METHODS OF USING CENSUS STATISTICS

for the calculation of life tables and other demographic measures

(WITH APPLICATIONS TO THE POPULATION OF BRAZIL)

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METHODS OF USING CENSUS STATISTICS

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(WITH APPLICATIONS TO THE POPULATION OF BRAZIL)

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FOREWORD

There are many countries where census data exist which are sufficiently accurate to be useful for demographic analysis, even though there are only very defective vital statistics or none at all. It may be expected that, as a result of the censuses to be taken in 1950 and 1951, there will be several other countries in this position. In such countries it is possible to make estimates, by the use of census statistics alone, of many demographic indices which are normally obtained with the aid of statistics derived from vital registration. Techniques for this purpose have been developed in several places and have made important contributions to demography, most notably, perhaps, in the series of life tables for India covering almost half-a century's mortality experience, for almost one-fifth of the world's population.

A co-ordinated investigation of the most important demographic characteristics of the population of Brazil has been conducted in recent years by Dr. Giorgio Mortara, Technical Advisor to the National Census Commission of Brazil. The present volume contains four articles by Dr. Mortara dealing with various aspects of this work. The first two articles describe techniques for computing life tables and estimating the birth-rate. The methods described require only census data on sex and age distribution. The last two articles deal with the computation of marriage rates, reproduction rates and other indices by methods requiring census data on marital status in the one case, and on the number of children born in the other.

All these articles have previously been published elsewhere. The first was originally prepared in English; the others have been translated by the United Nations from the Portuguese. These articles were written independently over a number of years and some topics are, therefore, dealt with more than once, from different points of view. The articles have been assembled, translated and published by the United Nations with the permission of the author in order to promote analysis of demographic trends in countries with deficient statistics. It is believed that technicians in different parts of the world who are faced with the problem of analysing census statistics such as those of Brazil, where vital statistics are not available, will find these articles useful.

CONTENTS

I. LIFE TABLES [1]

Generally the construction of mortality tables can be undertaken only for populations for which reliable census data on age distribution, and statistics on the number and age distribution of deaths, are available. But in some circumstances it may also be possible to construct mortality tables for populations lacking birth and death statistics, provided reliable data on the age distribution are available from two consecutive censuses. In order to demonstrate this possibility, it will be convenient to introduce greater simplification in the conditions of observation than obtains in reality.

First, let us suppose that we are dealing with a "closed" population, that is, a population not disturbed by immigration or emigration.

This hypothetical population lacks birth and death statistics, but it is described by two consecutive censuses, executed at the interval of ten years, both perfectly reliable in the total enumeration as well as in the particular classifications. These censuses give the age distribution of the population, by single years of age. In the conditions assumed, it will be possible to obtain from the census data approximate information on the number of births and on the number of deaths that occurred in this hypothetical population in the period between the two censuses, and hence to calculate natality and mortality rates.

Assuming, for the sake of simplicity, that the measurement of time begins with the instant of the first census, and that the unit of time is the year, we shall represent by ${}_0P'$ the total population, by ${}_0L'_x$ the population between ages x and $x+1$ according to the first census, and by ${}_{10}P'$ and ${}_{10}L'_x$ the corresponding data according to the second census, taken ten years later. By ${}_iN'$ and ${}_iM'$, respectively, we shall represent the number of live births and the number of deaths in the $(i+1)^{th}$ year following the instant of the first census.

We shall begin by determining the approximate number of live births. The number ${}_{10}L'_0$ of persons living in the first year of age according to the second census may be employed as a basis for the determination of the number of live births ${}_9N'$ in the year immediately preceding the instant of the second census. It is known, indeed, from wide international experience, that the number of persons living in the first year of age, ascertained by a census, is approximately equal to the number of live births in the year preceding the census, diminished by about two-thirds for the losses suffered by death through the first year of age. Therefore, representing by ${}_{9}\gamma_0$ the rate of mortality for the first year of age of the live births in the year preceding the second census, we may obtain the unknown initial number of them from the approximate relation

$$ {}_9N' = {}_{10}L'_0/(1 - 2_{9}\gamma_0/3). \qquad [1] $$

It is obvious that, owing to the lack of death statistics, the rate of mortality which appears in the formula [1] cannot be precisely known. But it is equally obvious that, assuming an approximate value for it, we shall be able to calculate approximately the number of live births.

No doubt the rate of mortality for the first year of age may differ very much in different populations and vary strongly in the course of time. But rarely does it reach levels higher than .27 or lower than .03, which, practically, may be considered the limits of its variation. Therefore, if we have no idea of the level of infant mortality in the supposed population, we may assume, as its approximate value, the average of the two extreme values indicated above, that is, .15. By the introduction of this value in the formula [1], the number of live births in the year preceding the census may be obtained by dividing the number of infants under one year of age enumerated at that census by .9. It may be easily verified that the number obtained in this way differs by less than 9 per cent from those that we should obtain by assuming the one or the other of the extreme mortality levels indicated above. Therefore, the number of live births can be determined with an approximation sufficient to disclose its order of magnitude. This knowledge, although only approximate, will constitute a great progress in comparison with our previous absolute ignorance.

[1] Reprint of "The Calculation of Life Tables for Populations Lacking Reliable Birth and Death Statistics, with Application to Brazil" by G. Mortara. (Paper presented to the Statistical Section of the Eighth American Scientific Congress, Washington, D. C., 1940, and previously published in the *Proceedings of the Eighth American Scientific Congress.)*

The number of births thus calculated may be checked, and the acquired information can be extended, by the application of a similar proceeding directed toward the calculation of the number of live births in the second, third, etc., year preceding the second census. Of course, the further we go from the census date, the greater the possible error introduced by the arbitrary hypotheses on mortality. But generally it will be possible to determine the total number of births occurring in the interval between the two censuses with an error of less than 10 per cent.

The knowledge of the population data ${}_0P'$ and ${}_{10}P'$ will permit the determination of the average population for each of the ten intercensal years on the ground of one of the usual hypotheses of regular increase. It will thus become possible to calculate the crude birth rates for each of these years. For instance, division of the assumed value of ${}_9N'$ by the mean population of the year immediately preceding the second census will give the birth rate for that year.

Now we shall pass to the approximate determination of the number of deaths. Our assumed population being "closed", the number of its components at the end of a given period will be equal to their initial number, increased by the number of births and diminished by the number of deaths that occurred in the same period. In particular, between the results of the two decennial censuses, there will exist the relation

$$ {}_{10}P' = {}_0P' + ({}_0N' + {}_1N' + \ldots + {}_9N') - ({}_0M' + {}_1M' + \ldots + {}_9M'). \qquad [2] $$

Therefore, having obtained the approximate number of the births that occurred in each of the intercensal years, we can immediately deduce from the preceding formula the total number of deaths in this interval and thence calculate the average death rate for the intercensal period.

The absolute error in the total number of deaths will be equal to that in the total number of births, but the *relative* error will be larger if, as normally occurs, the number of deaths is smaller than the number of births.[2] If k denotes the ratio of the total deaths to the total births and b the relative error of the total births, the relative error of the total deaths will be $\delta = b:k$. For instance, if $k = 0.70$ and $b = 0.07$ then $\delta = 0.07:0.70 = 0.10$.

The deaths occurring in the decennial interval between the two censuses may be divided into two classes.

[2] By "absolute error" I mean the difference between the known value and the true value of a statistic; by "relative error" the quotient of the absolute error by the true value of the statistic.

(1) The deaths of persons who were alive at the first census;

(2) The deaths of persons who were born after the first census.

The first class can be determined with absolute precision if the census data are exact; it is represented by the difference

$$ \sum_{x=0}^{x=\omega} {}_0L'_x \quad \text{——} \quad \sum_{x=10}^{x=\omega} {}_{10}L'_x \qquad [3] $$

between the total number living according to the first census and the number of survivors out of them according to the second census. ω stands for the extreme age in the population. The second class can be determined only with approximation. This second class is represented by the formula

$$ \sum_{i=0}^{i=9} {}_iN' \quad \text{——} \quad \sum_{x=0}^{x=9} {}_{10}L'_x \qquad [4] $$

which gives the difference between the calculated total number of live births in the intercensal period and the number of survivors out of them at the second census. This second class will thus be determined with an absolute error equal to that of the calculated total number of live births.

By the hypothesis of an exact census, we know *exactly* the number of deaths that occurred in ages over ten years, all these deaths belonging to the first class. On the other hand, we can know only *approximately* the deaths that occurred up to the age of ten years, since most of them belong to the second class, being deaths of persons born in the interval between the two censuses.

The comparison between the age distributions of living persons according to the two censuses allows the approximate reconstruction of the age distribution of the deaths belonging to the first class. Indeed, the difference

$$ {}_0L'_x - {}_{10}L'_{x+10} \qquad [5] $$

between the number living in a given year of age at the first census and the number of survivors out of them, in a ten years higher age, at the second census, represents deaths that occurred in ages comprised between x and $x + 11$. And the ratio

$$ ({}_0L'_x - {}_{10}L'_{x+10})/{}_0L'_x \qquad [6] $$

is the corresponding rate of mortality.

If we represent by d'_x the deaths from the observed group that occurred between the ages x and $x + 1$, by d'_{x+1} those that occurred between the ages $x + 1$ and $x + 2$, etc., we may assume, without risk of serious error for values of x greater

than 1, that about half of the d'_x deaths had occurred before the first census, and that about half of the number d'_{x+10} occurred after the second census.[3] We may thus write the approximate relation

$$ {}_0L'_x - {}_{10}L'_{x+10} = 0.5d'_x + d'_{x+1} + \cdots + d'_{x+9} + 0.5d'_{x+10}. \qquad [7] $$

By comparison between the two censuses, we are able to know the total number of deaths represented by the first member of the preceding relation. How can we now distribute it by years of age according to the scheme of the second member?

In the conditions assumed, the problem cannot be solved exactly, but generally we shall be able to get a good approximation if we make use of a life table for some other population, presenting rates of mortality only slightly different from those obtained for our supposed population by the formula [6]; that is, l_x being the number living at the precise age x, and $L_x = 0.5\ (l_x + l_{x+1})$ the average number living between the ages x and $x + 1$ according to the said table, the ratio

$$ (L_x - L_{x+10})/L_x \qquad [6 \text{ bis}] $$

is to differ only slightly from the corresponding ratio [6]. Representing by $d_x = l_x - l_{x+1}$, $d_{x+1} = l_{x+1} - l_{x+2}$, etc., the deaths that occurred in the various years of age according to the life table chosen, we can deduce from it:

$$ L_x - L_{x+10} = 0.5d_x + d_{x+1} + \cdots + d_{x+9} + 0.5d_{x+10}. \qquad [7 \text{ bis}] $$

Then, if we assume the age distribution of the deaths in [7] to be proportional to that in [7 bis], each of the unknown d'_x numbers will be obtained by multiplying the corresponding d_x number by the factor h

$$ h = ({}_0L'_x - {}_{10}L'_{x+10})/(L_x - L_{x+10}). \qquad [8] $$

Operating in this way for all the annual groups of age, beginning from ${}_0L'_0$, we shall be able to obtain the complete age distribution of the deaths of the first class occurring in the intercensal period.

The age distribution of the deaths of the second class remains to be reconstructed; that is, the deaths of persons born after the first census. We may employ to this end a parallel procedure to that applied for the first class. We shall begin by calculating the difference

$$ {}_1N' - {}_{10}L'_{9-1} \qquad [9] $$

between the initial number of live births and the corresponding number of survivors at the second census, which represents the number of deaths that occurred in the ${}_1N'$ group before the second census. Then we shall calculate the rate of mortality

$$ ({}_1N' - {}_{10}L'_{9-1})/{}_1N'. \qquad [10] $$

In case we should find among the existing life tables one in which the rate of mortality given by

$$ (l_0 - L_{9-1})/l_0 = [l_0 - \tfrac{1}{2}\ (l_{9-1} + l_{10-1})]/l_0 \qquad [10 \text{ bis}] $$

differs only slightly from the rate obtained from [10], we shall assume the distribution of the deaths in [9] by years of age to be proportional to that of the corresponding deaths represented by the numerator in [10 bis], deduced from the life table selected.

We have described how the complete distribution of the intercensal deaths by years of age can be estimated; it is also possible to extend the application of the process so as to obtain this classification separately for each year of the decennial period.[4] Since the initial age distribution of the population is known, thanks to the first census, the knowledge of the age distribution of deaths, and of the annual number of births, will permit the determination of the age distribution of the population at the end of each year between the two censuses, and thence the calculation of the average population of the decennial period by years of age. The problem of the calculation of a life table for the intercensal period can then be solved in the ordinary ways.

Sometimes it will only be possible, or more convenient for theoretical or practical reasons, to obtain the age distribution by larger age groups. Even in this case, the process described will be applicable with only a moderate loss of accuracy, provided the extension of the age groups does not exceed five years.

We can only hope to find that the hypothetical conditions which have been assumed are realized

[3] In the first two years of age, if we wish to obtain a better approximation than that offered by [7], we may use proportions differing from 0.5. For $x = 0$ we may assume that two-thirds of the d'_0 deaths had occurred before the first census; for $x = 1$ the corresponding proportion of the d'_1 may be assumed to be three-fifths. For $x = 0$ and $x = 1$, these assumptions would require modifications of the expression for L_x given in the text, and of the formula [7 bis].

[4] The separate determination of the deaths that occurred in each year of the decennial period between two censuses allows the calculation of the crude death rate for each of these years. But, owing to the process of determination, these rates will represent the trend rather than the real course of mortality.

with an approximation sufficient to allow the fruitful application of the process described. A "closed" population does not permanently exist, but there are populations "nearly closed", that is, with migratory movements of very little relative importance. Moreover, there are no "absolutely precise" censuses; but many of them indicate the total number of the inhabitants with good approximation. As to the age distribution of the population, each census presents more or less frequent and large errors, which, however, can mostly be approximately corrected with the help of demographic technique.

On the other hand, the conditions may in reality be more favourable than those assumed: for instance, although the number of births and deaths for the whole population is not known, the corresponding number for some section or group of the same population may be known or some objective birth or death rate estimate may be available. Those hypotheses concerning these phenomena which are required by our process, will then be founded on a more solid basis than that described so far.

The process outlined above was suggested by the needs of the investigation on mortality in Brazil and was tentatively applied to the Brazilian-born population, which may be regarded as "nearly closed", i.e., as providing only a negligible number of emigrants and receiving only a more negligible number of re-immigrants. Of this population, the fourth Brazilian census (1920) indicates the age distribution separately from that of the foreign-born inhabitants of this country. By the aid of some prudent hypotheses it was possible to deduce from the first (1872) and from the second (1890) census the corresponding age distributions in the years 1870 and 1890.

The total number of Brazilian-born inhabitants given by each census was taken to be reliable, though some necessary rectification was introduced in the age classification, which was affected by evident errors. The age distribution of the population was calculated by groups of five years and with intervals of time of five years. The age distribution of the deaths was calculated by groups of five years and for periods of time of five years. The distribution into groups of one year of age was obtained by interpolation when it became necessary.

After having calculated the age distribution of the average population and that of the average annual number of deaths for the periods 1 September 1870 - 31 August 1890 and 1 September 1890 - 31 August 1920, it was possible to undertake the construction of a life table for each of these periods.

This task was made less difficult by the circumstance that, although reliable data on births and deaths for the whole Brazilian population were lacking, there were available not only partial and local data, but also life tables for the population of the Brazilian capitals taken together and separately for the population of the federal capital (Rio de Janeiro).

As an illustration of the process described, we shall demonstrate its application to Brazil for the thirty-year period 1 September 1890 - 31 August 1920. It must be remembered that, since the formulae given in the text refer to a decennial period, they have to be modified for application to the longer period considered; but the preceding exposition can easily be adjusted to fit this longer interval of time.

Table 1 shows how the data on deaths of the first class (deaths of persons who were living in 1890) are determined from the age distribution of the Brazilian-born population at the two censuses. To this end, the number living in 1890 in the ages x to $x + 4$ is compared with the number living in 1920 in the ages $x + 30$ to $x + 34$; the difference between these two numbers represents the deaths that occurred in the intercensal period.

The ratio of deaths to the initial number living gives the rate of mortality, which in table 2 is compared with the corresponding rate deduced from the Bulhões Carvalho Life Table for Brazilian Capitals (1920), revised and adjusted by the author of the present paper (to be briefly designated as "1920 life table (BC Cap. Adjust.)"). The observed rates are lower than those deduced from this life table in the lower ages, and higher in the middle- and old-ages, but the relative differences between the two series of rates are small, as appears from the right-hand column of the same table.

Table 3 shows how the data on the deaths of the second class, that is, how deaths of persons born after the 1890 census are determined by the differences between the calculated numbers of births in quinquennial periods and the corresponding numbers of survivors in 1920. The ratios of these deaths to the initial numbers of live births give the rates of mortality, which are compared in table 4 with that deduced from the 1920 life table (BC Cap. Adjust.).

The subdivision of the 503,880 deaths observed during the thirty-year period by minor intervals of age, proportionate to the corresponding deaths of

the 1920 life table (BC Cap. Adjust.), is illustrated by an example given in table 5.

Table 6 shows, by quinquennial groups of age, the average population and the average annual number of deaths that occurred in the period 1 September 1890 - 31 August 1920, that is, the final elements on which the calculation of the life table was based.

Table 7 presents the fundamental data of the 1890-1920 life table for the Brazilian-born population, that is: the rate of mortality, the number of survivors and the complete expectation of life at each whole age x, and the number of deaths between the ages x and $x + 1$.

Finally, in table 8 the data of our Brazilian life table for some selected ages are compared with the corresponding data of other life tables. The most peculiar characteristic of the Brazilian mortality consists in the heavy tribute paid to death in the middle ages, as appears from an international comparison of the age distribution of deaths (see central section of table 8). Some comment on this subject and other characteristics of our life table would be highly interesting, but it would lead outside the scope of the present paper.

TABLE 1

Calculation of the number of deaths that occurred between the dates 1 September 1890 and 31 August 1920, for persons who were living at the earlier date

Age in 1890	*Number living in 1890 (in thousands)*	*Ages in 1920*	*Number living in 1920 (in thousands)*	*Deaths that occurred between 1890 and 1920 (in thousands)*
0– 4	2,378.3	30–34	1,759.5	618.8
5– 9	1,900.1	35–39	1,436.2	463.9
10–14	1,648.8	40–44	1,158.7	490.1
15–19	1,425.8	45–49	921.9	503.9
20–24	1,225.5	50–54	728.9	496.6
25–29	1,041.4	55–59	561.0	480.4
30–34	879.6	60–64	413.9	465.7
35–39	736.7	65–69	287.4	449.3
40–44	610.1	70–74	182.7	427.4
45–49	497.3	75–79	101.8	395.5
50–54	396.8	80–84	46.3	350.5
55–59	307.3	85–89	15.3	292.0
60–64	230.0	90–94	3.0	227.0
65–69	162.8	95–99	.2	162.6
70–ω	205.2	100–ω	...	205.2
0–ω	13,645.7	30–ω	7,616.8	6,028.9

Note. All data in this and the following tables relate to the Brazilian-born population of Brazil.

In the original calculations the data were approximated to the nearest tenth or, in the higher ages, to the nearest unit.

As we have said already, it will be extremely difficult to find real populations exactly fulfilling the conditions assumed in our theoretical treatment of the subject. But the success of our application to Brazil attests the possibility of practical and useful employment of the process devised, in conditions only approximately corresponding to the initial hypotheses. Particularly for Latin-American countries our process could find a wide field of application, for the past as well as for the near future when new censuses will supply the required data, that is, the age distributions of the locally-born populations.

TABLE 2

Mortality rates calculated on the data of table 1, and a comparison of them with those deduced from the 1920 life table (BC Cap. Adjust.)

Age in 1890	*Age in 1920*	*Rate of mortality between the age in 1890 and that in 1920 (per 1,000)*	*Corresponding rate from the table BC Cap. Adjust. (per 1,000)*	*Ratio of the first to the second rate*
0– 4.....	30–34	260.2	275.8	.943
5– 9.....	35–39	244.1	254.0	.961
10–14.....	40–44	297.2	299.0	.994
15–19.....	45–49	353.4	349.9	1.010
20–24.....	50–54	405.2	395.4	1.025
25–29.....	55–59	461.3	446.3	1.034
30–34.....	60–64	529.4	510.7	1.037
35–39.....	65–69	609.9	589.3	1.035
40–44.....	70–74	700.6	680.5	1.030
45–49.....	75–79	795.0	778.6	1.021
50–54.....	80–84	883.3	871.9	1.013
55–59.....	85–89	950.3	944.6	1.006
60–64.....	90–94	987.0	985.3	1.002
65–69.....	95–99	998.5	998.4	1.000
70–ω.....	100–ω	1,000.0	1,000.0	1.000

TABLE 3

Calculation of the number of deaths that occurred before 1 September 1920, of persons who were born after 1 September 1890

Years preceding the census of 1920	*Live births in the years stated (in thousands)*	*Age in 1920*	*Number living in 1920 (in thousands)*	*Deaths that occurred between the date of birth and 31 August 1920 (in thousands)*
0– 4.....	6,682.1	0– 4	5,375.6	1,306.5
5– 9.....	5,881.1	5– 9	4,328.8	1,552.3
10–14.....	5,176.4	10–14	3,710.4	1,466.0
15–19.....	4,556.0	15–19	3,172.0	1,384.0
20–24.....	4,009.9	20–24	2,653.5	1,356.4
25–29.....	3,529.3	25–29	2,188.0	1,341.3
0–30	29,834.8	0–29	21,428.3	8,406.5

TABLE 4

Mortality rates calculated on the data of table 3, and a comparison of them with those deduced from the 1920 life table (BC Cap. Adjust.)

Age in 1920	*Rate of mortality between birth and the age stated (per 1,000)*	*Corresponding rate from the table BC Cap. Adjust. (per 1,000)*	*Ratio of the first to the second rate*
0– 4.....	195.5	235.6	.830
5– 9.....	263.9	310.5	.850
10–14.....	283.2	325.5	.870
15–19.....	303.8	341.3	.890
20–24.....	338.3	371.7	.910
25–29.....	380.0	408.6	.930

TABLE 5

Example of the calculation of the age distribution of the observed deaths, proportionate to that of the 1920 life table (BC Cap. Adjust.)

(This example applies to population aged 15-19, living at the census of 1890)

Ages	*Deaths within age intervals*	
	Table BC Cap. Adjust.	*Brazilian-born population 1890–1920 (adjusted)*
15–19.....	1,257	27,480
20	1,782	38,960
20–24.....	1,836	40,140
25	1,857	40,600
25–29.....	1,868	40,840
30	1,906	41,670
30–34.....	1,931	42,220
35	1,995	43,620
35–39.....	2,032	44,430
40	2,123	46,410
40–44.....	2,172	47,490
45	2,288	50,020
Total	23,047	503,880

Note. The data represent the differences between the average number living in the ages from x to $x + 4$ and the number surviving at precise age $x + 5$, or the differences between the number surviving at precise age x and the average number living in the ages from x to $x + 4$. The data of the first column were deduced from the life table mentioned. Those of the second column were calculated according to the assumption that the age distribution of the total of 503,880 deaths (which appears rounded to 503.9 thousands in table 1) was proportional to that of the 23,047 deaths of the first column.

TABLE 6

Average population, average annual number of deaths, and central death rate by quinquennial age groups, for the Brazilian-born population in the period 1 September 1890—31 August 1920

Age	*Average population (1890–1920) of the given age group*	*Average annual number of deaths (1890–1920) in the given age group*	*Central death rate per 1,000 (see note)*
0– 4.....	3,700,562	253,270	68.44
5– 9.....	2,955,711	15,823	5.35
10–14.....	2,528,842	8,432	3.33
15–19.....	2,161,527	13,151	6.08
20–24.....	1,810,232	19,652	10.86
25–29.....	1,500,382	18,314	12.21
30–34.....	1,243,319	17,102	13.76
35–39.....	1,028,390	16,300	15.85
40–44.....	844,289	15,759	18.67
45–49.....	685,415	15,387	22.45
50–54.....	546,151	15,084	27.62
55–59.....	422,808	14,698	34.76
60–64.....	314,257	14,093	44.85
65–69.....	220,472	13,096	59.40
70–74.....	142,106	11,490	80.86
75–79.....	80,567	9,139	113.43
80–84.....	37,437	6,155	164.41
85–89.....	12,655	3,124	246.86
90–94.....	2,521	975	386.76
95–ω	204	129	632.35
0–ω	20,237,847	481,173	23.78

Note. The central death rate is the ratio of the average annual number of deaths to the average population.

