

Chapter III

TEMPO OF URBANIZATION AND URBAN CONCENTRATION

GENERAL CONSIDERATIONS

81. After examining the effect of definitions on the urban and rural population data and the likely influence of the components of population change, it is generally useful to study the trends in the two population categories (urban and rural) which have occurred in the past. In the absence of other criteria, it may sometimes be assumed in the population forecast that similar changes will continue in the future. There are several ways in which past urbanization trends can be measured, hence, depending on the measure used, the future forecast may differ. Different measurements can be relevant depending on whether the urbanization level is comparatively high, comparatively low or in some intermediate range.

82. Future assumptions, however, should not be derived uncritically from past observations. However it may be measured, the urbanization trend can vary from one period to another under the influence of numerous developments. The past period for which observations can be made may have been affected by special circumstances unlikely to be repeated in the future. Again, for future purposes government policy favouring either urban or rural developments (for instance, industry, agriculture or housing policy) may have effects on urbanization trends for which the past period does not constitute a suitable precedent. But even then the past period can provide a standard of comparison with assumptions drawn up more freely for the future. For instance, it may be assumed that the urbanization tempo (however measured) will either accelerate or slow down.

83. It should also be borne in mind that urbanization trends, despite their long-term inertia, are subject to fluctuations. While it is true that throughout the world the urbanization levels are continuously rising—apart from temporary effects of a disastrous period, e.g. a destructive war—it is also true that the rises differ among countries and from one decade to another.

84. There is no objective test for the distinction between trend and fluctuations in the urbanization tempo. If the past observations cover a long period, they may well be indicative of a trend; if they extend over a short period only, such as the most recent decade, they may be considerably influenced by temporary fluctuations; a very long past period, however, is not necessarily indicative of a future trend, because much time has elapsed since the beginning of the past period and relevant conditions may meanwhile have changed significantly.

85. Even for future periods it should be expected that the urbanization tempo will fluctuate again; but fluctua-

tions are clearly unpredictable. A useful forecast should reflect a future long-run trend which, with currently available judgment, appears plausible. Actual future developments, subject to fluctuations, should not be expected to coincide closely with the forecast. For such reasons, a forecast should preferably be made for a period of at least twenty years, and with three variants, "high", "low" and "medium", such that it appears more likely that future trends will lie within the range between the "high" and "low" variants than outside it. The "medium" series should be the one most recommended for actual use, and the "high" and "low" series should be presented to indicate probable errors of a magnitude that should be no cause for surprise.

CONCEPT OF A GROWTH RATE

86. Perhaps the best known concept of a growth rate is that of the annual, or compound-interest rate. Because of its widespread use, it will also be employed in the calculations of this manual unless otherwise stated.

87. The compound interest rate rests on an assumption that by the end of each year an increment is added to an initial amount (such as interest to capital) which stands in a fixed proportion to the size of that amount at the beginning of each year. Thus, if r is the proportional annual increment, and t the time interval measured in years, then $P_t = P_0(1 + r)^t$, P_0 and P_t being the magnitudes (of capital; of population; etc.) at the beginning and at the end of the time period. If r is unknown but P_0 and P_t are given, r can be determined by transposition

of that formula into the form $r = \sqrt[t]{\frac{P_t}{P_0}} - 1$. The

time interval t need not be an integral number of years and can also include fractions of a year. The relevant calculations are performed easily with reference to a conventional table of logarithms, i.e. logarithms with base 10, and such tables are widely available. It is not intended to discourage their use wherever appropriate.

88. For demographic purposes, however, the use of an annual rate is afflicted by some imperfections. First, it is to be noted that other demographic rates, such as birth or death rates, are not calculated with reference to the population at the beginning of each year but, ordinarily, with reference to mid-year or mid-period populations (being the best substitutes for period-mean populations). In growing populations, each mid-year figure is likely to be somewhat larger than the figure for the beginning of each year, some growth having occurred

already within the first half of each year. It follows that birth rates and death rates are ordinarily slightly smaller than they would have been if (which is not the practice) they had been calculated in proportion to the population at the beginning of each year. If a growth rate is to be consistent with other demographic rates, therefore, it should not be calculated on a compound interest basis.

