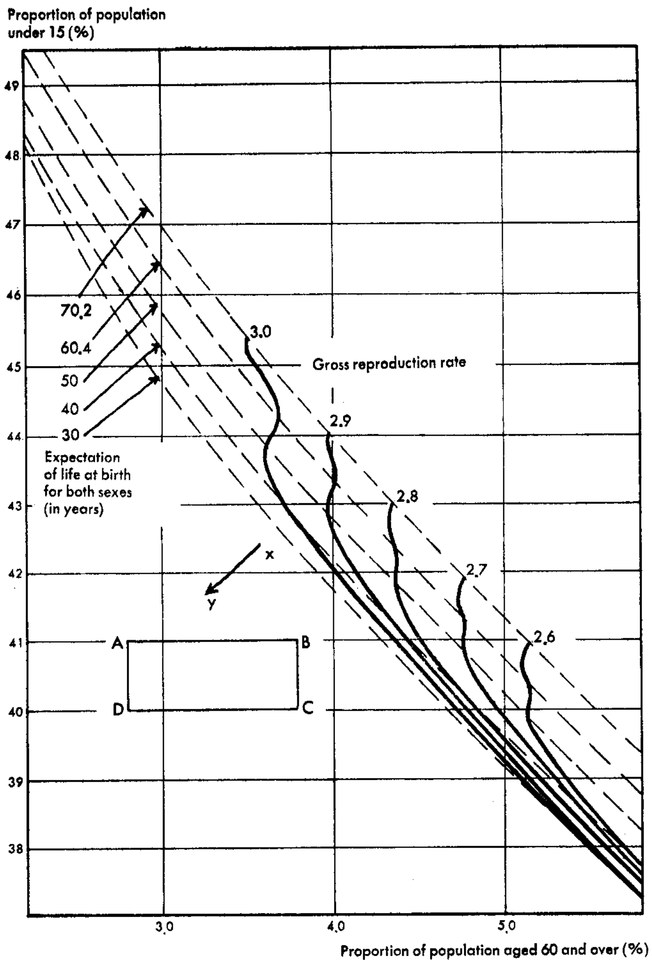
In the intermediate network, with graph VII.7, which is similar to graph VIII.8, we found a gross reproduction rate of between 2.84 and 3.10 and an expectation of life at birth of between 39 and 43, i.e., essentially the same values as with the upward-deviating network.

Graph VIII.9 is the same as VIII.8, but it represents the downward-deviating network. This time, the two curves of constant intrinsic rate of natural variation 22 and 23 do not pass through the rectangle ABCD. This was to be expected, since we have already seen that the assumption of a mortality with a maximum downward deviation is incompatible with Venezuela's age structure, and the addition of another factor—the rate of natural variation—cannot eliminate the incompatibility. However, as stated above, for mortality rates with less deviation than those used here there would be graphs similar to graph VIII.9, derived by a dilation in the direction indicated by the arrow XY in the graph,[4] the upper curve remaining unchanged. In this dilation, there would again be portions of the rectangle ABCD which would fall between the curves of constant intrinsic rate of natural variation 22 and 23. It can be seen from graph VIII.9 that the estimates in this case would give approximately the same values as those obtained with the other two networks.

[4] When there is no longer any deviation we must return to graph VII.7, relating to the intermediate network. This is how the direction of the dilation arrow is determined.



Graph VIII.6. Relations among age structure, mortality level and fertility level in the area of the upward-deviating network of model stable populations in which the population of Venezuela from 1936 to 1950 is situated (enlargement of graph VIII.3)

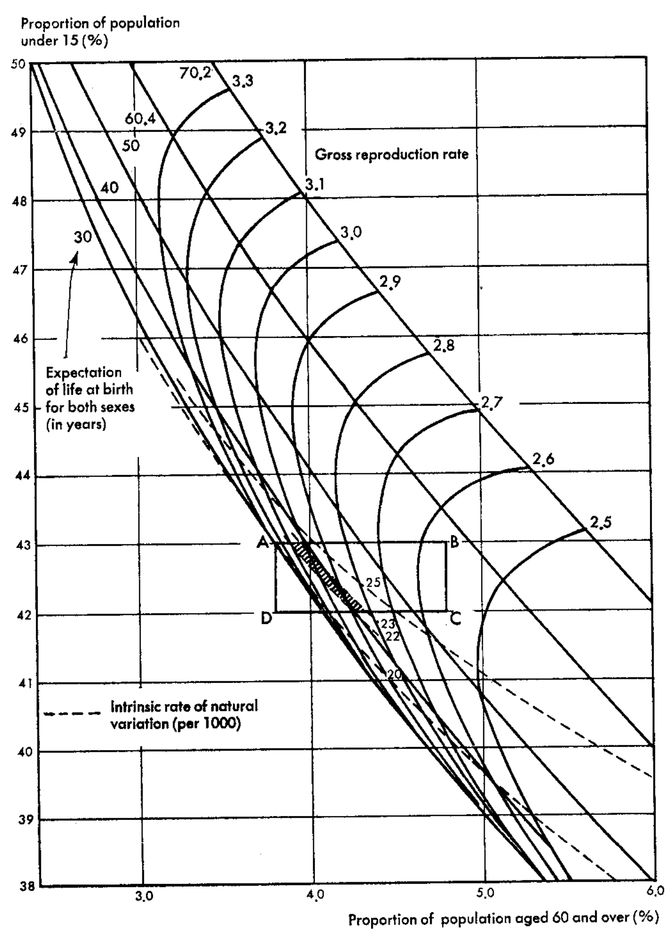


Graph VIII.7. Relations among age structure, mortality level and fertility level in the area of the downward-deviating network of model stable populations in which the population of Venezuela from 1936 to 1950 was situated (enlargement of graph VIII.3)

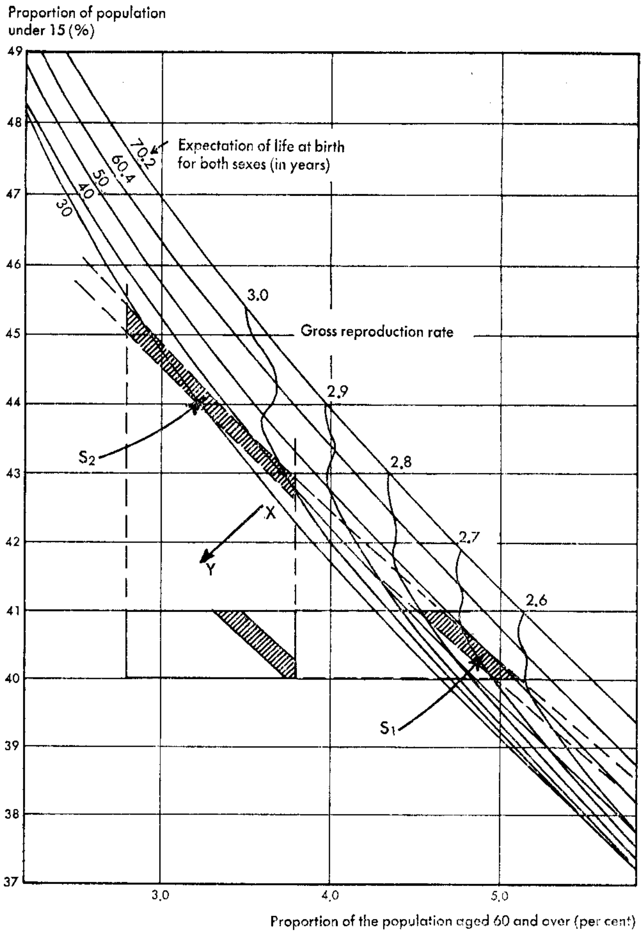
It is possible to make simpler graphs than graphs VIII.8 and VIII.9 by using only one characteristic of the age structure, instead of two. By indicating the proportion of persons aged 0-14 in the total population on the horizontal axis and the intrinsic rate of natural variation on the vertical axis, we obtain graphs similar to graph VII.8. These are shown in graph VIII.10, in which we have also reproduced graph VII.8. The graphs are very similar for all three networks. Graph VIII.11 gives an enlargement of graph VIII.10 for the two deviating networks in the area in which the population of Venezuela is situated (a graph similar to graph VII.9 for the intermediate network). With this enlarged graph, we get the following estimates:

Network	*Range of gross reproduction rate*	*Range of expectation of life at birth for both sexes (in years)*
Intermediate	2.8 and 3.0	39 and 44
Upward-deviating	2.8 and 3.1	37 and 42
Downward-deviating . .	2.6 and 2.8	45 and 50

This time, the shaded rectangle in which the population of Venezuela is found is crossed by the curves in all three networks, and one might gain the impression that it is possible to use the downward-deviating network, which had hitherto been found to deviate too greatly, in the case of Venezuela. This, in fact, is only an illusion. The downward-deviating network can be used on this occasion because, in discarding one characteristic of the age-structure, we actually opted for a less precise reconstruction of the population trends in Venezuela with the help of downward-deviating quasi-stable populations. It can readily be seen from graph VIII.9 how this sacrifice of precision enables us to use the downward-deviating network. If we use the proportion of persons under 15 and the intrinsic rate of natural variation as demographic characteristics, our estimates will lie within the small shaded area S_1 in graph VIII.9. If we had taken the proportion of persons aged 60 and over, instead of the proportion of persons under 15, as the characteristic of the age-structure, the estimates would have fallen within the small shaded area S_2. These two areas give very different estimates of fertility and mortality. With mortality rates having less downward deviation than those used here, in the dilation of graph VIII.9 described above, the two areas S_1 and S_2 are closer together, and so are the corresponding estimates. It is now clear why the use of the downward-deviating network in graph VIII.10 gives estimates differing from those given by the other networks. At the same time, it can be



Graph VIII.8. Relations among age structure, mortality level, fertility level and intrinsic rate of natural variation in the area of the upward-deviating network of model stable populations in which the population of Venezuela from 1936 to 1950 is situated (This graph is similar to graph VIII.6, with curves for constant intrinsic rates of natural variation added)



Graph VIII.9. Relations among age structure, mortality level, fertility level and intrinsic rate of natural variation in the area of the downward-deviating network of model stable populations in which the population of Venezuela from 1936 to 1950 is situated (This graph is similar to graph VIII.7, with curves for constant intrinsic rates of natural variation added)

seen how unsafe it would be to rely on a single graph in making use of the different networks. A number of these graphs must be used, and the convergence of the various estimates will indicate the degree of reliability.

The foregoing explanations lead to an important conclusion, the essence of which was indicated briefly by comparison of the three networks reflecting in tables VIII.6, VIII.7 and VIII.8. When we use as demographic characteristics the indices of age structure, the intrinsic rate of natural variation, the gross reproduction rate, the expectation of life at birth, the stable birth rate and the stable death rate, the trends of populations like that of Venezuela can be assimilated to that of any quasi-stable populations, whether intermediate, downward-deviating or upward-deviating. Whatever network of stable populations is utilized, we always arrive at roughly the same estimates.

G. Age structure of deaths

We shall encounter further difficulties with the use of the age distribution of deaths. The method, given a level of fertility, of relating the expectation of life at birth to the proportion of deaths at 60 and over in all deaths at 5 and over is obviously applicable to both deviating networks. A specific example will show the difficulties encountered in applying this method.

Graph VIII.12 shows, for a gross reproduction rate of 3.00, the curve of variation, in the three networks, in the proportion of deaths at 60 and over in all deaths at 5 and over, as a function of the expectation of life at birth. Between 1943 and 1960, this proportion increased in Venezuela from 0.2889 to 0.4518. This increase corresponds, on the three curves, to the following values of expectation of life at birth for both sexes:

Network	*Expectation of life at birth for both sexes (in years)* 1943	1960	*Gains in expectation* (a) *of life between 1942 and 1960 (in years)*
Intermediate. . . .	41.0 (point A)	61.4 (point A′)	20.4
Upward-deviating .	34.8 (point B)	58.4 (point B′)	23.6
Downward-deviating	53.0 (point C)	67.4 (point C′)	14.4

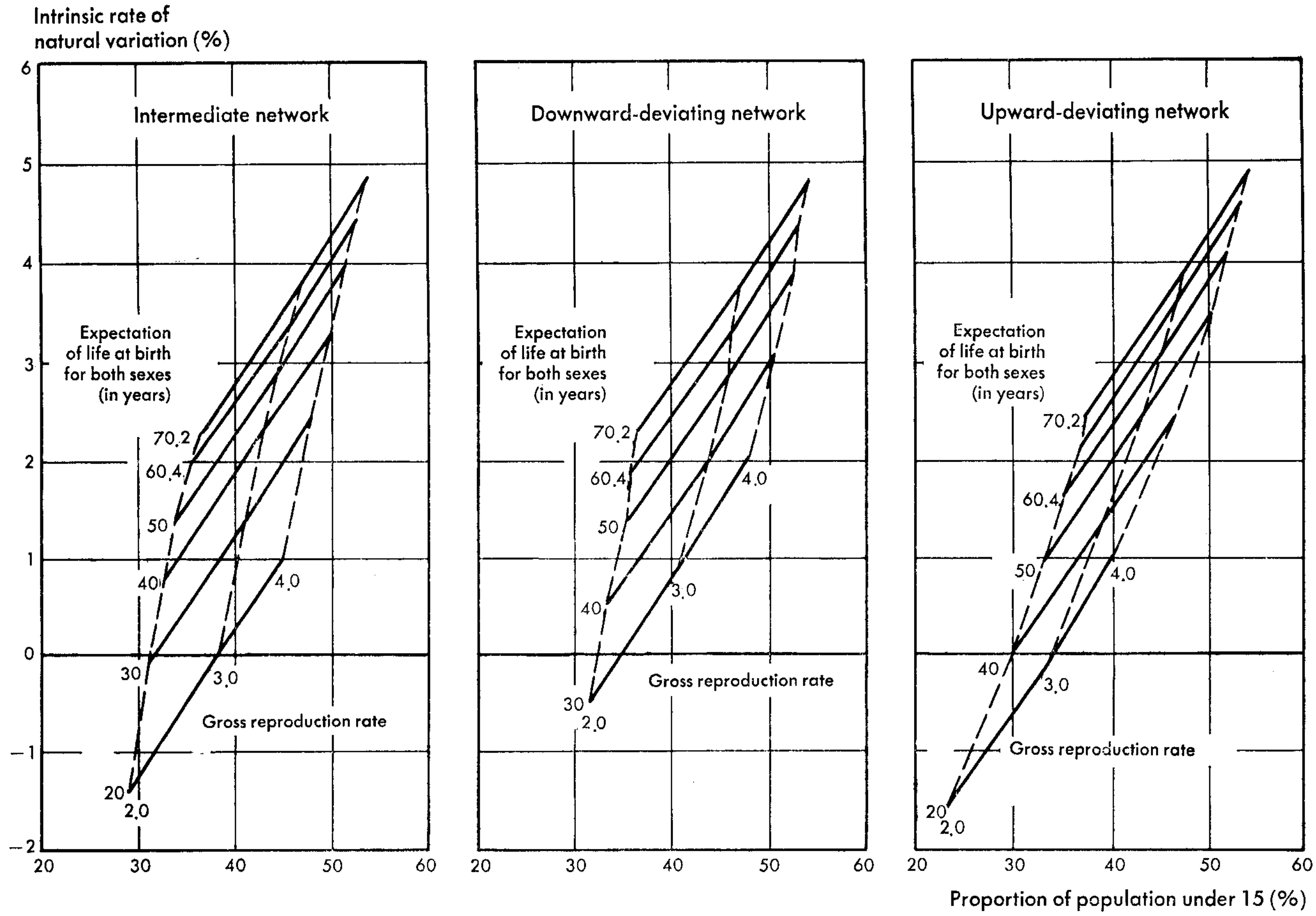
(a) Widely varying gains are obtained according to the network used.

The difficulty arises from the fact that one of the readings, corresponding to the lowest value for the proportion of deaths at 60 and over, is taken from the part of the three curves where the curvature is very different from the others. If the reading were taken on the part of the curves where they are almost parallel the results would be much better. For example, let us compare the two periods 1948-1950 and 1958-1960 for Venezuela. The ratio of deaths at 60 and over to the total number of deaths at 5 and over rises from 0.339 in one period to 0.439 in the other. Graph VIII.12 shows the following values for the expectation of life:

Network	*Expectation of life at birth for both sexes (in years)* 1948-1950	1958-1960	*Increase in expectation of life (in years)*
Intermediate. . . .	48.0	60.0	12.0
Upward-deviating .	46.4	57.2	10.8
Downward-deviating	58.2	66.4	8.2

Estimates of the same order of magnitude are obtained in all three networks. The estimates can also be improved by using other age groups. The proportion of deaths at 65 and over in total deaths at 5 and over, for example, gives good results. This is the ratio which was used for the construction of graph VIII.13. This graph was also used for the countries[5] in table VII.18, showing the estimated variation in expectation of life at birth over the past twenty years. Three periods have been considered: 1940-1942, 1948-1950 and 1958-1960. Table VIII.13 gives the proportions of deaths at 65 and over in all

[5] Some of the countries included in table VII.18 are not shown here because their statistics of deaths by age are prepared by age groups which cannot be used in computing the ratio under consideration. This applies to Honduras, India and Venezuela.



Graph VIII.10. Relations between the proportion of population under 15 and the intrinsic rate of natural variation in the three networks of model stable populations

TABLE VIII.13. RATIO OF DEATHS AT 65 AND OVER TO ALL DEATHS AT 5 AND OVER, OBSERVED IN FOURTEEN COUNTRIES DURING THREE PERIODS SPACED BETWEEN 1940-1960

	Period		
Country	*1940-1942*	*1948-1950*	*1958-1960*
Ceylon	0.2891	0.3610	0.4751
Chile	0.2814	0.3335	0.4037
Chine (Taiwan)	0.2582	0.2568	0.3820
Columbia	Not available ([a])	0.2945	0.3565
Dominican Republic	0.2478	0.2653	0.3593
El Salvador	0.1981	0.2288	0.3039
Guetemala	Not available ([a])	0.2037	0.2323
Mexico	0.2535	0.3192	0.3580
Nicaragua	0.2195	0.2308	0.2957
Panama	Not available ([a])	0.2727	0.4067
Paraguay	0.3338	0.3586	0.4815
Philippines	0.2900	0.2000	0.3451
Thailand	0.2147	0.2034	0.2670
Trinidad and Tobago	0.3498	0.3897	0.4894

([a]) Statistics of deaths are published for age groups which cannot be used in computing the ratio considered here.

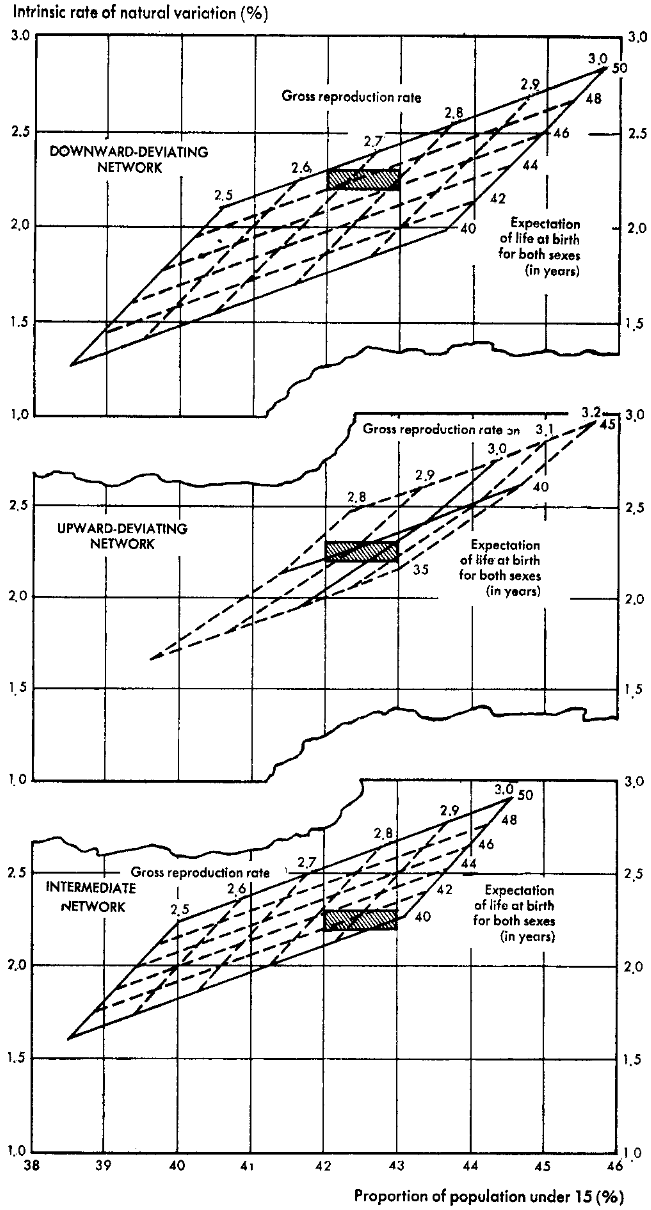
deaths at 5 and over[6] for these three periods, and table VIII.14 gives increases in expectation of life at birth for both sexes, in years corresponding to the ratios in table VIII.13, read from graph VIII.13. Generally speaking, similar estimates are obtained in all three networks. The exceptions, such as Guatemala and El Salvador, correspond to readings on the part of the curves having a very different curvature. Where the three networks give similar estimates, it is safe to assume that they indicate the order of magnitude of the variation in expectation of life at birth. If the estimates differ from network to network, this method cannot be used. This brings us to a particular feature of the use of the age distribution of deaths. With the characteristics used in the preceding graph, we could be satisfied with a reading in only one of the three networks. In this case, we must verify that the three networks give approximately the same result.

H. Estimated mortality by comparison of two census results

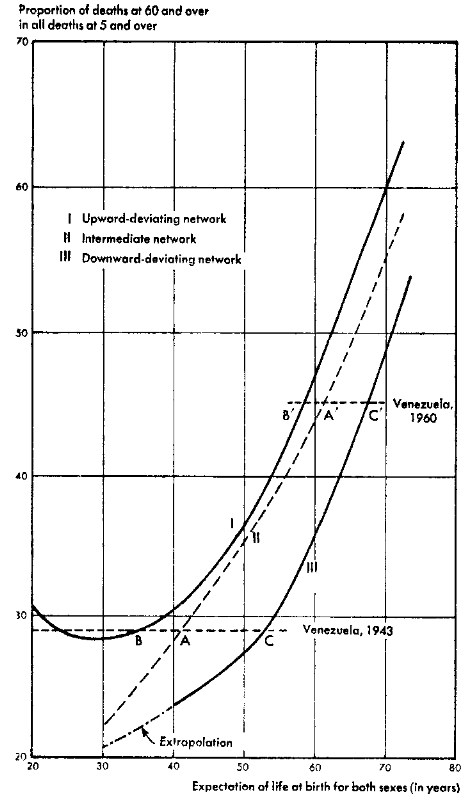
It was stated in chapter VII that the method of comparing corresponding age groups in two censuses in order to estimate mortality could be used without any reference to a network of stable populations. It was also shown that the use of networks could in some cases make applications of the method easier. We shall repeat the example given in chapter VII, with deviating networks, using a comparison of the total numbers of the population above certain age in two censuses.

Tables VIII.15 and VIII.16 give the mortality rates for the stable populations of the two networks above the

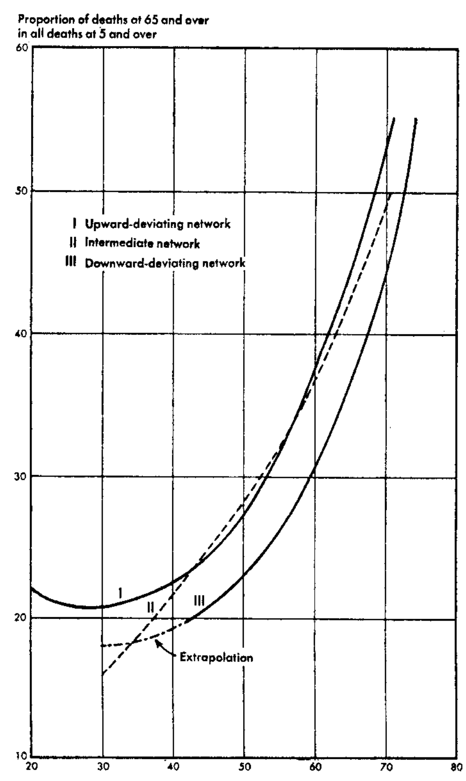
[6] The two graphs VIII.12 and VIII.13 are very similar. The improvement made in graph VIII.13 is due to the fact that the part of it which is used is not the same as in graph VIII.12. For the periods and countries considered in table VIII.13, with graph VIII.12 the readings are often found along a portion of the curves with different curvatures. With graph VIII.13 nearly all the readings happen to be along portions of the curves which are practically parallel to each other.



Graph VIII.11. Relations between the proportion of population under 15 and the intrinsic rate of natural variation in the region of the three networks of model stable populations in which the population of Venezuela is situated from 1936 to 1950 (enlargement of graph VIII.10)



Graph VIII.12. Variations in the ratio of deaths at 60 and over to all deaths at 5 and over in the three networks of model stable populations for different mortality levels and a fertility level corresponding to a gross reproduction rate of 3.0



Graph VIII.13. Variations in the ratio of deaths at 65 and over to all deaths at 5 and over in the three networks of model stable populations for different mortality rates and a fertility rate corresponding to a gross reproduction rate of 3.0

ages shown in the first column. By interpolation of the figures given in these two tables we determined the figures in table VIII.17, showing mortality rates for ages above 9.5 years in the two deviating networks; we have also reproduced figures corresponding to the intermediate network. This table is directly applicable to the case of Venezuela, since, as was shown in chapter VII, the mortality rate at the age of 9.5 and over of the population of Venezuela at mid-point in the period 1941-1950 can be estimated by comparing the results of the two censuses of 1941 and 1950 by age groups. A rate of 15.7 per thousand was obtained. Thus, with the figures in table VIII.17, we have the estimates in table VIII.18 for the expectation of life at birth for both sexes. These estimates vary according to the fertility level assumed, and for a given fertility level the estimates also vary according to the network. If it is assumed that the gross reproduction rate is between 2.8 and 3.2, the estimates vary between 35 and 45 years (table VIII.18). Owing to

TABLE VIII.14. GAINS IN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES (IN YEARS) CORRESPONDING, IN THE THREE NETWORKS OF MODEL STABLE POPULATIONS, TO THE VARIATION IN THE RATIO OF DEATHS AT 65 AND OVER TO ALL DEATHS AT 5 AND OVER, OBSERVED BETWEEN 1940 AND 1960 IN FOURTEEN COUNTRIES

	Network	*1940-1942 to 1948-1950 (8 calendar years)*	*1948-1950 to 1958-1960 (10 calendar years)*
Ceylon	Intermediate	7.9	8.1
	Upward-deviating	7.5	9.0
	Downward-deviating	6.2	7.1
Chile	Intermediate	6.6	5.6
	Upward-deviating	6.1	6.7
	Downward-deviating	5.2	5.6
China (Taiwan)	Intermediate	Variation negligible	13.8
	Upward-deviating		14.5
	Downward-deviating		12.1
Colombia	Intermediate	Figures not available	6.9
	Upward-deviating		6.5
	Downward-deviating		5.6
Dominican Republic	Intermediate	2.6	10.8
	Upward-deviating	3.0	11.0
	Downward-deviating	2.1	9.2
El Salvador	Intermediate	4.9	10.3
	Upward-deviating	Computation not possible	12.2
	Downward-deviating	8.9	9.3
Guatemala	Intermediate	Figures not available	4.2
	Upward-deviating		12.7
	Downward-deviating		6.0
Mexico	Intermediate	8.6	4.2
	Upward-deviating	9.0	4.0
	Downward-deviating	7.3	3.1
Nicaragua	Intermediate	3.9	10.5
	Upward-deviating	2.0	9.7
	Downward-deviating	2.3	8.2
Panama	Intermediate	Figures not available	13.6
	Upward-deviating		14.0
	Downward-deviating		12.2
Paraguay	Intermediate	0.4	8.0
	Upward-deviating	0.6	9.6
	Downward-deviating	0.3	7.9
Philippines	Intermediate	Figures not available	6.2
	Upward-deviating		5.6
	Downward-deviating		5.2
Thailand	Intermediate	Variation negligible	12.2
	Upward-deviating		8.6
	Downward-deviating		8.1
Trinidad and Tobago	Intermediate	5.2	6.0
	Upward-deviating	6.0	6.5
	Downward-deviating	4.6	6.0

their lack of precision, these estimates tell us very little about the mortality level. On the other hand, the estimates in table VIII.8, taken in conjunction with the preceding estimates, can provide information on the type of mortality in Venezuela.

Earlier, by using the age structure and the rate of natural variation, we obtained the following estimates for expectation of life at birth:

Network	*Expectation of life at birth for both sexes (in years)*
Intermediate	39-43
Upward-deviating	36-42
Downward-deviating	Under 43

Taking these results in conjunction with the figures in table VIII.18, one would be tempted to say that Venezuela's mortality in 1946 was rather of the intermediate type and should therefore be at a level that would give an expectation of life at birth for both sexes of about 40 years.

However, we must not overlook the fact that all these mortality estimates are very precarious, and this is indeed the conclusion which follows from the developments in chapters VII and VIII. When an actual population can legitimately be assimilated to a stable population, any network of stable populations will serve to provide a relatively precise estimate of the gross reproduction rate. The estimation of expectation of life at birth, however, remains very hazardous. In the first place, for a given network the estimation of expectation of life is less precise than the estimation of the gross reproduction rate; and, in addition, it varies according to the network used.

TABLE VIII.15. DEATH RATE (PER THOUSAND) ABOVE THE AGES INDICATED FOR DIFFERENT LEVELS OF FERTILITY AND MORTALITY IN THE UPWARD-DEVIATING NETWORK OF MODEL STABLE POPULATIONS

	Expectation of life at birth (in years)					
Age (in years)	*20*	*40*	*40*	*50*	*60.4*	*70.2*
Gross reproduction rate: 4.0						
0 and over	55.6	37.4	24.5	16.0	9.7	4.0
1 and over	31.1	21.1	13.7	8.6	4.6	2.4
5 and over	21.2	14.2	9.9	6.9	4.2	2.6
10 and over	22.8	15.8	11.2	8.0	4.9	3.3
15 and over	25.3	18.4	13.2	9.6	6.3	4.3
20 and over	28.6	21.2	15.6	11.5	7.8	5.2
25 and over	32.3	24.0	18.3	13.7	9.6	6.5
Gross reproduction rate: 3.0						
0 and over	50.5	34.7	23.5	19.6	9.0	4.6
1 and over	31.8	21.9	14.9	9.7	5.6	3.5
5 and over	24.4	16.7	11.7	8.5	5.5	3.8
10 and over	26.1	18.4	13.2	9.6	6.5	4.7
15 and over	29.2	21.1	15.6	11.5	8.2	5.6
20 and over	32.1	23.7	17.9	13.3	9.6	6.8
25 and over	35.1	26.6	20.7	15.7	11.5	8.3
Gross reproduction rate: 2.0						
0 and over	47.8	33.3	23.5	16.4	10.6	6.8
1 and over	35.1	25.1	17.7	12.5	8.3	5.8
5 and over	30.7	21.7	16.1	11.9	8.7	6.5
10 and over	32.6	23.6	17.7	13.4	9.9	7.5
15 and over	35.3	25.9	19.9	15.4	11.5	8.7
20 and over	38.1	28.7	22.3	17.6	13.2	10.2
25 and over	40.5	31.5	25.1	19.9	15.6	11.7

TABLE VIII.16. DEATH RATE (PER THOUSAND) ABOVE THE AGES INDICATED FOR DIFFERENT LEVELS OF FERTILITY AND MORTALITY IN THE DOWNWARD-DEVIATING NETWORK OF MODEL STABLE POPULATIONS

	Expectation of life at birth (in years)					
Age (in years)	*20*	*30*	*40*	*50*	*60.4*	*70.2*
Gross reproduction rate: 4.00						
0 and over	Rates not computed for these mortality levels		23.1	14.7	9.1	4.3
1 and over			18.3	11.1	6.5	3.1
5 and over			14.9	9.8	6.2	3.3
10 and over			15.7	11.5	6.8	3.9
15 and over			18.9	12.2	8.1	4.8
20 and over			20.3	13.8	9.3	5.9
25 and over			21.9	15.1	10.4	6.9
Gross reproduction rate: 3.00						
0 and over	Rates not computed for these mortality levels		23.4	15.0	9.5	4.8
1 and over			18.6	12.1	7.5	4.1
5 and over			16.5	11.2	7.4	4.6
10 and over			17.7	12.1	8.2	5.2
15 and over			19.8	13.8	9.5	6.1
20 and over			22.0	15.5	10.8	7.3
25 and over			23.6	17.0	12.2	8.7
Gross reproduction rate: 2.00						
0 and over	Rates not computed for these mortality levels		24.6	16.6	11.3	6.7
1 and over			21.2	14.7	10.0	6.3
5 and over			20.3	14.4	10.2	7.0
10 and over			21.4	15.5	11.2	7.8
15 and over			23.9	17.2	12.6	9.2
20 and over			25.8	19.0	14.1	10.5
25 and over			27.5	20.8	15.7	12.1

TABLE VIII.17. DEATH RATE (PER THOUSAND) AT THE AGE OF 9.5 AND OVER IN THE THREE NETWORKS OF MODEL STABLE POPULATIONS

		Expectation of life at birth for both sexes (in years)					
Gross reproduction rate	*Network*	*20*	*30*	*40*	*50*	*60.4*	*70.2*
4.0	Intermediate	27.6	18.5	12.6	8.6	5.6	3.5
	Upward-deviating	22.6	15.6	11.1	7.9	4.8	3.2
	Downward-deviating	(a)	(a)	15.6	11.3	6.7	3.8
3.0	Intermediate	30.7	20.9	14.7	10.2	7.0	4.8
	Upward-deviating	25.9	18.2	13.0	9.5	6.4	4.6
	Downward-deviating	(a)	(a)	17.6	12.0	8.1	5.1
2.0	Intermediate	36.4	25.7	18.6	13.8	10.1	7.6
	Upward-deviating	32.4	23.4	17.5	13.2	9.8	7.4
	Downward-deviating	(a)	(a)	21.3	15.4	11.1	7.7

(a) Rates not computed.

TABLE VIII.18. ESTIMATES IN THE THREE NETWORKS OF MODEL STABLE POPULATIONS OF EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES ACCORDING TO THE AVERAGE MORTALITY RATE, 1941-1950, AT THE AGE OF 9.5 YEARS AND OVER, OF THE POPULATION OF VENEZUELA

	Gross reproduction rate				
Network	*4.0*	*3.2*	*3.0*	*2.8*	*2.0*
Intermediate	35.3	38.2	38.9	40.5	46.7
Upward-deviating	30.0	35.4	36.8	38.3	44.2
Downward-deviating	40.0	42.7	43.4	44.6	49.5

We shall not pursue the subject of networks of stable populations any further. It was not our intention to make an exhaustive study of them, and many other examples of the use of these networks can readily be conceived. Those which we have chosen will suffice, however, to show how useful they can be, but they also show their limitations very clearly. In conclusion, we offer a few ideas regarding work which might usefully supplement this study.

1. First, the concept of quasi-stable populations would merit a more systematic study. We have indicated the properties of these populations on the basis of a small number of examples, using the intermediate model life tables. It would be useful to undertake other computations, first using the same tables and then using upward-deviating or downward-deviating model life tables.

2. It has been assumed that there was no migration. In other words, the factors determining population trends have been reduced to two: mortality and fertility. Since, in fact, there is always some migratory movement, we must begin by eliminating its effects before making any computations. This is what we did in the case of Venezuela, for example, when we distinguished between the alien population, the Indians in the jungle and the non-alien population. While it is often possible to eliminate the effects of migratory movement with respect to an entire country, it is much more difficult when a region within the country is being considered, in which case the necessary statistics are not available and as the migration is internal, its effects are greater than in the case of international migration. If the methods described in this work are to be applied to regions within a country or to sub-populations—e.g., the urban population and the rural population—the migration factor must be incorporated in the stable population and new model networks must be constructed.

3. Lastly, a basic assumption underlying all the methods of estimation proposed in this study is that fertility varies very little. This assumption has been more or less confirmed for the under-developed countries in the past and it still holds for most of those countries, but there is reason to believe that it will no longer reflect the true situation a few years hence. A decline in fertility seems likely sooner or later, and it is therefore necessary to compute networks of populations with declining fertility.

A coming publication of the Regional Centre for Demographic Training and Research in Latin America should meet these various needs, at least in part.

Annex I

CALCULATION OF THE TOTAL NUMBER OF THE POPULATION, ANNUAL BIRTHS, ANNUAL GROWTH AND ANNUAL DEATHS IN A STABLE POPULATION ON THE BASIS OF A GIVEN INITIAL POPULATION

A. Introduction

In chapter III, the concept of a stable population was defined as the limit of a demographic process, characterized by constant fertility and constant mortality, and evolved from a given initial population. It was recalled that Alfred J. Lotka had shown that in such a process the population approached a stable Malthusian population, or, in simpler terms, a stable population corresponding to the assumed constant laws of mortality and fertility. Formulae were established for computing the intrinsic rate of natural increase, the stable birth rate, the age distribution of the population and the age distribution of deaths for this stable population. Formulae were also given, *but not established*, for calculating the total number of the population and the absolute number of births and deaths in the limit stable population. The purpose of annex I is to establish the latter formulae.

B. Total number of the population

Let in the initial population—at time zero— the number of women of age a be equal to $K_f(a,0)da$. At time y the number has increased to:

$$\frac{K_f(a, 0)p_f(a + y)}{p_f(a)}\, da$$

where $p_f(a)$ is the female survivorship function assumed to be invariable over time. During the period dy, they give birth to dB daughters, so that:

$$dB = \frac{K_f(a, 0)}{p_f(a)}\, p_f(a + y)\varphi_f(a + y)dady$$

where $\varphi_f(a + y)$ is the fertility rate of the women of age $(a + y)$ calculated from the daughters. According to Lotka's theorem, when a sufficient length of time $(t - y)$ has elapsed, the population resulting from the birth element dB will have become stable. The total number of this population derived from dB will then be an exponential function of the time elapsed since birth and will be proportionate to the initial total number dB.

In other words, when time t is sufficiently large, the dB daughters will constitute a female population totalling:

$$Q\,\frac{K_f(a, 0)}{p_f(a)}\, p_f(a + y)\varphi_f(a + y)e^{r(t-y)}dady$$

where Q is a constant which remains the same irrespective of age a, depending only on the functions $p_f(a)$ and $\varphi_f(a)$, and r is the intrinsic rate of natural increase of the stable population corresponding to the laws of mortality and fertility $p_f(a)$ and $\varphi_f(a)$.

At time t, the total female population, offspring of the $K_f(a, 0)da$ women at time zero will be equal to:

$$dN = \frac{QK_f(a, 0)}{p_f(a)}\, da \int_0^{v-a} p_f(a + y)\varphi_f(a + y)e^{r(t-y)}dy$$

in which v is the age of menopause, after which the fertility function $\varphi_f(y)$ remains zero. If $a + y = x$, this can be written:

$$dN = \frac{QK_f(a, 0)}{p_f(a)}\, da \int_a^{v} p_f(x)\varphi_f(x)e^{r(t+a-x)}dx$$

or again

$$dN = \frac{QK_f(a, 0)}{p_f(a)}\, e^{r(t+a)}da \int_a^{v} p_f(x)\varphi_f(x)e^{-rx}dx$$

Let us assume that:

$$g(a) = \int_a^{v} p_f(x)\varphi_f(x)e^{-rx}dx$$

We can then write:

$$dN = Qe^{rt}\,\frac{K_f(a, 0)}{p_f(a)}\, e^{ra}g(a)da$$

At time t the total female population, progeny of the initial women, will be:

$$N_f(t) = Qe^{rt} \int_0^{v} \frac{K_f(a, 0)}{p_f(a)}\, e^{ra}g(a)da \qquad (1)$$

Computation of the constant Q

We shall now calculate Q. Let us assume, at time zero, a second female population *equal to unity* and with the age structure of the stable population corresponding to the laws $p_f(a)$ and $\varphi_f(a)$. In this population, the number of women of age a is equal to:

$$\frac{e^{-ra}p_f(a)}{\int_0^{\infty} e^{-ra}p_f(a)da} \times 1$$

At time t, the population in question has become $e^{rt} \times 1$ and formula (1) applies. Accordingly, we find that:

$$e^{rt} = Qe^{rt} \int_0^{v} \frac{e^{-ra}p_f(a)}{\int_0^{\infty} e^{-ra}p_f(a)da} \times \frac{1}{p_f(a)} \times e^{ra}g(a)da$$

This can be written:

$$\int_0^\omega e^{-ra}p_f(a)da = Q\int_0^v g(a)da$$

which finally gives:

$$Q = \frac{\int_0^\omega e^{-ra}p_f(a)da}{\int_0^v g(a)da}$$

If we introduce this value for Q into formula (1) we end with the following formula for the total number of the population in the stable state:

$$N_f(t) = e^{rt}\int_0^\omega e^{-ra}p_f(a)da\int_0^v \frac{K_f(a, 0)}{p_f(a)}\frac{g(a)}{\int_0^v g(a)da}e^{ra}da$$

or again:

$$N_f(t) = e^{rt}\int_0^\omega e^{-ra}p_f(a)da\int_0^v \frac{K_f(a, 0)}{p_f(a)}G(a)e^{ra}da \qquad (2)$$

assuming that:

$$G(a) = \frac{g(a)}{\int_0^v g(a)da} \qquad (3)$$

We can also write:

$$N_f(t) = e^{rt}\int_0^v \frac{K_f(a, 0)}{C_f(a)}G(a)da \qquad (4)$$

where $C_f(a)$ is the age distribution of the female stable population.

If the rate of increase r is zero, the population is a stationary one and formula 2 becomes:

$$N_0(t) = \int_0^\omega p_f(a)da\int_0^v \frac{K_f(a, 0)}{p_f(a)}G(a)da$$

or:

$$N_0(t) = e_0^0\int_0^v \frac{K_f(a, 0)}{p_f(a)}G(a)da \qquad (5)$$

Formula (4) is then written:

$$N_0(t) = \int_0^v \frac{K_f(a, 0)}{{}_0C_f(a)}G(a)da \qquad (6)$$

where ${}_0C_f(a)$ is the age structure of the stationary population.

Annual births, deaths and growth

The number of female births *per annum* is obtained by multiplying the total number of the population by the stable birth rate:

$$b_f = \frac{1}{\int_0^\omega e^{-ra}p_f(a)da}$$

or:

$$B_f(t) = e^{rt}\int_0^v \frac{K_f(a, 0)e^{ra}}{p_f(a)}G(a)da \qquad (7)$$

The annual increase in the female population is equal to the product of the total number of the population and the intrinsic rate of natural increase r. We therefore obtain:

$$A_f(t) = re^{rt}\int_0^\omega e^{-ra}p_f(a)da\int_0^v \frac{K_f(a, 0)e^{ra}}{p_f(a)}G(a)da \qquad (8)$$

The annual number of female deaths is:

$$D_f(t) = B_f(t) - A_f(t) \qquad (9)$$

The male population

The foregoing formulae relate to the female population. The characteristics of the male population can easily be deduced from them. We shall first compute the number of male births *per annum*. If m is the masculinity at birth, we obviously find that:

$$B_m(t) = mB_f(t)$$

whence

$$B_m(t) = me^{rt}\int_0^v \frac{K_f(a, 0)e^{ra}}{p_f(a)}G(a)da$$

The male population $N_m(t)$ is equal to the annual male births divided by the male birth rate:

$$b_m = \frac{1}{\int_0^\omega e^{-ra}p_m(a)da}$$

Accordingly,

$$N_m(t) = me^{rt}\int_0^\omega e^{-ra}p_m(a)da\int_0^v \frac{K_f(a, 0)e^{ra}}{p_f(a)}G(a)da$$

The male increase *per annum* is equal to the population multiplied by the intrinsic rate of natural increase:

$$A_m(t) = rN_m(t)$$

Finally, the annual male deaths are obtained by the difference between annual births and the annual increase:

$$D_m(t) = B_m(t) - A_m(t)$$

The reduced initial population

The initial female population $K_f(a, 0)$ from age zero to age v appears in all these formulae. The initial population above age v does not come into the reckoning, as might be expected, since it has no progeny. Moreover, this initial population is multiplied at each age by the coefficient

$$\frac{G(a)}{p_f(a)}e^{ra}$$

The series of quantitites:

$$\frac{K_f(a, 0)G(a)}{p_f(a)}e^{ra} \qquad (10)$$

will be called the "reduced initial net population". [1] The mean age of this reduced initial net population will be represented by γ.

[1] A definition of what is meant by the expression "reduced initial gross population" will be given later.

All the foregoing results apply, irrespective of the values of the functions $p_f(a)$, $p_m(a)$ and $\varphi_f(a)$. They can be greatly simplified if it is assumed that these functions represent mortality and fertility functions of the human species. This will be considered next.

STUDY OF THE FUNCTIONS $g(a)$ AND $G(a)$ IN THE CASE OF THE HUMAN SPECIES

The formula used was:

$$g(a) = \int_a^v p_f(x)\varphi_f(x)e^{-rx}dx$$

Its differential element is the product of three factors. $p_f(x)$ is the female survivorship function. It is equal to unity when $x = 0$ and to zero at the limit age of human life. In between, it is a decreasing function. $\varphi_f(x)$ is the female fertility function. It is zero before the age of puberty u and after the age of menopause v. In the intervening period (u, v) the fertility function at first increases, then decreases, with a maximum which varies between the ages of 20 and 25 years according to populations. For positive values of r, e^{-rx} decreases regularly between u and v; for negative values of r, it increases regularly between u and v.

The product $p_f(x)\varphi_f(x)e^{-rx}$ is therefore shaped as shown in graph A.I.1, and the function $g(a)$ is represented in the same graph by the shaded area. The function

$$G(a) = \frac{g(a)}{\int_0^\omega g(a)da}$$

is the distribution function of $g(a)$. It will be particularly noted that $G(a)$ depends only on the age distribution of the $\varphi_f(a)$ rates, and not on their value.

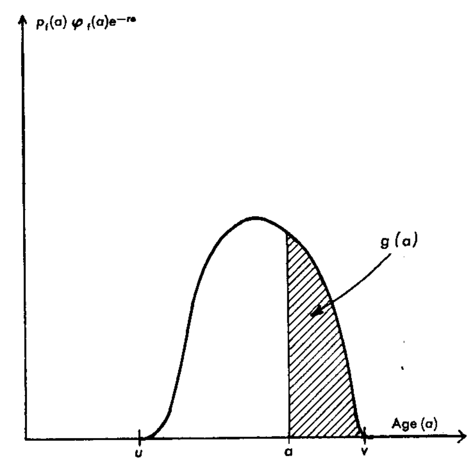
In tables A.I.1 and A.I.2, an example is given of the numerical computation of the function $G(a)$ under the following conditions: the female survivorship function corresponding to the model life table (intermediate series) with a life expectancy at birth for both sexes of 60.4 years was adopted. For the age distribution of fertility rates, the intermediate distribution was chosen. Lastly, an intrinsic rate of natural increase of 2 per cent was used. Table A.I.1 and table A.I.2 give the details of the computation of the function $p_f(a)\varphi_f(a)e^{-ra}$ and of the function $G(a)$ respectively.

Similar computations were made for other values of survivorship functions, age distributions of fertility and intrinsic rates of natural variation. The results of these computations are given in tables A.I.3 to A.I.5.

Table A.I.3 shows how $G(a)$ varies with the intrinsic rate of natural variation. Here, the same survivorship table and the same age distribution of female fertility rates are retained as for the computations in tables A.I.1 and A.I.2. The computations are confined to the values of the intrinlsc rate of natural variation that are encountered in practice, namely:

$$-0.02 < r < +0.04$$

It can be concluded from table A.I.3 that *in practice* $G(a)$ is virtually independent of the intrinsic rate of variation. Thus, it is sufficient simply to study the influence on $G(a)$ of variations in the survivorship function and in the age distribution of fertility rates, for a single value of the intrinsic rate of natural variation. This is what is done in tables A.I.4 and A.I.5, taking an intrinsic rate of natural variation equal to zero.



Graph A.I.1. Shape of the curve $p_f(a)\varphi_f(a)e^{-ra}$

TABLE A.I.1. EXAMPLE OF COMPUTATION OF THE FUNCTION $p_f(a)\varphi_f(a)e^{-ra}$

Age group (in years)	*Median age*	*Survivorship function* $p_f(a)$ (a)	$\varphi_f(a)$ (b)	*Product of the two preceding columns* $p_f(a)\varphi_f(a)$	e^{-ra} *for* $r = 0.02$	*Product of the two preceding columns* $p_f(a)\varphi_f(a)e^{-ra}$
15-19	17.5	0.879940	0.100	0.08799400	0.70469	0.062008492
20-24	22.5	0.868080	0.273	0.23698584	0.63763	0.151109281
25-29	27.5	0.854070	0.263	0.22462041	0.57695	0.129594746
30-34	32.5	0.839220	0.188	0.15777336	0.52205	0.082365583
35-39	37.5	0.823344	0.121	0.09962462	0.47237	0.047059682
40-44	42.5	0.805484	0.055	0.04430162	0.42742	0.018935398
			1.000			

(a) The survivorship function $p_f(a)$ is the female survivorship function from the model life table (intermediate series) with an expectation of life at birth for both sexes of 60.4 years.

(b) The female fertility rates $\varphi_f(a)$ are those which would be found with the intermediate age distribution if the gross reproduction rate was 5.00. For a reproduction rate of R′, the φ (a) rates would have to be multiplied by R′/5. The function G(a), which is expressed in the form of a fraction, would be multiplied (both numerator and denominator) by R′/5. It is therefore independent of R′ and depends only on the age distribution of fertility rates.

TABLE A.I.2. EXAMPLE OF COMPUTATION OF THE FUNCTIONS $g(a)$ AND $G(a)$
(for the values of $p_f(a)$, $\varphi_f(a)$ and r given in table A.I.1)

Median age	$p_f(a)\varphi_f(a)e^{-ra}$	Arithmetic mean of two (a) consecutive figures in the previous column	Cumulative totals 1/5g(a)	G(a) (distribution function of g(a))
2.5			0.491073180	0.1847
7.5			0.491073180	0.1847
12.5	0.000000000	0.031004246	0.491073180	0.1847
17.5	0.062008492	0.106558886	0.460068934	0.1729
22.5	0.151109281	0.140352013	0.353510048	0.1329
27.5	0.129594746	0.105980164	0.213158035	0.0802
32.5	0.082365583	0.064712632	0.107177871	0.0403
37.5	0.047059682	0.032997540	0.042465239	0.0160
42.5	0.018935398	0.009467699	0.009467699	0.0036
47.5	0.000000000			
			2.659067366	1.0000

(a) The method of computation amounts, in effect, to assuming low fertility from 12.5 to 15 years and from 45 to 47.5 years. The effect of this low fertility on the values found for G(a) is negligible.

TABLE A.I.3. THE FUNCTION $G(a)$ FOR VARIOUS VALUES OF THE INTRINSIC RATE OF NATURAL VARIATION *

Age group (in years)	Intrinsic rate of natural increase (per cent)												
	—2.0	—1.5	—1.0	—0.5	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0-4 . .	17.32	17.55	17.56	17.74	17.89	18.03	18.18	18.32	18.47	18.90	18.76	18.91	19.05
5-9 . .	17.32	17.55	17.56	17.74	17.89	18.03	18.18	18.32	18.47	18.90	18.76	18.91	19.05
10-14 . .	17.32	17.55	17.56	17.74	17.89	18.03	18.18	18.32	18.47	18.90	18.76	18.91	19.05
15-19 . .	16.60	16.70	16.74	16.87	16.96	17.05	17.14	17.22	17.30	17.65	17.46	17.52	17.60
20-24 . .	13.73	13.76	13.67	13.60	13.55	13.49	13.43	13.36	13.29	13.42	13.14	13.04	12.97
25-29 . .	9.35	9.24	9.11	8.87	8.70	8.53	8.36	8.19	8.02	6 42	7.67	7.50	7.32
30-34 . .	5.37	5.22	5.06	4.86	4.68	4.52	4.35	4.19	4.03	3.93	3.72	3.57	3.43
35-39 . .	2.41	2.31	2.21	2.09	1.98	1.88	1.72	1.69	1.60	1.53	1.43	1.35	1.27
40-44 . .	0.60	0.60	0.53	0.50	0.47	0.44	0.41	0.38	0.36	0.34	0.31	0.29	0.27
0-44 . .	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

* *Note*: The female survivorship function is that used in the model life table (intermediate series) with an expectation of life at birth for both sexes of 60.4 years, and the age distribution of female fertility rates is the intermediate distribution.

TABLE A.I.4. THE FUNCTION $G(a)$ IN A FEMALE STATIONARY POPULATION CORRESPONDING TO THE MODEL LIFE TABLE WITH AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS, FOR THREE AGE DISTRIBUTIONS OF FEMALE FERTILITY RATES

Age group (in years)	Age distribution of female fertility rates		
	Spain 1940	Intermediate	Jamaica 1951
0-4	16.43	17.89	18.82
5-9	16.43	17.89	18.82
10-14	16.43	17.89	18.82
15-19	16.28	16.96	15.29
20-24	14.49	13.55	13.39
25-29	9.55	8.70	8.25
30-34	5.88	4.68	4.37
35-39	3.68	1.98	1.83
40-44	0.88	0.47	0.42
0-44	100.00	100.00	100.00

TABLE A.I.5. THE FUNCTION $G(a)$ IN FIVE FEMALE STATIONARY POPULATIONS CORRESPONDING TO FIVE MODEL LIFE TABLES, THE INTERMEDIATE DISTRIBUTION BEING USED FOR THE AGE DISTRIBUTION OF FEMALE FERTILITY RATES

Age group (in years)	Expectation of life at birth for both sexes, in years				
	30	40	50	60.4	70.2
0-4	18.28	18.10	37.98	17.89	17.83
5-9	18.28	18.10	17.98	17.89	17.83
10-14	18.28	18.10	17.98	17.89	17.83
15-19	17.21	17.10	17.02	16.96	16.93
20-24	13.42	13.47	13.52	13.55	13.57
25-29	8.25	8.45	8.60	8.70	8.77
30-34	4.22	4.43	4.58	4.68	4.75
35-39	1.69	1.82	1.97	1.98	2.02
40-44	0.38	0.42	0.45	0.47	0.48
0-44	100.00	100.00	100.00	100.00	100.00

Table A.I.4 gives the function G(a) for three age distributions of fertility rates: that observed in Spain in 1940, that observed in Jamaica in 1951 and, lastly, the "intermediate" distribution already given in table A.I.3. It will be seen from table A.I.4 that variations in the age distribution of fertility rates do not greatly alter the function G(a).

Lastly, table A.I.5 demonstrates the effects on G(a) of variations in the survivorship function. G(a) was computed with the use of the intermediate age distribution of fertility rates for five stationary populations corresponding to the five model life tables (intermediate series) with expectations of life at birth for both sexes equal to 30, 40, 50, 60.4 and 70.2 years respectively. Table A.I.5 shows that the function G(a) varies little with the mortality level. To sum up, it may be said that in the human species the function G(a) shows little variation when fertility and mortality vary. In practice, therefore, the same function can always be used. For the computations which follow and for those in chapter II, the values shown in table A.I.6 were adopted for the function G(a).

TABLE A.I.6. ADOPTED VALUES FOR FUNCTION G(a) IN GENERAL COMPUTATIONS

Age group (in years)	G(a)
0-4	18
5-9	18
10-14	18
15-19	17
20-24	13
25-29	9
30-34	5
35-39	2
40-44	0
0-44	100

In the foregoing, the words "practically" or "in practice" have been stressed a number of times. It is wise to bear them constantly in mind.

In the numerical computations of the function G(a), it was first assumed that the mortality was that of the model life tables (intermediate series). The model tables represent only an average. Each specific case varies from the average to a greater or lesser degree, and the variations modify the function G(a). With human populations these variations in the function G(a) are negligible, but if the above results were to be applied to renewable resources other than human populations the numerical values of the function G(a) would have to be determined in each case.

Secondly, it was assumed that the age distribution of female fertility rates could vary only within very narrow limits. Distributions differing greatly from those limits might result in G(a) functions very different from the one chosen for the computation. Here again, so long as human populations are being studied there is no danger of going far outside the limits adopted, but this might not be so in the case of other populations.

Lastly, it was assumed in the above numerical computations that the intrinsic rate of natural variation was always relatively small, as it always is in the case of human populations. However, very high values of the intrinsic rate of natural variation will at times have to be considered below, in order to see how certain demographic curves behave. In such cases, it must be borne in mind that the curves thus obtained are valid only between certain limits. It is difficult to fix these limits in an absolute manner; they depend on the degree of imprecision one is prepared to accept.

Table A.I.7 shows:

(a) The extreme values of G(a) when r is infinite;[2]

(b) The function G(a) adopted for the calculations;

(c) The function G(a) for two relatively high values of the intrinsic rate of natural variation (+10 per cent and −10 per cent) computed from the intermediate age distribution of fertility rates and the model life table (intermediate series) with an expectation of life at birth for both sexes of 60.4 years.

It will be seen that, with intrinsic rates of natural variation of 10 per cent, there is a marked deviation from the function G(a) adopted for the computations.

Fortunately, the rates of natural variation in populations encountered in practice are always far below 10 per cent. A rate of between 4 and 5 per cent appears to be the maximum, and within those limits the function G(a) adopted for the computations is fully satisfactory.

TABLE A.I.7. VALUES OF THE FUNCTION G(a): (a) FOR INFINITELY LARGE VALUES OF THE INTRINSIC RATE OF NATURAL VARIATION; (b) FOR TWO RELATIVELY HIGH VALUES OF THE INTRINSIC RATE OF NATURAL VARIATION (+10 PER CENT AND −10 PER CENT) IN A FEMALE STABLE POPULATION CORRESPONDING TO THE MODEL LIFE TABLE WITH AN EXPECTATION OF LIFE AT BIRTH FOR BOTH SEXES OF 60.4 YEARS, USING THE "INTERMEDIATE" AGE DISTRIBUTION FOR THE RATES OF FEMALE FERTILITY; (c) AS ADOPTED FOR THE COMPUTATIONS

Age group (in years)	*Intrinsic rate of natural variation* $+\infty$	+10 *per cent*	−10 *per cent*	$-\infty$	*Function G(a) adopted for the computations*
0-4	33.33	20.73	15.26	11.11	18
5-9	33.33	20.73	15.26	11.11	18
10-14 . . .	33.33	20.73	15.26	11.11	18
15-19 . . .	00.00	18.24	15.04	11.11	17
20-24 . . .	00.00	11.68	13.86	11.11	13
25-29 . . .	00.00	5.28	11.37	11.11	9
30-34 . . .	00.00	1.95	8.10	11.11	5
35-39 . . .	00.00	0.57	4.51	11.11	2
40-44 . . .	00.00	0.10	1.34	11.11	0
0-44 . . .	100.00	100.00	100.00	100.00	100

C. Families of populations with constant mortality

From a given initial population, an infinite number of stable populations can be obtained, according to the combination $p_f(a)$, $\varphi_f(a)$ adopted for the computation. To be more precise, there is a double infinity of stable populations because there are an infinite number of ways of choosing either $p_f(a)$ or $\varphi_f(a)$. In this double infinity, "families of populations" can be distinguished by keeping either the survivorship function or the fertility function

[2] A proof of the values of G(a) when r is infinite appears at the end of the appendix.

constant. The families with a constant survivorship function are easier to study than those with a constant fertility function, since we have:

$$N_f(t) = e^{rt}\int_0^{\omega} p_f(a)e^{-ra}da \int_0^{v}\frac{K_f(a)}{p_f(a)}\,G(a)e^{ra}da$$

If the survivorship function is constant, the only variable is the intrinsic rate of natural variation because, as we have seen, G(a) is practically invariable. On the other hand, if the fertility function is constant, r and $p_f(a)$ are variable. Furthermore, those two quantities are linked by the relationship:

$$\int_0^{v} e^{-ra}p_f(a)\varphi_f(a)da = 1$$

It will be clear why the first case is easier to study than the second. Families of populations with a constant survivorship function will therefore be considered first.

We shall begin by stating the problem more precisely:

(*a*) An initial female population $K_f(a, 0)$ is assumed. For example, a female population of 1 million with its age distribution is given;

(*b*) A female mortality $p_f(a)$ is chosen;

(*c*) All the stable populations obtained from this initial population are computed by associating all the fertilities possible *in practice* with the chosen survivorship table. The "family" of populations obtained in this way is what we term a family of stable populations with constant mortality.

