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Population Studies, No. 25

*Manuals on methods of estimating population*

## MANUAL III

# Methods for Population Projections by Sex and Age

UNITED NATIONS

POPULATION STUDIES, No. 25

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UNITED NATIONS

Department of Economic and Social Affairs

New York, 1956

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# FOREWORD

In accordance with the recommendation of the Population Commission at its eighth session (E./CN.9/126), the Population Branch of the United Nations Bureau of Social Affairs has undertaken to prepare a series of manuals on methods of estimating population. Manual I dealt with the methods of estimating total population for current dates. Manual II described the procedures for appraising the quality of basic demographic data.

The present report, which is the third of this series of manuals, deals with methods of calculating future population estimates. Its main objective is to facilitate the making of future population estimates by sex and age for as many countries as possible, including countries where the available statistics are defective. A detailed description of the methods developed by the United Nations staff for the future population estimates published by the United Nations is included. Such estimates for countries in two regions of the world have been published in the series of Population Studies[1] and computations relating to future population in other regions are in process.

[1] *The Population of Central America (including Mexico), 1950-1980,* and *The Population of South America, 1950-1980.* (Population Studies, Nos. 16 and 21.)

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# I. GENERAL CONSIDERATIONS

## A. Purpose and scope of the manual

1. There is little need to emphasize the importance of future population estimates for countries attempting to plan their economic and social development. The primary needs of the people, which the development programmes aim to satisfy, cannot be gauged rationally without regard to the expected size and composition of the population, nor can the national resources be appraised adequately without considering labour, the supply of which depends primarily on population size and structure. If no estimates based on systematic analysis of population trends are available, the planners can only proceed with more or less vague assumptions or notions concerning the magnitude of needs and resources.

2. In spite of their importance, future population projections are rarely attempted in countries that do not have well-developed statistical apparatus furnishing detailed current information on population changes. One reason is that these countries generally also lack statisticians with much training and experience in the analysis of population trends, but there is also a mistaken idea that projections can only be useful if they are based on very detailed and accurate vital statistics and census materials. This idea reflects a misconception of the practical role of estimates as guides to action. Better statistics undoubtedly make better estimates possible, but any estimate, and especially one relating to the future, contains an element of uncertainty which cannot be overcome, no matter how abundant the statistical information on which it is based. A careful appraisal of the available statistics, even though they are fragmentary and defective, can yield future population estimates that have practical utility for planners, just as weather forecasts, fallible as they are, have a practical value for farmers even in areas where the most modern meteorological services have not been developed.

3. The making of future population estimates for countries with deficient statistics has been hindered also by the fact that rather little attention has so far been paid to the development of methods that are suitable for this purpose. The aim of this manual is to help in overcoming this difficulty, primarily by explaining the methods which have been developed by the United Nations staff for calculating future population estimates for various countries of the world, including those where demographic statistics are scanty and imperfect.

4. The estimates prepared by the United Nations staff are being published in a series of reports for the countries in various regions, two of which have been issued to date.[1] The methods employed for these projections have a wide range of usefulness, but many problems arise in their application to any particular country. In this manual, the methods are described in detail and their application under various conditions is illustrated. A specific example is presented and carried through all the successive steps in the making of a population projection.

5. As a preliminary to the detailed description of these methods, to be set forth in the subsequent chapters, a brief summary of the general principles of future population estimates, and alternative methods of computing them, will now be presented.

## B. Principles and methods in the calculation of future population estimates

### 1. *General principles*

6. It should be explained to begin with that not all calculations concerning possible future population trends are intended to provide estimates of the most probable future numbers. It is sometimes useful, for example, to calculate the growth of population that would result from the continuation during a specified future period of the current fertility or mortality rates, or from the admission of a stated number of immigrants; or to determine the rates of fertility, mortality, and migration that would be required to achieve a population of a stated size within a given length of time. Such calculations, made without regard to the probability that the stipulated conditions of fertility, mortality, or migration will actually materialize, may be very helpful in evaluating the merits of various proposals relating to population policy. This manual is not concerned with calculations of this variety, but with efforts to make the best possible estimates, in the light of available information, as to the size and composition of the population to be expected at a given future date.

7. Formerly, many scholars believed that a relatively simple "law" of population growth might be found which would suffice to predict future changes in almost any circumstances. The attempt to find such a formula has now been generally abandoned, since the accumulation of observations has shown that experience generally conflicts with any such theoretical expectations. As recently as the 1930's and early 1940's, it was nevertheless believed that future trends in any given area could be fairly reliably predicted on the basis of a careful study of detailed statistics relating to the current situation and past experience in that area. Unforeseen changes in the birth rates of many countries during the 1940's greatly undermined this confidence, and for a time caused many persons to question the utility of any future population estimates. It is now more generally realized that, while there can be no certainty in estimates of future population trends, they can serve a useful practical purpose by indicating the approximate numbers that appear most likely to be attained, in view of the information available.

[1] *The Population of Central America (including Mexico), 1950-1980*, and *The Population of South America, 1950-1980.* (Population Studies, No. 16 and No. 21.)

8. It is common to present two or more alternative estimates representing "high" and "low" expectations of future population growth. Of course, the purpose of any estimate would be defeated if it were attempted to make the alternatives embrace the entire range of possible future events. For practical purposes, the possibility of catastrophic events, such as a war or natural calamity, or the sudden appearance of important new factors without parallel in the current situation or past experience, has to be neglected, although it must always be recognized that such possibilities exist and that the trend of population growth may consequently be very different from anything which could have been foreseen. Within the framework of present and past experience, it is desirable to choose the upper and lower limits in such a way that the future trend will more probably lie between these limits than outside them. The limits may be revised from time to time in the light of changes being observed in the current situation.

9. There are numerous possible methods of calculating future population estimates in accordance with the principles stated above. Some of these are briefly cited in the following pages. Detailed description of them would be outside the scope of this manual, but a brief discussion will help to make clear in what situations the methods developed by the United Nations staff can be used with advantage.

### 2. *"Mathematical" methods*

10. The simplest method of estimating the future size of a population is to take the number of individuals as determined at a more or less recent date in the past and to apply to it an assumed rate of increase, as a function of time.[2] The rate may be derived from observations on the past growth of the population itself or by analogy with rates observed in other populations in similar circumstances. The calculations can be carried out directly with reference to the net rate of population growth, or the assumed birth rates, death rates, and rates of immigration and emigration may be calculated separately and added to obtain the rate of growth for each future period. Methods of this type are called "mathematical" because emphasis is placed upon the formulation of equations as expressing the rates as functions of time, instead of on particular factors which may influence the trend during any specific time period. However, all population projections embody this same principle at least in some degree. The really distinguishing feature of a "mathematical" projection lies in the fact that calculations are applied to the figure of total population only, rather than to population segments, or relations between the population and its environment. Nevertheless, subsidiary estimating techniques have been devised for dividing "mathematically" projected figures of total population according to sex, age, or other characteristics.

11. Not all "mathematical" projections are simple. Very elaborate methods have been invented for specific purposes. It is often assumed that population growth does not proceed at a constant rate but at varying rates, determined by a curve of regular form. The curve most widely use is the logistic growth curve. This curve represents a trend of growth with increasing annual increments until a point is reached where growth is at a maximum; beyond that point, growth diminishes until it becomes negligible. The logistic and other curves can be computed on the basis of population totals observed at various dates in the past.

12. At one time, certain scholars believed that the logistic curve expressed a universal biological law of growth. In fact, various biological phenomena, including the past growth of several human populations, have been found to follow a trend more or less closely resembling that of the logistic. However, since human behaviour changes constantly as a result of deliberate action, it should not ordinarily be expected that the growth of a human population will conform to a precise mathematical formula. Though no longer accepted as a universal law, the logistic curve is still employed in some population projections, since it usually portrays a trend which is at least within reason. Various adjustments have also been devised for modifying the shape of the logistic curve to achieve greater conformity with observed past trends.

### 3. *"Economic" methods*

13. Population growth can seldom, if ever, be expected to be completely independent of changing economic circumstances. Within limits, mortality and fertility are responsive to economic conditions. The same is true of migration; immigrants are attracted to areas of economic opportunity, while emigrants depart from areas where opportunities are more restricted. Within limits, a government may be able, by means of economic incentives and deterrents, to relate migratory movements and even the natural growth of the population to an economic plan.

14. A projection by "economic" methods is most obviously appropriate for an area subject to easy, unrestricted in-migration and out-migration, where an important new industry is being established which will dominate the economy of the area. The assumed future capacity of this industry to absorb labour is then the primary determinant of population growth. To this amount of labour, a certain multiple must be added to represent dependants, as well as the additional workers in subsidiary industries (trade, service, etc.) who will be required in the expanding community. The "economic" method can likewise be applied to an industrial area of free inward and outward movement where industrial employment opportunities are expected to change greatly. Under certain conditions, anticipated housing development may be regarded as a primary factor of expected population change. Expected changes in income, education or social status may also be listed among the factors influencing future population growth.

### 4. *"Component" projections*

15. The "component" method of population projections is usually understood to consist in the separate projection of numbers of males and females in each age group of the population. Separate projections for each of several ethnic or linguistic groups, of urban and rural

[2] This method is most frequently used in making current, or post-censal, estimates of total population. Its uses are discussed in *Methods of Estimating Total Population for Current Dates* (Manuals on methods of estimating population, No. I), ST/SOA/Ser. A, No. 10, United Nations, New York, 1952.

populations, or any other segments into which the population can be divided, might also be regarded as "component" projections.

16. It is most convenient to project the population by time-intervals equal to the age-intervals into which it has been divided. Thus, if the population is subdivided by five-year groups of ages, the projection can most easily be made for five-year intervals of time: at the end of a five-year period, all surviving members of one age group will have moved into the next subsequent age group.

17. The number of survivors from one date to another is calculated separately for each sex-age group by specific mortality rates selected for this purpose. Hence, the assumptions regarding mortality must be detailed enough to permit the derivation of survival ratios separately for each sex and age. Future mortality may be assumed either to be constant or to conform to some assumed trends. If the projection is made for five-year intervals of time, multiplication of original numbers in each sex-age group with the five-year survival ratios for each of these groups results in estimated numbers of persons five years older at a date which is five years later. Repetition of the procedure yields the estimated population aged ten years more than at the base date which should be expected ten years hence, and so forth.