89. Secondly, the compound interest rate lacks desirable mathematical properties. With this type of rate, for instance, different results are obtained if a given amount increases at a certain rate for two consecutive years, and if it increases at twice that rate for only one year. In the one instance, the relative increment becomes $(1 + r)^2$, which is $1 + 2r + r^2$, while in the other instance it becomes only $1 + 2r$, leaving a difference of r^2 , which may be quite small so long as the rate is not high. At higher rates, or over extended periods of time, however, this internal inconsistency can become considerable.

90. These drawbacks are avoided when growth is measured by instantaneous, also called "exponential" rates, as expressed in the formula $P_t = P_0 e^{ct}$, where c is the instantaneous rate and e is 2.71828, the basis of Napierian, also called "natural" logarithms. It may be noted that the relationship between the compound interest rate and the exponential rate takes the form $e^c = 1 + r$. The exponential increment, namely $c - 1$, if positive, is always at least slightly less than r , and if the rate is high it can be considerably less (in cases of decrease, the negative rate $c - 1$ is always at least slightly more than the rate r). This is so because in a steadily increasing population the amount of increment, per instant of time, rises continuously in proportion to the population increase itself. For some selected instantaneous increments the corresponding compound increments are as follows:

| Instantaneous increment ($c - 1$) | Compound increment (r) |
|-------------------------------------|----------------------------|
| 0.01 | 0.01005 |
| 0.02 | 0.02020 |
| 0.03 | 0.03045 |
| 0.04 | 0.04081 |
| 0.05 | 0.05127 |
| 0.06 | 0.06184 |
| 0.07 | 0.07251 |
| 0.08 | 0.08329 |
| 0.09 | 0.09417 |
| 0.10 | 0.10517 |

In some countries, urban populations grow at rates of 5, 6 or 7 per cent per year, and it is evident that an appreciable discrepancy can result between the two methods of measuring such a high growth rate.

91. The mathematical interaction between two or more instantaneous rates is evident from the fact that the variable is in the exponent. Thus, if c is the product of a and b , it must be true that e^{ct} equals e^{abt} . It also follows that the same increase is achieved whether growth occurs at a given rate over a given period of time, or at any multiple of that rate over a corresponding fraction of that time period, because nc times $\frac{1}{n}t$ is the same as ct . But to operate with the instantaneous, or exponential,

rate it is most convenient to make use of a table of the exponential function if available.⁴⁹ Since the user of this manual may find no table of the exponential function readily at hand, in so far as possible the calculations have been performed according to the compound interest rate, whose mathematical shortcomings are here admitted. Certain methods, however, will be explained further on, which are derived from an exponential function. In the use of those methods, however, only one derived function is needed, and this is tabulated in annex I to this manual.

92. For purposes of quick calculation, a third method is worth mentioning which is entirely independent of either logarithms or any other tabulated function, but it must be stressed that satisfactory results will be obtained only so long as the growth rate is not high and the time period under consideration is not long. Within such limits it can be demonstrated that a rough approximation

to a growth rate is obtained in the formula $P_t = P_0 \frac{2 + rt}{2 - rt}$ and its transformation $r = \frac{2(P_t - P_0)}{t(P_t + P_0)}$. The first

formula can serve to determine P_t when r is known, and the second to determine r when P_t is known. But despite its "mathematical" appearance, this formula is a simplification of only limited use. It should be employed only to establish quickly an approximate order of magnitude, for instance to check whether no error was made in the more precise calculation of either a compound interest or an exponential rate.

TEMPO OF URBANIZATION

93. The word "urbanization" has a double meaning: it can indicate a situation existing at any particular moment, in which instance it is best to speak of a *level of urbanization*; or it can be used to express the trend of urban developments, in which instance it is better to speak of the *tempo of urbanization*.

94. The level of urbanization is measured simply, and for present purposes satisfactorily, by the percentage of urban in the total population at any fixed date. True, some more complex concepts have been brought into use, such as indices of urbanization level weighted by the size classes of cities and towns,⁵⁰ but those special purpose measurements need not concern us here. It is also to be noted that different levels of urbanization will result where urban population is defined in two or more alternative ways.