Envelope of a family of populations with constant mortality

As they depend only on the single variable r, the populations in the family have an envelope. The abscissa of the point of contact of a population with this envelope is obtained by setting the derivative function dN/dr equal to zero, and this is written:

$$\frac{dN}{dr} = te^{rt}\int_0^{\omega} e^{-ra}p_f(a)da\int_0^{v}\frac{K_f(a,0)}{p_f(a)}\,e^{ra}G(a)da - e^{rt}\int_0^{\omega} ae^{-ra}p_f(a)da\int_0^{\omega}\frac{K_f(a,0)}{p_f(a)}\,e^{ra}G(a)da + e^{rt}\int_0^{\omega} e^{-ra}p_f(a)da\int_0^{v}\frac{aK_f(a,0)}{p_f(a)}\,e^{ra}G(a)da$$

or

$$\frac{dN}{dr} = e^{rt}\int_0^{\omega} e^{-ra}p_f(a)da\int_0^{v}\frac{K_f(a,0)}{p_f(a)}\,G(a)e^{ra}da(t-\alpha+\gamma)$$

and

$$\frac{dN}{dr} = N(t)(t-\alpha+\gamma)$$

assuming that

$$\alpha = \frac{\int_0^{\omega} ae^{-ra}p_f(a)da}{\int_0^{\omega} e^{-ra}p_f(a)da} \quad\text{and}\quad \gamma = \frac{\int_0^{v}\frac{aK_f(a,0)}{p_f(a)}\,e^{ra}G(a)da}{\int_0^{v}\frac{K_f(a,0)}{p_f(a)}\,e^{ra}G(a)da}$$

α is simply the mean age of the stable population and γ is the mean age of the reduced initial net population. At the point of contact, therefore: $t_c - \alpha + \gamma = 0$, whence $t_c = \alpha - \gamma$. The ordinate of the point of contact is obtained by introducing this value of $(\alpha - \gamma)$ into the formula giving the population:

$$N_c = e^{r(\alpha-\gamma)}\int_0^{\omega} p_f(a)e^{-ra}da\int_0^{v}\frac{K_f(a,0)}{p_f(a)}\,G(a)e^{ra}da$$

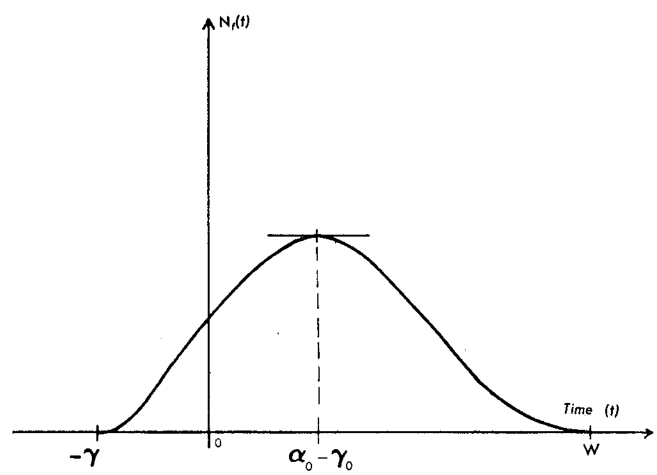
Clearly, the tangent to the envelope at the point of contact is also the tangent to the curve of the population. Its slope is obtained by replacing t by $\alpha - \gamma$ in the derivative function $dN/dt = rN$. The value for the slope at the point of contact is therefore $p_c = rN_c$.

Shape of the envelope

First, it will be seen that the slope p_c is zero when $r = 0$. The envelope reaches a maximum when the population is stationary. The abscissa of this maximum is $\alpha_0 - \gamma_0$, in which α_0 and γ_0 respectively indicate the values of α and γ corresponding to $r = 0$.

We have already said that, in practice, r can vary only within somewhat narrow limits. However, in order to learn more about the shape of the envelope, it is useful to compute the extreme values corresponding to infinitely great values of the intrinsic rate of natural variation. The computation shows [3] that, when r approaches $+\infty$, the ordinate of the point of contact t_c approaches zero and the abscissa approaches $-\gamma'$, the average age of mothers at the birth of their children in the initial population. When r tends towards $-\infty$, the ordinate of the point of contact approaches zero and the abscissa approaches the age at the end of life ω. Moreover, at these two points the tangents to the envelope are horizontal.

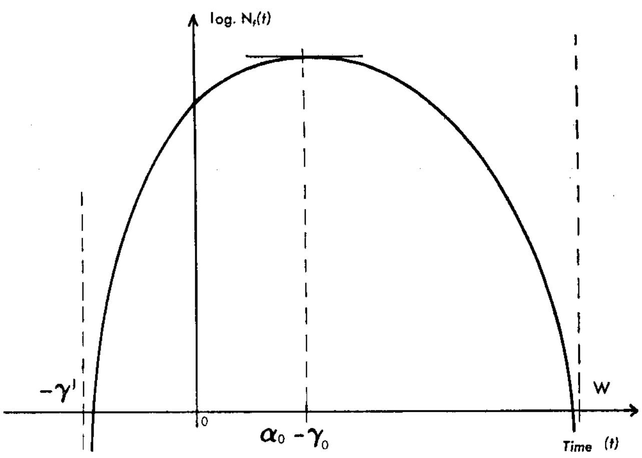
Finally, the shape of the envelope is as shown in graph A.I.2.



Graph A.I.2. Shape of the envelope of the population *curve* (metric scale for the horizontal and vertical axes)

As the total number of the population is an exponential function of time, it is convenient in graphic representations to use a semi-logarithmic graph in which time is indicated

[3] See the proof of these results at the end of annex I.



Graph A.I.3. Shape of the envelope of the population *lines* (metric scale for the horizontal axis, logarithmic scale for the vertical axis)

on the horizontal axis on a metric scale and the total number of the population on the vertical axis on a logarithmic scale. The total number of the population is then represented by a straight line with the slope r. In graphs of this kind the envelope always reaches a maximum for the stationary population, but as the logarithm of a number approaching zero increases indefinitely, the envelope has two vertical asymptotic lines of abscissa $-\gamma'$ and ω. The shape is shown in graph A.I.3. Graph A.I.4 shows the envelope obtained from the female population of Thailand, taking the female mortality to be that of the model life table (intermediate series) with an expectation of life at birth for both sexes of 60.4 years. The foregoing formulae were applied for the following seven values of the intrinsic rate of natural variation expressed as percentages: +10, +6, +0.87, 0, −1.57, −6, −10. The details of the computations, using an intrinsic rate of natural variation of +0.87 per cent, appear in chapter III. Here we simply give, in table A.I.8, the results of the computations for the other values of r.

At this point, we should remember what was stated above concerning the limits of possible variations of r. It was considered that 10 per cent appeared to be a value which could not be exceeded. Accordingly, only that part of the envelope drawn with a solid line should be considered.

If the lines of populations corresponding to higher values of r were calculated *directly*,[4] and not simply by applying formula 2, the results obtained would differ from those given by formula 2. Even for intrinsic rates of 10 per cent, there are bound to be marked divergences. As we have said, however, if we confine ourselves to rates encountered in practice, formula 2 gives very good results.

If the intrinsic rate of natural variation *is very small*, the envelope is reduced to its peak S and the population lines pivot around the point S.

Effects of changes in mortality on the envelope

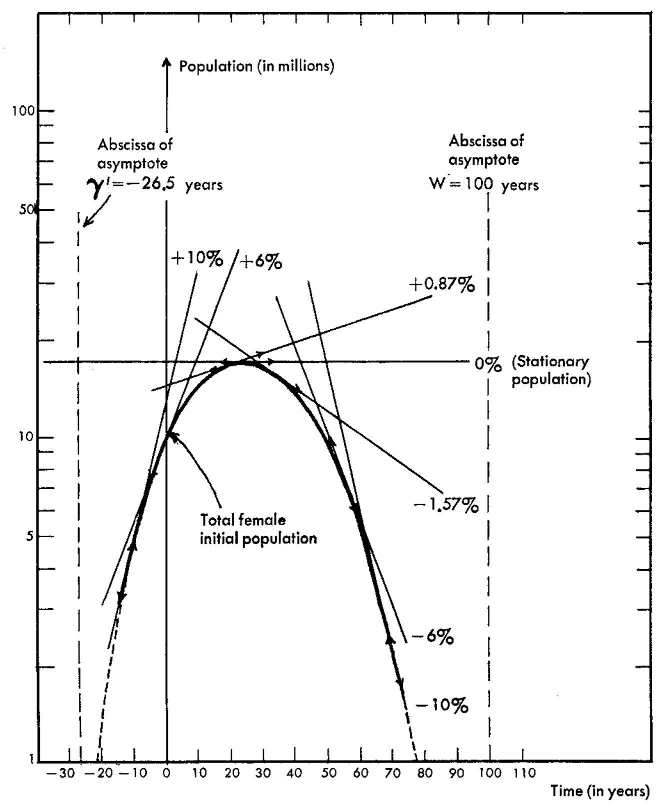
Each life table will, of course, have a different envelope corresponding to it. The variations in the peak S as a function of mortality provide information on the way in which the envelope is modified when mortality varies. The co-ordinates of the point S were calculated on the following assumptions:

(*a*) It was assumed that mortality corresponded to the series of model life tables (intermediate series), and six model tables were selected with expectations of life at birth for both sexes between 20 and 70.2 years;

(*b*) A female population of 1 million whose age distribution was the same as the population of Thailand in 1955, was taken as the initial population.

Table A.I.9 shows the results of the calculation, which is illustrated in graph A.I.5.

When mortality declines, $(\alpha_0 - \gamma_0)$ increases, and consequently the envelope is displaced to the right. Furthermore, the ordinate of the peak increases and the envelope moves upwards. If, starting from a given age structure, the aim is to arrive at a stationary state, the lower the mortality, the higher the level of this stationary state. This result is quite similar to that obtained for the net reproduction rate. We know that for a given fertility the net reproduction rate increases as mortality declines, and the next step is to define a gross reproduction rate representing the maximum net reproduction rate, which is reached when mortality is zero up to the limit age of procreation v. Can a gross stationary population, which would be the maximum attainable by the net stationary population, be defined in the same way? As with the definition of the gross reproduction rate, we can of course assume that mortality is zero up to age v, but that is not enough to determine the stationary population. We still have to make an assumption regarding mortality above age v, and all assumptions are *a priori* possible.



Graph A.I.4. Envelope of female population lines calculated from the population of Thailand in 1955 for a constant mortality corresponding to the model life table (intermediate series) with an expectation of life at birth for both sexes of 60.4 years and rates of variation between −10 per cent and +10 per cent

[4] For example, by using the method of population projections.

TABLE A.I.8. CHARACTERISTICS OF THE POPULATION LINES COMPUTED FROM THE POPULATION OF THAILAND IN 1955 FOR VARIOUS VALUES OF INTRINSIC RATE OF NATURAL VARIATION

Intrinsic rate of natural variation r (per cent) (1)	$\int_0^\infty e^{-ra}p_f(a)da$ (2)	$\int_0^v \frac{K_f(a, 0)}{p_f(a)} \times G(a)e^{ra}da$ (3)	*Product of the two preceding columns* (4)	*Age* a *(years)* (5)	*Age* γ *(years)* (6)	*Difference* $a - \gamma$ *(years)* (7)	$r(a - \gamma)$ (8)	$e^{r(a-\gamma)}$ (9)	*Ordinate of the point of contact of the envelope)*(f) *(in millions)* (10)
+∞ . . .	0	+∞ (a)	0	0	26.678 (e)	−26.678	+∞	+∞	0
+10 . . .	8.86396	1 493 319	13 236 720	9.863	21.553	−11.690	−1.1690	0.3106	4.111
+ 6 . . .	14.43263	680 205	9 817 147	15.405	17.767	− 2.362	−0.1416	0.8679	8.521
+ 0,87 . .	45.90099	308 380	14 154 947	32.470	13.220 (d)	−19.250	+0.1675	1.1820	16.732
0 . . .	62.04598	275 634	17 104 339	36.880	12.523 (c)	24.357	0	1.0000	17.104
− 1,57 . .	118.22975	228 479	27 013 000	45.270	11.370	33.900	−0.5323	0.5872	15.865
− 6 . . .	1 405.04816	146 963	206 490 100	64.876	8.711	56.165	−3.3699	0.03554	7.340
−10 . . .	23 389.03381	107 530	2 515 022 826	77.820	7.006	70.814	−7.0814	0.0008404	2.114
−∞ . . .	+∞ (b)	0	0	ω	0	ω	−∞	0	0

(a) This integral tends towards +∞ but the product

$$e^{-r\gamma'}\int_0^v \frac{K_f(a, 0)}{p_f(a)} G(a)e^{ra}da$$

tends towards zero.

(b) This integral tends towards +∞ but the product

$$e^{r\omega}\int_0^\infty e^{-ra}p_f(a)da$$

tends towards zero.

(c) Figure taken from the last line of table III.3 We have

$$\frac{34.5146}{2.75135} = 12.523$$

(d) Figure taken from the last line of table III.3. We have

$$\frac{40.7325}{3.0838} = 13.220$$

(e) It may be recalled that this is the average age γ' of mothers at the birth of their children in the initial population.

(f) Product of columns (4) and (9).

TABLE A.I.9. CHARACTERISTICS OF THE PEAK OF THE ENVELOPE OF A FAMILY OF POPULATIONS WITH CONSTANT MORTALITY FOR SIX MORTALITY LEVELS, CALCULATED FROM A FEMALE POPULATION OF 1 MILLION WITH THE SAME AGE DISTRIBUTION AS THE FEMALE POPULATION OF THAILAND IN 1955

Expectation of life at birth for both sexes (in years)	*Female expectation of life at birth (in years)*	*Reduced initial population*	*Product of the two columns (ordinate of the peak)*	*Mean age of the stationary population* (a_0) *(in years)*	*Mean age of the reduced initial population* (γ_0) *(in years)*	*Difference between the two preceding columns* ($a_0 - \gamma_0$) *(in years)*
20	20.197	51 741	1 045 013	24.159	14.168	9.991
30	30.402	39 223	1 192 458	28.306	13.469	14.837
40	40.743	33 100	1 348 593	31.745	13.044	18.701
50	51.308	29 261	1 501 323	34.599	12.736	21.863
60.4	62.046	26 403	1 638 201	36.880	12.523	24.357
70.2	71.803	24 375	1 750 198	38.549	12.376	26.173
Crude level.	80.000	23 513	1 881 040	40.573	12.303	28.270

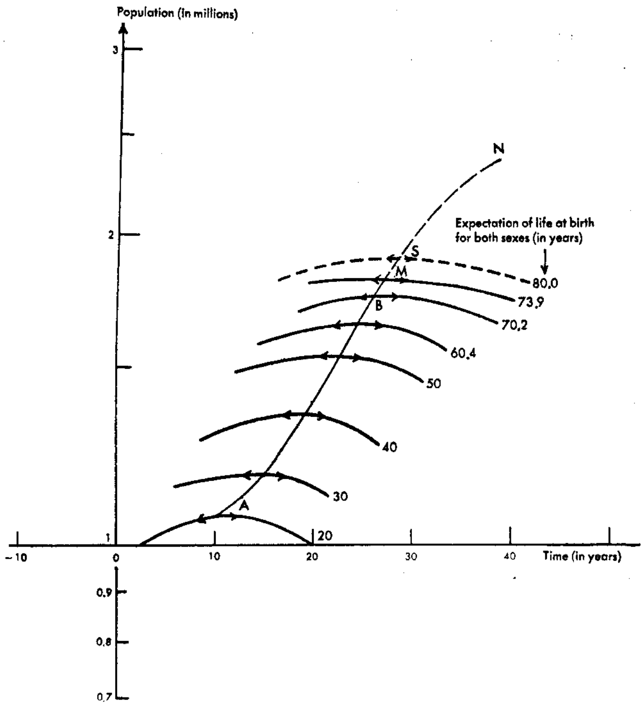
Graph A.I.6 shows the female survivorship curves which represent the extreme limits of the series of the model life tables. The intermediate survivorship curves come between these two extremes. As an example, the survivorship curve was drawn for a female expectation of life at birth of 51.31 years.

If mortality is assumed to be zero up to age v, the survivorship curve begins with the segment of straight line AB. It then declines from B to C. From point B there are, of course, an infinite number of paths for a declining curve to reach point C. One can only say that it should not decline more sharply than the survivorship curve corresponding to the upper limit of the model life tables. This condition merely means that it must remain within the shaded area of graph A.I.6. Obviously, for each possible survivorship curve there is a distinct stable population. In graph A.I.5, the peaks S corresponding to all these stable populations describe the segment of curve MN, point N being found by assuming that mortality is zero up to age ω and that everyone dies at that age.

Nevertheless, all these stable populations have one peculiar property. As they have the same intrinsic rate of natural variation, they have the same absolute number of births:

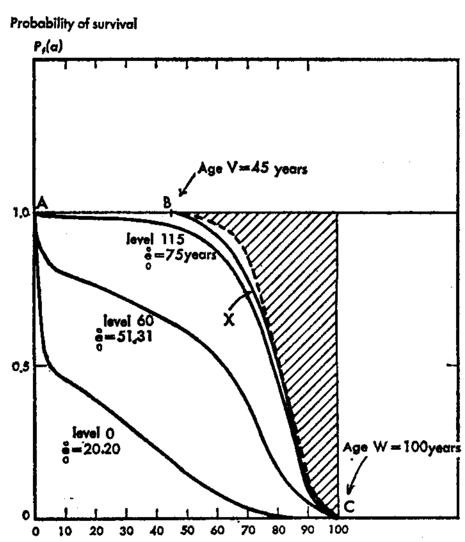
$$B = e^{rt}\int_0^v K_f(a, 0)G(a)e^{ra}da$$

Stable populations corresponding to the model life table would not have this property. The decline in mortality causing the survivorsip curve to pass through the shaded area in graph A.I.6 therefore introduces new factors which may bring into the theory of stable populations phenomena different from those observed when only the mortality levels noted in the model life tables are considered. In particular, these new phenomena may



Graph A.I.5. Characteristics of families of populations with constant mortality calculated from a population of one million with the same structure as the female population of Thailand in 1955

be expected to appear in families of populations with constant fertility, since the peculiarity mentioned above relates to births, and it will be seen in a moment that this is in fact so. Only stable populations calculated from the survivorship curves inside the shaded area in graph A.I.6 and near the curve BXC will be not too greatly affected by these new phenomena. Accordingly, out of all the possible assumptions concerning mortality beyond age v, we confine ourselves to those giving survivorship curves which do not deviate too greatly from BXC. An examination of families of populations with constant fertility will show that a female expectation of life at birth of eighty years provides a good extrapolation of the universe of the model life tables. If this value



Graph A.I.6. Extrapolation of the "universe" of female survivorship functions from the model life tables (intermediate series)

is adopted for the expectation of life, the survivorship curve beyond age v is determined with a very small degree of arbitrariness. Lastly, the following survivorship table was used to calculate the gross stationary population. The mean age of this stationary population is $a'_0 = 40.788$ years. The peak of the envelope corresponding to this survivorship table is the point S′ in graph A.I.5.

TABLE A.I.10. FEMALE SURVIVORSHIP TABLE USED TO CALCULATE THE GROSS STATIONARY POPULATION

Age (in years)	*Survivors per 1 000 at birth*
0	1 000
10	1 000
20	1 000
30	1 000
40	1 000
50	1 000
60	960
70	850
80	540
90	150
100	000

The following definitions can then be arrived at.

The ordinate of the point S′ will be the *gross stationary population:*

$$N_{s'} = 80\int_0^v K_f(a, 0)G(a)da \qquad (11)$$

The *gross reduced initial stationary population* is the quantity:

$$N_{s'_i} = \int_0^v K_f(a, 0)G(a)da \qquad (12)$$

The *net stationary population* is:

$$N_s = {}^0e_0\int_0^v \frac{K_f(a, 0)}{p_f(a)} G(a)da \qquad (13)$$

The *net reduced initial stationary population* is:

$$N_{s_i} = \int_0^v \frac{K_f(a, 0)G(a)}{p_f(a)} da \qquad (14)$$

In discontinuous notation we shall obtain the following formulae (for five-year age groups):

$$N_{s'} = 80\sum_0^v \frac{K_a}{5} G_a = 16\sum_0^v K_aG_a \qquad (15)$$

$$N_{s'_i} = \sum_0^v \frac{K_aG_a}{5} \qquad (16)$$

$$N_s = {}^0e_0\sum_0^v \frac{K_aG_a}{L_a} \qquad (17)$$

$$N_{s_i} = \sum_0^v \frac{K_aG_a}{L_a} \qquad (18)$$

The abscissa of the point S′ will be the *gross stationary time:*

$$t_{s'} = 40.788 - \gamma'_0$$

where γ'_0 is the mean age of the gross reduced initial stationary population. The abscissa of point S will be the *net stationary time:*

$$t_s = \alpha_0 - \gamma_0$$

α_0 being the mean age of the net stationary population and γ_0 the mean age of the net reduced initial stationary population.

The last line of table A.I.9 shows the results of the computations of the gross stationary population and the gross stationary time for the female population of Thailand in 1955.

Approximate formulae for the transition from gross to net characteristics

The foregoing formulae make it possible to change over without difficulty from gross to net values. However, much simpler approximate formulae can be developed.

The net stationary population is given by the formula:

$$N_s = \int_0^{\omega} p_f(a)da \int_0^{v} \frac{K_f(a)G(a)}{p_f(a)}\, da$$

This can be written:

$$N_s = \frac{{}^0e_0}{p_f(x)} \int_0^{v} K_f(a)G(a)da = \frac{{}^0e_0}{80p_f(x)} N_{s'}$$

where x is an age between u and v. The calculation shows that this age x varies only slightly with the mortality level. In the particular example taken, the following values are found.

Expectation of life at birth for both (sexes in years)	*20*	*30*	*40*	*50*	*60.4*	*70.4*
Age x	10.45	10.72	10.77	11.10	10.72	12.13

This gives the following approximate formula:

$$N_s = \frac{{}^0e_0}{80p_f(11)} N_{s'}$$

The net stationary population is equal to the gross stationary population multiplied by the coefficient:

$$\frac{{}^0e_0}{80p_f(11)}$$

The stationary time is $t_s = \alpha_0 - \gamma_0$. There is little variation in γ_0 with variations in the mortality level. Ignoring these variations and assuming that $\gamma_0 = \gamma'_0$, the approximate value of the net stationary time will be given by the expression:

$$t_s = t_s' - (40.788 - \alpha_0)$$

In the approximate formula, the net stationary population appears as the product of three variable factors. The first, ${}^0e_0$, is the expectation of life at birth. It depends only on mortality. The second, $N_{s'}$, is the gross stationary population, which depends only on the initial age structure. Lastly, there is the factor $1/p_f(x)$, which depends both on mortality, since it represents the probability of survival, and on the initial age structure, since the age x depends on that structure.

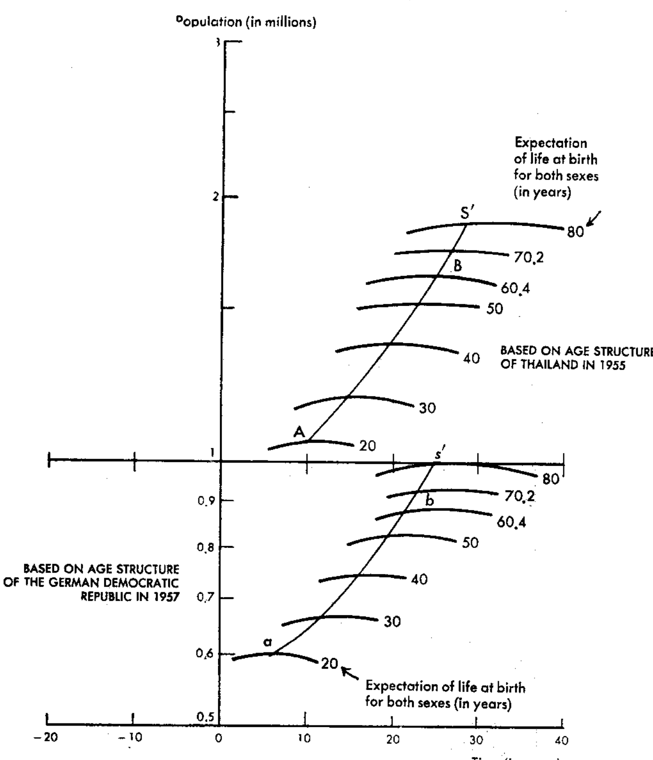
The net stationary time in the approximate formula appears as the difference between two quantities. The first, t_s, is the gross stationary time, which depends only on the initial age structure. The second $(40.788 - \alpha_0)$ depends only on mortality.

Effects of the initial age structure on the envelope

To see what variations appear in the envelope as a result of variations in the initial age structure, the foregoing calculations were repeated, taking a female population of 1 million with the age distribution of the female population of Eastern Germany. The results are shown in table A.I.11, which is similar to table A.I.9. The peaks have been plotted in graph A.I.7.[5] They form a line similar to that obtained from the age structure of Thailand but displaced downwards and to the left. The approximate formula giving the ordinate of the peak is applied, assuming that $x = 16$. In fact, the following ages x are obtained for the various mortality levels:

Expectation of life at birth for both sexes in years	*20*	*30*	*40*	*50*	*60.4*	*70.2*
Age x	15.94	16.20	16.24	16.17	16.23	16.87

[5] We have also plotted in graph A.I.7 the curve from graph A.I.5 obtained from the population of Thailand.



Graph A.I.7. Comparison of the characteristics of families of female population curves with constant mortality calculated from a population of one million: (A) with the age structure of the female population of Thailand in 1955; (*a*) with the age structure of the female population of Eastern Germany in 1957

We therefore have:

$$N_s = \frac{\overset{0}{e}_0}{80p_f(16)} N_{s'}$$

As in the previous case, the age x varies only slightly with mortality and the approximate formula giving the abscissa of the peak is still:

$$t_s = t_{s'} - (40.7875 - a_0)$$

As might be expected, the age x varies with the initial age structure. However, the two initial age structures used here differ greatly from each other, and the variation of five years in age x between them can be regarded as a maximum. Between the ages of 10 and 20, a variation of five years does not produce any large variation in the probability of survival. Indeed, for low mortality rates, this variation is negligible. A second approximate formula can therefore be derived by adopting for age x a "mean" value independent of the initial age structure. If, for example, we assume that $x = 12.5$ years, the second approximation formula is written as follows:

$$N_s = \frac{\overset{0}{e}_0}{80p_f(12.5)} N_{s'}.$$

Table A.I.12 shows the results given by the two approximation formulae and the results obtained from exact calculations. It is now possible to measure the effects of a variation in the initial age structure at various mortality levels on the co-ordinates of the stationary peaks. This is the object of table A.I.13.

TABLE A.I.11. CHARACTERISTICS OF THE PEAK OF THE ENVELOPE OF A FAMILY OF POPULATIONS WITH CONSTANT MORTALITY FOR SIX MORTALITY LEVELS CORRESPONDING TO EXPECTATIONS OF LIFE AT BIRTH FOR BOTH SEXES OF BETWEEN 20 AND 70.2 YEARS, CALCULATED FROM A FEMALE POPULATION OF ONE MILLION WITH THE SAME AGE DISTRIBUTION AS THE FEMALE POPULATION OF EASTERN GERMANY IN 1957

Expectation of life at birth for both sexes (in years)	*Female expectation of life at birth (in years)*	*Reduced initial population*	*Product of the two preceding columns (ordinate of the peak)*	*Mean age of the stationary population* a_0 *(in years)*	*Mean age of the reduced population* γ_0 *(in years)*	*Difference between the two preceding columns* $a_0 - \gamma_0$
20	20.197	29 876	603 406	24.159	18.188	6.971
30	30.422	22 039	610 050	28.306	16.521	11.785
40	40.743	18 281	744 823	31.745	16.085	15.660
50	51.308	15 967	819 235	34.599	15.777	18.822
60.4	62.046	14 281	886 079	36.880	15.557	21.323
70.2	71.803	13 104	940 907	38.549	15.404	23.145
Mortality zero up to age ν	80.000	12 603	1 008 240	40.7875	15.325	25.4625

TABLE A.I.12. RESULTS GIVEN BY THE VARIOUS FORMULAE PROPOSED FOR THE CALCULATION OF THE CO-ORDINATES OF THE STATIONARY PEAK

Expectation of life at birth (in years)	*Ordinate of the peak* N_s *(millions)*			*Abscissa of the peak* t_s *(years)*	
	Exact calculation	*First approximation*	*Second approximation*	*Exact calculation*	*Approximation*
I. Initial age structure identical with that of Thailand in 1955					
20	1.045	1.052	1.073	10.0	11.9
30	1.192	1.195	1.209	14.8	16.0
40	1.349	1.350	1.260	18.7	19.4
50	1.501	1.503	1.510	21.9	22.3
60.4	1.638	1.639	1.642	24.4	24.6
70.2	1.750	1.749	1.751	26.2	26.2
80.0 (a)	1.881	1.881	1.881	28.3	28.3
II. Initial age structure identical with that of Eastern Germany in 1957					
20	0.603	0.604	0.575	7.0	8.8
30	0.670	0.699	0.648	11.8	13.0
40	0.745	0.744	0.729	15.7	16.4
50	0.819	0.819	0.809	18.8	19.3
60.4	0.886	0.887	0.880	21.3	21.6
70.2	0.941	0.940	0.938	23.1	23.2
80.0 (a)	1.001	1.001	1.001	25.5	25.5

(a) Level adopted for the gross formulae.

TABLE A.I.13. COMPARISON BETWEEN THE CO-ORDINATES OF THE STATIONARY PEAKS CALCULATED ON THE BASIS OF THE INITIAL AGE STRUCTURE OF THAILAND IN 1955 AND THE CO-ORDINATES OF THE STATIONARY PEAKS CALCULATED ON THE BASIS OF THE INITIAL AGE STRUCTURE OF EASTERN GERMANY IN 1957

Expectation of life at birth (in years)	*Relationship between ordinates*			*Difference between the abscissae*	
	Exact calculation	*First approximation*	*Second approximation*	*Exact calculation*	*Approximation*
20	173.2	174.2	186.6	3.0	3.0
30	177.8	178.6	186.6	2.0	3.0
40	181.1	181.5	186.6	3.0	3.0
50	183.3	183.5	186.6	3.0	3.0
60.4	184.9	184.8	186.6	3.1	3.0
70.2	186.2	186.1	186.6	3.1	3.0
80.0 (a)	186.6	186.6	186.6	3.0	3.0

(a) Mortality level adopted for the gross formula.

At a mortality level corresponding to an expectation of life at birth for both sexes of 50 years, the transition from the age structure of Eastern Germany to that of Thailand increases the ordinate of the stationary peak by 83.3 per cent and its abscissa by three years. Table A.I.12 shows that the effects of a change in the initial age structure are approximately the same whatever the mortality level and that the approximate formulae enable these effects to be measured reasonably closely.

GROWTH POTENTIAL OF AN AGE STRUCTURE

In a semi-logarithmic graph, when the intrinsic rates of natural variation are small, the population lines pivot around the peak S. There is a temptation here to use the language of geometric optics and to say that the stationary point is the "image" of the initial age structure through the corresponding model life table.

If, after selecting an initial age structure and a model life table, we draw a large number of population lines on a semi-logarithmic graph for intrinsic rates of natural variation varying only slightly around zero, we shall also see all these lines diverge from a point which is none other than the stationary peak S, just as rays of light diverge from the image of a luminous point in an optical system. The requirement that the intrinsic rate of natural variation must be small adds still further to the analogy. We know that simple optical instruments do not give sharp images unless the light rays are inclined at a slight angle to the axes of the lenses and the aperture of the lenses is small. Here, the "image" of the initial age structure through a model life table is "sharp" only if the intrinsic rate of natural variation is small. Otherwise, the population lines deviate from the stationary peak and they have an envelope the characteristics of which were determined above. This envelope is also found in optical instruments. If the conditions for sharpness of image are not satisfied, the rays of light have what is called a "caustic envelope". The analogy must not be taken too far, of course, but the language of optics can make some explanations easier.

If we consider a series of initial age structures K_1, K_2, K_3, etc., there will be a corresponding series of "images" S_1, S_2, S_3, etc., through a given model life table. If the life table is changed, the image moves without becoming distorted. Thus the image *itself* does not depend on mortality. It depends solely on the initial age structures. Mortality merely fixes its position in the graph. Accordingly, the image formed by the profile S_1, S_2, S_3, etc. makes it possible to isolate the effect of each initial age structure on the growth potential of a population with that age structure. An example might make this point clearer. Successive computations of the *gross female stationary population* were made for 1 million persons having the same age structures as were observed recently in twenty-eight countries. The image thus obtained from the twenty-eight countries is reproduced in graph A.I.8. At the two extremes are Eastern Germany and Thailand, and this shows that, in choosing these two countries as examples, we have covered what might occur in reality. Variations in the abscissa of the stationary peak are slight and can be neglected in arriving at a first approximation. Variations in the ordinate are considerable, and the upper extreme is twice as large as the lower. In the case of Eastern Germany we find a gross stationary population of 1,008,000. This means that a population of 1 million, with the same age structure as the female population of East Germany in 1957, with a net reproduction rate equal to 1.00 (i.e., in this case, a gross reproduction rate equal to unity), and subject to the survivorship function shown in table A.I.10, would tend in time to become a stationary population of 1,008,000. For Taiwan, in 1959, we find a gross stationary population of 1,894,000.

If another survivalship function was taken instead of that used in table A.I.10, the net stationary populations calculated would differ from the gross stationary populations. The new broken line that would be obtained in graph A.I.8 for the twenty-eight countries in question would be derived by translation of the broken line corresponding to the gross stationary population. In particular, the ratio between the new ordinates for Taiwan and Eastern Germany would be the same as the ratio between 1,008,000 and 1,894,000. In other words, Taiwan is seen as having almost twice as much "growth capacity" as Eastern Germany, solely because the age structures are different. The gross stationary population therefore provides a measure of the "growth potential" of an age structure. This expression—the "growth potential" of a population—is the title of an article [6] submitted to the Société de statistique de Paris

[6] Paul Vincent, "Potentiel d'accroissement d'une population", *Journal de la Société de statistique de Paris*, January-February 1945, pp. 16 *et seq.*

by Mr. Paul Vincent in 1945. The problem is the same as was studied by Mr. Paul Vincent in his article, and the solution which he put forward then is very close to the one we now propose.

D. Families of populations with constant fertility

As was stated above, the mathematical study of families of populations with constant fertility is less simple than the study of families with constant mortality. The two categories of families are defined by the two formulae:

$$N_f(t) = e^{rt} \int_0^{\omega} p_f(a) e^{-ra} da \int_0^{v} \frac{K_f(a)}{p_f(a)} G(a) e^{ra} da$$

and

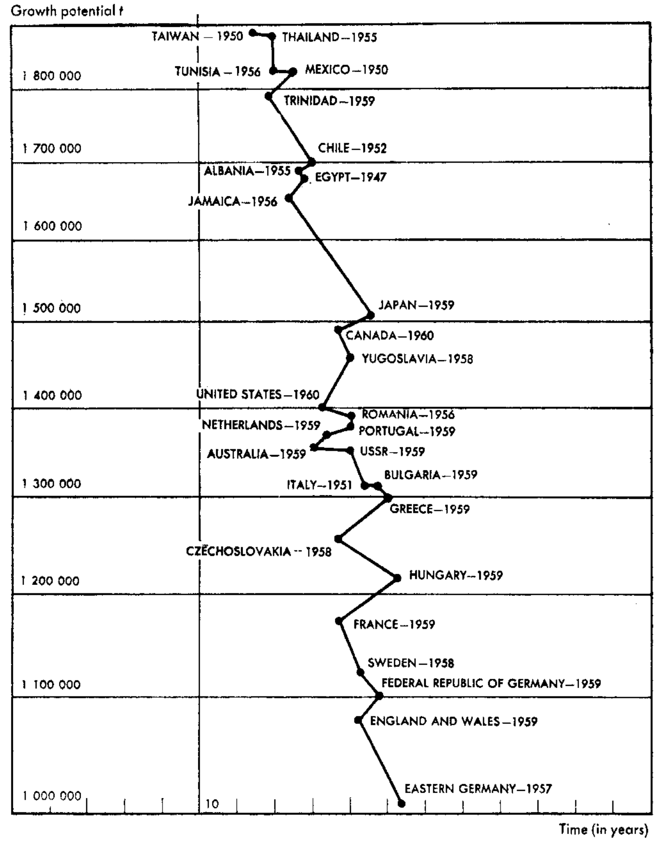
$$\int_0^{v} e^{-ra} p_f(a) \varphi_f(a) da = 1$$

In families with constant mortality, there is a single value of N corresponding to each value or r. In families with constant fertility, N is not determined when r is kwnon. Indeed, there are as many values of N as there are functions $p_f(a)$ satisfying the second condition. The problem becomes determinate upon the addition of the further condition that mortality should vary in conformity with a series of model life tables. Generally,[7] when this is the case, the second formula unambiguously defines the function $p_f(a)$ which is compatible with the values r and $\varphi_f(a)$, and the conditions are then the same as for the study of families of populations with constant mortality. N then depends only on a single parameter r. Clearly, however, the analysis we can make is then linked to the series of model tables selected; in other words, an *empirical* study of families of populations with constant fertility is warranted, once the series of model life tables has been adopted. The series of model life tables (intermediate series) has been used for this empirical study. We have already noted that it was possible to write:

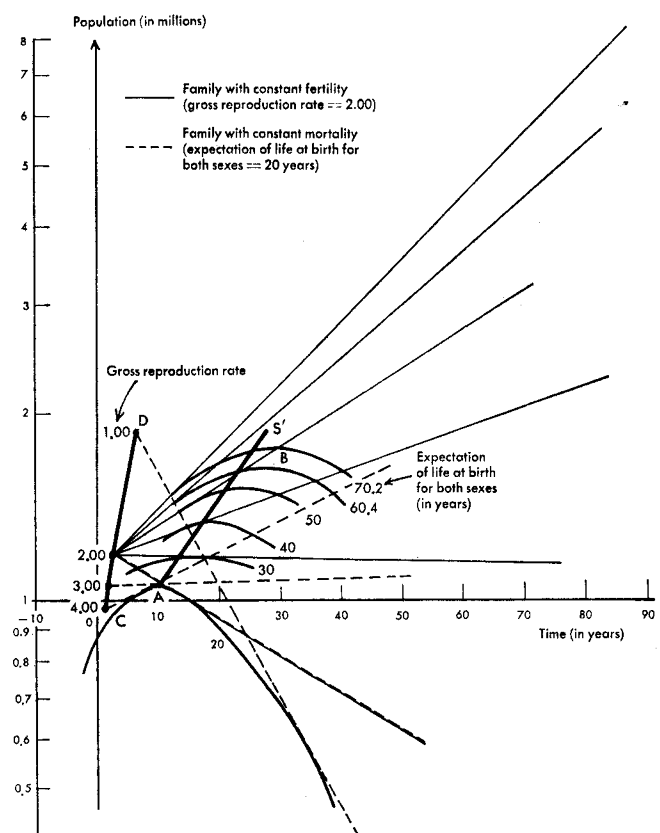
$$N_f(t) = e^{rt} \int_0^{v} \frac{K_f(a, 0) G(a)}{C_f(a)} da$$

where $C_f(a)$ is the age structure of the stable population. When mortality varies in conformity with the model life tables selected for the analysis, it has little effect on the age structure of stable populations. This structure is chiefly dependent on fertility. This means that for a given fertility the age structure does not vary greatly. Accordingly, when $t = 0$, the different values of $N_f(0)$ will be close to each other. In a semi-logarithmic graph and for a given fertility, therefore, we can expect to find lines which pivot around a point in the neighbourhood of the vertical axis. Graph A.I.9 confirms this. It will be seen that, for all values of the intrinsic rate of natural variation which are encountered in practice in human populations,

[7] The qualification "generally" is necessary because one could imagine a series of model life tables in which several values of $p_f(a)$ might correspond to a single pair of r, $\varphi_f(a)$. It should be made clear that this is not the case with the model tables published by the Secretariat of the United Nations. These tables fall within the general category. However, the life tables which were used in defining gross stationary populations and which correspond to the survivorship curves within the shaded area of graph A.I.6 are, in fact, tables where there is no longer any relation between $p_f(a)$, $\varphi_f(a)$ and r.



Graph A.I.8. Gross female stationary population for one million in twenty-eight countries at a recent date (growth potential)



Graph A.I.9. Characteristics of families of female populations with constant fertility and constant mortality calculated from a population of one million baving an age structure identical with that of the female population of Thailand in 1955

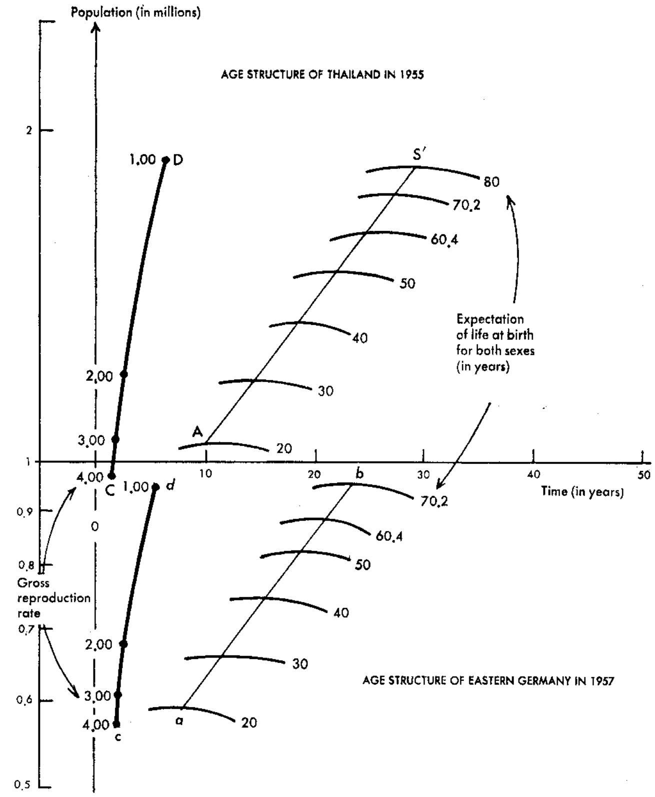
each family of populations with constant fertility is reduced on a semi-logarithmic graph to a bundle of straight lines.[8] Table A.I.14 shows the co-ordinates of the peak of the bundles of families of populations with constant fertility at four fertility levels (gross reproduction rates of 4.00, 3.00, 2.00 and 1.00) determined *empirically* on the basis of a population of 1 million with the respective age structures of Thailand in 1955 and Eastern Germany in 1957. In graph A.I.9, the peaks of the bundles determined on the basis of the age structure of Thailand form a curve CD. Each point on this curve corresponds to a fertility level, and this point may be said to be the "image" of the age structure of Thailand through this fertility level. *This time, however, the "image" is "sharp" even if the intrinsic rates of natural variation are high.* If, on the basis of the age structure of Thailand, and a given fertility, we plot a series of population lines on a semi-logarithmic graph, as mortality varies, these lines will diverge from a point located on the curve CD. It will be remembered that a similar result was obtained when studying the straight lines of populations with constant mortality. The lines obtained would diverge from a point located on the curve AB, provided that the intrinsic rate of natural variation was small. Thus the two results can now be combined. If we calculate a large number of stable populations on the basis of the age structure of Thailand while varying the mortality and the fertility, and if we draw the corresponding lines on a semi-logarithmic graph, one may note that the two curves AB and CD will appear on the graph. The first, curve AB, will be very poorly defined because only the lines of populations with a low intrinsic rate of natural variation will contribute to its formation. The second curve, CD, however, will emerge very clearly, since the envelope of a family with constant fertility is reduced to a point. The two sections of curve AB and CD are the "images" of the age structure of Thailand realized through the universe of mortality and fertility.

In graph A.I.9, *two examples of families* with a fertility corresponding to a gross reproduction rate of 2.00 have been plotted. The population lines calculated on the basis of the age structure of Thailand start from point M.

TABLE A.I.14. CO-ORDINATES OF THE PEAK OF THE BUNDLES OF FEMALE POPULATIONS WITH CONSTANT FERTILITY FOR 1 MILLION FEMALES AT TIME ZERO AND FOR VARIOUS FERTILITY LEVELS

Gross reproduction rate	*Abscissa of the peak (in years)*	*Ordinate of the peak*
Age structure of Thailand in 1955		
4.00	1.7	990 000
3.00	2.0	1 040 000
2.00	2.6	1 200 000
1.00	5.5	1 880 000
Age structure of Eastern Germany in 1957		
4.00	1.0	580 000
3.00	1.4	600 000
2.00	2.0	670 000
1.00	4.9	980 000

[8] On a metric graph each family would be represented by an ensemble of Malthusian curves passing through a fixed point.



Graph A.I.10. Comparison of the characteristics of families of female populations with constant fertility and constant mortality calculated on the basis of one million: (A) with the age structure of the female population of Thailand in 1955; (*a*) with the age structure of the female population of Eastern Germany in 1957

These are the solid straight lines on the graph. All the straight lines for female populations calculated on the basis of the same structure, but with the constant mortality of the model life table with an expectation of life at birth for both sexes of 20 years, will be tangential to the curve marked 20.[9] *These are the dotted lines on the graph.*

EFFECTS OF CHANGES IN THE AGE STRUCTURE ON THE BUNDLES OF POPULATIONS WITH CONSTANT FERTILITY

Each age structure has its corresponding curve CD. Graph A.I.10 shows the two curves obtained on the basis of the age structures of Thailand and Eastern Germany. It will be seen that the transposition from Thailand to Eastern Germany, as in the case of the curve of the stationary peaks, involves a shift downwards and to the left. Table A.I.15 shows that the shift downwards is of the same order of magnitude in both cases but that the shift to the left is markedly smaller for the image with constant fertility than for the image with constant mortality. If we consider ABCD as a whole, we can say that in the transposition from Thailand to Eastern Germany the image ABCD moves downwards and contracts.

DECLINE IN MORTALITY AT ADVANCED AGES

We saw that, in the model life table with the highest expectation of life, mortality was low up to age *v*, marking the end of the reproductive period, and that the series of

[9] This value for the expectation of life at birth was chosen for the sake of clarity in the graph. It is an extreme value which is scarcely ever encountered in practice.

model life tables could be extrapolated by conceiving a survivorship curve equal to unity up to age v and then declining from 1 to zero up to age ω, which represents the end of human life. At the same time, however, we saw that this single condition of a decline in mortality left a large degree of indeterminateness in the survivorship curve beyond age v. We shall now consider how the study of families of populations with constant fertility helps, at least in part, to reduce this indeterminateness.

Table A.I.15. Comparison of the "images" with constant fertility of the female age structures of Thailand in 1955 and of Eastern Germany in 1957 for four fertility levels

Gross reproduction rate	*Amount by which the abscissa for Thailand exceeds the abscissa for Eastern Germany (in years)*	*Ordinate for Thailand divided by the ordinate for Eastern Germany*
4.00	0.6	1.707
3.00	0.6	1.716
2.00	0.6	1.765
1.00	0.6	1.837

For a given fertility, there is an intrinsic rate of natural variation which cannot be exceeded. This is the value or r corresponding to zero mortality up to age v or, in other words, the value of r obtained by taking any one of the survivorship curves extrapolated as described above.[10] This maximum rate is the real solution to the equation:

$$\int_0^v \varphi_f(a)e^{-ra}da = 1$$

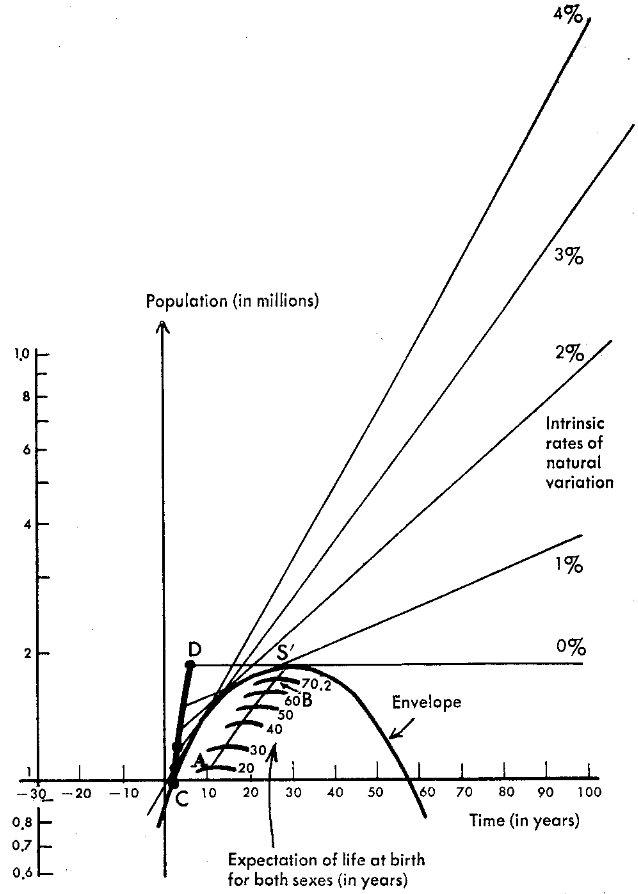
In table A.I.16, this maximum rate is computed for different values of gross reproduction rates, assuming that the age distribution of the fertility rates is the intermediate distribution.

Table A.I.16. Maximum values of the intrinsic rates of natural variation corresponding to different fertility levels

(Intermediate age distribution of the fertility rates)

Gross reproduction rate	*r (in percentages)*
1.00	0.00
1.32	1.00
1.74	2.00
2.00	2.53
2.28	3.00
2.97	4.00
3.00	4.04

If the "image" of the age structure is sharp for these maximum rates, the population lines will pass through the points on the curves CD corresponding to the values of the gross reproduction rate in table A.I.16. They can then be drawn, since their slope r is known. This has been done in graph A.I.11 for five intrinsic rates of natural variation: 0.00, 0.01, 0.02, 0.03 and 0.04. The envelope of these lines, drawn in by hand, is evidently a good extrapolation of the series of envelopes corresponding to the model life tables. On the curve AB there is a corresponding peak S′ the ordinate of which, as read from the graph, is 1,880,000. This ordinate divided by the gross reduced initial population gives 79.96, which we have rounded off to 80. This is the female expectation of life at birth which a life table with zero mortality up to age v should have if the corresponding envelope is to be that of the population lines having the maximum intrinsic rates of natural variation as their slopes. The degree of uncertainty in such a table is very small, and this is how the life table in table A.I.11 was worked out for the definition of gross characteristics. To sum up, it can be said that, for the series of model life tables extended by the life table in table A.I.11, the "image" of an age structure with constant fertility is "sharp" for all values of the intrinsic rate of natural variation encountered in practice.



Graph A.I.11. Envelope of the population lines corresponding to the maximum values of the intrinsic rates of natural variation for different fertility levels

The case of levels of mortality where the expectation of life at birth is over 80 years

Where the female expectation of life at birth is over 80 years, the "image" is no longer "sharp". As fertility is a known factor, the intrinsic rate of natural variation is also fixed. It is equal to the maximum rate computed above. As a result, all the population lines corresponding to a given fertility have the same slope, and they are therefore parallel. Thus they still pass through the same

[10] The indeterminate nature of the survivorship curve beyond age v does not affect the value of the maximum rate.

point, but that point is at infinity. It is as though the point on the curve CD around which the population lines pivot were suddenly thrust out to infinity when the slope of the population line reached the maximum rate, the direction of the thrust being towards the straight line having this maximum rate as its slope. For expectations of life at birth over 80 years, therefore, the phenomenon is of an entirely different kind.

RECONSIDERATION OF THE GROSS STATIONARY POPULATION

The gross stationary population defined above now appears in a new light. Its ordinate is none other than the ordinate of point D, around which the population lines pivot *when the gross reproduction rate is equal to unity.* When the age structure varies, the abscissa of point D varies very little (a difference of 0.6 year between structures as different as those of Thailand and Eastern Germany), and to all intents and purposes it is only its ordinate which varies. It may be said that, in practice, the abscissa of point D is always equal to 5 years. In other words, the "images" of a series of age structures through a constant gross reproduction rate equal to unity should all lie on a straight line parallel to the vertical axis situated five years after the point of origin, and the ordinates of those images would be equal to the gross stationary populations. Furthermore, the images would be "sharp" for all the intrinsic rates of natural variations encountered in practice. Thus, the gross stationary population does provide an index for measuring the growth potential of populations.

E. Families of births

Much space has been devoted to the total numbers of populations, but naturally the curves of births, population growth and deaths can also be studied. We shall confine ourselves to families with constant mortality, and we shall begin with births. The formula for births at time t is as follows:

$$B_f(t) = e^{rt} \int_0^u \frac{K_f(a, 0)}{p_f(a)} G(a) e^{ra} da$$

By varying the intrinsic rate of natural variation r, we obtain a series of curves with an envelope, and the abscissa of the point of contact is obtained by setting $dB_f(t)/dr$ equal to zero. We have

$$\frac{dB_f(t)}{dr} = te^{rt} \int_0^u \frac{K_f(a, 0)}{p_f(a)} G(a) e^{ra} da + e^{rt} \int_0^u \frac{aK_f(a, 0)}{p_f(a)} G(a) e^{ra} da = B_f(t)[t_c + \gamma]$$

This expression is equal to zero for $t_c = -\gamma$, γ being, as already mentioned, the mean age.

$$\gamma = \frac{\int_0^u \frac{aK_f(a, 0)}{p_f(a)} G(a) e^{ra} da}{\int_0^u \frac{K_f(a, 0)}{p_f(a)} G(a) e^{ra} da}$$

It was seen that, when r varied from $-\infty$ to $+\infty$, γ increased from zero to γ', γ' being the average age of mothers at the birth of their children in the initial population.

The ordinate of the point of contact is:

$$B_c = B_f(t_c) = e^{-r\gamma} \int_0^u \frac{K_f(a, 0)}{p_f(a)} G(a) e^{ra} da$$

and the slope of the tangent to the envelope at the point of contact is rB_c. This slope is equal to zero for $r = 0$. The envelope therefore reaches a maximum when the population is stationary. According to what we have already established, B_c and rB_c approach zero when r approaches $\pm\infty$. Finally, the shape of the envelope is as shown in graph A.I.12. On a semi-logarithmic graph, the envelope of birth straight lines has two vertical asymptotes of abscissæ 0 and $-\gamma$ (graph A.I.13).

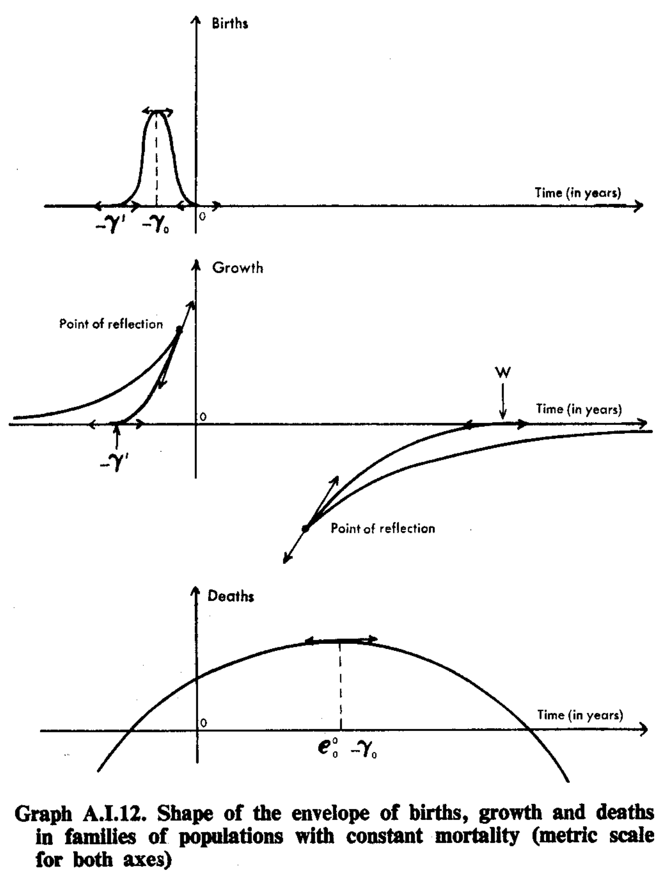
The envelope of the birth lines is therefore similar in shape to the envelope of the population lines. The two asymptotes of the envelope of birth lines are closer together than those of the envelope of population lines (the distance between them is γ' in the former case and $(\gamma' + \omega)$ in the second). The envelope of the birth lines is therefore more "pointed" than that of the population lines. Thus the "image" of births remains "sharp" over a wider range of variations of r than the population image.

F. Families of growth

Growth at time t is given by the expression:

$$A_f(t) = rN_f(t) = re^{rt} \int_0^\omega e^{-ra} p_f(a) da \int_0^\omega \frac{K_f(a, 0)}{p_f(a)} G(a) e^{ra} da$$

Where mortality is constant, the $A_f(t)$ curves have an envelope when the intrinsic rate of natural variation varies. The abscissa of the point of contact of the envelope



Graph A.I.12. Shape of the envelope of births, growth and deaths in families of populations with constant mortality (metric scale for both axes)

is obtained by winting the derivative of $A_f(t)$ with respect to r equal to zero:

$$\frac{dA_f(t)}{dr} = N_f(t) + \frac{r^{d}N_f(t)}{dr}$$

We have seen above that:

$$\frac{dN_f(t)}{dr} = N_f(t)\,(t - a + \gamma)$$

We therefore find that:

$$\frac{dA_f(t)}{dr} = N_f(t)[1 + r(t - a + \gamma)]$$

which is equal to zero for:

$$t_c = \alpha - \gamma - \frac{1}{r}$$

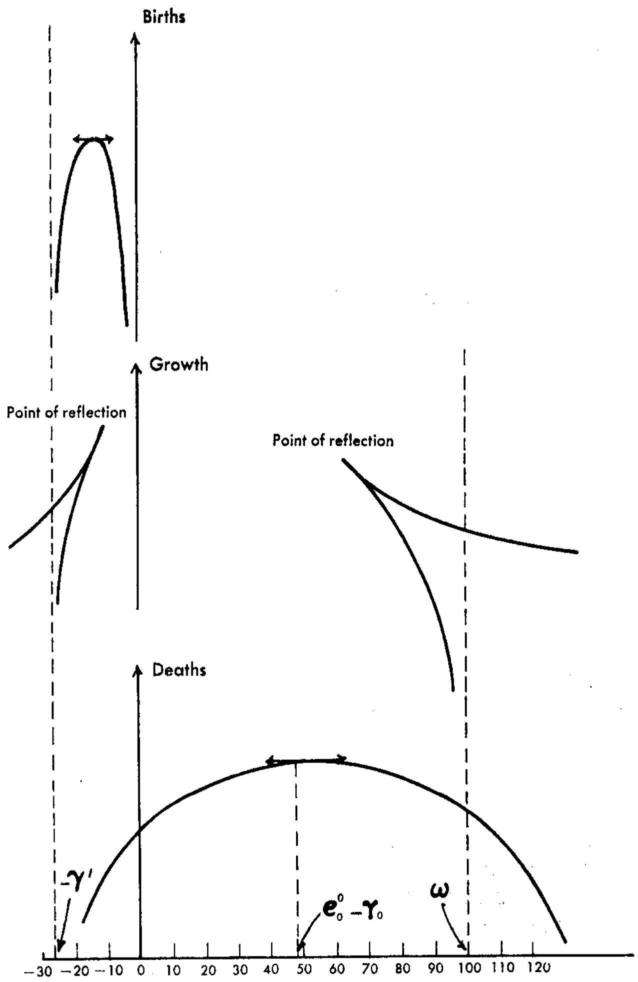
which is the abscissa of the point of contact of the envelope. The ordinate is:

$$A_f(t_c) = re^{rt_c}\int_0^{\omega} e^{-ra}p_f(a)da\int_0^{\omega}\frac{K_f(a, 0)}{p_f(a)}G(a)e^{ra}$$

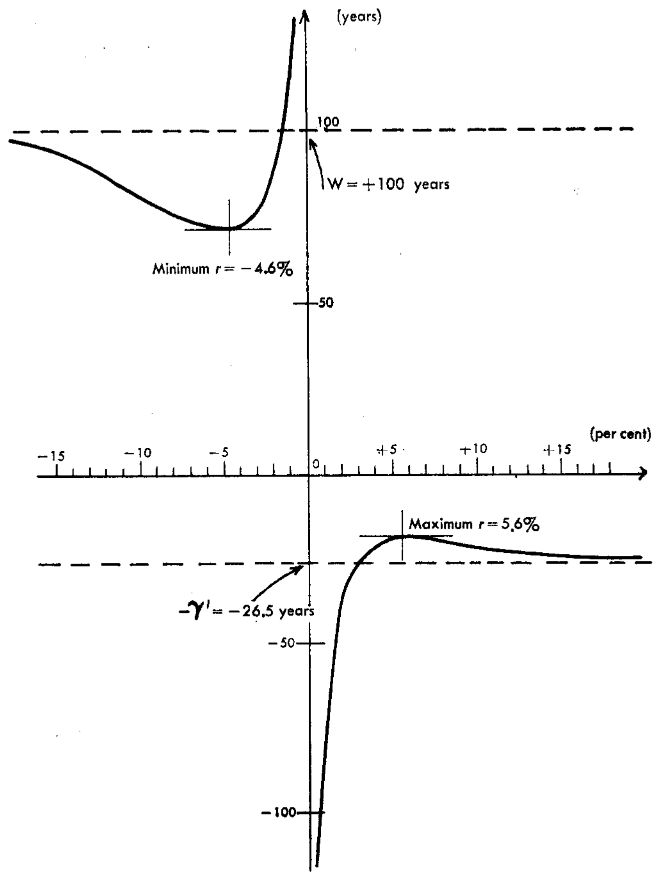
or

The slope of the tangent is

$$d\,[A_f(t_c)]/dt = r^2N_f(t_c) = A_f(t_c) = rN_f(t_c)$$
$$= r^2e^{rt_c}\int_0^{\omega} e^{-ra}p_f(a)da\int_0^{u}\frac{K_f(a, 0)}{p_f(a)}G(a)e^{ra}da$$



Graph A.I.13. Shape of the envelope of births, growth and deaths in families of populations with constant mortality (metric scale for the horizontal axis and logarithmic scale for the vertical axis)



Graph A.I.14. Variations in $t_c = \alpha - \gamma - 1/r$ as a function of r (intermediate model life table with an expectation of life at birth for both sexes of 60.4 years)

When r approaches zero, t_c increases indefinitely, and therefore the asymptote of the envelope is the horizontal axis. Moreover, we readily see from the earlier results that, when r approaches $+\infty$, t_c tends to $-\gamma'$, $A_f(t_c)$ tends to zero and the slope of the tangent to the point of contact of the envelope also tends to zero. When r approache $-\infty$, t_c tends to ω, $A_f(t_c)$ tends to zero and $rA_f(t_c)$ also tends to zero.