TABLE 7

Life table 1890-1920 for the Brazilian-born population[a]

Age x	*Rate of mortality at age x (per 1,000)*	*Survivors at age x*	*Deaths between ages x and x+1*	*Complete expectation of life at age x*
0	170.03	100,000	17,003	39.25
1	62.93	82,997	5,223	46.19
2	30.44	77,774	2,367	48.25
3	15.81	75,407	1,193	48.75
4	10.57	74,214	784	48.53
5	6.88	73,430	505	48.04
6	5.98	72,925	436	47.37
7	5.18	72,489	376	46.65
8	4.49	72,113	324	45.89
9	3.93	71,789	282	45.10
10	3.51	71,507	251	44.27
11	3.24	71,256	231	43.43
12	3.14	71,025	223	42.57
13	3.23	70,802	228	41.70
14	3.53	70,574	249	40.83
15	4.05	70,325	285	39.98
16	4.82	70,040	338	39.14
17	5.85	69,702	408	38.82
18	7.17	69,294	496	37.55
19	8.81	68,798	607	36.81
20	10.01	68,191	682	36.14
21	10.79	67,509	729	35.50
22	11.14	66,780	744	34.88
23	11.32	66,036	747	34.27
24	11.51	65,289	752	33.65
25	11.72	64,537	756	33.04
26	11.94	63,781	761	32.42
27	12.18	63,020	768	31.81
28	12.43	62,252	774	31.20
29	12.71	61,478	781	30.58
30	13.00	60,697	789	29.97
31	13.30	59,908	797	29.36
32	13.63	59,111	806	28.75
33	13.99	58,305	815	28.14
34	14.36	57,490	826	27.53
35	14.76	56,664	836	26.92
36	15.18	55,828	848	26.32
37	15.63	54,980	859	25.72
38	16.11	54,121	872	25.12
39	16.62	53,249	885	24.52
40	17.16	52,364	899	23.93
41	17.73	51,465	912	23.34
42	18.35	50,553	928	22.75
43	19.00	49,625	943	22.16
44	19.69	48,682	958	21.58
45	20.43	47,724	975	21.01
46	21.22	46,749	992	20.43
47	22.05	45,757	1,009	19.87
48	22.95	44,748	1,027	19.30
49	23.90	43,721	1,045	18.74
50	24.91	42,646	1,063	18.19
51	25.99	41,613	1,082	17.64
52	27.15	40,531	1,100	17.10
53	28.38	39,431	1,119	16.56
54	29.70	38,312	1,138	16.03

[a] This table will be cited hereafter as the "Brazil 1890-1920 life table".

TABLE 7 *(Continued)*

Life table 1890-1920 for the Brazilian-born population[a]

Age x	*Rate of mortality at age x (per 1,000)*	*Survivors at age x*	*Deaths between ages x and x+1*	*Complete expectation of life at age x*
55.....	31.11	37,174	1,156	15.51
56.....	32.62	36,018	1,175	14.99
57.....	34.23	34,843	1,193	14.48
58.....	35.95	33,650	1,210	13.97
59.....	37.80	32,440	1,226	13.48
60.....	39.78	31,214	1,242	12.99
61.....	41.91	29,972	1,256	12.50
62.....	44.19	28,716	1,268	12.03
63.....	46.64	27,448	1,281	11.56
64.....	49.27	26,167	1,289	11.10
65.....	52.10	24,878	1,296	10.65
66.....	55.14	23,582	1,301	10.21
67.....	58.42	22,281	1,301	9.78
68.....	61.95	20,980	1,300	9.35
69.....	65.76	19,680	1,294	8.94
70.....	69.87	18,386	1,285	8.53
71.....	74.31	17,101	1,271	8.13
72.....	79.10	15,830	1,252	7.75
73.....	84.28	14,578	1,228	7.37
74.....	89.89	13,350	1,200	7.00
75.....	95.96	12,150	1,166	6.64
76.....	102.54	10,984	1,127	6.30
77.....	109.67	9,857	1,081	5.96
78.....	117.42	8,776	1,030	5.63
79.....	125.83	7,746	975	5.31
80.....	134.96	6,771	914	5.01
81.....	144.91	5,857	848	4.71
82.....	155.73	5,009	780	4.42
83.....	167.52	4,229	709	4.15
84.....	180.37	3,520	635	3.88
85.....	194.39	2,885	561	3.63
86.....	209.70	2,324	487	3.38
87.....	226.44	1,837	416	3.15
88.....	244.74	1,421	348	2.92
89.....	264.78	1,073	284	2.70
90.....	286.72	789	226	2.50
91.....	310.79	563	175	2.30
92.....	337.19	388	131	2.11
93.....	366.18	257	94	1.93
94.....	398.05	163	65	1.76
95.....	433.10	98	42	1.59
96.....	471.68	56	27	1.41
97.....	514.19	29	15	1.26
98.....	561.07	14	8	1.07
99.....	612.80	6	4	.83
100.....	.....	2	2	.....

[a] This table will be cited hereafter as the "Brazil 1890–1920 life table".

Note. The rates of mortality were calculated from data for one-year age groups, obtained by interpolation from those of table 5 giving population and deaths by five-year groups. For the first five years of age a different process was adopted, it being there assumed that the rates of mortality are proportional to those of the table BC Cap. Adjust., and that the total number of deaths is equal to that observed. For the ages from 22 years onwards the calculated rates were graduated by a parabolic-logarithmic process which, however, only slightly altered the original values.

TABLE 8

International comparisons of life tables (reduced to the initial number of 1,000 survivors at age 0)

Age	*Brazil 1890–1920*	*Italy 1899–1902*	*England and Wales 1838–1854*	*France 1861–1865*	*Germany 1871–1880*	*American experience*
A. Survivors at age x						
0	1,000	1,000	1,000	1,000	1,000	1,000
5	734	717	737	694	665	732
10	715	695	703	668	637	695
20	682	666	663	629	608	644
40	524	575	539	524	502	543
60	312	440	370	385	337	403
80	68	95	90	88	58	101
B. Deaths in the indicated age intervals						
Age intervals						
0-19	318	334	337	371	392	356
20-59	370	226	293	244	271	241
60-ω	312	440	370	385	337	403
C. Complete expectation of life at age x						
Age						
0	39.25	43.00	40.90	39.85	37.02	41.45
10	44.27	51.10	47.35	48.70	47.38	48.72
20	36.14	43.10	39.90	41.40	39.36	42.20
60	12.99	13.60	13.90	13.75	12.46	14.10

Note. The European life tables cited are reproduced in the *Statistique internationale du mouvement de la population* edited by the Statistique Générale de la France (vol. 1, Paris, 1907, pp. 534, 546, 558, 560).

The "American experience" table is reproduced in the excellent book of L. I. Dublin and A. J. Lotka on *Length of Life* (New York, 1936), p. 376.

II. BIRTH RATES [1]

1. The population census as a supplement to the statistics of population movements

In a country possessing correct statistics on population movements, that is births, deaths, immigration and emigration, the population census is of great importance, both as a description of the state of the population at a given moment, and as a point of reference for the data on those movements and innumerable other economic and social phenomena occurring in periods of time close to the census. Only by means of this source of reference can the intensity of the various phenomena be measured, starting from simple methods, such as ascertaining the general death rate, birth rate, unemployment rate, criminality rate, etc., in order to arrive at more complex statistics such as life tables, fertility rates, etc.

In a country not possessing trustworthy statistics on population movements the importance of the population census is still greater, because it provides an approximate knowledge, at least in part, of certain fundamental data about such movements, always provided that recourse to sensible and reasonable hypotheses is allowed. For example, the number of persons living during the early years of life is less than the annual number of births during the period immediately preceding the census; comparison between the number of the living in successive age groups may give some indications concerning the number of deaths between the ages in question; the anomalies of the distribution of population in respect of age indicate the extent of migratory movements.

Investigation of this kind has not been lacking in Brazil, where accurate vital statistics of the population are a long-felt want which has not yet been filled. In the introduction to the volume on the census of 1920 referring to the age of the population, Dr. J. L. S. de Bulhões Carvalho, who has performed such outstanding work in Brazilian statistical organization and the use of national statistics, showed the possibility of calculating the number of births occurring in the year immediately preceding the census on the basis of the number of children of one year of age enumerated at the census, and explained some applications of the proposed method by calculating the number of births in various capitals and various states of Brazil. (See *Recenseamento do Brasil realizado em 1 setembro de 1920,* vol. IV, part 2, book I. pp. XLV *et seq.)*

While preparations are going on for the Brazilian census of 1940, it appears opportune to examine anew the problem of approximately determining measures on population movements, based on the results of a population census; for the next census should make it possible, among other things, to integrate, as far as possible, our very inadequate data on the population movement. Once figures for the past have been assembled, it may be easier, thanks to the possibility of more effective control, for the administrative bodies to obtain, in future, registrations of births and deaths. Thus it may be possible not only to complete statistics of births or deaths which are so important to obtain knowledge of the life of the country, but also to increase the efficiency of the civil registers —institutions of primary importance in the political, administrative, and juridical organization of the country.

The fresh examination of this old problem which we are about to attempt is directed towards the double purpose of determining the most suitable methods of obtaining acceptable results and applying these methods to a period previous to the 1920 census, in order to obtain data concerning the movement of the population comparable with those which it will be possible to obtain for the subsequent twenty years from the results of the next population census.

2. Relations between the census data and elements in the movement of the population

The first point which we shall establish is the method for deducing from a population census (and in order to simplify the explanation we shall refer solely to the Brazilian census of 1920) estimates of the numbers of births, not only in the year immediately preceding the census, but also in previous years.

[1] Translated from "Estudos sôbre a utilização do censo demográfico para a reconstrução das estatísticas do movimento da população do Brazil" (The use of census returns for reconstruction of missing data of vital statistics, with application to the population of Brazil), by G. Mortara. *Revista Brasileira de Estatistica.* vol. I, No. 1. January, March 1939.

If the census of 1 September 1920 had produced absolutely accurate results, the number of living persons, V, enumerated between the ages of x and $(x+1)$ would be equal to:

(*a*) The number N of live births occurring in Brazil in the period from 1 September $(1919 - x)$ to 31 August $(1920 - x)$;

(*b*) Plus the number I of immigrants arriving in Brazil up to 31 August 1920, born in the same period;

(*c*) Minus the number M of persons born in the same period and dying in Brazil before 31 August 1920;

(*d*) Minus the number E of emigrants from Brazil up to 31 August 1920, born in the same period.

We thus have the formula:

$$V = N + I - M - E. \qquad [1]$$

If four of the five magnitudes shown in the above equation are known, the fifth can be calculated. Thus, it would be possible to calculate the number of births if the numbers of persons enumerated, of deaths, of emigrants and of immigrants were known.

The difficulty arising from the lack of accurate data on immigrants and emigrants, classified by age, may be avoided by considering separately, as the results of the census permit, only those persons enumerated who are of Brazilian nationality.

3. Analysis of the terms of the relations established

The living persons (V) between the ages of x and $x+1$ may be divided according to nationality, into two groups: V′ Brazilian and V″ aliens, that is:

$$V = V' + V''. \qquad [2]$$

The group of Brazilians in its turn includes four sub-groups:

v_1' those born in Brazil, whether of Brazilian or foreign parents, who have Brazilian nationality by birth;

v_2' those born abroad, who, being children of Brazilian parents, have Brazilian nationality by right;

v_3' those born in Brazil, of foreign parents, who have at first retained their foreign nationality but have subsequently acquired Brazilian nationality;

v_4' those, born abroad of foreign parents, who have acquired Brazilian nationality since immigrating into Brazil.

That is:

$$V' = v_1' + v_2' + v_3' + v_4'. \qquad [3]$$

For the first ten years of age, sub-groups 2, 3 and 4 account for very small numbers and are thus only a small proportion of the number V′. At later ages sub-group 4 increases in size; sub-groups 2 and 3 continue very small.[2]

The group of foreigners may also be divided into the following two sub-groups:

v_1'' those born abroad, children of foreign parents who have immigrated to Brazil;

v''_2 those born in Brazil, children of foreign parents, who have retained their foreign nationality.

That is:

$$V'' = v_1'' + v_2''. \qquad [4]$$

The numbers in sub-group 2 are small, both in absolute value and in relation to sub-group 1, both in the first ten years of age and the later ages.

4. Simplification of the formulae

The quantity I in formula [1] obviously includes sub-groups 2 and 4 of the first, sub-group 1 of the second group and, in addition, a number M'' of individuals born in the year under consideration who immigrated to Brazil and died there before the date of the population census. This gives:

$$I = v_2' + v_4' + v_1'' + M''. \qquad [5]$$

Substituting V″ for I, we have the error ϵ which may be expressed by the difference:

$$\epsilon = V'' - I = v_2'' - v_2' - v_4' - M''. \qquad [6]$$

Given the absolute smallness of the four terms of the second member and the relative smallness of all of them in comparison with I, and further taking into account the partial compensation brought about by the positive and negative elements in the error, we may consider the error ϵ arising from the substitution of V″ for I as negligible in the first ten years of age. In respect of later ages this error may become of appreciable size and in general the negative elements prevail over the positive; in this case the substitution of V″ for I tends to produce an error of underestimation.

We may also consider that, for the first ten years of life, the magnitude E of formula [1] may be very small in comparison with the values V and V′. At higher ages, E acquires greater importance.

[2] In the Brazilian census of 1920, the 52,326 "foreigners who had Brazilian nationality", that is those belonging to sub-group 4, were classified as foreigners (group V″), though some of them may have been included in group V′ (Brazilians) by error of classification.

5. Formulae

Starting from these considerations and the equations given, we may rewrite formula [1] in the following manner:

$$V = N + I - M - E = N + V'' - \epsilon - M - E. \qquad [1\text{ bis}]$$

Abandoning the negative term $-E$ and the probably positive term $-\epsilon$, the foregoing formula can be reduced to the following, which is approximately accurate as a representation of the phenomenon under consideration:

$$V = N + V'' - M. \qquad [7]$$

From this the following relation may be deduced:

$$V - V'' = N - M, \qquad [8]$$

which, compared with formula [2], makes it possible to obtain, also approximately, the following formula:

$$V' = N - M. \qquad [9]$$

We thus reach the formula which was applied by Bulhões Carvalho in the above-mentioned calculation of live births, starting from the number of persons enumerated in the first year of their age. The foregoing account reveals the hypotheses implicit in the application of formula [9] and the consequent possibilities of error. In a general way we may expect that these possibilities will increase in proportion as the age of the group of enumerated persons under consideration increases.

6. Estimates of mortality data not available in Brazil

Knowing two of the three quantities linked in equation [9], we may make an approximate calculation of the third.

In the present case the population census indicates the value of V′ and the values of N and M are unknown.

Using the survival table produced by Bulhões Carvalho for the Brazilian capitals in 1920 (*op. cit.*, pp. LXIV - LXVI), in order to obtain an approximation for M, we may also obtain an approximation to N.

Assuming that the probabilities of death and survival indicated in this table are applicable to the total population of Brazil, and representing by $L_{x,x+1}$, the average number of living children[3] between the x^{th} and $(x+1)^{th}$ birthday, according to the above-mentioned table, and by l_0 the number of survivors of 0 years of age, that is, the number of live births in the generation to which the table refers, we may write the equations thus:

$$V' : N = L_{x,x+1} : l_0 \qquad [10]$$

from which we may deduce:

$$N = V' \frac{l_0}{L_{x,x+1}} = V' : \frac{L_{x,x+1}}{l_0}. \qquad [11]$$

The latter form is more convenient for computation. For example, if for the eighth year $V' = 936{,}936$ and $\frac{L_{7,8}}{l_0} = 0.6886$, then $N = \mathbf{1{,}360{,}639}$ is the calculated number of births in the eighth year prior to the date of the census, that is the year from 1 September 1912 to 31 August 1913.

7. Critical appraisal of the foregoing results

If the results of the 1920 census were perfectly accurate, the number N calculated by formula [11] would differ from the real number of live births in the period under consideration only as a result of:

(*a*) The simplifications introduced into the calculation (substitution of I for V″ and neglect of E);

(*b*) The differences between real Brazilian mortality in the various periods to which the calculation applies, and the mortality recorded in the capitals in 1920, which was used as a basis for the calculation.

The error due to (*a*) is certainly small as regards the first ten years of age.

As regards the error arising from (*b*), we find it very difficult to determine, *a priori* in a general way, its volume, and perhaps even its direction. Bulhões Carvalho stated, with indisputable authority, that in 1920 the mortality rate for the whole of Brazil must have been higher than that in the capitals (*op. cit.* p. LXXVI). If this opinion is right, there is even more reason to believe that the mortality rate in Brazil in the years prior to 1920 will have exceeded that in the capitals in 1920. According to this hypothesis the number of live births, calculated by formula [11], would be an underestimate.

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[3] The survival table indicates the numbers l_0, l_1, l_2, l_3, . . . of survivors on the dates of birth, the first birthday, the second, the third etc.

In order to calculate the average numbers $L_{0,1}$, $L_{1,2}$, $L_{2,3}$, . . . of those living between the date of birth and that of the first birthday, between the date of the first and second birthday, between the date of the second and third birthday etc. the following formulae are used:

$$L_{0,1} = \frac{l_0 + 2l_1}{3}, \quad L_{1,2} = \frac{2l_1 + 3l_2}{5}, \quad L_{2,3} = \frac{l_2 + l_3}{2}.$$

(After the fourth year the formulae can be reduced to the last of these, that is, the average number of persons living in each year of age is equal to half the survivors at the beginning and end of the same year.)

In the foregoing argument we have implied that the table quoted provided a fairly accurate picture of the real mortality in the Brazilian capitals. It would also be possible to base calculations upon a foreign life table, which might be regarded as an approximation to a life table which is not yet available for the whole of Brazil. We tested this process, and obtained results not widely different from those which we shall proceed to set out.

8. Effect of mis-statements of age of persons enumerated, in the first ten years of their age, illustrated by the application of the formulae to the Brazilian population enumerated in the census

There is another circumstance which has an influence perhaps greater than that examined in the preceding paragraph upon the results of the calculation of N.

The correct application of formula [11] requires that the census should have produced acceptable results in respect of the number of persons living in each year of age. Now the Brazilian census of 1920 does not fulfil this requirement, as it is affected by noticeable errors in the statements of age.

It would therefore not be advisable to apply formula [11] to the crude data of the census. This may easily be illustrated by applying the formula to the census data concerning persons in the first ten years of age which appear in column (b) of table 1.

If these data are divided by the corresponding co-efficients $\frac{L_{x,\,x+1}}{l_0}$ deduced from the 1920 life table (BC Cap. Adjust.) and reproduced in column (c) of table 1 (they have been multiplied by 100,000), the number of live births given in column (d) of the same table will be obtained.

TABLE 1

First calculation of the numbers of live births in Brazil in the ten years previous to the census of 1 September 1920

Years of age (*a*)	*Number of persons of Brazilian nationality enumerated* (*b*)	*Coefficient of survival (multiplied by 100,000)* (*c*)	*Calculated number of live births (rounded)* (*d*)	*Year preceding the census of 1 Sept. 1920* (*e*)
1	828,384	88,323	937,900	1
2	776,061	77,954	995,500	2
3	1,014,674	73,637	1,377,900	3
4	1,006,152	71,691	1,403,500	4
5	949,938	70,614	1,345,300	5
6	941,719	69,868	1,347,900	6
7	950,115	69,300	1,371,000	7
8	936,936	68,860	1,360,600	8
9	979,264	68,520	1,429,200	9
10	728,361	68,235	1,067,400	10
Total	9,111,604		12,636,200	

As a result of the obvious anomalies in the distribution of the persons enumerated according to age, the estimated numbers of births are in some cases too low, such as those for the first, second and tenth year previous to the census; in other cases the results do not accord with the most reasonable assumptions concerning the increase in the number of births over this period.

9. Attempts to correct errors and resulting calculation of the number of births in the ten years prior to the census

In order to achieve more accurate results it will be necessary first of all to correct the distribution by age of the persons enumerated. The results obtained in respect of live births will be noticeably different according to whether, for the purposes of this correction, it is assumed that the annual birth rate during the period varied or remained constant.

In a country where either the annual number of births or the probabilities of death in each year of age remain constant over the period, the number of persons enumerated by the census decreases gradually in proportion as the age increases. This decrease will be even more rapid in a country like Brazil where it is very likely that the actual number of births was tending to increase and the probability of death in the early years of life was tending to diminish in the ten years immediately prior to the 1920 census.

In a first estimate we may disregard these tendencies and suppose that the annual numbers of births and probabilities of death in the first ten

years of age have remained constant throughout the period. We may then calculate how the 9,111,604 persons enumerated in the census as included between the ages of 0 and 10 years are distributed, on the following assumptions:

(*a*) That the total number of persons enumerated, between the two age limits stated, is correct;

(*b*) That the probability of death in each year of age in this period is that indicated by the mortality and survival table for the Brazilian capitals previously mentioned;

(*c*) That consequently the distribution by single years of age of the 9,111,604 persons enumerated is in proportion to that deduced from this table.[4]

The results of the calculations based upon these hypotheses are shown in column (*b*) of table 2.

The distribution calculated shows a fairly wide difference from that recorded by the census, showing higher numbers for the ages between birth and the first birthday, between the second and the third and between the ninth and the tenth, but, on the other hand, showing lower numbers for the other ages.

The hypothesis of the constancy of the number of births appears less probable than that of a tendency to variation of the number within the period. On the assumption that in the ten years immediately prior to the 1920 census the absolute number of births had increased by 2 per cent annually,[5] a corrected distribution of children living at the date of the census will be obtained, as shown in column (*c*) of table 2.

Applying to the numbers of living children corresponding to the first or to the second hypothesis the survival coefficients already mentioned in column (*c*) of table 1, the numbers of births indicated respectively in columns (*d*) and (*e*) of table 2 will be obtained.

Despite the appreciable differences between the corrected distribution by age and that given by the census, the mean annual number of births in the ten years prior to 1 September 1920 is not very different from that shown in the different calculations. The results of those calculations are:

TABLE 2

Second and third calculations of the number of live births in Brazil in the ten years prior to the census of 1 September 1920

Year of age	*Number of persons alive at census corrected by means of the life table according to the hypothesis*		*Calculated number of live births (rounded)*		*Year prior to the census of 1 September 1920*
	1 of the constancy of the annual number of births	*2 of a mean annual geometric increase of 2 per cent in the number of births*	*First hypothesis*	*Second hypothesis*	
(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
1	1,106,969	1,203,552	1,253,300	1,362,700	1
2	977,037	1,041,456	1,253,300	1,336,000	2
3	898,495	964,463	1,253,300	1,309,800	3
4	922,914	920,545	1,253,300	1,284,100	4
5	885,010	888,928	1,253,300	1,258,900	5
6	875,625	862,322	1,253,300	1,234,200	6
7	868,518	838,541	1,253,300	1,210,000	7
8	863,051	816,855	1,253,300	1,186,300	8
9	858,769	796,901	1,253,300	1,163,000	9
10	855,216	778,041	1,253,300	1,140,200	10
Total	9,111,604	9,111,604	12,533,000	12,485,200	

[4] After calculating the number $L_{x,\,x+1}$ of the living, according to the life table, we deduce the estimated number $C'_{x,\,x+1}$ of the persons enumerated by means of the equation:

$$C'_{x,\,x+1} : 9{,}111{,}604 = L_{x,\,x+1} : \sum_{x=0}^{x=9} L_{x,\,x+1}$$

For example, for the first year of age:

$$C'_{0,1} : 9{,}111{,}604 = 88{,}323 : 726{,}999;$$

Whence:

$$C'_{0,1} = 1{,}106{,}969.$$

[5] In the case of the hypothesis set forth in the text, the numbers $L_{x,\,x+1}$ should be divided by 1.02^x, the former numbers being calculated by the method previously indicated, in order to obtain the numbers needed to calculate the corrected distribution of live children in accordance with this hypothesis.