95. There are several alternative ways in which the tempo of urbanization can be measured. The most

⁴⁹ See for instance, United States Department of Commerce, *Table of the Exponential Function e^x* (Washington, Government Printing Office, 1961). It is also possible to operate with conventional logarithms using the formula $\log_{10} P_t = \log_{10} P_0 + c \log_{10} e \cdot t$, where $\log_{10} e = 0.4342945$.

$$\frac{1}{P} \sum C_i^2$$

⁵⁰ For instance, $U = \frac{1}{P} \sum C_i^2$, where C_i is the population of city i , P is the total population of the country, and n is the number of cities. Eduardo E. Arriaga, "A New Approach to the Measurement of Urbanization", *Economic Development and Cultural Change* (Chicago), vol. 18, No. 2, January 1970, pp. 206-218.

TABLE 9. ALTERNATIVE MEASURES OF THE TEMPO OF URBANIZATION AS APPLIED TO ESTIMATES OF TOTAL AND URBAN POPULATION OF EIGHT MAJOR WORLD AREAS FOR 1960 AND 1970

| Major area | Total population (in thousands) | | Urban population (in thousands) | | Level of urbanization (per cent urban in total) | | Tempo of urbanization, as measured by (per cent per year): | | |
|----------------------|------------------------------------|-----------|------------------------------------|---------|----------------------------------------------------|-------|---------------------------------------------------------------|--------------------------------------------------------|----------------------------------------------------------------|
| | 1960 | 1970 | 1960 | 1970 | 1960 | 1970 | Annual rate of growth of urban population | Annual gain in percentage urban population | Annual rate of gain in percentage urban population |
| | | | | | | | | | |
| East Asia | 780,071 | 929,932 | 176,349 | 274,902 | 22.61 | 29.56 | 4.54 | 0.70 | 2.72 |
| South Asia | 865,320 | 1,125,843 | 155,889 | 233,052 | 18.02 | 20.70 | 4.10 | 0.27 | 1.40 |
| Europe | 424,563 | 462,117 | 247,400 | 293,700 | 58.19 | 63.56 | 1.73 | 0.54 | 0.89 |
| Soviet Union | 214,238 | 242,512 | 106,018 | 138,568 | 49.49 | 57.14 | 2.71 | 0.77 | 1.45 |
| Africa | 269,577 | 344,487 | 48,488 | 76,652 | 17.99 | 22.25 | 4.69 | 0.43 | 2.15 |
| Northern America ... | 198,765 | 227,572 | 138,387 | 169,117 | 69.65 | 74.31 | 2.03 | 0.47 | 0.65 |
| Latin America | 213,422 | 283,251 | 103,299 | 159,209 | 48.40 | 56.21 | 4.42 | 0.78 | 1.51 |
| Oceania | 15,755 | 19,370 | 10,321 | 13,124 | 65.51 | 67.75 | 2.43 | 0.22 | 0.34 |

Note: Estimates according to *Monthly Bulletin of Statistics*, November 1971 (United Nations publication).

common-sense measures might seem to be these: the annual rate at which the urban population is growing; the annual amount by which the (percentage) level of urbanization is rising; and the annual rate at which the level of urbanization is rising.⁵¹ When applied to several populations, differing in level of urbanization or in rate of population growth, comparative results will differ according to the measure used. Projections will also differ if the tempo of urbanization is assumed to continue but is measured in varied ways. The variety of results obtained by different measurements is illustrated in table 9.

96. Judging by rates of growth in urban population, the tempo of urbanization was highest in Africa (4.69), East Asia (4.54) and Latin America (4.42), and comparatively high also in South Asia (4.10), whereas in Europe it was the lowest (1.73). According to absolute rises in the percentage levels of urbanization, the tempo was highest in Latin America (0.78) and the Soviet Union (0.77), followed by East Asia (0.70) and Europe (0.54), whereas it was lowest in Oceania (0.22) and South Asia (0.27). According to rates of rise in those percentage levels, the tempo was highest in East Asia (2.72) and Africa (2.15), considerably lower in Latin America (1.51), and lowest in Oceania (0.34) and Northern America (0.65). Each of these different sets of measurement can have relevance, depending on the manner in which future projections are to be formulated. Their limitations in projections are illustrated in chapter IV.

97. If the United Nations method of projection is used, it is directly relevant to measure the tempo of urbanization as the net difference between the rate of growth in urban population and that in rural population, with the results shown in table 10. According to this measurement, the tempo is highest in East Asia (3.72), followed by Latin America (3.22) and the Soviet Union (3.10), being lowest in Oceania (1.03) and South Asia (1.78). The peculiar result for Oceania is to be attributed to the fact that most of the urban population is that of Australia and New Zealand (where natural increase is moderate),

while much of the rural population is that of Melanesia (where natural increase is high).