However, it should be noted that the expression $t_c = a - \gamma - 1/r$ has a minimum and a maximum. This can be seen in graph A.I.14, which shows the variations in t_c calculated from the female life table corresponding to an expectation of life at birth for both sexes of 60.4 years. The envelope shows points of reflection for the maximum and minimum of t_c. Lastly, the shape of the envelope is as shown in graph A.I.12.

In a semi-logarithmic graph, a new difficulty arises because a negative number has no logarithm. Instead of considering the growth of the population, we shall illustrate the excess of births over deaths for positive intrinsic rates of natural variation and the excess of deaths over births for negative intrinsic rates of natural variation. This can be seen in graph A.I.13.

G. Families of deaths

Deaths at time t are given by the expression

$$D_f(t) = B_f(t) - A_f(t)$$

in which we can simply replace $B_f(t)$ and $A_f(t)$ by the expressions given earlier. When r varies, the $D_f(t)$ curves and G_n is a constant equal to

$$\frac{1}{v+1}$$

Assuming that the last interval for which fertility is not zero is the interval immediately preceding the age of 45 years, we find that: $v.l = 45$.

For each of the five-year age groups, we have:

$$\underbrace{G_{0\text{-}4} = G_{5\text{-}9} = G_{10\text{-}14} = \ldots = G_{40\text{-}44}}_{9 \text{ groups}} = \frac{v}{9(v+1)} = \frac{45}{9(45+l)}$$

Returning to a continuous notation with l tending to zero, we find that:

$$G_{0\text{-}4} = G_{5\text{-}9} = G_{10\text{-}14} = \ldots = G_{40\text{-}44} = \frac{45}{9 \times 45} = \frac{1}{9} = 0.11111 \ldots$$

The reduced initial net population

The reduced initial net population is given in continuous notation by the formula:

$$\pi = \int_0^v \frac{K_f(a, 0)}{p_f(a)} G(a) e^{ra}$$

First case: r *approaches* $+\infty$

We shall now study the behaviour of the product $G(a)e^{ra}$ when r tends to $+\infty$. For this, the discontinuous notation will be used. For the first $(u + 1)$ intervals it is equal to:

$$\frac{e^{r\bar{n}}}{u+1}$$

For the following $(v - u)$ intervals, it is equal to:

$$\frac{p_f(\bar{n})F_n}{(u+1)p_f(\bar{u})F_u e^{-r\bar{u}'}}$$

The reduced initial net population can then be written:

$$\pi = \frac{1}{u+1} \sum_0^{u-1} \frac{K_n e^{r\bar{n}}}{p_f(n)} + \frac{e^{r\bar{u}'}}{(u+1)p_f(\bar{u})F_u} \sum_u^v K_n F_n$$

When r tends to $+\infty$, the expression π tends to $+\infty$.

Second case: r *approaches* $-\infty$

We saw that, when r tends towards $-\infty$, G_n is equivalent to $1/(v+1)$, so that the product $e^{r\bar{n}}G_n$ is equivalent to:

$$\frac{e^{r\bar{n}}}{v+1}$$

The reduced initial net population is thus equivalent to:

$$\pi = \frac{1}{v+1} \sum_0^v \frac{K_n e^{rn}}{p_f(\bar{n})}$$

When r tends to $-\infty$, it tends to zero.

The mean age γ of the reduced initial net population

First case: r *approaches* $+\infty$

In the expression π, the first u terms are negligible compared with the subsequent $v - u + 1$ terms. Consequently, the mean age of the reduced initial net population is equivalent to:

$$\gamma = \frac{\sum_u^v K_n F_n \times \bar{n}}{\sum_u^v K_n F_n}$$

This expression is simply the average age γ' of mothers at the birth of their children in the initial population. Therefore, when r tends to $+\infty$, γ tends to γ'.

Second case: r *approaches* $-\infty$

We start with:

$$\frac{\sum_0^v \frac{K_n e^{r\bar{n}} \times \bar{n}}{p_f(\bar{n})}}{\sum_0^v \frac{K_n e^{r\bar{n}}}{p_f(\bar{n})}}$$

In the numerator and the denominator of this formula, the first interval preponderates over all the others and γ is equal to:

$$\gamma = \frac{K_0 e^{r\bar{0}} \times \bar{0}}{K_0 e^{r\gamma}} = \bar{0}$$

$\bar{0}$ is the mean age of the first interval, and so we find that:

$$\gamma = \frac{l}{2}$$

When l tends to zero, γ tends to zero. Finally, when r tends to $-\infty$, γ tends to zero.

The envelope of families with constant mortality

We shall now investigate what happens to the point of contact of the envelope and the tangent to it in the case of infinitely great values of r. The ordinate of the point of contact N_c is given by the formula:

$$N_c = e^{rt^c} \int_0^\infty e^{-ra} p_f(a) da \int_0^v \frac{K_f(a)}{p_f(a)} G(a) e^{ra} da$$

where $tc = a - \gamma$ is the *abscissa of the point of contact.* Further, the slope of the tangent to the envelope is equal to rN_c.

First case: r *approaches* $+\infty$

We saw that a tends to zero[13] and γ to γ' when r approaches $+\infty$. The abscissa of the point of contact therefore tends to $-\infty$. In discontinuous notation, N_c is equal to:

---

[13] a is the mean age of the stable population. It tends to zero when r tends to $+\infty$, and to ω when r tends to $-\infty$.

The gross reduced initial population is obtained by multiplying the total of the last column by 16, i.e., 1,008,266 × 16 = 16,132,256. The 1947 census in Egypt yielded a figure of 9,575,039 women. The reduced initial population converted to 1 million is therefore equal to:

$$\frac{16\ 132\ 256}{9\ 575\ 039} = 1\ 684\ 824$$

Expressed per thousand, the growth potential of the population of Egypt in 1947 will be 1,685. Table A.I.19 shows the results of similar computations for the twenty-eight countries which were used to compile graph A.I.8.

TABLE A.I.19. GROWTH POTENTIAL OF TWENTY-SIX POPULATIONS AT A RECENT DATE[a]

(per thousand)

Country	*Year*	*Growth potential (per thousand)*
Taiwan	1959	1 894
Thailand	1955	1 881
Tunisia	1956	1 826
Mexico	1950	1 826
Trinidad and Tobago	1959	1 790
Chile	1952	1 698
Albania	1955	1 689
Egypt	1947	1 685
Jamaica	1956	1 659
Japan	1959	1 508
Canada	1960	1 486
Yugoslavia	1958	1 458
United States	1960	1 401
Romania	1956	1 387
Portugal	1959	1 371
Netherlands	1959	1 363
Australia	1959	1 363
USSR	1959	1 351
Italy	1951	1 312
Bulgaria	1959	1 312
Greece	1959	1 302
Czechoslovakia	1958	1 254
Hungary	1959	1 214
France	1959	1 174
Sweden	1958	1 121
Federal Republic of Germany	1959	1 098
England and Wales	1959	1 079
Eastern Germany	1957	1 008

[a] Using the age structure of the female populations published in the *Demographic Yearbook 1960* (United Nations publication, Sales No.: 61.XIII.1).

I. Note on the values used in the various formulae in annex I for infinitely great values of the intrinsic rate of natural increase r

THE FUNCTION G(a)

For a study of the function G(a) for infinitely great values of r, it is convenient to use a discontinuous notation. The successive age intervals may be designated by 0, 1, 2 . . . u, . . . n, v . . ., w. Thus u is the subscript of the first age interval for which fertility is not zero, v is the subscript of the last interval for which fertility is not zero, and w is the subscript of the last interval in which there are survivors. If $\bar{n}$ designates the median age of the interval of which the subscript is n, l the length of each interval, and F_n the function of the distribution female fertility rates by interval assumed to be independent of r, we can write:

$$G_n = \frac{\sum_n^v p_f(\bar{n}')F_n e^{-r\bar{n}} \times l}{\sum_0^v \sum_n^v p_f(\bar{n})F_n e^{-r\bar{n}} \times l}$$

First case: r *approaches* $+\infty$

When r approaches $+\infty$ in the sum of the numerator, the term $p_f(n)F_n e^{-r\bar{n}'} \times l$ preponderates over all the others, and consequently in each of the first $(u+1)$ intervals this sum is equivalent to: $p_f(\bar{u})F_u e^{-r\bar{u}} \times l$.

For the following v—u intervals, it is equivalent to:

$$p_f(\bar{n})F_n e^{-r\bar{n}} \times l$$

The sum of the denominator is then equivalent to:

$$(u+1)p_f(\bar{u}')Fe^{-r\bar{u}'} \times l + \sum_{u+1}^v p_f(\bar{n}')F_n e^{-r\bar{n}'} \times l$$

and the $e^{-r\bar{n}}$ term preponderates over all the others.

Thus, for the first $(u+1)$ intervals we find that:

$$G_n = \frac{p_f(\bar{u})F_u e^{-r\bar{u}} \times l}{(u+1)p_f(\bar{u})F_u e^{-r\bar{u}} \times l} = \frac{1}{u+1}$$

For the following $v-u$ intervals, we have:

$$G_n = \frac{p_f(\bar{n})F_n e^{-r\bar{n}}}{(u+1)p_f(\bar{u})F_u e^{-r\bar{u}}}$$

When r tends to $+\infty$, G_n tends to zero for the last $v-u$ intervals. Finally, G_n is a constant up to and including the interval u and is zero after that. Assuming that the first interval in which fertility is not zero is the interval after the age of 15 years, we find that $u \times l = 15$. For each of the three age groups of five successive years 0-4, 5-9, 10-14, we find that:

$$G_{0\text{-}4} = G_{5\text{-}9} = G_{10\text{-}14} = \frac{u}{3(u+1)} = \frac{15}{3(15+l)}$$

Returning to continuous notation with l tending to zero, we have:

$$G_{0\text{-}4} = G_{5\text{-}9} = G_{10\text{-}14} = \frac{15}{3 \times 15} = \frac{1}{3} = 0.333\ldots$$

Second case: r *approaches* $-\infty$

We revert to the expression:

$$G_n = \frac{\sum_n^v p_f(\bar{n})F_n e^{-r\bar{n}}}{\sum_0^v \sum_n^v p_f(\bar{n})F_n e^{-r\bar{n}}}$$

In the sum of the numerator, when r tends to $-\infty$, the term $p_f(\bar{v})F_v e^{-r\bar{v}}$ preponderates over all the others, and consequently this sum is equivalent to:

$$p_f(\bar{v})F_v e^{-r\bar{v}} \times l$$

The sum of the denominator is then equivalent to:

$$(V+1)p_f(\bar{v})F_v e^{-r\bar{v}} \times l$$

$$N_c = [e^{-r\gamma'}\sum_0^{\omega} e^{-r\bar{n}} p_f(\bar{n})] \times \left[\frac{e^{r\bar{u}}}{(u+1)p_f(\bar{u})F_u}\sum_u^v K_n F_n\right]$$

This can be written:

$$N_c = \frac{e^{-r\gamma'+r\bar{u}}}{(u+1)p_f(u')F_u} \times \frac{1}{b}\sum_u^v K_n F_n$$

where b is the gross birth rate of the stable population. As γ' is larger than u, $e^{-r\gamma'} + ^{ru}$ tends to zero when r tends to $+\infty$. The crude birth rate of the stable population tends to $+\infty$, and therefore $1/b$ tends to zero. Lastly, it will be seen that N_c *tends to zero.*

The slope of the tangent to the envelope rN_c also tends to zero, since the product $r \times e^{-r\gamma'} + ^{ru}$ *tends towards zero.*

Second case: r *approaches* $-\infty$

When r tends to $-\infty$, α tends[14] to ω and γ' to zero. Therefore, *the abscissa of the point of contact tends to* ω.

The ordinate of the point of contact is equal to:

$$N_c = e^{\bar{\omega}r'}\sum_0^{\bar{\omega}} e^{-r\bar{n}} p_f(\bar{n}) \frac{1}{v+1}\sum_0^v \frac{K_n e^{r\bar{n}}}{p_f(\bar{n})}$$

[14] α is the mean age of the stable population. It tends to zero when r tends to $+\infty$, and to ω when r tends to $-\infty$.

In the sum $\sum_0^{\omega} e^{-r\bar{n}} p_f(\bar{n})$, the term $e^{-r\bar{\omega}} p(\bar{\omega})$ preponderates over all the others when r tends to $-\infty$, and N_c is equal to:

$$\frac{e^{r\bar{\omega}}}{e^{r\bar{\omega}}} \frac{p_f(\bar{\omega})}{\omega+1}\sum_0^v \frac{K_n e^{r\bar{n}}}{p_f(\bar{n})} = \frac{p_f(\bar{\omega})}{\omega+1}\sum_0^v \frac{K_n e^{rn}}{p_f(\bar{n})}$$

This quantity tends to zero when r tends to $-\infty$. Finally, it will be seen that *the ordinate of the point of contact tends to zero.*

The slope of the tangent can be written:

$$rN_c = \frac{rp_f(\omega)}{v+1}\sum_0^v \frac{K_n e^{r\bar{n}}}{p_f(\bar{n})}$$

In the sum

$$\sum_0^v \frac{K_n e^{r\bar{n}}}{p_f(\bar{n})}$$

the term

$$\frac{Ke_0^{r\bar{0}}}{p_f(\bar{0})}$$

preponderates over all the others and we find that:

$$rN_c = \frac{p_f(\bar{\omega})}{v+1}\frac{K_0 e^{r\bar{0}}}{p_f(\bar{0})} = \frac{K_0 p_f(\bar{\omega}) r e^{rl/2}}{(v+1)p_f(\bar{0})}$$

This quantity tends to zero, once l has been fixed as small as is desired. Therefore, *the slope of the tangent tends to zero.*

Annex II

MODEL LIFE TABLES

A. Introduction

For a given health situation, a life table is an ensemble of numerical indices reflecting the mortality pattern of the members of a generation subject during their lives to the health conditions considered. The "way of dying" of the members of a generation can be presented specifically in the form of various numerical functions of age. There are certain links joining these functions together and which are easily expressed in mathematical language. The most widely used functions are the following:

The function of *probabilities of dying* ($_nq_x$), which show the proportion of survivors to age x who die within a time interval n after that age; this interval is one year in the case of annual quotients and five years in the case of quiquennial quotients. When the interval n is obvious, the notation used is often simply q_x.

The *survivorship* function ($p(x)$), which shows the proportion of survivors of a generation to age x. For example, if we take a cohort of 100,000 births, the function $p(x)$ decreases from 100,000 to zero. The survivorship function is also represented by l_x.

The function of *deaths by age* (d_x), which gives the age distribution of deaths in a generation.

Often, instead of considering probabilities of death, we calculate death rates indicating the ratio of deaths occurring between two given ages to the surviving population at the mid-point of the period. *This is the death rate function* (m_x).

It is sometimes convenient to conceive a population in which there are 100,000 births annually and which is subject to the health conditions in question. *This is a stationary population*, and the total number of persons of the successive ages is designated by L_x.

Lastly, we use the *expectation of life at age* x(^0e_x), which indicates the mean length of time which survivors to age x will continue to live.

These six functions—q_x, l_x, d_x, m_x, L_x and e_x—are equivalents, and if one of them is known the other five can be determined. They appear in the form of numerical tables of coefficients. Such tables are not easy to use. Attempts are therefore made to define general indices which summarize them without sacrificing the properties of the tables to too great an extent. The expectation of life at birth ($^0e_0$) is often used as a general index.

B. The principle of model life tables

When we compare life tables having similar expectations of life at birth, we find that each of the mortality functions defined above also has similar values. It would appear that for each value of the expectation of life at birth we can associate mortality functions around which the actual mortality functions, with the same value of the expectation of life at birth, are quite closely distributed. It is on this observation that the possibility of preparing model life tables is based.

The Secretariat of the United Nations has published a series of forty model life tables for expectations of life at birth ranging from 18 to 75 years.[1]

The various mortality functions, as defined above, corresponding to these life tables are given in the manual dealing with methods of computing population projections.[2] Whenever model mortality is mentioned below, the reference will be to the mortality corresponding to this series of forty tables. The universe of variations in each of the model mortality functions is stratified. If the curves for variations in one of these functions are drawn on a graph, the curves obtained do not cross each other. They are separate, one above the other. In the universe of model life tables, therefore, the notion of "mortality level" has a very specific meaning.

A series of model life tables is a very useful tool for the demographer. A whole range of applications is based on the following reasoning. Observations are made of the deaths in a population. For example, the crude death rate is observed. The series of model tables is then searched for the one which would give, for the population in question, the observed value for the crude death rate. It is assumed that this life table describes the health situation of the population in question. This, of course, presupposes that the mortality of the population in question does not deviate too greatly from the model pattern. Any attempt to work out a series of model life tables should therefore be accompanied by a study of the extent to which the mortality actually observed deviates from the model pattern.

C. Deviations of actual mortality from the model life table

At first sight, the deviations between the death rates actually observed and the model tables published by the Secretariat of the United Nations which are the subject of the two publications mentioned above are by no means negligible. Table A.II.1 exhibits the series of death rates m_x of model life tables. We have indicated for each sex and age group the position in the table of the death rates observed in British Guiana in 1953 and in Norway in 1926-1935. The points representing British Guiana have

[1] *Age and sex patterns of mortality. Model life tables for under-developed countries* (United Nations publication, Sales No.: 55.XIII.9).

[2] *Manuals on methods of estimating population. Manual III. Methods for population projections by sex and age* (United Nations publication, Sales No.: 56.XIII.3).

Sex and age (x) in years	*Level 0* (°e₀= 20)	*Level 5* (°e₀= 22.5)	*Level 10* (°e₀= 25)	*Level 15* (°e₀= 27.5)	*Level 20* (°e₀= 30)	*Level 25* (°e₀= 32.5)	*Level 30* (°e₀= 35)	*Level 35* (°e₀= 37.5)	*Level 40* (°e₀= 40)	*Level 45* (°e₀= 42.5)	*Level 50* (°e₀= 45)	*Level 55* (°e₀= 47.5)
MALES												
0	442.63	411.67	371.41	341.60	316.20	293.25	270.17	248.20	229.41	211.29	193.65	177.35
1- 4 . . .	77.96	67.11	58.27	50.89	44.99	39.73	35.13	30.97	27.55	24.31	21.44	18.85
5- 9 . . .	18.64	16.21	14.16	12.44	11.00	9.72	8.58	7.55	6.71	5.90	5.20	4.56
10-14 . . .	11.45	10.09	8.92	7.91	7.05	6.28	5.59	4.96	4.43	3.92	3.47	3.07
15-19 . . .	14.09	12.88	11.75	10.68	9.76	8.90	8.05	7.25	6.57	5.91	5.32	4.79
20-24 . . .	17.93	16.66	15.41	14.21	13.20	12.23	11.17	10.21	9.32	8.47	7.67	6.93
25-29 . . .	21.13	19.38	17.70	16.12	14.73	13.43	12.13	10.98	9.94	8.96	8.04	7.22
30-34 . . .	25.43	22.95	20.66	18.55	16.74	15.07	13.48	12.08	10.84	9.69	8.64	7.70
35-39 . . .	31.77	28.20	25.01	22.15	19.76	17.59	15.59	13.87	12.38	11.01	9.77	8.67
40-44 . . .	40.65	35.72	31.40	27.62	24.50	21.71	19.20	17.06	15.17	13.45	11.96	10.66
45-49 . . .	50.53	44.37	39.05	34.41	30.54	27.10	24.07	21.52	19.22	17.14	15.38	13.85
50-54 . . .	59.69	53.02	47.17	42.02	37.74	33.92	30.43	27.46	24.82	22.49	20.36	18.56
55-59 . . .	71.38	64.55	58.34	52.73	47.98	43.66	39.64	36.14	33.01	30.18	27.68	25.49
60-64 . . .	84.96	78.49	72.35	66.59	61.60	56.97	52.48	48.48	44.87	41.57	38.65	36.12
65-69 . . .	106.42	100.14	93.94	87.92	82.65	77.63	72.48	67.82	63.58	59.64	56.00	52.77
70-74 . . .	144.12	137.21	130.28	123.34	116.91	110.72	104.51	98.77	93.31	88.21	83.43	79.17
75-79 . . .	194.69	186.80	178.81	170.84	163.60	156.14	148.65	141.69	135.04	128.82	122.91	117.55
80-84 . . .	274.44	264.86	254.17	243.76	234.75	225.25	215.65	206.60	198.10	190.09	182.54	175.67
85 + . . .	511.36	460.12	418.80	387.42	363.99	344.56	327.70	313.93	302.48	292.38	284.10	276.97
FEMALES												
0	396.43	365.17	335.65	309.22	283.41	259.87	239.81	220.56	202.22	184.42	167.81	151.94
1- 4 . . .	79.80	68.66	59.60	52.13	45 75	40.16	35.51	31.29	27.71	24.33	21.37	18.69
5- 9 . . .	19.44	16.90	14.76	12.96	11.41	10.02	8.85	7.79	6.89	6.03	5.29	4.61
10-14 . . .	13.10	11.54	10.19	9.03	8.02	7.10	6.31	5.59	4.98	4.39	3.87	3.40
15-19 . . .	15.97	14.59	13.29	12.09	10.93	9.86	8.92	8.02	7.21	6.42	5.70	5.02
20-24 . . .	19.91	18.50	17.10	15.77	14.35	13.02	11.88	10.85	9.73	8.67	7.72	6.83
25-29 . . .	23.98	21.98	20.07	18.28	16.52	14.89	13.46	12.17	10.82	9.56	8.38	7.37
30-34 . . .	28.30	25.52	22.96	20.61	18.40	16.38	14.64	13.11	11.56	10.14	8.91	7.81
35-39 . . .	33.17	29.44	26.09	23.10	20.40	17.96	15.92	14.15	12.41	10.83	9.51	8.34
40-44 . . .	37.24	32.75	28.82	25.36	22.29	19.54	17.29	15.36	13.55	11.90	10.50	9.27
45-49 . . .	42.25	37.18	32.79	28.95	25.59	22.61	20.11	17.99	16.01	14.22	12.67	11.30
50-54 . . .	49.17	43.80	39.06	34.86	31.03	27.62	24.82	22.42	20.10	18.02	16.30	14.70
55-59 . . .	57.84	52.49	47.57	43.08	38.86	35.03	31.86	29.08	26.36	23.88	21.74	19.81
60-64 . . .	72.71	67.28	62.15	57.29	52.49	48.07	44.34	41.00	37.64	34.56	31.76	29.18
65-69 . . .	93.87	88.49	83.15	77.92	72.47	67.35	62.99	58.99	54.81	50.94	47.43	44.21
70-74 . . .	129.46	123.41	117.29	111.22	104.86	98.83	93.42	88.36	83.13	78.25	73.71	69.49
75-79 . . .	183.54	176.33	168.86	161.47	153.68	145.74	138.89	132.45	125.61	119.19	113.20	107.63
80-84 . . .	261.12	251.93	242.11	232.48	222.14	211.79	202.85	194.50	185.42	176.94	169.01	161.57
85 + . . .	456.14	417.09	396.93	362.32	341.38	323.96	310.57	299.33	288.89	279.84	272.19	265.53

[a] Equivalent values of $^{0}e_{0}$, shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

——————— British Guiana, 1953

- - - - - - - - - Norway, 1926-1935.

Level 60 ($°e_0$= 50)	Level 65 ($°e_0$= 52.5)	Level 70 ($°e_0$= 55)	Level 75 ($°e_0$= 57.6)	Level 80 ($°e_0$= 60.4)	Level 85 ($°e_0$= 63.2)	Level 90 ($°e_0$= 65.8)	Level 95 ($°e_0$= 68.2)	Level 100 ($°e_0$= 70.2)	Level 105 ($°e_0$= 71.7)	Level 110 ($°e_0$= 73.0)	Level 115 ($°e_0$= 73.9)	Sex and age (x) in years
												MALES
161.16	145.05	129.59	110.85	90.18	70.10	52.51	39.13	30.35	24.55	20.73	18.18	. . . 0
16.44	14.15	12.10	10.18	8.45	6.79	5.20	3.67	2.45	1.64	1.11	0.75	. . . 1- 4
3.98	3.44	2.96	2.51	2.12	1.76	1.45	1.15	0.81	0.56	0.39	0.28	. . . 5- 9
2.70	2.34	2.05	1.77	1.53	1.30	1.09	0.89	0.66	0.47	0.35	0.26	. . . 10-14
4.29	3.78	3.32	2.90	2.51	2.14	1.81	1.49	1.13	0.80	0.59	0.44	. . . 15-19
6.22	5.48	4.84	4.22	3.62	3.04	2.52	2.02	1.49	1.08	0.80	0.61	. . . 20-24
6.45	5.68	4.97	4.30	3.70	3.14	2.59	2.08	1.57	1.18	0.92	0.75	. . . 25-29
6.84	6.00	5.24	4.55	3.91	3.33	2.79	2.29	1.82	1.43	1.18	1.00	. . . 30-34
7.70	6.77	5.94	5.19	4.50	3.88	3.31	2.79	2.31	1.90	1.63	1.45	. . . 35-39
9.50	8.41	7.45	6.58	5.79	5.07	4.45	3.87	3.35	2.88	2.52	2.27	. . . 40-44
12.49	11.22	9.97	9.07	8.16	7.33	6.61	5.92	5.32	4.71	4.17	3.76	. . . 45-49
16.95	15.43	14.08	12.85	11.78	10.81	9.96	9.14	8.40	7.67	6.95	6.30	. . . 50-54
23.52	21.65	20.00	18.53	17.19	15.98	14.93	13.99	13.02	12.08	11.22	10.37	. . . 55-59
33.83	31.58	29.53	27.69	26.05	24.59	23.24	21.96	20.70	19.42	18.22	17.13	. . . 60-64
49.82	45.92	44.32	42.02	39.85	37.92	36.13	34.42	32.74	31.02	29.46	28.01	. . . 65-69
75.35	71.61	68.14	65.01	62.09	59.41	56.94	54.60	52.36	50.13	48.10	46.28	. . . 70-74
112.56	107.64	103.28	99.39	95.73	92.40	89.26	86.32	83.35	80.36	77.33	74.56	. . . 75-79
169.30	162.98	157.07	151.67	146.64	141.99	137.74	133.68	129.59	126.39	122.42	118.21	. . . 80-84
270.72	264.90	259.83	255.32	251.20	247.52	244.26	241.34	238.71	236.36	234.24	232.36	. . . 85 —
												FEMALES
136.41	121.72	107.62	91.92	74.54	57.57	42.65	31.27	23.79	18.84	15.57	13.39	. . . 0
16.19	13.87	11.65	9.80	7.92	6.14	4.55	3.09	2.06	1.38	0.93	0.63	. . . 1- 4
3.99	3.41	2.90	2.42	1.97	1.56	1.18	0.85	0.60	0.42	0.29	0.21	. . . 5- 9
2.96	2.55	2.17	1.82	1.49	[illegible]	0.91	0.67	0.50	0.36	0.26	0.20	. . . 10-14
4.38	3.79	3.27	2.77	2.30	1.86	1.45	1.10	0.84	0.60	0.44	0.33	. . . 15-19
5.97	5.19	4.46	3.76	3.13	2.54	1.99	1.49	1.11	0.80	0.60	0.47	. . . 20-24
6.45	5.58	4.80	4.06	3.38	2.73	2.19	1.70	1.29	0.97	0.76	0.62	. . . 25-29
6.80	5.89	5.09	4.34	3.64	3.00	2.47	1.97	1.56	1.23	1.00	0.85	. . . 30-34
7.28	6.31	5.48	4.70	4.00	3.37	2.84	2.35	1.95	1.60	1.38	1.22	. . . 35-39
8.15	7.15	6.29	5.49	4.78	4.14	3.58	3.06	2.65	2.31	2.04	1.87	. . . 40-44
10.06	8.95	8.00	7.11	6.33	5.62	5.02	4.45	4.00	3.62	3.28	3.02	. . . 45-49
13.22	11.91	10.78	9.74	8.80	7.94	7.20	6.51	5.98	5.51	5.15	4.83	. . . 50-54
18.05	16.48	15.05	13.71	12.53	11.45	10.45	9.52	8.87	8.33	7.92	7.62	. . . 55-59
26.79	24.70	22.82	21.04	19.41	17.89	16.58	15.33	14.47	13.78	13.10	12.59	. . . 60-64
41.19	38.47	35.91	33.46	31.26	29.21	27.41	25.71	24.47	23.39	22.33	21.29	. . . 65-69
65.38	61.59	58.10	54.80	51.80	48.96	46.46	44.08	42.31	40.53	38.68	36.89	. . . 70-74
102.30	97.39	92.63	88.03	83.82	79.85	76.31	72.96	70.52	68.24	65.40	62.47	. . . 75-79
154.48	147.96	141.94	136.17	130.92	125.92	121.48	117.21	113.98	110.99	107.61	104.01	. . . 80-84
259.56	254.36	249.76	245.57	241.69	238.20	235.22	232.54	230.61	228.97	227.41	225.98	. . . 85 —

been joined together with a solid line and those representing Norway with a broken line. If the death rates observed in these two countries conformed to the model pattern, the resulting profiles should scarcely deviate from one column of the table. As will be seen, this is not the case. From 5 to 34 years, the two death rates in question are close to the model table at level 80 (expectation of life at birth for both sexes of 60.4 years). For ages below 5 and over 34, they deviate greatly from it, in different directions. These are two of the death rates which show the greatest deviations from the model tables. If other series of observed death rates similar, between the ages of 5 and 34, to the two series considered here were superimposed on table A.II.1, the result would be a great variety of profiles covering the area bounded by the British Guiana and Norway profiles. However, they would follow very different courses across that area. Some would be to the left of the central column, like the profile for British Guiana, while others would be to its right, like that for Norway; some, however, would go first right and then left, or *vice versa*, from age group to age group.

D. Components of mortality

In an endeavour to introduce some order into this diversity, we shall utilize the results obtained by Ledermann and Breas in their study,[3] using the technique of factor analysis, of the 157 life tables from which the model life tables published by the Secretariat of the United Nations were prepared. An article published by the Secretariat of the United Nations in *Population Bulletin of the United Nations* No. 6 [4] presents a summary of the work of Ledermann and Breas and, taking that work as the basis for an analysis of series, of age-specific death rates (m_x), proposes a method about which we shall now say a few words. Factor analysis of the probabilities of death in the 157 life tables used for the computation of the model life tables shows that five independent factors are responsible for variations in the probabilities observed in these 157 tables. If C_1, C_2, C_3, C_4 and C_5 represent these five "components" of mortality, the death rate at age x for a given health situation takes the following form:

$$m_x = \mu_x(A_x)^{C_1}(B_x)^{C_2}(C_x)^{C_3}(D_x)^{C_4}(E_x)^{C_5}$$

C_1, C_2, C_3, C_4 and C_5 are coefficients which depend on the health situation and are the same for all ages x.

A_x, B_x, C_x, D_x and E_x are coefficients which depend on age, but remain the same regardless of health situations. The mortality level corresponding to the mean of the 157 tables is obtained when $C_1 = C_2 = C_3 = C_4 = C_5 = 0$. We have then $m_x = \mu_x$.

Each level of mortality is represented by the above equation in relation to the mean conditions represented by μ_x.

If only the first factor operated, we should have:

$$m_x = \mu_x(A_x)C_1$$

Experience shows that the values for m_x obtained in this way are very close to the death rates of the model life tables. We can therefore say that the model life tables show what would happen if the first component of mortality operated alone. It follows that the effects of the components other than the first provide a measure of the extent to which actual death rates deviate from the model death rates, and the problem of studying these deviations is therefore tantamount to studying the effects of the other components on the first component. These components have different effects at different ages. The second component operates only after the age of 25. Its effects, which are minor up to about 35, then increase, reaching a maximum at about 55, and then become negligible after 75. The third component operates mostly at advanced ages. The fourth component affects mortality below the age of 5. The fifth component relates only to male mortality between 5 and 70. Table A.II.2 summarizes the incidence of the five components by sex and by some age groups.

TABLE A.II.2. INCIDENCE OF THE COMPONENTS OF MORTALITY BY SEX AND BY SOME AGE GROUPS

Age group (in years)	Component number: Male	Female
0	1 and 4	1 and 4
1-4	1 and 5	1 and 4
5-34	1 and 5	1
45-64	1,2 and 5	1 and 2
70-84	1,2 and 3	1,2 and 3

Furthermore, once the effect of a component is known for a given age and sex, it is known for all ages and for the other sex. We indicate below how, in the case of the first, second and third components, we determine from the effect at a given age and for a given sex the effects at other ages and for the other sex.

The effects produced by the first component operating alone are presented, like the model life tables, in the form of a sequence of numerical series of probabilities of death (q_x) or death rates (m_x), each series corresponding to one value of the first component. Once we know the effect at a certain age, we can deduce the whole of the corresponding series from it by interpolation in the sequence.

Table A.II.3 shows, for various age groups and for both sexes, the effect of a second component modifying the first female component from 45 to 64 years by 10 per cent. Table A.II.4 gives the same data for a third component modifying the first female component from 70 to 84 years by 10 per cent. The same coefficients, modified proportionately, are used for other percentages.

E. Measuring the components of mortality

It is easy to see how the effects of the various components can be estimated in a given situation. According to table A.II.2:

(*a*) The female death rate between 5 and 34 depends only on the *first component*. To each value of this rate corresponds value for the expectation of life at birth of the first component;

[3] "Les dimensions de la mortalité", *Population* (Paris), 14[e] année, No. 4, October-December 1959.

[4] "Factor analysis of sex-age-specific death rates: a contribution to the study of the dimensions of mortality", *Population Bulletin of the United Nations* No. 6 (United Nations publication, Sales No.: 62.XIII.2).

TABLE A.II.3. PERCENTAGE VARIATION BY AGE GROUP AND SEX IN THE DEATH RATES OF THE FIRST COMPONENT WHEN THE SECOND COMPONENT CAUSES VARIATIONS OF 10 PER CENT IN THE MEAN DEATH RATE OF THE FIRST COMPONENT BETWEEN THE AGES OF 45 AND 64

Age group (in years)	*Percentage*	
	Male	*Female*
25-29	2.0	1.4
30-34	4.4	2.5
35-39	6.7	4.2
40-44	9.3	6.6
45-49	11.2	9.4
50-54	12.8	10.3
55-59	13.5	10.7
60-64	12.5	9.5
65-69	10.5	7.8
70-74	8.7	6.4
75-79	7.3	5.3
80-84	5.0	8.3
45-64	12.5	10.0

TABLE A.II.4. PERCENTAGE VARIATION BY AGE GROUP AND SEX IN THE DEATH RATE OF THE FIRST COMPONENT WHEN THE THIRD COMPONENT CAUSES VARIATIONS OF 10 PER CENT IN THE MEAN DEATH RATE OF THE FIRST COMPONENT BETWEEN THE AGES OF 70 AND 84

Age group (in years)	*Percentage*	
	Male	*Female*
55-59	0.6	1.6
60-64	1.4	3.0
65-69	2.9	4.4
70-74	4.8	7.1
75-79	7.4	10.6
80-84	10.5	12.4
70-84	7.6	10.0

(*b*) By dividing the observed female death rate between 45 and 64 by the female death rate of the first component, we obtain *the effect of the second female component between 45 and 64*;

(*c*) By dividing the observed female death rate under 1 year of age and between 1 and 4 respectively by the corresponding rates of the first component, we obtain *the effect of the fourth female component*;

(*d*) By dividing the observed female death rate between 70 and 84 by the female death rate derived from the first and second components, we obtain *the effect of the third female component between 70 and 84*;

(*e*) By dividing the observed male death rates by the male rate derived from the first, second and third components, we obtain *the effects of the fifth component.*

The details of the method of calculation will be found in the article in the *Population Bulletin of the United Nations* No. 6 mentioned above.

We indicated earlier what was meant by "mortality level" in the universe of model mortalities. We can also speak of mortality level in the universe of mortality of the first component, since the two universes coincide. We have seen above that, in actual mortalities, female mortality between 5 and 34 depends only on the first component. We can therefore associate with each actual mortality the mortality of the first component, or the life table mortality having, between the ages of 5 and 34, the same female mortality as the actual mortality. We can, therefore, speak of the level of mortality in the case of actual mortalities. This is the mortality level of the mortality of the first component (or of the model life table mortality) associated with it. In the case of females, however, the actual mortality will usually coincide with this level only between the ages of 5 and 34. At all other ages, and in the case of males we should expect to find deviations from this level due to the existence of the second, third, fourth and fifth components. It is these deviations that we wish to examine.

F. Practical limits of variations in the components of mortality

We cannot lay down in advance any limits to the variations in mortality due to the various components defined above. We can only learn from experience. We have applied the method of measuring the components which we have described in the principle of model life tables, using the death rates by age groups of various countries of the world, as published in four works.[5] The results appear in table A.II.5. The table gives the coefficients as defined above for the various components. Graphs A.II.1 to A.II.4 illustrate the table. In each graph, the expectation of life at birth of the first component is represented on the vertical axis and the multiplier of the various components on the horizontal axis, and the point thus obtained is indicated by a cross. For each component, the dispersion of the crosses around the vertical line equal to unity enables us to assess at a glance the value of that component in the populations concerned.

Let us first consider graph A.II.1, which relates to the fourth mortality component under 1 year. We find a cluster of crosses whose general direction is independent of the expectation of life at birth of the first component. This is proof that the fourth component, as we have defined it, really is independent of the first component. Apart from a few exceptional cases, we can say that the maximum effect of the fourth component is to multiply or divide by two the mortality under 1 year of the first component. Graph A.II.2, which relates to the fourth component of female mortality between 1 and 4, leads to the same conclusions and the same multipliers.

Graph A.II.3 (left-hand side) relates to the second female component. Here, the orientation of the cluster of crosses is not entirely independent of the expectation of life at birth of the first component. The leftward deviations from the 1.0 line are less for low and high mortalities than for average mortalities. In other words, instead of being enclosed inside a band of constant width, the crosses lie inside a band which narrows at each end.

[5] *Statistiques internationales du mouvement de la population d'après les registres de l'état civil jusqu'en 1905* (Paris, Imprimerie nationale, 1907); *Statistiques internationales du mouvement de la population d'après les registres de l'état civil (second volume), années* 1901 à 1910 (Paris, Imprimerie nationale, 1913); *Le mouvement naturel de la population dans le monde de 1906 à 1936* (Paris, Les Editions de l'Institut national d'études démographiques, 1954); and United Nations, *Demographic Yearbook (*years 1948 through 1960).

TABLE A.II.5. FIRST COMPONENT OF MORTALITY AND MULTIPLIERS OF THE SECOND, FOURTH AND FIFTH COMPONENTS FOR SOME AGE GROUPS IN FORTY-TWO COUNTRIES OR TERRITORIES ([a]) SINCE THE INCEPTION OF VITAL STATISTICS

Country and dates	First component expectation of life at birth for both sexes (in years)	Multipliers: Fourth component (female)		Multipliers: Second component (female)	Multipliers: Fifth component (male)	
		Under 1 year	*1-4 years*	*45-64 years*	*5-34 years*	*45-64 years*
AUSTRALIA						
1907-1915	60.4	0.883	0.863	0.965	0.954	1.082
1916-1925	62.2	...	...	0.910	0.974	1.106
1926-1935	65.2	0.902	0.933	0.897	0.875	1.130
1936-1939	67.1	0.746	0.909	1.015	0.963	1.071
1946-1949	71.5	0.847	0.955	1.202	1.070	1.067
1955-1959	74.5	0.925	1.247	1.249	1.446	1.151
AUSTRIA						
1866-1875	38.8	1.397	...	1.231	1.105	0.822
1876-1885	40.8	1.550	1.485	1.139	1.082	0.891
1886-1895	42.7	1.669	1.739	1.114	1.000	0.860
1896-1905	45.8	1.604	1.351	1.071	0.937	0.892
1906-1910	47.8	1.569	1.250	1.020	1.005	1.012
1926-1935	60.0	1.481	0.915	0.945	0.994	1.055
1936	63.0	1.257	0.959	1.012	0.969	1.043
1951	70.4	1.606	1.191	1.102	1.113	1.181
1955-1958	73.0	1.594	1.214	1.151	1.485	1.207
BELGIUM						
1846-1855	37.6	0.731	1.033	0.905	0.920	0.924
1856-1865	41.0	0.877	1.262	0.839	0.906	0.979
1866-1875	42.4	0.900	1.322	0.830	1.009	1.057
1876-1885	47.4	1.170	1.293	0.868	0.994	1.100
1886-1895	49.4	1.320	1.351	0.923	0.943	1.084
1896-1905	53.8	1.486	1.307	0.943	0.916	1.077
1906-1913	56.9	1.516	1.439	0.987	0.921	1.029
1919-1925	57.2	1.015	1.257	0.936	1.039	0.953
1926-1935	60.4	1.084	0.899	0.930	0.934	1.000
1937	63.6	1.231	0.895	1.008	0.999	1.012
1946-1949	68.0	1.389	0.898	1.049	1.159	1.130
1955	73.7	1.548	1.273	1.196	1.094	1.172
BULGARIA						
1888-1895	34.4	0.719	0.925	0.730	1.033	0.996
1896-1905	38.2	1.020	1.120	0.742	0.936	0.967
1906-1910	37.8	1.161	1.206	0.721	0.864	0.926
1918-1925	39.4	0.957	1.556	0.761	0.999	0.972
1926-1935	49.7	1.230	1.392	0.668	0.811	1.097
CANADA						
1921-1925 ([b])	60.4	...	1.238	0.923	0.896	0.809
1926-1935	61.8	...	0.884	0.905	0.852	0.761
1936-1939	64.5	1.036	0.949	0.978	0.870	0.897
1947	69.1	1.085	0.892	1.175	0.973	0.867
1955-1959	74.2	1.247	1.322	1.207	1.403	1.103
						0.940
CHILE						
1916-1925	37.6	1.697	1.218	1.127	1.090	0.940
1926-1935	43.0	1.527	1.168	1.022	1.049	0.102
CZECHOSLOVAKIA						
1919-1925	53.4	1.421	1.374	1.017	0.954	0.884
1926-1935	58.0	0.158	1.021	0.961	0.934	0.974
DENMARK						
1836-1845	45.2	...	...	1.023	0.983	0.986
1846-1855	45.2	0.941	1.016	1.000	1.045	0.993
1856-1865	43.2	0.794	0.969	0.848	0.987	1.087
1866-1875	44.6	0.955	0.858	0.839	1.068	1.081
1876-1885	45.2	0.974	0.787	0.758	0.881	1.132
1886-1895	48.8	1.081	0.929	0.810	0.956	1.041
1896-1905	54.8	1.200	0.855	0.851	0.910	1.068
1906-1915	59.0	1.160	0.770	0.879	0.892	1.042
1916-1925	59.6	0.886	0.694	0.872	1.028	0.854
1926-1935	65.0	1.291	0.870	0.958	0.828	0.778
1936-1939	67.8	1.257	0.884	1.157	0.890	0.771
1946-1949	71.7	1.070	0.935	1.131	0.959	0.799
1955-1958	75.1	1.065	1.046	1.231	1.109	0.897

TABLE A.II.5 (continued)

Country and dates	First component expectation of life at birth for both sexes (in years)	Multipliers: Fourth component (female) Under 1 year	Fourth component (female) 1-4 years	Second component (female) 45-64 years	Fifth component (male) 5-34 years	Fifth component (male) 45-64 years
EGYPT						
1931-1935	51	1.664	4.237	0.797	1.413	1.272
ENGLAND AND WALES						
1856-1865	43.5	0.969	1.389	0.946	1.006	0.868
1866-1875	45.6	1.066	1.317	1.028	1.086	0.862
1876-1885	49.3	1.156	1.440	1.113	1.028	0.846
1886-1895	52.8	1.387	1.680	1.240	0.985	0.818
1896-1905	56.5	1.645	1.918	1.250	1.016	0.812
1906-1915	59.6	1.354	1.824	1.218	1.047	0.952
1916-1925	58.3	0.831	1.419	0.954	1.061	1.021
1926-1935	63.1	0.922	1.254	0.984	0.955	1.006
1936-1939	65.3	0.908	0.960	1.020	0.925	1.071
1946-1949	69.6	0.928	0.703	1.000	0.856	1.067
1955-1958	75.1	1.089	1.078	1.294	1.096	1.151
FINLAND						
1866-1875	37.8	...	...	1.404	1.195	0.923
1876-1885	44.8	1.084	1.568	0.880	1.040	1.030
1886-1895	46.8	1.021	1.469	0.865	1.025	1.036
1896-1905	47.4	0.986	1.051	0.740	1.018	1.181
1906-1915	48.3	0.813	0.995	0.710	0.968	1.274
1916-1925	48.4	...	...	0.742	1.451	1.375
1926-1935	53.4	0.708	0.638	0.742	1.080	1.361
1937-1939	58.0	0.743	0.716	0.874	1.451	1.341
1946-1949	63.9	0.827	0.692	0.850	1.378	1.583
1955-1958	73.1	0.958	1.233	1.195	1.435	1.416
FRANCE						
1846-1855	40.5	1.181	1.173	0.937	1.125	0.858
1856-1865	42.0	1.157	1.210	0.887	1.027	0.884
1866-1875	41.2	1.249	1.119	0.889	1.133	0.961
1876-1885	44.9	1.322	1.104	0.903	1.025	0.951
1886-1895	47.6	1.403	1.212	0.970	1.048	0.960
1896-1905	50.0	1.232	0.907	0.932	1.034	1.040
1906-1913	52.4	1.063	0.843	0.933	1.029	1.117
1915-1918	48.6	1.015	0.686	0.879	1.804	1.097
1921-1925	55.4	0.838	1.115	0.916	1.024	0.113
1926-1935	57.6	0.802	0.680	0.903	0.988	1.189
1936-1939 (c)	62.0	0.842	0.727	0.974	1.008	1.235
1946-1949	67.8	1.287	1.093	1.014	1.024	1.074
1955-1958	73.7	1.349	1.338	1.138	1.284	1.280
GERMANY						
1906-1915	56.5	2.022	1.315	1.056	1.199	1.018
1922-1925	60	...	...	1.007	0.983	0.867
1926-1935	63.1	...	...	0.931	0.969	0.931
FEDERAL REPUBLIC OF GERMANY						
1938	65.5	...	0.899	1.078	1.070	0.913
1948	68.0	1.452	1.043	0.940	1.268	1.108
1955-1958	73.5	1.471	1.158	1.152	1.329	1.139
GERMAN DEMOCRATIC REPUBLIC						
1939	65.4	...	0.923	1.106	1.078	0.789
1949	66.1	1.483	0.754	0.999	1.185	1.193
1955-1957	72.1	1.570	1.375	1.062	1.155	1.095
BADEN						
1876-1885	47.0	2.042	1.190	1.146	0.971	0.843
1886-1895	48.2	2.052	1.148	1.150	0.937	0.837
1896-1905	49.7	1.786	0.930	1.062	0.811	0.929
BAVARIA						
1866-1875	46.4	3.212	1.390	1.196	1.063	0.852
1876-1885	49.1	2.911	1.386	1.130	1.000	0.900
1886-1895	49.6	2.504	1.553	1.148	0.936	0.886
1896-1905	53.3	2.615	1.391	1.158	0.921	0.945

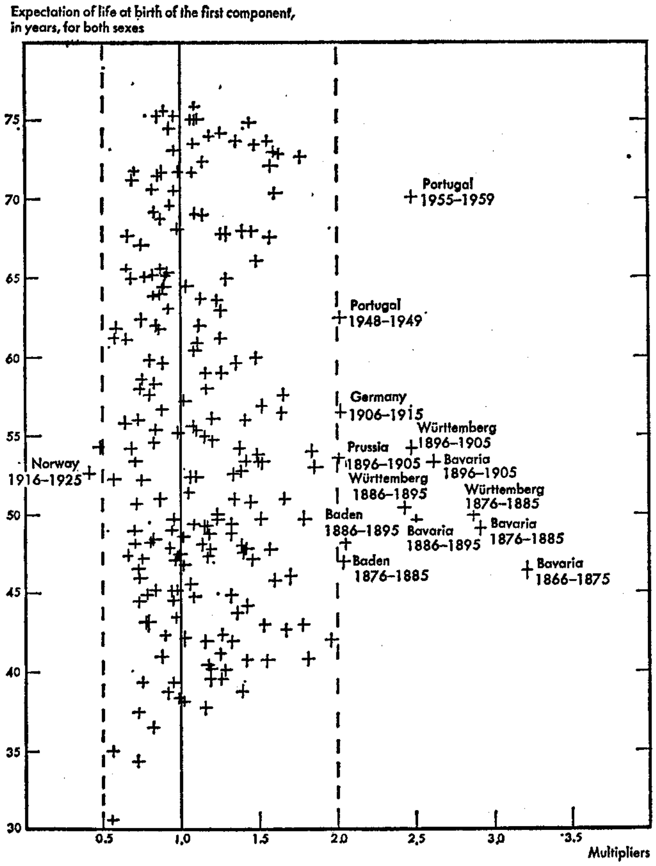
Country and dates	First component expectation of life at birth for both sexes (in years)	Multipliers: Fourth component (female) Under 1 year	Fourth component (female) 1-4 years	Second component (female) 45-64 years	Fifth component (male) 5-34 years	Fifth component (male) 45-64 years
PRUSSIA						
1876-1885	46.1	1.701	1.533	1.067	1.078	1.000
1886-1895	49.4	1.814	1.658	1.066	1.036	0.993
1896-1905	53.6	2.009	1.622	1.040	0.980	1.059
WÜRTEMTBERG						
1876-1885	49.7	2.976	1.516	1.218	0.997	0.823
1886-1895	50.4	2.431	1.225	1.189	0.969	0.832
1896-1905	54.2	2.476	1.211	1.157	0.934	0.903
GREECE						
1921-1925	46.0	0.741	1.709	0.739	1.101	0.121
1926-1935	49.7	0.960	1.511	0.715	0.997	1.231
1937-1938	56.1	1.199	1.412	0.707	1.005	1.319
1951	69.0	1.137	1.887	0.787	1.047	1.166
1955-1958	72.9	1.617	1.961	0.854	0.976	1.112
HUNGARY						
1896-1905	39.6	1.258	1.265	1.029	0.884	0.834
1906-1915	42.4	1.265	1.761	0.948	0.908	0.930
1919-1925	48.0	1.390	1.435	0.940	1.106	0.916
1926-1935	53.4	1.495	1.029	0.874	0.902	0.817
IRELAND						
1866-1875	49.0	0.708	1.000	0.939	1.070	0.824
1876-1885	47.2	0.754	0.864	1.063	0.997	0.677
1886-1895	46.6	0.736	0.799	1.053	0.959	0.775
1896-1905	48.4	0.828	0.832	1.080	0.957	0.766
1906-1915	51.4	1.051	0.866	1.195	0.933	0.753
IRELAND (EIRE)						
1926-1935	57.0	...	...	1.076	0.819	0.775
1936-1939	59.8	0.803	0.803	1.095	0.808	0.791
1946-1949	64.0	0.868	0.619	0.995	0.767	0.885
1955-1957	72.4	1.137	0.948	1.271	0.892	0.832
NORTHERN IRELAND						
1926-1935	58.2	0.826	0.888	1.313	0.844	0.683
1936-1939	62.0	1.117	1.078	1.279	0.905	0.772
1947-1949	68.1	0.975	0.811	1.188	0.816	0.885
1955-1958	74.9	1.433	1.179	1.532	1.024	0.832
ITALY						
1876-1885	40.8	1.417	1.921	0.928	0.963	0.896
1886-1895	43.8	1.360	2.033	0.883	0.915	0.896
1896-1905	47.8	1.190	1.860	0.856	0.889	0.939
1906-1915	51.0	1.345	1.917	0.834	1.002	0.958
1916-1925	48.2	...	...	0.755	0.964	0.972
1926-1935	58.0	...	...	0.863	0.918	0.994
1936	61.2	1.247	1.827	0.901	0.918	0.984
1948-1949	67.6	1.571	1.994	0.904	0.939	1.034
1955-1958	72.7	1.761	1.834	0.996	1.118	1.226
JAPAN						
1906-1915	42.2	1.025	0.940	0.830	0.845	1.079
1916-1925	38.4	0.995	0.788	0.810	0.878	1.153
1926-1935	45.2	0.843	0.866	0.828	0.910	1.223
1934-1936	44.5	0.735	0.849	0.821	1.033	1.403
1948-1949	55.7	0.646	0.954	0.872	1.062	1.022
1955-1959	68.8	0.872	1.168	1.037	1.032	1.092

Country and dates	First component expectation of life at birth for both sexes (in years)	Multipliers: Fourth component (female) Under 1 year	Fourth component (female) 1-4 years	Second component (female) 45-64 years	Fifth component (male) 5-34 years	Fifth component (male) 45-64 years
NETHERLANDS						
1846-1855	39.6	1.193	1.274	1.066	1.103	0.963
1856-1865	40.2	1.187	1.275	0.971	1.047	0.979
1866-1875	42.0	1.326	1.210	0.926	1.065	0.977
1876-1885	49.7	1.508	1.559	0.943	1.103	0.970
1886-1895	50.8	1.446	1.439	0.926	0.998	0.991
1896-1905	53.8	...	...	0.874	0.916	0.118
1906-1915	59.0	1.259	1.404	0.872	0.898	0.101
1916-1925	58.6	0.747	1.196	0.828	0.896	0.955
1926-1935	65.6	0.875	1.190	1.062	0.870	0.737
1937-1939	69.2	0.832	1.210	1.162	0.905	0.723
1946-1949	71.7	0.985	1.439	1.080	1.189	0.795
1955-1959	75.6	0.892	1.597	1.118	1.126	1.000
NEW ZEALAND						
1906-1915	61.8	0.859	0,706	0.934	0.978	1.048
1916-1925	61.8	0.588	0.676	0.912	1.073	0.939
1936-1939 (c)	67.7	0.663	0.739	1.044	0.941	0.933
1948-1949	71.8	0.702	0.897	1.133	0.962	0.967
1955-1959	75.3	0.954	1.444	1.305	1.434	1.005
NORWAY						
1876-1885	48.2	0.713	1.028	0.745	1.095	1.041
1886-1895	47.4	0.664	0.888	0.691	1.115	1.043
1896-1905	50.7	0.724	0.644	0.691	1.070	1.020
1906-1915	52.3	0.570	0.510	0.685	1.130	1.049
1916-1925	52.6	0.416	0.405	0.672	1.075	1.022
1926-1935	61.2	0.583	0.471	0.770	1.070	0.947
1936-1939	65.6	0.648	0.558	0.846	1.089	0.941
1946-1949	70.6	0.817	0.928	0.839	1.145	0.972
1955-1958	75.9	1.087	1.498	1.043	1.418	1.088
POLAND						
1927-1932	53.4	1.514	1.223	0.960	0.942	1.005
PORTUGAL						
1913-1915	53.0	1.855	2.143	0.898	1.109	1.119
1916-1925	44.2	1.422	1.658	0.818	1.159	1.189
1920-1935	54.0	1.837	1.444	0.744	1.082	1.303
1940	57.6	1.656	2.141	0.801	1.066	1.288
1948-1949	62.5	2.015	2.442	0.866	0.899	1.179
1955-1959	70.1	2.477	4.406	0.935	1.068	1.314
ROMANIA						
1896-1903	38.8	0.922	1.110	...	...	...
RUSSIA IN EUROPE						
1891-1900	40.9	1.816	1.805	1.110	1.039	0.795
1896-1905	42.1	1.959	2.279	1.087	1.019	0.845
SCOTLAND						
1856-1865	43.2	0.780	1.318	0.866	1.156	0.987
1866-1875	39.4	0.756	1.379	0.889	1.123	1.051
1876-1885	44.9	0.786	1.103	0.977	1.028	1.004
1886-1895	47.9	0.934	1.236	1.079	0.977	0.941
1896-1905	52.4	1.095	1.348	1.191	0.969	0.909
1906-1915	55.0	1.150	1.480	1.190	1.077	0.911
1916-1925	56.7	0.880	1.505	1.019	0.956	0.807
1926-1935	60.9	1.056	1.297	1.043	0.930	0.924
1936-1939	63.7	1.124	1.067	1.105	0.933	0.969
1946-1949	65.2	0.823	0.493	0.947	0.717	1.171
1955-1958	74.0	1.182	0.897	1.453	1.066	1.086
SERBIA						
1888-1895	30.6	0.556	0.780	1.044	0.996	0.794
1896-1905	35	0.566	0.902	1.083	0.870	0.793

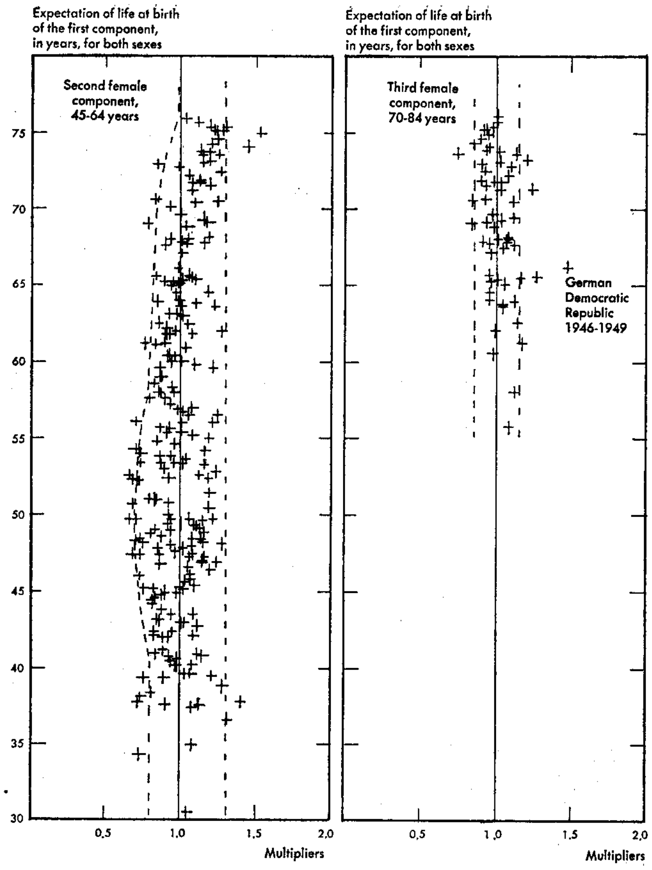
TABLE A.II.5 *(concluded)*

Country and dates	First component expectation of life at birth for both sexes (in years)	Multipliers: Fourth component (female) Under 1 year	Fourth component (female) 1-4 years	Second component (female) 45-64 years	Fifth component (male) 5-34 years	Fifth component (male) 45-64 years
SPAIN						
1878-1882	40.2	1.279	2.297	1.075	1.120	0.899
1900-1901	43.0	1.778	2.000	1.004	1.054	0.944
1911-1915	47.8	1.416	1.813	...	0.962	...
1916-1925	46.0	...	...	...	1.000	...
1926-1935	54.2	1.382	1.752	...	0.973	...
SWEDEN						
1756-1765	40.6	...	...	0.984	1.172	1.120
1766-1775	37.4	...	...	1.075	1.214	0.971
1776-1785	40.2	...	...	0.870	1.161	1.052
1786-1795	43.5	...	...	1.089	1.260	0.997
1796-1805	45.4	...	...	1.098	1.125	0.990
1806-1815	39.5	...	...	1.207	1.246	0.992
1816-1825	46.9	...	...	1.150	1.129	0.990
1826-1835	46.9	...	...	1.242	1.240	1.015
1836-1845	48.4	...	...	1.146	1.176	1.020
1846-1855	48.1	1.141	1.233	1.280	1.162	0.856
1856-1865	47.4	0.979	1.388	1.093	1.160	0.702
1866-1875	49.3	1.171	1.389	1.097	1.142	0.875
1876-1885	49.0	0.946	1.286	0.837	1.056	0.920
1886-1895	51.0	0.866	1.088	0.840	1.010	0.907
1896-1905	52.2	0.752	0.820	0.725	1.007	1.078
1906-1915	54.2	0.689	0.692	0.727	0.979	0.912
1916-1925	54.3	0.490	0.584	0.708	1.048	1.007
1926-1935	61.1	0.653	0.586	0.841	0.918	1.026
1936-1939	65.0	0.685	0.580	0,944	0.970	0.879
1946-1949	71.2	0.689	0.691	1.078	1.054	0.847
1955-1957	75.3	0.854	1.048	1.201	1.296	0.893
SWITZERLAND						
1876-1885	47.2	1.458	0.882	1.160	1.012	0.896
1886-1895	48.8	1.326	0.815	1.161	0.955	0.902
1896-1905	52.6	1.339	0.803	1.122	0.894	0.974
1906-1915	55.4	1.101	0.722	1.006	0.889	1.150
1916-1925	56.0	0.726	0,632	1.006	1.016	1.014
1926-1935	62.4	0,746	0.632	1,053	0.983	1.021
1937	65.1	0.770	0.755	0.998	0.988	1.075
1946-1949	71.7	0.881	0,905	1.080	1.050	1.024
1955-1958	75.1	1.097	1.508	1.245	1.614	1.045
UNITED STATES						
1936-1939	64.5	0.890	0.714	1.188	1.024	0.978
1946-1949	70.5	0.959	0.715	1.252	1.155	1.088
1955-1958	73.5	1.078	0.871	1.257	1.327	1.182
MASSACHUSETTS						
1906-1915	56.0	1.411	1.165	1.215	1.059	0.893
1918-1925	55.2	0.880	0.769	1.084	1.033	0.884
1926-1935	63.6	...	...	1.228	0.953	0.914
MICHIGAN						
1906-1912	55.6	1.082	0.814	0.933	0.942	0.874
1918-1925	54.6	0.828	0.615	0.965	0.958	0.835
1926-1935	61.8	...	...	1.080	0.994	0.925
YUGOSLAVIA						
1931-1935	47.1	0.965	0.977	...	...	...