In the first estimate, 1,263,620 (age distribution of children as given by the census);

In the second estimate, 1,253,300 (age distribution of children corrected on the hypothesis that the annual number of births remains constant);

In the third estimate, 1,248,520 (age distribution of children corrected on the hypothesis of a progressive increase in the annual number of births).

There are, however, considerable differences, according to the method of calculation followed, in the distribution of births over the ten years of the period under consideration.

If it is assumed, in accordance with the official figures, that the mean population of Brazil in the ten years prior to the 1920 census was about 26.5 million, the mean annual number of about 1.25 million births in this period would indicate a mean annual birth rate of 47.17 per 1,000 inhabitants.

This birth rate is exceptionally high, in comparison with the rates obtained in the same period in most countries of Western civilization. It would therefore appear timely and prudent to examine, by means of international comparisons and a more careful analysis of the Brazilian census of 1920 and preceding censuses, whether the results obtained are plausible and whether there are means of checking and, if necessary, correcting them. This will be the subject of another study.

III. MARRIAGE RATES BY AGE [1]

1. Introduction

Like the other elements of the movement of the population, the annual number of marriages celebrated in Brazil cannot be established with adequate exactitude owing to deficiencies in the statistics derived from civil registration.

For instance, according to civil register statistics, the number of marriages in 1938 was only 132,404 or 3.27 per 1,000 population. But the *Anuário Estatistico do Brasil* (Brazilian Statistical Yearbook) which gives that figure [2] states:

"The data relating to population movement as derived from the civil registers do not give a true demographic picture of the Brazilian population, as the figures appearing therein are very deficient, both as regards the movement of registrations and as regards the collection of the relevant information." The same source states that in 1938 only 2,205 registries provided complete information; 805 provided only incomplete information and 2,115 provided none [3] at all.

In these circumstances it would appear advisable to attempt to determine the annual number of marriages in Brazil by an indirect method based on data concerning the composition of the population according to conjugal status, obtained from a study of the 1940 population census.

The possibility of making use of census results for this purpose was already pointed out in a methodological note in 1943.[4] The procedure proposed in this note was applied to Brazilian and foreign census data as a test.[5] This procedure is now applied to Brazil as a whole, with the object of obtaining the approximate annual number of marriages in the year of the census (1940).

The method adopted in the calculations explained below is based on the principles set forth in the above-mentioned note, with some slight differences in application.

The study refers to the female population. A similar study for the male population would produce slightly different results but of the same order of magnitude.

2. Description of procedure used

The procedure adopted will be explained as simply as possible.

Suppose we observe a given group of women over a period of time, from the youngest age i at which marriage may occur, noting the number of these women who reach each subsequent birthday, and dividing them into two groups, those who remain unmarried and those who are or have been married.

If l_x represents the total number of survivors at the x^{th} birthday and m_x the number of married or formerly married women, the cumulative frequency of marriage [6] up to the x^{th} birthday, N_x, for the women then surviving will be expressed as follows:

$$N_x = \frac{m_x}{l_x}. \qquad [1]$$

If n_i, n_{i+1} . . . n_{x-1} denote the proportions of the females surviving at the x^{th} birthday, who have married for the first time in their $(i+1)^{th}$,

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[1] Translation from "Determinação de nupcialidade feminina, segundo a idade, no Brasil com base na apuração censitária do estado conjugal, e aplicações ao cálculo da tasea de nupcialidade" (Female marriage rates by age, based on marital condition of census returns in Brazil). By G. Mortara. *Revista Brasileira de Estatistica,* 1948. (Dr. Jose Etrog collaborated with Dr. Mortara in the compilation and preparation of the tables.)

[2] The figure 3.00, which appears in the Yearbook, has been corrected to 3.27 following a correction of the population estimate.

[3] Brazilian Statistical Yearbook, year V, 1939-1940, pp. 108, 105 and 106. The Yearbook also gives (pp. 1,167 and 1,168) the number of marriages celebrated by Roman Catholic priests (224,388) and by Protestant ministers (1,236) in 1937; these figures are also incomplete, but their total greatly exceeds the figure for that year given in the civil register (143,534).

[4] G. Mortara, "Note on making use of the Population Census to determine the Frequency of Variations in Civil Status in Terms of Age", published in *Revista Brasileira de Estatistica,* No. 15, 1943, pp. 435 to 438.

[5] These studies, which related to the populations of Mato Grosso and of Italy, have not yet been published.

[6] In this paragraph "marriage frequency" means "frequency of marriages of single women". Thus it does not include second and subsequent marriages. The latter will be taken into account in paragraph 8 below.

$(i+2)^{th}$, . . . x^{th} year of age, i.e., the proportions giving the frequency of marriage for each year of age for the women surviving at the x^{th} birthday, we have

$$N_x = n_i + n_{i+1} + \ldots + n_{x-1}. \qquad [2]$$

It is obvious, therefore, that

$$N_x - N_{x-1} = n_{x-1}, \qquad [3]$$

which shows that, by subtracting from the cumulative marriage rate up to the age of x that up to the age of $(x-1)$, we obtain the annual marriage rate in the x^{th} year of age of the females surviving at the x^{th} birthday.[7]

The same calculation can be made on the basis of the proportions of single women. If s_x be the number of single women surviving at the x^{th} birthday and

$$s_x + m_x = l_x,$$

equation [1] may be expressed thus:

$$N_x = \frac{l_x - s_x}{l_x}, \qquad [1a]$$

and [3] thus:

$$N_x - N_{x-1} = \frac{s_{x-1}}{l_{x-1}} - \frac{s_x}{l_x} = n_{x-1} \qquad [3a]$$

In other words the frequency of marriage at x years of age may be obtained by the difference between the proportions who are single among the survivors at the $(x-1)^{th}$ and x^{th} birthday.

Example: In a given generation of women, on their nineteenth birthday, 73.7 per cent are single and 26.3 per cent are not single; on their twentieth birthday, 64.6 per cent are single and 35.4 per cent are or have been married. The marriage rate,[8] at the nineteenth birthday of the women destined to survive to their twentieth birthday may be determined, according to [3], as follows:

$$35.4\% - 26.3\% = 9.1\%,$$

or, according to [3a],

$$73.7\% - 64.6\% = 9.1\%.$$

3. Conditions of applicability

The examples in the preceding paragraph relate to the women surviving at the x^{th} birthday. Under certain conditions they may be considered approximately valid for the whole of the generation, including those who died between the i^{th} and x^{th} birthdays.

They can also be applied, though generally with a wider margin of error, when the rates for the different ages have been determined by the observation (generally simultaneous) of different generations, instead of a single generation.[9]

If the results of this method are not to deviate too far from reality, it is essential for the course of the marriage rate in terms of age to be approximately the same for the different generations considered. For Brazil this may be assumed to be the case, and therefore application of this method appears justified. In other words it seems permissible to calculate the marriage rate at x years of age by comparing the proportion of single women in a generation aged $(x-1)$ at a given date, and one aged x on the same date.

Before proceeding to explain the results of these calculations, it seems fitting to examine critically the 1940 population census data which provided the basic data for this study.

4. Marital status of women in the 1940 census

Table 1 gives the distribution by marital status and years of age of the women present in Brazil in the 1 September 1940 census.[10]

[7] It should be noted that this is a marriage rate calculated on the basis of all the women in the group in question, considered at x years of age, including those who at the beginning of the year were or had been already married.

[8] In accordance with the nomenclature adopted in former studies by the present author "marriage rate" is used to denote marriage frequencies expressed in the form of proportions per 100 or other power of 10.

[9] The conditions governing the justifiability of applying this method which have already been indicated in the above-mentioned methodological note of 1943, are explained in a fuller and more general form in G. Mortara, *Analise Comparativa de Diversos Criterios Aplicáveis no Estudo Biométrico de Desenvolvimento de Caracteres Coletivamente Tipicos em Função da Idade*, published by the Brazilian Institute of Geography and Statistics (Rio de Janeiro, 1948).

[10] The figures given in table 1 do not include parts of the municipalities of Parintins (Amazonas) and Garça (Sao Paulo), which have approximately 8,100 female inhabitants, whose characteristics could not be tabulated, as the census schedules for those places did not reach the headquarters of the National Census Service.

TABLE 1

Distribution of the female population of Brazil on 1 September 1940 according to marital status and age

Age (completed years) (a)	*Marital status*					
	Single (b)	*Married* (c)	*Separated, judicially separated, divorced* (d)	*Widows* (e)	*Unknown* (f)	*Total* (g)
0	677,051	—	—	—	—	677,051
1	596,719	—	—	—	—	596,719
2	647,805	—	—	—	—	647,805
3	638,539	—	—	—	—	638,539
4	624,035	—	—	—	—	624,035
5	596,054	—	—	—	—	596,054
6	582,548	—	—	—	—	582,548
7	575,802	—	—	—	—	575,802
8	572,926	—	—	—	—	572,926
9	507,510	—	—	—	—	507,510
10	602,523	—	—	—	—	602,523
11	495,283	—	—	—	—	495,283
12	571,966	247	1	1	654	572,869
13	478,662	1,068	5	22	433	480,190
14	491,042	3,471	14	80	354	494,961
15	479,636	10,984	48	150	243	491,061
16	460,224	33,054	110	272	329	493,989
17	374,845	59,695	208	491	317	435,556
18	377,115	103,797	359	994	439	482,704
19	266,108	114,915	371	1,213	376	382,983
20	298,937	201,034	689	2,627	652	503,939
21	175,776	153,186	574	2,178	433	332,147
22	197,532	223,879	813	3,893	593	426,710
23	146,058	203,553	797	4,258	536	355,202
24	132,331	220,593	938	5,091	557	359,510
25	151,471	281,039	1,273	7,936	866	422,585
26	107,754	237,961	1,224	7,346	614	354,899
27	80,680	206,230	1,115	7,162	529	295,716
28	99,058	253,958	1,372	10,972	755	366,115
29	58,504	179,304	1,059	8,464	418	247,749
30	117,370	302,035	1,870	19,597	1,044	441,916
31	35,598	137,306	864	7,222	266	181,256
32	53,391	189,119	1,165	12,324	490	256,489
33	36,601	150,060	994	10,496	361	198,512
34	37,401	152,631	1,088	11,552	328	203,000
35	64,809	218,016	1,460	20,818	741	305,844
36	42,640	171,783	1,216	16,088	467	232,194
37	28,336	133,604	1,071	13,686	342	177,039
38	47,522	185,668	1,364	23,377	540	258,471
39	29,310	131,261	1,199	18,407	285	180,462
40	81,889	256,250	2,226	55,854	1,035	397,254
41	16,282	83,917	692	12,869	186	113,946
42	27,761	121,824	1,048	23,316	320	174,269
43	17,327	91,565	822	17,563	217	127,494
44	17,641	95,131	843	19,415	189	133,219
45	39,351	141,452	1,285	41,048	495	223,631
46	17,862	90,607	832	22,655	226	132,182
47	12,672	69,673	643	17,359	127	100,474
48	24,115	100,185	980	32,461	303	158,044
49	12,079	58,748	650	19,977	178	91,632
50	47,557	138,623	1,405	79,214	630	267,429
51	9,391	46,621	466	17,277	101	73,856
52	15,880	64,419	681	30,321	193	111,494
53	9,109	42,735	444	19,837	99	72,224
54	10,634	46,237	422	23,299	123	80,715

TABLE 1 *(Continued)*

Distribution of the female population of Brazil on 1 September 1940 according to marital status and age

Age (completed years) (a)	*Marital status*					
	Single (b)	*Married* (c)	*Separated, judicially separated, divorced* (d)	*Widows* (e)	*Unknown* (f)	*Total* (g)
55	17,481	61,347	590	39,251	192	118,861
56	10,649	42,697	448	26,897	113	80,804
57	6,339	28,910	301	16,949	85	52,584
58	11,941	40,857	393	30,602	121	83,914
59	6,195	24,067	242	19,005	64	49,573
60	29,551	60,006	689	89,014	421	179,681
61	3,841	15,815	161	13,195	38	33,050
62	6,948	21,333	193	23,625	67	52,166
63	5,377	18,193	192	20,839	71	44,672
64	5,017	16,978	139	20,334	58	42,526
65	10,712	23,266	215	39,396	142	73,731
66	4,198	12,967	109	18,047	40	35,361
67	2,916	9,401	107	14,009	44	26,477
68	5,725	12,867	112	23,246	74	42,024
69	2,715	6,704	77	12,277	23	21,796
70	13,784	17,256	184	54,755	183	86,162
71	1,748	4,229	38	8,103	18	14,136
72	3,157	5,702	48	14,107	34	23,048
73	1,991	4,059	37	10,633	23	16,743
74	1,886	3,896	36	10,122	26	15,966
75	4,678	5,447	67	20,056	63	30,311
76	1,906	3,081	28	9,895	24	14,934
77	1,186	1,984	14	6,727	22	9,933
78	2,266	2,477	35	10,814	29	15,621
79	1,048	1,335	8	5,404	9	7,804
80	6,554	3,897	49	26,587	98	37,185
81	621	659	5	3,280	6	4,571
82	1,036	910	8	5,004	16	6,974
83	564	529	6	3,367	10	4,476
84	695	528	6	3,791	15	5,035
85	1,615	821	11	6,545	30	9,022
86	640	406	4	3,156	13	4,219
87	342	250	3	1,880	3	2,478
88	499	294	5	2,501	1	3,300
89	326	164	1	1,521	—	2,012
90	1,894	594	8	7,136	44	9,676
91	123	67	1	594	1	786
92	261	101	1	1,036	4	1,403
93	125	54	1	579	1	760
94	127	55	2	627	—	811
95	510	168	5	1,750	13	2,446
96	221	78	2	920	—	1,221
97	69	48	—	329	2	448
98	311	110	2	1,072	3	1,498
99	133	47	2	483	—	665
100 and over	1,086	275	9	3,647	18	5,035
Unknown	8,734	5,556	100	2,633	579	17,602
Total	13,106,763	6,167,923	41,394	1,284,922	21,225	20,622,227

In interpreting these data, the observations made in study no. 344, paragraph 2, of the series "Analyses of Results of the Population Census",[11] should be borne in mind; above all it should be remembered that not all the women who were stated on the census schedules as married, were married in the legal sense of the term, and not all those declared as widows, separated, judicially separated or divorced had been previously married in the legal sense. However, the declaration "married", generally speaking, means a conjugal union of a stable nature, even though without legal sanction.

A summary analysis of the difference in the conjugal status of women according to age was made in the study referred to, the data being presented in groups extending over several years of age.

An examination of the data according to age reveals the irregularities in the trend of the proportions of the various categories of marital status according to age, caused by the errors in the statements of age, which affect the various categories to a different extent, as will be shown below.

Table 2 shows in percentage form, the different categories of conjugal status of the female population according to certain ages. The irregularities referred to above are brought out precisely in this table.

TABLE 2

Per cent distribution by marital status at each year of the female population of Brazil present on 1 September 1940, aged 12 years and upward and of known marital status [a]

Age (completed years) (a)	*Percentage of*				
	Single (b)	*Married* (c)	*Separated, judicially separated, divorced* (d)	*Widows* (e)	*Total* (f)
12	99.96	0.04	0.00	0.00	100.00
13	99.77	0.22	0.00	0.01	100.00
14	99.28	0.70	0.00	0.02	100.00
15	97.72	2.24	0.01	0.03	100.00
16	93.23	6.70	0.02	0.05	100.00
17	86.12	13.72	0.05	0.11	100.00
18	78.20	21.52	0.07	0.21	100.00
19	69.55	30.03	0.10	0.32	100.00
20	59.40	39.94	0.14	0.52	100.00
21	52.99	46.18	0.17	0.66	100.00
22	46.36	52.54	0.19	0.91	100.00
23	41.18	57.39	0.23	1.20	100.00
24	36.87	61.45	0.26	1.42	100.00

[a] The group of women of unknown age is not included in this table. Absolute figures concerning them are given in table 1.

[11] These observations were:

"In interpreting the data it should be remembered that, since no proof of marital status is required at the census, persons whose conjugal unions have received neither legal nor even religious sanction could declare themselves to be 'married'; similarly persons who have never been married in the legal sense of the term, could declare themselves separated, judicially separated (*desquitados*), divorced or widowed. This disadvantage is frequently found in all censuses but its effects are particularly marked in those countries in which free unions are most frequent.

"Moreover some confusion probably occurred in the declarations of the categories—separated, judicially separated and divorced. In addition to those persons who are legally in one of these categories, other husbands or wives who are only separated *de facto* may sometimes have declared themselves separated, judicially separated or even divorced. This last can only apply to nationals or former nationals of foreign countries, as divorce is not permitted under Brazilian legislation.

"It should be pointed out that although by virtue of the classification in decennial age groups the errors in the statements of age are reduced, since an approximate balance is reached between the concentration of declarations in the 'attractive' age groups and the avoidance of the 'unattractive' age groups, other types of errors persist. There are the errors of 'rejuvenation', characterized by the evasion of declaration, mainly in the 40 to 55 year groups in favour of the 20 to 39 year groups. These errors are particularly marked among women. There are 'old age' errors, caused by ignorance or senile vanity, leading people to give their age as older than it really is. There are also other possible errors, less important and more difficult to trace.

"It should also be noted that in the 12 to 14 year age group 391 males and 4,786 females stated themselves to be married, and 20 males and 103 females stated themselves to be widowed. As marriage at this age is not permitted either by civil or by canon law, the great majority of these declarations, if not all, are due to false statements. Probably the majority of the marriage declarations of those actually within these age groups were cases of free unions; and it is possible that some persons of these ages were referring to free unions already dissolved in declaring themselves to be widowed, separated, judicially separated or even divorced. It is also possible that the true age of some of these apparently precocious persons was older than the age declared."

TABLE 2 *(Continued)*

Per cent distribution by marital status at each year of the female population of Brazil present on 1 September 1940, aged 12 years and upward and of known marital status

Age (completed years) (a)	*Single* (b)	*Percentage of* *Married* (c)	*Separated, judicially separated, divorced* (d)	*Widows* (e)	*Total* (f)
25	34.29	63.62	0.29	1.80	100.00
26	30.41	67.17	0.35	2.07	100.00
27	27.33	69.86	0.38	2.43	100.00
28	27.11	69.51	0.38	3.00	100.00
29	23.65	72.50	0.43	3.42	100.00
30	26.62	68.51	0.42	4.45	100.00
31	19.67	75.86	0.48	3.99	100.00
32	20.86	73.87	0.46	4.81	100.00
33	18.47	75.73	0.50	5.30	100.00
34	18.45	75.31	0.54	5.70	100.00
35	21.24	71.46	0.48	6.82	100.00
36	18.40	74.13	0.53	6.94	100.00
37	16.04	75.61	0.61	7.74	100.00
38	18.43	71.98	0.53	9.06	100.00
39	16.27	72.85	0.66	10.22	100.00
40	20.67	64.67	0.56	14.10	100.00
41	14.31	73.77	0.61	11.31	100.00
42	15.97	70.03	0.60	13.40	100.00
43	13.61	71.94	0.65	13.80	100.00
44	13.26	71.51	0.63	14.60	100.00
45	17.63	63.39	0.58	18.40	100.00
46	13.54	68.66	0.63	17.17	100.00
47	12.63	69.43	0.64	17.30	100.00
48	15.29	63.51	0.62	20.58	100.00
49	13.21	64.24	0.71	21.84	100.00
50	17.82	51.96	0.53	29.69	100.00
51	12.73	63.21	0.63	23.43	100.00
52	14.27	57.88	0.61	27.24	100.00
53	12.63	59.25	0.62	27.50	100.00
54	13.20	57.37	0.52	28.91	100.00
55	14.73	51.69	0.50	33.08	100.00
56	13.20	52.91	0.56	33.33	100.00
57	12.07	55.07	0.57	32.29	100.00
58	14.25	48.76	0.47	36.52	100.00
59	12.51	48.61	0.49	38.39	100.00
60	16.49	33.47	0.38	49.66	100.00
61	11.63	47.91	0.49	39.97	100.00
62	13.33	40.95	0.37	45.35	100.00
63	12.06	40.79	0.43	46.72	100.00
64	11.81	39.98	0.33	47.88	100.00
65	14.56	31.62	0.29	53.53	100.00
66	11.89	36.71	0.31	51.09	100.00
67	11.03	35.57	0.40	53.00	100.00
68	13.65	30.67	0.27	55.41	100.00
69	12.47	30.79	0.35	56.39	100.00
70	16.03	20.07	0.21	63.69	100.00
71	12.38	29.95	0.27	57.40	100.00
72	13.72	24.77	0.21	61.30	100.00
73	11.91	24.28	0.22	63.59	100.00
74	11.83	24.44	0.23	63.50	100.00
75	15.47	18.01	0.22	66.30	100.00
76	12.78	20.66	0.19	66.37	100.00
77	11.97	20.02	0.14	67.87	100.00
78	14.53	15.89	0.22	69.36	100.00
79	13.44	17.13	0.10	69.33	100.00

TABLE 2 *(Continued)*

Per cent distribution by marital status at each year of the female population of Brazil present on 1 September 1940, aged 12 years and upward and of known marital status

Age (completed years) (a)	*Single* (b)	*Percentage of* *Married* (c)	*Separated, judicially separated, divorced* (d)	*Widows* (e)	*Total* (f)
80	17.67	10.51	0.13	71.69	100.00
81	13.60	14.44	0.11	71.85	100.00
82	14.89	13.08	0.11	71.92	100.00
83	12.63	11.85	0.13	75.39	100.00
84	13.84	10.52	0.12	75.52	100.00
85	17.96	9.13	0.12	72.79	100.00
86	15.22	9.65	0.10	75.03	100.00
87	13.82	10.10	0.12	75.96	100.00
88	15.13	8.91	0.15	75.81	100.00
89	16.20	8.15	0.05	75.60	100.00
90	19.66	6.17	0.08	74.09	100.00
91	15.67	8.53	0.13	75.67	100.00
92	18.66	7.22	0.07	74.05	100.00
93	16.47	7.11	0.13	76.29	100.00
94	15.66	6.78	0.25	77.31	100.00
95	20.96	6.90	0.21	71.93	100.00
96	18.10	6.39	0.16	75.35	100.00
97	15.47	10.76	—	73.77	100.00
98	20.80	7.36	0.13	71.71	100.00
99	20.00	7.07	0.30	72.63	100.00
100 and over	21.65	5.48	0.18	72.69	100.00

Below are shown, for instance, the percentages falling into the different marital status categories in the ages around 30, 40 and 50.

Age (completed years)	*Ratio per 100 women of known conjugal status* *Single*	*Married*	*Separated, judicially separated, divorced*	*Widowed*
29	23.65	72.50	0.43	3.42
30	26.62	68.51	0.42	4.45
31	19.67	75.86	0.48	3.99
39	16.27	72.85	0.66	10.22
40	20.67	64.67	0.56	14.10
41	14.31	73.77	0.61	11.31
49	13.21	64.24	0.71	21.84
50	17.82	51.96	0.53	29.69
51	12.73	63.21	0.63	23.43

At the ages which are multiples of ten there are concentrated many declarations of persons who had only a vague idea of their age. The concentration is relatively higher among single women and widows, who often lack the references and assistance for determining their age possessed by married women.