TABLE 10. TEMPO OF URBANIZATION AS MEASURED BY THE DIFFERENCE BETWEEN URBAN AND RURAL RATES OF POPULATION GROWTH (ACCORDING TO THE SAME ESTIMATES AS SHOWN IN TABLE 9)

| Major area | Rate of growth in urban population (per cent per year) | Rate of growth in rural population (per cent per year) | Urban rate minus rural rate (per cent per year) |
|------------------------|--------------------------------------------------------------------|--------------------------------------------------------------------|-------------------------------------------------------------|
| East Asia | 4.54 | 0.82 | 3.72 |
| South Asia | 4.10 | 2.32 | 1.78 |
| Europe | 1.73 | -0.50 | 2.23 |
| Soviet Union | 2.71 | -0.39 | 3.10 |
| Africa | 4.69 | 1.94 | 2.75 |
| Northern America | 2.03 | -0.31 | 2.34 |
| Latin America | 4.42 | 1.20 | 3.22 |
| Oceania | 2.43 | 1.40 | 1.03 |

REASONS FOR USE OF THE URBAN-RURAL GROWTH DIFFERENCE AS A MEASURE OF THE TEMPO OF URBANIZATION

98. The measure illustrated in table 10, for simplicity, will be referred to as the urban-rural growth difference, or URGD for short. It has several interesting advantages over the measures illustrated in table 9, especially in its range of applicability. In a wide variety of circumstances, comprising virtually all those which will ever occur, the assumption can be made that an URGD observed in the past may also be maintained for an indefinite future period without leading to absurd results. This remains true irrespective of the current level of urbanization, the rate of growth in total population⁵² or whether rural population is increasing or diminishing.

99. The avoidance of absurd results does not, of course, guarantee that such a projection will be an accurate prediction of the future course of events. No formula of any kind can offer a key to unlock the mystery of future human developments. These are always subject to the influence of unforeseeable changes in social

⁵¹ Annual, i.e. compound interest, rates are used here, for simplicity. For a further explanation, see beginning of chapter IV.

⁵² Assuming, of course, that the total population will not grow forever which, in the final result, eventually becomes impossible.

behaviour, difficult to identify and recognizable only in their consequences. But it can be argued on several grounds that the approach to the analysis of urbanization trends by means of the URGD is reasonable.

100. Let us first examine the three measures of the tempo of urbanization which have been illustrated in table 9.

101. Almost universally urban populations are now growing faster than the corresponding rural populations and therefore faster also than the total populations (urban and rural). If the rate of growth in urban population, higher than that in the total population, remains constant, sooner or later a time will come when the calculated future urban population begins to exceed the calculated future total population, a result which would obviously be absurd. Hence this measure of urbanization can have only limited use.

102. As regards the second measure (absolute rises, in the percentage of urban population), it is likewise evident that a trend, measured in these terms, cannot continue indefinitely, for eventually it would lead to the attainment and surpassing of a level of 100 per cent, which is inherently impossible.

103. As regards the third measure (relative rises in the percentage of urban population), an absurd result would appear at an even earlier date, because then the rises in percentage level would accelerate in proportion to the levels attained at each future moment.

104. Other measures, not illustrated here, can also be imagined. One might measure the tempo of urbanization, for instance, in terms of rural population growth, ordinarily slower than the growth in urban and total population. A constant rate of rural population growth, slower than the total, can in fact continue indefinitely, or at least as long as total growth continues, for it would remain consistent with rising levels of urbanization without ever reducing the rural population to zero or below. A constant amount of decrease in the percentage of rural population, however, would produce an impossible result, namely an eventual percentage of zero or less. On the other hand, no necessary absurdity is involved if the percentage of rural population is permitted to decrease at a constant rate: the absolute size of the rural population may eventually diminish—and this is not contradictory to observations already made in highly urbanized countries—but it would diminish by decreasing amounts without ever completely exhausting a rural residual.