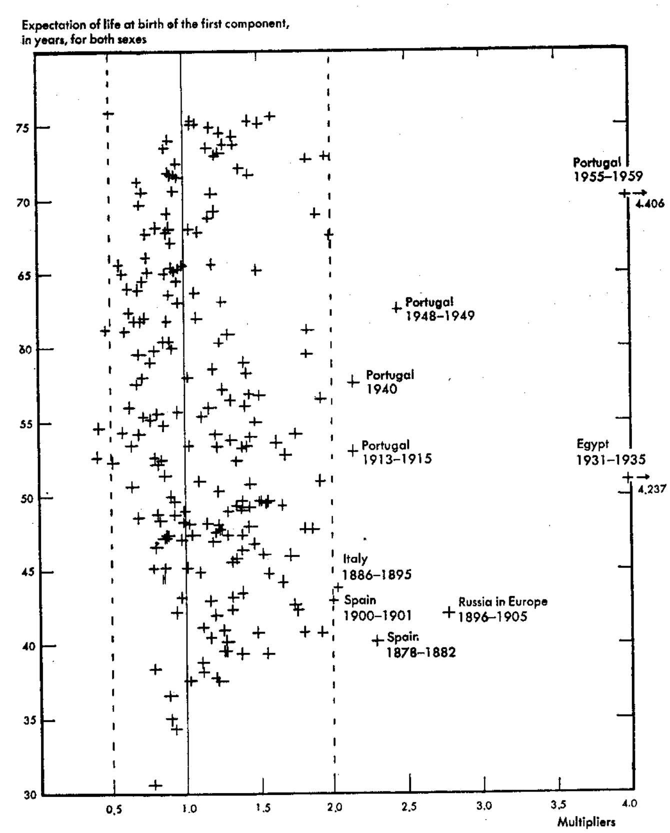
([a]) The frontiers of these countries or territories are those at the dates indicated in the first column.
([b]) Excluding the Province of Quebec.
([c]) Average of the three years 1936-1938 and 1939.



Graph A.II.1. Multipliers of the fourth female component under one year in forty-two countries or territories at various mortality levels



Graph A.II.3. Multipliers of the second female component between 45 and 64 years and of the third female component between 70 and 84 years in forty-two countries or territories at various mortality levels



Graph A.II.2. Multipliers of the fourth female component between 1 and 4 years in forty-two countries or territories at various mortality levels

Graph A.II.4 (left-hand side) relates to the fifth component between 5 and 34. Here, the orientation of the pattern of crosses is even more dependent than the preceding pattern on the expectation of life at birth of the first component. The band in which the crosses lie is markedly curved. The reason is that between 5 and 34 this fifth component is not a pure component so far as factor analysis is concerned.[6] Deaths by violence play a large part, and as there is little variation in such deaths over time their relative part in the general mortality depends on the level of the latter.

Graph A.II.4 (right-and side) is the graph of the fifth component between 45 and 64. Here, we again find a cluster whose direction is independent of the expectation of life at birth of the first component.

The third component presents special problems. It affects primarily advanced ages. The death rates at these ages are little known, however, particularly for the high mortalities generally observed in populations where the collection of data is defective. Ledermann and Breas found that, where a development over time could be traced in the third component, the variations were small. We therefore confined ourselves to a graph based on relatively recent observations in populations in Europe.

[6] Ledermann and Breas did not take their analysis beyond the third component. They simply outlined the calculation of the fourth and fifth components. The method of calculation adopted here for these two components is not, therefore, strictly speaking, a pure application of factor analysis.

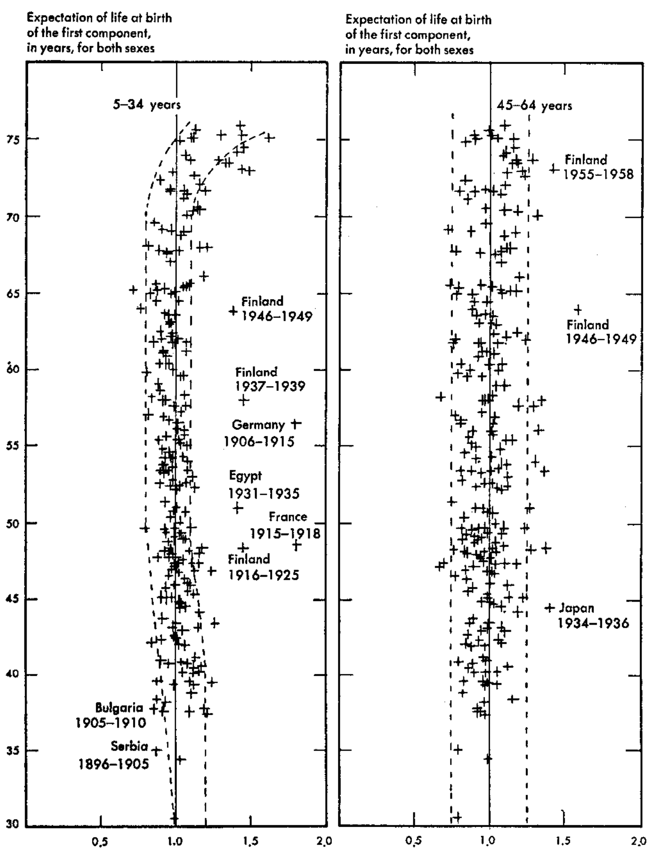
TABLE A.II.6. MAXIMUM UPWARD AND DOWNWARD DEVIATIONS DUE TO THE SECOND, THIRD, FOURTH AND FIFTH COMPONENTS AT VARIOUS MORTALITY LEVELS

(Multipliers for some age groups)

Level	Expectation of life at birth in years of the first component (both sexes)	Second female component between 45 and 64 years	Third female component between 70 and 84 years	Fourth female component under 1 year and between 1 and 4 years	Fifth male component at	
					5-34 years	45-64 years
			Upward deviation			
20	30	1.300	1.150	2.000	1.200	1.250
40	40	1.300	1.150	2.000	1.200	1.250
60	50	1.300	1.150	2.000	1.100	1.250
80	60.4	1.300	1.150	2.000	1.100	1.250
100	70.2	1.300	1.150	2.000	1.100	1.250
115	75.0	1.300	1.150	2.000	1.500	1.250
			Downward deviation			
20	30	0.800	0.850	0.500	1.000	0.750
40	40	0.800	0.850	0.500	0.900	0.750
60	50	0.700	0.850	0.500	0.800	0.750
80	60.4	0.800	0.850	0.500	0.800	0.750
100	70.2	0.850	0.850	0.500	0.800	0.750
115	75.0	1.000	0.850	0.500	1.000	0.750

or of European origin [7] (graph A.II.3 (right-hand side)). The result is a cluster of crosses of almost constant width situated between the vertical lines for 0.85 and 1.15. It was assumed that these multipliers were valid for all mortality levels.

[7] The multipliers of the third component used to construct this graph were taken from table 37 in the article on "Factor analysis of sex-age-specific death rates", published in *Population Bulletin of the United Nations* No. 6.



Graph A.II.4. Multipliers of the fifth component between 5 and 34 years and between 45 and 64 years in forty-two countries or territories at various mortality levels

The maximum and minimum deviations corresponding to each component for the age groups considered above and for six mortality levels of the first component (level determined by the expectation of life at birth of the first component) are shown in table A.II.6. The corresponding deviations by sex and five-year age groups are given in tables A.II.8 and A.II.9. For the second and third components, they were calculated on the basis of tables A.II.2 and A.II.3. For the fifth component, we have the six combinations in table A.II.7 with the six corresponding curves in graph A.II.5. For each combination, only the two points of the two age groups 5-34 and 45-64 are known. In the examples marked II, III, V and VI, these two points are sufficient to plot the curve without difficulty. The examples marked I and IV are more difficult. In these two cases we used the results of the fuller study of the fifth component in the article in the *Population Bulletin* referred to above.

TABLE A.II.7. MULTIPLIERS OF THE FIFTH COMPONENT FOR TWO AGE GROUPS AND VARIOUS MORTALITY LEVELS

Level	Expectation of life at birth of the first component (in years for both sexes)	Multiplier	
		5-34 years	45-64 years
	Upward deviation		
115	75.0	1.500	1.250
20 and 40 . . .	30.0 and 40.0	1.200	1.250
60, 80 and 100 .	50.0, 60.4 and 70.2	1.100	1.250
	Downward deviation		
20 and 115 . . .	30.0 and 75.0	1.000	0.750
40	40.0	0.900	0.750
60, 80 and 100 .	50.0, 60.4 and 70.2	0.800	0.750

TABLE A.II.8. MAXIMUM UPWARD DEVIATIONS DUE TO THE SECOND, THIRD, FOURTH AND FIFTH COMPONENTS FOR VARIOUS MORTALITY LEVELS

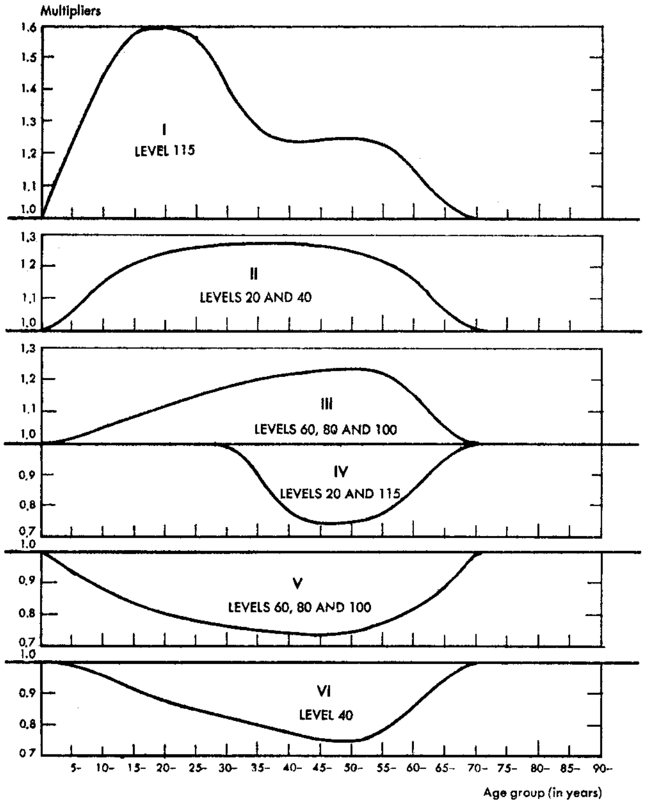
(Multipliers by five-year age groups)

Age group	*Second component*		*Third component*		*Fourth component*		*Fifth component*		
	All levels		*All levels*		*All levels*		*Level 115*	*Levels 20 and 40*	*Levels 60, 80 and 100*
	Male	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Male*	*Male*
0					2.000	2.000			
1-4					2.000	2.000			
5-9							1.220	1.020	1.050
10-14							1.450	1.050	1.150
15-19							1.580	1.080	1.200
20-24							1.600	1.120	1.250
25-29	1.060	1.042					1.550	1.150	1.250
30-34	1.132	1.075					1.400	1.180	1.250
35-39	1.201	1.126					1.280	1.200	1.250
40-44	1.279	1.198					1.240	1.220	1.250
45-49	1.336	1.282					1.240	1.230	1.250
50-54	1.384	1.309					1.250	1.250	1.250
55-59	1.405	1.321	1.009	1.024			1.230	1.230	1.250
60-64	1.375	1.285	1.021	1.045			1.150	1.150	1.150
65-69	1.315	1.234	1.044	1.066			1.050	1.050	1.050
70-74	1.261	1.192	1.072	1.107					
75-79	1.219	1.159	1.110	1.159					
80-84	1.150	1.099	1.158	1.186					

TABLE A.II.9. MAXIMUM DOWNWARD DEVIATIONS DUE TO THE SECOND, THIRD, FOURTH AND FIFTH COMPONENTS FOR VARIOUS MORTALITY LEVELS

(Multipliers by five-year age groups)

Age group (in years)	*Second component*						*Third component*		*Fourth component*		*Fifth component*		
											Levels		
	Level 60		*Level 100*		*Levels 20, 40 and 80*		*All levels*		*All levels*		*60, 80 100*	*40*	*20, 115*
	Male	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*		
0									0.500	0.500			
1-4									0.500	0.500			
5-9											0.94	0.99	
10-14											0.89	0.95	
15-19											0.84	0.91	
20-24											0.80	0.87	
25-29	0.940	0.958	0.970	0.979	0.960	0.972					0.79	0.85	
30-34	0.868	0.925	0.934	0.963	0.912	0.950					0.77	0.82	
35-39	0.799	0.874	0.900	0.937	0.866	0.916					0.76	0.80	0.90
40-44	0.721	0.802	0.860	0.901	0.814	0.868					0.75	0.78	0.75
45-49	0.664	0.718	0.832	0.859	0.776	0.812					0.75	0.75	0.75
50-54	0.616	0.691	0.808	0.846	0.744	0.794					0.76	0.75	0.75
55-59	0.595	0.679	0.798	0.840	0.730	0.786	0.991	0.976			0.78	0.78	0.75
60-64	0.625	0.715	0.813	0.857	0.750	0.810	0.979	0.955			0.84	0.85	0.85
65-69	0.685	0.766	0.842	0.883	0.790	0.844	0.956	0.934			0.88	0.95	0.95
70-74	0.739	0.808	0.869	0.904	0.826	0.872	0.928	0.893					
75-79	0.781	0.841	0.881	0.921	0.854	0.894	0.890	0.841					
80-84	0.850	0.901	0.925	0.950	0.900	0.934	0.842	0.814					



Graph A.II.5. Maximum and minimum values of the multiplier of the fifth component by five-year age groups for six mortality levels

G. Model life tables for extreme mortality levels

The maximum total effect of the second, third, fourth and fifth components on the first will be exerted when these components all operate to the maximum in the same direction, i.e., when either the multipliers in table A.II.8 or those in table A.II.9 are applied to the first component. In the first case the result will be a series of upward-deviating model life tables, and in the second case a series of downward-deviating model life tables. Tables A.II.10 to A.II.15 give the results of these calculations for the six mortality levels in table A.II.6. Viewed from this angle, the series of model life tables in the manual on methods of calculating population projections [8] appears as an intermediate model life table. We shall refer to it by that name from now on, in order to distinguish it from upward and downward-deviating model life tables. Each of the tables A.II.10 to A.II.15 provides the following information.

The last two column give, for the mortality level under consideration, the death rates (m_x) from the intermediate model life table. These rates are taken from table I in the annex to the aforementioned manual on methods for population projections by sex and age.

[8] *Manuals on methods of estimating population. Manual III. Methods for population projections by sex and age* (United Nations publication, Sales No.: 56.XIII.3).

TABLE AII.10. AGE-SPECIFIC DEATH RATES PER 1 000 (1 000 m_x) IN VARIOUS MODEL LIFE TABLES AT LEVEL 115

	Downward-deviating mortality				*Upward-deviating mortality*				*Mortality due to the first component*		*Intermediate model mortality*	
	Model mortality		*Norway 1958* (a)		*Model mortality*		*England and Wales* (a) *1958*					
Age group	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*
Under 1 year. . .	11.24	8.16	21.5	18.4	44.94	32.62	25.3	19.6	22.47	16.31	18.18	13.39
1-4	0.35	0.30	1.2	1.1	1.42	1.22	1.0	0.8	0.71	0.61	0.75	0.63
5-9	0.37	0.22	0.6	0.3	0.45	0.22	0.5	0.3	0.37	0.22	0.28	0.21
10-14	0.28	0.20	0.5	0.2	0.41	0.20	0.4	0.2	0.28	0.20	0.26	0.20
15-19	0.64	0.36	0.8	0.3	1.01	0.36	0.8	0.4	0.64	0.36	0.44	0.33
20-24	0.90	0.52	1.4	0.5	1.44	0.52	1.1	0.5	0.90	0.52	0.61	0.47
25-29	0.88	0.63	1.2	0.6	1.45	0.66	1.1	0.6	0.88	0.63	0.75	0.62
30-34	0.97	0.74	1.3	0.7	1.54	0.80	1.3	0.9	0.97	0.74	1.00	0.85
35-39	1.21	1.01	1.6	1.1	2.06	1.14	1.9	1.4	1.34	1.01	1.45	1.22
40-44	1.45	1.44	2.7	1.5	3.06	1.73	3.1	2.2	1.93	1.44	2.27	1.87
45-49	2.49	2.31	4.2	2.8	5.50	2.96	5.3	3.6	3.32	2.31	3.76	3.02
50-54	4.26	3.73	7.1	4.1	9.83	4.88	9.6	5.4	5.68	3.73	6.30	4.83
55-59	6.99	5.91	11.3	6.2	16.39	8.01	17.4	8.4	9.40	6.06	10.37	7.62
60-64	13.04	10.01	18.3	10.3	25.31	13.47	27.7	13.8	15.67	10.48	17.13	12.59
65-69	24.29	16.07	27.5	19.1	38.57	21.24	43.8	23.2	26.75	17.21	28.01	21.29
70-74	40.82	31.00	46.7	34.2	59.47	41.39	68.3	40.3	43.99	34.72	46.28	36.89
75-79	67.98	52.44	73.6	62.6	103.34	72.28	103.5	69.8	76.38	62.36	74.56	62.47
80-84	110.41	92.12	120.9	109.8	74.67	124.37	166.7	119.0	131.13	113.17	118.21	104.01
85 and over . . .	228.05	221.45	215.5	223.5	250.40	230.64	242.6	215.6	207.41	180.00	232.36	225.98
Average 5 to 34 years		0.44		0.43		0.49		0.48		0.45		0.45
Expectation of life at birth (in years)	74.049	77.356			66.942	73.118			72.537	75.156	72.646	75.191
Expectation of life at birth for both sexes	76.07				69.95				73.82		73.89	

(a) World Health Organization, *Annual Epidemiological and Vital Statistics 1958* (Geneva, 1961), p. 306.

The two columns immediately preceding the last two columns give the death rates of the first component at the same level as the intermediate model life table, i.e., having the same average for the female rates between 5 and 34 as for the intermediate model life table. A comparison of the intermediate model life table and the mortality due to the first component shows to what extent the two tables coincide. They coincide very closely except for level 20, i.e., for high mortalities. The reason for this is that, when the model life tables in the intermediate series were prepared, only a small number of life tables based on actual observations were available for high mortalities. Factor analysis has shown that these few tables were all of the upward-deviating type, with the result that the intermediate model mortality also tends to deviate upwards.

The multipliers in tables A.II.8 and A.II.9 were applied to the rates of this first component. Multiplying the rates of the first component by the multipliers in table A.II.9 we obtain a downward-deviating model life table, and multiplying the same rates by the multipliers in table A.II.8 we obtain an upward-deviating model life table. Next to each of these deviating model mortalities we show a mortality actually observed which is close to the model death rate. In the case of the tables with mortality at level 80, for instance, we show the mortalities for Norway in 1926-1935 and for British Guiana in 1953, which were used as examples at the beginning of this appendix to illustrate the deviations between actual death rates and intermediate model mortalities. We show in table A.II.12*bis*, as we did in table A.II.1, the downward-deviating and upward-deviating model mortality rates at level 80. A comparison of tables A.II.1 and A.II.12 *bis* shows that the rates observed in Norway in 1926-1935 and in British Guiana in 1953 are quite good representations of the upward-deviating and downward-deviating model life tables at level 80.

It should be remembered that each mortality level is "fixed" by the average of the female quinquennial rates (m_x) between 5 and 34; in other words *by definition*, this average is the same for the model life tables and the tables of mortality due to the first component. It is on this level, fixed in this way, that the components other than the first exert their influence. It will be noted that, between 5 and 34, for each model life table and first associated component only the averages are the same. The rates for each age group show small divergencies. The number attached to each level is that given in the series of model life tables (intermediate series) published in the manual on methods of calculating population projections.[9]

For other mortality levels it is not always possible to find such good examples. This is not surprising, because in order to calculate extreme model life tables we make the assumption that all the components other than the first operate in the same direction, whereas the opposite

[9] *Ibid.*

TABLE A.II.11. AGE-SPECIFIC DEATH RATES PER 1 000 (1 000 m_x) IN VARIOUS MODEL LIFE TABLES AT LEVEL 100

Age group	*Downward-deviating mortality*				*Upward-deviating mortality*				*Mortality due to the first component*		*Intermdiate model mortality*	
	Model mortality		*Norway 19[illegible] (a)*		*Model mortality*		*Finland 1953 (b)*					
	Male	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*
Under 1 year . . .	19.61	15.23	34.0(c)	23.6(c)	79.84	60.92	39.1	29.6	39.92	30.46	30.35	23.79
1-4	0.96	0.86	2.0(c)	1.6(c)	3.84	3.44	2.1	1.6	1.92	1.72	2.45	2.06
5-9	0.75	0.55	1.2	0.7	0.82	0.55	0.9	0.6	0.80	0.55	0.81	0.60
10-14	0.57	0.47	0.8	0.5	0.67	0.47	0.8	0.4	0.64	0.47	0.66	0.50
15-19	1.02	0.82	1.5	0.8	1.31	0.82	1.3	0.7	1.21	0.82	1.13	0.84
20-24	1.36	1.16	2.0	1.2	1.90	1.16	1.9	1.0	1.70	1.16	1.49	1.11
25-29	1.31	1.32	2.2	1.4	2.08	1.41	2.4	1.3	1.71	1.35	1.57	1.29
30-34	1.35	1.49	2.4	1.5	2.51	1.67	3.1	1.6	1.88	1.55	1.82	1.56
35-39	1.69	1.85	2.6	1.9	3.56	2.22	3.8	2.1	2.47	1.97	2.31	1.95
40-44	2.17	2.35	3.6	2.8	5.26	3.13	5.5	2.8	3.37	2.61	3.35	2.65
45-49	3.30	3.26	5.1	3.5	8.69	4.87	8.9	4.1	5.29	3.80	5.32	4.00
50-54	5.13	4.86	7.2	5.1	14.16	7.53	13.7	6.6	8.36	5.75	8.40	5.98
55-59	8.03	7.26	10.8	7.2	22.69	11.97	22.8	10.7	13.01	8.85	13.02	8.87
60-64	15.47	11.88	15.8	11.7	37.34	19.50	32.7	17.6	23.12	14.52	20.70	14.47
65-69	23.64	19.02	23.0	19.5	48.15	30.32	51.9	31.4	33.39	23.06	32.74	24.47
70-74	42.84	32.78	43.0	35.6	71.86	56.79	77.1	54.6	53.15	43.02	52.36	42.31
75-79	68.64	56.96	71.3	62.5	118.46	98.71	120.6	96.6	87.55	73.50	83.75	70.52
80-84	111.07	96.63	157.3	153.7	189.92	162.88	178.9	151.6	142.58	125.00	129.39	113.98
85 and over . . .	229.59	224.26			264.10	245.41	283.8	285.9	230.13	207.00	238.71	230.61
Average 5-34 years		0.97		1.02		1.01		0.97		0.98		0.98
Expectation of life at birth (in years)	72.730	74.540			61.003	66.870			68.092	71.523	68.555	71.803
Expectation of life at birth for both sexes	73.61				63.86				69.76		70.20	

(a) *Demographic Yearbook, 1957* (United Nations publication, Sales No.: 57.XIII.1), p. 308.
(b) *Ibid.*, p. 303.
(c) Estimated rates.

is more likely, since the components are independent. In other words, extreme mortalities are, by definition, a rare occurrence in actual practice. Indeed, we generally find that, if an actual death rate provides a good example of a downward-deviating or upward-deviating mortality above the age of 5, it does not provide a good example below that age, and *vice versa*. This is because the fourth component, on which mortality above the age of 5 depends a great deal, operates independently of the other components.

Thus, we now have four series of model life table age-specific death rates (m_x): (i) model life table upward-deviating death rates; (ii) model life table downward-deviating death rates; (iii) model life table death rates of the first component; (iv) intermediate model life table death rates.

The last two series are very similar, and we can retain only one of them. Theoretically, the death rates of the first component should have preference, but the use of the intermediate model life table is now so widespread that to replace it by the first component would lead to practical difficulties which the theoretical advantages of the first component would certainly not outweigh. Finally, we have the three upward-deviating, intermediate and downward-deviating series of model life tables. The various mortality functions corresponding to these three series and to the six mortality levels already considered are the subject of tables A.II.16 to A.II.19.

TABLE A.II.12. AGE-SPECIFIC DEATH RATES PER 1 000 (1 000 m_x) IN VARIOUS MODEL LIFE TABLES AT LEVEL 80

Age group	*Downward-deviating mortality*				*Upward-deviating mortality*				*Mortality due to the first component*		*Intermediate model mortality*	
	Model mortality		*Norway* [a] *1926-35*		*Model mortality*		*British Guiana* [b] *1953*					
	Male	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*
Under 1 year . . .	40.86	33.14	53.9	41.6	162.46	132.56	90.4	79.9	81.73	66.28	90.18	74.54
1-4	3.40	3.18	3.7	3.3	13.60	12.74	10.1	9.8	6.80	6.37	8.45	7.92
5-9	1.99	1.83	1.8	1.6	2.16	1.83	1.3	1.5	2.12	1.83	2.12	1.97
10-14	1.35	1.42	1.4	1.2	1.59	1.42	1.3	1.1	1.52	1.42	1.53	1.49
15-19	2.22	2.26	3.1	2.5	2.96	2.26	1.8	2.3	2.64	2.26	2.51	2.30
20-24	3.00	3.09	4.3	3.5	4.20	3.09	3.0	3.8	3.75	3.09	3.62	3.13
25-29	2.94	3.39	4.9	4.0	4.72	3.62	4.2	4.2	3.87	3.47	3.70	3.38
30-34	2.96	3.65	5.1	4.3	5.64	4.13	4.9	6.2	4.22	3.84	3.91	3.64
35-39	3.43	4.13	5.3	4.6	7.51	5.08	6.1	7.5	5.21	4.51	4.50	4.00
40-44	4.09	4.71	5.9	5.3	10.47	6.51	11.0	7.8	6.71	5.43	5.79	4.78
45-49	5.45	5.70	7.4	6.4	15.40	9.00	13.3	8.7	9.37	7.02	8.16	6.33
50-54	7.59	7.74	10.0	8.5	23.23	12.76	26.7	17.6	13.43	9.75	11.78	8.80
55-59	10.92	10.78	14.0	12.0	33.79	19.02	30.6	16.5	19.37	14.06	17.19	12.53
60-64	17.78	16.76	21.5	18.0	46.55	29.08	46.7	34.9	28.83	21.65	26.05	19.41
65-69	29.12	25.59	32.0	28.0	63.19	42.70	86.9	50.1	43.83	32.47	39.85	31.26
70-74	51.41	43.66	49.0	42.0	90.75	73.97	97.7	69.9	67.12	56.04	62.09	51.80
75-79	78.73	67.38	80.0	70.0	140.16	120.33	118.6	82.0	103.59	89.60	95.73	83.82
80-84	117.49	107.35	125.0	115.0	206.46	184.05	191.1	208.8	155.00	141.25	146.64	130.92
85 and over . . .	236.50	232.51	256.2	232.8	289.71	264.78	295.0	346.7	250.00	240.00	251.20	241.69
Average 5-34 years.		2.61		2.85		2.73		3.18		2.65		2.65
Expectation of life at birth (in years)	66.543	67.266			49.795	55.549			58.987	62.496	58.824	62.046
Expectation of life at birth for both sexes	66.90				52.60				60.48		60.40	

[a] *Le mouvement naturel de la population dans le monde de 1906 à 1936* (Paris, Les éditions de l'Institut national d'études démographique 1954), p. 106.

[b] *Demographic Yearbook 1957* (United Nations publication, Sales No.: 57.XIII.1), p. 296.

TABLE A.II.13. AGE-SPECIFIC DEATH RATES PER 1 000 (1 000 m_x) IN VARIOUS MODEL LIFE TABLES AT LEVEL 60

Age group	*Downward-deviating mortality*				*Upward-deviating mortality*				*Mortality due to the first component*		*Intermediate model mortality*	
	Model mortality		*Norway* [a] *1926-35*		*Model mortality*		*Sweden 1826-35*					
	Male	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*
Under 1 year. . .	66.59	56.21	58.9	47.2	266.84	224.83	206.36	173.92	133.19	112.42	161.16	136.41
1-4	7.94	7.68	6.9	6.2	31.78	30.70	29.25	26.98	15.89	15.35	16.44	16.19
5-9	3.79	3.85	2.7	2.8	4.11	3.85	6.2	5.7	4.03	3.85	3.98	3.99
10-14	2.41	2.79			2.85	2.79			2.71	2.79	2.70	2.96
15-19	3.74	4.45	6.8	5.9	4.81	4.45	6.9	5.9	4.45	4.45	4.29	4.38
20-24	5.06	5.94			7.09	5.94			6.33	5.94	6.22	5.97
25-29	4.94	6.24	7.7	6.7	8.11	6.78	11.7	8.9	6.65	6.51	6.45	6.45
30-34	4.84	6.49			9.69	7.55			7.25	7.02	6.84	6.80
35-39	5.20	6.83	7.0	6.7	12.35	8.79	17.1	12.6	8.57	7.81	7.70	7.28
40-44	5.72	7.08			16.49	10.58			10.57	8.83	9.50	8.15
45-49	6.83	7.58	9.9	8.9	22.54	13.54	26.1	18.1	13.72	10.56	12.49	10.06
50-54	8.62	9.59			31.87	18.17			18.42	13.88	16.95	13.22
55-59	11.65	12.70	18.5	15.3	44.16	25.92	43.6	33.4	25.32	19.16	23.52	18.05
60-64	18.56	19.33			58.32	38.01			36.11	28.30	33.83	26.79
65-69	30.32	29.18	41.1	35.1	75.91	53.67	83.5	71.9	52.64	40.81	49.82	41.19
70-74	53.92	48.39			106.27	88.47			78.60	67.02	75.35	65.38
75-79	80.70	72.59	105.4	93.0	157.10	137.89	173.2	155.5	116.11	102.67	112.56	102.30
80-84	121.27	112.46			225.60	199.92			169.37	153.43	169.30	154.48
85 and over . . .	249.92	239.06	248.0	231.0	323.45	287.87	340.2	331.11	285.48	260.00	270.72	259.56
Average 5-34 years.		4.96		5.13		5.23	6.83	6.83		5.09		5.09
Expectation of life at birth (in years)	60.745	60.067			38.391	43.809			49.595	52.401	48.721	51.308
Expectation of life at birth for both sexes	60.41				41.03				50.96		50.00	

([a]) See table A.II.12, foot-note ([a]).

([b]) *Statistiques internationales du mouvement de la population d'après les registres de l'état civil jusqu'en 1905* (Paris, Imprimerie nationale, 1907), p. 434.

TABLE A.II.12*bis*. AGE-SPECIFIC DEATH

Mortality level (or time-

Sex and age (x) in years	Level 0 (°e₀= 20)	Level 5 (°e₀= 22.5)	Level 10 (°e₀= 25)	Level 15 (°e₀= 27.5)	Level 20 (°e₀= 30)	Level 25 (°e₀= 32.5)	Level 30 (°e₀= 35)	Level 35 (°e₀= 37.5)	Level 40 (°e₀= 40)	Level 45 (°e₀= 42.5)	Level 50 (°e₀= 45)	Level 55 (°e₀= 47.5
MALES												
0	442.63	411.67	371.41	341.60	316.20	293.25	270.17	248.20	229.41	211.29	193.65	177.35
1- 4 . . .	77.96	67.11	58.27	50.89	44.99	39.73	35.13	30.97	27.55	24.31	21.44	18.85
5- 9 . . .	18.64	16.21	14.16	12.44	11.00	9.72	8.58	7.55	6.71	5.90	5.20	4.56
10-14 . . .	11.45	10.09	8.92	7.91	7.05	6.28	5.59	4.96	4.43	3.92	3.47	3.07
15-19 . . .	14.09	12.88	11.75	10.68	9.76	8.90	8.05	7.25	6.57	5.91	5.32	4.79
20-24 . . .	17.93	16.66	15.41	14.21	13.20	12.23	11.17	10.21	9.32	8.47	7.67	6.93
25-29 . . .	21.13	19.38	17.70	16.12	14.73	13.43	12.13	10.98	9.94	8.96	8.04	7.22
30-34 . . .	25.43	22.95	20.66	18.55	16.74	15.07	13.48	12.08	10.84	9.69	8.64	7.70
35-39 . . .	31.77	28.20	25.01	22.15	19.76	17.59	15.59	13.87	12.38	11.01	9.77	8.67
40-44 . . .	40.65	35.72	31.40	27.62	24.50	21.71	19.20	17.06	15.17	13.45	11.96	10.66
45-49 . . .	50.53	44.37	39.05	34.41	30.54	27.10	24.07	21.52	19.22	17.14	15.38	13.85
50-54 . . .	59.69	53.02	47.17	42.02	37.74	33.92	30.43	27.46	24.82	22.49	20.36	18.56
55-59 . . .	71.38	64.55	58.34	52.73	47.98	43.66	39.64	36.14	35.01	30.18	27.68	25.49
60-64 . . .	84.96	78.49	72.35	66.59	61.60	56.97	52.48	48.48	44.87	41.57	38.65	36.12
65-69 . . .	106.42	100.14	93.94	87.92	82.65	77.63	72.48	67.82	63.58	59.64	56.00	52.77
70-74 . . .	144.12	137.21	130.28	123.34	116.91	110.72	104.51	98.77	93.31	88.21	83.43	79.17
75-79 . . .	194.69	186.80	178.81	170.84	163.60	156.14	148.65	141.69	135.04	128.82	122.91	117.55
80-84 . .	274.44	264.86	254.17	243.76	234.75	225.25	215.65	206.60	198.10	190.09	182.54	175.67
85 + . . .	511.36	460.12	418.80	387.42	363.99	344.56	327.70	313.93	302.48	292.58	284.10	276.97
FEMALES												
0	396.43	365.17	335.65	309.22	283.41	259.87	239.81	220.56	202.22	184.42	167.81	151.94
1- 4 . . .	79.80	68.66	59.60	52.13	45.75	40.16	35.51	31.29	27.71	24.33	21.37	18.69
5- 9 . . .	19.44	16.90	14.76	12.96	11.41	10.02	8.85	7.79	6.89	6.03	5.29	4.61
10-14 . . .	13.10	11.54	10.19	9.03	8.02	7.10	6.31	5.59	4.98	4.39	3.87	3.40
15-19 . . .	15.97	14.59	13.29	12.09	10.93	9.86	8.92	8.02	7.21	6.42	5.70	5.02
20-24 . . .	19.91	18.50	17.10	15.77	14.35	13.02	11.88	10.85	9.73	8.67	7.72	6.83
25-29 . . .	23.98	21.98	20.07	18.28	16.52	14.89	13.46	12.17	10.82	9.56	8.38	7.37
30-34 . . .	28.30	25.52	22.96	20.61	18.40	16.38	14.64	13.11	11.56	10.14	8.91	7.81
35-39 . . .	33.17	29.44	26.09	23.10	20.40	17.96	15.92	14.15	12.41	10.83	9.51	8.34
40-44 . . .	37.24	32.75	28.82	25.36	22.29	19.54	17.29	15.36	13.55	11.90	10.50	9.27
45-49 . . .	42.25	37.18	32.79	28.95	25.59	22.61	20.11	17.99	16.01	14.22	12.67	11.30
50-54 . . .	49.17	43.80	39.06	34.86	31.03	27.62	24.82	22.42	20.10	18.02	16.30	14.70
55-59 . . .	57.84	52.49	47.57	43.08	38.86	35.03	31.86	29.08	26.36	23.88	21.74	19.81
60-64 . . .	72.71	67.28	62.15	57.29	52.49	48.07	44.34	41.00	37.64	34.56	31.76	29.18
65-69 . . .	93.87	88.49	83.15	77.92	72.47	67.35	62.99	58.99	54.81	50.94	47.43	44.21
70-74 . . .	129.46	123.41	117.29	111.22	104.86	98.83	93.42	88.36	83.13	78.25	73.71	69.49
75-79 . . .	183.54	176.33	168.86	161.47	153.68	145.74	138.89	132.45	125.61	119.19	113.20	107.63
80-84 . . .	261.12	251.93	242.11	232.48	222.14	211.79	202.85	194.50	185.42	176.94	169.01	161.57
85 + . . .	456.14	417.09	396.93	362.32	341.38	323.96	310.57	299.33	288.89	279.84	272.19	265.53

([a]) Equivalent values of $^{0}e_0$, shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

———————— First component modified to maximum extent *upwards* by the other components.

-- --------- First component modified to maximum extent *downwards* by the other components.

RATES (1 000 m_x) OF MODEL LIFE TABLES

reference in years) ([a])

Level 60 ($°e_0$= 50)	Level 65 ($°e_0$= 52.5)	Level 70 ($°e_0$= 55)	Level 75 ($°e_0$= 57.6)	Level 80 ($°e_0$= 60.4)	Level 85 ($°e_0$= 63.2)	Level 90 ($°e_0$= 65.8)	Level 95 ($°e_0$= 68.2)	Level 100 ($°e_0$= 70.2)	Level 105 ($°e_0$= 71.7)	Level 110 ($°e_0$= 73.0)	Level 115 ($°e_0$= 73.9)	Sex and age (x) in years
												MALES
161.16	145.05	129.59	110.85	90.18	70.10	52.51	39.13	30.35	24.55	20.73	18.18	. . . 0
16.44	14.13	12.10	10.18	8.45	6.79	5.20	3.6[illegible]	2.45	1.64	1.11	0.75	. . . 1- 4
3.98	3.44	2.96	2.51	2.12	1.76	1.45	1.15	0.81	0.56	0.39	0.28	. . . 5- 9
2.70	2.34	2.05	1.77	1.53	1.30	1.09	0.89	0.66	0.47	0.35	0.26	. . . 10-14
4.29	3.78	3.32	2.90	2.51	2.14	1.81	1.49	1.13	0 80	0.59	0.44	. . . 15-19
6.22	5.48	4.84	4.22	3.62	3.04	2,52	2.02	1.49	1.08	0.80	0.61	. . . 20-24
6.45	5.68	4.97	4.30	3.70	3.14	2.59	2.08	1.57	1.18	0.92	0.75	. . . 25-29
6.84	6.00	5.24	4.55	3.91	3.33	2.79	2.29	1.82	1.43	1.18	1.00	. . . 30-34
7.70	6.77	5.94	5.19	4.50	3.88	3.31	2.79	2.31	1.90	1.63	1.45	. . . 35-39
9.50	8.41	7.45	6.58	5.79	5.07	4.45	3.87	3.35	2.88	2.52	2.27	. . . 40-44
12.49	11.22	9.97	9.07	8.16	7.33	6.61	5.92	5.32	4.71	4.17	3.76	. . . 45-49
16.95	15.43	14.08	12.85	11.78	10.81	9.96	9.14	8.40	7.67	6.95	6.30	. . . 50-54
23.52	21.65	20.00	18.53	17.19	15.98	14.93	13.99	13.02	12.08	11.22	10.37	. . . 55-59
33.83	31.58	29.53	27.69	26.05	24.59	23.24	21.96	20.70	19.42	18.22	17.13	. . . 60-64
49.82	46.92	44.32	42.02	39.85	37.92	36.13	34.42	32.74	31.02	29.46	28.01	. . . 65-69
75.35	71.61	68.14	65.01	62.09	59.41	56.94	54.60	52.36	50.13	48.10	46.28	. . . 70-74
112.56	107.64	103.28	99.39	95.73	92.40	89.26	86.32	83.35	80.36	77.33	74.56	. . . 75-79
169.30	162.98	157.07	151.67	146.64	141.99	137.74	133.68	129.39	126.39	122.42	118.21	. . . 80-84
270.72	264.90	259.83	255.32	251.20	247.52	244.26	241.34	238.71	236.36	234.24	232.36	. . . 85 —
												FEMALES
136.41	121.72	107.62	91.92	74.54	57.57	42.65	31.27	23.79	18.84	15.57	13.39	. . . 0
16.19	13.87	11.65	9.80	7.92	6.14	4.55	3.09	2.06	1.38	0.93	0.63	. . . 1- 4
3.99	3.41	2.90	2.42	1.97	1.56	1.18	0.85	0.60	0.42	0.29	0.21	. . . 5- 9
2.96	2.55	2.17	1.82	1.49	1.18	0.91	0.67	0.50	0.36	0.26	0.20	. . . 10-14
4.38	3.79	3.27	2.77	2.30	1.86	1.45	1.10	0.84	0.60	0.44	0.33	. . . 15-19
5.97	5.19	4.46	3.76	3.13	2.54	1.99	1.49	1.11	0.80	0.60	0.47	. . . 20-24
6.45	5.58	4.80	4.06	3.28	2.73	2.19	1.70	1.29	0.97	0.76	0.62	. . . 25-29
6.80	5.89	5.09	4.34	3.64	3.00	2.47	1.97	1.56	1.23	1.00	0.85	. . . 30-34
7.28	6.31	5.48	4.70	4.00	3.37	2.84	2.35	1.95	1.60	1.38	1.22	. . . 35-39
8.15	7.15	6.29	5.49	4.78	4.14	3.58	3.06	2.65	2.31	2.04	1.87	. . . 40-44
10.06	8.95	8.00	7.11	6.33	5.62	5.02	4.45	4.00	3.62	3.28	3.02	. . . 45-49
13.22	11.91	10.78	9.74	8.80	7.94	7.20	6.51	5.98	5.51	5.15	4.83	. . . 50-54
18.05	16.48	15.05	13.71	12.53	11.45	10.45	9.52	8.87	8.33	7.92	7.62	. . . 55-59
26.79	24.70	22.82	21.04	19.41	17.89	16.58	15.33	14.47	13.78	13.10	12.59	. . . 60-64
41.19	38.47	35.91	33.46	31.26	29.21	27.41	25.71	24.47	23.39	22.33	21.29	. . . 65-69
65.38	61.59	58.10	54,80	51.80	48.96	46.46	44.08	42.31	40.53	38.68	36.89	. . . 70-74
102.30	97.39	92.63	88.03	83.82	79.85	76.31	72.96	70.52	68.24	65.40	62.47	. . . 75-79
154.48	147.96	141.94	136.17	130.92	125.92	121.48	117.21	113.98	110.99	107.61	104.01	. . . 80-84
259.56	254.36	249.76	245.57	241.69	238.20	235.22	232.54	230.61	228.97	227.41	225.98	. . . 85 +

TABLE A.II.14. AGE-SPECIFIC DEATH RATES PER 1 000 (1 000 m_x) IN VARIOUS MODEL LIFE TABLES AT LEVEL 40

Age group	*Downward-deviating mortality*				*Upward-deviating mortality*				*Mortality due to the first component*		*Intermediate model mortality*	
	Model mortality		*Scotland* [a] *1856-65*		*Model mortality*		*Finland* [b] *1866-75*					
	Male	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*
Under 1 year . . .	98.05	85.89	153.7	128.0	394.60	343.54	236.0 (c)	203.0 (c)	197.30	171.77	559.42	202.22
1-4	15.54	15.38	38.7	38.5	62.18	61.54	56.8 (c)	54.9 (c)	31.09	30.77	27.12	27.71
5-9	6.59	7.05	7.4	7.4	6.99	7.05	12.4	11.3	6.66	7.05	6.71	6.89
10-14	4.02	4.84			4.86	4.84			4.23	4.84	4.43	4.98
15-19	6.07	7.55	8.7	7.4	8.00	7.55	9.0	8.0	6.67	7.55	6.57	7.21
20-24	8.30	9.89			11.93	9.89			9.54	9.89	9.32	9.73
25-29	8.29	10.39	10.6	9.4	13.46	11.07	12.1	11.4	10.16	10.62	9.94	10.82
30-34	8.23	10.69			15.58	12.09			11.01	11.25	10.84	11.56
35-39	8.76	11.00	12.6	11.4	18.97	13.52	18.6	17.0	12.64	12.01	12.38	12.41
40-44	9.64	11.22			24.27	15.49			15.18	12.93	15.17	13.55
65-49	10.77	11.81	17.5	14.4	30.90	18.64	31.1	24.3	18.50	14.54	19.22	16.01
60-54	13.08	14.52			40.55	23.94			23.44	18.29	24.82	20.10
75-59	17.57	18.74	29.1	24.4	55.26	33.05	55.7	45.9	31.17	24.43	33.01	26.36
70-64	26.92	27.06			69.67	46.95			43.14	34.96	44.87	37.64
85-69	43.64	38.53	63.4	54.5	87.76	64.30	11.9	98 8	60.86	48.90	63.58	54.81
0-74	68.27	60.19			120.50	101.98			89.13	77.26	93.31	83.13
5-79	96.63	86.04	138.0	125.7	172.02	153.67	214.1	193.3	127.14	114.42	135.04	125.61
0-84	135.68	124.52			238.43	213.48			179.00	163.84	198.10	185.42
85 and over . . .	257.57	253.73	312.9	283.1	371.97	320.69	342.9	346.6	306.07	245.56	302.48	288.89
Average 5-34 years .		8.40		8.07		8.75		10.27		8.53		8.53
Expectation of life at birth (in years)	50.656	50.392			26.876	30.924			40.054	41.994	39.243	40.7
Expectation of life at birth for both sexes	50.53				28.85				41.00		40.00	

(a) See table A.II.13, foot-note (b), p. 431.
(b) *Ibid.*, p. 434.
(c) Estimated rates.

TABLE A.II.15. AGE-SPECIFIC DEATH RATES PER 1 000 (1 000 m_x) IN VARIOUS MODEL LIFE TABLES AT LEVEL 20

Age group	*Downward-deviating mortality*				*Upward-deviating mortality*				*Mortality due to the first component*		*Intermediate model mortality*	
	Model mortality		*Bulgaria, 1888-95*		*Model mortality*		*Mauritius* ([a]), *1942-46*					
	Male	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*
Under 1 year . . .	139.55	124.76	195.3	168.6	538.20	499.02	220	219	279.10	249.51	316.20	283.41
1-4	27.99	28.32	48.3	46.7	111.96	113.26	32	35	55.98	56.63	44.99	45.75
5-9	102.10	11.80	12.5	12.0	10.72	11.80	6	6	10.21	11.80	11.00	11.41
10-14	6.17	7.71			7.10	7.71	5	5	6.17	7.71	7.05	8.02
15-19	9.41	11.83	10.4	10.7	11.29	11.83	10	13	9.41	11.83	9.76	10.93
20-24	13.53	15.29			16.91	15.29	15	21	13.53	15.29	13.20	14.35
25-29	14.01	15.79	11.0	14.2	19.33	16.83	18	21	14.59	16.15	14.73	16.52
30-34	14.45	15.99			22.41	18.09	21	21	15.84	16.83	16.74	18.40
35-39	13.73	15.88	12.2	13.6	26.44	19.52	24	21	17.62	17.34	19.76	20.40
40-44	12.59	15.54			32.96	21.44	32	23	20.61	17.90	24.50	22.29
45-49	13.90	15.50	17.4	15.5	39.90	24.47	42	26	23.89	19.09	30.54	25.59
50-54	16.24	18.38			50.30	30.30	53	33	29.10	23.15	37.74	31.03
55-59	20.21	22.52	27.1	24.5	66.12	39.72	72	44	37.29	29.36	47.98	38.86
60-64	31.36	32.43			81.15	56.27	96	63	50.25	41.90	61.60	52.49
65-69	49.43	45.02	45.6	45.1	99.41	75.43	124	88	68.94	57.13	82.65	72.47
70-74	75.52	68.04			133.29	115.29	155	126	98.59	87.34	116.91	104.86
75-79	104.48	94.48	67.5	67.7	186.00	168.73	197	133	137.47	125.64	163.60	153.68
80-84	142.17	131.74			249.83	225.86	232	185	187.56	173.34	234.75	222.14
85 and over . . .	270.64	268.62	83.5	90.5	454.82	375.68			325.03	266.39	363.99	341.38
Average 5-34 years.		13.07		12.3		13.59		14.5		13.27		13.27
Expectation of life at birth (in years)	42.584	41.172			15.797	18.196			30.393	31.290	29.575	30.402
Expectation of life at birth for both sexes	41.89				16.97				30.83		29.98	

([a]) Estimate based on the 1942-1946 life table, *Demographic Yearbook 1951* (United Nations publication, Sales No.: 52.XIII.1).

TABLE A.II.16. AGE-SPECIFIC DEATH RATES (1 000 m_x) FOR SIX MORTALITY LEVELS IN FOUR SERIES OF MODEL LIFE TABLES ([a])

	Level 20 *Expectation of life at birth in years*				*Level 40* *Expectation of life at birth in years*			
Both sexes . . .	41.89	30.83	29.98	16.97	50.81	41.00	40.00	28.85
Male.	42.58	30.39	29.57	15.80	51.21	40.05	39.24	26.88
Female.	41.17	31.29	30.40	18.20	50.39	41.99	40.74	30.92
Sex and age group (in years)	*First component modified* ([a]) *downwards*	*First component*	*Model table* ([a])	*First component modified* ([a]) *upwards*	*First component modified* ([a]) *downwards*	*First component*	*Model table* ([a])	*First component modified* ([a]) *upwards*
MALE								
0	139.55	279.10	316.20	558.20	98.65	197.30	229.41	394.60
1-4	27.99	55.98	44.99	111.96	15.54	31.09	27.55	62.18
5-9	10.21	10.21	11.00	10.72	6.59	6.66	6.71	6.99
10-14	6.17	6.17	7.05	7.10	4.02	4.23	4.23	4.86
15-19	9.41	9.41	9.76	11.29	6.07	6.67	6.57	8.00
20-24	13.53	13.53	13.20	16.91	8.30	9.54	9.32	11.93
25-29	14.01	14.59	14.73	19.33	8.29	10.16	9.94	13.46
30-34	14.45	15.84	16.74	22.41	8.23	11.01	10.84	15.58
35-39	13.73	17.62	19.76	26.44	8.76	12.64	12.38	18.97
40-44	12.59	20.61	24.50	32.96	9.64	15.18	15.17	24.27
45-49	13.90	23.89	30.54	39.90	10.77	18.50	19.22	30.90
50-54	16.24	29.10	37.74	50.30	13.08	23.44	24.82	40.55
55-59	20.21	37.29	47.98	66.12	17.57	31.17	33.01	55.26
60-64	31.36	50.25	61.60	81.15	26.92	43.14	44.87	69.67
65-69	49.43	68.94	82.65	99.41	43.64	60.86	63.58	87.76
70-74	75.52	98.59	116.91	133.29	68.27	89.13	93.31	120.50
75-79	104.48	137.47	163.60	186.00	96.63	127.14	135.04	172.02
80-84	142.17	187.56	234.75	249.83	135.68	179.00	198.10	238.43
85 + ...	270.64	325.03	363.99	454.82	256.79	306.07	302.48	371.77

	Level 60 *Expectation of life at birth in years*				*Level 80* *Expectation of life at birth in years*			
Both sexes . . .	60.41	50.96	50.00	41.03	66.90	60.48	60.40	52.60
Male.	60.74	49.60	48.72	38.39	66.54	58.99	58.82	49.79
Female.	60.07	52.40	51.31	43.81	67.27	62.50	62.05	55.55
Sex and age group (in years)	*First component modified* ([a]) *downwards*	*First component*	*Model table* ([a])	*First component modified* ([a]) *upwards*	*First component modified* ([a]) *downwards*	*First component*	*Model table* ([a])	*First component modified* ([a]) *upwards*
MALE								
0	66.59	133.19	161.16	266.38	40.86	81.73	90.18	162.46
1-4	7.94	15.89	16.44	31.78	3.40	6.80	8.45	13.60
5-9	3.79	4.03	3.98	4.11	1.99	2.12	2.12	2.16
10-14	2.41	2.71	2.70	2.85	1.35	1.52	1.53	1.59
15-19	3.74	4.45	4.29	4.81	2.22	2.64	2.51	2.96
20-24	5.06	6.33	6.22	7.09	3.00	3.75	3.62	4.20
25-29	4.94	6.65	6.45	8.11	2.94	3.87	3.70	4.72
30-34	4.84	7.25	6.84	9.69	2.96	4.22	3.91	5.64
35-39	5.20	8.57	7.70	12.35	3.43	5.21	4.50	7.51
40-44	5.72	10.57	9.50	16.49	4.09	6.70	5.79	10.47
45-49	6.83	13.72	12.49	22.54	5.45	9.37	8.16	15.40
50-54	8.62	18.42	16.95	31.87	7.59	13.43	11.78	23.23
55-59	11.65	25.32	23.52	44.16	10.92	19.37	17.19	33.79
60-64	18.56	36.11	33.83	58.32	17.78	28.83	26.05	46.55
65-69	30.32	52.64	49.82	75.91	29.12	43.83	39.85	63.19
70-74	53.92	78.60	75.35	106.27	51.41	67.12	62.09	90.75
75-79	80.70	116.10	112.56	157.10	78.73	103.59	95.73	140.16
80-84	121.27	169.40	169.30	225.60	117.49	155.00	146.64	206.46
85 + ...	240.92	285.50	270.72	323.45	236.50	265.52	251.20	289.71

TABLE A.II.16 (*continued*)

	Level 100 Expectation of life at birth in years				Level 115 Expectation of life at birth in years			
Both sexes	73.61	69.76	70.20	63.86	76.07	73.82	73.89	69.95
Male	72.73	68.09	68.56	61.00	74.85	72.54	72.65	66.94
Female	74.54	71.52	71.80	66.87	77.36	75.16	75.19	73.12
Sex and age group (in years)	*First component modified* ([a]) *downwards*	*First component*	*Model table* ([a])	*First component modified* ([a]) *upwards*	*First component modified* ([a]) *downwards*	*First component*	*Model table* ([a])	*First component modified* ([a]) *upwards*
MALE								
0	19.61	39.92	30.35	79.84	11.24	22.47	18.18	44.94
1-4	0.96	1.92	2.45	3.84	0.35	0.71	0.75	1.42
5-9	0.75	0.80	0.81	0.82	0.37	0.37	0.28	0.45
10-14	0.57	0.64	0.66	0.67	0.28	0.28	0.26	0.41
15-19	1.02	1.21	1.13	1.31	0.64	0.64	0.44	1.01
20-24	1.36	1.70	1.49	1 90	0.90	0.90	0.61	1.44
25-29	1.31	1.71	1.57	2.08	0.88	0.88	0.75	1.45
30-34	1.35	1.88	1.82	2.51	0.97	0.97	1.00	1.54
35-39	1.69	2.47	2.31	3.56	1.21	1.34	1.45	2.06
40-44	2.17	3.37	3.35	5.26	1.45	1.93	2.27	3.06
45-49	3.30	5.29	5.32	8.69	2.49	3.32	3.76	5.50
50-54	5.13	8.36	8.40	14.16	4.26	5.68	6.30	9.83
55-59	8.03	13.01	13.02	22.69	6.99	9.40	10.37	16.39
60-64	15.47	23.12	20.70	37.34	13.04	15.67	17.13	25.31
65-69	23.64	33.39	32.74	48.15	24.29	26.75	28.01	38.57
70-74	42.84	53.15	52.36	71.86	40.82	43.99	46.28	59.47
75-79	68.64	87.55	83.35	118.46	67.98	76.38	74.56	103.34
80-84	111.04	142.58	129.39	189.92	110.41	131.13	118.21	174.67
85 +	229.59	230.13	238.71	264.10	228.05	207.41	232.36	250.40

	Level 20 Expectation of life at birth in years				Level 40 Expectation of life at birth in years			
Both sexes	41.89	30.83	29.98	16.97	50.81	41.00	40.00	28.85
Male	42.58	30.39	29.57	15.80	51.21	40.05	39.24	26.88
Female	41.17	31.29	30.40	18.20	50.39	41.99	40.74	30.92
Sex and age group (in years)	*First component modified* ([a]) *downwards*	*First component*	*Model table* ([a])	*First component modified* ([a]) *upwards*	*First component modified* ([a]) *downwards*	*First component*	*Model table* ([a])	*First component modified* ([a]) *upwards*
FEMALE								
0	124.76	249.51	283.41	499.02	85.89	171.77	202.22	343.54
1-4	28.32	56.63	45.75	113.26	15.38	30.77	27.71	61.54
5-9	11.80	11.80	11.41	11.80	7.05	7.05	6.89	7.05
10-14	7.71	7.71	8.02	7.71	4.84	4.84	4.98	4.84
15-19	11.83	11.83	10.93	11.83	7.55	7.55	7.21	7.55
20-24	15.29	15.29	14.35	15.29	9.89	9.89	9.73	9.89
25-29	15.79	15.15	16.52	16.83	10.39	10.62	10.82	11.07
30-34	15.99	16.83	18.40	18.09	10.69	11.25	11.56	12.09
35-39	15.88	17.34	20.40	19.52	11.00	12.01	12.41	13.52
40-44	15.54	17.90	22.29	21.44	11.22	12.93	13.55	15.49
45-49	15.50	19.09	25.59	24.47	11.81	14.54	16.01	18.64
50-54	18.38	23.15	31.03	30.30	14.52	18.29	20.10	23.94
55-59	22.52	29.36	38.86	39.72	18.74	24.43	26.36	33.05
60-64	32.43	41.90	52.49	56.27	27.06	34.96	37.64	46.95
65-69	45.02	57.13	72.47	75.13	38.53	48.90	54.81	64.30
70-74	68.04	87.34	104.86	115.29	60.19	77.26	83.13	101.98
75-79	94.48	125.64	153.68	168.73	86.04	114.42	125.61	153.67
80-84	131.74	173.34	222.14	225.86	124.52	163.84	185.42	213.48
85 +	268.62	266.39	341.38	375.68	253.73	245.56	288.89	320.69

TABLE A.II.16 *(concluded)*

	Level 60 Expectation of life at birth in years				*Level 80 Expectation of life at birth in years*			
Both sexes	60.41	50.96	50.00	41.03	66.90	60.48	60.40	52.60
Male	60.74	49.60	48.72	38.39	66.54	58.99	58.82	49.79
Female	60.07	52.40	51.31	43.81	67.27	62.50	62.05	55.55
Sex and age group (in years)	*First component modified (a) downwards*	*First component*	*Model table (a)*	*First component modified (a) upwards*	*First component modified (a) downwards*	*First component*	*Model table (a)*	*First component modified (a) upwards*
FEMALE								
0	56.21	112.42	136.41	224.84	33.14	66.28	74.54	132.56
1-4	7.68	15.35	16.19	30.70	3.18	6.37	7.92	12.74
5-9	3.85	3.85	3.99	3.85	1.83	1.83	1.97	1.83
10-14	2.79	2.79	2.96	2.79	1.42	1.42	1.49	1.42
15-19	4.45	4.45	4.38	4.45	2.26	2.26	2.30	2.26
20-24	5.94	5.94	5.97	5.94	3.09	3.09	3.13	3.09
25-29	6.24	6.51	6.45	6.78	3.39	3.47	3.38	3.62
30-34	6.49	7.02	6.80	7.55	3.65	3.84	3.64	4.13
35-39	6.83	7.81	7.28	8.79	4.13	4.51	4.00	5.08
40-44	7.08	8.83	8.15	10.58	4.71	5.43	4.78	6.51
45-49	7.58	10.56	10.06	13.54	5.70	7.02	6.33	9.00
50-54	9.59	13.88	13.22	18.17	7.74	9.75	8.80	12.76
55-59	12.70	19.16	18.05	25.92	10.78	14.06	12.53	19.02
60-64	19.32	28.30	26.79	38.01	16.76	21.65	19.41	29.08
65-69	29.18	40.81	41.19	53.67	25.59	32.47	31.26	42.70
70-74	48.39	67.02	65.38	88.47	43.66	56.04	51.80	73.97
75-79	72.59	102.67	102.30	137.89	67.38	89.60	83.82	120.33
80-84	112.46	153.43	154.48	199.92	107.35	141.25	130.92	184.05
85 +	239.06	259.00	259.56	287.87	232.51	260.76	241.69	264.78

	Level 100 Expectation of life at birth in years				*Level 115 Expectation of life at birth in years*			
Both sexes	73.61	69.76	70.20	63.86	76.07	73.82	73.89	69.95
Male	72.73	68.09	68.56	61.00	74.85	72.54	72.65	66.94
Female	74.54	71.52	71.80	66.87	77.36	75.16	75.19	73.12
Sex and age group (in years)	*First component modified (a) downwards*	*First component*	*Model table (a)*	*First component modified (a) upwards*	*First component modified (a) downwards*	*First component*	*Model table (a)*	*First component modified (a) upwards*
FEMALE								
0	15.23	30.46	23.79	60.92	8.16	16.31	13.39	32.62
1-4	0.86	1.72	2.06	3.44	0.30	0.61	0.63	1.22
5-9	0.55	0.55	0.60	0.55	0.22	0.22	0.21	0.22
10-14	0.47	0.47	0.50	0.47	0.20	0.20	0.20	0.20
15-19	0.82	0.82	0.84	0.82	0.36	0.36	0.33	0.36
20-24	1.16	1.16	1.11	1.16	0.52	0.52	0.47	0.52
25-29	1.32	1.35	1.29	1.41	0.63	0.63	0.62	0.66
30-34	1.49	1.55	1.56	1.67	0.74	0.74	0.85	0.80
35-39	1.85	1.97	1.95	2.22	1.01	1.01	1.22	1.14
40-44	2.35	2.61	2.65	3.13	1.44	1.44	1.87	1.73
45-49	3.26	3.80	4.00	4.87	2.31	2.31	3.02	2.96
50-54	4.86	5.75	5.98	7.53	3.73	3.73	4.83	4.88
55-59	7.26	8.85	8.87	11.97	5.91	6.06	7.62	8.01
60-64	11.88	14.52	14.47	19.50	10.01	10.48	12.59	13.47
65-69	19.02	23.06	24.47	30.32	16.07	17.21	21.29	21.24
70-74	32.78	43.02	42.31	56.79	31.00	34.72	36.89	41.39
75-79	56.96	73.50	70.52	98.71	52.44	62.36	62.67	72.28
80-84	96.63	125.00	113.98	162.88	92.12	113.17	104.01	124.37
85 +	186.30	230.00	230.01	264.50	157.58	193.59	225.98	230.94

(a) The series in these tables are designated respectively by the expressions: model life tables (downward-deviating series), model life tables (first component), model life tables (intermediate series), model life tables (upward-deviating series).