The increase can be seen by comparing the crude and graduated data for ages ending in zero. For instance, for age 50 the data are as follows:

Marital status	*Number of women* *Crude*	*Graduated*	*Difference between crude and graduated data* *Absolute*	*Per cent*
Single	47,557	18,772	28,785	153.34
Married	138,623	75,925	62,698	82.58
Separated, judicially separated, divorced	1,405	746	659	88.34
Widows	79,214	30,255	48,959	161.82

It will be seen that the relative excess is almost twice as large among the single women and widows as among the married.

As a result of the differing degrees of error in the various categories of conjugal status, the apparent proportion of married women decreases in the ages which are multiples of ten and in other "attractive" ages, in which there is a concentration of the age statements of uneducated women, and increases in the "unattractive" ages.

5. Graduation of percentages

The irregularities referred to under 4 are so great that it is obviously impossible to use the percentages given in table 2 for further study without appropriate graduation to eliminate these irregularities.

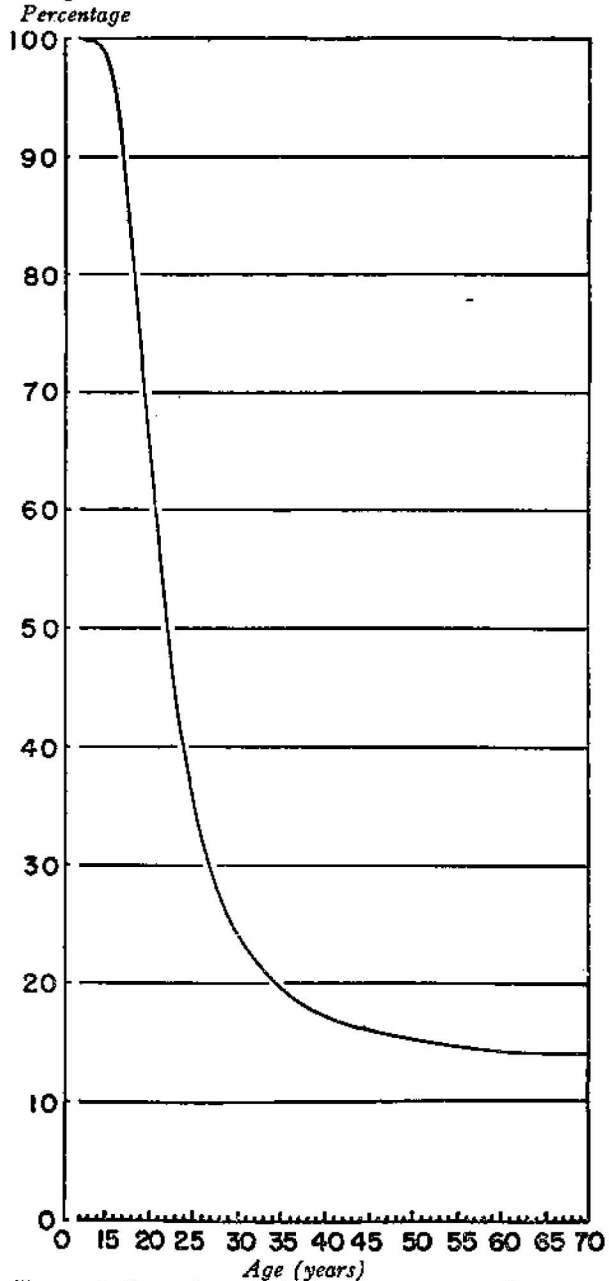

FIGURE 1. Percentage of single women among all women by age (graduated data); Brazil, 1940

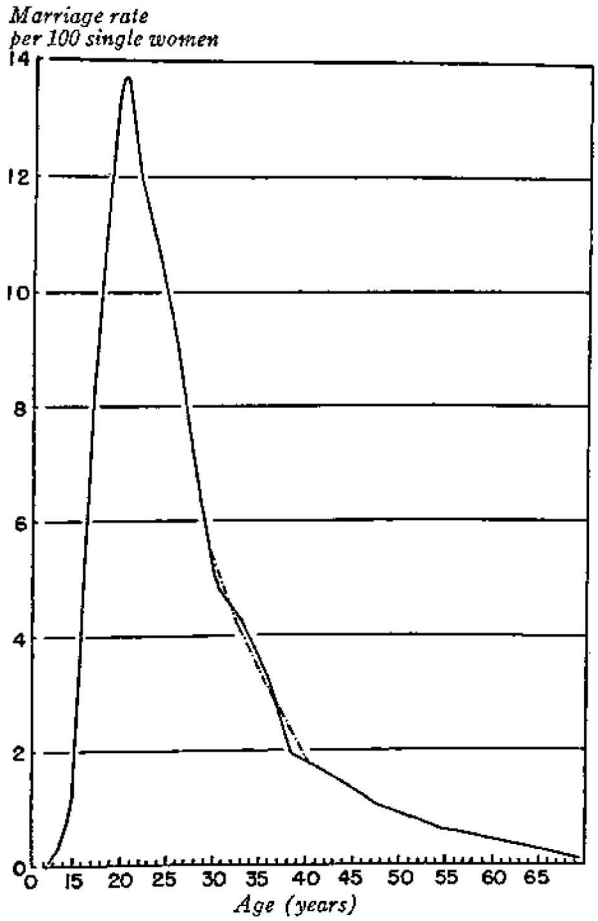

FIGURE 2. Marriage rate per 100 single women according to age (computed from census of Brazil, 1940)

For the purposes of the present study it seems advisable to confine graduation to the percentages of single women between the ages of 12 and 69. Below the age of 12 the problem of graduating the percentages of single women does not arise, as below that age the percentage is always 100. The ages 70 and above make an insignificant contribution to the marriages of single women; moreover, classification of women according to marital status in these ages is strongly affected by the increased frequency of "old age errors" in the age declarations of single women, so that any correction of the percentage may more conveniently be done by extrapolation than interpolation.

The graduation was made by a graphical process, which smoothed the curve representing the cumulative marriage rates without changing its shape and maintained approximately unaltered the proportion of single women in the total population between the ages of 12 and 69.

Column (*g*) of table 3 shows the graduated percentages of single women for the ages 12 to 69 as compared with the crude figures, shown in column (*f*).

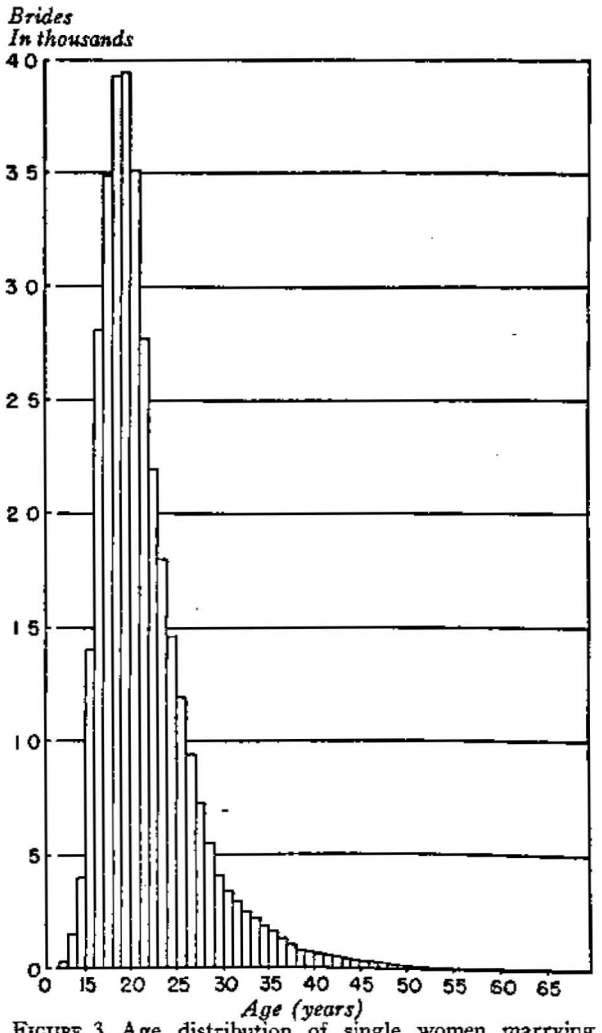

FIGURE 3. Age distribution of single women marrying between the ages of 12 and 70 years computed for census of Brazil, 1940

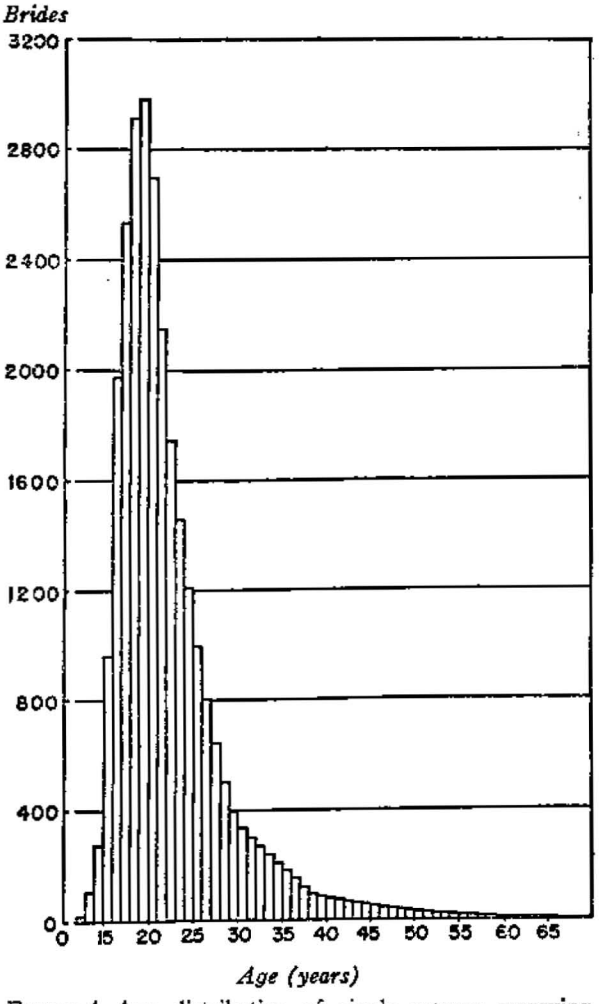

FIGURE 4. Age distribution of single women marrying between the ages of 12 and 70 years in stationary population according to 1920 life table (BC Cap. Adjust.) and marriage rates derived from census of Brazil, 1940

TABLE 3

Total female population of known marital status. Number and per cent of single women by age. Crude and graduated data, Brazil, 1940

Age (completed years) (a)	All women		Single women		Percentage of single women[a]	
	Crude figures (b)	Graduated figures (c)	Crude figures (d)	Graduated figures (e)	Crude figures (f)	Graduated figures (g)
12	572,869	520,924	571,966	520,924	99.96	100.00
13	480,190	509,418	478,662	508,399	99.77	99.80
14	494,961	498,696	491,042	495,704	99.28	99.40
15	491,061	487,557	479,636	478,293	97.72	98.10
16	493,989	474,491	460,224	443,649	93.23	93.50
17	435,556	459,049	374,845	396,159	86.12	86.30
18	482,704	445,512	377,115	348,836	78.20	78.30
19	382,983	432,509	266,108	299,729	69.55	69.30

[a] Percentage of single women in the population whose marital status is known.

TABLE 3 *(Continued)*

Total female population of known marital status. Number and per cent of single women by age. Crude and graduated data, Brazil, 1940

Age (completed years) (a)	All women		Single women		Percentage of single women[a]	
	Crude figures (b)	Graduated figures (c)	Crude figures (d)	Graduated figures (e)	Crude figures (f)	Graduated figures (g)
20	503,939	423,326	298,937	255,901	59.40	60.45
21	332,147	413,962	175,776	219,193	52.99	52.95
22	426,710	400,037	197,532	187,417	46.36	46.85
23	355,202	387,234	146,058	161,748	41.18	41.77
24	442,585	364,813	132,331	140,498	36.87	37.50
25	354,899	354,183	151,471	123,745	34.29	33.92
26	295,716	338,152	107,754	109,690	30.41	30.97
27	359,510	374,661	80,680	96,610	27.33	28.57
28	366,115	321,830	99,058	85,768	27.11	26.65
29	247,749	306,193	58,504	76,916	23.65	25.12
30	441,916	291,553	117,370	69,594	26.62	23.87
31	181,256	278,599	35,598	63,437	19.67	22.77
32	256,489	266,548	53,391	58,027	20.86	21.77
33	198,512	255,249	36,601	53,270	18.47	20.87
34	203,000	246,516	37,401	49,476	18.45	20.07
35	305,844	240,927	64,809	46,668	21.24	19.37
36	232,194	235,337	42,640	44,173	18.40	18.77
37	177,039	227,871	28,336	41,655	16.04	18.28
38	258,471	220,220	47,522	39,397	18.43	17.89
39	180,462	213,191	29,310	37,436	16.27	17.56
40	397,254	205,603	81,889	35,467	20.67	17.25
41	113,946	196,505	16,282	33,327	14.31	16.96
42	174,269	187,690	27,761	31,325	15.97	16.69
43	127,494	178,853	17,327	29,403	13.61	16.44
44	133,219	169,404	17,641	27,460	13.26	16.21
45	223,631	158,487	39,351	25,358	17.63	16.00
46	132,182	150,004	17,862	23,716	13.54	15.81
47	100,474	144,868	12,672	22,657	12.63	15.64
48	158,044	138,975	24,115	21,513	15.29	15.48
49	91,632	133,594	12,079	20,480	13.21	15.33
50	267,429	125,743	47,557	19,113	17.82	15.20
51	73,856	117,947	9,391	17,786	12.73	15.08
52	111,494	112,991	15,880	16,915	14.27	14.97
53	72,224	106,899	9,109	15,896	12.63	14.87
54	80,715	101,098	10,634	14,932	13.20	14.77
55	118,861	94,617	17,481	13,890	14.73	14.68
56	80,804	88,199	10,649	12,868	13.20	14.59
57	52,584	83,200	6,339	12,072	12.07	14.51
58	83,914	78,863	11,941	11,388	14.25	14.44
59	49,573	75,581	6,195	10,869	12.51	14.38
60	179,681	71,421	29,551	10,227	16.49	14.32
61	33,050	66,899	3,841	9,540	11.63	14.26
62	52,166	63,326	6,948	8,992	13.33	14.20
63	44,672	59,932	5,377	8,480	12.06	14.15
64	42,526	56,453	5,017	7,966	11.81	14.11
65	73,731	50,398	10,712	7,091	14.56	14.07
66	35,361	44,784	4,198	6,288	11.89	14.04
67	26,477	42,386	2,916	5,943	11.03	14.02
68	42,024	39,538	5,725	5,535	13.65	14.00
69	21,796	36,817	2,715	5,147	12.47	13.98
12 to 69	13,149,151[b]	13,169,633	5,929,832	5,943,956	45.10	45.13

[a] Percentage of single women in the population whose marital status is known.
[b] Including 19,937 women of unknown marital status.

By applying these percentages to the graduated numbers of women in each age group which are given in column (*c*) (explained in study No. 341 in the "Analyses of Population Census Results"), the numbers of single women were obtained, (column (*e*)). Out of the total population between 12 and 69 years of age the proportion of single women, calculated from the graduated data, is approximately 45 per cent, the same as that calculated from the crude data.[12]

By means of the same type of graphical process as that which served to determine the graduated percentages of single women by age, the graduated percentages of single women at each birthday between 12 and 70 were determined and are shown in column (*b*) of table 4.

Lastly, from the difference between the percentages thus determined for successive ages, the marriage rates at different ages were calculated by an approximate application of formula [3a]. For example, if the proportion of single women aged 24 is 39.45 per cent and that aged 25 is 35.55 per cent, the marriage rate of women aged 25 is 3.90 per cent. The rates thus calculated are shown in column (*c*) of table 4.

TABLE 4

Graduated percentages of single women among women who have completed each year of age between the 12th and 70th (derived from 1940 census of Brazil)

Age years x (*a*)	*Percentage of single women in total number of women aged x years* (*b*)	*Difference between percentages for ages x and (x+1)* (*c*)	*Age years x* (*a*)	*Percentage of single women in total number of women aged x years* (*b*)	*Difference between percentages for ages x and (x+1)* (*c*)
12	100.00	0.05	42	16.82	0.26
13	99.95	0.30	43	16.56	0.24
14	99.65	0.80	44	16.32	0.22
15	98.85	2.85	45	16.10	0.20
16	96.00	5.90	46	15.90	0.18
17	90.10	7.60	47	15.72	0.16
18	82.50	8.80	48	15.56	0.15
19	73.70	9.10	49	15.41	0.14
20	64.60	8.30	50	15.27	0.13
21	56.30	6.70	51	15.14	0.12
22	49.60	5.50	52	15.02	0.11
23	44.10	4.65	53	14.91	0.10
24	39.45	3.90	54	14.81	0.09
25	35.55	3.25	55	14.72	0.085
26	32.30	2.65	56	14.635	0.080
27	29.65	2.15	57	14.555	0.075
28	27.50	1.70	58	14.480	0.070
29	25.80	1.35	59	14.410	0.065
30	24.45	1.15	60	14.345	0.060
31	23.30	1.05	61	14.285	0.055
32	22.25	0.95	62	14.230	0.050
33	21.30	0.85	63	14.180	0.045
34	20.45	0.75	64	14.135	0.040
35	19.70	0.65	65	14.095	0.035
36	19.05	0.55	66	14.060	0.030
37	18.50	0.44	67	14.030	0.025
38	18.06	0.34	68	14.005	0.020
39	17.72	0.32	69	13.985	0.015
40	17.40	0.30	70	13.970	...
41	17.10	0.28			

[12] By grouping the data in table 3 in decennial age groups the following percentages of single women may be calculated according to crude and graduated data respectively.

Age (Completed years)	*Percentage of single women calculated according to*	
	Crude data	*Graduated data*
15 to 24	68.22	68.20
25 to 34	26.03	26.01
35 to 44	17.78	17.65
45 to 54	15.14	15.37
55 to 64	14.01	14.39

The percentage of single women computed from the graduated data differs little from that computed from the crude data. It should be noted that for the ages 50 and above the graduated proportions were purposely maintained slightly above the crude data, to compensate for the greater frequency of "old age errors" in the age statements of unmarried women.

6. Calculation of marriage rates

The n_x rates, defined in formulae [3] and [3a], are calculated, as explained in the note to section 2, in relation to the total number of women of each age, regardless of marital status. They can serve as a basis for determining the marriage rates of single women according to age.

Applying the symbols used in section 2 and representing the marriage rate of single women in their x^{th} year by c_{x-1}, the rate may be determined by an approximate elementary process, according to the following formula:

$$c_{x-1} = \frac{(N_x - N_{x-1})\ (l_{x-1} + l_x)}{l_{x-1}(1 - N_{x-1}) + l_x(1 - N_x)}. \qquad [4]$$

On the right-hand side of the equation, half the numerator represents the number of marriages of single women in their x^{th} year and half the denominator for the average number of single women living in their x^{th} year.

To express the marriage rate independently of the numbers l_x which vary in different cases, we may substitute the term $\frac{1}{2}(l_{x-1} + l_x)$ for both l_x and l_{x-1} in the denominator with a slight loss of accuracy[13]; thus we obtain the final formula:

$$c_{x-1} = \frac{2(N_x - N_{x-1})}{(1 - N_{x-1}) + (1 - N_x)} \qquad [4a]$$

For example, for the twenty-fifth year of life, applying, respectively, formulae [4] and [4a] to the graduated figures for all woman and for single women (table 3), we have:

$$\frac{3.90\%\ (374{,}661)}{140{,}498} = 10.40\%$$

$$\frac{2\ (3.90\%)}{75\%} = 10.40\%.$$

The results coincide approximately. And the same is true of the marriage rate at age 50.

$$\frac{0.14\%\ (133{,}594)}{20{,}480} = 0.91\%$$

$$\frac{2\ (0.14\%)}{30.68\%} = 0.91\%.$$

The marriage rates of single women calculated according to the formula [4a] are shown in column (c) of table 5.

In analysing the trends of these marriage rates of single women by age, the meaning to be attached to the declaration "married" in the census (see paragraph 4 above) must not be forgotten. The appreciable number of married women—and therefore of marriages, in the present calculations—below the age of 16, which is the minimum legal age for marriage, represents partly religious unions and partly *de facto* unions.[14]

Rising rapidly after the fifteenth birthday, the marriage rate of single women reaches its maximum, slightly under 14 per cent, at the age of 21, then declines continuously to below 5 per cent at 31, 2 per cent at 39, 1 per cent at 49 and 0.5 per cent at 59.

7. Calculation of number of marriages of single women

By means of the present calculations both the graduated number of single women in each age group from 13 to 70 on 1 September 1940, and the marriage rate of single women for each age group within those limits, were determined.[15]

It was thus possible, on the basis of these statistics of single women and marriage rates, to calculate the number of marriages of single women occurring in a year in the population of Brazil.

[13] The loss of accuracy is negligible in the ages in which the difference $(l_{x-1} - l_x)$ is relatively small, as it is in in all ages having a high marriage rate.

[14] It is also possible, as indicated in Section 4, that some women who declared themselves to be married and under 16 years of age, have given ages lower than their actual age.

[15] The calculation may be completed by taking into account the group of single women aged 70 and over, numbering about 42,000, and, taking their estimated marriage rate (0.05 per cent), assuming an additional 21 marriages, which, together with the 340,134 calculated in the preceding table, gives a total of 340,155.

TABLE 5

Calculation of the number of single women who marry during one year between the ages of 12 and 69, based on the returns of the 1940 census of Brazil

Age x years (*a*)	*Single women in their $(x+1)^{th}$ year (graduated data)* (*b*)	*Marriage rate per 100 single women in their $(x+1)^{th}$ year* (*c*)	*Number of single women marrying in their $(x+1)^{th}$ year* (*d*)
12	520,924	0.05	260
13	508,399	0.30	1,525
14	495,704	0.81	4,015
15	478,293	2.93	14,014
16	443,649	6.34	28,127
17	396,159	8.81	34,902
18	348,836	11.27	39,314
19	299,729	13.16	39,444
20	255,901	13.73	35,135
21	219,193	12.65	27,728
22	187,417	11.74	22,003
23	161,748	11.13	18,003
24	140,498	10.40	14,612
25	123,745	9.58	11,855
26	109,690	8.56	9,389
27	96,610	7.52	7,265
28	85,768	6.38	5,472
29	76,916	5.37	4,130
30	69,594	4.82	3,354
31	63,437	4.61	2,924
32	58,027	4.36	2,530
33	53,270	4.07	2,168
34	49,476	3.74	1,850
35	46,668	3.35	1,563
36	44,173	2.91	1,285
37	41,655	2.41	1,004
38	39,397	1.90	749
39	37,436	1.82	681
40	35,467	1.74	617
41	33,327	1.65	550
42	31,325	1.56	489
43	29,403	1.46	429
44	27,460	1.36	373
45	25,358	1.25	317
46	23,716	1.14	270
47	22,657	1.02	231
48	21,513	0.97	209
49	20,480	0.91	186
50	19,113	0.85	162
51	17,786	0.80	142
52	16,915	0.74	125
53	15,896	0.67	107
54	14,932	0.61	91
55	13,890	0.58	81
56	12,868	0.55	71
57	12,072	0.52	63
58	11,388	0.48	55
59	10,869	0.45	49
60	10,227	0.42	43
61	9,540	0.39	37
62	8,992	0.35	31
63	8,480	0.32	27
64	7,966	0.28	22
65	7,091	0.25	18
66	6,288	0.21	13
67	5,943	0.18	11
68	5,535	0.14	8
69	5,147	0.11	6
12 to 69	5,943,956	—	340,134

The results of this calculation are shown in column (*d*) of table 5, by individual years of age, and summarized in table 6 by quinquennial age groups.[16] Adding to this the marriages of single women aged 70 and over, as calculated in footnote [15], we have an annual total of 340,155 marriages of single women.

The distribution by age at marriage of single women marrying may be summarized as follows:

Arithmetic mean of ages, 21.54 years;

Median age, 20.24 years;

Modal (i.e., most frequent) age, about 19 years.