105. Two measures in terms of rural population, therefore, are theoretically compatible with possible developments up to an indefinite future. But it has to be admitted that the measurement of urbanization in terms of rural population change does not suggest a common-sense approach. Except under very special conditions, unlikely to persist, it would be difficult to argue that the tempo of rural population change, by itself, determines how urbanization might progress. It is true, for instance, that in several Latin American countries, with accelerating growth in total population, rural populations in recent decades have grown at comparatively constant rates, whereas urban growth was all the more accelerated. But these observations may merely

reflect a fortuitous coincidence because, as the level of urbanization rises very high, or the acceleration in the growth of total population subsides, rural population growth is bound to slow down. In fact, in nearly all highly urbanized countries the previous growth in rural population has come to an end and has given way to rural population decrease, a possibility not allowed for in an assumption of constant rates of rural population growth.

106. A measure might also be devised relating growth in the urban population to growth in the total population. For instance, there are now many countries, with high as well as low rates of growth in total population, where the rate of growth in urban population is approximately twice that in total population. But again it is evident that such a situation cannot continue indefinitely because an urban population growing faster than the total would eventually come to exceed the total.

107. All the alternative measures of urbanization tempo, discussed in the foregoing, have limitations either in terms of eventually absurd consequences or in terms of common sense. This is not to say that they are useless. In the analysis of past or current observations they may have much illustrative value. They may also serve in simple population projections over limited future periods during which unlikely eventual consequences would not yet occur. The remote future is extremely uncertain, no matter what method of projection may be used, and simplicity of future assumptions is in any event desirable. In many situations, and for limited periods, the use of one of those measures need not be discouraged.

108. By contrast it can be argued that the URGD measure offers at least three types of advantage: the avoidance of absurd consequences, the compatibility with common sense and the consistency with a logistic curve describing the rise of percentage in urban population. These three advantages are now discussed, each in turn.

109. In the most diverse combinations of population trends, indefinite continuation of a given level of URGD never produces an absurd result. While the percentage level of urbanization is low, the rural population increases almost as rapidly as the total population, and the corresponding urban population can increase considerably more rapidly than the total; when the percentage level of urbanization rises high, the urban population increases only slightly more rapidly than the total, and the rural population can increase considerably less rapidly, and possibly decrease. While total population increases at a fast rate, urban population may grow with very great speed, and rural population may also grow considerably; when growth in total population slows down, the growth in urban population may also slow down, while rural population growth may be much reduced, or give way to decrease. Such would be the consequences of an indefinite continuance of a given URGD, in diverse situations; they are at least possible, and also in rough conformity with observations actually made.

110. In terms of common sense, the urbanization process is influenced by comparative advantages and disadvantages perceived in both the urban and the rural environment, often referred to as relative "push" and

“pull”. This makes it reasonable to imagine the usually greater attraction of urban over rural areas as a residual force reflected in the difference of growth rates; this can, of course, fluctuate within short time periods, but on an average is likely to change only very slowly within extended periods of time. A temporary unusually rapid transfer of people from rural to urban areas may produce an increased pressure on urban employment opportunities and other facilities while reducing that on rural facilities, and the opposite may happen in the contrary event. Regulatory mechanisms may then operate to maintain urban-rural growth differences more nearly at constant levels over the long run.

111. From another point of view, it can be argued that there are varying restraints on the rate of rise in the percentage level of urbanization, depending on the level attained at any moment. The level is likely to rise by smaller amounts per unit of time when it is either very low or very high, than when it is in some intermediate range. Rises in level, therefore, are apt to accelerate at first, to reach a maximum rate, and then to slow down again. Perhaps the simplest curve in which such a relationship between given level and rate of rise can be expressed is a logistic curve.

112. An analogy may help to suggest why rises in a growing proportion could generally be expected to be slow at first, then more rapid and eventually again slow. Let us consider the rates at which the percentage of literates in a population (say, aged 10 years and over) may rise. When the level is very low, only few literate persons will be available to teach literacy to others, hence a considerable effort is then involved in raising the general literacy level by any considerable amount. When a substantial proportion of the population, perhaps about one-half, are literate, the remaining illiterates that can be reached are still numerous, but so are also the potentially available teachers, hence, at this level, progress can be quite rapid. At a high level of literacy, most of the remaining illiterates will be those difficult to reach, whether because of their geographic remoteness, cultural resistance or physical and mental handicaps; at such a level, a large effort can secure only a comparatively small amount of additional progress.