18. An additional procedure is required to estimate the future numbers of persons not yet born at the base date. Fertility can be defined in several different ways and its probable future trend can be estimated under a variety of assumptions. Usually, the number of children to be born in the future is conceived as a function of the number of women of various ages, but there are situations in which a different relation of births to women appears more pertinent. The number of children surviving to the end of a given time-interval is determined by multiplying the number of births expected during the interval with the appropriate survival ratio.

19. The effects of immigration and emigration can also be calculated, on the assumption of a certain volume of future migration, divided according to sex and age, with the use of appropriate survival ratios and assumptions as regards the fertility of the migrants.

20. Sex-age projections like these can also be made by single years of age and for various intervals of time. The procedure is reversible, permitting an estimation of the population in the past, by sex and age. Many theoretical applications of this method are possible, since it permits calculation of changes in age structure of the population which result from any combination of hypothetical assumptions as regards the initial age structure, mortality, fertility and migration.

21. "Component" projections of a population with respect to characteristics other than sex and age do not require any departure in basic method. The procedures for the projection of any one population segment can very well be identical with those for projecting the combined population, except where transfers and interactions between one segment and another must be taken into account, as in the case of changes of religion on the part of individuals, mixed marriages between ethnic groups, or migration between the rural and the urban habitat.

### 5. *Relevance of various methods to different situations*

22. Each of the three types of methods reviewed involves some basic assumption without which its use would not be justifiable. The implicit conditions might be stated as follows.

(*a*) The use of a "mathematical" method presupposes that the trend of total population growth is fairly regular and that relevant features of the economic and social setting of the future will be the same as those of the past, or will represent the result of a gradual evolution. Consistent trends of change can reasonably be expected in populations with fairly regular age composition where the social and economic environment exerts a steady or only gradually changing influence. "Mathematical" methods are not appropriate for use where the present age structure of the population exhibits marked peculiarities. Moreover, where important and rapid changes in social and economic conditions relevant to population growth can be expected on grounds of current indications or the previous experience of other populations similarly situated, it is not appropriate to use a "mathematical" method that implies the continuation, without modification, of the growth tendencies observed in the past. In such cases, it may be possible to devise a "mathematical" method which takes account of the expected changes in conditions—for example, with a future arithmetic or geometric rate of growth assumed to be higher or lower, in a certain ratio, than the rates observed in a past period. The logistic curve, however, does not readily lend itself to such modifications.

(*b*) The use of an "economic" method is justified where some major economic variable, such as the rise or decline of an industry, can exert an overriding influence on population change, particularly through migration. Birth rates and death rates are only moderately responsive to economic change, since they depend also on a variety of other factors in the cultural and social environment, some of which are statistically imponderable. The "economic" method is most pertinent where migration is sufficiently free of legal barriers or other impediments to become a major element in the trend of population.

(*c*) The "component" method is superior to "mathematical" methods in that it involves a separate analysis of the changes affecting each component of the population.

23. If plausible assumptions can be made with respect to each of a number of population segments, the risk of error in the resulting total is thereby decreased. Furthermore, useful information is then obtained with regard to expected future changes in population composition. Unfortunately, statistical information is often not sufficiently detailed or accurate to permit the formulation of the specific assumptions needed in the projection of each component. Some populations are not large enough to permit the expectation of regularity of trends in each of their components. Accurate and detailed statistics relating to sex-age composition, mortality and fertility of migrants are particularly scarce; this is an impediment to "component" projections, particularly where migration is likely to have substantial effects; trends of migration, moreover, are notorious for their irregularity.

24. It follows that a "component" method is generally preferable to others except where there are important obstacles to its application (inaccuracy or lack of detailed statistics, smallness of the population), or where an economic or other factor in the environment assumes an overriding importance, with its influence exerted primarily through migration. If one or more of these reasons prejudice the use of a "component" projection, then a "mathematical" or "economic" method is to be preferred, depending on the conditions of the particular case. Sometimes a combination of two or more methods yields the best results.

### C. Principles underlying the methods developed by the United Nations staff

25. The methods developed by the United Nations staff have as their main objective to facilitate the making of "component" projections for as many countries as possible, including those where the available statistics are defective in several respects. In the latter instances, the procedures include the use of certain general schemes by which the missing figures can be approximated. Since the projections themselves are in the nature of estimates, they are still justified where a part of their statistical basis is likewise estimated. The latter estimates should, of course, be realistic.

#### 1. *Brief summary of the methods*

26. Two types of procedures are involved in these methods. One set of procedures has been devised for the purpose of estimating those elements in the *current*, or recent, sex-age distribution of the population, or its rates of fertility and mortality, for which the available statistical documentation is not adequate. Another set of procedures consists in the evaluation of probable *future* trends in fertility and mortality and the calculation of their effects on the size and composition of the population.

27. The current structure of a population is the result of past rates of fertility, mortality and migration; these same factors will also determine the future size and structure of the population. The three elements of population change, however, cannot all be evaluated with the same degree of confidence. Comparatively speaking, future mortality trends can be treated with the greatest and future trends in migration with the least assurance, while the degree of confidence with which fertility can be predicted is of an intermediate order. The difficulties of statistical analysis of these three factors also vary in the same order. Mortality is readily represented by death rates according to sex and age,[3] which have usually exhibited fairly stable trends.[4] Birth rates according to age of mother are easy to compute where the data required are available, but are not in all instances the most pertinent indicators of fertility.[5] The volume of a migratory movement and its distribution by sex and age can often vary abruptly; the available statistics of migration are very deficient in many cases; moreover, there is very little knowledge concerning the rates of fertility and mortality among migrants.

28. The numerical importance of the three factors of population change is also different. Disregarding some of the more extreme cases, we find that the birth rates in most populations now lie between 18 and 50 per 1,000 inhabitants; death rates of most countries range between 8 and 25; and there are now few countries where the annual rate of net immigration or emigration even approaches 10 per 1,000 inhabitants. By and large—though not in every single instance—fertility is most important, mortality is second in importance, and migration is the least important factor of population change. Trends in fertility, however, cannot usually be estimated with as much assurance as trends in mortality.

29. Because of these considerations, different principles had to be adopted in the treatment of each of the three demographic factors.

30. The comparative stability observed in mortality trends makes it possible to estimate future mortality rates with the smallest risk of error. Such errors as may be made, moreover, do not have a very great effect on the future population estimates, since the magnitude of mortality is generally of less importance than that of fertility. It was therefore found sufficient to base the estimates of mortality, current and future, on some generalized scheme in which various patterns of mortality rates and changes therein are reflected. This scheme, which has been elaborated on the basis of actual past experience in death rates, constitutes the backbone of the methods, as will be shown later.

31. The greater instability of fertility trends and difficulties in their measurement involve more serious risk of error, especially in view of the fact that fertility is usually the most important element of population change. Some procedures have been developed which permit the computation of a fairly simple yet sensitive index of fertility even in the absence of accurate, detailed statistics. As regards its possible future trend, reasoned judgement must be exercised in each individual instance. Moreover, it is considered necessary to make more than one assumption as to future changes in birth rates in each country, and to compute the effects of plausible "high" and "low" expectations of future fertility.

32. In the case of a population which has not been subjected to major changes in its vital rates or to large changes through migration, the sex-age structure, the rates of mortality and the rates of fertility constitute a system in which a measure of any two factors is sufficient to determine the third. If the rates of mortality and fertility are known, the resulting age structure can be computed. Given a certain age structure, a certain level of mortality corresponds to a certain level of fertility, and *vice versa*. Various factors such as irregularity of past trends, migration, and inaccuracies in the data usually interfere so that the system cannot be reconstructed perfectly. Nevertheless, these relationships often permit the estimation of certain factors given the remaining factors, or fairly reliable estimates of the latter. The scheme of mortality rates already referred

[3] Mortality of different social groups, and according to causes of deaths, though relevant, would introduce an unwarranted degree of refinement for the purposes here contemplated.

[4] A sudden rise or sharp fall in mortality may sometimes occur in connexion with rather extraordinary events. Such events, however, are not within the sphere of projections.

[5] Other relevant indicators are birth rates by duration of marriage and/or by number of children already born to the same woman. The relevance of these and other fertility measures varies in different situations.

to, in combination with statistics of the population by sex and age, often permits a fairly reasonable estimate of the level of fertility.

33. The combination of one assumption as to future mortality trends (derived from the general scheme) with two or more assumptions as to future fertility results in population projections which differ only in respect of those future age groups whose members were still unborn at the base date from which the calculation proceeds. Thus, if the computation departs from the year 1950, alternative projections for the year 1960 are obtained for children less than 10 years old; persons aged 10 years or more, however, are estimated uniformly, irrespective of fertility.

34. Because of the great uncertainty attached to future migration, it is advisable in any case to compute first a population projection in which the possible effects of international movement are not taken into account. Such a projection will suffice for a country in which migration is not expected to have much importance. For other countries, separate treatment of the migration factor is useful because the expected effects of migration can then be singled out and assessed separately from the natural growth of the resident population.

35. Since comparatively little is known concerning the composition and vital rates of migratory contingents, the effects of only one characteristic type of international movements have been worked out in detail for the United Nations estimates. These calculations constitute a model which, with suitable modifications, can be applied in certain analogous situations.

### 2. *Relevance of generalized assumptions to actual situations*

36. It is plausible to assume, where there is no specific indication to the contrary, that the trend of a given demographic factor in a certain instance will correspond to a general pattern which has been observed in other instances. For example, unless there is some reason to suppose otherwise, the mortality rates of a particular population may reasonably be assumed to decline in the future at a rate representing estimated average expectations of mortality decline throughout the world, according to recent experience. This assumption is probably better than one of constant mortality, in view of the almost universal experience of decreasing mortality in modern times. Likewise, where it is not possible to assess specific factors which might affect the trend in fertility, the best that can be done is to roughly evaluate the type of situation that exists in the given country and to assume that future developments will be like those observed elsewhere in similar situations in the past.