TABLE A.II.17. AGE-SPECIFIC PROBABILITIES OF DEATH (1 000 q_x) FOR SIX MORTALITY LEVELS IN THREE SERIES OF MODEL LIFE TABLES [a]

	Level 20 Expectation of life at birth in years			Level 40 Expectation of life at birth in years			Level 60 Expectation of life at birth in years		
Both sexes . . .	4189	29.98	16.97	50.81	40.00	28.85	60.41	50.00	41.03
Male	42.58	29.57	15.80	51.21	39.24	26.88	60.74	48.72	38.39
Female	41.17	30.40	18.20	50.39	40.74	30.92	60.07	51.31	43.81
Sex and age (in years)	*First component modified down-wards* [a]	*Model table* [a]	*First component modified upwards* [a]	*First component modified down-wards* [a]	*Model table* [a]	*First component modified upwards* [a]	*First component modified down-wards* [a]	*Model table* [a]	*First component modified upwards* [a]
MALE									
0	115.20	255.59	438.68	84.92	195.73	307.18	59.25	143.78	214.35
1-4	98.12	164.43	377.81	57.24	104.14	213·32	30.16	63.56	109.75
5-9	49.87	53.53	52.30	32.50	33.00	34.39	18.78	19.72	20.36
10-14	30.42	34.66	34.92	19.83	21.89	24.03	11.98	13.40	14.16
15-19	46.04	47.65	55.00	29.93	32.34	39.27	18.54	21.22	23.78
20-24	65.58	63.91	81.34	40.71	45.52	58.04	25.00	30.63	34.88
25-29	67.84	71.05	92.46	40.67	48.50	65.25	24.42	31.74	39.80
30-34	69.89	80.35	106.45	40.38	52.79	75.16	23.93	33.61	47.38
35-39	66.52	94.14	124.44	42.93	60.05	90.82	25.69	37.75	60.02
40-44	61.16	115.45	152.86	47.14	73.06	114.80	28.22	46.41	79.39
45-49	67.32	141.87	182.16	52.53	91.67	143.97	33.62	60.58	107.03
50-54	78.23	172.43	224.33	63.47	116.84	184.86	42.25	81.28	148.17
55-59	96.48	214.20	284.64	84.38	152.50	243.73	56.71	111.05	199.68
60-64	145.96	266.90	337.90	126.57	201.75	297.56	88.94	155.94	255.47
65-69	220.88	342.45	397.66	197.56	274.30	360.07	141.46	221.49	319.76
70-74	318.39	452.34	495.51	292.49	378.29	460.45	238.53	317.01	418.79
75-79	413.34	580.59	618.86	388.89	504.82	589.22	336.36	439.18	555.22
80-84	518.60	739.73	728.33	501.83	662.40	713.21	462.62	594.78	692.38

	Level 80 Expectation of life at birth in years			Level 100 Expectation of life at birth in years			Level 115 Expectation of life at birth in years		
Both sexes . . .	66.90	60.40	52.60	73.61	70.20	63.86	76.07	73.89	69.55
Male	66.54	58.82	49.79	72.73	68.56	61.00	74.85	72.65	66.94
Female	67.27	62.05	55.55	74.54	71.80	66.87	77.36	75.19	73.12
Sex and age (in years)	*First component modified down-wards* [a]	*Model table* [a]	*First component modified upwards* [a]	*First component modified down-wards* [a]	*Model table* [a]	*First component modified upwards* [a]	*First First modified down-wards* [a]	*Model table* [a]	*First component modified upwards* [a]
MALE									
0	37.34	84.47	131.02	18.32	29.67	70.07	10.60	17.94	40.90
1-4	13.15	33.21	50.48	3.75	9.75	14.83	1.37	3.00	5.53
5-9	9.90	10.55	10.75	3.74	4.06	4.09	1.85	1.40	2.24
10-14	6.73	7.61	7.92	2.84	3.31	3.34	1.40	1.31	2.04
15-19	11.04	12.45	14.70	5.09	5.64	6.53	3.19	2.21	5.04
20-24	14.90	17.94	20.80	6.78	7.44	9.46	4.49	3.07	7.17
25-29	14.60	18.35	23.34	6.53	7.83	10.35	4.39	3.75	7.22
30-34	14.70	19.36	27.84	6.73	9.05	12.48	4.84	5.01	7.67
35-39	17.01	22.26	36.90	8.42	11.50	17.65	6.03	7.22	10.25
40-44	20.26	28.52	51.10	10.79	16.61	25.98	7.22	11.30	15.19
45-49	26.91	40.00	74.33	16.37	26.23	42.59	12.38	18.61	27.15
50-54	37.29	57.21	110.14	25.35	41.11	68.54	21.09	31.01	48.05
55-59	53.25	82.41	156.41	39.42	63.06	107.71	34.39	50.54	78.93
60-64	85.35	122.30	209.36	74.65	98.40	171.46	63.28	82.15	119.43
65-69	136.23	181.21	273.81	111.98	151.32	215.78	114.88	130.87	176.62
70-74	228.71	268.74	369.97	194.29	231.51	305.43	185.98	207.39	259.84
75-79	329.58	386.23	513.46	293.84	344.87	454.65	291.44	314.24	409.85
80-84	451.87	536.52	658.68	433.17	490.21	626.81	431.21	456.23	595.00

TABLE A.II.17 *(continued)*

	Level 20 Expectation of life at birth in years			*Level 40 Expectation of life at birth in years*			*Level 60 Expectation of life at bierth in years*		
Both sexes . . .	41.89	29.98	16.97	50.81	40.00	28.85	60.41	50.00	41.03
Male	42.58	29.57	15.80	51.21	39.24	26.88	60.74	48.72	38.39
Female	41.17	30.40	18.20	50.39	40.74	30.92	60.07	51.31	43.81
Sex and age (in years)	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])
FEMALE									
0	104.55	233.73	395.75	74.91	175.59	273.28	50.56	123.75	178.36
1-4	99.14	166.95	381.71	56.69	104.75	211.22	29.20	62.64	106.47
5-9	57.42	55.45	57.42	34.68	33.85	34.68	19.08	19.73	19.08
10-14	37.87	39.29	37.87	23.93	24.58	23.93	13.86	14.68	13.86
15-19	57.56	53.21	57.57	37.10	35.42	37.10	22.02	21.65	22.02
20-24	73.82	69.27	73.82	48.34	47.48	48.34	29.30	29.42	29.30
25-29	76.14	79.35	80.96	50.72	52.67	53.96	30.75	31.75	33.38
30-34	77.07	87.96	86.78	53.60	56.19	58.80	31.97	33.45	37.10
35-39	76.56	97.05	93.33	53.63	60.19	65.53	33.62	35.73	43.91
40-44	74.98	105.55	102.06	54.67	65.52	74.75	34.83	39.94	51.63
45-49	74.79	120.27	115.69	57.47	76.96	89.30	37.24	49.06	65.63
50-54	88.11	144.01	141.37	70.22	95.70	113.32	46.90	63.98	87.14
55-59	106.94	177.08	181.41	89.76	123.64	153.24	61.68	86.36	122.14
60-64	150.58	232.02	247.62	127.18	172.01	210.97	92.42	125.54	174.28
65-69	203.18	306.78	318.08	176.45	241.02	277.93	136.49	186.71	237.56
70-74	291.58	415.44	445.53	262.56	344.11	405.66	216.74	280.98	362.48
75-79	382.04	555.05	581.98	354.42	477.96	547.04	308.03	407.31	507.60
80-84	491.38	714.27	692.82	471.71	633.46	671.42	437.26	557.17	646.39

	Level 80 Expectation of life at birth in years			*Level 100 Expectation of life at birth in years*			*Level 115 Expectation of life at birth in years*		
Both sexes . . .	66.90	60.40	52.60	73.61	70.20	63.86	76.07	73.89	69.95
Male	66.54	58.82	49.79	72.73	68.56	61.00	74.85	72.65	66.94
Female	67.27	62.05	55.55	74.54	71.80	66.87	77.36	75.19	73.12
Sex and age (in years)	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])
FEMALE									
0	30.53	70.59	110.21	14.30	23.37	54.53	7.72	13.26	30.07
1-4	12.31	31.17	47.46	3.36	8.22	13.30	1.17	2.52	4.76
5-9	9.11	9.82	9.11	2.74	3.01	2.74	1.10	1.05	1.10
10-14	7.07	7.40	7.07	2.34	2.49	2.34	1.00	1.01	1.00
20-24	11.24	11.43	11.24	4.09	4.17	4.09	1.80	1.67	1.80
20-24	15.34	15.54	15.34	5.78	5.51	5.78	2.59	2.35	2.59
25-29	16.82	16.74	17.95	6.58	6.41	7.02	3.14	3.08	3.29
30-34	18.10	18.04	20.45	7.42	7.79	8.32	3.69	4.26	3.99
35-39	20.45	19.81	25.10	9.21	9.70	11.04	5.04	6.09	5.68
40-44	23.29	23.62	32.06	11.68	13.19	15.54	7.17	9.29	8.61
45-49	28.13	31.17	44.08	16.18	19.79	24.08	11.49	14.99	14.70
50-54	38.02	43.06	61.96	24.03	29.47	37.00	18.49	23.85	24.13
55-59	52.58	60.76	91.05	35.70	43.37	58.23	29.15	37.38	39.32
60-64	80.64	92.57	136.05	57.80	69.81	93.24	48.91	61.01	65.20
65-69	120.68	144.98	193.72	91.05	115.32	141.46	77.44	101.05	101.16
70-74	197.64	229.29	312.79	152.08	191.33	249.62	144.41	168.88	188.33
75-79	289.25	346.50	459.97	250.27	299.75	395.46	232.75	270.15	306.93
80-84	422.05	493.16	614.85	388.89	443.51	568.69	374.42	412.71	471.29

([a]) The series in these tables are designated respectively by the expressions: model life tables (downward-deviating series), model life table (intermediate series), model life tables (upward-deviating series).

TABLE A.II.18. SURVIVORS OF A GIVEN AGE (l_x) FOR SIX MORTALITY LEVELS IN THREE SERIES OF MODEL LIFE TABLES [a]

	Level 20 Expectation of life at birth in years			Level 40 Expectation of life at birth in years			Level 60 Expectation of life at birth in years		
Both sexes	41.89	29.98	16.97	50.81	40.00	28.85	60.41	50.00	41.03
Male	42.58	29.57	15.80	51.21	39.24	26.88	60.74	48.72	38.39
Female	41.17	30.40	18.20	50.39	40.74	30.92	60.07	51.31	43.81
Sex and age (in years)	*First component modified downwards* [a]	*Model table* [a]	*First component modified upwards* [a]	*First component modified downwards* [a]	*Model table* [a]	*First component modified upwards* [a]	*First component modified downwards* [a]	*Model table* [a]	*First component modified upwards* [a]
MALE									
0	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
1	88 480	74 441	56 132	91 508	80 427	69 282	94 075	85 622	78 565
5	79 798	62 201	34 925	86 270	72 050	54 503	91 238	80 180	69 942
10	75 819	58 871	33 098	83 466	69 672	52 628	89 524	78 599	68 518
15	73 512	56 831	31 942	81 811	68 147	51 364	88 452	77 546	67 548
20	70 128	54 123	30 186	79 363	65 943	49 347	86 812	75 900	65 942
25	65 529	50 664	27 730	76 132	62 941	46 483	84 642	73 575	63 642
30	61 083	47 064	25 166	73 035	59 888	43 450	82 575	71 240	61 109
35	56 814	43 282	22 487	70 086	56 727	40 184	80 599	68 846	58 214
40	53 035	39 207	19 689	67 077	53 321	36 534	78 528	66 247	54 720
45	49 791	34 681	16 679	63 915	49 425	32 340	76 312	63 172	50 375
50	46 440	29 761	13 641	60 558	44 894	27 684	73 746	59 345	44 984
55	42 807	24 629	10 581	56 714	39 649	22 567	70 631	54 521	38 318
60	38 676	19 353	7 569	51 929	33 603	17 066	66 625	48 466	30 667
65	33 031	14 188	5 012	45 356	26 824	11 988	60 699	40 908	22 833
70	25 735	9 329	3 019	36 396	19 466	7 672	52 113	31 847	15 532
75	17 542	5 109	1 523	25 750	12 102	4 139	39 682	21 751	9 027
80	10 291	2 143	580	15 736	5 993	1 700	26 335	12 198	4 015
85	4 954	558	158	7 839	2 023	488	14 152	4 943	1 235

	Level 80 Expectation of life at birth in years			Level 100 Expectation of life at birth in years			Level 115 Expectation of life at birth in years		
Both sexes	66.90	60.40	52.60	73.61	70.20	63.86	76.07	73.89	69.95
Male	66.54	58.82	49.79	72.73	68.56	61.00	74.85	72.65	66.94
Female	67.27	62.05	55.55	74.54	71.80	66.87	77.36	75.19	73.12
Sex and age (in years)	*First component modified downwards* [a]	*Model table* [a]	*First component modified upwards* [a]	*First component modified downwards* [a]	*Model table* [a]	*First component modified upwards* [a]	*First component modified downwards* [a]	*Model table* [a]	*First component modified upwards* [a]
MALE									
0	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
1	96 266	91 553	86 898	98 168	97 033	92 993	98 940	98 206	95 910
5	95 000	88 513	82 511	97 800	96 087	91 614	98 804	97 911	95 380
10	94 060	87 579	81 624	97 434	95 697	91 239	98 622	97 774	95 166
15	93 427	86 913	80 978	97 157	95 380	90 934	98 484	97 646	94 972
20	92 395	85 831	79 788	96 663	94 842	90 341	98 169	97 430	94 493
25	91 018	84 291	78 128	96 007	94 136	89 486	97 729	97 131	93 816
30	89 690	82 744	76 304	95 381	93 399	88 560	97 300	96 767	93 138
35	88 371	81 142	74 180	94 739	92 554	87 455	96 829	96 282	92 424
40	86 868	79 336	71 443	93 941	91 490	85 911	96 245	95 587	91 477
45	85 108	77 073	67 792	92 927	89 970	83 679	95 550	94 507	90 087
50	82 818	73 990	62 753	91 406	87 610	80 115	94 367	92 748	87 641
55	79 729	69 757	55 842	89 089	84 008	74 624	92 377	89 872	83 430
60	75 537	64 008	47 107	85 577	78 710	66 586	89 200	85 330	76 845
65	69 090	56 180	37 245	79 189	70 965	55 170	83 555	78 320	67 667
70	59 678	46 000	27 046	70 321	60 227	43 265	73 957	68 070	55 716
75	46 029	33 638	17 041	56 658	46 284	30 051	60 202	53 953	41 239
80	30 859	20 646	8 291	40 010	30 322	16 388	42 657	36 999	24 337
85	16 914	9 569	2 830	22 679	15 458	6 116	24 263	20 119	9 856

TABLE A.II.18 *(continued)*

	Level 20 *Expectation of life at birth in years*			*Level 40* *Expectation of life at birth in years*			*Level 60* *Expectation of life at birth in years*		
Both sexes . . .	41.89	29.98	16.97	50.81	40.00	28.85	60.41	50.00	41.03
Male	42.58	29.57	15.80	51.21	39.24	26.88	60.74	48.72	38.39
Female	41.17	30.40	18.20	50.39	40.74	30.92	60.07	51.31	43.81
Sex and age (in years)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)
FEMALE									
0	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
1	89 545	76 627	60 425	92 509	82 441	72 267	94 944	87 625	82 164
5	80 668	63 834	37 360	87 265	73 805	57 322	92 172	82 136	73 416
10	76 036	60 294	35 215	84 238	71 307	55 334	90 413	80 515	72 015
15	73 156	57 925	33 881	82 222	69 554	54 010	89 160	79 333	71 017
20	68 945	54 843	31 931	79 172	67 090	52 006	87 197	77 615	69 453
25	63 856	51 044	29 574	75 345	63 905	49 492	84 642	75 332	67 418
30	58 993	46 994	27 179	71 523	60 539	46 822	82 039	72 940	65 168
35	54 447	42 860	24 281	67 690	57 137	44 069	79 416	70 500	62 750
40	50 279	38 700	22 504	64 060	53 698	41 181	76 746	76 981	59 995
45	46 509	34 615	20 207	60 557	50 180	38 103	74 073	65 266	56 897
50	43 030	30 452	17 870	57 077	46 318	34 700	71 315	62 064	53 163
55	39 239	26 067	15 343	53 069	41 885	30 768	67 970	58 093	48 531
60	35 043	21 451	12 560	48 306	36 706	26 053	63 778	53 076	42 603
65	29 766	16 474	9 450	42 162	30 392	20 556	57 883	46 413	35 178
70	23 718	11 420	6 444	34 723	23 067	14 843	49 983	37 747	26 221
75	16 802	6 676	3 573	25 606	12 129	8 822	39 149	27 141	17 099
80	10 383	2 970	1 494	16 531	7 898	3 996	27 090	16 086	8 420
85	5 281	849	459	8 733	2 895	1 313	15 245	7 123	2 977

	Level 80 *Expectation of life at birth in years*			*Level 100* *Expectation of life at birth in years*			*Level 115* *Expectation of life at birth in years*		
Both sexes . . .	66.90	60.40	52.60	73.61	70.20	63.86	76.07	73.89	69.95
Male	66.54	58.82	49.79	72.73	68.56	61.00	74.85	72.65	66.94
Female	67.27	62.05	55.55	74.54	71.80	66.87	77.36	75.19	73.12
Sex and age (in years)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)	*First modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)
FEMALE									
0	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
1	96 947	92 941	88 979	98 570	97 663	94 547	99 228	98 674	96 993
5	95 754	90 044	84 756	98 239	96 860	93 290	99 112	98 425	96 531
10	94 881	89 160	83 984	97 970	96 568	93 034	99 003	98 322	96 425
15	94 210	88 500	83 390	97 740	96 328	92 816	98 904	98 223	96 329
20	93 152	87 488	82 453	97 341	95 926	92 437	98 726	98 059	96 155
25	91 723	86 128	81 188	96 778	95 397	91 902	98 470	97 829	95 906
30	90 180	84 686	79 731	96 141	94 786	91 257	98 161	97 528	95 591
35	88 548	83 158	78 100	95 428	94 048	90 498	97 799	97 113	95 209
40	86 737	81 511	76 140	94 549	93 136	89 499	97 306	96 522	94 669
45	84 717	79 586	73 699	93 445	91 908	88 108	96 608	95 625	93 853
50	82 334	77 105	70 450	91 933	90 089	85 986	95 498	94 192	92 474
55	79 203	73 785	66 085	89 724	87 434	82 805	93 732	91 946	90 242
60	75 039	69 302	60 068	86 520	83 642	77 983	91 000	88 509	86 694
65	68 988	62 887	51 896	81 520	77 803	70 712	86 549	83 109	81 042
70	60 662	53 770	41 843	74 097	68 831	60 709	79 847	74 711	72 843
75	48 672	41 441	28 754	62 828	35 662	45 555	68 316	62 094	59 125
80	34 594	27 082	15 528	47 104	38 977	27 540	52 416	45 319	40 978
85	19 994	13 120	5 981	28 786	21 690	11 878	32 790	26 615	21 665

(a) The series in these tables are designated respectively by the expressions: model life tables (downward-deviating series), model life tables (intermediate series), model life tables (upward-deviating series).

TABLE A.II.19. SURVIVORS IN EACH AGE GROUP (L_x) FOR SIX MORTALITY LEVELS IN THREE SERIES OF MODEL LIFE TABLES ([a])

	Level 20 Expectation of life at birth in years			*Level 40 Expectation of life at birth in years*			*Level 60 Expectation of lift at bierth in years*		
Both sexes	41.89	29.98	16.97	50.81	40.00	28.85	60.41	50.00	41.03
Male	42.58	29.57	15.80	51.21	39.24	26.88	60.74	48.72	38.39
Female	41.17	30.40	18.20	50.39	40.74	30.92	60.07	51.31	43.81
Sex and age (in years)	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])
MALE									
T_0	4 258 397	2 957 495	1 579 657	5 120 556	3 924 285	2 687 575	6 074 491	4 872 148	3 839 057
0-4	427 048	352 891	247 092	448 663	389 437	323 054	465 899	420 276	380 075
5-9	389 042	302 680	170 058	424 340	354 305	267 828	451 905	396 948	346 150
10-14	373 328	289 255	162 600	413 192	344 548	259 980	444 940	390 362	340 165
15-19	359 100	277 385	155 320	402 935	335 225	251 778	438 160	383 615	333 725
20-24	339 142	261 968	144 790	388 738	322 210	239 575	428 635	373 688	323 960
25-29	316 530	244 320	132 240	372 918	307 072	224 832	418 042	362 038	311 878
30-34	294 742	225 865	119 132	397 802	291 538	209 085	407 935	350 215	298 308
35-39	274 622	206 222	105 440	342 908	275 120	191 795	397 818	337 732	282 335
40-44	257 065	184 720	90 920	327 480	256 865	172 185	387 100	323 548	262 738
45-49	240 578	161 105	75 800	311 182	235 798	150 060	375 145	306 292	238 398
50-54	223 118	135 975	60 555	293 180	211 358	125 628	360 942	284 665	208 255
55-59	203 708	109 955	45 375	271 608	183 130	99 082	343 140	257 468	172 462
60-64	179 268	83 852	31 452	243 212	151 068	72 635	318 310	223 435	133 750
65-69	146 915	58 792	20 078	204 380	115 725	49 150	282 030	181 888	95 912
70-74	108 192	36 095	11 355	155 365	78 920	29 528	229 488	133 995	61 398
75-79	69 582	18 130	5 258	103 715	45 238	14 598	165 042	84 872	32 605
80-84	38 112	6 752	1 845	58 938	20 040	5 470	101 218	42 852	13 125
85 +	18 305	1 533	347	30 527	6 688	1 312	58 742	18 259	3 818

	Level 80 Expectation of life at birth in years			*Level 100 Expectation of life at birth in years*			*Level 115 Expectation of life at birth in years*		
Both sexes	66.90	60.40	52.60	73.61	70.20	63.86	76.07	73.89	69.95
Male	66.54	58.82	49.79	72.73	68.56	61.00	74.85	72.65	66.94
Female	67.27	62.05	55.55	74.54	71.80	66.87	77.36	75.19	73.12
Sex and age (in years)	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])	*First component modified down-wards* ([a])	*Model table* ([a])	*First component modified upwards* ([a])
MALE									
T_0	6 654 293	5 882 432	4 979 486	7 272 999	6 855 509	6 100 326	7 484 953	7 264 597	6 694 152
0-4	479 605	453 493	428 553	490 525	483 920	463 821	494 679	490 859	479 460
5-9	472 650	440 230	410 338	488 085	479 460	457 132	493 565	489 212	476 365
10-14	468 718	436 230	406 505	486 478	477 692	455 432	492 765	488 550	475 345
15-19	464 555	431 860	401 915	484 550	475 555	453 188	491 632	487 690	473 662
20-24	458 532	425 305	394 790	481 675	472 445	449 568	489 745	486 402	470 772
25-29	451 770	417 588	386 080	478 470	468 838	445 115	487 572	484 745	467 385
30-34	445 152	409 715	376 210	475 300	464 882	440 038	485 322	482 622	463 905
35-39	438 098	401 195	364 058	471 700	460 110	433 415	482 685	479 672	459 752
40-44	429 940	391 022	348 088	467 170	453 650	423 975	479 488	475 235	453 910
45-49	419 815	377 658	326 362	460 832	443 950	409 485	474 792	468 138	444 320
50-54	406 368	359 368	296 488	451 238	429 045	386 848	466 860	456 550	427 678
55-59	388 165	334 412	257 372	436 665	406 795	353 025	453 942	438 005	400 688
60-64	361 568	300 470	210 880	411 915	374 188	304 390	431 888	409 125	361 280
65-69	321 920	255 450	160 728	373 775	327 980	246 088	393 780	365 975	308 458
70-74	264 268	199 095	110 218	317 448	266 278	183 290	335 398	305 058	242 388
75-79	192 220	135 710	63 330	241 670	191 515	116 098	257 148	227 380	163 940
80-84	119 432	75 538	27 802	156 722	114 450	56 260	167 300	142 795	85 482
85 +	71 517	38 093	9 769	98 781	64 758	23 158	106 392	86 584	39 362

	Level 20 Expectation of life at birth in years			*Level 40 Expectation of life at birth in years*			*Level 60 Expectation of life at birth in years*		
Both sexes . . .	41.89	29.98	16.97	50.81	40.00	28.85	60.41	50.00	41.03
Male	42.58	29.57	15.80	51.21	39.24	26.88	60.74	48.72	38.39
Female	41.17	30.40	18.20	50.39	40.74	30.92	60.07	51.31	43.81
Sex and age (in years)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)
FEMALE									
T_0	4 117 237	3 040 235	1 819 580	5 039 221	4 074 257	3 092 413	6 006 747	5 130 792	4 380 872
0-4	431 697	362 113	263 582	453 405	398 459	337 957	470 163	429 692	396 908
5-9	391 760	310 320	181 438	428 758	362 780	281 640	456 462	406 628	363 578
10-14	372 980	295 548	172 740	416 150	352 152	273 360	448 932	399 620	357 580
15-19	355 252	281 920	164 530	403 485	341 610	265 040	440 892	392 370	351 175
20-24	332 002	264 718	153 762	386 292	327 488	253 745	429 598	382 368	342 178
25-29	307 122	245 095	141 882	367 170	311 110	240 785	416 702	370 680	331 465
30-34	283 600	224 635	130 000	348 032	294 190	227 228	403 638	358 600	319 795
35-39	261 815	203 900	118 312	329 375	277 088	213 125	390 405	346 202	306 862
40-44	241 970	183 288	106 778	311 542	259 695	198 210	377 048	333 118	292 230
45-49	223 848	162 668	95 192	294 085	241 245	182 008	363 470	318 325	275 150
50-54	205 672	141 298	83 032	275 365	220 508	163 670	348 212	300 392	254 235
55-59	185 705	118 795	69 758	253 438	196 478	142 052	329 370	277 922	227 835
60-64	162 022	94 812	55 025	226 170	167 745	116 522	304 152	248 722	194 452
65-69	133 710	69 735	39 735	192 212	133 648	88 498	269 665	210 400	154 998
70-74	101 300	45 240	25 042	150 822	95 490	59 162	222 830	162 220	109 800
75-79	67 962	24 115	12 668	105 342	57 568	32 045	165 598	108 068	63 798
80-84	39 160	9 548	4 882	63 160	26 982	13 272	105 838	58 022	28 492
85 + ...	19 660	2 487	1 222	34 418	10 021	4 094	63 772	27 443	10 341

	Level 80 Expectation of life at birth in years			*Level 100 Expectation of life at birth in years*			*Level 115 Expectation of life at birth in years*		
Both sexes . . .	66.90	60.40	52.60	73.61	70.20	63.86	76.07	73.89	69.95
Male	66.54	58.82	49.79	72.73	68.56	61.00	74.85	72.65	66.94
Female	67.27	62.05	55.55	74.54	71.80	66.87	77.36	75.19	73.12
Sex and age (in years)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)	*First component modified downwards* (a)	*Model table* (a)	*First component modified upwards* (a)
FEMALE									
T_0	6 726 563	6 204 598	5 554 759	7 454 024	7 180 317	6 686 986	7 735 591	7 519 058	7 311 846
0-4	482 993	460 386	438 782	492 512	487 213	471 459	496 089	493 179	484 747
5-9	476 588	448 010	421 850	490 522	483 570	465 810	495 288	491 868	482 390
10-14	472 728	444 150	418 435	489 275	482 240	464 625	494 768	491 362	481 885
15-19	468 405	439 970	414 608	487 702	480 635	463 132	494 075	490 705	481 210
20-24	462 188	434 040	409 102	485 298	478 308	460 848	492 990	489 720	480 152
25-29	454 758	427 035	402 298	482 298	475 458	457 898	491 578	488 392	478 742
30-34	446 820	419 610	394 578	478 922	472 085	454 388	489 900	486 602	477 000
35-39	438 212	411 672	385 600	474 942	467 960	449 992	487 762	484 088	474 695
40-44	428 635	402 742	374 598	469 985	462 610	444 018	484 785	480 368	471 305
45-49	417 628	391 728	360 372	463 445	454 992	435 235	480 265	474 542	465 818
50-54	403 842	371 275	341 338	454 142	443 808	421 978	473 075	465 345	456 790
55-59	385 605	357 718	315 382	440 610	427 690	401 970	461 830	451 138	442 340
60-64	360 068	330 472	279 910	420 100	403 612	371 738	443 872	429 045	419 340
65-69	324 125	291 642	234 348	389 042	366 585	328 552	415 990	394 550	384 712
70-74	275 220	238 028	176 492	342 312	311 232	265 660	370 408	342 012	329 920
75-79	208 168	171 308	110 705	274 830	236 598	182 738	301 830	268 532	250 258
80-84	136 470	102 020	53 772	189 725	151 668	98 545	213 015	179 835	156 608
85 + ...	85 992	56 792	22 380	128 362	94 053	48 400	148 071	117 775	93 934

(a) The series in these tables are designated respectively by the expressions: model life tables (downward-deviating series), model life tables (intermediate series), model life tables (upward-deviating series).

Annex III

STABLE POPULATIONS
(INTERMEDIATE SERIES)

TABLE A.III.1. — DISTRIBUTION BY SEX AND AGE GROUPS OF THIRTY-SIX STABLE POPULATIONS CALCULATED BY ASSOCIATING SIX LEVELS OF INTERMEDIATE MODEL LIFE TABLE WITH SIX FERTILITY LEVELS

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 1.00						*Gross reproduction rate: 1.50*					
MALES												
All ages	499 504	499 935	498 872	496 221	497 112	500 065	503 752	503 503	502 350	500 180	501 208	503 962
0-4	25 203	27 079	28 337	29 665	31 415	33 086	41 205	44 201	46 350	48 489	51 094	53 496
5-14	50 124	55 207	57 893	60 308	63 388	66 348	73 254	80 609	84 747	88 254	92 332	96 101
15-29	102 367	101 014	99 095	98 212	99 041	100 482	124 177	122 599	120 656	119 599	120 091	121 184
30-44	128 463	116 927	109 479	104 447	101 762	100 499	125 353	113 962	106 950	102 006	98 941	97 176
45-59	117 531	112 964	108 150	102 966	98 797	95 874	92 551	88 643	84 947	80 792	77 142	74 425
60-74	65 072	71 828	76 007	76 601	75 673	74 517	41 539	45 668	48 303	48 566	47 690	46 649
75 +	10 744	14 916	19 911	24 022	27 036	29 259	5 673	7 821	10 397	12 474	13 918	14 931
FEMALES												
All ages	500 497	500 066	501 131	503 777	502 888	499 933	496 245	496 497	497 651	499 821	498 790	496 038
0-4	24 784	26 464	27 612	28 886	30 373	31 725	40 520	43 196	45 165	47 215	49 400	51 296
5-14	48 947	53 810	56 404	58 816	61 451	63 757	71 542	78 575	82 569	86 072	89 510	92 350
15-29	97 719	97 139	95 897	95 717	96 273	96 887	118 601	117 032	116 780	116 557	116 724	116 836
30-44	118 912	110 446	105 215	102 080	99 512	97 381	116 053	107 651	102 780	99 682	96 738	94 155
45-59	117 300	112 064	107 711	103 705	98 977	94 649	92 133	87 794	84 506	81 295	77 223	73 427
60-74	77 423	80 621	83 297	84 173	82 168	79 265	49 277	51 139	52 831	53 261	51 680	49 529
75 +	15 412	19 522	24 995	30 400	34 134	36 269	8 119	10 210	13 020	15 739	17 515	18 445
BOTH SEXES												
A llages (a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	49 987	53 543	55 949	58 551	61 788	64 811	81 725	87 397	91 515	95 704	100 494	104 792
5-14	99 071	109 017	114 297	119 124	124 839	130 105	144 796	159 184	167 316	174 326	181 842	188 451
15-29	200 086	198 153	194 992	193 929	195 314	197 369	242 778	240 531	237 436	236 156	236 815	238 020
30-44	247 375	227 373	214 694	206 527	201 274	197 880	241 406	221 613	209 730	201 688	195 679	191 331
45-59	234 831	225 028	215 861	206 671	197 774	190 523	184 684	176 437	169 453	162 087	154 365	147 852
60-74	142 495	152 449	159 304	160 774	157 841	153 782	90 816	96 807	101 134	101 827	99 370	96 178
75 +	26 156	34 438	44 906	54 422	61 710	65 528	13 792	18 031	23 417	28 213	31 433	33 376

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

Table A.III.1 *(continued)*

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	Mortality level 0	20	40	60	80	100	Mortality level 0	20	40	60	80	100
Expectation of life at birth for both sexes (in years)	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 2.00						*Gross reproduction rate: 2.50*					
Males												
All ages	505 740	505 203	504 077	502 212	503 308	505 914	506 754	506 106	505 038	503 394	504 523	507 030
0-4	55 440	59 185	61 945	64 614	67 749	70 577	67 884	72 089	75 248	78 248	81 695	84 765
5-14	91 099	99 812	104 776	108 818	113 308	117 364	104 986	114 466	119 868	124 132	128 731	132 823
15-29	135 495	133 325	131 078	129 626	129 570	130 135	141 193	138 364	135 754	133 891	133 316	133 401
30-44	117 211	106 069	99 385	94 538	91 267	89 212	108 350	97 562	91 180	86 481	83 154	80 982
45-59	74 326	70 742	67 621	64 107	60 902	58 466	61 057	57 751	55 020	51 988	49 178	47 027
60-74	28 740	31 387	33 082	33 127	32 339	31 453	21 029	22 815	23 949	23 887	23 205	22 474
75 +	3 429	4 683	6 190	7 382	8 173	8 707	2 255	3 059	4 019	4 767	5 244	5 558
Females												
All ages	494 261	494 796	495 921	497 789	496 694	494 085	493 246	493 893	494 962	496 604	495 480	492 969
0-4	54 519	57 840	60 364	62 916	65 503	67 676	66 756	70 450	73 327	76 190	78 986	81 280
5-14	88 977	97 296	102 085	106 128	109 844	112 781	102 547	111 586	116 792	121 063	124 795	127 637
15-29	129 459	128 281	126 879	126 326	125 929	125 460	134 943	133 153	131 414	130 481	129 563	128 599
30-44	108 529	100 202	95 506	92 375	89 227	86 430	100 335	92 168	87 622	84 497	81 292	78 451
45-59	73 857	69 983	67 215	64 468	60 932	57 655	60 589	57 080	54 656	52 251	49 185	46 362
60-74	34 021	35 090	36 133	36 280	34 997	33 352	24 855	25 474	26 132	26 130	25 087	23 802
75 +	4 899	6 104	7 739	9 295	10 262	10 731	3 221	3 982	5 019	5 992	6 572	6 838
Both sexes												
All ages (a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	109 959	117 025	122 309	127 530	133 252	138 253	134 640	142 539	148 575	154 438	160 681	166 045
5-14	180 076	197 108	206 861	214 946	223 152	230 145	207 533	226 052	236 660	245 195	253 526	260 460
15-29	264 954	261 606	257 957	255 952	255 499	255 595	276 136	271 517	267 168	264 372	262 879	262 000
30-44	225 740	206 271	194 891	186 913	180 494	175 642	208 685	189 730	178 803	170 978	164 446	159 433
45-59	148 183	140 725	134 836	128 575	121 834	116 121	121 646	114 831	109 676	104 239	98 363	93 389
60-74	62 761	66 477	69 215	69 407	67 336	64 805	45 884	48 289	50 081	50 017	48 292	46 276
75 +	8 328	10 787	13 929	16 678	18 435	19 438	5 476	7 041	9 038	10 759	11 816	12 396

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.1 *(continued)*

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	Mortality level 0	20	40	60	80	100	Mortality level 0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 3.00						*Gross reproduction rate: 4.00*					
MALES												
All ages	507 302	506 615	505 625	504 134	505 297	507 724	507 754	507 099	506 237	504 978	506 183	508 520
0-4	78 777	83 252	86 659	89 857	93 489	96 698	96 922	101 593	105 242	108 618	112 421	115 755
5-14	115 980	125 880	131 483	135 799	140 361	144 385	132 108	142 283	147 951	152 141	156 464	160 256
15-29	143 736	140 305	137 352	135 131	134 122	133 818	144 044	139 665	136 175	133 430	131 804	130 953
30-44	100 019	89 636	83 557	79 038	75 754	73 554	85 887	76 368	70 858	66 735	63 648	61 533
45-59	51 177	48 128	45 711	43 058	40 591	38 699	37 739	35 153	33 202	31 128	29 188	27 703
60-74	16 035	17 291	18 086	17 975	17 392	16 786	10 182	10 874	11 301	11 164	10 740	10 312
75 +	1 578	2 123	2 777	3 276	3 588	3 784	872	1 163	1 508	1 762	1 918	2 008
FEMALES												
All ages	492 700	493 383	494 376	495 869	494 706	492 278	492 249	492 900	493 764	495 020	493 816	491 480
0-4	77 467	81 358	84 445	87 497	90 391	92 722	95 311	99 284	102 552	105 764	108 695	110 995
5-14	113 293	122 716	128 110	132 443	136 069	138 747	129 056	138 716	144 160	148 380	151 679	153 995
15-29	137 407	135 040	132 967	131 689	130 341	128 998	137 750	134 453	131 844	130 030	128 080	126 229
30-44	92 626	84 682	80 292	77 221	74 048	71 252	79 550	72 149	68 087	65 195	62 210	59 606
45-59	50 728	47 535	45 382	43 262	40 580	38 140	37 343	34 680	32 941	31 254	29 169	27 292
60-74	18 928	19 290	19 717	19 644	18 786	17 768	11 995	12 110	12 303	12 186	11 586	10 900
75 +	2 251	2 762	3 463	4 113	4 491	4 651	1 244	1 508	1 877	2 211	2 397	2 463
BOTH SEXES												
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	156 244	164 610	171 104	177 354	183 880	189 420	192 233	200 877	207 794	214 382	221 116	226 750
5-14	229 273	248 596	259 593	268 242	276 430	283 132	261 164	280 999	292 111	300 521	308 143	314 251
15-29	281 143	275 345	270 319	266 820	264 463	262 816	281 794	274 118	268 019	263 460	259 884	257 182
30-44	192 645	174 318	163 849	156 259	149 802	144 806	165 437	148 517	138 945	131 930	125 858	121 139
45-59	101 905	95 663	91 093	86 320	81 171	76 839	75 082	69 833	66 143	62 382	58 357	54 995
60-74	34 963	36 581	37 803	37 619	36 178	34 554	22 177	22 984	23 604	23 350	22 326	21 212
75 +	3 829	4 885	6 240	7 389	8 079	8 435	2 116	2 671	3 385	3 973	4 315	4 471

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.1 (continued)

(For 1 million of both sexes and all ages)

MALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 1.00						*Gross reproduction rate: 1.50*					
All ages	499 504	499 935	498 872	496 221	497 112	500 065	503 752	503 503	502 350	500 180	501 208	503 962
0-4	25 203	27 079	28 337	29 665	31 415	33 096	41 205	44 201	46 350	48 489	51 094	53 496
5-9	23 541	26 441	28 102	29 586	31 384	33 083	35 752	40 092	42 700	44 922	47 416	49 689
10-14	26 583	28 766	29 791	30 722	32 004	33 265	37 502	40 517	42 047	43 332	44 916	46 412
15-19	30 389	31 404	31 596	31 878	32 605	33 419	39 825	41 089	41 426	41 767	42 508	43 313
20-24	34 188	33 763	33 106	32 790	33 046	33 506	41 619	41 037	40 320	39 908	40 020	40 341
25-29	37 790	35 847	34 393	33 544	33 390	33 557	42 733	40 473	38 910	37 924	37 563	37 530
30-34	41 002	37 727	35 595	34 264	33 713	33 581	43 071	39 568	37 408	35 984	35 232	34 886
35-39	43 338	39 214	36 617	34 890	33 973	33 542	42 288	38 205	35 747	34 038	32 980	32 370
40-44	44 123	39 986	37 267	35 293	34 076	33 376	39 994	36 189	33 795	31 984	30 729	29 920
45-49	42 880	39 701	37 293	35 279	33 869	32 963	36 104	33 377	31 416	29 699	28 373	27 450
50-54	39 717	38 147	36 439	34 622	33 167	32 149	31 065	29 791	28 513	27 074	25 810	24 869
55-59	34 934	35 116	34 418	33 065	31 761	30 762	25 382	25 475	25 018	24 019	22 959	22 106
60-64	28 842	30 486	30 949	30 298	29 369	28 557	19 466	20 544	20 897	20 445	19 721	19 062
65-69	21 861	24 334	25 845	26 044	25 696	25 262	13 705	15 234	16 211	16 325	16 028	15 665
70-74	14 369	17 008	19 213	20 259	20 608	10 698	8 368	9 890	11 195	11 796	11 941	11 922
75-79	7 531	9 726	12 004	13 549	14 457	15 023	4 074	5 254	6 497	7 328	7 782	8 038
80-84	2 795	4 124	5 798	7 232	8 281	9 062	1 404	2 070	2 915	3 629	4 140	4 504
85 +	418	1 066	2 109	3 250	4 298	5 174	195	497	985	1 517	1 996	2 389

TABLE A.III.1 *(continued)*

(For 1 million of both sexes and all ages)

MALE

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate : 2.00						*Gross reproduction rate : 2.50*					
All ages	505 740	505 203	504 077	502 212	503 308	505 914	506 754	506 106	505 038	503 394	504 523	507 030
0-4	55 440	59 185	61 945	64 614	67 749	70 577	67 884	72 089	75 248	78 248	81 695	84 765
5-9	45 652	50 948	54 161	56 811	59 667	62 216	53 676	59 588	63 175	66 062	69 089	71 751
10-14	45 447	48 864	50 615	52 007	53 641	55 148	51 310	54 878	56 693	58 070	59 642	61 072
15-19	45 803	47 028	47 326	47 576	48 178	48 844	49 655	50 716	50 902	51 010	51 437	51 942
20-24	45 426	44 575	43 715	43 142	43 048	43 174	47 289	46 160	45 148	44 416	44 131	44 084
25-29	44 266	41 722	40 037	38 908	38 344	38 117	44 249	41 488	39 704	38 465	37 748	37 375
30-34	42 343	38 710	36 531	35 036	34 131	33 626	40 643	36 963	34 787	33 259	32 263	31 661
35-39	39 455	35 472	33 129	31 453	30 323	29 612	36 365	32 524	30 293	28 670	27 552	26 772
40-44	35 413	31 887	29 725	28 049	26 813	25 974	31 342	28 075	26 100	24 552	23 369	22 549
45-49	30 340	27 911	26 224	24 718	23 495	22 616	25 785	23 598	22 110	20 776	19 664	18 854
50-54	24 775	23 643	22 588	21 384	20 283	19 446	20 218	19 195	18 287	17 259	16 300	15 566
55-59	19 211	19 188	18 809	18 005	17 124	16 404	15 054	14 958	14 623	13 953	13 214	12 607
60-64	13 983	14 685	14 911	14 546	13 960	13 424	10 522	10 993	11 130	10 823	10 344	9 908
65-69	9.343	10 335	10 977	11 022	10 767	10 468	6 751	7 428	7 868	7 877	7 661	7 420
70-74	5 414	6 367	7 194	7 559	7 612	7 561	3 756	4 394	4 951	5 187	5 200	5 146
75-79	2 502	3 210	3 962	4 457	4 709	4 838	1 666	2 128	2 618	2 936	3 088	3 163
80-84	819	1 200	1 687	2 095	2 377	2 573	523	764	1 071	1 326	1 498	1 614
85 +	108	273	541	830	1 087	1 296	66	167	330	505	658	781

TABLE A.III.1 *(continued)*

(For 1 million of both sexes and all ages)

MALE

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate : 3.00						*Gross reproduction rate : 4.00*					
All ages	507 302	506 615	505 625	504 134	505 297	507 724	507 754	507 099	506 237	504 978	506 183	508 520
0-4	78 777	83 252	86 659	89 857	93 489	96 698	96 922	101 593	105 242	108 618	112 421	115 755
5-9	60 257	66 570	70 382	73 390	76 487	79 183	70 359	77 096	81 119	84 193	87 286	89 957
10-14	55 723	59 310	61 101	62 409	63 874	65 202	61 749	65 187	66 832	67 948	69 178	70 299
15-19	52 168	53 025	53 071	53 033	53 290	53 644	54 863	55 310	55 091	54 796	54 774	54 891
20-24	48 062	46 687	45 538	44 673	44 232	44 048	47 971	46 217	44 862	43 805	43 148	42 771
25-29	43 506	40 593	38 743	37 425	36 600	36 126	41 210	38 138	36 222	34 829	33 882	33 291
30-34	38 658	34 986	32 837	31 307	30 264	29 603	34 751	31 195	29 137	27 649	26 589	25 890
35-39	33 462	29 781	27 663	26 106	24 976	24 218	28 547	25 201	23 294	21 883	20 824	20 098
40-44	27 899	24 869	23 057	21 625	20 514	19 733	22 589	19 972	18 427	17 203	16 235	15 545
45-49	22 203	20 221	18 895	17 703	16 698	15 962	17 061	15 411	14 330	13 366	12 541	11 932
50-54	16 843	15 912	15 120	14 228	13 391	12 747	12 282	11 509	10 882	10 195	9 544	9 043
55-59	12 131	11 995	11 696	11 127	10 502	9 990	8 396	8 233	7 990	7 567	7 103	6 728
60-64	8 203	8 527	8 611	8 351	7 952	7 594	5 387	5 555	5 584	5 387	5 105	4 853
65-69	5 092	5 574	5 889	5 879	5 698	5 502	3 174	3 447	3 623	3 600	3 471	3 336
70-74	2 740	3 190	3 586	3 745	3 742	3 690	1 621	1 872	2 094	2 177	2 164	2 123
75-79	1 177	1 494	1 834	2 051	2 150	2 194	660	833	1 017	1 130	1 180	1 918
80-84	358	519	726	895	1 009	1 084	190	274	383	468	526	562
85 +	43	110	217	330	429	506	22	56	108	164	212	248

TABLE A.III.1 *(continued)*

(For 1 million of both sexes and all ages)

FEMALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate : 1.00						*Gross reproduction rate : 1.50*					
All ages	500 497	500 066	501 131	503 777	502 888	499 933	496 245	496 497	497 651	499 821	498 790	496 038
0-4	24 784	26 464	27 612	28 886	30 373	31 725	40 520	43 196	45 165	47 215	49 400	51 296
5-9	23 062	25 818	27 405	28 864	30 418	31 776	35 025	39 147	41 639	43 826	45 956	47 728
10-14	25 885	27 992	28 999	29 952	31 033	31 981	36 517	39 428	40 930	42 246	43 554	44 622
15-19	29 334	30 397	30 665	31 054	31 636	32 169	38 442	39 772	40 205	40 686	41 244	41 692
20-24	32 685	32 493	32 046	31 954	32 117	32 307	39 789	39 492	39 029	38 891	38 896	38 896
25-29	35 700	34 249	33 186	32 709	32 520	32 411	40 370	38 668	37 546	36 980	36 584	36 248
30-34	38 185	35 734	34 209	33 413	32 884	32 476	40 111	37 478	35 951	35 091	34 365	33 740
35-39	39 921	36 925	35 123	34 061	33 201	32 490	38 954	35 975	34 288	33 229	32 230	31 356
40-44	40 806	37 787	35 883	34 606	33 427	32 415	36 988	34 198	32 541	31 362	30 143	29 059
45-49	40 782	38 178	36 338	34 919	33 458	32 174	34 338	32 097	30 611	29 396	28 027	26 793
50-54	39 573	37 753	36 206	34 794	33 162	31 672	30 952	29 484	28 332	27 208	25 806	24 500
55-59	36 945	36 133	35 167	33 992	32 357	30 803	26 843	26 213	25 563	24 691	23 390	22 134
60-64	32 582	32 831	32 730	32 122	30 764	29 336	21 990	22 125	22 100	21 675	20 657	19 583
65-69	26 307	27 489	28 426	28 692	27 938	26 890	16 493	17 208	17 830	17 985	17 426	16 675
70-74	18 534	20 301	22 141	23 359	23 466	23 039	10 794	11 806	12 901	13 601	13 597	13 271
75-79	10 469	12 320	14 550	16 432	17 380	17 677	5 663	6 655	7 876	8 888	9 355	9 458
80-84	4 173	5 554	7 435	9 316	10 652	11 435	2 097	2 787	3 738	4 680	5 326	5 683
85 +	770	1 648	3 010	4 652	6 102	7 157	359	768	1 406	2 171	2 834	3 304

TABLE A.III.1 (continued)

(For 1 million of both sexes and all ages)

FEMALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 2.00						*Gross reproduction rate: 2.50*					
All ages	494 261	494 796	495 921	497 789	496 694	494 085	493 246	493 893	494 962	496 604	495 580	492 969
0-4	54 519	57 840	60 364	62 916	65 503	67 676	66 756	70 450	73 327	76 190	78 986	81 280
5-9	44 724	49 746	52 816	55 424	57 830	59 759	52 585	58 184	61 606	64 448	66 963	68 919
10-14	44 253	47 550	49 269	50 704	52 014	53 022	49 962	53 402	55 186	56 615	57 832	58 718
15-19	44 211	45 521	45 931	46 344	46 745	47 017	47 930	49 092	49 399	49 689	49 908	49 995
20-24	43 430	42 898	42 315	42 043	41 838	41 626	45 211	44 423	43 703	43 284	42 891	42 506
25-29	41 818	39 862	38 633	37 939	37 346	36 815	41 802	39 639	38 312	37 508	36 764	36 098
30-34	39 433	36 666	35 107	34 167	33 292	35 521	37 850	35 011	33 433	32 434	31 471	30 620
35-39	36 345	33 402	31 778	30 705	29 634	28 682	33 499	30 626	29 058	27 990	26 896	25 931
40-44	32 751	30 134	28 621	27 503	26 301	25 227	28 986	26 531	25 131	24 073	22 925	21 900
45-49	28 856	26 841	25 551	24 467	23 209	22 074	24 523	22 693	21 543	20 562	19 425	18 402
50-54	24 685	23 399	22 444	21 492	20 279	19 156	20 145	18 996	18 771	17 345	16 298	15 335
55-59	20 316	19 743	19 220	18 509	17 444	16 425	15 921	15 391	14 942	14 344	13 462	12 625
60-64	15 795	15 815	15 769	15 421	14 622	13 791	11 886	11 838	11 772	11 474	10 835	10 178
65-69	11 243	11 674	12 073	12 143	11 707	11 144	8 124	8 390	8 654	8 676	8 330	7 897
70-74	6 983	7 601	8 291	8 716	8 668	8 417	4 845	5 246	5 706	5 980	5 922	5 727
75-79	3 478	4 066	4 803	5 405	5 659	5 694	2 317	2 695	3 174	3 561	3 714	3 721
80-84	1 222	1 616	2 164	2 701	3 059	3 246	781	1 029	1 374	1 708	1 925	2 037
85 +	199	422	772	1 190	1 544	1 791	123	258	471	723	933	1 080

TABLE A.III.1 *(continued)*

(For 1 million of both sexes and all ages)

FEMALE

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 3.00						*Gross reproduction rate: 4.00*					
All ages	492 700	493 383	494 376	495 869	494 706	492 278	492 249	492 900	493 764	495 020	493 816	491 480
0-4	77 467	81 358	84 445	87 497	90 391	92 722	95 311	99 284	102 552	105 764	108 695	110 995
5-9	59 033	65 001	68 634	71 597	74 133	76 059	68 929	75 280	79 104	82 136	84 599	86 407
10-14	54 260	57 715	59 476	60 846	61 936	62 688	60 127	63 436	65 056	66 244	67 080	67 588
15-19	50 356	51 326	51 505	51 660	51 707	51 636	52 957	53 538	53 466	53 379	53 146	52 834
20-24	45 950	44 931	44 079	43 535	42 990	42 471	45 862	44 479	43 426	42 688	41 935	41 239
25-29	41 101	38 783	37 383	36 494	35 644	34 891	38 931	36 436	34 952	33 963	32 999	32 156
30-34	36 001	33 138	31 558	30 528	29 520	28 631	32 363	29 547	28 000	26 964	25 936	25 039
35-39	30 823	28 043	26 534	25 487	24 406	23 457	26 296	23 730	22 345	21 362	20 350	19 469
40-44	25 802	23 501	22 200	21 206	20 122	19 164	20 891	18 872	17 742	16 869	15 924	15 098
45-49	21 117	19 444	18 410	17 524	16 494	15 580	16 226	14 820	13 964	13 230	12 388	11 647
50-54	16 781	15 748	15 023	14 299	13 388	12 558	12 238	11 389	10 813	10 245	9 544	8 910
55-59	12 830	12 343	11 949	11 439	10 698	10 002	8 879	8 471	8 164	7 779	7 237	6 735
60-64	9 266	9 184	9 108	8 852	8 331	7 802	6 086	5 983	5 905	5 713	5 347	4 986
65-69	6 127	6 298	6 477	6 475	6 194	5 857	3 818	3 893	3 985	3 965	3 775	3 551
70-74	3 535	3 808	4 132	4 317	4 261	4 109	2 091	2 234	2 413	2 508	2 464	2 363
75-79	1 636	1 893	2 224	2 486	2 584	2 583	918	1 053	1 233	1 373	1 419	1 409
80-84	534	699	930	1 156	1 298	1 367	285	369	490	604	677	710
85 +	81	170	309	471	609	701	41	86	154	234	301	344

TABLE A.III.1 *(continued)*

(For 1 million of both sexes and all ages)

BOTH SEXES

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 1.00						*Gross reproduction rate: 1.50*					
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	49 987	53 543	55 949	58 551	61 788	64 811	81 725	87 397	91 515	95 704	100 494	104 792
5-9	46 603	52 259	55 507	58 450	61 802	64 859	70 777	79 239	84 339	88 748	93 372	97 417
10-14	52 468	56 758	58 790	60 674	63 037	65 246	74 019	79 945	82 977	85 578	88 470	91 034
15-19	59 723	61 801	62 261	62 932	64 241	65 588	78 267	80 861	81 631	82 453	83 752	85 005
20-24	66 873	66 256	65 152	64 744	65 163	65 813	81 408	80 529	79 349	78 799	78 916	79 237
25-29	73 490	70 096	67 579	66 253	65 910	65 968	83 103	79 141	76 456	74 904	74 147	73 778
30-34	79 187	73 461	69 804	67 677	66 597	66 057	83 182	77 046	73 359	71 075	69 597	68 626
35-39	83 259	76 139	71 740	68 951	67 174	66 032	81 242	74 180	70 035	67 267	65 210	63 726
40-44	84 929	77 773	73 150	69 899	67 503	65 791	76 982	70 387	66 336	60 346	60 872	58 979
45-49	83 662	77 879	73 631	70 198	67 327	65 137	70 442	65 474	62 027	59 095	56 400	54 243
50-54	79 290	75 900	72 645	69 416	66 329	63 821	62 017	59 275	56 845	54 282	51 616	49 369
55-59	71 879	71 249	69 585	67 057	64 118	61 565	52 225	51 688	50 581	48 710	46 349	44 240
60-64	61 424	63 317	63 679	62 420	60 133	57 893	41 456	42 669	42 997	42 120	40 378	38 645
65-69	48 168	51 823	54 271	54 736	53 634	52 152	30 198	32 442	34 041	34 310	33 454	32 340
70-74	32 903	37 309	41 354	43 618	44 074	43 737	19 162	21 696	24 096	25 397	25 538	25 193
75-79	18 000	22 046	26 554	29 981	31 837	32 700	9 737	11 909	14 373	16 216	17 137	17 496
80-84	6 968	9 678	13 233	16 539	18 933	20 497	3 501	4 857	6 653	8 309	9 466	10 187
85 +	1 188	2 714	5 119	7 902	10 400	12 331	554	1 265	2 391	3 688	4 830	5 693

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.1 (*continued*)

(For 1 million of both sexes and all ages)

BOTH SEXES

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 2.00						*Gross reproduction rate: 2.50*					
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	109 959	117 025	122 309	127 530	133 252	138 253	134 640	142 539	148 575	154 438	160 681	166 045
5-9	90 376	100 694	106 977	112 235	117 497	121 975	106 261	117 772	124 781	130 510	136 052	140 670
10-14	89 700	96 414	99 884	102 711	105 655	108 170	101 272	108 280	111 879	114 685	117 474	119 790
15-19	90 014	92 549	93 257	93 920	94 923	95 861	97 585	99 808	100 301	100 699	101 345	101 937
20-24	88 856	87 473	86 030	85 185	84 886	84 802	92 500	90 583	88 851	87 700	87 022	86 590
25-29	86 084	81 584	78 670	76 847	75 690	74 932	86 051	81 126	78 016	75 973	74 512	73 473
30-34	81 776	75 376	71 638	69 203	67 423	66 147	78 493	71 974	68 220	65 693	63 734	62 281
35-39	75 800	68 874	64 907	62 158	59 957	58 294	69 864	63 150	59 351	56 660	54 418	52 703
40-44	68 164	62 021	58 346	55 552	53 114	51 201	60 328	54 606	51 231	48 625	46 294	44 449
45-49	59 196	54 752	51 775	49 185	46 704	44 690	50 308	46 291	43 653	41 338	39 089	37 256
50-54	49 460	47 042	45 032	42 876	40 562	38 602	40 363	38 191	36 458	34 604	32 598	30 901
55-59	39 527	38 931	38 029	36 514	34 568	32 829	30 975	30 349	29 565	29 297	26 676	25 232
60-64	29 778	30 500	30 680	29 967	28 582	27 215	22 408	22 831	22 902	22 297	21 179	20 086
65-69	20 586	22 009	23 050	23 165	22 474	21 612	14 875	15 818	16 522	16 553	15 991	15 317
70-74	12 397	13 968	15 485	16 275	16 280	15 978	8 601	9 640	10 657	11 167	11 122	10 873
75-79	5 980	7 276	8 765	9 862	10 368	10 532	3 983	4 823	5 792	6 497	6 802	6 884
80-84	2 041	2 816	3 851	4 796	5 436	5 819	1 304	1 793	2 445	3 034	3 423	3 651
85 +	307	695	1 313	2 020	2 631	3 087	189	425	801	1 228	1 591	1 861

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

Table A.III.1 (*concluded*)

(For 1 million of both sexes and all ages)

Both sexes

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate : 3.00						*Gross reproduction rate : 4.00*					
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	156 244	164 610	171 104	177 354	183 880	189 420	192 233	200 877	207 794	214 382	221 116	226 750
5-9	119 290	131 571	139 106	144 987	150 620	155 242	139 288	152 376	160 223	166 329	171 885	176 364
10-14	109 983	117 025	120 577	123 255	125 810	127 890	121 876	128 623	131 888	134 192	136 258	137 887
15-19	102 524	104 351	104 576	104 693	104 997	105 280	107 820	108 848	108 557	108 175	107 920	107 725
20-24	94 012	91 618	89 617	88 208	87 222	86 519	93 833	90 696	88 288	86 493	85 083	84 010
25-29	84 607	79 376	76 126	73 919	72 244	71 017	80 141	74 574	71 174	68 792	66 881	65 447
30-34	74 659	68 124	64 395	61 835	59 784	58 234	67 114	60 742	57 137	54 613	52 525	50 929
35-39	64 285	57 824	54 197	51 593	49 382	47 675	54 843	48 931	45 639	43 245	41 174	39 567
40-44	53 701	48 370	45 257	42 831	40 636	38 897	43 480	38 844	36 169	34 072	32 159	30 643
45-49	43 320	39 665	37 305	35 227	33 192	31 542	33 287	30 231	28 294	26 596	24 929	23 579
50-54	33 624	31 660	30 143	28 527	26 779	25 305	24 520	22 898	21 695	20 440	19 088	17 953
55-59	24 961	24 338	23 645	22 566	21 200	19 992	17 275	16 704	16 154	15 346	14 340	13 463
60-64	17 469	17 711	17 719	17 203	16 283	15 396	11 473	11 538	11 489	11 100	10 452	9 839
65-69	11 219	11 872	12 366	12 354	11 892	11 359	6 992	7 340	7 608	7 565	7 246	6 887
70-74	6 275	6 998	7 718	8 062	8 003	7 799	3 712	4 106	4 507	4 685	4 628	4 486
75-79	2 813	3 387	4 058	4 537	4 734	4 777	1 578	1 886	2 250	2 503	2 599	2 607
80-84	892	1 218	1 656	2 051	2 307	2 451	475	643	873	1 072	1 203	1 272
85 +	124	280	526	801	1 038	1 207	63	142	262	398	513	592

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.2. — DISTRIBUTION BY SEX AND AGE GROUPS OF FORTY-EIGHT STABLE POPULATIONS CALCULATED BY ASSOCIATING SIX LEVELS OF INTERMEDIATE MODEL LIFE TABLE WITH EIGHT LEVELS OF THE INTRINSIC RATE OF NATURAL VARIATION