113. Similarly the growth of urban residence will depend both on urban facilities already in existence (these would be in the role of “potential teachers”, to use the analogy) and the comparative size of the rural reservoir from which additional urban population can be recruited (in the analogy, the remaining “illiterates”). Though the subject matter is entirely different, varied rates of rise in urbanization can be expected as logically as varied rates of rise in literacy, depending on levels already attained.⁵³

⁵³ The notion that the interaction of “push” and “pull” factors in the process of urbanization may tend to be reflected in the rise of the urbanization level in accordance with a logistic curve has also been discussed in another source, including some empirical evidence. It was noted that other variables, such as school enrolment ratios or labour force participation ratios may also be considered from this point of view. M. Sivamurthy and K. V. Ramachandran, “An empirical investigation into the evolution of certain demographic variables”, *Journal of Social Sciences, Karnatak University (Dharwar)*, vol. IV, April 1968.

114. The relationship between the assumption of a constant URGD and that of a logistic curve in the percentage of urban population can be demonstrated as follows.

The general formula for a logistic curve is

$$Y_o = \frac{1}{k + ab^x},$$

which can also be put in the form

$$Y_o = \frac{k}{1 + e^{a+bx}};$$

in either case k is a constant, and e is the basis of natural logarithms, leaving a and b to be determined.^{53a}

115. Now, let T_o , U_o and R_o be the total urban and rural populations at the beginning of a period ($t = 0$), T_t , U_t and R_t the same populations after t years, u and r the exponential rates of growth in the urban and rural population, and d the URGD, i.e. the difference between the rates, namely $u-r$. Then, at any given moment $U_t = U_o e^{ut}$, and $R_t = R_o e^{rt}$, so that

$$\frac{U_t}{R_t} = \frac{U_t}{T_t - U_t} = \frac{U_o}{R_o} e^{(u-r)t} = \frac{U_o}{R_o} e^{dt}.$$

Therefore,

$$100 \frac{U_t}{T_t} = \frac{100 \frac{U_o}{R_o} e^{dt}}{1 + \frac{U_o}{R_o} e^{dt}},$$

which is a simple formula for the logistic curve. In the

above, of course, $100 \frac{U_t}{T_t}$ is the urbanization level (urban

as a percentage of total population). The formula holds true in the course of time though both u and r may vary with time, provided that the difference $u-r$ remains constant. The formula is further simplified if we take the logistic’s point of origin ($t = 0$) at the point where the urbanization level is 50 per cent. This is the point of inflexion, or of maximum rate of rise in the urbanization level, and about this point the logistic curve is symmetrical.

Here, $U_o = R_o$, and the fraction $\frac{U_o}{R_o}$ is unity. The

formula then becomes

$$100 \frac{U_t}{T_t} = \frac{100 e^{dt}}{1 + e^{dt}},$$

t being positive where the urbanization level is higher than 50 per cent, and negative where it is lower. If it is further assumed that d (the level of URGD, in per cent per year) is unity, the formula is reduced to

$$100 \frac{U_t}{T_t} = \frac{100 e^t}{1 + e^t}$$

Values of this curve ranging from less than one per cent to more than 99 per cent are tabulated in annex I.

^{53a} For methods of computation see Croxton and Cowden, *Applied General Statistics*, 2d. ed. (Englewood Cliffs, N.J., Prentice, Hall Inc.), p. 310.

116. An additional observation is worth recording, making it possible to determine the value of e^{at} or, in other terms, $(1 + \text{URGD})$, without use of the logistic reference table. This can be shown as follows. One of the formulas in paragraph 115 was

$$\frac{U_t}{R_t} = \frac{U_o}{R_o} e^{at},$$

from which it also follows that

$$\frac{\frac{U_t}{T_t}}{\frac{R_t}{T_t}} = \frac{\frac{U_o}{T_o}}{\frac{R_o}{T_o}} e^{at}$$

expressing the same relationship in terms of proportions of the total population urban and rural, instead of absolute numbers urban and rural. By cross-multiplication from both sides of the equation we obtain both that

$$\frac{U_t \cdot R_o}{R_t \cdot U_o} = e^{at},$$

and that

$$\frac{\frac{U_t}{T_t} \cdot \frac{R_o}{T_o}}{\frac{R_t}{T_t} \cdot \frac{U_o}{T_o}} = e^{at}.$$

In either instance the exponential growth-difference rate e^{at} is obtained as the t -th root of the expression to the left side of the equation. This mathematical relationship may in some instances find a useful practical application. The extraction of the root, of course, will usually have to be performed with the use of conventional logarithms.