37. The procedures suggested here would be misused if they were applied as a rule of thumb in every situation, without regard to such pertinent information as could be obtained with regard to each case. In each instance, before the application of these procedures is considered, all available information relevant to the situation and prospects in the given area should be studied. Very often the result of such a study will be a departure from or modification of the generalized assumptions.

## D. Arrangements of the detailed description of the United Nations methods

38. The calculation of a population projection by the "component" method requires, first, the establishment of base figures for the population, classified by sex and age groups, at the date from which the projection departs. The problem of establishing these base figures is discussed in chapter II. With these figures as the starting point, projections are carried out by means of estimates of current fertility and mortality rates and assumptions as to their future trends. Chapters III, IV, and V deal with the techniques of estimation and assumptions concerning these factors of population change, and chapter VI describes the method of computing a projection when these factors have been established.

39. In most situations, the treatment of the mortality factor involves the use of certain life-table functions. Model life-tables containing these functions are tabulated in appendix A, and the concepts involved are explained in chapter III. Chapter IV deals with the application of these and other functions in estimates and assumptions relevant to the mortality factor. Estimates and assumptions pertaining to fertility are taken up in chapter V.

40. Chapter VII describes how the effects of international migration can be taken into account in a population projection. A model, representing characteristic effects of transoceanic movements, is tabulated in appendix B. Methods for developing an alternative model, when needed, are also described.

41. Chapter VIII deals with efficient modes of presentation and with provisions for quick revision of figures, which should be considered when the results of a projection are being published.

42. Various illustrative examples are presented in the text of each of the chapters. In each case, the examples were selected to illustrate the use of a particular procedure that is pertinent if the statistics or demographic trends are of a certain kind. In addition, in order to give a concrete illustration of the entire series of operations required for an estimate of future population, at the end of each chapter one example is carried through all the stages. For this example, a projection of the population of Costa Rica was chosen. Many other national populations for which at least a certain minimum of statistics exists could reasonably be projected by the methods applied to Costa Rica, though not always by the same detailed procedures.

43. An examination of the statistics of Costa Rica indicates that they are of an intermediate order of detail and quality. The data are not as accurate or abundant as the statistics of some countries of Western Europe, Northern America, Australia and New Zealand. On the other hand, especially since the taking of the 1950 census, statistics on the population of Costa Rica are considerably more advanced than the data for many populations to which the methods described here are also applicable. The example selected may be regarded as one which illustrates the "average" conditions under which the present methods can be used with advantage.

44. The methods described here have been developed to project populations classified by five-year groups of ages over successive five-year intervals of time. Such projections must proceed from an enumeration or estimate of the population, divided according to sex and five-year age groups, for the date which has been selected as the starting point.

45. In some countries, estimates of the population by sex and age are made currently with the help of vital statistics showing the changes since the latest census date. In that case, no problem exists, provided that the estimates are sufficiently reliable and are presented in the form, with regard to age grouping, that is desired for projection. Likewise, there is no problem if a census has been taken quite recently, providing reliable data by sex and age groups in the desired form, and if the census date is a suitable starting point for the projection. This chapter deals with situations in which these ideal conditions are not met.

46. The first case to be considered is that of a country for which population data by sex and age in the desired form, and meeting the required standard of accuracy,[6] are available for a prior date. In this case the problem consists in bringing the data forward to the starting date for the projection. This problem is taken up in part A of the chapter.

47. Part B deals with the methods of adjusting inaccurate data. Such adjustments are indispensable if distortion of the projection is to be avoided. The adjustment can be made either before or after the data have been brought forward to the starting date of the projection. A different kind of adjustment is required if the data are not tabulated according to the desired grouping of ages; procedures for this kind of adjustment are described in part C.

48. Finally, it is necessary to consider the case of a country which lacks suitable data on population by sex and age to form the basis for a projection. In this case, it may nevertheless be possible to make an estimate that will satisfy the purpose. Some methods of dealing with this problem are taken up in part D.

## A. Bringing statistics forward to the starting date for the projection

### 1. *"Pro-rating"*

49. The simplest method to carry forward census data on sex and age to a more recent date is to distribute the population total estimated for the latter date[7] by sex and age in the same proportions as those observed at the census. This is done by determining first the ratio between the total populations at the two dates and then multiplying all the observed numbers in each sex-age category by this ratio.

50. The method is illustrated in table 1 by statistics for Barbados (although, as will be shown later, the application of the method in this case is questionable). A sex-age distribution is available from the census of 9 April 1946, for an enumerated population of 192,800. For seven individuals no age was returned, so that the sex-age data refer to a population of 192,793. If the population projection is to start from mid-year 1950—for which date the total population of Barbados was estimated at 209,239—the required base population can be estimated by multiplying all numbers of the 1946 census by the ratio of 192,793 to 209,239, i.e., by 1.0853.

TABLE 1. SEX-AGE DISTRIBUTION OF POPULATION OF BARBADOS AT 1946 CENSUS "PRO-RATED" TO MID-YEAR 1950

| Ages | *Census, 1946* | | *Estimate, 1950* | |
|---|---|---|---|---|
| | *Males* | *Females* | *Males* | *Females* |
| 0–4.......... | 11,297 | 11,580 | 12,261 | 12,568 |
| 5–9.......... | 10,695 | 10,943 | 11,607 | 11,876 |
| 10–14......... | 9,578 | 9,926 | 10,395 | 10,773 |
| 15–19......... | 8,751 | 9,551 | 9,497 | 10,366 |
| 20–24......... | 7,079 | 8,227 | 7,683 | 8,929 |
| 25–29......... | 6,494 | 7,863 | 7,048 | 8,534 |
| 30–34......... | 6,416 | 7,902 | 6,963 | 8,576 |
| 35–39......... | 6,562 | 7,954 | 7,122 | 8,632 |
| 40–44......... | 5,438 | 6,716 | 5,902 | 7,289 |
| 45–49......... | 3,959 | 5,994 | 4,297 | 6,505 |
| 50–54......... | 2,723 | 4,639 | 2,955 | 5,035 |
| 55–59......... | 1,689 | 3,866 | 1,833 | 4,196 |
| 60–64......... | 1,493 | 3,369 | 1,620 | 3,656 |
| 65–69......... | 1,301 | 3,046 | 1,412 | 3,306 |
| 70–74......... | 1,173 | 2,564 | 1,273 | 2,783 |
| 75–79......... | 665 | 1,638 | 722 | 1,778 |
| 80–84......... | 272 | 809 | 295 | 878 |
| 85 and over.... | 136 | 485 | 148 | 526 |
| TOTAL | 85,721 | 107,072 | 93,033 | 116,206 |
| Age unknown.. | 6 | 1 | — | — |

51. In this example, the number of persons of unknown ages was so small that it could be safely neglected. Methods of adjustment where the numbers of persons of unknown age are important are taken up in part C. If separate estimates of total numbers of males and females are available for the latter date, "pro-rating" should be done for each sex separately.

52. This simple method is quite adequate in almost any situation for a short time interval such as a year or less, unless a high degree of accuracy can be achieved in other steps of the estimating procedure. If the interval between the census and the date of the estimate is longer,

[6] Methods of testing the accuracy of population statistics by sex and age groups are described in: United Nations, *Methods of Appraisal of Quality of Basic Data for Population Estimates* (Manuals on methods of estimating population, No. II), ST/SOA/Series A, Population Studies, No. 23.

[7] Methods of estimating population totals for dates since the latest census are described in: United Nations, *Methods of Estimating Total Population for Current Dates* (Manuals on methods of estimating population, No. I), ST/SOA/Series A, No. 10.

"pro-rating" may still be adequate, provided (*a*) that there were no serious irregularities in the age structure at the time of the census, and (*b*) that no important changes occurred in the birth rate, mortality conditions affecting particular age groups, or the flow of migration during the post-censal intervals.

53. The figures for Barbados provide an example of an irregularity of age structure which is sufficient to distort considerably the "pro-rated" estimate for an interval as long as four years. According to the 1946 census, the numbers of both males and females 35 to 39 years old exceeded the numbers 25 to 29 and 30 to 34 years, possibly because of emigration of young adults during the years prior to the census. On the assumption that the census figures are correct, the "pro-rated" figures for the group 35 to 39 years old in 1950 are presumably excessive, since this group would be made up primarily of the survivors from the relatively small group 30 to 34 years old in 1946. The 1950 figures for persons 30 to 34 years old are also likely to be excessive, and those for persons 40 to 44 years old underestimated. Such errors can sometimes be avoided by making separate estimates for the age groups affected, and "pro-rating" the remainder.

54. Changes in mortality during a fairly short interval between the census date and the date of the estimate will ordinarily have little effect on the age distribution of the population, except that a sharp reduction of infant mortality may substantially increase the proportion of young children. The effect of changes in the birth rate will be limited to the youngest age groups. It is often desirable to make separate estimates, as explained further on, for the numbers of small children, and confine the "prorating" to the higher age groups. The effects of a moderate amount of migration during a relatively short period can often be safely disregarded where the scope and quality of the available data do not justify an attempt at a high standard of accuracy. However, in a situation such as the figures for Barbados imply, migration may be sufficient to invalidate any "pro-rated" estimates for young adults. Unless the previous heavy emigration suggested by the 1946 census results had ceased, not only the residual effect of that migration upon the 1950 age distribution but also the further effects of migration between 1946 and 1950 would have to be taken into account for a valid estimate.

### 2. *Utilization of data on births and deaths by age*

55. If adequate statistics on births and on deaths by age are available, the sex-age data of a census can be brought forward to a more recent date by adding the births and subtracting the deaths during each year from the numbers of persons of corresponding age in that year. This procedure ignores migration, which, if important, would have to be dealt with separately. The calculation can be carried to a great degree of refinement if data by single year of age are used, but it is laborious, and few countries have accurate enough statistics to warrant such meticulous calculation. A procedure utilizing data in five-year groups of ages for five-year intervals of time will ordinarily suffice, only the age group under one year being treated separately.

56. Deaths are tabulated according to the ages of the decedents at death, whereas the classification desired for the present purpose refers to their ages at the beginning of the period for which the calculations are made. Only a part of the deaths occurring in a given five-year age group during a period of five years belongs to the cohort[8] of that age group at the beginning of the period; the remainder is attributable to the next younger cohort. However, with the exception of infants and small children, annual numbers of deaths do not ordinarily change greatly from one five-year age group to the next. Hence, no serious error is incurred in calculations for the groups over five years of age by assigning one-half of the tabulated deaths of the same age group and one-half of those of the next age group to a given cohort. The procedure will presently be illustrated with data for Portugal (see tables 2 and 3).