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						Expectation of life at birth for both sexes (in years)					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: — 1 per cent						Intrinsic rate of natural variation: 0 per cent					
MALES												
All ages	506 328	503 728	500 741	496 494	495 682	497 329	507 272	505 299	502 820	499 267	498 867	500 625
0-4	61 891	45 788	36 554	30 642	26 735	24 139	78 020	60 293	49 899	43 067	38 459	35 338
5-14	98 481	82 762	70 704	61 879	55 704	51 471	15 250	101 134	89 544	80 679	74 329	69 897
15-29	138 864	124 034	110 508	99 665	91 813	86 362	143 624	133 893	123 583	114 702	108 107	103 464
30-44	112 746	113 284	109 553	104 602	100 556	97 618	100 606	105 384	105 518	103 652	101 931	100 676
45-59	67 164	86 597	97 214	101 779	104 014	105 172	51 816	69 543	80 759	86 942	90 865	93 457
60-74	24 426	43 867	61 563	74 760	84 758	92 085	16 339	30 538	44 296	55 266	64 030	70 721
75 +	2 756	7 396	14 645	23 167	32 102	40 482	1 617	4 514	9 221	14 959	21 146	27 072
FEMALES												
All ages	493 672	496 270	499 262	503 507	504 319	502 672	492 726	494 700	497 180	500 733	501 132	499 375
0-4	60 863	44 747	35 620	29 837	25 848	23 143	76 723	58 922	48 624	41 935	37 184	33 885
5-14	96 193	80 675	68 886	60 350	54 002	49 463	112 578	98 586	87 243	78 684	72 058	67 170
15-29	132 699	119 316	106 948	97 134	89 248	83 273	137 295	128 828	119 614	111 786	105 082	99 759
30-44	104 399	107 013	105 284	102 232	98 333	94 595	93 170	99 554	101 403	101 294	99 670	97 552
45-59	66 690	85 754	96 765	102 504	104 228	103 864	51 365	68 791	80 323	87 506	91 003	92 255
60-74	28 894	49 111	67 400	82 139	92 091	98 057	19 289	34 136	48 432	60 640	69 472	75 210
75 +	3 934	9 654	18 359	29 311	40 569	50 277	2 306	5 883	11 541	18 888	26 663	33 544
BOTH SEXES												
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	122 754	90 535	72 174	60 479	52 583	47 282	154 743	119 215	98 523	85 002	75 643	69 223
5-14	194 674	163 437	139 590	122 229	109 706	100 934	227 828	199 720	176 787	159 363	146 387	137 067
15-29	271 563	243 350	217 456	196 799	181 061	169 635	280 919	262 721	243 197	266 488	213 189	203 223
30-44	217 145	220 297	214 837	206 834	198 889	192 213	193 776	204 938	206 921	204 946	201 601	198 228
45-49	133 854	172 351	193 979	204 283	208 242	209 036	103 181	138 334	161 082	174 448	181 868	185 712
60-74	53 320	92 978	128 963	156 899	176 849	190 142	35 628	64 674	92 728	115 906	133 502	145 931
75 +	6 690	17 050	33 004	52 478	72 671	90 759	3 923	10 397	20 762	33 847	47 809	60 616

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.2 *(continued)*

(Summary table for 1 million of both sexes and all age)

Sex and age group (in years)	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 0.5 per cent						*Intrinsic rate of natural variation: 1 per cent*					
MALES												
All ages	507 553	505 877	503 648	500 433	500 228	502 041	507 733	506 333	504 349	501 449	501 434	503 301
0-4	86 550	68 188	57 343	50 143	45 251	41 914	95 307	76 423	65 217	57 719	52 594	49 079
5-14	123 211	110 206	99 139	90 495	84 248	79 860	130 778	119 032	108 649	100 367	94 343	90 092
15-29	144 406	137 187	128 624	120 925	115 151	111 077	144 185	139 359	132 545	126 085	121 210	117 772
30-44	93 956	100 236	101 920	101 398	100 739	100 280	87 136	94 523	97 470	98 102	98 388	98 652
45-59	44 995	61 457	72 444	78 967	83 366	86 410	38 799	53 846	64 341	70 936	75 584	78 905
60-74	13 210	25 126	36 979	46 695	54 628	60 788	10 607	20 495	30 563	39 018	46 056	51 603
75 +	1 225	3 477	7 199	11 810	16 845	21 712	921	2 655	5 564	9 222	13 259	17 198
FEMALES												
All ages	492 448	494 122	496 351	499 566	499 769	497 956	492 269	493 669	405 649	498 553	498 565	496 698
0-4	85 111	66 637	55 878	48 825	43 751	40 191	93 723	74 685	63 550	56 202	50 851	47 060
5-14	120 360	107 431	96 592	88 256	81 673	76 744	127 755	116 038	105 859	97 885	91 461	86 577
15-29	138 067	132 011	124 501	117 848	111 926	107 095	137 880	134 115	128 302	122 875	117 812	113 549
30-44	87 017	94 693	97 945	99 085	98 500	97 168	80 706	89 298	93 666	95 861	96 195	95 585
45-59	44 567	60 759	72 026	79 453	83 471	85 280	38 397	53 205	63 945	71 350	75 660	77 858
60-74	15 580	28 064	40 405	51 201	59 231	64 605	12 496	22 873	33 374	42 758	49 905	54 808
75 +	1 746	4 527	9 004	14 898	21 217	26 873	1 312	3 455	6 953	11 622	16 681	21 261
BOTH SEXES												
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	171 661	134 825	113 221	98 968	89 002	82 105	189 030	151 108	128 767	113 921	103 445	96 139
5-14	243 571	217 637	195 731	178 751	165 921	156 604	258 533	235 070	214 508	198 252	185 804	176 669
15-29	282 473	269 198	253 125	238 773	227 077	218 172	282 065	273 474	260 847	248 960	239 022	231 321
30-44	180 973	194 929	199 865	200 483	199 239	197 448	167 842	183 821	191 136	193 963	194 583	194 237
45-59	89 562	122 216	144 470	158 420	166 837	171 690	77 690	107 051	128 286	142 286	151 244	156 763
60-74	28 790	53 190	77 384	97 896	113 859	125 393	23 103	43 368	63 937	81 776	95 961	106 411
75 +	2 971	8 004	16 203	26 708	38 062	48 585	2 233	6 110	12 517	20 844	29 940	38 459

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.2 (continued)

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 1.5 per cent						*Intrinsic rate of natural variation: 2 per cent*					
MALES												
All ages	507 823	506 692	504 930	502 323	502 487	504 404	507 845	506 935	505 405	503 070	503 395	505 355
0-4	104 230	84 933	73 450	65 720	60 416	56 762	113 265	93 651	81 969	74 071	68 639	64 888
5-14	137 879	127 505	117 928	110 128	104 434	100 405	144 461	135 527	126 853	119 630	114 351	110 615
15-29	143 039	140 430	135 312	130 099	126 155	123 392	141 054	140 460	136 936	132 933	129 912	127 832
30-44	80 289	88 421	92 344	93 927	95 012	95 901	73 542	82 098	86 727	89 054	90 782	92 182
45-59	33 241	46 799	56 610	63 058	67 757	71 197	28 307	40 369	49 376	55 510	60 100	63 523
60-74	8 458	16 583	25 025	32 266	38 394	43 287	6 706	13 318	20 312	26 422	31 666	35 901
75 +	687	2 011	4 261	7 125	10 319	13 460	510	1 512	3 232	5 450	7 945	10 414
FEMALES												
All ages	492 178	493 318	495 068	497 674	497 511	495 595	492 153	493 062	494 593	496 930	496 606	494 648
0-4	102 498	83 001	71 574	63 991	58 413	54 429	111 382	91 520	79 876	72 124	66 365	62 218
5-14	134 697	124 299	114 902	107 405	101 242	96 487	141 133	132 124	123 598	116 672	110 855	106 297
15-29	136 806	135 161	130 985	126 789	122 613	118 963	134 933	135 203	132 563	129 550	126 261	123 242
30-44	74 369	83 535	88 739	91 778	92 888	92 913	68 124	77 564	83 340	87 013	88 754	89 309
45-59	32 871	46 217	56 240	63 408	67 809	70 238	27 969	39 849	49 033	55 801	60 129	62 652
60-74	9 958	18 492	27 308	35 333	41 575	45 945	7 888	14 841	22 149	28 914	34 268	38 081
75 +	979	2 613	5 320	8 970	12 971	16 620	724	1 961	4 034	6 856	9 974	12 849
BOTH SEXES												
All ages[a]	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	206 728	167 934	145 024	129 711	118 829	111 191	224 647	185 171	161 845	146 195	135 004	127 106
5-14	272 576	251 804	232 830	217 533	205 676	196 892	285 594	267 651	250 451	236 302	225 206	216 912
15-29	279 845	275 591	266 297	256 888	248 768	242 355	275 987	275 663	269 499	262 483	256 173	251 074
30-44	154 658	171 956	181 083	185 705	187 900	188 814	141 666	159 662	170 067	176 067	179 536	181 491
45-49	66 112	93 016	112 850	126 466	135 566	141 435	56 276	80 218	98 409	111 311	120 229	126 175
60-74	18 416	35 075	52 333	67 599	79 969	89 232	14 594	28 159	42 461	55 336	65 934	73 982
75 +	1 666	4 624	9 581	16 095	23 290	30 080	1 234	3 473	7 266	12 306	17 919	23 263

[a] Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.2 *(continued)*

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						Expectation of life at birth for both sexes (in years)					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 3 per cent						Intrinsic rate of natural variation: 4 per cent					
MALES												
All ages	507 736	507 231	506 084	504 431	505 105	507 155	507 502	507 319	506 490	505 007	505 829	507 889
0-4	131 463	111 463	99 583	91 548	86 010	82 167	149 554	129 428	117 550	109 466	103 939	100 117
5-14	155 947	149 971	143 249	137 417	133 157	130 163	165 106	162 007	157 272	152 805	149 635	147 562
15-29	134 988	137 728	136 970	135 206	133 910	133 118	126 822	131 972	133 333	133 262	133 336	133 598
30-44	60 714	69 372	74 712	77 991	80 558	82 633	49 206	57 280	62 635	66 183	69 055	71 392
45-59	20 202	29 445	36 685	41 905	45 958	49 064	14 152	20 984	26 524	30 653	33 946	36 521
60-74	4 147	8 417	13 067	17 256	20 937	23 962	2 516	5 195	8 181	10 922	13 373	15 414
75 +	275	835	1 818	3 108	4 575	6 048	146	453	995	1 716	2 545	3 385
FEMALES												
All ages	492 263	492 768	493 913	495 568	494 897	492 848	492 495	492 680	493 509	494 992	494 171	492 111
0-4	129 278	108 928	97 038	89 142	83 159	78 788	147 069	126 486	114 545	106 589	100 495	95 999
5-14	152 367	146 209	139 577	134 021	129 085	125 080	161 327	157 952	153 245	149 035	145 059	141 704
15-29	129 178	132 605	132 610	131 764	130 136	128 328	121 406	127 089	129 099	129 865	129 572	128 788
30-44	56 249	65 543	71 790	76 194	78 749	80 052	45 594	54 123	60 189	64 654	67 495	69 160
45-59	19 929	29 033	36 402	42 100	45 958	48 373	13 940	20 672	26 301	30 777	33 934	35 992
60-74	4 869	9 366	14 231	18 861	22 627	25 387	2 952	5 775	8 896	11 923	14 432	16 310
75 +	393	1 084	2 265	3 486	5 183	6 840	207	583	1 234	2 149	3 184	4 158
BOTH SEXES												
All ages[a]	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	260 741	220 391	196 621	180 690	169 169	160 955	296 623	255 914	232 095	216 055	204 434	196 116
5-14	308 314	296 180	282 826	271 438	262 242	255 243	326 433	319 959	310 517	301 840	294 694	289 166
15-29	264 166	270 333	269 580	266 970	264 046	261 446	248 228	259 061	262 432	263 127	262 908	262 386
30-44	116 963	134 915	146 502	154 185	159 307	162 685	94 800	111 403	122 824	130 837	136 550	140 552
45-49	40 131	58 478	73 087	84 005	91 916	97 437	28 092	41 656	52 825	61 430	67 880	72 513
60-74	9 016	17 783	27 298	36 117	43 564	49 349	5 468	10 970	17 077	22 845	27 805	31 724
75 +	668	1 919	4 083	6 594	9 758	12 888	353	1 036	2 229	3 865	5 729	7 543

[a] Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.2 (continued)

(For 1 million of both sexes and all ages)

MALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: — 1 per cent						*Intrinsic rate of natural variation: 0 per cent*					
All ages	506 328	503 728	500 741	496 494	495 682	497 329	507 272	505 299	502 820	499 267	498 867	500 625
0-4	61 891	45 788	36 554	30 642	26 735	24 139	78 020	60 293	49 899	43 067	38 459	35 338
5-9	49 880	41 285	34 962	30 425	27 283	25 140	59 813	51 714	45 397	40 677	37 334	35 013
10-14	48 601	41 477	35 742	31 454	28 421	26 331	55 437	49 420	44 147	40 002	36 995	34 884
15-19	47 941	41 815	36 559	32 495	29 579	27 557	52 017	47 392	42 953	39 310	36 624	34 727
20-24	46 537	41 515	36 940	33 277	30 624	28 780	48 031	44 758	41 285	38 293	36 069	34 500
25-29	44 386	40 704	37 009	33 893	31 610	30 025	43 576	41 743	39 345	37 099	35 414	34 237
30-34	41 555	39 559	36 939	34 467	32 604	31 298	38 809	38 590	37 355	35 888	34 746	33 948
35-39	37 898	37 970	36 646	34 943	33 563	32 566	33 666	35 234	35 251	34 609	34 024	33 600
40-44	33 293	35 755	35 968	35 192	34 389	33 754	28 133	31 560	32 912	33 155	33 161	33 128
45-49	27 918	32 782	34 712	35 023	34 916	34 726	22 440	27 525	30 213	31 387	32 028	32 420
50-54	22 312	29 088	32 708	34 219	34 929	35 281	17 060	23 232	27 081	29 171	30 477	31 331
55-59	16 934	24 727	29 794	32 537	34 169	35 165	12 316	18 786	23 465	26 384	28 360	29 706
60-64	12 063	19 823	25 837	29 683	32 276	34 006	18 346	14 326	19 356	22 896	25 482	27 325
65-69	7 889	14 613	20 808	25 403	28 847	31 335	5 192	10 045	14 828	18 639	21 664	23 951
70-74	4 474	9 431	14 918	19 674	23 635	26 744	2 801	6 167	10 112	13 737	16 884	19 445
75-79	2 024	4 980	8 989	13 100	16 936	20 221	1 205	3 098	5 796	8 697	11 509	13 985
80-84	648	1 950	4 187	6 953	9 911	12 704	367	1 154	2 568	4 391	6 406	8 358
85 +	84	466	1 469	3 114	5 255	7 557	45	262	857	1 871	3 231	4 729

TABLE A.III.2 (continued)

(For 1 million of both sexes and all ages)

MALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 0.5 per cent						*Intrinsic rate of natural variation: 1 per cent*					
All ages	507 553	505 877	503 648	500 433	500 228	502 041	507 733	506 333	504 349	501 449	501 434	503 301
0-4	86 550	68 188	57 343	50 143	45 251	41 914	95 307	76 423	65 217	57 719	52 594	49 879
5-9	64 713	57 041	50 881	46 192	42 843	40 503	69 502	62 352	56 440	51 857	48 565	46 255
10-14	58 498	53 165	48 258	44 303	41 405	39 357	61 276	56 680	52 209	48 510	45 778	43 637
15-19	53 534	49 724	45 794	42 463	39 979	38 214	54 691	51 704	48 319	45 345	43 108	41 512
20-24	48 212	45 802	42 929	40 343	38 400	37 026	48 038	46 448	44 178	42 018	40 385	39 229
25-29	42 660	41 661	39 901	38 119	36 772	35 837	41 456	41 207	40 048	38 722	37 717	37 031
30-34	37 054	37 564	36 948	35 965	35 188	34 656	35 119	36 236	36 169	35 631	35 201	34 927
35-39	31 351	33 450	34 006	33 827	33 606	33 454	28 980	31 472	32 467	32 685	32 788	32 884
40-44	25 551	29 222	30 966	31 606	31 945	32 170	23 037	26 815	28 834	29 786	30 399	30 841
45-49	19 878	24 857	27 725	29 182	30 092	30 705	17 479	22 247	25 179	26 822	27 928	28 710
50-54	14 739	20 462	24 237	26 451	27 928	28 942	12 640	17 861	21 468	23 712	25 280	26 392
55-59	10 378	16 138	20 482	23 334	25 346	26 763	8 680	13 738	17 694	20 402	22 376	23 803
60-64	6 859	12 002	16 478	19 749	22 212	24 011	5 595	9 965	13 883	16 840	19 124	20 827
65-69	4 162	8 209	12 312	15 680	18 417	20 525	3 312	6 648	10 118	13 040	15 466	17 365
70-74	2 189	4 915	8 189	11 266	13 999	16 252	1 700	3 882	6 562	9 138	11 466	13 411
75-79	919	2 408	4 578	6 959	9 307	11 400	695	1 855	3 578	5 506	7 434	9 175
80-84	273	875	1 978	3 427	5 053	6 645	202	658	1 508	2 644	3 936	5 216
85 +	33	194	643	1 424	2 485	3 667	24	142	478	1 072	1 889	2 807

TABLE A.III.2 (continued)

(For 1 million of both sexes and all ages)

MALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 1.5 per cent						*Intrinsic rate of natural variation: 2 per cent*					
All ages	507 823	506 682	504 930	502 323	502 487	504 404	507 845	506 935	505 405	503 070	503 395	505 355
0-4	104 230	84 933	73 450	65 720	60 416	56 762	113 265	93 651	81 969	74 071	68 639	64 888
5-9	74 134	67 585	61 996	57 588	54 412	52 177	78 570	72 681	67 478	63 302	60 292	58 172
10-14	63 745	59 920	55 932	52 540	50 022	48 228	65 891	62 846	59 375	56 328	54 059	52 443
15-19	55 491	53 309	50 487	47 901	45 943	44 543	55 943	54 533	52 273	50 085	48 424	47 239
20-24	47 537	46 708	45 020	43 289	41 976	41 052	46 741	46 601	45 462	44 148	43 152	42 464
25-29	40 011	40 413	39 805	38 909	38 236	37 797	38 370	39 326	39 201	38 700	38 336	38 130
30-34	33 058	34 661	35 061	34 919	34 805	34 770	30 919	32 896	33 677	33 874	34 033	34 211
35-39	26 605	29 361	30 695	31 242	31 618	31 927	24 271	27 176	28 757	29 558	30 155	30 638
40-44	20 626	24 399	26 588	27 766	28 589	29 204	18 352	22 026	24 293	25 622	26 594	27 333
45-49	15 264	19 742	22 644	24 387	25 618	26 514	23 244	17 382	20 179	21 948	23 241	24 202
50-54	10 766	15 459	18 829	21 027	22 615	23 772	9 111	13 275	16 366	18 457	20 010	21 161
55-59	7 211	11 598	15 137	17 644	19 524	20 911	5 952	9 712	12 831	15 105	16 849	18 157
60-64	4 533	8 205	11.584	14 205	16 275	17 845	3 649	6 703	9 578	11 861	13 699	15 112
65-69	2 676	5 337	8 233	10 729	12 837	14 512	2 054	4 252	6 638	8 737	10 537	11 985
70-74	1 309	3 041	5 208	7 332	9 282	10 930	1 003	2 363	4 096	5 824	7 430	8 804
75-79	523	1 417	2 769	4 309	5 870	7 293	391	1 074	2 124	3 338	4 584	5 729
80-84	147	490	1 139	2 018	3 030	4 044	107	363	851	1 524	2 308	3 098
85 +	17	104	353	798	1 419	2 123	12	75	257	588	1 053	1 587

TABLE A.III.2 *(continued)*

(For 1 million of both sexes and all ages)

MALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 3 per cent						*Intrinsic rate of natural variation: 4 per cent*					
All ages	507 736	507 231	506 084	504 431	505 105	507 155	507 502	507 319	506 490	505 007	505 829	507 889
0-4	131 463	111 463	99 583	91 548	86 010	82 167	149 554	129 428	117 550	109 466	103 939	100 117
5-9	86 747	82 288	77 979	74 423	71 865	70 074	93 873	90 892	87 560	84 650	82 612	81 214
10-14	69 200	67 683	65 270	62 994	61 292	60 089	71 233	71 115	69 712	68 155	67 023	66 248
15-19	55 887	55 865	54 659	53 281	52 226	51 487	54 723	55 834	55 534	54 837	54 323	53 996
20-24	44 416	45 411	45 219	44 673	44 271	44 026	41 370	43 172	43 700	43 735	43 803	43 920
25-29	34 685	36 452	37 092	37 252	37 413	37 605	30 729	32 966	34 099	34 690	35 210	35 682
30-34	26 586	29 005	30 310	31 017	31 593	32 093	22 405	24 951	26 504	27 476	28 283	28 969
35-39	19 951	22 794	24 618	25 745	26 628	27 339	15 914	18 652	20 479	21 692	22 677	23 474
40-44	14 277	17 573	19 784	21 229	22 337	23 201	10 887	13 677	15 652	17 015	18 095	18 949
45-49	9 802	13 192	15 632	17 297	18 569	19 542	7 110	9 767	11 765	13 189	14 307	15 183
50-54	6 415	9 583	12 059	13 837	15 208	16 256	4 426	6 750	8 633	10 034	11 147	12 014
55-59	3 985	6 670	8 994	10 771	12 181	13 266	2 616	4 467	6 126	7 430	8 492	9 324
60-64	2 325	4 378	6 385	8 045	9 421	10 504	1 451	2 790	4 137	5 281	6 249	7 023
65-69	1 244	2 642	4 210	5 637	6 892	7 922	739	1 601	2 596	3 520	4 349	5 041
70-74	578	1 397	2 742	3 574	4 624	5 536	326	804	1 448	2 121	2 775	3 350
75-79	214	604	1 220	1 948	2 712	3 426	114	332	680	1 101	1 547	1 973
80-84	56	193	465	848	1 299	1 762	30	102	247	455	705	964
85 +	5	38	133	312	564	860	2	19	68	160	293	448

TABLE A.III.2 (continued)

(For 1 million of both sexes and all ages)

FEMALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: — 1 per cent						*Intrinsic rate of natural variation: 0 per cent*					
All ages	493 672	496 270	499 262	503 507	504 319	502 672	492 726	494 700	497 180	500 733	501 132	499 375
0-4	60 863	44 747	35 620	29 837	25 848	23 143	76 723	58 922	48 624	41 935	37 184	33 885
5-9	48 867	40 313	34 094	29 683	26 443	24 147	58 597	50 495	44 270	39 684	36 185	33 631
10-14	47 326	40 362	34 792	30 667	27 559	25 316	53 981	48 091	42 973	39 000	35 873	33 539
15-19	46 276	40 474	35 480	31 655	28 699	26 525	50 210	45 873	41 686	38 293	35 535	33 427
20-24	44 492	39 953	35 757	32 429	29 764	27 749	45 920	43 074	39 963	37 317	35 056	33 265
25-29	41 931	38 889	35 711	33 050	30 785	28 999	41 165	39 881	37 965	36 176	34 491	33 067
30-34	38 699	37 470	35 500	33 612	31 801	30 269	36 140	36 552	35 900	34 997	33 891	32 832
35-39	34 910	35 755	35 151	34 113	32 799	31 544	31 012	33 178	33 813	33 787	33 250	32 546
40-44	30 790	33 788	34 633	34 507	33 733	32 782	26 018	29 824	31 690	32 510	32 529	32 174
45-49	26 551	31 524	33 822	34 666	34 493	33 895	21 342	26 469	29 439	31 067	31 639	31 644
50-54	22 231	28 787	32 500	34 390	34 924	34 757	16 998	22 992	26 908	29 316	30 472	30 866
55-59	17 908	25 443	30 443	33 448	34 811	35 212	13 025	19 330	23 976	27 123	28 892	29 745
60-64	13 628	21 348	27 324	31 470	33 815	34 932	9 428	15 428	20 470	24 274	26 692	28 070
65-69	9 494	16 506	22 886	27 985	31 365	33 355	6 248	11 347	16 309	20 534	23 555	25 495
70-74	5 772	11 257	17 190	22 684	26 911	29 770	3 613	7 361	11 653	15 832	19 225	21 645
75-79	2 813	6 308	10 894	15 886	20 361	23 792	1 675	3 924	7 025	10 547	13 836	16 455
80-84	967	2 626	5 369	8 967	12 748	16 033	548	1 554	3 293	5 663	8 240	10 548
85 +	154	720	2 096	4 458	7 460	10 452	83	405	1 223	2 678	4 587	6 541

TABLE A.III.2 (continued)

(For 1 million of both sexes and all ages)

FEMALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 0.5 per cent						*Intrinsic rate of natural variation: 1 per cent*					
All ages	492 448	494 122	496 351	499 566	499 769	497 956	492 269	493 669	495 649	498 553	498 565	496 698
0-4	85 111	66 637	55 878	48 825	43 751	40 191	93 723	74 685	63 550	56 202	50 851	47 060
5-9	63 398	55 696	49 617	45 063	41 524	38 904	68 089	60 882	55 038	50 590	47 071	44 430
10-14	56 962	51 735	46 975	43 193	40 149	37 840	59 666	55 156	50 821	47 295	44 390	42 147
15-19	51 674	48 131	44 444	41 364	38 790	36 782	52 791	50 047	46 895	44 171	41 827	39 958
20-24	46 093	44 078	41 555	39 314	37 322	35 701	45 926	44 700	42 763	40 946	39 251	37 825
25-29	40 300	39 802	38 502	37 170	35 814	34 612	39 163	39 368	38 644	37 758	36 734	35 766
30-34	34 507	35 580	35 510	35 072	24 322	33 518	32 705	34 323	34 760	34 746	34 335	33 780
35-39	28 880	31 499	32 619	33 023	32 841	32 406	26 696	29 635	31 142	31 909	32 042	31 853
40-44	23 630	27 614	29 816	30 990	31 337	31 244	21 305	25 340	27 764	29 206	29 818	29 952
45-49	18 905	23 903	27 015	28 883	29 727	29 970	16 623	21 393	24 534	26 548	27 589	28 023
50-54	14 686	20 251	24 083	26 583	27 923	28 512	12 594	17 676	21 331	23 830	25 275	26 000
55-59	10 976	16 605	20 928	23 987	25 821	26 798	9 180	14 136	18 080	20 972	22 796	23 835
60-64	7 748	12 925	17 427	20 937	23 266	24 665	6 320	10 732	14 683	17 854	20 033	21 395
65-69	5 008	9 272	13 541	17 274	20 025	21 849	3 984	7 508	11 128	14 367	16 816	18 485
70-74	2 824	5 867	9 437	12 990	15 940	18 091	2 192	4 633	7 563	10 537	13 056	14 928
75-79	1 277	3 050	5 548	8 440	11 189	13 414	967	2 350	4 336	6 677	8 937	10 795
80-84	408	1 178	2 537	4 420	6 499	8 387	301	885	1 934	3 411	5 063	6 583
85 +	61	299	919	2 038	3 529	5 072	44	220	683	1 534	2 681	3 883

TABLE A.III.2 *(continued)*

(For 1 million of both sexes and all ages)

FEMALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 1.5 per cent						*Intrinsic rate of natural variation: 2 per cent*					
All ages	492 178	493 318	495 068	497 674	497 511	495 595	492 153	493 062	494 593	496 930	496 606	494 648
0-4	102 498	83 001	71 574	63 991	58 413	54 429	111 382	91 520	79 876	72 124	66 365	62 218
5-9	72 626	65 991	60 457	56 182	52 737	50 118	76 973	70 968	65 803	61 756	58 436	55 877
10-14	62 071	58 308	54 445	51 223	48 505	46 369	64 160	61 156	57 795	54 916	52 419	50 420
15-19	53 563	51 600	48 998	46 661	44 576	42 875	53 999	52 785	50 730	48 791	46 983	45 471
20-24	45 446	44 950	43 579	42 187	40 797	39 583	44 686	44 847	44 006	43 021	41 940	40 944
25-29	37 797	38 611	38 408	37 941	37 240	36 505	36 248	37 571	37 827	37 738	37 338	36 827
30-34	30 786	32 832	33 695	34 053	33 947	33 626	28 794	31 159	32 366	33 033	33 197	33 086
35-39	24 508	27 647	29 443	30 499	30 897	30 925	22 357	25 591	27 583	28 857	29 469	29 677
40-44	19 075	23 056	25 601	27 226	28 044	28 362	16 973	20 814	23 391	25 123	26 088	26 546
45-49	14 518	18 985	22 063	24 138	25 306	25 880	12 597	16 716	19 661	21 724	22 958	23 623
50-54	10 727	15 299	18 710	21 131	22 612	23 419	9 078	13 138	16 261	18 549	20 007	20 849
55-59	7 626	11 933	15 467	18 139	19 891	20 939	6 294	9 995	13 111	15 528	17 164	18 180
60-64	5 121	8 836	12 250	15 060	17 048	18 331	4 122	7 217	10 128	12 575	14 350	15 524
65-69	3 148	6 028	9 055	11 819	13 958	15 447	2 472	4 804	7 301	9 624	11 456	12 757
70-74	1 689	3 628	6 003	8 454	10 569	12 167	1 294	2 820	4 720	6 715	8 462	9 800
75-79	727	1 794	3 358	5 225	7 057	8 580	543	1 360	2 575	4 048	5 511	6 743
80-84	221	660	1 459	2 603	3 900	5 103	160	487	1 091	1 967	2 968	3 911
85 +	31	159	503	1 142	2 014	2 937	21	114	368	841	1 495	2 190

Table A.III.2 (continued)

(For 1 million of both sexes and all ages)

Female

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 3 per cent						*Intrinsic rate of natural variation: 4 per cent*					
All ages	492 263	492 768	493 913	495 568	494 897	492 848	492 495	492 680	493 509	494 992	494 171	492 111
0-4	129 278	108 928	97 038	89 142	83 159	78 788	147 069	126 486	114 545	106 589	100 495	95 999
5-9	84 983	80 347	76 043	72 606	69 652	67 307	91 965	88 749	85 386	82 585	80 070	78 011
10-14	67 384	65 862	63 534	61 415	59 433	57 773	69 362	69 203	67 859	66 450	64 989	63 693
15-19	53 946	54 075	53 047	51 904	50 673	49 560	52 823	54 045	53 896	53 418	52 707	51 976
20-24	42 465	43 704	43 772	43 534	43 027	42 449	39 553	41 549	42 302	42 620	42 572	42 348
25-29	32 767	34 826	35 791	36 326	36 436	36 319	29 030	31 495	32 901	33 827	34 293	34 464
30-34	24 759	27 473	29 129	30 247	30 815	31 038	20 865	23 635	25 473	26 793	27 587	28 017
35-39	18 286	21 463	23 613	25 133	26 022	26 482	14 660	17 563	19 644	21 178	22 160	22 738
40-44	13 204	16 607	19 048	20 814	21 912	22 532	10 069	12 925	15 072	16 683	17 748	18 405
45-49	9 323	12 685	15 230	17 120	18 342	19 075	6 762	9 393	11 463	13 054	14 134	14 819
50-54	6 391	9 485	11 982	13 906	15 206	16 015	4 411	6 681	8 579	10 085	11 147	11 836
55-59	4 215	6 863	9 190	11 074	12 410	13 283	2 767	4 598	6 259	7 638	8 653	9 337
60-64	2 626	4 715	6 752	8 530	9 867	10 790	1 640	3 004	4 374	5 599	6 544	7 214
65-69	1 498	2 985	4 631	6 209	7 495	8 434	890	1 810	2 853	3 877	4 728	5 367
70-74	745	1 666	2 848	4 122	5 265	6 163	422	967	1 669	2 447	3 160	3 729
75-79	298	765	1 478	1 948	2 712	3 426	159	4 20	823	1 334	1 861	2 320
80-84	84	261	596	1 093	1 671	2 226	42	135	315	587	908	1 218
85 +	11	58	191	445	800	1 188	6	28	96	228	415	620

TABLE A.III.2 *(continued)*

(For 1 million of both sexes and all ages)

BOTH SEXES

Age group	*Mortality evel*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: — 1 per cent						*Intrinsic rate of natural variation: 0 per cent*					
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	122 754	90 535	72 174	60 479	52 583	47 282	154 743	119 215	98 523	85 002	75 643	69 223
5-9	98 747	81 598	69 056	60 108	53 726	49 287	118 410	102 209	89 667	80 361	73 519	68 644
10-14	95 927	81 839	70 534	62 121	55 980	51 647	109 418	97 511	87 120	79 002	72 868	68 423
15-19	94 217	82 289	72 039	64 150	58 278	54 082	102 227	93 265	84 639	77 603	72 159	68 154
20-24	91 029	81 468	72 697	65 706	60 388	56 529	93 951	87 832	81 248	75 610	71 125	67 765
25-29	86 317	79 593	72 720	66 943	62 395	59 024	84 741	81 624	77 310	73 275	69 905	67 304
30-34	80 254	77 029	72 439	68 079	64 405	61 567	74 947	75 142	73 255	70 885	58 637	66 780
35-39	72 808	73 725	71 797	69 056	66 362	64 110	64 678	68 412	69 064	68 396	62 274	66 146
40-44	64 083	69 543	70 601	69 699	68 122	66 536	54 151	61 384	64 602	65 665	65 690	65 302
45-49	54 469	64 306	68 534	69 689	69 409	68 621	43 782	53 994	59 652	62 454	63 667	64 064
50-54	44 543	57 875	65 208	68 609	69 853	70 038	34 058	46 224	53 989	58 487	60 949	62 197
55-59	34 842	50 170	60 237	65 985	68 980	70 377	25 341	38 116	47 441	53 507	57 252	59 451
60-64	25 691	41 171	53 161	61 153	66 091	68 938	17 774	29 754	39 826	47 170	52 174	55 395
65-69	17 383	31 119	43 694	53 388	60 212	64 690	11 440	21 392	31 137	39 173	45 219	49 446
70-74	10 246	20 688	32 108	42 358	50 546	56 514	6 414	13 528	21 765	29 563	36 109	41 090
75-79	4 837	11 288	19 883	28 986	37 297	44 103	2 880	7 022	12 821	19 244	25 345	30 440
80-84	1 615	4 576	9 556	15 920	22 659	28 737	915	2 708	5 861	10 054	14 646	18 906
85 + ...	238	1 186	3 565	7 572	12 715	18 009	128	667	2 080	4 549	7 818	11 270

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.2 (*continued*)

(*For 1 million of both sexes and all ages*)

BOTH SEXES

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 0.5 per cent						*Intrinsic rate of natural variation: 1 per cent*					
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	171 661	134 825	113 221	98 968	89 002	82 105	189 030	151 108	128 767	113 921	103 445	96 139
5-9	128 111	112 737	100 498	91 255	84 367	79 407	137 591	123 234	111 478	102 447	95 636	90 685
10-14	115 460	104 900	95 233	87 496	81 554	77 197	120 942	111 836	103 030	95 805	90 168	85 984
15-19	105 208	97 855	90 238	83 827	78 769	74 996	107 482	101 751	95 214	89 516	84 935	81 470
20-24	94 305	89 880	84 484	79 657	75 722	72 727	93 960	91 148	86 941	82 964	79 636	77 054
25-29	82 960	81 463	78 403	75 289	72 586	70 449	80 619	80 575	78 692	76 480	74 451	72 797
30-34	71 561	73 144	72 458	71 037	69 510	68 174	67 824	70 559	70 929	70 377	69 536	68 707
35-39	60 231	64 949	66 625	66 850	66 447	65 860	55 676	61 107	63 609	64 594	64 830	64 737
40-44	49 181	56 836	60 782	62 596	63 282	63 414	44 342	52 155	56 598	58 992	60 217	60 793
45-49	38 783	48 760	54 740	58 065	59 819	60 675	34 102	43 640	49 713	53 370	55 517	56 733
50-54	29 425	40 713	48 320	53 034	55 851	57 454	25 234	35 537	42 799	47 542	50 555	52 392
55-59	21 354	32 743	41 410	47 321	51 167	53 561	17 860	27 874	35 774	41 374	45 172	47 638
60-64	14 607	24 927	33 905	40 686	45 478	48 676	11 915	20 697	28 566	34 694	39 157	42 222
65-69	9 270	17 481	25 853	32 954	38 442	42 374	7 296	14 156	21 246	27 407	32 282	35 850
70-74	5 013	10 782	17 626	24 256	29 939	34 343	3 892	8 515	14 125	19 675	24 522	28 339
75-79	2 196	5 458	10 126	15 399	20 496	24 814	1 662	4 205	7 914	12 183	16 372	19 970
80-84	681	2 053	4 515	7 847	11 552	15 032	503	1 543	3 442	6 055	8 999	11 799
85 +	94	493	1 562	3 462	6 014	8 739	68	362	1 161	2 606	4 570	6 690

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.2 *(continued)*

(For 1 million of both sexes and all ages)

BOTH SEXES

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 1.5 per cent						*Intrinsic rate of natural variation: 2 per cent*					
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	206 728	167 934	145 024	129 711	118 829	111 191	224 647	185 171	161 845	146 195	135 004	127 106
5-9	146 760	133 576	122 453	113 770	107 149	102 295	155 543	143 649	133 281	125 058	118 728	114 049
10-14	125 816	118 228	110 377	103 763	98 527	94 597	130 051	124 002	117 170	111 244	106 478	102 863
15-19	109 054	104 909	99 485	94 562	90 519	87 418	109 942	107 318	103 003	98 876	95 407	92 710
20-24	92 983	91 658	88 599	85 476	82 773	80 635	91 427	91 448	89 468	87 169	85 092	83 407
25-29	77 808	79 024	78 213	76 850	75 476	74 302	74 618	76 897	77 028	76 438	75 674	74 957
30-34	63 844	67 493	68 756	68 972	68 752	68 396	59 713	64 055	66 043	66 907	67 230	67 297
35-39	51 113	57 008	60 138	61 741	62 515	62 852	46 628	52 767	56 340	58 415	59 624	60 315
40-44	39 701	47 455	52 189	54 992	56 633	57 566	35 325	42 840	47 684	50 745	52 682	53 879
45-49	29 782	38 727	44 707	48 525	50 924	52 394	25 841	34 098	39 840	43 672	46 199	47 825
50-54	21 493	30 758	37 539	42 158	45 227	47 191	18 189	26 413	32 627	37 006	40 017	42 013
55-59	14 837	23 531	30 604	35 783	39 415	41 850	12 206	19 707	25 942	30 633	34 013	36 337
60-64	9 654	17 041	23 834	29 265	33 323	36 176	7 771	13 920	19 706	24 436	28 049	30 636
65-69	5 764	11 365	17 288	22 548	26 795	29 959	4 526	9 056	13 939	18 361	21 993	24 742
70-74	2 998	6 669	11 211	15 786	19 851	23 097	2 297	5 183	8 816	12 539	15 892	18 604
75-79	1 250	3 211	6 127	9 534	12 927	15 873	934	2 434	4 699	7 386	10 095	12 472
80-84	368	1 150	2 598	4 621	6 930	9 147	267	850	1 942	3 491	5 276	7 009
85 +	48	263	856	1 940	3 433	5 060	33	189	625	1 429	2 548	3 782

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.2 (*concluded*)

(For 1 million of both sexes and all ages)

BOTH SEXES

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 3 per cent						*Intrinsic rate of natural variation: 4 per cent*					
All ages(a)	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	260 741	220 391	196 621	180 690	169 169	160 955	296 623	255 914	232 095	216 055	204 434	196 116
5-9	171 730	162 635	154 022	147 029	141 517	137 381	185 838	179 641	172 946	167 235	162 682	159 225
10-14	136 584	133 545	128 804	124 409	120 725	117 862	140 595	140 318	137 571	134 605	132 012	129 941
15-19	109 833	109 940	107 706	105 185	102 899	101 047	107 546	109 879	109 430	108 255	107 030	105 972
20-24	86 881	89 115	88 991	88 207	87 298	86 475	80 923	84 721	86 002	86 355	86 375	86 268
25-29	67 452	71 278	72 883	73 578	73 849	73 924	59 759	64 461	67 000	68 517	69 503	70 146
30-34	51 345	56 478	59 439	61 264	62 408	63 131	43 270	48 586	51 977	54 269	55 870	56 986
35-39	38 137	44 257	48 231	50 878	52 650	53 821	30 574	36 215	40 123	42 870	44 837	46 212
40-44	27 481	34 180	38 832	42 043	44 249	45 733	20 956	26 602	30 724	33 698	35 843	37 354
45-49	19 125	25 877	30 862	34 417	36 911	38 617	13 872	19 160	23 228	26 243	28 441	30 002
50-54	12 806	19 068	24 041	27 743	30 414	32 271	8 837	13 431	17 212	20 119	22 294	23 850
55-59	8 200	13 533	18 184	21 845	24 591	26 549	5 383	9 065	12 385	15 068	17 145	18 661
60-64	4 951	9 093	13 137	16 575	19 288	21 294	3 091	5 794	8 511	10 880	12 793	14 237
65-69	2 742	5 627	8 841	11 846	14 387	16 356	1 629	3 411	5 449	7 397	9 077	10 408
70-74	1 323	3 063	5 320	7 696	9 889	11 699	748	1 765	3 117	4 568	5 935	7 079
75-79	512	1 369	2 698	3 896	5 424	6 852	273	752	1 503	2 435	3 408	4 293
80-84	140	454	1 061	1 941	2 970	3 988	72	237	562	1 042	1 613	2 182
85 +	16	96	324	757	1 364	2 048	8	47	164	388	708	1 068

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.3. — DISTRIBUTION OF DEATHS BY SEX AND AGE GROUPS IN THIRTY-SIX STABLE POPULATIONS CALCULATED BY ASSOCIATING SIX LEVELS OF INTERMEDIATE MODEL LIFE TABLE WITH SIX FERTILITY LEVELS

(Summary table for 100 000 deaths of both sexes and all ages)

Sex and age group (in years)	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 1.00						*Gross reproduction rate: 1.50*					
MALES												
All ages	50 295	50 431	50 489	50 520	50 779	51 079	50 798	50 940	51 940	51 461	51 940	52 323
0-4	7 499	7 047	6 458	5 685	4 146	1 766	13 572	13 360	12 953	12 173	9 705	4 687
5-14	1 377	1 238	1 040	828	605	324	2 212	2 078	1 846	1 567	1 253	759
15-29	3 406	3 208	2 785	2 297	1 707	930	4 458	4 402	4 044	3 559	2 891	1 791
30-44	7 798	5 977	4 557	3 454	2 530	1 658	8 197	6 587	5 315	4 298	3 439	2 540
45-59	13 014	10 863	8 944	7 440	6 370	5 598	11 073	9 625	8 350	7 386	6 892	6 838
60-74	12 675	14 718	15 652	15 862	16 111	16 557	8 710	10 524	11 771	12 678	14 029	16 294
75 +	4 526	7 380	11 053	14 954	19 310	24 246	2 576	4 364	6 847	9 800	13 731	19 414
FEMALES												
All ages	49 704	49 570	49 514	49 480	49 221	48 921	49 200	49 057	48 874	48 541	48 065	47 680
0-4	6 955	6 434	5 771	4 884	3 424	1 349	12 573	12 184	11 565	10 447	8 005	3 580
5-14	1 458	1 300	1 080	839	557	233	2 339	2 181	1 916	1 588	1 152	543
15-29	3 657	3 418	2 894	2 217	1 486	690	4 788	4 690	4 203	3 437	2 520	1 324
30-44	7 265	5 644	4 276	3 120	2 165	1 325	7 674	6 251	5 009	3 889	2 949	2 032
45-49	10 744	8 898	7 254	5 873	4 773	3 913	9 126	7 879	6 770	5 831	5 170	4 785
60-74	13 398	14 636	15 020	14 711	14 103	13 614	9 166	10 427	11 255	11 709	12 221	13 328
75 +	6 227	9 240	13 219	17 836	22 713	27 797	3 534	5 445	8 156	11 630	16 048	22 088
BOTH SEXES												
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0-4	14 454	13 381	12 229	10 569	7 570	3 115	26 145	25 544	24 518	22 620	17 710	8 267
5-14	2 835	2 538	2 120	1 667	1 162	557	4 551	4 259	3 762	3 155	2 405	1 302
15-29	7 063	6 626	5 679	4 514	3 193	1 620	9 246	9 092	8 247	6 996	5 411	3 115
30-44	15 063	11 621	8 833	6 574	4 695	2 983	15 871	12 838	10 324	8 197	6 388	4 572
45-59	23 758	19 761	16 198	13 313	11 143	9 511	20 199	17 504	15 120	13 217	12 062	11 623
60-74	26 073	29 354	30 672	30 573	30 214	30 171	17 876	20 951	23 026	24 387	26 250	29 622
75 +	10 753	16 620	24 272	32 790	42 023	52 043	6 110	9 809	15 003	21 430	29 779	41 502
	Gross reproduction rate: 2.00						*Gross reproduction rate: 2.50*					
MALES												
All ages	51 031	51 204	51 491	52 063	52 748	53 316	51 151	51 350	51 709	52 441	53 301	53 134
0-4	18 840	18 959	18 939	18 518	15 814	8 655	23 070	23 458	23 802	23 828	21 371	13 079
5-14	2 822	2 709	2 477	2 188	1 869	1 280	3 239	3 141	2 916	2 632	2 365	1 803
15-29	4 927	4 977	4 709	4 315	3 757	2 636	5 061	5 167	4 966	4 658	4 262	3 350
30-44	7 754	6 376	5 303	4 462	3 817	3 180	7 060	5 869	4 958	4 270	3 835	3 558
45-59	9 016	7 980	7 109	6 528	6 506	7 295	7 307	6 511	5 878	5 517	5 765	7 212
60-74	6 095	7 491	8 596	9 605	11 357	14 913	4 391	5 432	6 314	7 202	8 929	13 093
75 +	1 577	2 712	4 358	6 447	9 628	15 357	1 023	1 772	2 875	4 334	6 774	12 039
FEMALES												
All ages	48 970	48 794	48 511	47 938	47 249	46 681	48 848	48 646	48 291	47 558	46 696	45 868
0-4	17 438	17 277	16 895	15 880	13 036	6 609	21 339	21 362	21 220	20 421	17 608	9 987
5-14	2 982	2 839	2 568	2 211	1 715	930	3 419	3 288	3 019	2 661	2 167	1 289
15-29	5 294	5 303	4 896	4 167	3 273	1 944	5 439	5 508	5 166	4 501	3 711	2 465
30-44	7 285	6 070	5 011	4 054	3 282	2 546	6 650	5 603	4 700	3 889	3 299	2 856
45-59	7 421	6 526	5 764	5 155	4 883	5 111	6 008	5 325	4 767	4 354	4 329	5 065
60-74	6 394	7 402	8 201	8 846	9 861	12 159	4 596	5 359	6 011	6 618	7 733	10 635
75 +	2 156	3 377	5 176	7 625	11 199	17 382	1 397	2 201	3 408	5 114	7 849	13 571
BOTH SEXES												
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0-4	36 278	36 236	35 834	34 398	28 850	15 264	44 409	44 820	45 022	44 249	38 979	23 066
5-14	5 804	5 548	5 045	4 399	3 584	2 210	6 658	6 429	5 935	5 293	4 532	3 092
15-29	10 221	10 280	9 605	8 482	7 030	4 580	10 500	10 675	10 132	9 159	7 973	5 815
30-44	15 039	12 446	10 314	8 516	7 099	5 726	13 710	11 472	9 658	8 159	7 134	6 414
45-59	16 437	14 506	12 873	11 683	11 389	12 406	13 315	11 836	10 645	9 871	10 094	12 277
60-74	12 489	14 893	16 797	18 451	21 218	27 072	8 987	10 791	12 325	13 820	16 662	23 728
75 +	3 733	6 089	9 534	14 072	20 827	32 739	2 420	3 973	6 283	9 448	14 623	25 610

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.3 *(continued)*

(Summary table for 100 000 deaths of both sexes and all ages)

Sex and age group (in years)	GRR 3.00 — Mortality level 0 — e₀ 20	20 — 30	40 — 40	60 — 50	80 — 60.4	100 — 70.2	GRR 4.00 — Mortality level 0 — e₀ 20	20 — 30	40 — 40	60 — 50	80 — 60.4	100 — 70.2
MALES												
All ages	51 220	51 450	51 859	52 689	53 665	54 764	51 309	51 573	52 034	52 976	54 082	55 598
0-4	26 405	26 977	27 598	28 011	25 998	17 459	31 164	31 927	32 865	33 793	32 648	25 060
5-14	3 516	3 425	3 202	2 929	2 720	2 278	3 820	3 726	3 505	3 245	3 134	2 989
15-29	5 022	5 155	4 996	4 750	4 494	3 884	4 738	4 877	4 756	4 580	4 518	4 451
30-44	6 346	5 306	4 520	3 945	3 661	3 720	5 124	4 300	3 687	3 260	3 147	3 635
45-59	5 973	5 336	4 849	4 604	4 976	6 807	4 152	3 703	3 374	3 239	3 636	5 662
60-74	3 261	4 043	4 722	5 450	6 983	11 215	1 947	2 410	2 821	3 287	4 378	8 007
75 +	697	1 208	1 972	3 000	4 833	9 401	364	630	1 026	1 572	2 621	5 794
FEMALES												
All ages	48 776	48 552	48 144	47 311	46 336	45 237	48 690	48 427	47 968	47 024	45 916	44 403
0-4	24 411	24 557	24 590	23 992	21 408	13 321	28 787	29 038	29 261	28 924	26 869	19 120
5-14	3 709	3 584	3 315	2 961	2 495	1 633	4 027	3 894	3 625	3 277	2 872	2 147
15-29	5 398	5 495	5 197	4 591	3 914	2 850	5 092	5 200	4 949	4 433	3 939	3 265
30-44	5 992	5 075	4 290	3 600	3 156	2 995	4 855	4 126	3 510	2 980	2 715	2 923
45-59	4 909	4 361	3 931	3 637	3 735	4 782	3 407	3 027	2 736	2 558	2 732	3 977
60-74	3 406	3 981	4 490	4 999	6 037	9 093	2 028	2 365	2 676	3 008	3 772	6 479
75 +	951	1 499	2 331	3 531	5 591	10 563	494	777	1 211	1 844	3 017	6 492
BOTH SEXES												
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0-4	50 816	51 534	52 188	52 003	47 406	30 780	59 951	60 965	62 126	62 717	59 517	44 180
5-14	7 225	7 009	6 517	5 890	5 215	3 911	7 847	7 620	7 130	6 522	6 006	5 136
15-29	10 420	10 650	10 193	9 341	8 408	6 734	9 830	10 077	9 105	9 013	8 457	7 716
30-44	12 338	10 381	8 810	7 545	6 817	6 715	9 979	8 426	7 197	6 240	5 862	6 558
45-59	10 882	9 697	8 780	8 241	8 711	11 589	7 559	6 730	6 110	5 797	6 368	9 639
60-74	6 667	8 024	9 212	10 449	13 020	20 308	3 975	4 775	5 497	6 295	8 150	14 486
75 +	1 648	2 707	4 303	6 531	10 424	19 964	858	1 407	2 237	3 416	5 638	12 286

MALE

Age group	GRR 1.00 — Mortality level 0	20	40	60	80	100	GRR 1.50 — Mortality level 0	20	40	60	80	100
All ages	50 295	50 431	50 489	50 520	50 779	51 079	50 798	50 940	51 126	51 461	51 940	52 323
0	4 704	4 664	4 463	4 093	3 037	1 337	8 629	8 952	9 054	8 854	7 180	3 580
1-4	2 795	2 383	1 995	1 592	1 109	429	4 943	4 408	3 899	3 319	2 525	1 107
5-9	813	730	612	486	349	178	1 345	1 261	1 119	948	745	432
10-14	564	508	428	342	256	146	867	817	727	619	508	327
15-19	793	768	674	564	429	251	1 132	1 147	1 064	949	788	522
20-24	1 135	1 117	1 001	841	629	331	1 505	1 550	1 468	1 315	1 073	643
25-29	1 478	1 323	1 110	892	649	348	1 821	1 705	1 512	1 295	1 030	626
30-34	1 930	1 582	1 252	965	692	404	2 209	1 894	1 584	1 302	1 022	676
35-39	2 548	1 941	1 471	1 107	803	514	2 709	2 158	1 729	1 387	1 100	797
40-44	3 320	2 454	1 834	1 382	1 035	740	3 279	2 535	2 002	1 609	1 317	1 067
45-49	4 010	3 037	2 325	1 817	1 452	1 160	3 679	2 914	2 359	1 965	1 715	1 553
50-54	4 388	3 606	2 933	2 418	2 051	1 786	3 740	3 216	2 764	2 430	2 252	2 222
55-59	4 616	4 220	3 686	3 205	2 867	2 652	3 654	3 495	3 227	2 991	2 925	3 063
60-64	4 536	4 702	4 505	4 224	4 016	3 912	3 336	3 618	3 663	3 662	3 805	4 197
65-69	4 307	5 037	5 331	5 347	5 376	5 473	2 942	3 601	4 027	4 308	4 731	5 456
70-74	3 832	4 979	5 816	6 291	6 719	7 172	2 432	3 305	4 081	4 708	5 493	6 641
75-79	2 714	3 984	5 259	6 286	7 266	8 286	1 599	2 458	3 428	4 369	5 518	7 127
80-84	1 417	2 424	3 725	5 041	6 376	7 787	776	1 389	2 255	3 255	4 498	6 222
85 +	395	972	2 069	3 627	5 668	8 173	201	517	1 164	2 176	3 715	6 065

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.3 (continued)

(For 100 000 deaths of both sexes and all ages)

MALE

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						Expectation of life at birth for both sexes (in years)					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 2.00						Gross reproduction rate: 2.50					
All ages	51 031	51 204	51 491	52 063	52 748	53 316	51 151	51 350	51 709	52 441	53 301	54 134
0	12 092	12 812	13 341	13 564	11 777	6 653	14 915	15 956	16 867	17 548	15 999	10 101
1-4	6 748	6 147	5 598	4 954	4 037	2 002	8 155	7 502	6 935	6 280	5 372	2 978
5-9	1 751	1 678	1 532	1 351	1 135	744	2 041	1 975	1 832	1 651	1 458	1 067
10-14	1 071	1 031	945	837	734	536	1 198	1 166	1 084	981	907	736
15-19	1 328	1 375	1 312	1 217	1 083	811	1 426	1 491	1 445	1 371	1 282	1 074
20-24	1 675	1 762	1 718	1 601	1 399	948	1 729	1 835	1 816	1 733	1 589	1 202
25-29	1 924	1 840	1 679	1 497	1 275	877	1 906	1 841	1 705	1 554	1 391	1 074
30-34	2 215	1 940	1 670	1 429	1 198	899	2 107	1 863	1 628	1 424	1 256	1 053
35-39	2 577	2 089	1 730	1 443	1 226	1 005	2 355	1 935	1 620	1 383	1 233	1 128
40-44	2 962	2 338	1 903	1 590	1 393	1 276	2 598	2 071	1 710	1 463	1 346	1 377
45-49	3 153	2 552	2 126	1 842	1 721	1 763	2 656	2 170	1 835	1 627	1 597	1 830
50-54	3 042	2 672	2 364	2 161	2 144	2 396	2 460	2 181	1 959	1 834	1 908	2 384
55-59	2 821	2 756	2 619	2 525	2 641	3 136	2 191	2 160	2 084	2 056	2 260	2 998
60-64	2 444	2 707	2 822	2 933	3 262	4 075	1 823	2 038	2 157	2 294	2 680	3 741
65-69	2 046	2 556	2 943	3 275	3 851	5 027	1 465	1 848	2 161	2 460	3 036	4 436
70-74	1 605	2 228	2 831	3 397	4 244	5 811	1 103	1 546	1 996	2 448	3 213	4 916
75-79	1 002	1 571	2 257	2 991	4 046	5 918	661	1 048	1 528	2 072	2 943	4 808
80-84	461	843	1 410	2 115	3 130	4 903	293	540	916	1 407	2 185	3 828
85 +	114	298	691	1 341	2 452	4 536	69	184	431	855	1 646	3 403
	Gross reproduction rate: 3.00						Gross reproduction rate: 4.00					
All ages	51 220	51 450	51 859	52 689	53 665	54 764	51 309	51 573	52 034	52 976	54 082	55 598
0	17 170	18 447	19 650	20 720	19 543	13 530	20 449	22 012	23 576	25 166	24 698	19 542
1-4	9 235	8 530	7 948	7 291	6 455	3 929	10 715	9 915	9 289	8 627	7 950	5 518
5-9	2 242	2 180	2 036	1 858	1 700	1 361	2 482	2 417	2 271	2 098	1 996	1 830
10-14	1 274	1 245	1 166	1 071	1 020	917	1 338	1 309	1 234	1 147	1 138	1 159
15-19	1 468	1 540	1 505	1 447	1 397	1 289	1 463	1 539	1 511	1 471	1 477	1 554
20-24	1 720	1 835	1 830	1 767	1 677	1 397	1 627	1 738	1 743	1 704	1 687	1 593
25-29	1 834	1 780	1 661	1 536	1 420	1 198	1 648	1 600	1 502	1 405	1 354	1 304
30-34	1 961	1 743	1 534	1 362	1 240	1 134	1 672	1 488	1 317	1 182	1 121	1 172
35-39	2 121	1 751	1 477	1 276	1 176	1 189	1 715	1 418	1 203	1 053	1 010	1 159
40-44	2 264	1 812	1 509	1 307	1 245	1 397	1 737	1 394	1 167	1 025	1 016	1 304
45-49	2 239	1 838	1 566	1 407	1 429	1 797	1 631	1 341	1 148	1 046	1 103	1 580
50-54	2 006	1 787	1 618	1 533	1 654	2 260	1 387	1 237	1 126	1 081	1 214	1 896
55-59	1 728	1 711	1 655	1 664	1 893	2 750	1 134	1 125	1 100	1 112	1 319	2 186
60-64	1 391	1 562	1 665	1 796	2 168	3 321	866	975	1 045	1 140	1 436	2 502
65-69	1 082	1 371	1 614	1 861	2 380	3 811	639	812	961	1 122	1 494	2 726
70-74	788	1 110	1 443	1 793	2 435	4 083	442	623	815	1 025	1 448	2 779
75-79	457	727	1 069	1 467	2 155	3 866	243	388	574	798	1 220	2 489
80-84	196	362	621	965	1 548	2 976	99	184	316	495	829	1 817
85 +	44	119	282	568	1 130	2 559	22	58	136	279	572	1 488

TABLE A.III.3 *(continued)*

(For 100 000 deaths of both sexes and all ages)

FEMALE

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 1.00						*Gross reproduction rate: 1.50*					
All ages	49 704	49 570	49 514	49 480	49 221	48 921	49 200	49 057	48 874	48 541	48 065	47 680
0	4 135	4 062	3 813	3 535	2 417	1 003	7 586	7 796	7 736	7 258	5 713	2 686
1-4	2 820	2 372	1 958	1 529	1 007	346	4 987	4 388	3 829	3 189	2 292	894
5-9	830	738	612	474	315	127	1 374	1 277	1 120	925	672	306
10-14	628	562	468	365	242	106	965	904	796	663	480	237
15-19	867	832	718	560	382	178	1 238	1 243	1 133	943	703	370
20-24	1 205	1 168	1 011	787	528	237	1 598	1 620	1 483	1 230	902	458
25-29	1 585	1 418	1 165	870	576	275	1 952	1 827	1 587	1 264	915	496
30-34	2 000	1 647	1 284	937	629	336	2 289	1 973	1 625	1 265	927	562
35-39	2 451	1 887	1 415	1 021	697	419	2 606	2 098	1 662	1 281	955	650
40-44	2 814	2 110	1 577	1 162	839	570	2 779	2 180	1 722	1 353	1 067	820
45-49	3 189	2 447	1 887	1 448	1 112	851	2 926	2 349	1 914	1 566	1 315	1 138
50-54	3 601	2 934	2 360	1 896	1 532	1 254	3 069	2 617	2 224	1 905	1 683	1 558
55-59	3 954	3 517	3 007	2 529	2 129	1 808	3 131	2 913	2 632	2 360	2 172	2 089
60-64	4 386	4 316	3 996	3 546	3 135	2 808	3 225	3 321	3 249	3 075	2 970	3 014
65-69	4 572	4 989	5 054	4 870	4 586	4 355	3 123	3 566	3 817	3 924	4 035	4 341
70-74	4 440	5 331	5 970	6 295	6 382	6 451	2 818	3 540	4 189	4 710	5 216	5 973
75-79	3 558	4 742	5 928	6 928	7 648	8 249	2 098	2 925	3 863	4 815	5 808	7 094
80-84	2 016	3 091	4 471	5 930	7 321	8 626	1 104	1 771	2 707	3 820	5 165	6 890
85 +	653	1 407	2 820	4 978	7 744	10 922	332	749	1 586	2 985	5 075	8 104
	Gross reproduction rate: 2.00						*Gross reproduction rate: 2.50*					
All ages	48 970	48 794	48 511	47 938	47 249	46 681	48 848	48 646	48 291	47 558	46 696	45 868
0	10 631	11 158	11 398	11 121	9 372	4 992	13 112	13 896	14 410	14 387	12 731	7 576
1-4	6 807	6 119	5 497	4 759	3 664	1 617	8 227	7 466	6 810	6 034	4 877	2 411
5-9	1 789	1 698	1 534	1 317	1 022	545	2 084	1 999	1 833	1 610	1 312	756
10-14	1 193	1 141	1 034	894	693	385	1 335	1 289	1 186	1 051	855	533
15-19	1 452	1 489	1 398	1 210	965	576	1 560	1 616	1 538	1 364	1 143	763
20-24	1 779	1 842	1 736	1 497	1 176	673	1 835	1 919	1 837	1 619	1 335	858
25-29	2 063	1 972	1 762	1 460	1 132	695	2 044	1 973	1 791	1 518	1 233	844
30-34	2 295	2 020	1 713	1 386	1 088	744	2 184	1 940	1 671	1 383	1 140	871
35-39	2 480	2 040	1 663	1 332	1 064	819	2 265	1 882	1 559	1 227	1 068	925
40-44	2 510	2 010	1 635	1 336	1 130	983	2 201	1 781	1 470	1 229	1 091	1 060
45-49	2 508	2 056	1 725	1 468	1 319	1 293	2 112	1 749	1 489	1 296	1 222	1 344
50-54	2 496	2 174	1 903	1 694	1 602	1 683	2 019	1 775	1 577	1 436	1 428	1 675
55-59	2 417	2 296	2 136	1 993	1 962	2 135	1 877	1 801	1 701	1 622	1 679	2 046
60-64	2 363	2 485	2 504	2 464	2 548	2 928	1 762	1 872	1 913	1 928	2 092	2 687
65-69	2 172	2 532	2 791	2 983	3 284	4 004	1 555	1 831	2 049	2 239	2 590	3 525
70-74	1 859	2 385	2 906	3 399	4 029	5 227	1 279	1 656	2 049	2 451	3 051	4 423
75-79	1 313	1 871	2 544	3 297	4 257	5 891	867	1 248	1 722	2 282	3 096	4 787
80-84	656	1 075	1 691	2 488	3 592	5 431	416	688	1 099	1 656	2 508	4 240
85 +	187	431	941	1 840	3 350	6 060	114	265	587	1 176	2 245	4 544