117. To provide a simple numerical example, let us suppose that a population is exactly 20 per cent urban at the date of one census, and exactly 25 per cent urban at the next census, taken ten years later. In this case

$\frac{U_t}{T_t} = 0.25$, $\frac{R_t}{T_t} = 0.75$, $\frac{R_o}{T_o} = 0.8$, $\frac{U_o}{T_o} = 0.2$ and $t = 10$. We find that, in this case, e^{at} is the 10th root of $\frac{0.25 \times 0.8}{0.75 \times 0.2}$, i.e. the tenth root of $\frac{0.2}{0.15}$, or the tenth root of 1.333333. With the help of logarithms we find that this comes to 1.0292, and we can say that the URGD, as an exponential rate, comes to 2.92 per cent per year.

118. As a final note, it is admitted that the assumption that urbanization may ordinarily proceed according to a logistic curve is arbitrary, and that it commends itself chiefly because of its simplicity of application. If there were more knowledge on the subject one might perhaps postulate, that urbanization tends to progress, say, like a normal probability curve (which resembles the logistic rather closely), or like some asymmetric curve, whether an asymmetric logistic, a modified exponential or a Gompertz curve. Any of these curves, and perhaps some others as well, might be compatible with the observations made so far. But the fact is that our knowledge is not so precise, hence it is advisable to seek the solution to the problem in the simplest terms.

119. Generalizing further from the observations already made, we may also subdivide the urban population into two subpopulations, for instance the population of cities (larger than some minimum size, or a given list of cities) and that of towns (smaller than that size, or all urban localities outside a given list). The percentage of city population in the combined urban population, at any given time, may be called the *level of concentration of the urban population*, and increases (possibly also declines) in that level may be considered as measurable in terms of a *tempo of concentration* (or, in the event of decline, deconcentration). It is evident that similar types of relationships can be assumed to exist between city population and towns population within the combined urban population, as between urban population and rural population within the combined total population.

120. It is evident also that one can subdivide the urban population into that of the country's largest, or capital, city and the remainder of the urban population. Additional steps are possible, making it feasible to measure successively the tempos of concentration for each of a list of cities, for instance beginning with the largest city, then, after eliminating it from the total, proceeding to the next largest city and so forth. One may also deal similarly dividing a single city into segments, for instance distinguishing its core and its suburbs.

121. Where the URGD may serve in the projection of urban, relative to an already projected total, population, an analogous type of measurement can serve likewise in the projection of city, relative to an already projected urban population. If individual cities are taken up successively, this leads to the possibility of projecting their populations individually. The advantage of the URGD type of measurement is again that absurd results can be avoided, and that projections for individual cities remain consistent with a projection for the combined urban population.

122. Where such detailed use is made of this method of measurement it is necessary, of course, to bear in mind how the urban population and that of individual cities are defined, and whether the definitions are comparable in their geographic rigidity or flexibility. It would be unreasonable, for instance, to compare the growth of cities larger than, say, 100,000 inhabitants with the growth of the urban population of a country if the number of such cities is small: the fortuitous attainment of the 100,000 size limit by an additional previously smaller, city is then a discontinuous event, by which such measurement might be easily upset. In a large country having numerous cities of such size, the inclusion of additional cities attaining that size is a fairly continuous process, and there is no incongruity in assuming that an observed tempo of concentration, for such cities within the combined urban population, can continue.

123. On the other hand, the boundaries of some cities may have been widened, while in other cities this was not the case. In such instances it would not be reasonable to assume that cities of the first type will continue to grow faster than cities of the second type. The opposite may

happen. Cities with recently widened boundaries may for some future time experience few territorial additions, whereas in cities where boundary changes have not occurred recently the likelihood of this happening in the near future may be greater. Such considerations have implications for the validity of calculated differential rates of the URGD type. The tempo of concentration

in large cities or in individual cities of the urban population may also fluctuate in time, depending on local developments, for instance government policies affecting the geographic distribution of investments which, because of resulting employment opportunities, may cause the population of some cities to grow faster than that of some others, at least at certain times.