The absolute mean deviation from the arithmetic mean is 3.81 years, or about 18 per cent of the mean.

The age at the first quartile is 18.06 and at the third, 23.48. In other words, about 50 per cent of single women who marry are between the ages of 18 and 23.5 years.

By relating the number of marriages of single women to the population of single women in quinquennial age groups, the average marriage rates of single women in these groups were calculated (see table 6); these data can be used for international comparisons.[17]

[16] Except for the first group, 12 to 14 years, which is triennial.

[17] In order not to interrupt the description of the calculations in question, some international comparisons of the marriage rate of single women are given in an appendix.

TABLE 6

Summary by age groups of the marriages of single women as derived from the 1940 census of Brazil

Age (completed years) (*a*)	*Single women in the age group (graduated data)* (*b*)	*Marriage rate per 100 single women in the age group* (*c*)	*Number of single women who marry* (*d*)
12 to 14	1,525,027	0.38	5,800
15 to 19	1,966,666	7.92	155,801
20 to 24	964,757	12.18	117,481
25 to 29	492,729	7.73	38,111
30 to 34	293,804	4.37	12,826
35 to 39	209,329	2.52	5,282
40 to 44	156,982	1.57	2,458
45 to 49	113,724	1.07	1,213
50 to 54	84,642	0.74	627
55 to 59	61,087	0.52	319
60 to 64	45,205	0.35	160
65 to 69	30,004	0.19	56
12 to 69	5,943,956	5.72	340,134

8. Calculation of marriages of widows

To complete the calculation of the annual number of marriages in Brazil, the marriages of widows must also be taken into account. At present their number can only be estimated with a wide margin of error as complete data on the subject are lacking.[18]

International population statistics show that the number of marriages of widows is generally small in relation to those of single women (3.7 per cent in Chile, 3.4 per cent in Portugal, 3.3 per cent in Italy, 3.1 per cent in Argentina, 3.0 per cent in Colombia, 2.8 per cent in Spain etc.[19]). On a rough estimate it may be assumed that the proportion in Brazil is about 4 per cent.

It should be taken into account also that the marriage rate of widows is presumably lower than that of single women in the younger age groups and higher in the older groups. Assuming that the ratio between the marriage rate of widows and single women is 0.51 in the age group 15 to 19, 1.00 in the age group 50 to 54 and 1.35 in the age group 65 to 69, a regular succession of coefficients was determined; this enabled the marriage rate of widows to be calculated, as shown in column (*c*) of table 7, from the marriage rates of single women by age groups as given in column (*c*) of table 6.

These rates were then applied to the crude statistics of the number of widows, and thus the annual number of widows who marry was obtained (column (*d*) of table 7). In order to make this figure comparable with the figure for single women, graduated (and not the crude) figures for widows should be taken, but in view of the wide margin of error in the computation, this particular adjustment was not made as it would add but little to the degree of accuracy.

[18] Partial data, relatively reliable, exist. In 1940 the ratio of widowed brides to single brides among registered marriages was 3.4 per cent in the Federal District and 2.7 per cent in the State of Sao Paulo.

[19] The data are for 1929-37 (Italy) or years within that period (1929-36, Chile; 1929-35, Portugal; 1929-34, Spain; 1936, Argentina and Colombia).

TABLE 7

Estimated annual number of widows who remarry

Age (completed years) (a)	*Widows in the age group (crude data)* (b)	*Marriage rate per 100 widows in the age group* (c)	*Number of widows who remarry* (d)
12 to 14	103	0.19	...
15 to 19	3,120	4.04	126
20 to 24	18,047	6.70	1,209
25 to 29	41,880	4.64	1,943
30 to 34	61,191	2.88	1,762
35 to 39	92,376	1.84	1,700
40 to 44	129,017	1.27	1,639
45 to 49	133,500	0.96	1,282
50 to 54	169,948	0.74	1,258
55 to 59	132,704	0.58	770
60 to 64	167,007	0.43	718
65 to 69	106,975	0.26	278
70 and over	226,421	0.10	226
12 and over	1,282,289	1.01	12,911

The total number of marriages of widows is 12,911. The average age at which widows remarry is slightly over 39.

According to this calculation the annual number of marriages of widows, 12,911, is 3.8 per cent of the marriages of single women, which is 340,155.

Table 8 gives a summary, by age groups, of the distribution of marriages according to the marital status of the bride from data given in tables 6 and 7.

TABLE 8

Summary by age groups, of the women marrying in the course of one year, classified according to marital status prior to the marriage

Age (completed years) (a)	*Marriages*		
	Single women (b)	*Widows* (c)	*Total* (d)
12 to 19	161,601	126	161,727
20 to 29	155,592	3,152	158,744
30 to 39	18,108	3,462	21,570
40 to 49	3,671	2,921	6,592
50 to 59	946	2,028	2,974
60 to 69	216	996	1,212
70 and over	21	226	247
12 and over	340,155	12,911	353,066

In the younger age groups the marriages of single women preponderate; as the ages rise the proportion of marriages of widows rises, becoming in its turn preponderant, as the following data show:

Age (completed years)	*Ratio between the number of marriages of widows and those of single women*
12 to 19	0.001
20 to 29	0.02
30 to 39	0.19
40 to 49	0.80
50 to 59	2.14
60 to 69	4.61
70 and over	10.76

9. General marriage rates

Taking the round figure 340,000 as the annual estimated number of marriages of single women, and 13,000 as that of the marriages of widows, we have a total of 353,000 marriages per year.

In a population of approximately 41,250,000, this corresponds to a crude marriage rate of 8.56 per 1,000 inhabitants.

In the period 1936-1940, the rates in some American countries were as follows: United States, 10.9 per 1,000 inhabitants; Canada, 8.6; Chile, 8.4; Argentina, 7.1; Mexico, 7.0; Colombia, 4.9.[20]

[20] Rates given in the League of Nations *Statistical Yearbook, 1942-44*, p. 35.

For comparisons with European countries, it seems advisable to consider the period 1936-1938, excluding later years during which the marriage rate was affected by the war. During this period the crude marriage rate was 9.2 per 1,000 inhabitants in Germany, 8.8 in England and Wales, 8.2 in Poland, 7.8 in Italy, 6.6 in France and 6.4 in Portugal.[20]

The crude marriage rate computed for Brazil does not differ greatly from those of other countries; but it cannot for that reason be considered reliable, since, as the foregoing data show, the range of variation in the marriage rate is very wide.

Another international comparison may be made for the marriage rates of single and of widowed women aged 15 and above. The data given below[21] show that in this comparison Brazil is among the first. In this connexion it should be noted that in comparisons of the crude marriage rate, Brazil takes a lower place on account of the high proportion of children in her population who do not contribute to the marriage rate.

In this comparison also, the rates for Brazil do not differ greatly from those for other countries; this suffices to show that they are not improbable but does not prove that they are approximately exact.

		Marriage rate per 100 women aged 15 and above	
Country[a]	*Period*	*Single women*	*Widows*
Poland	1929-32	7.52	0.92
Brazil	1940	7.50	1.01
France	1929-32	7.36	0.61
Germany	1932-34	7.07	0.50
Canada	1929-32	5.65	1.38
Chile	1929-32	5.64	0.77
Australia	1932-34	5.40	0.72
Italy	1929-32	5.29	0.54
England and Wales	1929-32	5.15	0.89
Portugal	1929-32	4.54	0.51

[a] The countries are listed in descending order of the marriage rates of single women.

The crude marriage rates for these countries in the periods specified above are as follows: Poland 9.0, Brazil 8.6, France 7.9, Germany 9.6, Canada 6.8, Chile 8.1, Australia 7.1, Italy 6.9, England and Wales 7.8, Portugal 6.8.

On account of the considerable differences in the composition of the populations as regards age and marital status, the order of countries according to specific marriage rates is different from that according to crude marriage rates.

[20] Rates given in the League of Nations *Statistical Yearbook, 1942-44*, p. 35.

[21] Data taken from the *Aperçu de la Démographie des Divers Pays du Monde, 1929-1936*, of the International Institute of Statistics, p. 160, with slight corrections to obtain rounded figures.

10. Calculation of table of first marriages in a female generation

Another interesting application of the marriage rates of single women, as given in column (*c*) of table 4, is to determine the frequency of first marriage for women in a hypothetical generation, assumed to be constantly subject throughout its existence to the mortality and marriage rates ascertained for Brazil.

Taking as a basis for calculation the life table constructed by Bulhoes Carvalho for the capitals of Brazil, as corrected and adjusted by G. Mortara,[22] column (*d*) of table 9 gives the number of single women surviving at each birthday from 12 to 50, as obtained by applying to the total number of survivors the graduated percentages of single women given in column (*b*) of table 4.

By taking one-half of the total number of single women surviving at two consecutive birthdays, the average number of single women living in each year from 13 to 50 (column (*b*) of table 10) was then calculated; and by applying to the number of single women living at each year of age the appropriate marriage rate (column (*c*)), the number of marriages of single women (column (*d*)) was obtained.

For age 50 and over the calculation was made in larger age groups, as indicated in the note to table 10.

The total number of marriages of single women is 27,000.[23] This figure may be compared either with the initial number of women composing a generation (48,544 out of a total of 100,000 of both sexes at age 0), or with the number of survivors at the twelfth birthday, taking that as the initial *de facto* marriageable age.

We then have the following comparison:

Out of 48,544 female live births: 14,421 (29.71 per cent) died before reaching their 12th birthday; 34,123 (70.29 per cent) survived their 12th birthday; 27,000 (55.62 per cent) married after their 12th birthday; 7,123 (14.67 per cent) died unmarried after their 12th birthday.

Out of 34,123 women surviving their 12th birthday: 27,000 (79.13 per cent) married subsequently; 7,123 (20.87 per cent) died unmarried subsequently.

It will be observed that more than one-half of the women of the generation, and rather less than

[22] *Revista Brasileira de Estatistica*, No. 4, 1940, pages 683 and 684.

[23] The fact that this total is a multiple of 1,000 is a coincidence and not the result of rounding the figure.

four-fifths of those who survived their 12th birthday, married.

The average ages of single women who marry may be summarized as follows:

Arithmetic mean, 22.37 years;

Median, 20.65 years;

Mode, slightly over 19 years.

These averages relate to a stationary population with mortality and marriage rates constantly equal to those found in Brazil. They are slightly higher than those calculated above (section 7) for the actual population of Brazil which is increasing and not stationary, and thus more numerous in the low-age groups and less numerous in the old-age groups than a stationary population.

TABLE 9

Calculation of single women surviving to each age between 12 and 50, according to the 1920 life table (BC Cap. Adjust.) and the percentages of single women according to the 1940 census of Brazil

Age x years (a)	*Women surviving at x years* (b)	*Percentage graduated of single women among women aged x years* (c)	*Single women surviving at x years* (d)	*Index numbers of women surviving*	
				Total (e)	*Single women* (f)
12	34,123	100.00	34,123	—	—
13	34,001	99.95	33,984	—	—
14	33,873	99.65	33,754	—	—
15	22,739	98.85	33,351	10,000	10,000
16	33,591	96.00	32,247	9,956	9,669
17	33,413	90.10	30,105	9,903	9,027
18	33,196	82.50	27,387	9,839	8,212
19	32,931	73.70	24,270	9,761	7,277
20	32,629	64.60	21,078	9,671	6,320
21	32,288	56.30	18,178	9,570	5,451
22	31,920	49.60	15,832	9,461	4,747
23	31,547	44.10	13,912	9,350	4,171
24	31,174	39.45	12,298	9,240	3,687
25	30,801	35.55	10,950	9,129	3,283
26	30,428	32.30	9,828	9,019	2,947
27	30,053	29.65	8,911	8,907	2,672
28	29,677	27.50	8,161	8,796	2,447
29	29,300	25.80	7,559	8,684	2,266
30	28,920	24.45	7,071	8,572	2,120
31	28,539	23.30	6,650	8,459	1,994
32	28,154	22.25	6,264	8,345	1,878
33	27,767	21.30	5,914	8,230	1,773
34	27,377	20.45	5,599	8,114	1,679
35	26,983	19.70	5,316	7,998	1,594
36	26,585	19.05	5,064	7,880	1,518
37	26,183	18.50	4,844	7,760	1,452
38	25,777	18.06	4,655	7,640	1,396
39	25,366	17.72	4,495	7,518	1,348
40	24,949	17.40	4,341	7,395	1,302
41	24,527	17.10	4,194	7,270	1,258
42	24,100	16.82	4,054	7,143	1,216
43	23,666	16.56	3,919	7,014	1,175
44	23,226	16.32	3,790	6,884	1,136
45	22,780	16.10	3,668	6,752	1,100
46	22,326	15.90	3,550	6,617	1,064
47	21,866	15.72	3,437	6,481	1,031
48	21,398	15.56	3,330	6,342	998
49	20,922	15.41	3,224	6,201	967
50	20,439	15.27	3,121	6,058	936

TABLE 10

Nuptiality table for single women between the ages of 12 and 50 years[a]

Age x years (a)	*Single women in their $(x+1)^{th}$ year of age* (b)	*Marriage rate per 100 single women in the $(x+1)^{th}$ year of age* (c)	*Number of single women who marry in their $(x+1)^{th}$ year of age* (d)	*Difference between numbers of single women surviving to x^{th} and to $(x+1)^{th}$ year of age* (e)	*Deaths of single women in $(x+1)^{th}$ year of age* (f)
12	34,054	0.05	17	139	122
13	33,869	0.30	102	230	128
14	33,553	0.81	272	403	131
15	32,799	2.93	961	1,104	143
16	31,176	6.34	1,977	2,142	165
17	28,746	8.81	2,533	2,718	185
18	25,829	11.27	2,911	3,117	206
19	22,674	13.16	2,984	3,192	208
20	19,628	13.73	2,695	2,900	205
21	17,005	12.65	2,151	2,346	195
22	14,872	11.74	1,746	1,920	174
23	13,105	11.13	1,459	1,614	155
24	11,624	10.40	1,209	1,348	139
25	10,389	9.58	995	1,122	127
26	9,369	8.56	802	917	115
27	8,536	7.52	642	750	108
28	7,860	6.38	501	602	101
29	7,315	5.37	393	488	95
30	6,861	4.82	331	421	90
31	6,457	4.61	298	386	88
32	6,089	4.36	265	350	85
33	5,756	4.07	234	315	81
34	5,458	3.74	204	283	79
35	5,190	3.35	174	252	78
36	4,954	2.93	145	220	75
37	4,749	2.41	114	189	75
38	4,575	1.90	87	160	73
39	4,418	1.82	80	154	74
40	4,268	1.74	74	147	73
41	4,124	1.65	68	140	72
42	3,986	1.56	62	135	73
43	3,855	1.46	56	129	73
44	3,729	1.36	51	122	71
45	3,609	1.25	45	118	73
46	3,493	1.14	40	113	73
47	3,384	1.02	35	107	72
48	3,277	0.97	32	106	74
49	3,172	0.91	29	103	74
12 to 49	—	—	26,774	31,002	4,228

[a] The calculation may be completed for age 50 and over as follows:

Age Completed years	*Single women at specified ages*	*Marriage rate per 100 single women*	*Number of single women who marry*
50 to 59	26,432	0.64	169
60 to 69	17,164	0.30	51
70 and over	11,046	0.05	6

If the ages 50 and over are included, the total number of marriages of single women becomes 27,000 and the number of deaths after the 12th birthday, 7,123.

11. Supplementary calculation of second and subsequent marriages

To complete the calculation of the marriages of women in the generation considered, an estimate of the marriages of widows was made, assuming that the ratio of such marriages to those of single women in that generation, was the same as that calculated for the population of Brazil in each age group as given in table 8.

The results of this estimate are shown in column (*c*) of table 11. The total number of marriages of widows in the generation under consideration was 1,687, or 6.2 per cent of the marriages of single women.[24]

TABLE 11

Distribution of marriages of women in one generation, by age groups and marital status prior to marriage

Age (completed years) (*a*)	*Marriages*		
	Single women (*b*)	*Widows* (*c*)	*Total* (*d*)
12 to 19	11,757	9	11,766
20 to 29	12,593	256	12,849
30 to 39	1,932	369	2,301
40 to 49	492	391	883
50 to 59	169	362	531
60 to 69	51	235	286
70 and over	6	65	71
12 and over	27,000	1,687	28,687

12. Additional observations

Tables 9 and 10 present some additional calculations.

Columns (*e*) and (*f*) of table 9 show in index form, based on the exact age of 15 years (15th birthday), the total numbers of women surviving and the number of single women in particular, as given in columns (*b*) and (*d*) of the table.

These data can be used for international comparisons; it was precisely with this possible use in view that the age of 15 instead of 12 years, was taken as a basis.

In the generation of women as a whole, 60.58 per cent of the women who survived their fifteenth birthday also survived their fiftieth birthday. As regards single women the proportion is only 9.36 per cent in the whole generation. If we assume that mortality is the same in the various marital status categories, it may be calculated that of 100 women surviving to the age of fifteen years, 60.58 per cent were still alive at 50, 9.36 per cent of whom were single while 51.22 per cent had been married, whereas 39.42 per cent died between their fifteenth and fiftieth birthdays, 11.53 per cent of whom were single while 27.89 per cent had been married.

The distribution by age of deaths of single women occurring between their twelfth and fiftieth birthdays is shown in column (*f*) of table 10. As column (*d*) of table 9 gives the numbers of single women, in the generation under consideration, surviving at subsequent birthdays, it is easy to calculate the difference between the number of women surviving at two consecutive birthdays (column (*e*) of table 10). As the difference between the number of single women surviving at x years and those surviving at $(x+1)$ years is equal to the sum of the numbers of single women who married at the age of $(x+1)$ and those who died at the same age, and as the total and the first factor are known, the second factor can be calculated by subtraction.[25]

[24] The proportion of the total marriages of widows of all ages to those of single women of all ages is greater in a hypothetical stationary population than in a real increasing population on account of the higher proportion of persons in the mature and old-age groups in the former.

[25] The number of single women dying in each age group may be calculated directly by taking the average number of single women alive at each age and applying the general mortality rate. This calculation was in fact made in order to check the method described in the text.

13. Conclusion

The results of the present study may be judged either from the methodological or the demographic point of view.

From the technical standpoint, the process which has been used to deduce the marriage rates of single women by age from the crude census figures of the female population, tabulated by age and marital status, is undoubtedly interesting.

However, it is easy to explain why, so far as is known, this process has not been applied in other countries. In the first place, all the countries advanced in population studies possess reliable marriage statistics and by co-ordinating these with the crude census figures they can obtain direct and almost exact marriage rate figures, instead of indirect and largely approximate figures as in the process described. In the second place, a relative stability in the vital rates of the population is an essential condition for the useful application of this process; and this condition is not fulfilled in most of the countries of the western world, where these rates have changed rapidly and to some extent irregularly during recent decades.

Brazil is one of the rare countries in which the process here described and adopted can be used with the expectation of obtaining sufficiently exact results. It would, however, be going too far to attempt a quantitative measure of the degree of approximation; the crude marriage rate, which was calculated to be 8.56 per 1,000 inhabitants, may in reality be over 9 or below 8.

From the demographic point of view, however, it marks a great advance from the previous situation to have verified that the crude marriage rate in Brazil is of the order of 8 to 9 per 1,000 inhabitants.[26] It was hardly possible from the civil register statistics to state that the rate was above 3 per 1,000.

It should be added that the classification, according to age, of single women who marry and the data summarizing this classification, together with the estimate of the classification according to age, of widows who remarry, though not representing the real picture as precisely as do the classifications established by other countries from complete marriage registers, gives a preliminary picture of previously unknown characteristics of the marriage rate in Brazil.

Further detail is also added to the picture by the analysis of the marriage rate tabulation showing the trend of the marriage rate in a given female generation.

The elements presented in this study may serve a useful purpose not only for the demographer but also for the sociologist until such time as the civil register statistics will supply direct and complete data on the matter—a time which does not yet seem near at hand, much as one would wish it so.

APPENDIX

Some international comparisons of the marriage rates of single women by age groups

1. Taking the average annual marriage statistics for a period for which the census figures may be considered to give the mean population, and comparing them with the crude census population figures, the general marriage rate, that is, the marriage rates for the population as a whole, or the special marriage rates, that is, for a given group of inhabitants, may be calculated.

Table 12 shows the basic data and results of such calculations of the marriage rates of single women by age groups, for seven foreign countries.[27] The comparative data for Brazil given in the same table differ from those for the other countries, since the marriage figures were obtained from indirect calculations, as explained in the foregoing study, instead of being obtained directly from civil registration.

To facilitate comparison, table 12 below gives the marriage rates, extracted from table 15.

[26] It should be remembered that by reason of the wide interpretation given to the status "married" in the census declarations, the calculations made in the present study produce a marriage rate appreciably higher than that which would be obtained if the computations were based on the data for married women in the restricted legal sense of that term.

[27] The comparison of marriage rates for single women aged fifteen and above given in section 9 of the present paper included Canada and Chile in addition to the eight countries considered here. For Canada and Chile the sources utilized do not give the distribution of brides by age.

TABLE 12

International comparison of marriage rates per 100 single women according to age groups

Age (Completed years) (a)	*Poland 1929-34* (b)	*Brazil 1940* (c)	*France 1929-32* (d)	*Germany 1932-34* (e)	*Australia 1932-34* (f)	*Italy 1929-32* (g)	*England and Wales 1929-32* (h)	*Portugal 1929-32* (i)
15 to 19	4.08	7.92	4.07	2.27	2.63	2.48	1.66	2.42
20 to 24	12.05	12.18	18.12	11.56	10.54	10.80	10.58	9.56
25 to 29	12.23	7.73	14.15	13.85	11.26	9.72	11.90	7.94
30 to 34	8.34	4.37	6.62	7.32	6.15	5.16	5.72	4.63
35 to 39	4.97	2.52	3.60	3.52	3.13	2.96	2.70	2.73
40 to 44	2.66	1.57	1.99	1.86	1.63	1.63	1.48	1.66
45 to 49	1.51	1.07	1.16	0.97	0.94	0.93	0.96	1.11
50 and over	0.44	0.45	0.28	0.17	0.27	0.26	0.31	0.28
15 and over[a]	7.52	7.50	7.36	7.07	5.40	5.29	5.15	4.54

[a] Including the cases in which the age was not reported.

A cursory examination of table 12 suffices to show that the graph of the marriage rate in terms of age differs greatly in the various countries.

The general shape of the curve is always similar. The marriage rate rises rapidly from the initial marriageable age to a maximum which, in all countries, is still a young age, and descends gradually thereafter to very low values in the higher ages.

But in spite of the general similarity the graphs for the various countries differ noticeably.

The maximum marriage rate in the different countries is reached at different ages. In Brazil it is reached at 21; in France, at 23; in Italy and Portugal, at 25; in Poland, Australia, England and Wales, at 26; in Germany, at 27.[28]

The maximum figure is different, however, varying from 11 or 12 per cent (Portugal) to 19 or 20 per cent (France).

In the 15-19 age group, the highest marriage rate is that of Brazil, nearly 8 per cent; the lowest, that of England and Wales, below 2 per cent.

In the 20-24 age group, the highest rate is in France, 18 per cent; and the lowest in Portugal, under 10 per cent.

In the 25-29 year group, France retains the first place, with a 14 per cent rate, but the rate for Germany is not much less; the lowest rates—below 8 per cent—are those of Portugal and Brazil.

In the 30-34 age group, Poland has a higher marriage rate, above 8 per cent, whereas the lowest is Brazil's, slightly above 4 per cent.

In the 35-39 year age group, the highest rate is that of Poland, with nearly 5 per cent, almost double the minimum, which is Brazil's.

In the higher ages the interest of the comparison diminishes on account of the low level of the marriage rates.

To sum up the general impression resulting from the comparisons made in table 12, it may be said that Brazil's main characteristic is the early age of marriage.

* * *

2. This characteristic is brought out more clearly in the comparison in table 13 of the percentage distributions by age of single women who marry.