Table A.III.3 *(continued)*

(For 100 000 deaths of both sexes and all ages)

Female

Age group	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate: 3.00						*Gross reproduction rate: 4.00*					
All ages	48 776	48 552	48 144	47 311	46 336	45 237	48 690	48 427	47 968	47 024	45 916	44 403
0	15 095	16 066	16 787	16 986	15 551	10 145	17 979	19 169	20 142	20 632	19 654	14 656
1-4	9 316	8 491	7 803	7 006	5 857	3 176	10 808	9 869	9 119	8 292	7 215	4 464
5-9	2 290	2 207	2 030	1 816	1 530	971	2 536	2 446	2 274	2 050	1 798	1 304
10-14	1 419	1 377	1 277	1 415	965	662	1 491	1 448	1 351	1 227	1 074	843
15-19	1 604	1 669	1 602	1 439	1 245	917	1 599	1 666	1 609	1 464	1 319	1 106
20-24	1 826	1 919	1 850	1 653	1 410	989	1 727	1 818	1 762	1 596	1 418	1 132
25-29	1 968	1 907	1 745	1 499	1 259	944	1 766	1 716	1 578	1 373	1 202	1 027
30-34	2 033	1 815	1 574	1 322	1 126	944	1 732	1 549	1 351	1 147	1 016	974
35-39	2 041	1 702	1 419	1 179	1 024	971	1 650	1 379	1 157	972	876	948
40-44	1 918	1 558	1 297	1 099	1 006	1 080	1 473	1 198	1 002	861	823	1 001
45-49	1 781	1 480	1 271	1 122	1 093	1 316	1 298	1 081	933	833	846	1 159
50-54	1 647	1 453	1 302	1 202	1 236	1 588	1 138	1 008	906	847	905	1 330
55-59	1 481	1 428	1 358	1 313	1 406	1 878	971	938	897	878	981	1 488
60-64	1 345	1 435	1 479	1 507	1 695	2 387	837	895	928	959	1 121	1 804
65-69	1 148	1 358	1 530	1 696	2 031	3 031	679	803	911	1 021	1 273	2 173
70-74	913	1 188	1 481	1 796	2 311	3 675	512	667	837	1 028	1 378	2 502
75-79	599	866	1 203	1 619	2 269	3 848	319	461	646	878	1 284	2 489
80-84	278	461	743	1 133	1 778	3 294	140	233	378	586	951	2 015
85 + ...	74	172	385	779	1 544	3 421	35	83	187	380	782	1 988

Both sexes

	Gross reproduction rate: 1.00						*Gross reproduction rate: 1.50*					
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0	8 839	8 726	8 276	7 448	5 454	2 340	16 215	16 748	16 790	16 112	12 893	6 266
1-4	5 615	4 755	3 953	3 121	2 116	775	9 930	8 796	7 728	6 508	4 817	2 001
5-9	1 643	1 468	1 224	960	664	305	2 719	2 538	2 239	1 873	1 417	738
10-14	1 192	1 070	896	707	498	252	1 832	1 721	1 523	1 282	988	564
15-19	1 660	1 600	1 392	1 124	811	429	2 370	2 390	2 197	1 892	1 491	892
20-24	2 340	2 285	2 012	1 628	1 157	568	3 103	3 170	2 951	2 545	1 975	1 101
25-29	3 063	2 741	2 275	1 762	1 225	623	3 773	3 532	3 099	2 559	1 945	1 122
30-34	3 930	3 229	2 536	1 902	1 321	740	4 498	3 867	3 209	2 567	1 949	1 238
35-39	4 999	3 828	2 886	2 128	1 500	933	5 315	4 256	3 391	2 668	2 055	1 447
40-44	6 134	4 564	3 411	2 544	1 874	1 310	6 058	4 715	3 724	2 962	2 384	1 887
45-49	7 199	5 484	4 212	3 265	2 564	2 011	6 605	5 263	4 273	3 531	3 030	2 691
50-54	7 989	6 540	5 293	4 314	3 583	3 040	6 809	5 833	4 988	4 335	3 935	3 780
55-59	8 570	7 737	6 693	5 734	4 996	4 460	6 785	6 408	5 859	5 351	5 097	5 152
60-64	8 922	9 018	8 501	7 770	7 151	6 720	6 561	6 939	6 912	6 737	6 775	7 211
65-69	8 879	10 026	10 385	10 217	9 962	9 828	6 065	7 167	7 844	8 232	8.766	9 797
70-74	8 272	10 310	11 786	12 586	13 101	13 623	5 250	6 845	8 270	9 418	10 709	12 614
75-79	6 272	8 726	11 187	13 214	14 914	16 535	3 697	5 383	7 291	9 184	11 326	14 221
80-84	3 433	5 515	8 186	10 971	13 697	16 413	1 880	3 160	4 962	7 085	9 663	13 112
85 + ...	1 048	2 379	4 889	8 605	13 412	19 095	533	1 266	2 750	5 161	8 790	14 169

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.3 *(concluded)*

(For 100 000 deaths of both sexes and all ages)

BOTH SEXES

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Gross reproduction rate : 2.00						*Gross reproduction rate : 2.50*					
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0	22 723	23 970	24 739	24 685	21 149	11 645	28 027	29 852	31 277	31 935	28 730	17 677
1-4	13 555	12 266	11 095	9 713	7 701	3 619	16 382	14 968	13 745	12 314	10 249	5 389
5-9	3 540	3 376	3 066	2 668	2 157	1 289	4 125	3 974	3 665	3 261	2 770	1 823
10-14	2 264	2 172	1 979	1 731	1 427	921	2 533	2 455	2 270	2 032	1 762	1 269
15-19	2 780	2 864	2 710	2 427	2 048	1 387	2 986	3 107	2 983	2 735	2 425	1 837
20-24	3 454	3 604	3 454	3 098	2 575	1 621	3 654	3 754	3 653	3 352	2 924	2 060
25-29	3 987	3 812	3 441	2 957	2 407	1 572	3 950	3 814	3 496	3 072	2 624	1 918
30-34	4 510	3 960	3 383	2 815	2 286	1 643	4 291	3 803	3 299	2 807	2 396	1 924
35-39	5 057	4 138	3 393	2 775	2 290	1 824	4 620	3 817	3 179	2 660	2 301	2 053
40-44	5 472	4 348	3 538	2 926	2 523	2 259	4 799	3 852	3 180	2 692	2 437	2 437
45-49	5 661	4 608	3 851	3 110	3 040	3 056	4 768	3 919	3 324	2 923	2 819	3 174
50-54	5 538	4 846	4 267	3 855	3 746	4 079	4 479	3 956	3 536	3 270	3 336	4 059
55-59	5 238	5 052	4 755	4 518	4 603	5 271	4 068	3 961	3 785	3 678	3 939	5 044
60-64	4 807	5 192	5 326	5 397	5 810	7 003	3 585	3 910	4 070	4 222	4 772	6 428
65-69	4 218	5 088	5 734	6 258	7 135	9 031	3 020	3 679	4 210	4 699	5 626	7 961
70-74	3 464	4 613	5 737	6 796	8 273	11 038	2 382	3 202	4 045	4 899	6 264	9 339
75-79	2 315	3 442	4 801	6 288	8 303	11 809	1 528	2 296	3 250	4 354	6 039	9 595
80-84	1 117	1 918	3 101	4 603	6 722	10 334	709	1 228	2 015	3 063	4 693	8 068
85 + ...	301	729	1 632	3 181	5 802	10 596	183	449	1 018	2 031	3 891	7 947
	Gross reproduction rate : 3.00						*Gross reproduction rate : 4.00*					
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0	32 265	34 513	36 437	37 706	35 094	23 675	38 428	41 181	43 718	45 798	44 352	34 198
1-4	18 551	17 021	15 751	14 297	12 312	7 105	21 523	19 784	18 408	16 919	15 165	9 982
5-9	4 532	4 387	4 074	3 674	3 230	2 332	5 018	4 863	4 545	4 148	3 794	3 134
10-14	2 693	2 622	2 443	2 216	1 985	1 579	2 829	2 757	2 585	2 374	2 212	2 002
15-19	3 072	3 209	3 107	2 886	2 642	2 206	3 062	3 205	3 120	2 935	2 796	2 660
20-24	3 546	3 754	3 680	3 420	3 087	2 386	3 354	3 556	3 505	3 300	3 105	2 725
25-29	3 802	3 687	3 406	3 035	2 679	2 142	3 414	3 316	3 080	2 778	2 556	2 331
30-34	3 994	3 558	3 108	2 684	2 366	2 078	3 404	3 037	2 668	2 329	2 137	2 146
35-39	4 162	3 453	2 896	2 455	2 200	2 160	3 365	2 797	2 360	2 025	1 886	2 107
40-44	4 182	3 370	2 806	2 406	2 251	2 477	3 210	2 592	2 169	1 886	1 839	2 305
45-49	4 020	3 318	2 837	2 529	2 522	3 113	2 929	2 422	2 081	1 879	1 949	2 739
50-54	3 653	3 240	2 920	2 735	2 890	3 848	2 525	2 245	2 032	1 928	2 119	3 226
55-59	3 209	3 139	3 023	2 977	3 299	4 628	2 105	2 063	1 997	1 990	2 300	3 674
60-64	2 736	2 997	3 144	3 303	3 863	5 708	1 703	1 870	1 973	2 099	2 557	4 306
65-69	2 230	2 729	3 144	3 557	4 411	6 842	1 318	1 615	1 872	2 143	2 767	4 899
70-74	1 701	2 298	2 924	3 589	4 746	7 758	954	1 290	1 652	2 053	2 826	5 281
75-79	1 056	1 593	2 272	3 086	4 424	7 714	562	849	1 220	1 676	2 504	4 978
80-84	474	823	1 364	2 098	3 326	6 270	239	417	694	1 081	1 780	3 832
85 + ...	118	291	667	1 347	2 674	5 980	57	141	323	659	1 354	3 476

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.4. — DISTRIBUTION OF DEATHS BY SEX AND AGE GROUPS IN FORTY-EIGHT STABLE POPULATIONS CALCULATED BY ASSOCIATING SIX LEVELS OF INTERMEDIATE MODEL LIFE TABLE WITH EIGHT LEVELS OF THE INTRINSIC RATE OF NATURAL VARIATION

(Summary table for 100 000 deaths of both sexes and all ages)

Sex and age group (in years)	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: — 1 per cent						*Intrinsic rate of natural variation: 0 per cent*					
MALES												
All ages	51 099	50 980	50 816	50 576	50 467	50 499	51 220	51 219	51 221	51 218	51 219	51 223
0-4	21 085	13 962	9 298	5 978	3 156	992	26 186	19 360	14 316	10 151	5 884	2 005
5-14	3 052	2 152	1 410	864	478	196	3 500	2 751	1 999	1 349	819	362
15-29	5 024	4 483	3 419	2 369	1 427	626	5 029	5 003	4 231	3 230	2 135	1 015
30-44	7 418	6 593	5 023	3 517	2 256	1 264	6 398	6 342	5 360	4 132	2 905	1 757
45-59	8 110	9 465	8 837	7 472	6 064	4 835	6 061	7 851	8 104	7 532	6 692	5 768
60-74	5 150	10 172	13 904	15 726	16 332	16 120	3 330	7 295	11 013	13 683	15 555	16 609
75 +	1 260	4 153	8 925	14 650	20 754	26 466	716	2 617	6 198	11 141	17 229	23 707
FEMALES												
All ages	48 902	49 021	49 185	49 424	49 533	49 504	48 782	48 781	48 780	48 782	48 778	48 780
0-	19 509	12 731	8 305	5 137	2 606	758	24 209	17 641	12 778	8 715	4 856	1 532
5-14	3 222	2 256	1 465	877	441	140	3 692	2 883	2 074	1 368	753	259
15-29	5 399	4 777	3 553	2 287	1 243	464	5 405	5 332	4 398	3 119	1 860	752
30-44	6 979	6 257	4 723	3 176	1 927	1 008	6 040	6 039	5 054	3 743	2 487	1 404
45-59	6 671	7 746	7 166	5 898	4 545	3 376	4 980	6 422	6 572	5 946	5 017	4 032
60-74	5 398	10 073	13 320	14 581	14 313	13 294	3 479	7 207	10 525	12 651	13 590	13 649
75 +	1 724	5 181	10 653	17 468	24 458	30 464	977	3 257	7 379	13 240	20 215	27 152
BOTH SEXES												
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0-4	40 594	26 693	17 603	11 115	5 762	1 750	50 395	37 001	27 094	18 866	10 740	3 537
5-14	6 274	4 408	2 875	1 741	919	336	7 192	5 634	4 073	2 717	1 572	621
5-29	10 423	9 260	6 972	4 656	2 670	1 090	10 434	10 335	8 629	6 349	3 995	1 767
30-44	14 397	12 850	9 746	6 693	4 183	2 272	12 438	12 381	10 414	7 875	5 392	3 161
45-59	14 781	17 209	16 003	13 370	10 609	8 211	11 041	14 273	14 678	13 478	11 709	9 800
60-74	10 548	20 245	27 224	30 307	30 645	29 414	6 809	14 502	21 538	26 334	29 145	30 258
75 +	2 984	9 334	19 578	32 118	45 212	56 930	1 693	5 874	13 577	24 381	37 444	50 859
	Intrinsic rate of natural variation: 0.5 per cent						*Intrinsic rate of natural variation: 1 per cent*					
MALES												
All ages	51 263	51 315	51 395	51 529	51 618	51 623	51 301	51 394	51 550	51 822	52 019	52 053
0-4	28 565	22 143	17 183	12 807	7 833	2 812	30 779	24 870	20 169	15 773	10 211	3 899
5-14	3 669	3 021	2 303	1 632	1 044	486	3 801	3 260	2 595	1 929	1 306	647
15-29	4 927	5 135	4 554	3 652	2 548	1 277	4 771	5 179	4 798	4 036	2 979	1 586
30-44	5 819	6 044	5 359	4 337	3 217	2 046	5 230	5 660	5 240	4 451	3 489	2 353
45-59	5 133	6 949	7 511	7 326	6 857	6 217	4 294	6 041	6 811	6 960	6 886	6 627
60-74	2 622	6 004	9 486	12 364	14 808	16 639	2 041	4 855	7 993	10 910	13 814	16 483
75 +	528	2 019	4 999	9 411	15 311	22 146	385	1 529	3 944	7 763	13 334	20 458
FEMALES												
All ages	48 737	48 683	48 606	48 473	48 385	48 378	48 696	48 607	48 450	48 177	47 982	47 946
0-4	26 398	20 168	15 331	10 990	6 462	2 148	28 433	22 644	17 989	13 531	8 422	2 977
5-14	3 869	3 164	2 388	1 655	960	348	4 005	3 412	2 690	1 954	1 201	463
15-29	5 297	5 473	4 736	3 529	2 221	947	5 130	5 521	4 988	3 897	2 596	1 173
30-44	5 503	5 764	5 059	3 936	2 756	1 635	4 956	5 407	4 955	4 043	2 992	1 884
45-59	4 214	5 682	6 093	5 784	5 142	4 349	3 522	4 939	5 523	5 494	5 163	4 638
60-74	2 736	5 923	9 056	11 415	12 916	13 650	2 126	4 784	7 622	10 060	12 030	13 497
5 +	720	2 509	5 943	11 164	17 928	25 301	524	1 900	4 683	9 198	15 578	23 314
BOTH SEXES												
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0-4	54 963	42 311	32 514	23 797	14 295	4 960	59 212	47 514	38 158	29 304	18 633	6 876
5-14	7 538	6 185	4 691	3 287	2 004	834	7 806	6 672	5 285	3 883	2 507	1 110
15-29	10 224	10 608	9 290	7 181	4 769	2 224	9 901	10 700	9 786	7 933	5 575	2 759
30-34	11 322	11 808	10 418	8 273	5 973	3 681	10 186	11 067	10 195	8 494	6 481	4 237
45-59	9 347	12 631	13 604	13 110	11 999	10 566	7 816	10 980	12 334	12 454	12 049	11 265
60-74	5 358	11 927	18 542	23 779	27 724	30 289	4 167	9 639	15 615	20 970	25 844	29 980
5 +	1 248	4 528	10 942	20 575	33 239	47 447	909	3 429	8 627	16 961	28 912	43 772

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.4 *(continued)*

(Summary table for 100 000 deaths of both sexes and all ages)

Sex and age group (in years)	Mortality level						Mortality level					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						Expectation of life at birth for both sexes (in years)					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 1.5 per cent						*Intrinsic rate of natural variation: 2 per cent*					
MALES												
All ages	51 340	51 462	51 683	52 094	52 413	52 516	51 370	51 523	51 798	52 338	52 789	52 991
0-4	32 813	27 476	23 170	18 957	13 011	5 333	34 653	29 911	26 086	22 234	16 162	7 169
5-14	3 896	3 458	2 861	2 226	1 598	846	3 955	3 619	3 093	2 505	1 903	1 088
15-29	4 574	5 140	4 947	4 353	3 404	1 946	4 342	5 028	5 004	4 581	3 795	2 350
30-44	4 654	5 216	5 018	4 455	3 699	2 674	4 104	4 737	4 710	4 354	3 825	2 982
45-59	3 555	5 171	6 046	6 452	6 755	6 969	2 918	4 362	5 265	5 842	6 472	7 209
60-7	1 571	3 861	6 595	9 400	12 594	16 106	1 198	3 029	5 336	7 904	11 204	15 484
75 +	277	1 140	3 046	6 251	11 352	18 642	200	837	2 304	4 918	9 428	16 709
FEMALES												
All ages	48 662	48 537	48 321	47 908	47 587	47 485	48 631	48 480	48 200	47 663	47 211	48 007
0-4	30 298	25 008	20 659	16 256	10 726	4 071	31 986	27 216	23 250	19 057	13 323	5 473
5-14	4 103	3 619	2 965	2 253	1 467	605	4 164	3 783	3 203	2 534	1 748	778
15-29	4 917	5 479	5 145	4 206	2 965	1 437	4 672	5 361	5 202	4 426	3 308	1 735
30-44	4 415	4 991	4 754	4 051	3 176	2 141	3 900	4 540	4 469	3 964	3 285	2 389
45-49	2 916	4 225	4 904	5 095	5 068	4 879	2 393	3 563	4 270	4 612	4 854	5 049
60-74	1 635	3 802	6 281	8 655	10 952	13 164	1 245	2 978	5 073	7 266	9 725	12 634
75 +	378	1 413	3 613	7 392	13 233	21 188	271	1 039	2 733	5 804	10 968	18 949
BOTH SEXES												
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0-4	63 111	52 484	43 829	35 213	23 737	9 404	66 639	57 127	49 336	41 291	29 485	12 642
5-14	7 999	7 077	5 826	4 479	3 065	1 451	8 119	7 402	6 296	5 039	3 651	1 866
15-29	9 491	10 619	10 092	8 559	6 369	3 383	9 014	10 389	10 206	9 007	7 103	4 085
30-44	9 069	10 207	9 772	8 506	6 875	4 815	8 004	9 277	9 179	8 318	7 110	5 371
45-59	6 471	9 396	10 950	11 547	11 823	11 848	5 311	7 925	9 535	10 454	11 326	12 258
60-74	3 206	7 663	12 876	18 055	23 546	29 270	2 443	6 007	10 409	15 170	20 929	28 118
75 +	655	2 553	6 659	13 643	24 585	39 830	471	1 876	5 037	10 722	20 396	35 658
	Intrinsic rate of natural variation: 3 per cent						*Intrinsic rate of natural variation: 4 per cent*					
MALES												
All ages	51 436	51 628	51 988	52 719	53 446	53 988	51 497	51 715	52 127	52 997	53 918	54 939
0-4	37 787	34 168	31 385	28 571	23 070	12 183	40 264	37 581	35 727	34 026	29 820	18 793
5-14	3 989	3 816	3 434	2 965	2 499	1 702	3 930	3 878	3 606	3 255	2 976	2 408
15-29	3 834	4 641	4 858	4 749	4 370	3 215	3 312	4 134	4 473	4 569	4 562	4 003
30-44	3 121	3 774	3 949	3 889	3 790	3 505	2 325	2 895	3 132	3 227	3 405	3 739
45-59	1 922	2 992	3 790	4 477	5 494	7 265	1 240	1 982	2 586	3 185	4 223	6 646
60-74	683	1 800	3 316	5 230	8 203	13 470	379	1 027	1 955	3 214	5 447	10 659
75 +	100	437	1 256	2 838	6 020	12 648	47	218	648	1 521	3 485	8 691
FEMALES												
All ages	48 566	48 374	48 013	47 284	46 553	46 012	48 502	48 285	47 872	47 005	46 082	45 063
0-4	34 850	31 065	27 948	24 472	19 000	9 300	37 108	34 143	31 794	29 117	24 548	14 339
5-14	4 195	3 988	3 550	2 999	2 293	1 212	4 131	4 045	3 726	3 287	2 726	1 723
15-29	4 125	4 950	5 054	4 589	3 806	2 374	3 564	4 408	4 655	4 414	3 973	2 946
30-44	2 978	3 627	3 756	3 551	3 262	2 807	2 223	2 794	2 985	2 950	2 938	2 995
45-59	1 575	2 444	3 073	3 536	4 122	5 093	1 016	1 618	2 096	2 514	3 172	4 669
60-74	708	1 760	3 149	4 793	7 100	10 958	394	1 006	1 850	2 936	4 705	8 623
75 +	135	540	1 483	3 344	6 970	14 268	66	271	766	1 787	4 020	9 768
BOTH SEXES												
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0-4	72 637	56 233	59 333	53 043	42 070	21 483	77 372	71 724	67 521	63 143	54 368	33 132
5-14	8 184	7 804	6 984	5 964	4 792	2 914	8 061	7 923	7 332	6 542	5 702	4 131
15-29	7 959	9 591	9 912	9 338	8 176	5 589	6 876	8 542	9 128	8 983	8 535	6 949
30-44	6 099	7 401	7 705	7 440	7 052	6 312	4 548	5 689	6 117	6 177	6 343	6 734
45-59	3 497	5 436	6 863	8 013	9 616	12 358	2 256	3 600	4 682	5 699	7 395	11 315
60-74	1 391	3 560	6 465	10 023	15 303	24 428	773	2 033	3 805	6 150	10 152	19 282
75 +	235	977	2 739	6 182	12 990	26 916	113	489	1 414	3 308	7 505	18 459

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.4 *(continued)*

(For 100 000 deaths of both sexes and all ages)

MALES

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: — 1 per cent						*Intrinsic rate of natural variation: 0 per cent*					
All ages	51 099	50 980	50 816	50 576	50 476	50 499	51 220	51 219	51 221	51 218	51 219	51 223
0	13 585	9 364	6 462	4 307	2 306	747	17 021	13 091	10 025	7 364	4 327	1 520
1-4	7 500	4 598	2 836	1 671	850	245	9 165	6 269	4 291	2 787	1 557	485
5-9	1 909	1 309	842	508	273	106	2 230	1 706	1 218	810	478	200
10-14	1 143	843	568	356	205	90	1 270	1 045	781	539	341	162
15-19	1 387	1 176	863	584	350	161	1 466	1 387	1 129	843	554	276
20-24	1 712	1 579	1 235	868	524	222	1 722	1 772	1 538	1 191	789	362
25-29	1 925	1 728	1 321	917	553	243	1 841	1 844	1 564	1 196	792	377
30-34	2 169	1 908	1 437	988	602	293	1 973	1 937	1 619	1 226	821	433
35-39	2 471	2 161	1 628	1 127	714	388	2 138	2 087	1 745	1 331	925	545
40-44	2 778	2 524	1 958	1 402	940	583	2 287	2 318	1 996	1 575	1 159	779
45-49	2 895	2 884	2 394	1 834	1 346	951	2 267	2 520	2 321	1 960	1 579	1 209
50-54	2 734	3 163	2 913	2 431	1 943	1 525	2 036	2 629	2 686	2 471	2 168	1 845
55-59	2 481	3 418	3 530	3 207	2 775	2 359	1 758	2 702	3 097	3 101	2 945	2 714
60-64	2 104	3 517	4 161	4 209	3 971	3 625	1 418	2 645	3 472	3 871	4 009	3 967
65-69	1 723	3 479	4 748	5 304	5 429	5 283	1 105	2 489	3 769	4 641	5 214	5 500
70-74	1 323	3 176	4 995	6 213	6 932	7 212	807	2 161	3 772	5 171	6 332	7 142
75-79	808	2 346	4 356	6 181	7 657	8 680	469	1 519	3 129	4 893	6 654	8 176
80-84	365	1 319	2 975	4 934	6 864	8 496	201	812	2 033	3 716	5 674	7 613
85 + ...	87	488	1 594	3 535	6 233	9 290	46	286	1 036	2 532	4 901	7 918
	Intrinsic rate of natural variation: 0.5 per cent						*Intrinsic rate of natural variation: 1 per cent*					
All ages	51 263	51 315	51 395	51 529	51 618	51 623	51 301	51 394	51 550	51 822	52 019	52 053
0	18 648	15 033	12 078	9 322	5 779	2 138	20 181	16 952	14 229	11 520	7 558	2 973
1-4	9 917	7 110	5 105	3 485	2 054	674	10 598	7 918	5 940	4 253	2 653	926
5-9	2 359	1 891	1 417	990	616	272	2 465	2 060	1 612	1 181	778	366
10-14	1 310	1 130	886	642	428	214	1 336	1 200	983	748	528	281
15-19	1 475	1 463	1 249	980	680	357	1 467	1 515	1 351	1 113	816	456
20-24	1 690	1 822	1 659	1 350	944	456	1 638	1 841	1 752	1 495	1 106	568
25-29	1 762	1 850	1 646	1 322	924	464	1 666	1 823	1 695	1 428	1 057	562
30-34	1 842	1 895	1 662	1 322	935	519	1 699	1 822	1 669	1 393	1 041	615
35-39	1 946	1 992	1 748	1 400	1 027	638	1 750	1 866	1 710	1 439	1 117	737
40-44	2 031	2 157	1 949	1 615	1 255	889	1 781	1 972	1 861	1 619	1 331	1 001
45-49	1 964	2 287	2 210	1 962	1 667	1 344	1 680	2 039	2 058	1 917	1 724	1 479
50-54	1 720	2 328	2 495	2 412	2 232	2 001	1 435	2 024	2 267	2 299	2 253	2 145
55-59	1 449	2 334	2 806	2 952	2 958	2 872	1 179	1 978	2 486	2 744	2 909	3 003
60-64	1 139	2 228	3 068	3 594	3 927	4 093	904	1 843	2 650	3 258	3 768	4 175
65-69	866	2 045	3 248	4 203	4 981	5 536	671	1 649	2 737	3 715	4 661	5 506
70-74	617	1 731	3 170	4 567	5 900	7 010	466	1 363	2 606	3 937	5 385	6 802
75-79	349	1 187	2 565	2 414	6 046	7 827	257	911	2 057	3 544	5 381	7 407
80-84	146	619	1 626	3 122	5 029	7 109	105	463	1 271	2 560	4 366	6 560
85 + ...	33	213	808	2 075	4 236	7 210	23	135	616	1 659	3 587	6 491

TABLE A.III.4 *(continued)*

(For 100 000 deaths of both sexes and all ages)

MALES

Age group	*Intrinsic rate of natural variation: 1.5 per cent*						*Intrinsic rate of natural variation: 2 per cent*					
Mortality level	0	20	40	60	80	100	0	20	40	60	80	100
Expectation of life at birth for both sexes (in years)	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
All ages	51 340	51 462	51 683	52 094	52 413	52 516	51 370	51 523	51 798	52 338	52 789	52 991
0	21 607	18 803	16 406	13 893	9 663	4 079	22 916	20 550	18 538	16 348	12 041	5 499
1-4	11 206	8 673	6 764	5 064	3 348	1 254	11 737	9 361	7 548	5 886	4 121	1 670
5-9	2 549	2 205	1 794	1 376	961	484	2 610	2 328	1 957	1 563	1 155	628
10-14	1 347	1 253	1 067	850	637	362	1 345	1 291	1 136	942	748	460
15-19	1 443	1 544	1 431	1 232	958	573	1 405	1 549	1 487	1 332	1 096	709
20-24	1 572	1 829	1 809	1 616	1 267	697	1 493	1 792	1 832	1 702	1 414	844
25-29	1 559	1 767	1 707	1 505	1 179	676	1 444	1 687	1 685	1 547	1 285	797
30-34	1 550	1 722	1 639	1 431	1 134	719	1 401	1 603	1 578	1 435	1 206	826
35-39	1 558	1 721	1 639	1 441	1 186	841	1 373	1 563	1 539	1 410	1 228	939
40-44	1 546	1 773	1 740	1 583	1 379	1 114	1 330	1 571	1 593	1 509	1 391	1 217
45-49	1 422	1 789	1 876	1 827	1 741	1 603	1 193	1 546	1 677	1 700	1 717	1 710
50-54	1 184	1 731	2 015	2 137	2 218	2 268	968	1 459	1 756	1 940	2 133	2 360
55-59	949	1 651	2 155	2 488	2 796	3 098	757	1 357	1 832	2 202	2 622	3 139
60-64	710	1 499	2 242	2 882	3 532	4 200	552	1 202	1 858	2 487	3 230	4 154
65-69	513	1 308	2 257	3 205	4 261	5 400	389	1 023	1 825	2 697	3 800	5 210
70-74	348	1 054	2 096	3 313	4 801	6 506	257	804	1 653	2 720	4 174	6 120
75-79	188	687	1 613	2 908	4 682	6 911	136	511	1 240	2 330	3 969	6 339
80-84	74	342	973	2 049	3 703	5 971	53	247	728	1 601	3 064	5 342
85 +	15	111	460	1 294	2 967	5 760	11	79	336	987	2 395	5 028

Age group	*Intrinsic rate of natural variation: 3 per cent*						*Intrinsic rate of natural variation: 4 per cent*					
Mortality level	0	20	40	60	80	100	0	20	40	60	80	100
Expectation of life at birth for both sexes (in years)	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
All ages	51 436	51 628	51 988	52 719	53 446	53 988	51 497	51 715	52 127	52 997	53 918	54 939
0	25 199	23 658	22 464	21 146	17 296	9 401	27 073	26 220	25 753	25 346	22 495	14 584
1-4	12 588	10 510	8 921	7 425	5 774	2 782	13 191	11 361	9 974	8 680	7 325	4 209
5-9	2 677	2 499	2 213	1 886	1 550	1 005	2 681	2 583	2 364	2 107	1 878	1 449
10-14	1 312	1 317	1 221	1 079	949	697	1 249	1 295	1 242	1 148	1 098	959
15-19	1 303	1 504	1 519	1 455	1 331	1 024	1 181	1 408	1 470	1 471	1 459	1 341
20-24	1 318	1 655	1 781	1 767	1 631	1 155	1 136	1 471	1 638	1 700	1 703	1 439
25-29	1 213	1 482	1 558	1 527	1 408	1 036	995	1 255	1 365	1 398	1 400	1 223
30-34	1 118	1 341	1 390	1 347	1 258	1 024	873	1 079	1 156	1 173	1 188	1 155
35-39	1 043	1 244	1 290	1 259	1 217	1 112	774	952	1 019	1 043	1 093	1 194
40-44	960	1 189	1 269	1 283	1 315	1 369	678	864	957	1 011	1 124	1 390
45-49	819	1 112	1 269	1 373	1 542	1 827	550	770	909	1 029	1 252	1 772
50-54	633	998	1 265	1 492	1 822	2 399	404	658	862	1 064	1 406	2 212
55-59	470	882	1 256	1 612	2 130	3 039	286	554	815	1 092	1 565	2 662
60-64	326	745	1 210	1 732	2 495	3 818	188	443	747	1 117	1 745	3 191
65-69	219	604	1 130	1 787	2 791	4 559	120	341	663	1 096	1 856	3 621
70-74	138	451	976	1 711	2 917	5 093	71	243	545	1 001	1 846	3 847
75-79	69	273	696	1 393	2 641	5 018	34	139	370	773	1 591	3 602
80-84	26	125	389	912	1 939	4 025	11	61	197	481	1 109	2 750
85 +	5	39	171	533	1 440	3 605	2	18	81	267	785	2 339

TABLE A.III.4 *(continued)*

(For 100 000 deaths of both sexes and all ages)

FEMALES

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: — 1 per cent						*Intrinsic rate of natural variation: 0 per cent*					
All ages	48 902	49 021	49 185	49 424	49 533	49 504	48 782	48 781	48 780	48 782	48 778	48 780
0	11 944	8 155	5 521	3 531	1 834	560	14 964	11 401	8 565	6 037	3 443	1 140
1-4	7 565	4 576	2 784	1 606	772	198	9 245	6 240	4 213	2 678	1 413	392
5-9	1 950	1 324	843	496	247	75	2 278	1 727	1 219	791	431	142
10-14	1 272	932	622	381	194	65	1 414	1 156	855	577	322	117
15-19	1 517	1 274	918	581	312	114	1 603	1 503	1 202	838	494	196
20-24	1 818	1 652	1 248	812	440	158	1 828	1 853	1 554	1 114	663	258
25-29	2 064	1 851	1 387	894	491	192	1 974	1 976	1 642	1 167	703	298
30-34	2 248	1 987	1 473	958	547	244	2 045	2 017	1 660	1 190	745	360
35-39	2 377	2 101	1 566	1 040	619	316	2 057	2 029	1 678	1 229	803	445
40-44	2 354	2 169	1 684	1 178	761	448	1 938	1 993	1 716	1 324	939	599
45-49	2 302	2 234	1 943	1 462	1 032	697	1 803	2 031	1 884	1 562	1 210	887
50-54	2 244	2 574	2 344	1 905	1 452	1 071	1 671	2 139	2 162	1 937	1 620	1 295
55-59	2 125	2 848	2 879	2 531	2 061	1 608	1 506	2 252	2 526	2 447	2 187	1 850
60-64	2 034	3 228	3 691	3 533	3 099	2 602	1 371	2 428	3 080	3 250	3 129	2 848
65-69	1 830	3 445	4 501	4 831	4 631	4 205	1 173	2 465	3 573	4 227	4 447	4 377
70-74	1 534	3 400	5 128	6 217	6 583	6 487	935	2 314	3 872	5 174	6 014	6 424
75-79	1 060	2 793	4 910	6 812	8 060	8 640	615	1 808	3 527	5 393	7 004	8 139
80-84	519	1 681	3 571	5 805	7 882	9 411	286	1 035	2 440	4 372	6 515	8 433
85 +	145	707	2 172	4 851	8 516	12 413	76	414	1 412	3 475	6 696	10 580
	Intrinsic rate of natural variation: 0.5 per cent						*Intrinsic rate of natural variation: 1 per cent*					
All ages	48 737	48 683	48 606	48 473	48 385	48 378	48 696	48 607	48 450	48 177	47 982	47 946
0	16 395	13 092	10 319	7 642	4 598	1 604	17 742	14 763	12 156	9 445	6 015	2 230
1-4	10 003	7 076	5 012	3 348	1 864	544	10 691	7 881	5 833	4 086	2 047	747
5-9	2 410	1 914	1 418	967	556	193	2 518	2 085	1 613	1 154	702	260
10-14	1 459	1 250	970	688	404	155	1 487	1 327	1 077	800	499	203
15-19	1 613	1 585	1 330	975	607	254	1 604	1 642	1 439	1 105	729	324
20-24	1 795	1 906	1 678	1 263	793	326	1 740	1 926	1 770	1 399	929	405
25-29	1 889	1 982	1 728	1 291	821	367	1 786	1 953	1 779	1 393	938	444
30-34	1 909	1 973	1 704	1 284	848	432	1 761	1 896	1 710	1 352	945	511
35-39	1 873	1 936	1 680	1 293	892	520	1 685	1 815	1 645	1 329	969	602
40-44	1 721	1 855	1 675	1 359	1 016	683	1 510	1 696	1 600	1 362	1 078	771
45-49	1 562	1 844	1 795	1 563	1 277	986	1 336	1 644	1 672	1 527	1 320	1 085
50-54	1 411	1 894	2 009	1 891	1 668	1 405	1 117	1 646	1 824	1 802	1 682	1 506
55-59	1 241	1 944	2 289	2 330	2 197	1 958	1 009	1 649	2 027	2 165	2 161	2 047
60-64	1 102	2 045	2 721	3 018	3 065	2 939	875	1 692	2 352	2 736	2 941	2 997
65-69	919	2 025	3 080	3 827	4 248	4 405	711	1 633	2 595	3 384	3 975	4 383
70-74	715	1 853	3 255	4 570	5 603	6 306	540	1 459	2 675	3 940	5 114	6 117
75-79	458	1 413	2 891	4 644	6 365	7 791	337	1 084	2 318	3 907	5 666	7 374
80-84	208	789	1 951	3 672	5 775	7 875	149	591	1 525	3 013	5 012	7 268
85 +	54	307	1 101	2 848	5 788	9 635	38	225	840	2 278	4 900	8 672

TABLE A.III.4 (continued)

(For 100 000 deaths of both sexes and all ages)

FEMALES

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 1.5 per cent						*Intrinsic rate of natural variation: 2 per cent*					
All ages	48 662	48 537	48 321	47 908	47 587	47 485	48 631	48 480	48 200	47 663	47 211	47 007
0	18 995	16 376	14 017	11 390	7 687	3 057	20 146	17 898	15 839	13 402	9 582	4 125
1-4	11 303	8 632	6 642	4 866	3 039	1 014	11 840	9 318	7 411	5 665	3 741	1 348
5-9	2 604	2 233	1 796	1 344	866	343	2 667	2 356	1 959	1 527	1 043	446
10-14	1 499	1 386	1 169	909	601	262	1 497	1 427	1 244	1 007	705	332
15-19	1 577	1 673	1 524	1 226	855	408	1 537	1 679	1 582	1 325	978	504
20-24	1 668	1 913	1 829	1 511	1 064	497	1 586	1 874	1 851	1 592	1 189	603
25-29	1 672	1 893	1 792	1 469	1 046	532	1 549	1 808	1 769	1 509	1 141	628
30-34	1 606	1 793	1 682	1 389	1 030	597	1 452	1 670	1 619	1 392	1 093	687
35-39	1 499	1 673	1 576	1 331	1 030	687	1 322	1 519	1 481	1 303	1 065	767
40-44	1 310	1 525	1 496	1 331	1 116	857	1 126	1 351	1 369	1 269	1 127	935
45-49	1 131	1 441	1 524	1 456	1 334	1 176	948	1 245	1 362	1 354	1 315	1 253
50-54	972	1 408	1 622	1 675	1 658	1 592	796	1 188	1 414	1 520	1 594	1 655
55-59	813	1 376	1 758	1 964	2 076	2 111	649	1 130	1 494	1 738	1 945	2 141
60-64	687	1 376	1 989	2 420	2 756	3 014	534	1 104	1 647	2 087	2 518	2 981
65-69	545	1 297	2 140	2 920	3 636	4 298	413	1 013	1 730	2 457	3 241	4 147
70-74	403	1 129	2 152	3 315	4 560	5 852	298	861	1 696	2 722	3 966	5 506
75-79	246	818	1 819	3 205	4 927	6 879	178	609	1 399	2 567	4 180	6 314
80-84	106	434	1 167	2 410	4 252	6 611	75	316	876	1 883	3 516	5 919
85 +	26	161	627	1 777	4 054	7 698	18	114	458	1 354	3 272	6 716
	Intrinsic rate of natural variation: 3 per cent						*Intrinsic rate of natural variation: 4 per cent*					
All ages	48 566	48 374	48 013	47 284	46 553	46 012	48 502	48 285	47 872	47 005	46 082	45 063
0	22 153	20 603	19 191	17 336	13 762	7 052	23 802	22 835	22 001	20 777	17 902	10 933
1-4	12 697	10 462	8 757	7 136	5 238	2 248	13 306	11 308	9 793	8 340	6 646	3 406
5-9	2 734	2 530	2 213	1 842	1 396	710	2 739	2 613	2 366	2 058	1 692	1 028
10-14	1 461	1 458	1 337	1 157	897	502	1 392	1 432	1 360	1 229	1 034	695
15-19	1 425	1 631	1 617	1 446	1 185	728	1 292	1 524	1 565	1 461	1 300	949
20-24	1 399	1 730	1 799	1 653	1 371	823	1 206	1 539	1 656	1 591	1 432	1 028
25-29	1 301	1 589	1 638	1 490	1 250	823	1 066	1 345	1 434	1 362	1 241	969
30-34	1 159	1 396	1 424	1 309	1 140	854	904	1 124	1 185	1 138	1 077	959
35-39	1 004	1 209	1 240	1 163	1 059	904	745	926	980	962	949	969
40-44	815	1 022	1 092	1 079	1 063	1 049	574	744	820	850	912	1 067
45-39	652	895	1 030	1 096	1 181	1 338	438	621	739	822	960	1 302
50-54	520	813	1 019	1 169	1 359	1 683	333	535	694	832	1 050	1 556
55-59	403	736	1 024	1 271	1 582	2 072	245	462	663	860	1 162	1 811
60-64	316	682	1 074	1 452	1 947	2 744	183	407	663	938	1 363	2 290
65-69	233	596	1 074	1 627	2 382	3 630	128	339	629	997	1 586	2 878
70-74	159	482	1 001	1 714	2 771	4 584	83	260	558	1 001	1 756	3 455
75-79	90	325	785	1 536	2 779	4 999	45	166	417	853	1 676	3 592
80-84	36	160	466	1 073	2 233	4 459	18	78	236	565	1 273	3 044
85 +	9	55	232	735	1 968	4 810	3	27	113	369	1 071	3 132

TABLE A.III.4 *(continued)*

(For 100 000 deaths of both sexes and all ages)

BOTH SEXES

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: — 1 per cent						*Intrinsic rate of natural variation: 0 per cent*					
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0	25 529	17 519	11 983	7 838	4 140	1 307	31 985	24 492	18 590	13 401	7 770	2 660
1-4	15 065	9 174	5 620	3 277	1 622	443	18 410	12 509	8 504	5 465	2 970	877
5-9	3 859	2 633	1 685	1 004	520	181	4 508	3 433	2 437	1 601	909	342
10-14	2 415	1 775	1 190	737	399	155	2 684	2 201	1 636	1 116	663	279
15-19	2 904	2 450	1 781	1 165	662	275	3 069	2 890	2 331	1 681	1 048	472
20-24	3 530	3 231	2 483	1 680	964	380	3 550	3 625	3 092	2 305	1 452	620
25-29	3 989	3 579	2 708	1 811	1 044	435	3 815	3 820	3 206	2 363	1 495	675
30-34	4 417	3 895	2 910	1 946	1 149	537	4 018	3 954	3 279	2 416	1 566	793
35-39	4 848	4 262	3 194	2 167	1 333	704	4 195	4 116	3 423	2 560	1 728	990
40-44	5 132	4 693	3 642	2 580	1 701	1 031	4 225	4 311	3 712	2 899	2 098	1 378
45-49	5 197	5 206	4 337	3 296	2 378	1 648	4 070	4 551	4 205	3 522	2 789	2 096
50-54	4 978	5 737	5 257	4 336	3 395	2 596	3 707	4 768	4 850	4 408	3 788	3 140
55-59	4 606	6 266	6 409	5 738	4 836	3 967	3 264	4 954	5 623	5 548	5 132	4 564
60-64	4 138	6 745	7 852	7 742	7 070	6 227	2 789	5 073	6 552	7 121	7 138	6 815
65-69	3 553	6 924	9 249	10 135	10 060	9 488	2 278	4 954	7 342	8 868	9 661	9 877
70-74	2 857	6 576	10 123	12 430	13 515	13 699	1 742	4 475	7 644	10 345	12 346	13 566
75-79	1 868	5 139	9 266	12 993	15 717	17 320	1 084	3 327	6 656	10 286	13 658	16 315
80 84	884	3 000	6 546	10 739	14 746	17 907	487	1 847	4 473	8 088	12 189	16 046
85 +	232	1 195	3 766	8 386	14 749	21 703	122	700	2 448	6 007	11 597	18 498
	Intrinsic rate of natural variation: 0.5 per cent						*Intrinsic rate of natural variation: 1 per cent*					
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0	35 043	28 125	22 397	16 964	10 377	3 742	37 923	31 715	26 385	20 965	13 573	5 203
1-4	19 920	14 186	10 117	6 833	3 918	1 218	21 289	15 799	11 773	8 339	5 060	1 673
5-9	4 769	3 805	2 835	1 957	1 172	465	4 983	4 145	3 225	2 335	1 480	626
10-14	2 769	2 380	1 856	1 330	832	369	2 823	2 527	2 060	1 548	1 027	484
15-19	3 088	3 048	2 579	1 955	1 287	611	3 071	3 157	2 790	2 218	1 545	780
20-24	3 485	3 728	3 337	2 613	1 737	782	3 378	3 767	3 522	2 894	2 035	973
25-29	3 651	3 832	3 374	2 613	1 745	831	3 452	3 776	3 474	2 821	1 995	1 006
30-34	3 751	3 868	3 366	2 606	1 783	951	3 460	3 718	3 379	2 745	1 986	1 126
35-39	3 819	3 928	3 428	2 693	1 919	1 158	3 435	3 681	3 355	2 768	2 086	1 339
40-44	3 752	4 012	3 624	2 974	2 271	1 572	3 291	3 668	3 461	2 981	2 409	1 772
45-49	3 526	4 131	4 005	3 525	2 944	2 330	3 016	3 683	3 730	3 444	3 044	2 564
50-54	3 131	4 222	4 504	4 303	3 900	3 406	2 612	3 670	4 091	4 101	3 935	3 651
55-59	2 690	4 278	5 095	5 282	5 155	4 830	2 188	3 627	4 513	4 909	5 070	5 050
60-64	2 241	4 273	5 789	6 612	6 992	7 032	1 779	3 535	5 002	5 994	6 709	7 172
65-69	1 785	4 070	6 328	8 030	9 229	9 941	1 382	3 282	5 332	7 099	8 636	9 889
70-74	1 332	3 584	6 425	9 137	11 503	13 316	1 006	2 822	5 281	7 877	10 499	12 919
75-79	807	2 600	5 456	8 858	12 411	15 618	594	1 995	4 375	7 451	11 047	14 781
80-84	354	1 408	3 577	6 794	10 804	14 984	254	1 054	2 796	5 573	9 378	13 828
85 +	87	520	1 909	4 923	10 024	16 845	61	380	1 456	3 937	8 487	15 163

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

TABLE A.III.4 *(concluded)*

(For 100 000 deaths of both sexes and all ages)

BOTH SEXES

Age group	*Mortality level*						*Mortality level*					
	0	20	40	60	80	100	0	20	40	60	80	100
	Expectation of life at birth for both sexes (in years)						*Expectation of life at birth for both sexes (in years)*					
	20	30	40	50	60.4	70.2	20	30	40	50	60.4	70.2
	Intrinsic rate of natural variation: 1.5 per cent						*Intrinsic rate of natural variation: 2 per cent*					
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0	40 602	35 179	30 423	25 283	17 350	7 136	43 064	38 448	34 377	29 750	21 623	9 624
1-4	22 509	17 305	13 406	9 930	6 387	2 268	23 577	18 679	14 959	11 541	7 862	3 018
5-9	5 153	4 438	3 590	2 720	1 827	827	5 277	4 684	3 916	3 090	2 198	1 074
10-14	2 846	2 639	2 236	1 759	1 238	624	2 842	2 718	2 380	1 949	1 453	792
15-19	3 020	3 217	2 995	2 458	1 813	981	2 942	3 228	3 069	2 657	2 074	1 213
20-24	3 240	3 742	3 638	3 127	2 331	1 194	3 079	3 666	3 683	3 294	2 603	1 447
25-29	3 231	3 660	3 499	2 974	2 225	1 208	2 993	3 495	3 454	3 056	2 426	1 425
30-34	3 156	3 515	3 321	2 820	2 164	1 316	2 853	3 273	3 197	2 827	2 299	1 513
35-39	3 057	3 394	3 215	2 722	2 216	1 528	2 695	3 082	3 020	2 713	2 293	1 706
40-44	2 856	3 298	3 236	2 914	2 495	1 971	2 456	2 922	2 962	2 778	2 518	2 152
45-49	2 553	3 230	3 400	3 283	3 075	2 779	2 141	2 791	3 039	3 054	3 032	2 963
50-54	2 156	3 139	3 637	3 812	3 876	3 860	1 764	2 647	3 170	3 460	3 727	4 015
55-59	1 762	3 027	3 913	4 452	4 872	5 209	1 406	2 487	3 326	3 940	4 567	5 280
60-64	1 397	2 875	4 231	5 302	6 288	7 214	1 086	2 306	3 505	4 574	5 748	7 135
65-69	1 058	2 605	4 397	6 125	7 897	9 698	802	2 036	3 555	5 154	7 041	9 357
70-74	751	2 183	4 248	6 628	9 361	12 358	555	1 665	3 349	5 442	8 140	11 626
75-79	434	1 505	3 432	6 113	9 609	13 790	314	1 120	2 639	4 897	8 149	12 653
80-84	180	776	2 140	4 459	7 955	12 582	128	563	1 604	3 484	6 580	11 261
85 +	41	272	1 087	3 071	7 021	13 458	29	193	794	2 341	5 667	11 744
	Intrinsic rate of natural variation: 3 per cent						*Intrinsic rate of natural variation: 4 per cent*					
All ages (a)	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
0	47 352	44 261	41 655	38 482	31 058	16 453	50 875	49 055	47 754	46 123	40 397	25 517
1-4	25 285	20 972	17 678	14 561	11 012	5 030	26 497	22 669	19 767	17 020	13 971	7 615
5-9	5 411	5 029	4 426	3 728	2 946	1 715	5 420	5 196	4 730	4 165	3 570	2 477
10-14	2 773	2 775	2 558	2 236	1 846	1 199	2 641	2 727	2 602	2 377	2 132	1 654
15-19	2 728	3 135	3 136	2 901	2 516	1 752	2 473	2 932	3 035	2 932	2 759	2 290
20-24	2 717	3 385	2 580	3 420	3 002	1 978	2 342	3 010	3 294	3 291	3 135	2 467
25-29	2 514	3 071	3 196	3 017	2 658	1 859	2 061	2 600	2 799	2 760	2 641	2 192
30-34	2 277	2 737	2 814	2 656	2 398	1 878	1 777	2 203	2 341	2 311	2 265	2 114
35-39	2 047	2 453	2 530	2 422	2 276	2 016	1 519	1 878	1 999	2 005	2 042	2 163
40-44	1 775	2 211	2 361	2 362	2 378	2 418	1 252	1 608	1 777	1 861	2 036	2 457
45-49	1 471	2 007	2 299	2 469	2 723	3 165	988	1 391	1 648	1 851	2 212	3 074
50-54	1 153	1 811	2 284	2 661	3 181	4 082	737	1 193	1 556	1 896	2 456	3 768
55-59	873	1 618	2 280	2 883	3 712	5 111	531	1 016	1 478	1 952	2 727	4 473
60-64	642	1 427	2 284	3 184	4 442	6 562	371	850	1 410	2 055	3 108	5 481
65-69	452	1 200	2 204	3 414	5 173	8 189	248	680	1 292	2 093	3 442	6 499
70-74	297	933	1 977	3 425	5 688	9 677	154	503	1 103	2 002	3 602	7 302
75-79	159	598	1 481	2 929	5 420	10 017	79	305	787	1 626	3 267	7 194
80-84	62	285	855	1 985	4 162	8 484	29	139	433	1 046	2 382	5 794
85 +	14	94	403	1 268	3 408	8 415	5	45	194	636	1 856	5 471

(a) Because of the rounding-off of figures, the total may not be exactly 1 000 000.