57. Since infant deaths are always far more numerous than deaths at ages past infancy, the cohort aged initially less than five years, and the five-year cohort born during the period for which the calculation is made, must be dealt with in a different fashion. If high accuracy were desired, the separation of numbers of infant deaths and deaths of small children would have to be calculated with a somewhat complicated formula in which the distribution of infant deaths by month of age is also taken into account.[9] Moderately accurate results, however, can be derived by a much simpler formula which, for the present purpose, is quite adequate.

58. Since more infant deaths occur shortly after birth than during the remainder of the infants' first year of life, a great portion of infant deaths in the first year after the census will occur to infants born during that same year than to infants already born before the date of the census. A rough rule which has been widely employed and is here recommended is to attribute two-thirds of infant deaths occurring in a year to infants born during that year; and one-third to infants born during the preceding year.[10] This rule will have to be applied only with respect to infant deaths occurring during the first year after the census. None of the infants dying in subsequent years could have been born prior to the census.

59. There is always a considerable decrease in age-specific death rates from age 1 to age 4; more deaths occur to children aged 1 and 2 than to children aged 3 and 4. If deaths were uniformly distributed among ages 1-4 (which is never the case), then five-eighths of these deaths would be ascribed to the cohort aged 0-4 at the census and three-eighths to the cohort born in the subsequent 5 years.[11] Experimental calculations have shown, however, that it is usually fairly accurate to assume that

[8] The term "cohort" is used throughout this manual in the technical sense of a group of persons born during the same period (in this case a five-year period), who consequently belong to the same age group at any given time.

[9] See: W. P. D. Logan, "The measurement of infant mortality", *Population Bulletin of the United Nations,* No. 3 (October, 1953); also United Nations, *Foetal, Infant and Early Childhood Mortality, Vol. 1. The Statistics.* (ST/SOA/Series A., No. 13.)

[10] This rule is more accurate where mortality is high than where it is low. Where infant mortality is extremely low, up to 80 per cent of infant deaths occur to infants born during the same year. For the present purpose, however, great accuracy is not needed where mortality is low since the number of deaths involved is then comparatively small.

[11] This would be so because no children born after the census would reach age 1 until the second year following the census.

one-half of deaths occurring at ages 1-4 belong to the cohort initially aged 0-4 and one-half to the cohort born in the course of the five-year period.[12]

60. In the following example, the data of the Portuguese census of 12 December 1940 and of Portuguese vital statistics for the period 1941-1945 are employed. Both the census data and the death statistics include returns for which ages were not reported. Owing to their small numbers (0.3 per cent of the census population and 0.2 per cent of recorded deaths), these are neglected at the present stage of the computation.[13] The

[12] The calculations have shown this to be equally accurate where mortality is high and where it is low. This accuracy, though not perfect, is quite sufficient for the present purposes.

[13] Otherwise, "pro-rating" of the groups of unknown age would be recommended.

TABLE 2. DEATHS IN PORTUGAL, 1941–1945, ATTRIBUTABLE TO COHORTS OF GIVEN AGES ON 31 DECEMBER 1940

| *Sex and age group (years)* | *Deaths reported in this age group* 1941 | 1942 | 1943 | 1944 | 1945 | *Totals, 1941–1945* | *Deaths attributable to cohort initially in this age group*[a] |
|---|---|---|---|---|---|---|---|
| ***Males*** | | | | | | | |
| 0 | 14,934 | 13,482 | 14,339 | 13,535 | 13,191 | 69,481 | 27,999[b] |
| 1– 4 | 9,443 | 8,212 | 7,386 | 7,068 | 6,319 | 38,428 | |
| 5– 9 | 1,631 | 1,749 | 1,572 | 1,405 | 1,257 | 7,614 | 6,155 |
| 10–14 | 956 | 1,061 | 974 | 860 | 845 | 4,696 | 5,864 |
| 15–19 | 1,381 | 1,516 | 1,496 | 1,388 | 1,251 | 7,032 | 7,809 |
| 20–24 | 1,658 | 1,706 | 1,695 | 1,743 | 1,784 | 8,586 | 8,180 |
| 25–29 | 1,648 | 1,556 | 1,618 | 1,448 | 1,503 | 7,773 | 8,004 |
| 30–34 | 1,729 | 1,652 | 1,601 | 1,643 | 1,609 | 8,234 | 8,556 |
| 35–39 | 1,780 | 1,757 | 1,769 | 1,753 | 1,818 | 8,877 | 9,138 |
| 40–44 | 1,849 | 1,808 | 1,853 | 1,915 | 1,974 | 9.399 | 9,788 |
| 45–49 | 2,068 | 1,957 | 1,929 | 2,093 | 2,129 | 10,176 | 10,966 |
| 50–54 | 2,419 | 2,274 | 2,261 | 2,423 | 2,378 | 11,755 | 12,764 |
| 55–59 | 2,778 | 2,720 | 2,601 | 2,870 | 2,804 | 13,773 | 15,686 |
| 60–64 | 3,855 | 3,641 | 3,288 | 3,456 | 3,358 | 17,598 | 19,186 |
| 65–69 | 4,416 | 4,245 | 4,010 | 4,173 | 3,931 | 20,775 | 21,668 |
| 70–74 | 4,903 | 4,586 | 4,275 | 4,529 | 4,268 | 22,561 | 21,879 |
| 75–79 | 4,622 | 4,334 | 4,056 | 4,195 | 3,990 | 21,197 | 18,344 |
| 80–84 | 3,385 | 3,125 | 3,019 | 3,026 | 2,936 | 15,491 | 18,569 |
| 85 and over | 2,433 | 2,136 | 2,004 | 2,088 | 2,162 | 10,823 | |
| ***Females*** | | | | | | | |
| 0 | 12,860 | 11,147 | 11,939 | 11,068 | 10,843 | 57,857 | 25,467[c] |
| 1– 4 | 8,745 | 7,698 | 6,892 | 6,351 | 5,860 | 35,546 | |
| 5– 9 | 1,445 | 1,595 | 1,397 | 1,353 | 1,024 | 6,814 | 5,686 |
| 10–14 | 939 | 1,039 | 929 | 839 | 812 | 4,558 | 5,612 |
| 15–19 | 1,296 | 1,480 | 1,422 | 1,262 | 1,206 | 6,666 | 7,018 |
| 20–24 | 1,438 | 1,479 | 1,483 | 1,475 | 1,495 | 7,370 | 7,080 |
| 25–29 | 1,432 | 1,473 | 1,355 | 1,312 | 1,219 | 6,791 | 6,863 |
| 30–34 | 1,431 | 1,426 | 1,366 | 1,341 | 1,371 | 6,935 | 6,972 |
| 35–39 | 1,497 | 1,457 | 1,481 | 1,301 | 1,272 | 7,008 | 6,960 |
| 40–44 | 1,445 | 1,385 | 1,316 | 1,415 | 1,351 | 6,912 | 6,927 |
| 45–49 | 1,492 | 1,417 | 1,336 | 1,318 | 1,379 | 6,942 | 7,854 |
| 50–54 | 1,860 | 1,878 | 1,774 | 1,572 | 1,681 | 8,765 | 9,644 |
| 55–59 | 2,251 | 2,170 | 2,008 | 2,049 | 2,045 | 10,523 | 12,635 |
| 60–64 | 3,205 | 3,180 | 2,834 | 2,801 | 2,727 | 14,747 | 16,839 |
| 65–69 | 4,041 | 3,960 | 3,771 | 3,631 | 3,528 | 18,931 | 21,383 |
| 70–74 | 5,235 | 5,032 | 4,711 | 4,500 | 4,357 | 23,835 | 24,770 |
| 75–79 | 5,878 | 5,285 | 4,925 | 4,815 | 4,802 | 25,705 | 24,683 |
| 80–84 | 5,217 | 4,937 | 4,626 | 4,485 | 4,396 | 23,661 | 34,927 |
| 85 and over | 5,090 | 4,769 | 4,363 | 4,486 | 4,389 | 23,097 | |

[a] For initial ages 5 and over, half the deaths at the same ages and half those at the next higher ages were attributed to the given cohort. For initial ages 0–4, the deaths attributed to the cohort comprise: one-third of infant deaths (age 0) in the year 1941 only, one-half of all deaths at ages 1–4 and one-half of all deaths at ages 5–9. For those born after 1940, the following deaths have been attributed: all infant deaths except one-third of those occurring in 1941, and one-half of all deaths at ages 1–4.

[b] The balance of 83,717 is attributable to the male cohort born 1941–1945.

[c] The balance of 71,343 is attributable to the female cohort born 1941–1945.