The number of brides in the age group 15-19 in Brazil is 46.60 per cent of the total, whereas in the other countries considered the percentages are much lower, varying between a maximum of 22.23 per cent for Poland and a minimum of 7.68 per cent for Germany.

The number of brides in the age group 20-24 is only 35.14 per cent in Brazil, whereas in the other countries it varies between a maximum of 50.58 per cent for Italy and a minimum of 46.21 per cent for Australia.

[28] The determinations of the ages at which the maximum marriage rate is reached are approximate for all the foreign countries.

TABLE 13

International comparison of the percentages of single brides in each age group.

Percentages per 100 brides of known age

Age (Completed years) (a)	*Poland 1929-34* (b)	*Brazil 1940* (c)	*France 1929-32* (d)	*Germany 1932-34* (e)	*Australia 1932-34* (f)	*Italy 1929-32* (g)	*England and Wales 1929-32* (h)	*Portugal 1929-32* (i)
15 to 19	22.23	46.60	18.98	7.68	17.07	17.53	9.59	18.44
20 to 24	46.63	35.14	50.52	47.75	46.21	50.58	48.18	48.30
25 to 29	20.83	11.40	19.31	31.09	24.18	20.60	28.57	20.50
30 to 34	6.73	3.84	6.17	8.84	7.34	6.56	7.89	6.81
35 to 39	2.28	1.58	2.68	2.87	2.83	2.68	2.89	2.92
40 to 44	0.78	0.73	1.19	1.12	1.25	1.13	1.31	1.52
45 to 49	0.29	0.36	0.59	0.43	0.60	0.49	0.75	0.79
50 and over	0.23	0.35	0.56	0.22	0.52	0.43	0.82	0.72
15 and over	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

In the age group 25-29 Brazil again figures last, with 11.40 per cent, much below the other countries, which vary from 31.09 per cent (Germany) to 19.31 per cent (France).

The proportion of brides aged 30 and above is only 6.86 per cent in Brazil, varying in the other countries between a maximum of 13.48 per cent in Germany and a minimum of 10.31 per cent in Poland.

* * *

3. The comparisons made in the preceding paragraphs refer, for the foreign countries, to a short period around 1930.

An analysis of census figures would provide a more extensive retrospective picture.

In table 14 the experiment of using the census statistics for this purpose was tried and the proportions of women remaining single at the age of 50 are given for the same group of countries.

These proportions were calculated for a decennial age group, and not for a one-year group, in order to diminish the influence of errors in age declarations, as described above.

TABLE 14

International comparison of the proportion of single women in the female population between the ages of 45 and 54

		Women between the ages of 45 and 54		
			Single women	
Country (a)	*Year* (b)	*Total* (c)	*Absolute figure* (d)	*Per cent* (e)
Poland	1931	1,388,893	93,753	6.75
Brazil	1940	1,311,681	198,650	15.14
France	1931	2,721,729	296,949	10.91
Germany	1933	4,048,356	464,369	11.47
Australia	1933	360,839	51,930	14.39
Italy	1931	2,111,469	261,011	12.36
England and Wales	1931	2,632,703	430,817	16.36
Portugal	1930	354,859	60,450	17.03

Poland and France, with the highest marriage rates, similarly have the lowest proportions of single women at 50; and England and Portugal with the lowest marriage rates have the highest proportion.

The proportion of single women in Brazil seems high in view of the high marriage rate in the younger age groups; but the low marriage rate in the age groups over 25 must be taken into account.[29]

[29] Moreover, the number of "single women" in the higher age groups includes many women living in conjugal unions of a stable nature, as was indicated in other studies of the 1940 population census figures.

TABLE 15

Marriage rates of single women by age for various countries[a]

Age (completed years) (a)	Single women (b)	Marriages of single women (c)	Marriage rate per 100 single women (d)	Single women (b)	Marriages of single women (c)	Marriage rate per 100 single women (d)
	POLAND			BRAZIL		
15 to 19	1,460,791	59,663	4.08	1,966,666	155,801	7.92
20 to 24	1,038,493	125,121	12.05	964,757	117,481	12.18
25 to 29	456,877	55,884	12.23	492,729	38,111	7.73
30 to 34	216,654	18,062	8.34	293,804	12,826	4.37
35 to 39	122,986	6,107	4.97	209,329	5,282	2.52
40 to 44	78,925	2,101	2.66	156,982	2,458	1.57
45 to 49	52,207	787	1.51	113,724	1,213	1.07
50 and above	140,124	610	0.44	262,938	1,183	0.45
Unknown	5,558	186	...	—	—	—
15 and above	3,572,615	268,521	7.52	4,460,929	334,355	7.50
	FRANCE			GERMANY		
15 to 19	1,406,560	57,309	4.07	1,996,232	45,260	2.27
20 to 24	841,761	152,501	18.12	2,434,349	281,431	11.56
25 to 29	412,050	58,303	14.15	1,322,974	183,210	13.85
30 to 34	281,665	18,640	6.62	712,075	52,090	7.32
35 to 39	224,761	8,089	3.60	481,133	16,921	3.52
40 to 44	180,113	3,580	1.99	353,415	6,575	1.86
45 to 49	153,864	1,783	1.16	261,655	2,550	0.97
50 and above	602,808	1,694	0.28	771,525	1,323	0.17
Unknown	—	—	—	—	—	—
15 and above	4,103,582	301,899	7.36	8,333,358	589,360	7.07
	AUSTRALIA			ITALY		
15 to 29	289,949	7,613	2.63	1,931,289	47,846	2.48
20 to 24	195,535	20,617	10.54	1,277,936	138,040	10.80
25 to 29	95,863	10,790	11.26	578,356	56,206	9.72
30 to 34	53,249	3,275	6.15	347,132	17,899	5.16
35 to 39	40,360	1,263	3.13	247,240	7,321	2.96
40 to 44	34,178	558	1.63	187,907	3,067	1.63
45 to 49	28,519	268	0.94	144,719	1,348	0.93
50 and above	86,027	230	0.27	443,893	1,170	0.26
Unknown	1,768	—	—	3,558	349	—
15 and above	825,448	44,614	5.40	5,162,030	273,246	5.29
	ENGLAND AND WALES			PORTUGAL		
15 to 19	1,693,820	28,048	1.66	333,306	8,074	2.42
20 to 24	1,332,051	140,897	10.58	221,176	21,153	9.56
25 to 29	702,110	83,526	11.90	113,036	8,979	7.94
30 to 34	403,283	23,080	5.72	64,516	2,985	4.63
35 to 39	313,030	8,453	2.70	46,816	1,277	2.73
40 to 44	260,015	3,836	1.48	39,941	665	1.66
45 to 49	229,213	2,193	0.96	31,190	346	1.11
50 and above	768,853	2,401	0.31	110,947	314	0.28
Unknown	—	1,523	—	3,663	—	—
15 and above	5,702,375	293,957	5.15	964,591	43,793	4.54

[a] The census figures from which the population data given in column (*b*) were taken are given in table 14; the periods to which the average annual number of marriages in column (*c*) refer are given in table 12.

The data for the foreign countries were from the *Aperçu de la Démographie des Divers Pays du Monde, 1929-1936,* except those for Italy which are from the *Annuario Statistico Italiano* of 1934.

IV. FERTILITY AND REPRODUCTION RATES [1]

1. Various methods of computing fertility rates

The study of the fertility of women, which is fundamental for the knowledge of a country's demographic characteristics, is normally based upon a combined use of civil registration and of population censuses.

The former gives the numbers of confinements or births occurring in a period of one or more years close to the date of the census; the latter provide material for calculating the average number of women of child-bearing age during that period.

Births may be divided into live births and still births, legitimate and illegitimate, etc.; or grouped according to the characteristics of the mother, such as age, colour, nationality, occupation, etc.; or classified according to their place in the total sequence of children born of the mother or of the marriage, etc.

Women may be classified according to marital status, nationality, occupation, etc.; divided according to colour, age, etc.; or grouped by the number of children ever born, etc.

Fertility rates are obtained by comparing the groups of births with the corresponding groups of women—for instance, the annual number of children born of mothers of 25 completed years of age with the average number of women of that age alive in a given year, and the number of second children of a given group of women with the number of women in the group having one child.

The rate is the value of the ratio expressed in the form of a percentage or some other power of ten.

Fertility rates are based on the total number of women, usually including those as yet childless. For instance, in the above example, rates may be obtained by a comparison between the number of children born of mothers aged 25 and the total number of women of that age.

If appropriately classified birth statistics only were available, it would be possible approximately to calculate the fertility rates, in spite of the partial lack of the corresponding census data regarding women. Much conjectural work of this kind has already been done for countries which possess reliable civil register statistics but lack adequate census data.

The opposite is the case in Brazil, as civil registration statistics cannot be used because of their serious deficiencies, whereas the 1940 population census affords copious and, what is more, reliable data.

To illustrate the possibilities of utilizing these data, a brief preliminary exposition of methods may not be out of place.

2. Derivation of fertility rates by age from data about a generation of women

Suppose we follow a given group of women of the same age, and observe them over a period of time, noting the number who reach each successive birthday and the number of children they have during each year.

Taking l_x as the number of women surviving to the x^{th} birthday, the quantity L_x which denotes the average number of women alive during the $(x+1)^{th}$ year of age may be obtained by the approximate formula

$$L_x = \frac{l_x + l_{x+1}}{2}.$$

And, since f_x, the number of children born of the women under observation during the $(x+1)^{th}$ year is known, we may compute the ratio:

$$b_x = \frac{f_x}{L_x}, \qquad [1]$$

and this, multiplied by 100, gives the fertility rate of the women at the $(x+1)^{th}$ year of age.

If women's initial reproductive age is fixed as the i^{th} birthday, the sum of the fertility ratios

$$b_i + b_{i+1} + \ldots + b_{x-2} + b_{x-1} = B_x \qquad [2]$$

gives the cumulative fertility ratio B_x up to the

[1] Translated from section I of "Estudos sôbre a fecundidade e a prolificidade da mulher no Brazil, no conjunto da populaçao e nos diversos grupos de côr". Instituto Brasileira de Geografia e Estatistica. *Estudos de Estatistica Téorica e Aplicada; Estatistica Demográfica. No. 5.* Rio de Janeiro. 1949. By G. Mortara, Dr. José Etrog collaborated in the calculations and preparation of the tables.

x^{th} birthday, which, multiplied by 100, is the cumulative fertility rate as at the x^{th} birthday.

Hence

$$B_{x+1} - B_x = b_x \qquad [3]$$

by which, if the cumulative fertility ratios B_x are known, the fertility ratios according to years of age, b_x, may be calculated.

By means of a comparison between the data on births taken from the civil register statistics, and those on the number of women taken from the census, it is possible to calculate the ratios between the number, f'_x, of children born within a one-year period (i.e., within one of the years observed, or for an average of two or more years) and the average, L'_x, of women aged x completed years alive at that period:

$$b'_x = \frac{f'_x}{L'_x} \qquad [1a]$$

The ratios of type [1a] differ from those of type [1] on account of the difference in the composition of the groups observed.

The data which appear in formula [1] are obtained by observation of a single group of women, throughout the whole reproductive period of their life.

The data which appear in formula [1a] are obtained by the simultaneous observation of various groups of women, each group being observed for one year.[2]

In a population with demographic characteristics which remain strictly stable over a period of time, the equation

$$b'_x = b_x \qquad [4]$$

remains true for all values of x, and similarly, by virtue of formula [2]

$$B'_x = B_x \qquad [5]$$

for all values of x.

3. Application of this process using census data for a population with stable vital rates

If, in the population census, women were requested to state the number of children they had borne as at the date of the census, as was done in the Brazilian census schedules in 1940[3]—cumulative fertility ratios could be calculated from the data received.

In effect, the census figures would give both L'_x, the number of women aged x completed years, and the total number of children born to them as at the date of the census. These women would be between their x^{th} and $x + 1^{th}$ birthdays, hence their average age could be considered approximately $(x + \frac{1}{2})$ years; and the number of children born to them could be taken to represent the reproductive activity of that group of women up to that age.

The ratio between the number of children born and the number of women is a cumulative fertility ratio analogous to B_x in formula [2], differing from the latter only in that it indicates the fertility of the group considered up to the exact age $(x + \frac{1}{2})$ instead of up to the exact age x.

From such ratios it is possible to ascertain, for instance, the cumulative fertility ratio at 39.5 years of the group of women who at the time of the observations were passing through their 40th year and at 40.5 for those who were passing through their 41st year.

In a population with demographic characteristics remaining strictly stable over a period, the cumulative fertility ratios at each given age would be the same for various groups of women, so that a mean of the ratios at 39.5 years and at 40.5 years would represent the ratio at the exact age of 40, or, in other words, up to the 40th birthday. This would thus be a ratio of the type B_x in formula [2]. And the difference between this B_{40} ratio and the ratio B_{39}, calculated similarly in relation to the exact age of 39, would give, by formula [3], the value of b_{39}, the fertility ratio in the 40th year.

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[2] The expression used in the text is not strictly correct. If the census were taken at the middle of the year observed, the group of women aged x completed years, i.e. in their $(x + 1)^{th}$ year of age, would include some women who at the beginning of the year considered were still in their x^{th} year and some women who at the end of the year in question would already be in their $(x + 2)^{th}$ year, and would not include some women who at the beginning of the year considered were still in their $(x + 1)^{th}$ year or some women who at the end of that year would already be in their $(x + 1)^{th}$ year. Therefore, the group of "women aged x completed years", appearing in a census taken in the middle of the year considered, only partly coincides with the group who are the mothers of the "children born of women aged x completed years" in the course of the year considered. But, as a rule, the difference between the numbers forming these two groups is relatively small, so that the number representing one of them may be taken as a sufficiently accurate figure for the other.

The preceding considerations may also readily be applied to the case of a census taken at the beginning or end of the year observed, or at some other date not in the middle of the year. They may also be extended to the case in which the number of births represents the annual average, calculated on the basis of observations taken over two or more years, and the number of women represents the average of those alive during the period; in this case, however, the degree of accuracy is generally higher.

[3] Women were asked to state the number of children they had had, specifying whether born alive or still-born.

The characteristics of the Brazilian population as regards the frequency of births are not strictly stable. However, the rate at which the frequency changes over a period of time is relatively slow, as previous studies have indicated.[4] It seems justifiable, therefore, to take formula [3] as applicable, not strictly but approximately, to the B_x type of ratios which can be calculated from the 1940 crude census data. And hence from the results of those calculations it seems possible to calculate the b_x fertility ratios according to age.

If it were correct to assume that the demographic characteristics of the Brazilian population are approximately constant, and if the census returns were not affected by systematic errors, it would be relatively simple to determine the age-specific fertility rates of the female population.

It would suffice to calculate the fertility rates as at the exact ages of $(i+0.5)$, $(i+1.5)$, $(i+2.5)$ etc.; to estimate from these the fertility rates as at the exact ages $(i+1)$, $(i+2)$, $(i+3)$; and finally, from the difference between each term of the latter series and the preceding term, to calculate the fertility ratios in the $(i+1)^{th}$, $(i+2)^{th}$, and $(i+3)^{th}$, etc. years of age.

But the real calculation is much more laborious owing to the errors which affect the crude data.

The successive stages of the procedure applied will be explained in the following paragraphs.

4. Calculation of cumulative fertility rates from crude data

Table 1 gives the crude data[5] on the number of women in the *de facto* population—classified according to single years of age from 15 to 49, and according to decennial age groups from 50 upwards, and also indicating separately those who stated that they had one or more live births (called for the sake of brevity "fertile women").

The table also gives the number of live or still-born children, born to the women who stated that they had had live-born children.[6]

According to the declarations, still births represent 7.54 per cent of the total of children born to women who stated that they had borne live-born children.[7] As explained in previous studies, this apparent proportion is rather high because in many cases children dying shortly after birth are registered as still-born. To compensate for the consequences of this mis-statement the true percentage of still births was taken as 5 per cent, a fairly high figure, which makes the proportion of live births 95 per cent of the total.

The corrected numbers of still births thus calculated are given in column (c) of table 2. Column (b) gives the crude numbers of women; and column (d), the cumulative fertility rates, calculated from the ratios between the number of children born alive and the number of women. The rate given for the age of x completed years may be taken as the cumulative fertility rate as at the exact age of $(x+\frac{1}{2})$ years, as explained in paragraph 5.

[4] See in particular studies 25A and 36 in the above-mentioned series "Applications of the Demographic Census". In addition, this stability is confirmed by the results of the investigations made in the present study for the ages 50 and above.

[5] Excluding 5,100 women of 12 years and above in parts of the municipalities of Parintins (Amazon) and Garça (Sao Paulo), enumerated in the census, who could not be taken into account at the time of the mechanical sorting of the individual schedules, as the information concerning them had not reached the headquarters of the National Census Service. The tables in the present study do not include about 17,600 women enumerated in the census, whose ages were not stated.

[6] The still births stated to have been born by women who had no live births, are not included. In other investigations the still births were also included, but in any case they represent a negligible fraction of the total number of births declared.

[7] The proportion would rise to about 8 per cent if the still births born to women who had not had a live birth were included.

TABLE 1

Number of women, aged 12 and over, number of mothers having one or more live births and number of children born alive or dead, by age of women (Brazil, 1940)

Age (completed years) (a)	Women[a]		Children born to women who have had live-born children		
	Total (b)	Those who have had live-born children (c)	Live births (d)	Still births (e)	Total (f)
12	572,869	32	37	1	38
13	480,190	259	340	31	371
14	494,961	1,130	1,458	107	1,565
15	491,061	3,995	4,922	298	5,220
16	493,989	12,981	15,728	702	16,430
17	435,556	30,615	38,712	2,026	40,738
18	482,704	65,728	89,985	4,969	94,954
19	382,983	84,755	128,165	7,744	135,909
20	503,939	168,285	295,335	19,212	314,547
21	332,147	132,067	253,771	17,236	271,007
22	426,710	205,920	442,313	30,300	472,613
23	355,202	192,687	452,030	32,444	484,474
24	359,510	215,900	561,442	40,533	601,975
25	442,585	287,290	840,974	61,076	902,050
26	354,899	243,805	772,143	57,204	829,347
27	295,716	210,794	722,082	54,133	776,215
28	366,115	270,306	1,021,203	76,061	1,097,264
29	247,749	187,432	746,042	57,659	803,701
30	441,916	336,594	1,473,973	111,880	1,585,853
31	181,256	142,846	642,914	49,537	692,451
32	256,489	205,795	1,004,391	76,699	1,081,090
33	198,512	161,289	821,068	64,017	885,085
34	203,000	165,865	880,219	68,341	948,560
35	305,844	249,231	1,385,864	108,647	1,494,511
36	232,194	193,898	1,132,602	89,232	1,221,834
37	177,039	148,451	890,995	71,617	962,612
38	258,471	217,091	1,350,826	109,048	1,459,874
39	180,462	151,683	959,761	82,593	1,042,354
40	397,254	327,658	2,132,770	177,305	2,310,075
41	113,946	95,940	649,133	54,573	703,706
42	174,269	147,227	1,022,453	85,522	1,107,975
43	127,494	108,766	770,102	65,330	835,432
44	133,219	113,823	820,827	70,314	891,141
45	223,631	188,555	1,337,909	112,743	1,450,652
46	132,182	113,547	833,082	71,334	904,416
47	100,474	85,866	631,481	55,588	687,069
48	158,044	135,101	993,639	85,370	1,079,009
49	91,632	77,986	573,207	52,169	625,376
50 to 59	991,454	835,031	6,143,161	538,696	6,681,857
60 to 69	551,484	463,544	3,464,744	294,409	3,759,153
70 to 79	234,658	196,093	1,464,853	120,850	1,585,703
80 and over	104,021	85,185	631,586	51,609	683,195
12 and over	13,487,830	7,261,046	38,398,242	3,129,159	41,527,401

[a] Excluding women whose ages were not stated.

5. Effects of misreporting of age of women at the census

Before analysing the results of the calculations of the fertility rates, it is necessary to examine the crude data and to point out certain irregularities appearing in them which are indications of errors in the census returns.

The most obvious irregularities are those arising from mis-statements of age in the form of concentration in the "attractive" ages (multiples of 10, 5 and 2) and avoidance of the "unattractive" ages.

In previous studies [8] these irregularities were measured and approximately corrected and there is no need to repeat here the observations already made. It suffices to recall that these errors of concentration and evasion alter the age distribution of the population to a significant degree.

For example, the following data [9] illustrate the concentration of statements in the ages which are multiples of ten.

Age (completed years)	*Women enumerated*		*Differences between crude and graduated figures*	
	Crude figures	*Graduated figures*	*Absolute*	*Per cent*
20	503,939	423,326	+ 80,613	+ 19.0
30	441,916	291,553	+150,363	+ 51.6
40	397,254	205,603	+191,651	+ 93.2
50	267,429	125,743	+141,686	+112.7
60	179,681	71,421	+108,260	+151.6

It will be seen that the consequences of the errors arising from the concentration of statements in the ages which are multiples of ten are fairly serious. The number of women giving their age as 40 is almost twice the estimated number; and the number of those giving their age as 50 is more than twice.

The influence of these errors on the calculation of the fertility rates is due mainly to the presence of a contingent of a different age among the women classified, according to the data, in the "attractive" ages. For example, according to the above calculation, about 150,000 women giving their age as 30, over 190,000 giving their age as 40 and over 140,000 giving their age as 50, were not of those ages.

The real age of the women who are wrongly placed in the census returns by reason of mis-statements, may be, and in fact is,[10] in some cases higher and in some cases lower than the age indicated. Consequently, if the age interval considered is within the female reproductive period, the number of children born and declared by the women in question may be greater or less than the real number of children which they have had or will have had at the age appearing in the returns, i.e. the mis-stated age.[11] In the total number of children declared by women who state themselves to be of the age under consideration, overstatements in the number of births are obviously partially compensated by the understatements; however, it would be too optimistic to assume that this compensation is complete.

It should also be noted that the frequency of the omissions in the statements of the number of children is probably greater among uneducated women who do not know their own age well, than among the more educated. Hence the concentration of these women in the "attractive" ages[12] may tend to cause an understatement of the total number of children born to women of these ages.

Another contributing factor is the greater fre-

[8] See, in particular, in the series of "Analyses of Results of the Population Census" compiled and issued by the Technical Office of the National Census Service, No. 341 "Adjustment of the Age Distribution of the Population in Brazil", and No. 358 "Statements of Age according to the Census Form in the Census of 1 September 1940".

[9] Taken from Study No. 341, mentioned in the preceding footnote.

[10] Calculations made in the above-mentioned Study No. 341 show that the attraction towards a given age occurs on both sides of it, i.e., from below as well as above (see in particular table III, pages 7 and 8 of that study).

[11] Mis-statements of age by women who have already reached the end of their reproductive period do not appreciably influence the calculation of the fertility rates of a country in which fertility varies very little over a period of time. If women aged 56 or 64 give their age as 60, this has little effect upon the calculation, because, as pregnancies in the case of women over 50 are very rare, the cumulative fertility rate remains almost stationary with the increase in age above this limit.

[12] As examples of the greater concentration of uneducated women in the "attractive" ages, note the following comparisons in the percentages of illiterate women:

In the "attractive" ages: aged 30, 71.1 per cent; aged 40, 77.0 per cent;

In the "unattractive" ages: aged 29 and 31, 52.9 per cent; aged 39 and 41, 57.9 per cent.

quency of mis-statements, taking the form of concentrations in the "attractive" ages, by single women as compared with married or formerly married women,[13] as the average number of children born to the former is less, at all ages, than the number born to the latter.

Another circumstance which exercises a similar influence is the higher concentration in the "attractive" ages, of Negro women,[14] whose fertility is generally lower than that of the other colour groups.

An irregularity in the age distribution of women which does not appear at first sight in examining the figures but is revealed by a comparative analysis of sex distribution, is caused by the tendency on the part of women, both the young and the more mature, to give ages less than their real ones.[15] As a result of errors of this kind, the crude figures for the women in the younger age groups are increased by comparison with the real numbers and the crude figures for those in the higher age groups are lessened.