Annex IV

STABLE POPULATIONS CALCULATED BY ASSOCIATING THREE FERTILITY LEVELS WITH SIX LEVELS OF UPWARD-DEVIATING MODEL LIFE TABLE AND SIX LEVELS OF DOWNWARD-DEVIATING MODEL LIFE TABLE

TABLE A.IV.1. — DISTRIBUTION BY SEX AND AGE GROUPS OF EIGHTEEN STABLE POPULATIONS CALCULATED BY ASSOCIATING SIX LEVELS OF UPWARD-DEVIATING MODEL LIFE TABLE WITH THREE FERTILITY LEVELS

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	*Mortality level*					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 2.00					
MALES						
All ages	460 360	476 206	484 940	493 471	498 769	500 692
0-4	41 952	55 279	62 615	67 303	69 801	70 443
5-14	66 152	91 253	104 692	112 732	116 569	117 286
15-29	110 987	125 929	130 294	131 823	131 170	129 935
30-44	109 900	102 831	97 272	93 674	90 826	88 981
45-49	85 526	68 606	61 426	58 709	58 071	57 859
60-74	39 671	28 244	24 940	25 069	26 988	29 122
75 +	6 172	4 064	3 701	4 161	5 344	6 766
FEMALES						
All ages	539 640	523 794	515 060	506 529	501 231	499 308
0-4	42 630	55 075	62 269	65 623	67 557	67 824
5-14	67 068	91 384	104 765	110 440	113 187	113 169
15-29	112 551	127 200	131 136	130 094	128 042	126 183
30-44	118 095	109 139	100 869	94 588	89 852	87 503
45-59	111 900	85 063	71 388	64 418	60 440	58 979
60-74	72 503	46 985	37 353	34 055	33 633	34 627
75 +	14 893	8 948	7 280	7 311	8 520	11 023
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	84 582	110 354	124 884	132 926	137 358	138 267
5-14	133 220	182 637	209 457	223 172	229 756	230 455
15-29	223 538	253 129	261 430	261 917	259 212	256 118
30-44	227 995	211 970	198 141	188 262	180 678	176 484
45-59	197 426	153 669	132 814	123 127	118 511	116 838
60-74	112 174	75 229	62 293	59 124	60 621	64 049
75 +	21 065	13 012	10 981	11 472	13 864	17 789
	Gross reproduction rate: 3.00					
MALES						
All ages	472 885	484 868	490 911	498 218	502 470	504 268
0-4	64 439	79 263	87 071	92 365	95 387	96 463
5-14	90 723	117 098	130 456	138 746	142 931	144 130
15-29	126 729	134 819	135 576	135 553	134 416	133 466
30-44	100 860	88 420	81 250	77 284	74 640	73 279
45-59	63 285	47 536	41 320	38 974	38 354	38 269
60-74	23 802	15 858	13 584	13 464	14 407	15 706
75 +	3 047	1 874	1 654	1 832	2 335	2 955
FEMALES						
All ages	527 115	515 132	509 089	501 782	497 530	495 732
0-4	65 470	78 961	86 581	90 054	92 307	92 876
5-14	91 987	117 266	130 545	135 923	138 781	139 069
15-29	128 475	136 124	136 421	133 743	131 189	129 587
30-44	108 186	93 715	84 180	77 987	73 814	72 046
45-59	82 399	58 706	47 868	42 657	39 855	38 962
60-74	43 273	26 251	20 254	18 213	17 882	18 416
75 +	7 325	4 109	3 240	3 205	3 702	4 776
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	129 909	158 224	173 652	182 419	187 694	189 339
5-14	182 710	234 364	261 001	274 669	281 712	283 199
15-29	255 204	270 943	271 997	269 296	265 605	263 053
30-44	209 046	182 135	165 430	155 271	148 454	145 325
45-59	145 684	106 242	89 188	81 631	78 209	77 231
60-74	67 075	42 109	33 838	31 677	32 289	34 122
75 +	10 372	5 983	4 894	5 037	6 037	7 731

TABLE A.IV.1 *(continued)*

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	Mortality level 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 4.00					
MALES						
All ages	479 619	489 270	493 805	500 462	504 151	505 849
0-4	82 951	97 822	105 484	110 958	114 189	115 483
5-14	107 830	133 663	146 309	154 387	158 542	159 901
15-29	132 446	135 510	133 977	132 956	131 448	130 558
30-44	90 270	76 072	68 698	64 831	62 410	61 282
45-59	48 614	35 087	29 961	28 020	27 448	27 394
60-74	15 753	10 081	8 479	8 327	8 868	9 656
75 +	1 755	1 035	897	983	1 246	1 575
FEMALES						
All ages	520 381	510 730	506 195	499 538	495 849	494 151
0-4	84 267	97 442	104 883	108 179	110 489	111 187
5-14	109 339	133 854	146 407	151 243	153 936	154 285
15-29	134 241	136 731	134 788	131 157	128 279	126 747
30-44	96 708	80 548	71 133	65 396	61 706	60 241
45-59	63 081	43 211	34 633	30 616	28 488	27 866
60-74	28 537	16 630	12 600	11 232	10 978	11 294
75 +	4 208	2 264	1 751	1 715	1 973	2 531
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	167 218	195 264	210 367	219 137	224 678	226 670
5-14	217 169	267 517	292 716	305 630	312 478	314 186
15-29	266 687	272 291	268 765	264 113	259 727	257 305
30-44	186 978	156 620	139 831	130 227	124 116	121 523
45-59	111 695	78 298	64 594	58 636	55 936	55 260
60-74	44 290	26 711	21 079	19 559	19 846	20 950
75 +	5 963	3 299	2 648	2 698	3 219	4 106

(For 1 million of both sexes and all ages)

	Gross reproduction rate: 2.00					
MALES						
All ages	460 360	476 206	484 940	493 471	498 769	500 692
0	10 970	13 135	14 104	14 651	14 893	14 924
1-4	30 982	42 144	48 511	52 652	54 908	55 519
5-9	32 115	46 149	54 144	59 066	61 596	62 165
10-14	34 037	45 104	50 548	53 666	54 973	55 121
15-19	36 041	43 981	47 111	48 663	49 002	48 806
20-24	37 242	42 135	43 447	43 840	43 546	43 103
25-29	37 704	39 813	39 736	39 320	38 622	38 026
30-34	37 652	37 279	36 106	35 139	34 203	33 537
35-39	36 940	34 430	32 465	31 187	30 178	29 534
40-44	35 308	31 122	28 701	27 348	26 445	25 910
45-49	32 630	27 308	24 741	23 516	22 880	22 536
50-54	28 895	23 019	20 532	19 594	19 362	19 276
55-59	24 001	18 279	16 153	15 599	15 829	16 047
60-64	18 441	13 492	11 901	11 722	12 226	12 857
65-69	13 049	9 192	8 108	8 194	8 854	9 754
70-74	8 181	5 560	4 931	5 153	5 908	6 817
75-79	4 199	2 768	2 487	2 716	3 352	4 093
80-84	1 633	1 044	951	1 093	1 455	1 897
85 +	340	252	263	352	537	776

TABLE A.IV.1 *(continued)*

(For 1 million of both sexes and all ages)

Age group (in years)	Mortality level 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 3.00					
MALES						
All ages	472 885	484 868	490 911	498 218	502 470	504 268
0	17 313	19 368	20 179	20 693	20 948	21 037
1-4	47 126	59 895	66 892	71 672	74 439	75 426
5-9	45 715	61 376	69 866	75 242	78 145	79 033
10-14	45 008	55 722	60 590	63 504	64 786	65 097
15-19	44 270	50 473	52 458	53 492	53 646	53 543
20-24	42 495	44 919	44 939	44 765	44 285	43 926
25-29	39 964	39 427	38 179	37 296	36 485	35 997
30-34	37 073	34 293	32 227	30 963	30 015	29 492
35-39	33 787	29 422	26 917	25 527	24 600	24 126
40-44	30 000	24 705	22 106	20 794	20 025	19 661
45-49	25 754	20 137	17 701	16 610	16 094	15 886
50-54	21 185	15 768	13 646	12 856	12 652	12 622
55-59	16 346	11 631	9 973	9 508	9 608	9 761
60-64	11 667	7 795	6 825	6 637	6 894	7 265
65-69	7 669	5 047	4 319	4 309	4 638	5 120
70-74	4 466	2 836	2 440	2 518	2 875	3 321
75-79	2 129	1 311	1 144	1 233	1 515	1 854
80-84	769	460	406	461	611	798
85 +	149	103	104	138	209	303
	Gross reproduction rate: 4.00					
MALES						
All ages	479 619	489 270	493 805	500 462	504 151	505 849
0	22 715	24 379	24 941	25 366	25 593	25 704
1-4	60 236	73 443	80 543	85 592	88 596	89 779
5-9	55 744	71 799	80 255	85 724	88 730	89 747
10-14	52 086	61 864	66 054	68 663	69 812	70 154
15-19	48 621	53 179	54 274	54 891	54 861	54 762
20-24	44 292	44 916	44 126	43 595	42 980	42 636
25-29	39 533	37 415	35 577	34 470	33 607	33 160
30-34	34 803	30 885	28 500	27 157	26 236	25 783
35-39	30 102	25 148	22 591	21 248	20 409	20 017
40-44	25 365	20 039	17 607	16 426	15 765	15 482
45-49	20 666	15 502	13 381	12 453	12 025	11 872
50-54	16 134	11 520	9 790	9 147	8 972	8 952
55-59	11 814	8 065	6 790	6 420	6 451	6 570
60-64	8 002	5 248	4 410	4 253	4 403	4 641
65-69	4 992	3 152	2 649	2 621	2 811	3 104
70-74	2 759	1 681	1 420	1 453	1 654	1 911
75-79	1 248	738	632	675	827	1 102
80-84	428	245	213	240	316	414
85 +	79	52	52	68	103	149

TABLE A.IV.1 (*continued*)

(For 1 million of both sexes and all ages)

Age group (in years)	Mortality level: 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 2.00					
FEMALES						
All ages	539 640	523 794	515 060	506 529	501 231	499 308
0	10 949	12 923	13 864	14 195	14 154	14 332
1-4	31 681	42 152	48 405	51 428	53 403	53 492
5-9	32 631	46 218	54 161	57 831	59 776	59 952
10-14	34 437	45 166	50 604	52 609	53 411	53 217
15-19	36 359	44 092	47 213	47 809	47 692	47 221
20-24	37 666	42 501	43 704	43 265	42 512	41 868
25-29	38 526	40 607	40 219	39 020	37 838	37 094
30-34	39 129	38 583	36 863	35 100	33 636	32 841
35-39	39 475	36 437	33 604	31 459	29 840	29 041
40-44	39 491	34 119	30 402	28 029	26 376	25 621
45-49	39 026	31 544	27 194	24 730	23 160	22 501
50-54	37 734	28 561	23 871	21 483	20 115	19 607
55-59	35 140	24 958	20 323	18 205	17 165	16 871
60-64	30 726	20 613	16 478	14 818	14 220	14 212
65-69	24 595	15 763	12 478	11 378	11 258	11 586
70-74	17 182	10 609	8 397	7 859	8 155	8 829
75-79	9 635	5 786	4 635	4 521	5 025	5 951
80-84	4 116	2 413	1 967	2 014	2 427	3 309
85 + ...	1 142	749	678	776	1 068	1 763
	Gross reproduction rate: 3.00					
FEMALES						
All ages	527 115	515 132	509 089	501 782	497 530	495 732
0	17 279	19 055	19 835	20 048	19 908	20 203
1-4	48 191	59 906	66 746	70 006	72 399	72 673
5-9	46 450	61 467	69 887	73 669	75 836	76 220
10-14	45 537	55 799	60 658	62 254	62 945	62 849
15-19	44 661	50 600	52 572	52 533	52 211	51 805
20-24	42 978	45 310	45 205	44 178	43 233	42 667
25-29	40 873	40 124	38 644	37 012	35 745	35 115
30-34	38 127	35 494	32 902	30 927	29 517	28 880
35-39	36 105	31 137	27 862	25 749	24 325	23 724
40-44	33 554	27 084	23 416	21 311	19 972	19 442
45-49	30 801	23 261	19 456	17 467	16 292	15 861
50-54	27 665	19 564	15 865	14 095	13 144	12 839
55-59	23 933	15 881	12 547	11 095	10 419	10 262
60-64	19 439	12 184	9 450	8 390	8 018	8 030
65-69	14 454	8 655	6 648	5 984	5 896	6 081
70-74	9 380	5 412	4 156	3 839	3 968	4 305
75-79	4 886	2 741	2 131	2 052	2 271	2 695
80-84	1 939	1 062	840	849	1 019	1 392
85 + ...	500	306	269	304	412	689

TABLE A.IV.1 *(continued)*

(For 1 million of both sexes and all ages)

Age group (in years)	Mortality level 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 4.00					
FEMALES						
All ages	520 381	510 730	506 195	499 538	495 849	494 151
0	22 671	23 984	24 516	24 576	24 322	24 685
1-4	61 596	73 458	80 367	83 603	86 167	86 502
5-9	56 641	71 905	80 280	83 931	86 107	86 553
10-14	52 698	61 949	66 127	67 312	67 829	67 732
15-19	49 051	53 314	54 391	53 927	53 394	52 985
20-24	44 796	45 306	44 387	43 023	41 960	41 414
25-29	40 394	38 161	36 010	34 207	32 925	32 348
30-34	36 169	31 966	29 098	27 127	25 802	25 249
35-39	32 168	26 613	23 384	21 434	20 180	19 683
40-44	28 371	21 969	18 651	16 835	15 724	15 309
45-49	24 716	17 907	14 708	13 096	12 172	11 853
50-54	21 068	14 293	11 382	10 028	9 321	9 106
55-59	17 297	11 011	8 543	7 492	6 995	6 907
60-64	13 333	8 017	6 106	5 377	5 121	5 130
65-69	9 409	5 405	4 076	3 639	3 574	3 687
70-74	5 795	3 208	2 418	2 216	2 283	2 477
75-79	2 865	1 542	1 177	1 124	1 240	1 471
80-84	1 079	567	440	441	528	721
85 +	264	155	134	150	205	339
	Gross reproduction rate: 2.00					
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0	21 919	26 058	27 968	28 846	29 047	29 256
1-4	62 663	84 296	96 916	104 080	108 311	109 011
5-9	64 746	92 367	108 305	116 897	121 372	122 117
10-14	68 474	90 270	101 152	106 275	108 384	108 338
15-19	72 400	88 073	94 324	96 472	96 694	96 027
20-24	74 908	84 636	87 151	87 105	86 058	84 971
25-29	76 230	80 420	79 955	78 340	76 460	75 120
30-34	76 781	75 862	72 969	70 239	67 839	66 378
35-39	76 415	70 867	66 069	62 646	60 018	58 575
40-44	74 799	65 241	59 103	55 377	52 821	51 531
45-49	71 656	58 852	51 935	48 246	46 040	45 037
50-54	66 629	51 580	44 403	41 077	39 477	38 883
55-59	59 141	43 237	36 476	33 804	32 994	32 918
60-64	49 167	34 105	28 379	26 540	26 446	27 069
65-69	37 644	24 955	20 586	19 572	20 112	21 340
70-74	25 363	16 169	13 328	13 012	14 063	15 640
75-79	13 834	8 554	7 122	7 237	8 377	10 044
80-84	5 749	3 457	2 918	3 107	3 882	5 206
85 +	1 482	1 001	941	1 128	1 605	2 539

TABLE A.IV.1 *(concluded)*

(For 1 million of both sexes and all ages)

Age group (in years)	Mortality level					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 3.00					
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0	34 592	38 423	40 014	40 741	40 856	41 240
1-4	95 317	119 801	33 638	141 678	146 838	148 099
5-9	92 165	122 843	139 753	148 911	153 981	155 253
10-14	90 545	111 521	121 248	125 758	127 731	127 946
15-19	88 931	101 073	105 030	106 045	105 857	105 348
20-24	85 473	90 229	90 144	88 943	87 518	87 593
25-29	80 800	79 641	76 823	74 308	72 230	71 112
30-34	75 600	69 787	65 129	61 890	59 532	58 372
35-39	69 892	60 559	54 779	51 276	48 925	47 850
40-44	63 554	51 789	45 522	42 105	39 997	39 103
45-49	56 555	43 398	37 157	34 077	32 386	31 747
50-54	48 850	35 392	29 511	26 951	25 796	25 461
55-59	40 279	27 512	22 520	20 603	20 027	20 023
60-64	31 106	20 159	16 275	15 027	14 912	15 295
65-69	22 123	13 702	10 967	10 293	10 534	11 201
70-74	13 846	8 248	6 596	6 357	6 843	7 626
75-79	7 015	4 052	3 275	3 285	3 786	4 549
80-84	2 708	1 522	1 246	1 310	1 630	2 190
85 +	649	409	373	442	621	992
	Gross reproduction rate: 4.00					
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0	45 386	48 363	49 457	49 942	49 915	50 389
1-4	121 832	146 901	160 910	169 195	174 763	176 281
5-9	112 385	143 704	160 535	169 655	174 837	176 300
10-14	104 784	123 813	132 181	135 975	137 641	137 886
15-19	97 672	106 493	108 665	108 818	108 255	107 747
20-24	89 088	90 222	88 513	86 618	84 940	84 050
25-29	79 927	75 576	71 587	68 677	66 532	65 508
30-34	70 972	62 851	57 598	54 284	52 038	51 032
35-39	62 270	51 761	45 975	42 682	40 589	39 700
40-44	53 736	42 008	36 258	33 261	31 489	30 791
45-49	45 382	33 409	28 089	25 549	24 197	23 725
50-54	37 202	25 813	21 172	19 175	18 293	18 058
55-59	29 111	19 076	15 333	13 912	13 446	13 477
60-64	21 335	13 265	10 516	9 630	9 524	9 771
65-69	14 401	8 557	6 725	6 260	6 385	6 791
70-74	8 554	4 889	3 838	3 669	3 937	4 288
75-79	4 113	2 280	1 809	1 799	2 067	2 483
80-84	1 507	812	653	681	314	1 135
85 +	343	207	186	218	308	488

TABLE A.IV.2. — DISTRIBUTION OF DEATHS BY SEX AND AGE GROUPS OF EIGHTEEN STABLE POPULATIONS CALCULATED BY ASSOCIATING SIX LEVELS OF UPWARD-DEVIATING MODEL LIFE TABLE WITH THREE FERTILITY LEVELS

(Summary table for 100 000 of both sexes and all ages)

Sex and age group (in years)	Mortality level					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 2.00					
MALES						
All ages	48 777	50 967	51 893	53 548	54 636	55 226
0-4	19 098	22 410	22 547	20 402	15 292	10 932
5-14	1 167	1 557	1 562	1 403	961	743
15-29	3 516	3 994	3 646	3 387	2 480	2 420
30-44	5 944	5 711	5 207	4 732	3 626	2 799
45-59	8 645	8 003	8 191	8 856	9 088	8 395
60-74	7 731	6 940	7 799	10 096	14 287	16 120
75 +	2 676	2 352	2 941	4 672	8 902	13 817
FEMALES						
All ages	51 223	49 033	48 107	46 452	45 364	44 774
0-4	18 023	20 199	19 586	16 718	11 426	7 768
5-14	1 297	1 565	1 489	1 192	663	349
15-29	3 293	3 454	3 161	2 524	1 540	919
30-44	4 631	4 273	3 808	3 209	2 240	1 501
45-59	6 956	6 022	6 166	5 556	5 122	4 344
60-74	11 080	8 799	8 676	9 872	11 808	11 689
75 +	5 943	4 721	5 221	7 381	12 595	18 204
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0-4	37 121	42 609	42 133	37 120	26 718	18 700
5-14	2 464	3 122	3 051	2 595	1 594	1 092
15-29	6 809	7 488	6 807	5 911	2 020	3 339
30-44	10 575	9 984	9 015	7 941	5 866	4 300
45-59	15 601	14 025	14 357	14 412	14 210	12 739
60-74	18 811	15 739	16 475	19 968	26 095	27 809
75 +	8 619	7 073	8 162	12 053	21 497	32 021
	Gross reproduction rate: 3.00					
MALES						
All ages	50 339	52 092	52 935	54 710	56 410	57 571
0-4	28 284	31 415	31 980	30 807	26 698	21 668
5-14	1 534	1 935	1 962	1 875	1 459	1 298
15-29	3 770	4 064	3 756	3 708	3 135	3 481
30-44	5 136	4 676	4 302	4 156	3 655	3 193
45-59	6 011	5 262	5 432	6 222	7 323	7 642
60-74	4 362	3 706	4 199	5 754	9 367	11 906
75 +	1 242	1 034	1 304	2 188	4 773	8 383
FEMALES						
All ages	49 661	47 908	47 065	45 290	43 590	42 429
0-4	26 658	28 281	27 751	25 217	19 934	15 407
5-14	1 702	1 943	1 868	1 584	982	618
15-29	3 545	3 524	3 262	2 770	1 950	1 318
30-44	4 016	3 510	3 159	2 827	2 263	1 730
45-59	4 813	3 943	4 106	3 893	4 131	3 955
60-74	6 182	4 646	4 622	5 568	7 636	8 547
75 +	2 745	2 061	2 297	3 431	6 694	10 854
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0-4	54 942	59 696	59 731	56 024	46 632	37 075
5-14	3 236	3 878	3 830	3 459	2 441	1 916
15-29	7 315	7 588	7 018	6 478	5 085	4 759
30-44	9 152	8 186	7 461	6 983	5 918	4 923
45-59	10 824	9 205	9 538	10 115	11 454	11 597
60-74	10 544	8 352	8 821	11 322	17 003	20 453
75 +	3 987	3 095	3 601	5 619	11 467	19 237

TABLE A.IV.2 *(continued)*

(Summary table for 100 000 of both sexes and all ages)

Sex and age group (in years)	Mortality level 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 4.00					
MALES						
All ages	50 951	52 472	53 305	55 092	57 181	58 772
0-4	33 926	36 597	37 404	37 072	34 682	30 517
5-14	1 690	2 071	2 105	2 063	1 746	1 642
15-29	3 602	3 779	3 507	3 563	3 260	3 904
30-44	4 212	3 725	3 434	3 402	3 230	3 047
45-59	4 223	3 591	3 714	4 356	5 487	6 213
60-74	2 643	2 182	2 475	3 479	6 084	8 355
75 +	665	527	666	1 157	2 692	5 094
FEMALES						
All ages	49 049	47 528	46 695	44 908	42 819	41 228
0-4	31 945	32 915	32 426	30 325	25 877	21 685
5-14	1 876	2 082	2 004	1 753	1 150	785
15-29	3 398	3 283	3 048	2 659	2 022	1 476
30-44	3 301	2 862	2 532	2 323	1 994	1 619
45-59	3 371	2 682	2 808	2 722	3 101	3 189
60-74	3 715	2 712	2 703	3 331	4 919	5 952
75 +	1 443	1 052	1 174	1 795	3 756	6 522
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0-4	65 871	69 512	69 830	67 397	60 559	52 202
5-14	3 566	4 153	4 109	3 816	2 896	2 427
15-29	7 000	7 062	6 555	6 222	5 282	5 380
30-44	7 513	6 527	5 966	5 725	5 224	4 666
45-59	7 594	6 273	5 622	7 078	8 588	9 402
60-74	6 358	4 894	5 178	6 810	11 003	14 307
75 +	2 098	1 579	1 840	2 952	6 448	11 616

(For 100 000 of both sexes and all ages)

	Gross reproduction rate: 2.00					
MALES						
All ages	48 777	50 967	51 893	53 548	54 636	55 226
0	12 191	14 883	15 986	15 864	12 987	9 780
1-4	6 907	7 527	6 561	4 718	2 305	1 152
5-9	685	928	949	843	557	408
10-14	482	629	613	560	404	335
15-19	810	1 011	966	949	699	714
20-24	1 254	1 444	1 310	1 212	907	904
25-29	1 452	1 539	1 370	1 226	874	802
30-34	1 681	1 668	1 489	1 305	939	758
35-39	1 945	1 875	1 706	1 542	1 169	889
40-44	2 318	2 168	2 012	1 885	1 518	1 152
45-49	2 592	2 424	2 374	2 385	2 174	1 807
50-54	2 893	2 679	2 783	2 998	2 993	2 755
55-59	3 160	2 900	3 034	3 473	3 921	3 833
60-64	2 979	2 699	2 953	3 598	4 992	4 737
65-69	2 582	2 317	2 617	3 414	4 653	5 480
70-74	2 170	1 924	2 229	3 084	4 642	5 903
75-79	1 555	1 367	1 664	2 511	4 336	6 165
80-84	812	715	915	1 489	3 015	4 824
85 +	309	270	362	672	1 551	2 828

TABLE A.IV.2 *(continued)*

(For 100 000 of both sexes and all ages)

Age group (in years)	Mortality level 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 3.00					
MALES						
All ages	50 339	52 092	52 935	54 710	56 410	57 571
0	18 296	21 123	22 916	23 881	22 798	19 464
1-4	9 988	10 292	9 064	6 926	3 900	2 204
5-9	928	1 186	1 224	1 158	873	742
10-14	606	749	738	717	586	556
15-19	946	1 116	1 074	1 122	954	1 112
20-24	1 361	1 481	1 360	1 336	1 145	1 298
25-29	1 463	1 467	1 322	1 250	1 036	1 071
30-34	1 573	1 476	1 330	1 234	1 023	927
35-39	1 691	1 542	1 416	1 364	1 200	1 030
40-44	1 872	1 658	1 556	1 549	1 432	1 236
45-49	1 946	1 719	1 701	1 818	1 909	1 792
50-54	2 018	1 766	1 855	2 124	2 441	2 544
55-59	2 047	1 777	1 876	2 280	2 973	3 296
60-64	1 793	1 537	1 697	2 195	3 504	3 790
65-69	1 443	1 224	1 398	1 932	3 041	4 058
70-74	1 126	945	1 104	1 627	2 822	4 058
75-79	750	625	767	1 229	2 441	3 955
80-84	383	304	392	675	1 582	2 863
85 +	129	105	145	284	750	1 565
	Gross reproduction rate: 4.00					
MALES						
All ages	50 951	52 472	53 305	55 092	57 181	58 772
0	22 147	24 816	27 000	28 907	29 734	27 494
1-4	11 779	11 781	10 404	8 165	4 948	3 023
5-9	1 044	1 295	1 341	1 298	1 062	952
10-14	646	776	764	765	684	690
15-19	959	1 096	1 061	1 136	1 048	1 309
20-24	1 308	1 383	1 272	1 284	1 193	1 452
25-29	1 335	1 300	1 174	1 143	1 019	1 143
30-34	1 362	1 241	1 122	1 073	960	952
35-39	1 390	1 230	1 134	1 122	1 062	976
40-44	1 460	1 254	1 178	1 207	1 208	1 119
45-49	1 441	1 236	1 227	1 347	1 514	1 547
50-54	1 418	1 205	1 268	1 487	1 848	2 095
55-59	1 364	1 150	1 219	1 522	2 125	2 571
60-64	1 134	944	1 044	1 389	2 387	2 785
65-69	866	714	817	1 164	1 965	2 856
70-74	643	524	614	926	1 732	2 714
75-79	405	328	402	666	1 426	2 499
80-84	187	150	195	351	873	1 714
85 +	63	49	69	140	393	881

TABLE A.IV.2 (continued)

(For 100 000 of both sexes and all ages)

Age group (in years)	Mortality level 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 2.00					
FEMALES						
All ages	51 223	49 033	48 107	46 452	45 364	44 774
0	10 879	12 750	13 263	12 402	9 416	6 821
1-4	7 144	7 449	6 323	4 316	2 010	947
5-9	767	936	889	698	360	189
10-14	530	629	600	494	273	160
15-19	856	956	893	712	426	248
20-24	1 147	1 206	1 106	883	535	321
25-29	1 290	1 292	1 162	929	579	350
30-34	1 410	1 338	1 183	956	612	379
35-39	1 535	1 416	1 255	1 054	721	481
40-44	1 686	1 519	1 370	1 199	907	641
45-49	1 901	1 689	1 566	1 470	1 234	977
50-54	2 276	1 964	1 847	1 806	1 649	1 399
55-59	2 779	2 369	2 753	2 280	2 239	1 968
60-64	3 443	2 780	2 664	2 840	3 026	2 784
65-69	2 693	2 912	2 851	3 203	3 725	3 585
70-74	3 944	3 107	3 161	3 829	5 057	5 320
75-79	3 237	2 553	2 719	3 585	5 481	6 267
80-84	1 852	1 479	1 672	2 445	4 315	6 005
85 +	854	689	830	1 351	2 862	5 932
	Gross reproduction rate: 3.00					
FEMALES						
All ages	49 661	47 908	47 065	45 290	43 590	42 429
0	16 325	18 091	19 015	18 881	16 539	13 574
1-4	10 333	10 190	8 736	6 336	3 395	1 833
5-9	1 037	1 197	1 147	959	573	360
10-14	665	746	721	625	409	263
15-19	1 000	1 056	998	845	586	391
20-24	1 244	1 238	1 147	973	682	453
25-29	1 301	1 230	1 117	952	682	474
30-34	1 320	1 186	1 057	909	668	474
35-39	1 335	1 163	1 045	931	736	556
40-44	1 361	1 161	1 057	987	859	700
45-49	1 427	1 199	1 121	1 115	1 077	963
50-54	1 586	1 293	1 228	1 279	1 350	1 298
55-59	1 800	1 451	1 757	1 499	1 704	1 689
60-64	2 071	1 581	1 531	1 733	2 127	2 224
65-69	2 064	1 539	1 522	1 818	2 441	2 657
70-74	2 047	1 526	1 569	2 017	3 068	3 666
75-79	1 560	1 163	1 253	1 755	3 054	4 016
80-84	829	627	716	1 108	2 263	3 563
85 +	356	271	328	568	1 377	3 275

TABLE A.IV.2 *(continued)*

(For 100 000 of both sexes and all ages)

Age group (in years)	Mortality level					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 4.00					
FEMALES						
All ages	49 049	47 528	46 695	44 908	42 819	41 228
0	19 760	21 253	22 400	22 854	21 569	19 162
1-4	12 185	11 662	10 026	7 471	4 308	2 523
5-9	1 167	1 308	1 256	1 080	684	452
10-14	709	774	748	673	466	333
15-19	1 013	1 039	983	856	640	452
20-24	1 197	1 156	1 073	933	713	524
25-29	1 188	1 088	992	870	669	500
30-34	1 142	996	894	786	626	476
35-39	1 097	929	837	765	655	524
40-44	1 062	877	801	772	713	619
45-49	1 057	861	809	828	859	833
50-54	1 114	882	841	898	1 019	1 047
55-59	1 200	939	1 158	996	1 223	1 309
60-64	1 310	970	943	1 094	1 455	1 643
65-69	1 238	898	890	1 087	1 572	1 857
70-74	1 167	844	870	1 150	1 892	2 452
75-79	844	611	658	947	1 776	2 523
80-84	426	312	358	568	1 252	2 142
55 +	173	129	158	280	728	1 857
	Gross reproduction rate: 2.00					
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0	23 070	27 633	29 249	28 086	22 403	16 601
1-4	14 051	14 976	12 884	9 034	4 315	2 099
5-9	1 452	1 864	1 838	1 541	917	597
10-14	1 012	1 258	1 213	1 054	677	495
15-19	1 666	1 967	1 859	1 661	1 125	962
20-24	2 401	2 650	2 416	2 095	1 442	1 225
25-29	2 742	2 831	2 532	2 155	1 453	1 152
30-34	3 091	3 006	2 672	2 261	1 551	1 137
35-39	3 480	3 291	2 961	2 596	1 890	1 370
40-44	4 004	3 687	3 382	3 084	2 425	1 793
45-49	4 493	4 113	3 940	3 885	3 408	2 784
50-54	5 169	4 643	4 630	4 804	4 642	4 154
55-59	5 939	5 269	5 787	5 753	6 160	5 801
60-64	6 422	5 479	5 617	6 438	8 018	7 521
65-69	6 275	5 229	5 468	6 617	8 378	9 065
70-74	6 114	5 031	5 390	6 913	9 699	11 223
75-79	4 792	3 920	4 383	6 096	9 754	12 432
80-84	2 664	2 194	2 587	3 934	7 330	10 829
85 +	1 163	959	1 192	2 023	4 413	8 760

TABLE A.IV.2 (*concluded*)

(*For 100 000 of both sexes and all ages*)

Age group (in years)	Mortality level 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	16.97	28.85	41.03	52.60	63.86	69.95
	Gross reproduction rate: 3.00					
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0	34 621	39 214	41 931	42 762	39 337	33 038
1-4	20 321	20 482	17 800	13 262	7 295	4 037
5-9	1 965	2 383	2 371	2 117	1 446	1 092
10-14	1 271	1 495	1 459	1 342	995	824
15-19	1 946	2 172	2 072	1 967	1 540	1 503
20-24	2 605	2 719	2 507	2 309	1 827	1 751
25-29	2 764	2 697	2 439	2 202	1 718	1 545
30-34	2 893	2 662	2 387	2 152	1 691	1 401
35-39	3 026	2 705	2 461	2 295	1 936	1 586
40-44	3 233	2 819	2 613	2 536	2 291	1 936
45-49	3 373	2 918	2 822	2 933	2 986	2 760
50-54	3 604	3 059	3 083	3 403	3 791	3 852
55-59	3 847	3 228	3 633	3 779	4 677	4 985
60-64	3 864	3 118	3 228	3 928	5 631	6 014
65-69	3 507	2 763	2 920	3 750	5 482	6 715
70-74	3 173	2 471	2 673	3 644	5 890	7 724
75-79	2 310	1 788	2 020	2 984	5 495	7 971
80-84	1 192	931	1 108	1 783	3 845	6 426
85 +	485	376	473	852	2 127	4 840
	Gross reproduction rate: 4.00					
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0	41 907	46 069	49 400	51 761	51 303	46 656
1-4	23 964	23 443	20 430	15 636	9 256	5 546
5-9	2 211	2 603	2 597	2 378	1 746	1 404
10-14	1 355	1 550	1 512	1 438	1 150	1 023
15-19	1 972	2 135	2 044	1 992	1 688	1 761
20-24	2 505	2 539	2 345	2 217	1 906	1 976
25-29	2 523	2 388	2 166	2 013	1 688	1 643
30-34	2 504	2 237	2 016	1 859	1 586	1 428
35-39	2 487	2 159	1 971	1 887	1 717	1 500
40-44	2 522	2 131	1 979	1 979	1 921	1 738
45-49	2 498	2 097	2 036	2 175	2 373	2 380
50-54	2 532	2 087	2 109	2 385	2 867	3 142
55-59	2 564	2 089	2 377	2 518	3 348	3 880
60-64	2 444	1 914	1 987	2 483	3 842	4 428
65-69	2 104	1 612	1 707	2 251	3 537	4 713
70-74	1 810	1 368	1 484	2 076	3 624	5 166
75-79	1 249	939	1 060	1 613	3 202	5 022
80-84	613	462	553	919	2 125	3 856
85 +	236	178	227	420	1 121	2 738

TABLE A.IV.3. — DISTRIBUTION BY SEX AND AGE GROUPS OF EIGHTEEN STABLE POPULATIONS CALCULATED BY ASSOCIATING SIX LEVELS OF DOWNWARD-DEVIATING MODEL LIFE TABLE WITH THREE FERTILITY LEVELS

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	*Mortality level*					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 2.00					
MALES						
All ages	519 457	516 201	513 659	510 395	509 112	508 408
0-4	64 120	66 797	68 042	69 826	70 411	70 629
5-14	108 250	112 357	114 060	116 572	117 117	117 299
15-29	131 416	131 498	129 915	130 135	129 221	128 909
30-44	95 675	94 164	91 295	89 710	88 405	87 860
45-59	69 086	65 121	62 580	59 952	59 002	58 659
60-74	40 352	36 601	36 675	34 042	33 993	34 029
75 +	10 558	9 663	11 092	10 158	10 963	11 023
FEMALES						
All ages	480 543	483 799	486 341	489 605	490 888	491 592
0-4	61 729	64 286	65 392	66 968	67 327	67 457
5-14	103 422	107 953	109 664	111 956	112 135	112 131
15-29	122 669	124 460	123 989	124 889	123 962	123 563
30-44	86 836	86 300	85 401	85 488	84 775	84 533
45-59	60 667	58 277	57 507	56 757	56 578	56 636
60-74	35 113	32 876	33 516	32 711	33 720	34 243
75 +	10 107	9 647	10 872	10 836	12 391	13 029
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	125 849	131 083	133 434	136 794	137 738	138 086
5-14	211 672	220 310	223 724	228 528	229 252	229 430
15-29	254 085	255 958	253 904	255 024	253 183	252 472
30-44	182 511	180 464	176 696	175 198	173 180	172 393
45-59	129 753	123 398	120 087	116 709	115 580	115 295
60-74	75 465	69 477	70 191	66 753	67 713	68 272
75 +	20 665	19 310	21 964	20 994	23 354	24 052
	Gross reproduction rate: 3.00					
MALES						
All ages	519 318	514 658	512 656	510 533	510 001	509 725
0-4	93 563	92 555	94 122	95 992	96 851	97 178
5-14	135 354	139 640	141 556	143 810	144 563	144 850
15-29	137 292	136 554	134 745	134 184	133 343	133 066
30-44	80 144	78 403	75 910	74 162	73 131	72 703
45-59	46 386	43 513	41 747	39 768	39 158	38 940
60-74	21 905	19 746	19 721	18 198	18 171	18 196
75 +	4 674	4 247	4 855	4 419	4 764	4 792
FEMALES						
All ages	480 682	485 342	487 344	489 467	489 999	490 275
0-4	86 091	89 074	90 455	92 062	92 609	92 811
5-14	129 335	134 174	136 104	138 115	138 429	138 467
15-29	128 214	129 298	128 628	128 781	127 910	127 537
30-44	72 779	71 894	71 037	70 683	70 131	69 947
45-59	40 752	38 952	38 353	37 649	37 547	37 591
60-74	19 045	17 718	18 015	17 473	18 002	18 279
75 +	4 466	4 232	4 752	4 704	5 371	5 643
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	179 654	181 629	184 577	188 054	189 460	189 989
5-14	264 689	273 814	277 660	281 925	283 012	283 317
15-29	265 506	265 852	263 373	262 965	261 253	260 603
30-44	152 923	150 297	146 947	144 845	143 262	142 650
45-59	87 138	82 465	80 100	77 417	76 705	76 531
60-74	40 950	37 464	37 736	35 671	36 173	36 475
75 +	9 140	8 479	9 607	9 123	10 135	10 435

TABLE A.IV.3 *(continued)*

(Summary table for 1 million of both sexes and all ages)

Sex and age group (in years)	*Mortality level*					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 4.00					
MALES						
All ages	516 022	513 701	512 048	510 372	510 355	510 273
0-4	108 887	111 570	113 273	115 149	116 110	116 496
5-14	152 627	155 929	157 843	159 859	160 636	160 931
15-19	136 462	134 416	132 383	131 517	130 629	130 357
30-44	68 104	65 981	63 793	61 807	61 245	60 885
45-59	33 736	31 324	30 008	28 502	28 047	27 888
60-74	13 681	12 210	12 161	11 191	11 163	11 117
75 +	2 525	2 271	2 587	2 347	2 525	2 539
FEMALES						
All ages	483 978	486 299	487 952	489 628	489 645	489 727
0-4	104 825	107 372	108 858	110 433	111 024	111 260
5-14	145 856	149 832	151 766	153 528	153 799	153 839
15-29	127 483	127 311	126 393	126 222	125 305	124 933
30-44	61 867	60 526	59 713	59 229	58 734	58 576
45-59	29 649	28 048	27 585	26 984	26 892	26 919
60-74	11 889	10 951	11 108	10 738	11 049	11 216
75 +	2 409	2 259	2 529	2 494	2 842	2 984
BOTH SEXES						
All ages	1000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0-4	213 712	218 942	222 131	225 582	227 134	227 756
5-14	298 483	305 761	309 609	313 387	314 435	314 770
15-29	263 945	261 727	258 776	257 739	255 934	255 290
30-44	129 971	126 507	123 506	121 036	119 979	119 461
45-59	63 385	59 372	57 593	55 486	54 939	54 807
60-74	25 570	23 161	23 269	21 929	22 212	22 393
75 +	4 934	4 530	5 116	4 841	5 367	5 523

(For 1 million of both sexes and all ages)

	Gross reproduction rate: 2.00					
MALES						
All ages	519 457	516 201	513 659	510 395	509 112	508 408
0	13 920	14 329	14 478	14 775	14 845	14 873
1-4	50 200	52 468	53 564	55 051	55 566	55 756
5-9	56 252	58 887	60 119	61 696	62 149	62 298
10-14	51 998	53 470	53 941	54 876	54 968	55 001
15-19	48 179	48 623	48 407	48 783	48 585	48 526
20-24	43 831	43 744	43 154	43 188	42 858	42 748
25-29	39 406	39 131	38 354	38 164	37 778	37 635
30-34	35 346	35 011	34 107	33 730	33 302	33 128
35-39	31 724	31 289	30 310	29 773	29 328	29 137
40-44	28 605	27 864	26 878	26 207	25 775	25 595
45-49	25 787	24 690	23 737	22 952	22 562	22 412
50-54	23 038	21 692	20 812	19 927	19 605	19 489
55-59	20 261	18 739	18 031	17 073	16 835	16 758
60-64	17 175	15 647	15 242	14 264	14 092	14 099
65-69	13 559	12 262	12 307	11 391	11 348	11 368
70-74	9 618	8 692	9 126	8 387	8 553	8 562
75-79	5 959	5 411	5 981	5 471	5 778	5 805
80-84	3 144	2 867	3 343	3 049	3 325	3 340
85 +	1 455	1 385	1 768	1 638	1 860	1 878

TABLE A.IV.3 *(continued)*

(For 1 million of both sexes and all ages)

Age group (in years)	Mortality level					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 3.00					
MALES						
All ages	519 318	514 658	512 656	510 533	510 001	509 725
0	19 980	20 436	20 614	20 908	21 019	21 066
1-4	73 583	72 119	73 508	75 084	75 832	76 112
5-9	72 823	75 748	77 206	78 746	79 371	79 582
10-14	62 531	63 892	64 350	65 064	65 212	65 268
15-19	53 822	53 970	53 644	53 728	53 543	53 493
20-24	45 484	45 104	44 424	44 185	43 874	43 774
25-29	37 986	37 480	36 677	36 271	35 926	35 799
30-34	31 651	31 150	30 297	29 778	29 418	29 272
35-39	26 389	25 860	25 011	24 418	24 066	23 916
40-44	22 104	21 393	20 602	19 966	19 647	19 515
45-49	18 510	17 609	16 902	16 243	15 976	15 875
50-54	15 326	14 371	13 766	13 100	12 895	12 823
55-59	12 550	11 533	11 079	10 425	10 287	10 242
60-64	9 882	8 946	8 700	8 091	7 999	8 005
65-69	7 247	6 512	6 526	6 002	5 983	5 996
70-74	4 776	4 288	4 495	4 105	4 189	4 195
75-79	2 748	2 479	2 736	2 488	2 629	2 642
80-84	1 347	1 220	1 421	1 288	1 405	1 412
85 +	579	548	698	643	730	738
	Gross reproduction rate: 4.00					
MALES						
All ages	516 022	513 701	512 048	510 372	510 355	510 273
0	24 826	25 140	25 319	25 598	25 719	25 775
1-4	84 061	86 430	87 954	89 551	90 391	90 721
5-9	84 096	86 604	88 131	89 600	90 258	90 495
10-14	68 531	69 325	69 712	70 259	70 378	70 436
15-19	55 980	55 576	55 153	55 063	54 839	54 786
20-24	44 897	44 079	43 346	42 974	42 648	42 548
25-29	35 585	34 761	33 884	33 480	33 142	33 023
30-34	28 140	27 419	26 626	26 086	25 756	25 626
35-39	22 265	21 602	20 860	20 299	19 996	19 870
40-44	17 699	16 960	16 307	15 422	15 493	15 389
45-49	14 067	13 248	12 697	12 162	11 956	11 879
50-54	11 079	10 261	9 815	9 309	9 158	9 106
55-59	8 590	7 815	7 496	7 031	6 933	6 903
60-64	6 419	5 753	5 586	5 178	5 117	5 120
65-69	4 468	3 974	3 976	3 646	3 632	3 640
70-74	2 794	2 483	2 599	2 367	2 414	2 417
75-79	1 526	1 363	1 502	1 361	1 437	1 444
80-84	710	637	740	669	729	732
85 +	289	271	345	317	359	363

TABLE A.IV.3 *(continued)*

(For 1 million of both sexes and all ages)

Age group (in years)	*Mortality level* 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 2.00					
FEMALES						
All ages	480 543	483 799	486 341	489 605	490 888	491 592
0	13 373	13 756	13 882	14 145	14 181	14 196
1-4	48 356	50 530	51 510	52 823	53 146	53 261
5-9	53 947	56 666	57 832	59 247	59 484	59 537
10-14	49 475	51 287	51 832	52 709	52 651	52 594
15-19	45 392	46 370	46 389	46 844	46 572	46 444
20-24	40 864	41 398	41 190	41 458	41 123	40 982
25-29	36 413	36 692	36 410	36 587	36 267	36 137
30-34	32 389	32 433	32 140	32 243	31 957	31 847
35-39	28 804	28 622	28 328	28 362	28 123	28 041
40-44	25 643	25 245	24 933	24 883	24 695	24 645
45-49	22 851	22 222	21 902	21 745	21 609	21 591
50-54	20 225	19 403	19 122	18 860	18 791	18 808
55-59	17 591	16 652	16 483	16 152	16 178	16 237
60-64	14 784	13 858	13 870	13 528	13 688	13 800
65-69	11 752	10 982	11 207	10 922	11 249	11 437
70-74	8 577	8 036	8 439	8 261	8 783	9 006
75-79	5 543	5 234	5 715	5 643	6 257	6 489
80-84	3 076	2 926	3 329	3 318	3 833	4 050
85 + ...	1 488	1 487	1 828	1 875	2 301	2 490
	Gross reproduction rate: 3.00					
FEMALES						
All ages	480 682	485 342	487 344	489 467	489 999	490 275
0	19 195	19 618	19 766	20 017	20 079	20 106
1-4	66 896	69 456	70 689	72 045	72 530	72 705
5-9	69 838	72 890	74 270	75 620	75 967	76 055
10-14	59 497	61 284	61 834	62 495	62 462	62 412
15-19	50 708	51 469	51 407	51 593	51 324	51 198
20-24	42 405	42 685	42 403	42 416	42 098	41 965
25-29	35 101	35 144	34 818	34 772	34 488	34 374
30-34	29 004	28 856	28 550	28 466	28 230	28 140
35-39	23 960	23 656	23 376	23 260	23 077	23 016
40-44	19 815	19 382	19 111	18 957	18 824	18 791
45-49	16 402	15 849	15 577	15 388	15 302	15 293
50-54	13 454	12 854	12 648	12 398	12 360	12 375
55-59	10 896	10 249	10 128	9 863	9 885	9 923
60-64	8 506	7 922	7 917	7 674	7 769	7 835
65-69	6 281	5 832	5 942	5 755	5 931	6 032
70-74	4 258	3 964	4 156	4 044	4 302	4 412
75-79	2 556	2 398	2 615	2 566	2 847	2 953
80-84	1 318	1 246	1 415	1 402	1 620	1 712
85 + ...	592	588	722	736	904	978

TABLE A.IV.3 *(continued)*

(For 1 million of both sexes and all ages)

Age group (in years)	Mortality level					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 4.00					
FEMALES						
All ages	483 978	486 299	487 952	489 628	489 645	489 727
0	23 850	24 134	24 277	24 506	24 569	24 600
1-4	80 975	83 238	84 581	85 927	86 455	86 660
5-9	80 650	83 337	84 779	86 043	86 388	86 485
10-14	65 206	66 495	66 987	67 485	67 411	67 354
15-19	52 742	53 001	52 853	52 874	52 567	52 435
20-24	41 858	41 715	41 374	41 253	40 922	40 790
25-29	32 883	32 595	32 166	32 095	31 816	31 708
30-34	25 786	25 399	25 090	24 936	24 716	24 636
35-39	20 215	19 761	19 496	19 337	19 174	19 123
40-44	15 866	15 366	15 127	14 956	14 844	14 817
45-49	12 465	11 924	11 716	11 522	11 451	11 443
50-54	9 726	9 179	9 017	8 811	8 778	8 788
55-59	7 458	6 945	6 852	6 651	6 663	6 688
60-64	5 526	5 095	5 083	4 911	4 970	5 012
65-69	3 872	3 560	3 621	3 496	3 600	3 662
70-74	2 491	2 296	2 404	2 331	2 479	2 542
75-79	1 419	1 318	1 435	1 403	556	1615
80-84	694	650	737	728	841	888
85 +	296	291	357	363	445	481
	Gross reproduction rate: 2.00					
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0	27 293	28 085	28 360	28 920	29 026	29 906
1-4	98 556	102 998	105 074	107 874	108 712	109 017
5-9	110 199	115 553	177 951	120 943	121 633	121 835
10-14	101 473	104 757	105 773	107 585	107 619	107 595
15-19	93 571	94 993	94 796	95 627	95 157	94 970
20-24	84 695	85 142	84 344	84 646	83 931	83 730
25-29	75 819	75 823	74 764	74 751	74 045	73 772
30-34	67 735	67 444	66 247	65 973	65 259	64 975
35-39	60 528	59 911	58 638	58 135	57 451	57 178
40-44	54 248	53 109	51 811	51 090	50 470	50 240
45-49	48 638	46 912	45 639	44 697	44 171	44 003
50-54	43 263	41 095	39 934	38 787	38 396	38 297
55-59	37 852	35 391	34 514	33 225	33 013	32 995
60-64	31 959	29 505	29 112	27 792	27 780	27 899
65-69	25 311	23 244	23 514	22 313	22 597	22 805
70-74	18 195	16 728	17 565	16 648	17 336	17 568
75-79	11 502	10 645	11 696	11 114	12 035	12 294
80-84	6 220	5 793	6 672	6 367	7 158	7 390
85 +	2 943	2 872	3 596	3 513	4 161	4 368

TABLE A.IV.3 *(concluded)*

(For 1 million of both sexes and all ages)

Age group (in years)	*Mortality level*					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 3.00					
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0	39 175	40 054	40 380	40 925	41 098	41 172
1-4	140 479	141 575	144 197	147 129	148 362	148 817
5-9	142 661	148 638	151 476	154 366	155 338	155 637
10-14	122 028	125 176	126 184	127 559	127 674	127 680
15-19	104 530	105 439	105 051	105 321	104 867	104 691
20-24	87 889	87 789	86 827	86 601	85 972	85 739
25-29	73 087	72 624	71 495	71 043	70 414	70 173
30-34	60 655	60 006	58 847	58 244	57 648	57 412
35-39	50 349	49 516	48 387	47 678	47 143	46 932
40-44	41 919	40 775	39 713	38 923	38 471	38 306
45-49	34 912	33 458	32 479	31 631	31 278	31 168
50-54	28 780	27 225	26 414	25 498	25 255	25 198
55-59	23 446	21 782	21 207	20 288	20 172	20 165
60-64	18 388	16 868	16 617	15 765	15 768	15 840
65-69	13 528	12 344	12 468	11 757	11 914	12 028
70-74	9 034	8 252	8 651	8 149	8 491	8 607
75-79	5 304	4 877	5 351	5 054	5 476	5 595
80-84	2 665	2 466	2 836	2 690	3 025	3 124
85 +	1 171	1 136	1 420	1 379	1 634	1 716
	Gross reproduction rate: 4.00					
BOTH SEXES						
All ages	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
0	48 676	49 274	49 596	50 104	50 288	50 375
1-4	165 036	169 668	172 535	175 478	176 846	177 381
5-9	164 746	169 941	172 910	175 643	176 646	176 980
10-14	133 737	135 820	136 699	137 744	137 789	137 790
15-19	108 722	108 577	108 006	107 937	107 406	107 221
20-24	86 755	85 794	84 720	84 227	83 570	83 338
25-29	68 468	67 356	66 050	65 575	64 958	64 731
30-34	53 926	52 818	51 716	51 022	50 472	50 262
35-39	42 480	41 363	40 356	39 636	39 170	38 993
40-44	33 565	32 326	31 434	30 378	30 337	30 206
45-49	26 532	25 172	24 413	23 684	23 407	23 322
50-54	20 805	19 440	18 832	18 120	17 936	17 894
55-59	16 048	14 760	14 348	13 682	13 596	13 591
60-64	11 945	10 848	10 669	10 089	10 087	10 132
65-69	8 340	7 534	7 597	7 142	7 232	7 302
70-74	5 285	4 779	5 003	4 698	4 893	4 959
75-79	2 945	2 681	2 937	2 764	2 993	3 059
80-84	1 404	1 287	1 477	1 397	1 570	1 620
85 +	585	562	702	680	804	844

TABLE A.IV.4. — DISTRIBUTION OF DEATHS BY SEX AND AGE GROUPS IN EIGHTEEN STABLE POPULATIONS CALCULATED BY ASSOCIATING SIX LEVELS OF DOWNWARD-DEVIATING MODEL LIFE TABLE WITH THREE FERTILITY LEVELS

(Summary table for 100 000 of both sexes and all ages)

Sex and age group (in years)	*Mortality level*					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 2.00					
MALES						
All ages	51 048	50 982	51 039	51 739	52 447	52 996
0-4	14 622	13 624	12 171	9 513	5 848	3 685
5-14	3 909	3 686	3 137	2 369	1 326	749
15-29	6 980	6 002	5 152	4 210	2 669	2 010
30-44	5 708	5 079	4 179	3 717	2 567	2 050
45-59	4 984	5 373	4 827	5 556	5 270	5 046
60-74	8 451	9 467	10 059	12 232	14 484	15 964
75 +	6 394	7 751	11 514	14 142	20 283	23 492
FEMALES						
All ages	48 952	49 018	48 961	48 261	47 553	47 004
0-4	13 264	11 973	10 304	7 660	4 454	2 601
5-14	4 446	3 955	3 224	2 201	986	473
15-29	7 586	6 968	5 941	4 305	2 278	1 202
30-44	5 997	5 776	5 073	4 233	2 686	1 715
45-59	4 901	5 232	4 889	5 340	4 726	4 257
60-74	6 953	7 835	8 788	10 427	11 306	11 845
75 +	5 805	7 279	10 742	14 095	21 117	24 911
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0-4	27 886	25 597	22 475	17 173	10 302	6 286
5-14	8 355	7 641	6 361	4 570	2 312	1 222
15-29	14 566	12 970	11 093	8 515	4 947	3 212
30-44	11 705	10 855	9 252	7 950	5 253	3 765
45-59	9 885	10 605	9 716	10 896	9 996	9 303
60-74	15 404	17 302	18 847	22 659	25 790	27 809
75 +	12 199	15 030	22 256	28 237	41 400	48 403
	Gross reproduction rate: 3.00					
MALES						
All ages	51 049	50 884	51 161	52 268	53 487	54 438
0-4	22 190	21 167	20 301	17 575	12 855	9 032
5-14	5 172	5 101	4 647	3 883	2 571	1 608
15-29	7 566	6 836	6 297	5 690	4 294	3 592
30-44	5 021	4 649	4 098	4 026	3 287	2 908
45-59	3 478	3 921	3 765	4 787	5 354	5 713
60-74	4 710	5 519	6 234	8 399	11 767	14 403
75 +	2 912	3 691	5 819	7 908	13 359	17 242
FEMALES						
All ages	48 951	49 116	48 839	47 732	46 513	45 502
0-4	19 631	18 576	17 158	14 136	9 753	6 364
5-14	5 873	5 472	4 762	3 597	1 882	993
15-29	8 248	7 936	7 241	5 800	3 631	2 122
30-44	5 273	5 296	4 979	4 579	3 419	2 429
45-59	3 414	3 820	3 817	4 596	4 823	4 789
60-74	3 885	4 575	5 467	7 179	9 170	10 673
75 +	2 627	3 441	5 415	7 845	13 835	18 132
BOTH SEXES						
All ages	1 00 000	100 000	100 000	100 000	100 000	100 000
0-4	41 821	39 743	37 459	31 711	22 608	15 396
5-14	11 045	10 573	9 409	7 480	4 453	2 601
15-29	15 814	14 772	13 538	11 490	7 925	5 714
30-44	10 294	9 945	9 077	8 605	6 706	5 337
45-59	6 892	7 741	7 582	9 383	10 177	10 502
60-74	8 595	10 094	11 701	15 578	20 937	25 076
75 +	5 539	7 132	11 234	15 753	27 194	35 374

TABLE A.IV.4 (*continued*)

(*Summary table for 100 000 of both sexes and all ages*)

Sex and age group (in years)	20	40	60	80	100	115
	Mortality level					
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 4.00					
MALES						
All ages	50 760	50 962	51 409	52 719	54 295	55 733
0-4	26 390	26 180	25 933	23 906	19 674	15 196
5-14	5 816	5 821	5 461	4 834	3 596	2 501
15-29	7 409	6 786	6 440	6 180	5 226	4 813
30-44	4 246	3 958	3 590	3 720	3 429	3 351
45-59	2 495	2 835	2 817	3 790	4 726	5 521
60-74	2 871	3 410	3 970	5 667	8 922	11 937
75 +	1 533	1 972	3 198	4 622	8 722	12 364
FEMALES						
All ages	49 240	49 038	48 591	47 281	45 705	44 267
0-4	23 903	22 961	21 919	19 213	14 913	10 713
5-14	6 601	6 231	5 580	4 480	2 663	1 510
15-29	8 088	7 888	7 419	6 286	4 394	2 832
30-44	4 446	4 527	4 384	4 269	3 562	2 737
45-59	2 450	2 767	2 849	3 648	4 261	4 672
60-74	2 368	2 828	3 481	4 834	6 924	8 872
75 +	1 384	1 836	2 959	4 551	8 988	12 931
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0-4	50 293	49 141	47 852	43 119	34 587	25 909
5-14	12 417	12 052	11 041	9 314	6 259	4 011
15-29	15 497	14 674	13 859	12 466	9 620	7 645
30-44	8 692	8 485	7 974	7 989	6 991	6 088
45-59	4 945	5 602	5 666	7 438	8 987	10 193
60-74	5 239	6 238	7 451	10 501	15 846	20 859
75 +	2 917	3 808	6 157	9 173	17 710	25 295

(*For 100 000 of both sexes and all ages*)

	Gross reproduction rate: 2.00					
MALES						
All ages	51 048	50 982	51 039	51 739	52 447	52 996
0	8 486	8 643	8 447	7 264	4 947	3 291
1-4	6 136	4 981	3 724	2 249	901	394
5-9	2 507	2 372	1 998	1 479	799	453
10-14	1 402	1 314	1 139	890	527	296
15-19	1 979	1 803	1 586	1 299	850	611
20-24	2 590	2 219	1 910	1 564	986	749
25-29	2 411	1 980	1 656	1 347	833	650
30-34	2 232	1 760	1 446	1 203	765	631
35-39	1 904	1 657	1 384	1 227	850	690
40-44	1 572	1 644	1 349	1 287	952	729
45-49	1 564	1 626	1 419	1 503	1 258	1 104
50-54	1 634	1 736	1 568	1 816	1 717	1 636
55-59	1 786	2 011	1 840	2 237	2 295	2 306
60-64	2 354	2 573	2 480	3 055	3 706	3 626
65-69	2 926	3 270	3 268	3 993	4 556	5 440
70-74	3 171	3 624	4 311	5 184	6 222	6 898
75-79	2 721	3 197	4 232	5 183	6 750	7 785
80-84	1 952	2 378	3 549	4 305	6 273	7 272
85 +	1 721	2 176	3 733	4 654	7 260	8 435

TABLE A.IV.4 *(continued)*

(For 100 000 of both sexes and all ages)

Age group (in years)	Mortality level 20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 3.00					
MALES						
All ages	51 049	50 884	51 161	52 268	53 487	54 499
0	12 761	13 603	14 243	13 534	10 920	8 108
1-4	9 429	7 564	6 058	4 041	1 935	924
5-9	3 405	3 367	3 039	2 488	1 590	992
10-14	1 767	1 734	1 608	1 395	981	616
15-19	2 316	2 213	2 085	1 886	1 458	1 163
20-24	2 815	2 524	2 334	2 108	1 590	1 334
25-29	2 435	2 099	1 878	1 696	1 246	1 095
30-34	2 092	1 727	1 525	1 395	1 060	958
35-39	1 657	1 532	1 349	1 331	1 087	992
40-44	1 272	1 390	1 224	1 300	1 140	958
45-49	1 176	1 282	1 193	1 411	1 405	1 368
50-54	1 140	1 269	1 234	1 569	1 749	1 882
55-59	1 162	1 370	1 338	1 807	2 200	2 463
60-64	1 419	1 626	1 670	2 282	3 286	3 558
65-69	1 639	1 916	2 054	2 773	3 737	4 995
70-74	1 652	1 977	2 510	3 344	4 744	5 850
75-79	1 314	1 620	2 292	3 106	4 771	6 158
80-84	879	1 120	1 784	2 393	4 135	5 337
85 + ...	719	951	1 743	2 409	4 453	5 747
	Gross reproduction rate: 4.00					
MALES						
All ages	50 760	50 962	51 409	52 719	54 295	55 733
0	15 715	16 983	18 340	18 523	16 778	13 686
1-4	10 675	9 197	7 593	5 383	2 896	1 510
5-9	3 897	3 910	3 633	3 152	2 264	1 557
10-14	1 919	1 911	1 828	1 682	1 332	944
15-19	2 391	2 308	2 241	2 161	1 864	1 652
20-24	2 754	2 506	2 382	2 284	1 931	1 794
25-29	2 264	1 972	1 817	1 735	1 431	1 367
30-34	1 846	1 548	1 403	1 364	1 165	1 180
35-39	1 388	1 294	1 175	1 240	1 132	1 133
40-44	1 012	1 116	1 012	1 116	1 132	1 038
45-49	889	979	946	1 169	1 298	1 416
50-54	817	918	925	1 257	1 564	1 340
55-59	789	938	946	1 364	1 864	2 265
60-64	912	1 061	1 131	1 629	2 630	3 162
65-69	1 002	1 185	1 316	1 877	2 863	4 153
70-74	957	1 164	1 523	2 161	3 429	4 672
75-79	721	904	1316	1 895	3 296	4 625
80-84	458	589	979	1 399	2 696	3 822
85 + ...	354	479	903	1 328	2 730	3 917

TABLE A.IV.4 *(continued)*

(For 100 000 of both sexes and all ages)

Age group (in years)	*Mortality level*					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 2.00					
FEMALES						
All ages	48 952	49 018	48 961	48 261	47 553	47 004
0	7 285	7 224	6 834	5 640	3 672	2 286
1-4	5 979	4 749	3 740	2 020	782	315
5-9	2 782	2 439	1 954	1 289	561	256
10-14	1 664	1 516	1 270	902	425	217
15-19	2 345	2 139	1 805	1 275	646	335
20-24	2 730	2 500	2 147	1 539	816	414
25-29	2 511	2 329	1 989	1 491	816	453
30-34	2 263	2 121	1 831	1 419	816	473
35-39	1 996	1 925	1 691	1 407	884	552
40-44	1 738	1 730	1 551	1 407	986	690
45-49	1 546	1 601	1 454	1 497	1 190	985
50-54	1 625	1 724	1 604	1 755	1 547	1 380
55-59	1 730	1 907	1 831	2 098	1 989	1 892
60-64	2 092	2 292	2 348	2 730	2 771	2 720
65-69	2 310	2 585	2 865	3 355	3 638	3 626
70-74	2 551	2 958	3 575	4 342	4 897	5 499
75-79	2 289	2 750	3 636	4 570	6 053	6 701
80-84	1 769	2 225	3 277	4 281	6 291	7 351
85 +	1 747	2 304	3 829	5 244	8 773	10 859
	Gross reproduction rate: 3.00					
FEMALES						
All ages	48 951	49 116	48 839	47 732	46 513	45 502
0	10 962	11 370	11 525	10 507	8 110	5 611
1-4	8 669	7 206	5 633	3 629	1 643	753
5-9	3 772	3 468	2 967	2 187	1 113	582
10-14	2 101	2 004	1 795	1 410	769	411
15-19	2 746	2 625	2 376	1 854	1 113	616
20-24	2 966	2 848	2 614	2 076	1 299	753
25-29	2 536	2 463	2 251	1 870	1 219	753
30-34	2 124	2 078	1 919	1 648	1 113	718
35-39	1 739	1 754	1 660	1 521	1 140	787
40-44	1 410	1 464	1 400	1 410	1 166	924
45-49	1 163	1 262	1 224	1 395	1 325	1 197
50-54	1 130	1 262	1 255	1 521	1 590	1 574
55-59	1 121	1 296	1 338	1 680	1 908	2 018
60-64	1 263	1 444	1 587	2 044	2 438	2 668
65-69	1 295	1 518	1 795	2 330	2 995	3 318
70-74	1 327	1 613	2 085	2 805	2 737	4 687
75-79	1 103	1 390	1 971	2 742	4 294	5 303
80-84	796	1 046	1 649	2 393	4 161	5 405
85 +	728	1 005	1 795	2 710	5 380	7 424

TABLE A.IV.4 *(continued)*

(For 100 000 of both sexes and all ages)

Age group (in years)	*Mortality level*					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 4.00					
FEMALES						
All ages	49 240	49 038	48 591	47 281	45 705	44 267
0	13 501	14 196	14 848	14 379	12 450	9 486
1-4	10 402	8 765	7 071	4 834	2 463	1 227
5-9	4 319	4 026	3 546	2 780	1 598	897
10-14	2 282	2 205	2 034	1 700	1 065	613
15-19	2 831	2 739	2 556	2 107	1 431	897
20-24	2 903	2 828	2 676	2 249	1 565	991
25-29	2 354	2 321	2 187	1 930	1 398	944
30-34	1 869	1 863	1 773	1 612	1 232	849
35-39	1 456	1 486	1 447	1 417	1 165	897
40-44	1 121	1 178	1 164	1 240	1 165	991
45-49	876	966	968	1 169	1 232	1 227
50-54	812	911	935	1 204	1 431	1 557
55-59	762	890	946	1 275	1 598	1 888
60-64	812	945	1 066	1 452	1 964	2 360
65-69	789	938	1 153	1 576	2 264	2 784
70-74	767	945	1 262	1 806	2 696	3 728
75-79	608	774	1 131	1 682	2 963	4 011
80-84	413	555	903	1 381	2 696	3 870
85 +	363	507	925	1 488	3 329	5 050
	Gross reproduction rate: 2.00					
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0	15 771	15 867	15 281	12 904	8 619	5 577
1-4	12 115	9 730	7 194	4 269	1 683	709
5-9	5 289	4 811	3 952	2 778	1 360	709
10-14	3 066	2 839	2 409	1 792	952	513
15-19	4 324	3 942	3 391	2 574	1 496	946
20-24	5 320	4 719	4 057	3 103	1 802	1 163
25-29	4 922	4 309	3 645	2 838	1 649	1 103
30-34	4 495	3 881	3 277	2 622	1 581	1 104
35-39	3 900	3 600	3 075	2 634	1 734	1 242
40-44	3 310	3 374	2 900	2 694	1 938	1 419
45-49	3 110	3 227	2 873	2 994	2 448	2 089
50-54	3 259	3 460	3 172	3 572	3 264	3 016
55-59	3 516	3 918	3 671	4 330	4 284	4 198
60-64	4 446	4 865	4 828	5 785	6 477	6 346
65-69	5 236	5 855	6 133	7 348	8 194	9 066
70-74	5 722	6 582	7 886	9 526	11 119	12 397
75-79	5 010	5 947	7 868	9 753	12 803	14 486
80-84	3 721	4 603	6 826	8 586	12 564	14 623
85 +	3 468	4 480	7 562	9 898	16 033	19 294

TABLE A.IV.4 *(concluded)*

(For 100 000 of both sexes and all ages)

Age group (in years)	Mortality level					
	20	40	60	80	100	115
	Expectation of life at birth for both sexes (in years)					
	41.89	50.81	60.41	66.90	73.61	76.07
	Gross reproduction rate: 3.00					
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0	23 723	24 973	25 768	24 041	19 030	13 719
1-4	18 098	14 770	11 691	7 670	3 578	1 677
5-9	7 177	6 835	6 006	4 675	2 703	1 574
10-14	3 868	3 738	3 403	2 805	1 750	1 027
15-19	5 062	4 838	4 461	3 740	2 571	1 779
20-24	5 781	5 372	4 948	4 184	2 889	2 087
25-29	4 971	4 562	4 129	3 566	2 465	1 848
30-34	4 216	3 805	3 444	3 043	2 173	1 676
35-39	3 396	3 286	3 009	2 852	2 227	1 779
40-44	2 682	2 854	2 624	2 710	2 306	1 882
45-49	2 339	2 544	2 417	2 806	2 730	2 565
50-54	2 270	2 531	2 489	3 090	3 339	3 456
55-59	2 283	2 666	2 676	3 487	4 108	4 481
60-64	2 682	3 070	3 257	4 326	5 724	6 226
65-69	2 934	3 434	3 849	5 103	6 732	8 313
70-74	2 979	3 590	4 595	6 149	8 481	10 537
75-79	2 417	3 010	4 263	5 848	9 065	11 461
80-84	1 675	2 166	3 433	4 786	8 296	10 742
85 +	1 447	1 956	3 538	5 119	9 833	13 171
	Gross reproduction rate: 4.00					
BOTH SEXES						
All ages	100 000	100 000	100 000	100 000	100 000	100 000
0	29 216	31 179	33 188	32 902	29 228	23 172
1-4	21 077	17 962	14 664	10 217	5 359	2 737
5-9	8 216	7 936	7 179	5 932	3 862	2 454
10-14	4 201	4 116	3 862	3 382	2 397	1 557
15-19	5 222	5 047	4 797	4 268	3 295	2 549
20-24	5 657	5 334	5 058	4 533	3 496	2 784
25-29	4 618	4 293	4 004	3 665	2 829	2 312
30-34	3 715	3 411	3 176	2 976	2 397	2 029
35-39	2 844	2 789	2 622	2 657	2 297	2 030
40-44	2 133	2 294	2 176	2 356	2 297	2 029
45-49	1 765	1 945	1 914	2 338	2 530	2 643
50-54	1 629	1 829	1 860	2 461	2 995	3 397
55-59	1 551	1 828	1 892	2 639	3 462	4 153
60-64	1 724	2 006	2 197	3 081	4 594	5 522
65-69	1 791	2 123	2 469	3 453	5 127	6 937
70-74	1 724	2 109	2 785	3 967	6 125	8 400
75-79	1 329	1 678	2 447	3 577	6 259	8 636
80-84	871	1 144	1 882	2 780	5 392	7 692
85 +	717	986	1 828	2 816	6 059	8 967

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