TABLE 3. UTILIZATION OF STATISTICS ON BIRTHS AND DEATHS IN BRINGING PORTUGUESE SEX-AGE DISTRIBUTION FORWARD FROM 1940 TO 1945

| *Age of cohort as of 31 December 1940 (years)* | *Size of cohort enumerated as of 12 December 1940* [a] | *Births, 1941–1945* [b] | *Deaths attributed to cohort, 1941–1945* | *Size of cohort estimated as of 31 December 1945* | *Age of cohort as of 31 December 1945 (years)* |
|---|---|---|---|---|---|
| *Males* | | | | | |
| Unborn | — | 463,256 | 83,717 | 379,539 | 0– 4 |
| 0– 4 | 425,058 | — | 27,999 | 397,059 | 5– 9 |
| 5– 9 | 426,760 | — | 6,155 | 420,605 | 10–14 |
| 10–14 | 408 344 | — | 5 864 | 402,480 | 15–19 |
| 15–19 | 373,088 | — | 7,809 | 365,279 | 20–24 |
| 20–24 | 314,829 | — | 8,180 | 306,649 | 25–29 |
| 25–29 | 297,457 | — | 8,004 | 289,453 | 30–34 |
| 30–34 | 267,017 | — | 8,556 | 258,461 | 35–39 |
| 35–39 | 230,164 | — | 9,138 | 221,026 | 40–44 |
| 40–44 | 194,495 | — | 9,788 | 184,707 | 45–49 |
| 45–49 | 170,842 | — | 10,966 | 159,876 | 50–54 |
| 50–54 | 154,011 | — | 12,764 | 141,247 | 55–59 |
| 55–59 | 125,538 | — | 15,686 | 109,852 | 60–64 |
| 60–64 | 114,063 | — | 19,186 | 94,877 | 65–69 |
| 65–69 | 82,026 | — | 21,668 | 60,358 | 70–74 |
| 70–74 | 57,148 | — | 21,879 | 35,269 | 75–79 |
| 75–79 | 33,848 | — | 18,344 | 15,504 | 80–84 |
| 80–84 | 16,860 | — | 18,569 | 7,836 | 85 and over |
| 85 and over | 9,545 | | | | |
| *Females* | | | | | |
| Unborn | — | 441,188 | 71,343 | 369,845 | 0– 4 |
| 0– 4 | 404,083 | — | 25,467 | 378,616 | 5– 9 |
| 5– 9 | 408,961 | — | 5,686 | 403,275 | 10–14 |
| 10–14 | 395,012 | — | 5,612 | 389,400 | 15–19 |
| 15–19 | 375,394 | — | 7,018 | 368,376 | 20–24 |
| 20–24 | 315,853 | — | 7,080 | 308,773 | 25–29 |
| 25–29 | 311,429 | — | 6,863 | 304,566 | 30–34 |
| 30–34 | 289,619 | — | 6,972 | 282,647 | 35–39 |
| 35–39 | 265,763 | — | 6,960 | 258,803 | 40–44 |
| 40–44 | 227,893 | — | 6,927 | 220,966 | 45–49 |
| 45–49 | 204,339 | — | 7,854 | 196,485 | 50–54 |
| 50–54 | 192,155 | — | 9,644 | 182,511 | 55–59 |
| 55–59 | 159,884 | — | 12,635 | 147,249 | 60–64 |
| 60–64 | 148,265 | — | 16,839 | 131,426 | 65–69 |
| 65–69 | 112,244 | — | 21,383 | 90,861 | 70–74 |
| 70–74 | 82,996 | — | 24,770 | 58,226 | 75–79 |
| 75–79 | 53,126 | — | 24,683 | 28,443 | 80–84 |
| 80–84 | 30,575 | — | 34,927 | 15,572 | 85 and over |
| 85 and over | 19,924 | | | | |

[a] It is assumed for the purpose of the calculation, that the enumerated population as of 12 December 1940 equals the actual population as of 31 December 1940.

[b] There were 904,444 registered births, of which it was assumed that 51.22 per cent were male and 48.78 per cent female.

census having been taken so near the end of the year, the census population is assumed to equal the population at the end of the year as of 31 December 1940.

61. The results of these computations for Portugal would have to be further "pro-rated" to agree with an available estimate of total population for the end of 1945.

62. With modifications, this method is also applicable in different situations. If the date of the census was not so near the end of the year, numbers of births and deaths would have to be calculated for a following five-year period not coinciding with calendar years. Statistics on births and deaths for each month or quarter, if available, may be helpful in this connexion but, except where

high accuracy is attainable, it is sufficient to estimate numbers of births and deaths for fractions of years from the annual figures, by assuming that these numbers are distributed uniformly throughout the year. Though seasonal fluctuations introduce an error in the estimate for the fraction of the first year, this error is mostly compensated by a similar error for the remaining fraction of the last year.

63. If deaths of infants are not distinguished in the statistics from deaths of children aged 1-4, another, though less accurate, rough formula can be used to separate deaths attributable to the cohort initially aged 0-4 years from those to children born during the period. Experimental calculations have shown that, on an average, about four-fifths of all deaths at ages 0-4 occur to children born within the five-year period, and only one-fifth to the initial cohort of small children. The fraction tends to be lower where mortality is high and higher where mortality is low. It is preferable to use the formulas previously mentioned wherever the statistics permit a distinction of infant deaths from deaths at ages 1-4.

64. Where the statistics of the population and of deaths are not sufficiently accurate, or where they are not available in the required five-year age groups, modifications of the data are required before the method can be applied. Some of these modifications will be discussed further on in this chapter. Where an estimate of migration is also needed, methods such as those suggested in chapter VI also have to be used.

### 3. *Interpolation with respect to time*

65. The procedure which has been illustrated with statistics for Portugal serves to bring the census statistics forward through a time-interval of exactly five years. By repetition, the data can also be brought forward by 10 or 15 years. The time-interval between the census and the base date for the projection, however, is not necessarily an exact multiple of five years.

66. As already indicated, "pro-rating" can be used to make the estimates agree with an estimate of total population for some different date. However, if there is a considerable interval between that date and the one to which the detailed estimates refer, "pro-rating" results in some loss of accuracy, Since the procedure which has been described is designed to maintain a fairly high degree of accuracy, "pro-rating" of its results for a time-interval of one year or more is not desirable. It is greatly preferable to interpolate the figures estimated both for the beginning and the end of the five-year period in question, with respect to numbers in each sex-age group. "Pro-rating" implies an assumption that the proportionate numbers in the different sex-age groups remain unchanged; this assumption is not justified unless demographic conditions have been very stable. Interpolation, on the other hand, depends on the assumption that the relevant conditions change in a gradual, regular fashion; under ordinary conditions, this assumption is usually approximately correct and the errors to which it may give rise within a five-year period are usually very slight.

67. Using once more the data for Portugal, let us assume, by way of an example, that an estimate is required for the middle of 1944. This date is 3½ years after the date of the census (end of 1940) and 1½ years before the date of the detailed estimates (end of 1945). Interpolation, therefore, must be carried out in the ratio of 3½ to 1½, or 7 to 3. This is done, as in table 4, by multiplying all estimated figures for 1945 by 7, adding 3 times the census figures for the same age groups, and dividing the sums by 10. The results are then "pro-rated" to the mid-year 1944 official estimate of total population.

TABLE 4. INTERPOLATION OF PORTUGUESE SEX-AGE DISTRIBUTIONS OF END–1940 AND END–1945 TO OBTAIN ESTIMATE FOR MID-1944

| *Sex and age group (years)* | *Enumerated, 12 December 1940* | *Estimated, 31 December 1945* | *Interpolated, mid-1944* | |
|---|---|---|---|---|
| | | | *Not "pro-rated"* | *"Pro-rated" to 1944 estimate of total population* |
| *Males* | | | | |
| 0–4 | 425,058 | 379,539 | 393,195 | 396,878 |
| 5–9 | 426,760 | 397,059 | 405,969 | 409,771 |
| 10–14 | 408,344 | 420,605 | 416,927 | 420,832 |
| 15–19 | 373,088 | 402,480 | 393,662 | 397,349 |
| 20–24 | 314,829 | 365,279 | 350,144 | 353,424 |
| 25–29 | 297,457 | 306,649 | 303,891 | 306,737 |
| 30–34 | 267,017 | 289,453 | 282,722 | 285,370 |
| 35–39 | 230,164 | 258,461 | 249,972 | 252,313 |
| 40–44 | 194,495 | 221,026 | 213,067 | 215,063 |
| 45–49 | 170,842 | 184,707 | 180,548 | 182,239 |
| 50–54 | 154,011 | 159,876 | 158,116 | 159,597 |
| 55–59 | 125,538 | 141,247 | 136,534 | 137,813 |
| 60–64 | 114,063 | 109,852 | 111,115 | 112,156 |
| 65–69 | 82,026 | 94,877 | 91,021 | 91,875 |
| 70–74 | 57,148 | 60,358 | 59,395 | 59,951 |
| 75–79 | 33,848 | 35,269 | 34,843 | 35,169 |
| 80–84 | 16,860 | 15,504 | 15,911 | 16,060 |
| 85 and over | 9,545 | 7,836 | 8,349 | 8,427 |
| TOTAL | | | 3,805,382 | 3,841,024 |
| *Females* | | | | |
| 0–4 | 404,083 | 369,845 | 380,116 | 383,676 |
| 5–9 | 408,961 | 378,616 | 387,720 | 391,351 |
| 10–14 | 395,012 | 403,275 | 400,796 | 404,550 |
| 15–19 | 375,394 | 389,400 | 385,198 | 388,806 |
| 20–24 | 315,853 | 368,376 | 352,619 | 355,922 |
| 25–29 | 311,429 | 308,773 | 309,570 | 312,469 |
| 30–34 | 289,619 | 304,566 | 300,082 | 302,893 |
| 35–39 | 265,763 | 282,647 | 277,582 | 280,182 |
| 40–44 | 227,893 | 258,803 | 249,530 | 251,867 |
| 45–49 | 204,339 | 220,966 | 215,978 | 218,001 |
| 50–54 | 192,155 | 196,485 | 195,186 | 197,014 |
| 55–59 | 159,884 | 182,511 | 175,723 | 177,369 |
| 60–64 | 148,265 | 147,249 | 147,554 | 148,936 |
| 65–69 | 112,244 | 131,426 | 125,671 | 126,848 |
| 70–74 | 82,996 | 90,861 | 88,502 | 89,331 |
| 75–79 | 53,126 | 58,226 | 56,696 | 57,227 |
| 80–84 | 30,575 | 28,443 | 29,083 | 29,355 |
| 85 and over | 19,924 | 15,572 | 16,878 | 17,036 |
| TOTAL | | | 4,094,484 | 4,132,833 |

68. The difference between the total of the interpolated estimates and the independent estimate of total population is partly accounted for, in this instance, by the fact that persons whose ages were not reported in the census were excluded from the computations. Differences may also arise because of the fluctuation of the number of births during the five-year interval, or because of migratory movements. If an examination of the data shows that the latter factors are important, a modified estimate for the age group 0-4 may be substituted and estimates of the effects of migration added or subtracted, before "pro-rating" is done.

69. It is also possible to make a short-range projection in order to move the estimates beyond the date of the most recent vital statistics to a date considered more appropriate as the starting point for long-range projections. For example, if estimates have been made for 1952 by bringing forward the data of a 1947 census, and if the desired starting point is 1955, the 1952 estimates may be projected to 1957 with suitable assumptions—preferably simple extrapolation of trends in the vital rates—and the results interpolated with the 1952 figures to obtain estimates for 1955.