For example, it may be assumed that some of the women giving their age as 18 were in fact aged 19, 20, 21 or above, and consequently, had an average number of children greater than they had when they were really aged 18.

As in this example, so in general, rejuvenation errors in the age declarations of women tend to cause errors of over-estimation in the calculation of the cumulative fertility rates.

On the other hand "old age overestimates", found in the exaggerated figures for women aged 80 and above in the crude data, do not exercise an appreciable influence on this calculation in a country with constant or slowly changing fertility. In the great majority of cases of "old age overestimates", both the stated and the real age are above the limit of the female reproductive period.

Errors due to failure to declare the number of children they ever had must be more frequent among women of advanced age than among those in early or middle life, owing to the difficulty of recalling long past events.

[13] As examples of the greater concentration of single women in the "attractive" ages, note the following comparisons in the percentages of these women:

In the "attractive" ages: aged 30, 26.6 per cent; aged 40, 20.6 per cent;

In the "unattractive" ages: aged 29 and 31, 21.9 per cent; aged 31 and 41, 15.5 per cent.

[14] As examples of the greater concentration of Negro women in the "attractive" ages, note the following comparisons of the percentages of this colour group:

In the "attractive" ages: aged 30, 18 per cent; aged 40, 20 per cent;

In the "unattractive" ages: aged 29 and 31, 12.6 per cent; aged 39 and 51, 13.8 per cent.

[15] Concerning these "rejuvenation errors", see Study No. 338 "The Population of Brazil according to Nationality, in terms of Sex and Age Groups" (paragraph 10, pp. 6 and 7) in the above-mentioned series of analyses.

TABLE 2

Cumulative fertility rates of women, for each age 12 to 49, and for decennial age groups above the age of 50 (Brazil, 1940)

Age (completed years) (a)	*Total number of women* (b)	*Live births (corrected figures)* (c)	*Live births per 100 women* (d)
12	572,869	36	0.01
13	480,190	352	0.07
14	494,961	1,487	0.30
15	491,061	4,959	1.01
16	493,989	15,609	3.16
17	435,556	38,701	8.89
18	482,704	90,206	18.69
19	382,983	129,114	33.71
20	503,939	298,820	59.30
21	332,147	257,457	77.51
22	426,710	448,982	105.22
23	355,202	460,250	129.57
24	359,510	571,876	159.07
25	442,585	856,948	193.62
26	354,899	787,880	222.00
27	295,716	737,404	249.36
28	366,115	1,042,401	284.72
29	247,749	763,516	308.18
30	441,916	1,506,560	340.92
31	181,256	657,828	362.93
32	256,489	1,027,036	400.42
33	198,512	840,831	423.57
34	203,000	901,132	443.91
35	305,844	1,419,785	464.22
36	232,194	1,160,742	499.90
37	177,039	914,481	516.54
38	258,471	1,386,880	536.57
39	180,462	990,236	548.72
40	397,254	2,194,571	552.44
41	113,946	668,521	586.70
42	174,269	1,052,576	603.99
43	127,494	793,660	622.51
44	133,219	846,584	635.48
45	223,631	1,378,120	616.25
46	132,182	859,195	650.01
47	100,474	652,716	649.64
48	158,044	1,025,059	648.59
49	91,632	594,107	648.36
50 to 59	991,454	6,347,764	640.25
60 to 69	551,484	3,571,196	647.56
70 to 79	234,658	1,506,418	641.96
80 and over	104,021	649,036	623.95
12 and over	13,487,830	39,451,032	292.49

The observations in the preceding paragraph will facilitate analysis of the fertility rates given in table 2.

Since these rates are cumulative—in other words, indicative of the proportion between the number of children born to women up to the age of $(x+\frac{1}{2})$ and the number of women of that age—they tend to increase as the age increases up to the upper limit of the female reproductive period, and to remain stationary above that limit.

The following data, which relate to certain selected five-year age intervals, bring out the first of these tendencies.

Age x	*Live births, as at the exact age specified, per 100 women attaining that age*
12.5	0.01
17.5	8.89
22.5	105.22
27.5	249.36
32.5	400.55
37.5	516.54
42.5	603.99
47.5	649.64

In the first intervals the cumulative fertility rate rises rapidly with the increase in age; in the later ones it rises gradually, until it becomes almost stationary in the advanced age groups, as is shown by the following data.

Age (completed years)	*Live births per 100 women of the ages specified*
45 to 49	638.73
50 to 59	640.25
60 and over	643.33

The rates for ages 50 and over are probably slightly lower than the true rates, as a result of errors through failure of women of advanced age to declare the number of children born. This discrepancy, however, appears to be relatively small.

The clearly visible trend in the curve of the cumulative fertility rates in terms of age does not appear to be without irregularities; the irregularities occurring in the crude data owing to misstatements in the census returns seem to influence the calculation of these rates.

Let us look, for instance, at the rates for the years near the "attractive" age 40.

Age (completed years)	*Live births per 100 women of the ages specified*
37	516.54
38	536.57
39	548.72
40	552.44
41	586.70
42	603.99
43	622.51

The fertility rate tends to increase between the ages 37 and 43; its total increases by 105.97, its annual average, by 17.66.

But at 40 the rate is only 3.72 above that at 39, and less by 34.26 than that at 41. The rise is almost broken at an age which is a multiple of 10, owing to the concentration, at that point, of mis-statements of age by women with an average number of children less than that of women really aged 40.

If the average fertility rate is calculated successively for the age of 40 and for 3-, 5- and 7-year groups round the central age of 40, the fall in the rate lessens but does not disappear.

Age (completed years)	*Live births per 100 women of the ages specified*
40	552.44
39 to 41	552.11
38 to 42	559.66
37 to 43	559.92

A similar, though less marked, irregularity, occurs with regard to the "attractive" age 35. The cumulative fertility rate for 35 is only 20.31 more than that for 34, but that for 36 is 35.68 more than that for 35.

Again, a similar much more marked irregularity occurs with the "attractive" age of 45. Instead of increasing, the cumulative fertility rate decreases by 19.23 for age 45 as compared with 44, then rises by 33.76 for 46 as compared with 45. The rate for 45 is only 616.25, whereas for the quinquennial period of which 45 is the central age, the figure is 631.84.

In the examples given the main, if not the only, reason for the relatively low level of the cumulative fertility rate for the "attractive" age, as already pointed out, is the concentration, in that age, of declarations by women of other ages having an average number of children less than the true average for the "attractive" age.

The same factor contributes to the relatively high level of the cumulative fertility rate in the "unattractive" ages, as illustrated in the following comparisons between the rates calculated for given ages and the average rate for the quinquennial period for which those ages are central.

Age (completed years)	*Live births per 100 women*	
	At x years	*At (x-2) to (x + 2) years*
39	548.72	546.03
41	586.70	573.73
44	635.48	623.44
46	650.01	636.97

Perhaps this factor also contributes to raise above the true level the cumulative fertility rates for the ages 47 and 48, which are close to the "highly attractive" age of 50.[16]

In the higher ages the cumulative fertility rates show an appreciable decline, mainly owing to the fall in the proportion of women who state that they have had one or more live births, as the following data show.

Age (completed years)	*Live births per 100 women of the ages specified*	*Women who state that they have had live births per 100 women of the ages specified*
50 to 59	640.25	84.22
60 to 69	647.56	48.05
70 to 79	641.96	83.57
80 and over	623.95	81.89

If the greater proportion of women who have not had children, in the upper ages, were real, it would indicate a greater survival rate for women not exposed to causes of death connected with maternity.

But it is very probable that this higher percentage is only apparent. It may result from a greater frequency of overestimates in the age declarations of childless women, whose childlessness deprives them of useful points of reference for estimating their ages.

For the ages 50 and above as a whole a cumulative fertility rate between 640 and 650 per 100 women may be taken as approximately correct.

[16] It is not possible to calculate the fertility rate for the quinquennial period centring round the age of 48, because above the age of 50 figures are only available in ten-year and not in single-year age groups.

TABLE 3

Graduation of cumulative fertility rates of women and calculation of fertility rates according to age, from 15 to 49 (Brazil, 1940)

Age x (a)	*Live births per 100 women, at the exact age of $(x + ½)$ years*		*Live births per 100 women, at the exact age of x years*		*Live births per 100 women between the exact ages of x and $(x + 1)$ years*[e] (f)
	Original calculation[a] (b)	*1st graduation*[b] (c)	*1st graduation*[c] (d)	*2nd graduation*[d] (e)	
14	0.3	0.3	0.0	0.0	0.6
15	1.0	1.0	0.5	0.6	1.2
16	3.2	3.0	1.8	1.8	3.5
17	8.9	9.0	5.5	5.3	7.4
18	18.7	19.0	13.4	12.7	12.8
19	33.7	34.0	26.2	25.5	16.9
20	59.3	54.0	44.0	42.4	20.5
21	77.5	76.0	65.0	62.9	23.6
22	105.2	100.0	88.0	86.5	26.2
23	129.6	126.0	113.0	112.7	28.3
24	159.1	154.0	140.0	141.0	29.9
25	193.6	184.0	169.0	170.9	31.0
26	222.0	216.0	200.0	201.9	31.6
27	249.4	248.0	232.0	233.5	31.1
28	284.7	279.0	263.5	264.6	30.5
29	308.2	309.0	294.0	295.1	29.8
30	340.9	338.0	323.5	324.9	29.0
31	362.9	366.0	352.0	353.9	28.1
32	400.4	393.0	379.5	382.0	27.1
33	423.6	419.0	406.0	409.1	26.0
34	443.9	444.0	431.5	435.1	24.8
35	464.2	468.0	456.0	459.9	23.5
36	499.9	491.0	479.5	483.4	22.1
37	516.5	513.0	502.0	505.5	20.6
38	536.6	534.0	523.5	526.1	19.1
39	548.7	554.0	544.0	545.2	17.5
40	552.4	572.0	563.0	562.7	15.9
41	586.7	588.0	580.0	578.6	14.3
42	604.0	602.0	595.0	592.9	12.6
43	622.5	614.0	608.0	605.5	10.9
44	635.5	624.0	619.0	616.4	9.2
45	616.3	632.0	628.0	625.6	7.5
46	650.0	638.0	635.0	633.1	5.7
47	649.6	642.0	640.0	638.8	3.9
48	648.6	644.0	643.0	642.7	2.1
49	648.4	645.0	644.5	644.8	0.3
50	...	...	645.0	645.1	—

[a] Data from table 2, column (d) (after rounding).
[b] Graphic-numerical graduation of data in column (b).
[c] Data calculated from those of column (c).
[d] Graphic-numerical graduation of the data in column (d).
[e] Difference between successive figures in column (e).

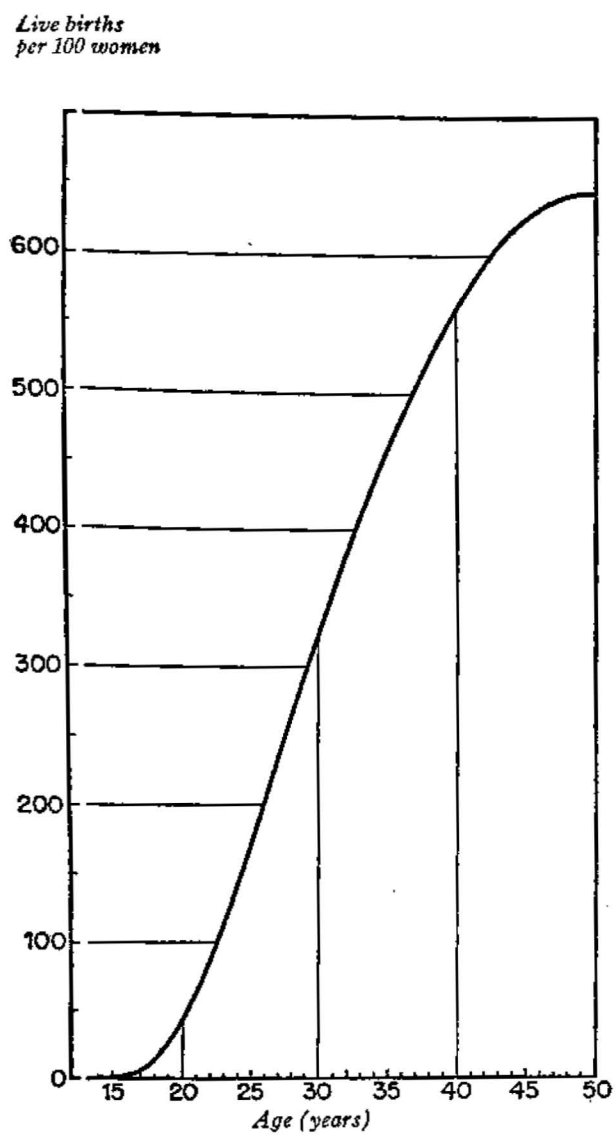

FIGURE 1. Live births, per 100 women, as at x years of age (cumulative fertility rate) according to column (e) of table 3

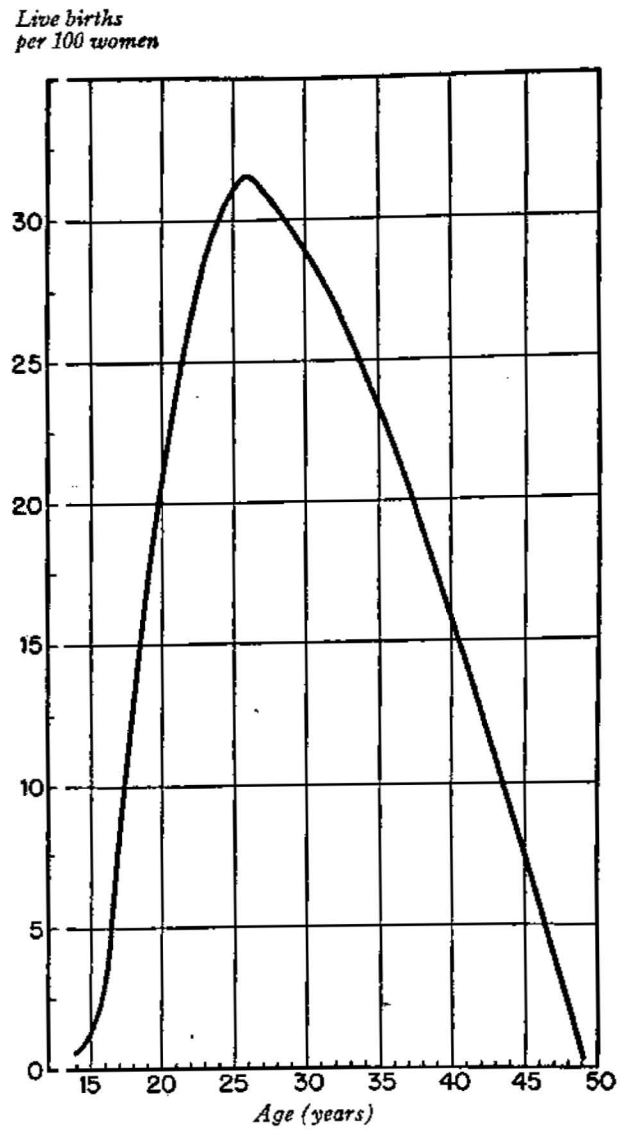

FIGURE 2. Live births, per 100 women, occurring between the ages of x and $(x + 1)$ (annual fertility rates) according to column (f) of table 3

6. Graduation of fertility rates

An analysis of the irregularities apparent in the curve of the cumulative fertility rates in relation to age leads to the conclusion that these irregularities are for the most part systematic and not accidental. Consequently, in order to graduate the curve, it is advisable to apply a process of graphical adjustment instead of an analytical process, because in the former type of adjustment the operator determines and applies the appropriate rectifications without remaining bound to the rigidity of a formula as in the second method.

Such a graduation was therefore applied to smooth the curve of the series of cumulative fertility rates calculated for the ages 14 to 49 (ages below 14 were excluded in view of the very low rates and the well-grounded suspicion that most of the women who stated themselves to be under 14 and to have borne children had given an age lower than their real age; the ages of 50 and above were excluded, because at these ages the cumulative fertility rate remains approximately constant).

The graduation was made with the following general purposes in mind:

To limit the deviations from the observed data to a minimum compatible with a satisfactory "smoothing" of the curve of rates in relation to age;

To eliminate the oscillations caused by misstatements leading to a concentration of the

declarations of age into certain years;

To maintain the total number of children approximately unchanged.

Concerning the last point it should be explained that, in calculating the total number of children according to the graduated rates, these rates were applied to the graduated numbers of women at the different ages,[17] and not to the crude numbers which, as was pointed out, deviate greatly from reality. It would be illogical to do otherwise.

The average number of children born alive to women aged between 19 and 49 is 334 per 100 women, according to the crude data, and is also 334 according to the graduated data.

The graduated rates, like the original rates, represent the percentage of children born per 100 women as at the age of $(x+\frac{1}{2})$ years. In order to calculate the percentage as at x years, the simple average of those born as at ages $(x-\frac{1}{2})$ and $(x+\frac{1}{2})$ may be taken. This procedure was applied from the age of 20 upwards; for the lower ages in which the simple average gives too high a result owing to the increasingly rapid rise in the rates in relation to age, weighted averages were taken, the rate for $(x-\frac{1}{2})$ being the more heavily weighted.[18] The rates given in column (d) of table 3 were calculated in this manner.

If the cumulative fertility ratios at the exact age of x years ($x = 14, 15, 16 \ldots, 50$) are known, it is possible, as explained under section 5, to calculate the fertility rates for different years of age. For instance, if 563.0 per 100 women is the fertility rate as at the exact age of 40, and 544.0 as at the exact age of 39, the difference of 19.0 per 100 women gives the fertility rate in the fortieth year.

The fertility rates for the years 15 to 50 were calculated in this way. In order to eliminate some minor irregularities brought to light by this calculation and to correct the effects of the "errors of rejuvenation" in the age declarations of women, as they affected the fertility rates in the younger ages,[19] it was found advisable to make a second graduation, by means of which the final figures for the fertility rates for different years of age, given in column (f) of table 3, were obtained. By successively adding together the annual rates the final figures for the cumulative fertility rates as at x years, given in column (e) of the same table, were obtained. This second graduation left unaltered the average number of children born to women between the ages of 19 and 49, namely 334 per 100 women (calculated according to the graduated age distribution as explained).

7. The variation of fertility rates with age

According to the final graduation, the average cumulative fertility rate for women as at their fiftieth birthday is 645 live-born children per 100 women.

The graph showing the graduated cumulative rates according to age faithfully reproduces the trend indicated by the rates calculated from the crude data,[20] as the comparison given in table 4 between the two series of rates shows.

In the ages below 30, the graduated cumulative rates generally remain slightly below the original ones, owing to the adjustment to eliminate the effect of the errors of understatements of age. From 30 upwards they are sometimes above and sometimes below.

The following summarized comparison shows the general concordance between the two series:

Age x	*Live births as at the exact age of x per 100 women attaining that age*	
	Original figure	*Graduated figure*
15	0.6	0.6
20	46.5	42.4
25	176.3	170.9
30	324.6	324.9
35	454.1	459.9
40	550.6	562.7
45	625.9	625.6
50	646.9	645.1

[17] The graduated numbers of women are those calculated in the above-mentioned Study No. 341 in the series of "analyses" (table 3, column (c)). They are shown in column (b) of table 5 of the present study.

[18] For the age of 15 the weights adopted were 0.65 and 0.35; for 16, 0.62 and 0.38; for 17, 0.59 and 0.41; for 18, 0.56 and 0.44; for 19, 0.53 and 0.47.

[19] See the remarks on the influence of these errors under section 51.

[20] The cumulative fertility rates as at the exact age of x, according to the crude data, were calculated on those as at the exact age of $(x + \frac{1}{2})$ given in column (d) of table 2 by taking simple averages from the age of 20 upwards, and weighted averages for the ages 14 to 19.

TABLE 4

Comparison between the cumulative fertility rates as at the exact age of x, calculated according to crude data and according to the final graduation (Brazil, 1940)

Age x (a)	*Live births per 100 women at exact age of x years*		*Difference between graduated and original calculation* (d)
	Original calculation[a] (b)	*Second graduation*[b] (c)	
14	0.1	0.0	— 0.1
15	0.6	0.6	—
16	1.8	1.8	—
17	5.5	5.3	— 0.2
18	13.2	12.7	— 0.5
19	25.7	25.5	— 0.2
20	46.5	42.4	— 4.1
21	68.4	62.9	— 5.5
22	91.4	86.5	— 4.9
23	117.4	112.7	— 4.7
24	144.3	141.0	— 3.3
25	176.3	170.9	— 5.4
26	207.8	201.9	— 5.9
27	235.7	233.5	— 2.2
28	267.0	264.6	— 2.4
29	296.5	295.1	— 1.4
30	324.6	324.9	+ 0.3
31	351.9	353.9	+ 2.1
32	381.7	382.0	+ 0.3
33	412.0	409.1	— 2.9
34	433.7	435.1	+ 1.4
35	454.1	459.9	+ 5.8
36	482.1	483.4	+ 1.3
37	508.2	505.5	— 2.7
38	526.6	526.1	— 0.5
39	542.6	545.2	+ 2.6
40	550.6	562.7	+ 12.1
41	569.6	578.6	+ 9.0
42	595.3	592.9	— 2.4
43	613.3	605.5	— 7.8
44	629.0	616.4	— 12.6
45	625.9	625.6	— 0.3
46	633.1	633.1	—
47	649.8	638.8	— 11.0
48	649.1	642.7	— 6.4
49	648.5	644.8	— 3.7
50	646.9	645.1	— 1.8

[a] Data calculated according to the rates as at $(x + \frac{1}{2})$ years given in column (d) of table 2.
[b] Data from table 3, column (e).

TABLE 5

Calculation of the annual number of live births according to the number of women alive as at 1 September 1940 and the fertility rates for each year of age calculated in table 3 (Brazil, 1940)

Age (completed years) (a)	*Women living*[a] (b)	*Fertility rates per 100 women*[b] (c)	*Calculated live births*[c] (d)
14	498,696	0.6	2,992
15	487,557	1.2	5,851
16	474,491	3.5	16,607
17	459,049	7.4	33,970
18	445,512	12.8	57,026
19	432,509	16.9	73,094
20	423,326	20.5	86,782
21	413,962	23.6	97,695
22	400,037	26.2	104,810
23	387,234	28.3	109,587
24	374,661	29.9	112,024
25	364,813	31.0	113,092
26	354,183	31.6	111,922
27	338,152	31.1	105,165
28	321,830	30.5	98,158
29	306,193	29.8	91,246
30	291,553	29.0	84,550
31	278,599	28.1	78,286
32	266,548	27.1	72,235
33	255,249	26.0	66,365
34	246,516	24.8	61,136
35	240,927	23.5	56,618
36	235,337	22.1	52,009
37	227,871	20.6	46,941
38	220,220	19.1	42,062
39	213,191	17.5	37,308
40	205,603	15.9	32,691
41	196,505	14.3	28,100
42	187,690	12.6	23,649
43	178,853	10.9	14,495
44	169,404	9.2	15,585
45	158,487	7.5	11,887
46	150,004	5.7	8,550
47	144,868	3.9	5,650
48	138,975	2.1	2,918
49	133,594	0.3	401
14 to 49	10,622,199	—	1,966,457

[a] Graduated data.
[b] Data from table 3, column (*f*).
[c] Column (*d*) = (*b*). (*c*)/100.

The graph of the fertility rates by age as finally graduated (column (*f*) of table 3), shows a rapid rise in the interval between the initial age of the reproductive period and the first years of the sixth quinquennial period, and a constant decline, slow at first and then more rapid, until the final year of the reproductive period.