70. The validity of interpolated estimates by five-year age groups depends on the assumption that the population in each group is distributed fairly evenly by single-year ages within the group. (The same assumption is involved where statistics of deaths of five-year age groups are used to bring forward population figures for five-year age groups over an interval of less than five years.) Where this assumption does not hold, for example because of large annual variations in the number of births during the period when a given five-year cohort was born, it may be necessary to make the calculations separately for single-year age groups. Census data classified by single-year age groups are often too unreliable for this purpose, because of errors in age reporting. The distribution of single-year groups within a five-year age group can often be estimated, however, by reference to the numbers of births in the corresponding years.

## B. Adjustment for errors due to inaccurate age reports or faulty enumeration

71. Both census data on the age distribution of population and statistics of deaths by age are subject to errors due to inaccurate reporting of ages and to under-enumeration or under-registration. Those errors which have very little effect on the numbers in the five-year age groups can generally be ignored for the present purpose, even though they may affect substantially the distribution by single-year age groups. In the case of errors which substantially affect one or more of the five-year groups, at least a rough adjustment is necessary if a useful population projection is to be obtained.

72. The problem of errors due to incomplete census enumeration arises most commonly in the case of the age group 0-4. Children of this age are often very incompletely enumerated in the census. In the registration of deaths also, omissions are generally most likely to occur in this same age group. Incomplete enumeration and registration may also affect other age groups, and in some censuses the military population, which consists predominantly of young men, is either excluded or treated separately.

73. If incomplete enumeration of a particular age group must be assumed, unless there are compensating errors in the enumeration of other age groups, it follows that the total population as shown by the census is understated. Nevertheless, it may be desirable to keep the projections in line with the published census total. Otherwise, unwarranted comparisons might be made, leading to confusion. Therefore, if an adjustment is made with respect to some age group, it is usually recommendable to "pro-rate" the resulting figures once more so that they fall in line again with the totals established by the census or current official estimates.

74. In the case of data substantially affected by errors in age reporting, smoothing is indicated. Such errors, both in census enumerations and in the registration of deaths, commonly result in alternating understatement and overstatement of the figures for successive five-year groups, caused by the different attraction of the various terminal digits for statements of age. This has been called the "saw-tooth" effect of age errors.[14] A simple moving average would not suffice to smooth out such irregularities. Nevertheless, for the present purposes, the simplest possible formula is recommended. It cannot be claimed for any graduation formula, no matter how elaborate, that it precisely corrects the errors in the data. Though a more refined formula[15] may produce smoother and apparently more consistent results, it remains doubtful whether a closer approximation to reality is thereby attained.

75. Procedures for smoothing age distributions affected by errors in age statements, and for correcting the effects of under-enumeration or under-registration in certain age groups, are described below, separately with reference to age groups in the range from 10 to 74 years, ages 0-4, 5-9, and 75 and over.

### 1. *Ages from 10 to 74*

76. Theoretically, different formulae should be used to smooth data exhibiting different degrees of inaccuracy, so that fairly accurate statistics would be modified only slightly while less accurate data would be more radically transformed. Further discussion of this question would lead far afield. In the interest of simplicity, only one smoothing formula for age groups in the range from 10 to 74 years is described here. This formula is appropriate for use where the data are markedly inaccurate. It is derived from a simple parabola and has been widely used for relatively simple work.

77. The formula employs five terms; that is, in order to adjust the figure for one five-year age group, data for the two preceding and the two following age groups must also be inserted in the formula. If the age statistics

[14] The characteristic effects of age errors are discussed in further detail in chapter III of *Methods of Appraisal of Quality of Basic Data for Population Estimates*, ST/SOA/Series A, Population Studies, No. 23. See also "Acurracy tests for census age distributions tabulated in five-year and ten-year groups", *Population Bulletin* No. 2, October 1952.

[15] A suitable procedure for more refined graduation of age data, by means of "pivotal values" and "osculatory interpolation" has been very well described by T. N. E. Greville in ***United States Life Tables and Actuarial Tables, 1939-1941***, United States Bureau of the Census, Washington, D.C., 1946.

are tabulated by five-year groups up to age 85, smoothing can be effected by such a formula for all groups between the ages of 10 and 75. The numbers at the youngest and the oldest ages have to be dealt with separately.

The formula may be stated as follows:
$\Sigma = 1/16\,(-S_{-2} + 4\,S_{-1} + 10\,S + 4\,S_1 - S_2)$ where $\Sigma$ is the adjusted number of persons in one five-year group, to be computed, S is the reported number of persons in the same five-year group, $S_{-2}$ and $S_{-1}$ are reported numbers in the two preceding five-year groups, and $S_1$ and $S_2$ are reported numbers in the two subsequent five-year groups.

78. An example of the use of this formula is given in table 5 by its application to the age statistics of the 1950 census of the Dominican Republic. Inspection of the census figures reveals marked irregularities which increase as age advances. In particular, it seems unreasonable that there should have been more females aged 60-64 than 55-59, and more aged 70-74 than 65-69. These and other irregularities are largely—if not entirely—overcome by use of the formula.

TABLE 5. "SMOOTHING" OF SEX-AGE DATA OF THE 1950 CENSUS OF THE DOMINICAN REPUBLIC

| | *Males* | | *Females* | |
|---|---|---|---|---|
| *Age (years)* | *Population according to census** | *Figures adjusted by formula* | *Population according to census* | *Figures adjusted by formula* |
| All ages... | 1,070,065 | — | 1,065,088 | — |
| 0-4...... | 189,383 | — | 186,458 | — |
| 5-9...... | 150,704 | — | 147,061 | — |
| 10-14..... | 141,661 | 133,194 | 135,179 | 133,819 |
| 15-19..... | 101,552 | 110,903 | 124,194 | 124,585 |
| 20-24..... | 105,152 | 97,933 | 109,241 | 106,924 |
| 25-29..... | 77,620 | 79,599 | 79,196 | 80,580 |
| 30-34..... | 59,618 | 62,180 | 60,011 | 61,666 |
| 35-39..... | 60,137 | 57,150 | 55,494 | 53,567 |
| 40-44..... | 47,183 | 48,016 | 42,978 | 42,833 |
| 45-49..... | 36,551 | 37,244 | 30,621 | 32,710 |
| 50-54..... | 30,712 | 29,414 | 28,908 | 25,588 |
| 55-59..... | 21,049 | 22,884 | 14,894 | 18,769 |
| 60-64..... | 19,716 | 17,530 | 18,760 | 15,212 |
| 65-69..... | 9,502 | 11,350 | 8,688 | 11,342 |
| 70-74..... | 8,176 | 7,007 | 9,628 | 7,740 |
| 75-79..... | 3,941 | — | 4,073 | — |
| 80-84..... | 3,697 | — | 4,725 | — |
| 85 and over | 3,711 | — | 4,979 | — |

As applied to females aged 70-74, for instance, the formula works as follows:

| | |
|---|---|
| $-S_{-2}$ .................... | − 18,760 |
| $+4\,S_{-1}$ .................. | + 34,752 |
| $+10\,S$ .................... | + 96,280 |
| $+4\,S_1$ .................... | + 16,292 |
| $-S_2$ ...................... | − 4,725 |
| TOTAL | 123,839 |
| SAME, divided by 16 | 7,740 |

79. Nevertheless, after the adjustment, some irregularity remains. It will be noted that the adjusted figures for females exceed considerably those for males at ages between 15 and 25 and in the group 70 to 74, whereas they are appreciably smaller than corresponding numbers of males between the ages of 35 and 65. A very plausible explanation is the tendency of many women of young and middle ages to understate their age; the resulting error cannot be removed by graduation. There are also probably some tendencies among men to misstate their ages, and there is a bias towards the exaggeration of age among the older persons of either sex. The correction of errors due to these biases would require the reconstruction of each sex-age group on the basis of statistics on births, deaths and migration over a period of almost a century. This is not possible in the case of the Dominican Republic. In fact, few countries possess reliable statistics covering such a long period of time.

80. If inspection of the data justifies a presumption that the age reports for one sex are more accurate than those for the other sex, an adjustment of the more faulty distribution can be made by reference to the more accurate one. The evidence is seldom clear, but occasionally the presumption as to relative accuracy is quite strong. The procedure is somewhat arbitrary and is recommended only if the data for one sex are obviously much more accurate than those for the other sex. This is not necessarily the case in the present example; nevertheless, the method is illustrated with reference to the Dominican statistics in table 6.

TABLE 6. ESTIMATE OF FEMALE POPULATION BY AGE GROUPS ON THE BASIS OF AGE DISTRIBUTION OF MALES, FOR THE DOMINICAN REPUBLIC, 1950

| *Age (years)* | *Adjusted number of males* | *Estimated ratio of females to males* | *Number of females: first estimate* | *Number of females: "pro-rated" estimate* |
|---|---|---|---|---|
| 10-14...... | 133,194 | 1.005 | 133,860 | 130,468 |
| 15-19...... | 110,903 | 1.010 | 112,012 | 109,173 |
| 20-24...... | 97,933 | 1.015 | 99,402 | 96,883 |
| 25-29...... | 79,599 | 1.020 | 81,191 | 79,133 |
| 30-34...... | 62,180 | 1.025 | 63,734 | 62,119 |
| 35-39...... | 57,150 | 1.035 | 59,150 | 57,651 |
| 40-44...... | 48,016 | 1.045 | 50,177 | 48,905 |
| 45-49...... | 37,244 | 1.055 | 39,292 | 38,296 |
| 50-54...... | 29,414 | 1.065 | 31,326 | 30,532 |
| 55-59...... | 22,884 | 1.075 | 24,600 | 23,977 |
| 60-64...... | 17,530 | 1.085 | 19,020 | 18,538 |
| 65-69...... | 11,350 | 1.095 | 12,428 | 12,113 |
| 70-74...... | 7,007 | 1.105 | 7,743 | 7,547 |
| TOTAL | | | 733,935 | 715,335 |

The adjusted figures for ages 10-14 (see table 5) indicate a ratio of females to males of about 1.005; those for ages 70-74 give a ratio of 1.105. According to the experience of most life-tables, the ratio of females to males tends to rise very gradually with age, slowly at young ages and more rapidly at old ages. A smooth series of hypothetical ratios can be inserted,[16] rising from 1.005 at ages 10-14 to 1.105 at ages 70-74. The numbers of males, as adjusted by the smoothing formula, are then multiplied by these ratios to obtain first estimates of corresponding numbers of females of the same ages. Initially, the adjusted figures for females between ages 10 and 75 totalled 715,335, whereas the newly computed numbers of females total 733,935. "Pro-rating" is used to bring the total back to 715,335.

81. When deciding whether to smooth census age data, one should always recall that no formula has the power of correcting errors precisely. All that any formula can do is to eliminate irregularities and put the statistics in a plausible form. Irregularities may, however, be the result of actual past fluctuations or discontinuities in demographic trends as well as of errors in the data. Therefore, smoothing should not be done if the statistics are fairly accurate: in such a case, a smoothing formula, by eliminating irregularities in population composition which the data truly reflect, would introduce greater errors than those which it corrects. If the data are inaccurate but past demographic trends are known to have been irregular, it may be appropriate to smooth the figures for some, but not all, age groups.

82. A situation of the latter type was encountered in preparing a population projection for Mexico.[17] The census age data for 1940 were found to be affected by irregularities, evidently due to inaccurate age reporting, similar to those observed in the Dominican Republic. In addition, however, the numbers of both sexes reported at ages 20-24 and 25-29 were considerably smaller than might have been expected. A plausible explanation for this circumstance was found in the Mexican Civil War, which lasted from 1910 to 1921 and severely disrupted life in various areas of the country at various times; as a result, birth rates during that period may have been abnormally low.[18]

83. Before applying the smoothing formula to the Mexican census data, therefore, it was necessary to insert, somewhat arbitrarily, such numbers for age groups 20-24 and 25-29 as might have survived to 1940 if birth rates during the Civil War period had remained fairly constant. The numbers inserted were obtained by linear interpolation between reported numbers at ages 15-19 and 30-34. The smoothing formula was then applied to obtain adjusted estimates for ages between 10 and 20, and between 30 and 75. For ages 20-24 and 25-29, the numbers originally reported were retained in the adjusted age distribution, on the assumption that they probably did not stray very much from the true numbers.

84. The smoothing formula is less well suited to statistics of deaths by age, in view of the large numbers of deaths in infancy and early childhood, followed by comparatively few deaths during later childhood and adolescence. With respect to deaths at ages 20 and over, however, the smoothing formula gives fairly satisfactory results.

### 2. *Ages 0-4*

85. As already stated, children under five years of age are incompletely enumerated in the censuses of many countries. Where the census figure for this age group is found to be seriously deficient, it should be replaced by an independent estimate. The deficiency is sometimes found to be important only in the case of infants under one year of age; if so, the independent estimate may be limited to these infants, and combined with the census figure for ages 1-4.

86. The method of deriving an estimate of the number of children under five years of age from the number of births during the preceding five-year time period, less the deaths among these children, has been described in part A of this chapter. This method depends upon the availability of accurate vital statistics. A modification of the method is necessary where nearly accurate statistics of births are available but the statistics of infant deaths are deficient. It must be noted in this connexion that infant deaths are commonly less completely registered than deaths in other age groups. The modification consists in substitution for the registered numbers of infant deaths, of an estimate based on assumed mortality conditions. Needless to say, the assumption as to mortality used for this purpose should correspond to the assumptions concerning this factor of population change to be made in later steps of the projection.

87. Where sufficiently reliable statistics of births also are not available, the number of births, too, may be estimated by means of appropriate assumptions. The relevant methods are discussed in subsequent chapters of this manual. Even though the estimate of the population 0-4 years of age obtained by such methods is very uncertain, it may still be much to be preferred over the census figure.

### 3. *Ages 5-9*

88. The numbers of children aged 5-9 as reported in the census can ordinarily be accepted, unless there are special reasons to doubt their accuracy. Examination of census data for many countries yields little evidence that numbers aged 5-9 years are subject to any special inaccuracy of reporting; in fact, even where the age data are generally very defective, the figures for this age group commonly appear to be more accurate than others. Experience has demonstrated that children of this age are not nearly as likely to be overlooked in a census enumeration as younger children. It is easier to obtain from their parents at least a nearly accurate estimate of age for children in this group than to get accurate reports on older groups. Any gross mis-statement of the age of a child in this group would ordinarily be apparent if the child were in sight, and there is usually no such obvious motive for mis-stating the age as may apply, for example, to a youth just under the legal age for employment or for marriage.

[16] This rough series suffices at the present stage; eventual "pro-rating" of the resulting estimates will result in a somewhat different series of ratios.

[17] See *The Population of Central America (including Mexico), 1950-1980,* ST/SOA/Ser. A, Population Studies No. 16. At the time when the calculations were made, the age statistics of the Mexican census of 1950 had not yet become available; hence, data of the 1940 census had to be used. Later, when the detailed statistics for 1950 were received, they were compared with the estimates derived from the data for 1940 and found to be quite consistent. The discrepancies between 1950 census data and estimates based on 1940 census data could be quite satisfactorily explained in terms of inaccurate age reporting at the 1950 census.

[18] Vital statistics of this disturbed period have, unfortunately, not become available.

89. For the development of certain estimates, especially those pertaining to fertility, it is often indispensable to assume that at least the data for one age group have relatively high accuracy, so that other quantities can be estimated by reference to these data. In such cases, it may be well to choose the figures for the 5-9 group.

90. In cases of extreme inaccuracy of age statistics, however, the accuracy of data for the 5-9 group is also subject to considerable doubt. In such cases, the reports at all age levels are usually strongly affected by attraction of even numbers (e.g., ages ending in 6, 8 and 10), and by the fact that the question relating to age in "completed years" is often misunderstood as referring to age "at the nearest (possibly the next-following) birthday". Allowance can be made for these influences by using as an estimate for the 5-9 group, the sum of one-half the numbers reported at ages 5 and 10, plus the reported numbers at ages 6, 7, 8 and 9.

### 4. *Ages 75 and over*

91. Owing to the tendency of older persons to exaggerate their ages, the numbers of persons reported as aged 75 and over are excessive in most censuses and greatly in excess in some. To estimate the correct numbers for the advanced ages would in any case be difficult, whether by extrapolation or by reconstruction from birth and death statistics pertaining to a distant period of the past.

92. For the purpose of a population projection, errors in numbers at the most advanced ages are only of small importance, because the numbers in these cohorts are rapidly depleted by deaths. In the population projections so far undertaken by the United Nations staff, no adjustments have been attempted with respect to persons aged 75 years or more though their numbers were admittedly quite inaccurate in most instances. However, in cases where the data are patently inaccurate, more reasonable figures may be substituted with the aid of observations derived from stationary populations.[19]

93. Table 7 shows, for a model stationary population, the percentages in each of the advanced age groups corresponding to a given percentage of total population at ages 70 and over. Given a certain percentage aged 70-74, corresponding percentages aged 75-79, 80-84 and 85 and over can be derived by interpolation.

94. The Dominican Republic census showed 1,070,742 males and 1,065,130 females, and our adjusted estimates for the age group 70-74 were 7,007 males, i.e., 0.654 per cent of all males, and 7,743 females, i.e., 0.73 per cent of all females. In accordance with the table, only about 0.3 per cent of males should be aged 75-79, 0.1 per cent 80-84, and 0.01 per cent aged 85 and over; of females, about 0.34 per cent should be aged 75-79, 0.11 per cent 80-84, and 0.02 per cent 85 and over. Accordingly, the following figures for these age groups may be substituted for the reported numbers:

| Age (years) | Males | | Females | |
|---|---|---|---|---|
| | *Reported* | *Estimated* | *Reported* | *Estimated* |
| 70-74........ | 8,176 | 7,007 | 9,628 | 7,743 |
| 75-79........ | 3,941 | 3,210 | 4,073 | 3,621 |
| 80-84........ | 3,697 | 1,070 | 4,725 | 1,172 |
| 85 and over... | 3,711 | 107 | 4,979 | 213 |

TABLE 7. PERCENTAGES OF PERSONS AT ADVANCED AGES IN STATIONARY POPULATIONS[a]

| *Percentage of total population aged 70 and over* | *Estimated percentage of total population in specified age group* | | | | |
|---|---|---|---|---|---|
| | *70–74* | *75–79* | *80–84* | *85 and over* | *75 and over* |
| 1.0................ | 0.62 | 0.28 | 0.09 | 0.01 | 0.38 |
| 1.5................ | 0.90 | 0.43 | 0.14 | 0.03 | 0.60 |
| 2.0................ | 1.16 | 0.58 | 0.21 | 0.05 | 0.84 |
| 2.5................ | 1.41 | 0.73 | 0.29 | 0.07 | 1.09 |
| 3.0................ | 1.64 | 0.89 | 0.37 | 0.10 | 1.36 |
| 3.5................ | 1.86 | 1.05 | 0.45 | 0.14 | 1.64 |
| 4.0................ | 2.08 | 1.20 | 0.54 | 0.18 | 1.92 |
| 4.5................ | 2.28 | 1.36 | 0.63 | 0.23 | 2.22 |
| 5.0................ | 2.48 | 1.51 | 0.73 | 0.28 | 2.52 |

[a] These stationary populations are interpolated from the stationary populations corresponding to the model life tables used for the projections (see following chapter and appendix).

Numbers at advanced ages, though estimated only very roughly, are evidently much smaller than reported numbers.

### 5. *Final adjustment of estimates*

95. The various adjustments by procedures described in this section result in figures by age groups which, if added up, do not agree with the initial figure of total population. Since it is usually expedient to maintain consistency with published figures on total population, "pro-rating" is indicated.

96. It is unnecessary to "pro-rate" after each stage of the calculations since "pro-rating" at the final stage usually has practically the same effect. It is usually sufficient to "pro-rate" only once, after the last adjustment has been carried out.

[19] Though actual populations are not identical with stationary ones, the percentages of total population at the most advanced ages in fact often conform reasonably well with those of stationary populations. The assumption is admittedly imperfect but suffices to effect a considerable improvement on inaccurate data. Better approximations may sometimes be obtained by means of stable population models.