The course of this curve is shown in the following partial summary:

Age x	*Live births per 100 women between the exact ages of x and x + 1*
16	3.5
21	23.6
26	31.6
31	28.1
36	22.1
41	14.3
46	5.7

8. Calculation of number of live births in one year

The cumulative and annual fertility rates obtained by calculations based on the census returns do not indicate fertility at the time of the census, but are instead determined by the number of children who had been born to the women alive at that time; some of these children had been born long before.

However, as regards the women under 50 years of age, it may be assumed that the large majority of their children—almost 90 per cent—were born within the 20 years prior to the date of the census.[21] Thus, the calculated fertility rates are determined largely from births taking place not very long before 1940 and may be considered representative of a situation not very different from that of 1940, as, in Brazil, these rates vary little in the course of time.

If the calculated rates are applied to the number of women of each age alive in 1940, the numbers of births are obtained, and their sum should not differ greatly from the real figure for that year. This calculation was made and its components and results are given in table 5.

The total number of live births calculated is 1,966,457 which, for a population of 41,236,315, indicates a crude birth rate of 47.69 per 1,000 inhabitants.

The real birth rate for 1940 is probably slightly lower, because—as indicated—the calculated fertility rates in part reflect the situation at earlier dates when the birth rate was somewhat, though not much, higher than in 1940.

The distribution of calculated live births in table 5, by age of the mother at the time of the birth, is shown in the following data:

Age of mother (completed years)	*Proportional distribution of 100,000 live births by age of the mother at the time of the birth, from the calculation for Brazil in 1940*
14	152
15 to 19	9,487
20 to 24	25,981
25 to 29	26,422
30 to 34	18,438
35 to 39	11,947
40 to 44	6,078
45 to 49	1,495
14 to 49	100,000

These figures may be summarized to show the average ages of mothers at the birth of a live-born child:

Arithmetic mean of ages, 28.6 years;
Median age, 27.5 years;
Modal (i.e., most frequent) age, 25.5 years.

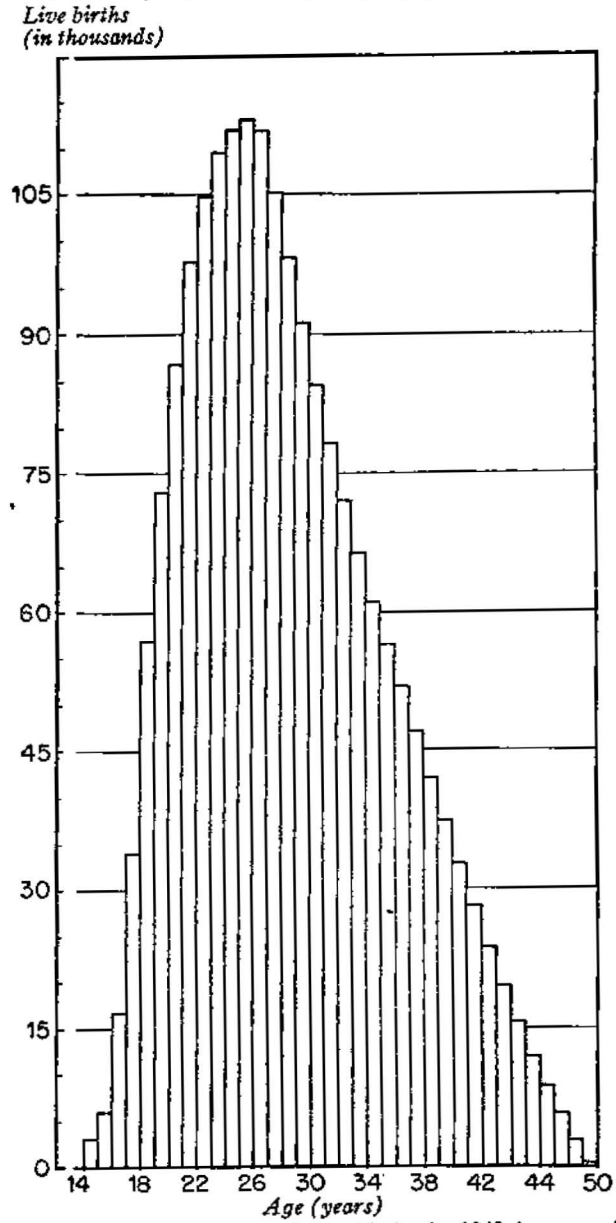

FIGURE 3. Distribution of live births in 1940 by age of mother (estimated figure) (data from column (*d*) of table 5)

[21] This approximate calculation is based on their cumulative fertility rates.

In fact, the mean number of children born in the last 20 years to women aged x completed years at the date of the census is approximately equal to the difference between the mean number of children born to these women and the number to women aged $(x-20)$ completed years.

For instance, if, according to the final graduation, the cumulative fertility rate for age 45 completed years is 629.4 and for 25 completed years 186.4, the difference between these two rates, i.e. 443.0, represents the approximate number of children born in the 20 years prior to the date of the census, to 100 women aged 45 completed years at that date.

9. Fertility rates by age groups and international comparisons

By grouping the data in columns (*b*) and (*d*) in table 5 into quinquennial age groups, the fertility rates for such groups which are useful for comparison may be calculated.

Table 6 is a comparison between these rates and various rates, calculated by comparing the numbers of registered live births and the number of women according to the census, for some countries with a high birth rate, together with the estimated figures for Brazil in 1920 based on a conjectural analysis.

TABLE 6

International comparison of fertility rates in quinquennial age groups
Fertility rate per 100 women

Age (completed years) (*a*)	*Brazil 1940 census*[a] (*b*)	*Brazil 1920*[b] (*c*)	*Ukraine 1896-97*[c] (*d*)	*Bulgaria 1901-05*[c] (*e*)	*Croatia-Slovenia 1900-01*[c] (*f*)	*Mexico 1929-31*[d] (*g*)	*Chile 1930-31*[d] (*h*)	*Japan 1930*[d] (*i*)
15 to 19	8.11	8.86	3.20	2.35	4.52	11.74	6.96	3.15
20 to 24	25.56	27.68	32.80	28.86	25.06	22.85	19.04	20.02
25 to 29	30.83	31.00	35.50	31.22	28.79	24.29	21.34	24.86
30 to 34	27.09	28.78	23.20	30.94	24.57	21.70	21.23	21.70
35 to 39	20.65	19.93	13.80	20.43	17.20	15.61	14.91	16.31
40 to 44	12.73	11.07	6.30	12.10	8.73	8.34	7.89	7.16
45 to 49	4.05	3.32		5.55	1.75	0.49	3.06	1.03

[a] Data calculated from the ratio between the 5-yearly totals of live births and the 5-yearly totals of women alive at that time, as given in table 5.

[b] Data estimated according to a conjectural process by G. Mortara, "Fecundidade de Mulher Brasileira e a Capacidade de Reprodução da População do Brasil" ("The Fertility of Brazilian Women and the Reproductive Capacity of the Population of Brazil") in the *Revista Brasileira de Estatistica*, No. 9, 1942.

[c] Data calculated by R. R. Kuczynski, *The Balance of Births and Deaths*, vol. II, pp. 163-164 (Washington, Brookings Institution, 1931).

[d] Data calculated by the International Statistical Institute, *Aperçu de la Démographie des Divers Pays du Monde*, 1929-36, p. 167 (The Hague, 1939).

The comparison shows a notable similarity between the results of the two calculations for Brazil. It should be pointed out that these calculations were made by completely different and independent processes.

In view of what has been said respecting the situation of Brazil as compared with that of other countries, the Brazilian fertility rates, though high, cannot be considered improbable since even higher rates are to be found at all ages in other populations: in the age group 15-19 in Mexico; in the age groups 35 to 39 and 40 to 44 in Ukraine; and in the other groups, in Ukraine and Bulgaria.

An over-all comparison may be made by calculating the cumulative fertility rate at the fiftieth birthday, the approximate figure for which is obtained by multiplying by five the sum of the rates for the quinquennial age groups given in table 6. For Brazil this approximate calculation gives figures similar to those obtained by adding together the annual rates.

Country and date	*Number of live births per 100 women at the exact age of 50*
Brazil (census of 1940)	645
Brazil (1920)	653
Ukraine (1896-97)	750
Bulgaria (1901-05)	657
Croatia-Slovenia (1900-01)	553
Mexico (1929-31)	525
Chile (1930-31)	472
Japan (1930)	471

The fertility rate of women in Brazil, according to the 1940 census figures and the estimated figures for 1920, is lower than that for the Ukraine in the last years of the nineteenth century, almost equal to that for Bulgaria for the first years of the twentieth century [22] and higher than those for the years round 1930 for Mexico, Chile and Japan.

[22] The fertility rates in these East European countries are *at present* much lower. By 1926-1927 the mean number of children which had been born to women aged 50 as calculated from the fertility rates of that time had already fallen to 513 per 100 in the Ukraine and 460 per 100 in Bulgaria (Kuczynski, *The Balance of Births and Deaths*, vol. II, pp. 163-164 (Washington, Brookings Institution, 1931)).

TABLE 7

Fertility table according to fertility rates calculated from the 1940 census and the 1890-1920 life table for Brazil

Age x (*a*)	*Women in the* $(x+1)^{th}$ *year of age*[a] (*b*)	*Fertility rate per 100 women in the* $(x+1)^{th}$ *year of age*[b] (*c*)	*Live births to women*	
			In the $(x+1)^{th}$ *year of age*[c] (*d*)	*As at the end of the* $(x+1)^{th}$ *year of age*[d] (*e*)
14	35,577	0.6	213	213
15	35,442	1.2	425	638
16	35,285	3.5	1,235	1,873
17	35,096	7.4	2,597	4,470
18	34,868	12.8	4,463	8,933
19	34,590	16.9	5,846	14,779
20	34,264	20.5	7,024	21,803
21	33,908	23.6	8,002	29,805
22	33,536	26.2	8,786	38,591
23	33,159	28.3	9,384	47,975
24	32,781	29.9	9,802	57,777
25	32,400	31.0	10,044	67,821
26	32,018	31.6	10,118	77,939
27	31,631	31.1	9,837	87,776
28	31,242	30.5	9,529	97,305
29	30,849	29.8	9,193	106,498
30	30,453	29.0	8,831	115,329
31	30,052	28.1	8,445	123,774
32	29,648	27.1	8,035	131,809
33	29,238	26.0	7,602	139,411
34	28,824	24.8	7,148	146,559
35	28,404	23.5	6,675	153,234
36	27,979	22.1	6,183	159,417
37	27,548	20.6	5,675	165,092
38	27,111	19.1	5,178	170,270
39	26,668	17.5	4,667	174,937
40	26,216	15.9	4,168	179,105
41	25,759	14.3	3,684	182,789
42	25,295	12.6	3,187	185,976
43	24,823	10.9	2,706	188,682
44	24,343	9.2	2,240	190,922
45	23,854	7.5	1,789	192,711
46	23,358	5.7	1,331	194,042
47	22,853	3.9	891	194,933
48	22,338	2.1	469	195,402
49	21,815	0.3	65	195,467
14 to 49	1,063,225	—	195,467	—

[a] L_x of the female sex, the number at l_0 of both sexes together being 10,000.
[b] Rates calculated in table 3, column (*f*).
[c] Column (*d*) $=(b)\cdot(c)/100$.
[d] Column (*e*) = sum of figures in (*d*) up to and including the line in question.

TABLE 8

Fertility table according to fertility rates calculated from the 1940 census of Brazil and the 1920 life table (BC Cap. Adjust.)

Age x (*a*)	*Women in the* $(x+1)^{th}$ *year of age*[a] (*b*)	*Fertility rate per 100 women in the* $(x+1)^{th}$ *year of age*[b] (*c*)	*Live births to women*	
			In the $(x+1)^{th}$ *year of age*[c] (*d*)	*As at the end* $(x+1)^{th}$ *of the year of age*[d] (*e*)
14	33,806	0.6	203	203
15	33,665	1.2	404	607
16	33,502	3.5	1,173	1,780
17	33,305	7.4	2,465	4,245
18	33,064	12.8	4,232	8,477
19	32,780	16.9	5,540	14,017
20	32,458	20.5	6,654	20,671
21	32,104	23.6	7,577	28,248
22	31,733	26.2	8,314	36,562
23	31,361	28.3	8,875	45,437
24	30,988	29.9	9,265	54,702
25	30,615	31.0	9,491	64,193
26	30,240	31.6	9,556	73,749
27	29,865	31.1	9,288	83,037
28	29,488	30.5	8,994	92,031
29	29,110	29.8	8,675	100,706
30	28,729	29.0	8,331	109,037
31	28,347	28.1	7,966	117,003
32	27,960	27.1	7,577	124,580
33	27,572	26.0	7,169	131,749
34	27,180	24.8	6,741	138,490
35	26,784	23.5	6,294	144,784
36	26,384	22.1	5,831	150,615
37	25,980	20.6	5,352	155,967
38	25,571	19.1	4,884	160,851
39	25,157	17.5	4,402	165,253
40	24,738	15.9	3,933	169,186
41	24,313	14.3	3,477	172,663
42	23,883	12.6	3,009	175,672
43	23,447	10.9	2,556	178,228
44	23,003	9.2	2,116	180,344
45	22,553	7.5	1,691	182,035
46	22,096	5.7	1,259	183,294
47	21,632	3.9	844	184,138
48	21,160	2.1	444	184,582
49	20,681	0.3	62	184,644
14 to 49	1,005,254	—	184,644	—

[a] L_x of the female sex, the number at l_0 of both sexes together being 100,000.
[b] Rates calculated in table 3, column (*f*).
[c] Column (*d*) = (*b*). (*c*)/100.
[d] Column (*e*) = the sum of the figures in (*d*) up to and including the line in question.

10. Calculation of fertility table and reproduction rate

From the results of the calculation of the fertility rates by years of age, in combination with life table data it is possible to work out the fertility table, from which the reproductive capacity of the population under consideration can be measured.

For Brazil no life table for the period 1920-1940 is available, which would have been suitable for use in combination with the fertility rates representing the position for that period. Two tables

referring to previous periods were used, namely, that calculated for the whole national population of Brazil, according to the mortality figures for the period 1890-1920,[23] and that calculated for the principal capital cities according to the mortality figures for 1920.[24]

The life table shows the average number of survivors at each year of age in a hypothetical generation initially composed of 100,000 live births, distributed in normal proportions between the two sexes and subject in each year of age to a probability of death equal to that of the population at the period to which the life table refers.

If the life table refers to both sexes together, as in the present case, it is possible to calculate the number of females in the total number of survivors at each year of age.[25]

By this means the average number [26] of women, in a hypothetical generation, alive at each year of age of the female reproductive period, can be calculated. Columns (*b*) of tables 7 and 8 give these averages calculated respectively from the 1890-1920 life table for Brazil as a whole and the 1920 life table for the capitals (BC Cap. Adjust.).

By applying the fertility rate determined for the year of age in question to the average number of women alive at that age, we obtain the number of live births, given in columns (*d*) of tables 7 and 8 (the rates applied are given in columns (*c*)).

Lastly, the sum of these numbers gives the total number of children who would be born alive to the women of the hypothetical generation in the course of their existence: 195,467 for the 1890-1920 Brazil life table and 184,644 for the 1920 life table (BC Cap. Adjust.).

Dividing these figures by 100,000 we obtain the corresponding net reproduction rates: 1.955 and 1.846.

The meaning of these rates is very simple: they indicate that ratio between each new generation and the original parent generation, which would obtain in a population whose death rates and female fertility rates for each year of age are permanently equal to those ascertained for the population and the period investigated.

In the case in point the new generation would be slightly less than double the original generation. It would exceed it by 95 to 85 per cent.

Before presenting a few international comparisons, the purpose and meaning of the dual calculation should be explained.

A priori, taking into account the tendency for mortality to decline in civilized countries, one might suppose that mortality in Brazil during the period 1920-1940 was lower than during the period 1890-1920. Then, if the 1890-1920 life table for Brazil faithfully describes the situation for the period to which it refers, the numbers of female survivors calculated according to that table would be lower than those which would be obtained from the actual mortality of 1920-1940; and the net reproduction rate of 1.955, though so high, should be presumed to be below the real figure.

But research subsequent to the computation of the 1890-1920 life table for Brazil (see in particular the analyses in Study No. 30 of the above-mentioned series of "applications") showed that the overestimates which affected the results of the 1920 population census, had their repercussions upon the calculations in that table resulting in underestimation of the mortality rates and overestimation of the survival rates. It was thus ascertained that both for the period 1890-1920 and for the following period 1920-1940, the table referred to gives figures for the survival rate and the numbers of survivors that are too high.

It was therefore thought wise to calculate a second fertility table, based on survivors derived from the 1920 life table (BC Cap. Adjust.), which has a higher mortality level than that of the Brazil 1890-1920 life table,[27] and is probably approximate to that effectively true for Brazil for the period 1920-1940. The net reproduction rate of 1.846 thus computed should not deviate greatly from the true value, but for greater prudence, it could be rounded to 1.80.[28]

By way of comparison, the net reproduction rates calculated for the two countries with the highest fertility rate among those considered in

[23] Tabulation calculated by G. Mortara, *Revista Brasileira de Estatística,* No. 7, 1941, page 504 and No. 9, 1942, pp. 83 and 84). [Translator's note—This life table is that described above, pp. 1-10.]

[24] Life table calculated by Bulhões Carvalho and adjusted by G. Mortara (*Revista Brasileira de Estatística,* No. 4, 1940, pp. 683 and 684). Abbreviated title: 1920 life table (BC Cap. Adjust.).

[25] In the case in point this was done by applying to the total number of living persons the co-efficient 0.505 suggested by the results of the 1920 census. See the above-mentioned study in No. 9 of *Revista Brasileira de Estatística,* pp. 78 and 81.

[26] L_x in the conventional symbols for life tables.

[27] According to the 1920 life table (BC Cap. Adjust.), the average annual life span of the new-born child is only 37.43, as compared with 39.25 according to the 1890-1920 Brazil life table.

[28] This figure was previously adopted, as a wise reduction, to summarize the calculations for the year 1920. See the above-mentioned study in *Revista Brasileira de Estatística,* No. 9, p. 90.

the former comparisons may be recalled: 1.96 for the Ukraine in 1896-1897 and 1.84 for Bulgaria in 1901-1905.[29]

The two fertility tables calculated give proportionate distributions of births by age of mother at the time of birth which differ but little, as the following data indicate.

Age of mother (completed years)	*Proportionate distribution of 100,000 live births, by age of mother at the time of birth, according to the fertility tables based on the life tables stated*	
	Brazil 1890-1920	*BC Cap. Adjust. 1920*
14	109	110
15 to 19	7,452	7,481
20 to 24	21,998	22,034
25 to 29	24,925	24,915
30 to 34	20,495	20,463
35 to 39	14,518	14,495
40 to 44	8,178	8,173
45 to 49	2,325	2,329
14 to 49	100,000	100,000

The following averages of the ages of women at birth of their children summarize the above figures:

Arithmetic mean of ages, 29.8 years;

Median age, 29.0 years;

Modal (i.e., most frequent) age, 26.5 years.

It should be remembered that fertility tables and hence the data given in the present paragraph refer to a hypothetical generation, subject at each year of age to a mortality rate equal to that ascertained for Brazil, and having the same fertility rate for each year of the female reproductive period.

On the other hand, the data given in section 7 refer to the real population of Brazil, which is growing rapidly and in which each generation is usually larger than the preceding one.

By virtue of that difference, the contribution from the women of the younger ages to the total of live births is greater, and that from the more advanced ages less, in the real population than in the hypothetical generation. The arithmetic mean age of the mothers at birth of their children is 28.6 for the real population, whereas it is 29.8 for the hypothetical generation.

[29] Calculations by Kuczynski (*op. cit.*, pp. 12, 31). By the period 1926-1927 the net reproduction rate had fallen to 1.68 in the Ukraine and 1.45 in Bulgaria, according to calculations by the same author (*op. cit.* pp. 20, 31).

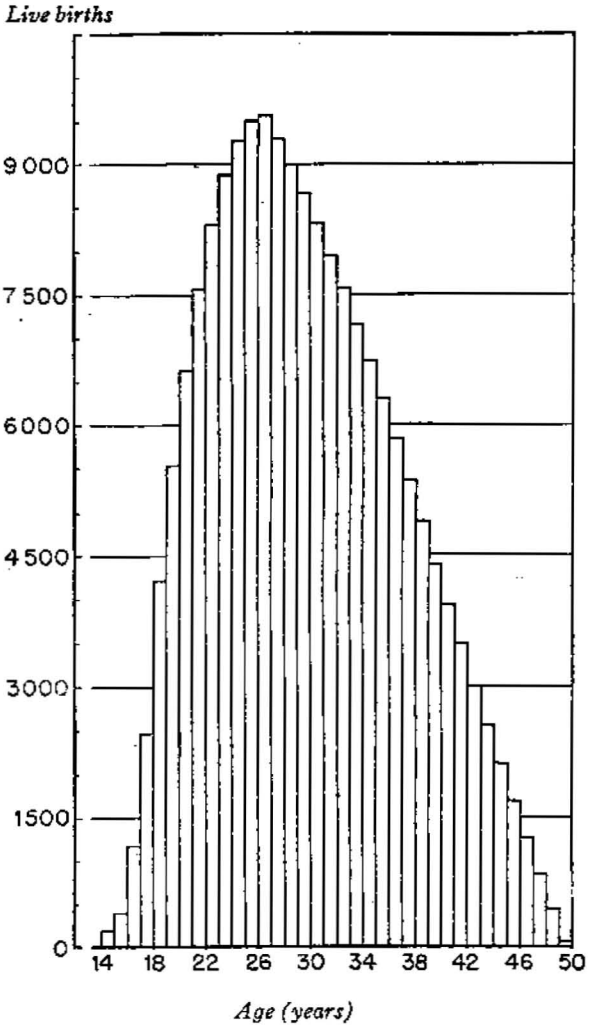

FIGURE 4. Distribution by age of mother, of the live births in the fertility table based on the life table for 1920 (BC Cap. Adjust.) (data from column (*d*) of table 8)

11. Calculation of intrinsic rate of natural increase

With the aid of a life table and the corresponding fertility table, it is possible to calculate the constant annual rate of natural increase which would occur in a hypothetical closed population,[30] in which the mortality rate at each year of age and the fertility rate at each year of age of the female reproductive period remained constantly equal to those indicated in the tables in question.

This rate (the "intrinsic rate of natural increase" developed by Lotka) is 0.02295 if calculated on the 1890-1920 life table for Brazil or 0.02097 if calculated on the BC Cap. Adjust. 1920 life table. In other words, the rate of natural in-

[30] That is, not subject to migration out of or into the country.

crease of the hypothetical population, whose age composition is constant over time, would be about 23 or 21 per 1,000, according to the mortality rate assumed. As already indicated, the second of the life tables seems to be closer to reality, so that it would be wise to adopt the second figure as the most reliable value of the intrinsic rate of natural increase corresponding to the mortality and fertility situation in Brazil in the period 1920-1940.[81]

12. Summary

Cumulative and annual female fertility rates for Brazil have been compared in the present study exclusively from the results of the 1940 census.

From the annual fertility rates it was possible to calculate the birth rate.

With the aid of the life tables which had previously been calculated fertility tables according to the two different death-rate hypotheses were then constructed.

Lastly, from the fertility tables, the net reproduction rate and the intrinsic rate of natural increase for Brazil were calculated.

These calculations demonstrate the very high level of the birth rate, fertility rate and reproductive capacity of the population of Brazil.

[81] In a previous study ("Life Tables and Their Applications to Population," *Revista Brasileira de Estatística*, No. 19, 1944), G. Mortara calculated the intrinsic rate of natural increase for the period 1890-1920 as 0.02381.

This figure, based on the Brazil 1890-1920 life table, which subsequent analyses show to have been optimistic, is an appreciable overestimate. The correct figure should be a little less than the figure 0.021 adopted in the present study for the period 1920-1940, but definitely above that of 0.018 adopted for the period 1870-1890 (*op. cit.* p. 347, note 82).

Concerning the significance and process of calculating the intrinsic rate of natural increase, the above-mentioned study may be consulted.

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