## C. Adjustment of data not presented by conventional five-year age groups

97. For the purpose of the methods of projection discussed here, it is necessary that the population be classified by five-year age groups up to the age of 85. If the statistics are not presented in this form, adjustments are required.

98. In many cases, although a suitable classification by age for most of the population is given, the ages of some individuals remain unreported. Statistics are sometimes presented only by ten-year age groups for part or all of the age scale. Furthermore, the age groups may be irregular and may not fit into the required five-year, or even ten-year groups. Finally, in some cases only a few broad age groups are distinguished and these are sometimes identified only by vague criteria such as "infants", "children", "adults", etc. In the first three instances, adjustments are possible as described in the following pages. The last case can be dealt with by special methods which will be explained in another publication.

### 1. *Persons whose ages were not reported*

99. The persons whose ages are not reported in a census are usually only a small minority. This does not imply that the ages of other persons, which were reported, were always well ascertained. "Pro-rating" of the population of reported ages to total population is usually quite sufficient. The "pro-rating" should be done for each sex separately.

100. In some censuses ages are not ascertained or not tabulated in detail in respect of a certain segment of the population; for example, aboriginal tribes, other minorities, or even majority groups, the enumeration of which involves special difficulties. If the groups not classified by age are very small, no problem is presented; "pro-rating" or any reasonable estimate of the age distribution of these groups will be adequate. Where they represent a substantial fraction of the population, care must be taken to make the best possible estimates for them. It cannot ordinarily be assumed that such a population segment resembles the remainder of the population in its age structure. Estimates may be made by such methods as are described in part D of this chapter, for use where adequate statistics on age compositions are not available.

### 2. *The halving of ten-year age groups*

101. To reduce data presented by ten-year age groups to the required five-year groups, Newton's formula can be used:

$$f_{na} = 1/2\ [f_n + 1/8\ (f_{n-1} - f_{n+1})]$$

where $f_{na}$ is the number in the first half of the given 10-year group, to be computed, $f_n$ is the given number in the entire 10-year group, and $f_{n-1}$ and $f_{n+1}$ are numbers in the preceding and following 10-year groups.

102. As an example in the application of this formula, we may consider the official mid-year 1953 estimate of the Pacific Islands Trust Territory, which was given by sex and age, as follows:

| *Age (years)* | *Population* Male | *Population* Female |
|---|---|---|
| Total | 25,956 | 24,929 |
| 0–4 | 4,640 | 4,310 |
| 5–9 | 2,245 | 2,148 |
| 10–14 | 2,093 | 1,978 |
| 15–19 | 2,015 | 2,037 |
| 20–24 | 2,025 | 2,026 |
| 25–34 | 3,233 | 3,124 |
| 35–44 | 2,651 | 2,637 |
| 45–54 | 2,209 | 2,155 |
| 55–64 | 1,758 | 1,605 |
| 65–74 | 1,126 | 1,024 |
| 75 and over | 606 | 521 |
| Age unknown | 1,355 | 1,364 |

103. For the number of males aged 25-29, Newton's formula gives:

$$f_{na} = 1/2\ [3{,}233 + 1/8\ (4{,}040 - 2{,}651] = 1{,}703.$$

The number of males aged 30-34 is obtained by subtracting 1,703 from 3,233, i.e., 1,530. This method makes it possible to construct the estimates for five-year age groups up to age 64. To halve the 65-74 group, estimates for age groups over 75 years of age are required. These can be obtained by reference to table 7 above. In this example, 2.46 per cent of males and 2.21 per cent of females of known age are aged 75 years and over. By interpolation, about 1.48 per cent of males should be aged 75-79, 0.71 per cent 80-84, and 0.27 per cent 85 and over; 1.36 per cent of females should be aged 75-79, 0.63 per cent 80-84, and 0.23 per cent 85 and over. The corresponding numbers are:

| *Age (years)* | *Males* | *Females* |
|---|---|---|
| 75–79 | 364 | 320 |
| 80–84 | 175 | 148 |
| 85 and over | 66 | 54 |

104. It is now possible to enter an estimate for the 75-84 group in Newton's formula and thus to halve the 65-74 group. The following results are obtained for the five-year age groups from 25 to 74:[20]

| *Age (years)* | *Males* | *Females* |
|---|---|---|
| 25–29 | 1,703 | 1,651 |
| 30–34 | 1,530 | 1,473 |
| 35–39 | 1,390 | 1,379 |
| 40–44 | 1,261 | 1,258 |
| 45–49 | 1,160 | 1,142 |
| 50–54 | 1,049 | 1,013 |
| 55–59 | 947 | 873 |
| 60–64 | 811 | 732 |
| 65–69 | 639 | 583 |
| 70–74 | 487 | 441 |

[20] In the application of formulas using several successive age groups, one must bear in mind that such formulas cannot be used where there have been past irregularities in the annual numbers of births. The number of the people affected by these irregularities have to be replaced by proper numbers before applying the formulas. See for example what is said about the population of Mexico in part B of this chapter.

105. In view of the large number of persons whose ages were not reported, it is important in this case to take account of any clues which may be available concerning possible peculiarities of the age distribution of this group. In case it can be assumed that their age distribution was similar to that of the remainder of the population, the estimates for all age groups should be "pro-rated" to the total numbers of each sex.

### 3. *Interpolation for ages tabulated by irregular groupings*

106. A very rough method for dealing with statistics by age in irregular groups which require transformation into conventional five-year groups comprises the following steps. First, the number in each group is multiplied by a factor to make it equal roughly the number that would be found in a five-year group having the same central age. The numbers resulting from this calculation are interpolated, with reference to the central ages of adjacent groups, to obtain the numbers which may be expected in five-year groups centring on exact ages 2½, 7½, 12½, 17½, etc., which are central in the conventional grouping.

107. This method is rough because it must be assumed that ages are distributed, by single years, in a linear form. Such perfect distribution of ages cannot be assumed with regard to the entire age scale but may, in fact, be approximated within relatively short ranges of ages if the birth rates have not fluctuated greatly in the past. This procedure also tends to reduce somewhat the errors in the original statistics, though it does not eliminate them.

108. The following statistics of the 1930 census of El Salvador may be taken as an illustration:

| *Age (years)* | *Enumerated population* | | *Exact central age (years)* | *Width of age group (years)* |
|---|---|---|---|---|
| | *Male* | *Female* | | |
| 0-1 | 39,510 | 38,649 | 1 | 2 |
| 2-4 | 64,533 | 64,049 | 3 | 3 |
| 5-7 | 62,125 | 60,361 | 6.5 | 3 |
| 8-14 | 129,666 | 118,109 | 11.5 | 7 |
| 15-17 | 40,057 | 44,959 | 16.5 | 3 |
| 18-22 | 78,347 | 85,107 | 20.5 | 5 |
| 23-29 | 80,994 | 85,933 | 26.5 | 7 |
| 30-39 | 96,327 | 93,515 | 35 | 10 |

109. The age group 0-4 can be obtained directly by addition. It is not necessary to interpolate to obtain age group 35-39, since this will be obtained by subtraction from the total for 30-39, once the 30-34 group has been calculated.

110. Estimates for five-year age groups, having the same central ages as the given groups, are obtained by multiplying the figures of each given group by five and dividing by the number of years comprised in the original group. Five times the numbers in age groups 2-4, 5-7, and 15-17 are divided by three, and five times the numbers in age groups 8-14 and 23-29 are divided by seven. No multiplication is needed for the 18-22 group, which is already a five-year group. The numbers in the 30-39 group are divided by two to obtain an estimate of a five-year group centring around the exact age of 35 years. The results of the multiplications are as follows:

| *Central age (years)* | *Estimated number in corresponding five-year group* | |
|---|---|---|
| | *Males* | *Females* |
| 3.5 | 107,555 | 106,748 |
| 6.5 | 103,542 | 100,602 |
| 11.5 | 92,619 | 84,364 |
| 16.5 | 66,762 | 74,932 |
| 20.5 | 78,347 | 85,107 |
| 26.5 | 57,853 | 61,381 |
| 35 | 48,164 | 46,758 |

111. To interpolate these figures with respect to the central ages of conventional five-year groups, note must be taken of the distances, in years, of the latter central ages from the central ages in the five-year groups listed above. The interpolation is then carried out by multiplying each of the above figures in inverse proportion to its distance from the required central age. The procedure can be illustrated, as follows, with respect to males aged 5-9. The exact central age of this group is 7.5 years; that is, one year more than 6.5 and 4 years less than 11.5. The figure relating to central age 6.5 is multiplied by four-fifths, and that relating to central age 11.5 by one-fifth, and the two results are added. The result is 101,358, that is, the required estimate for the age group 5-9. When the other interpolations have been made by the same method, the distribution is obtained:

| *Age group (years)* | *Central age (years)* | *Estimated population* | |
|---|---|---|---|
| | | *Male* | *Female* |
| 0-4 | 2.5 | 104,043 | 102,698 |
| 5-9 | 7.5 | 101,358 | 97,355 |
| 10-14 | 12.5 | 87,447 | 82,477 |
| 15-19 | 17.5 | 69,658 | 77,476 |
| 20-24 | 22.5 | 71,516 | 77,198 |
| 25-29 | 27.5 | 56,713 | 59,661 |
| 30-34 | 32.5 | 51,014 | 51,059 |
| 35-39 | 37.5 | 45,313 | 42,456 |

112. The results are not entirely satisfactory. In particular, there appear to be relatively too few females aged 10-14 and too few males aged 15-19. This is probably a reflection of inaccuracies in the original data. Before using these results, it might be desirable to apply some of the corrections and adjustments discussed in part B of this chapter.

## D. Estimation of age distribution of a population for which statistics are not available

113. A different order of estimation is required to obtain the basis for population projections where no census has been taken, or the latest census figures are too old to be brought forward to a suitable starting point, or where the census provides no age classification, or a classification that is too inaccurate or too general in its groupings to serve as a basis for the estimates. In such situations any estimate of the age distribution will necessarily be questionable, but estimates may nevertheless be made, in many such cases, with sufficient accuracy to provide the basis for usable projections.

### 1. *Analogy with another similar population*

114. If the statistics of country A are insufficient for an estimate of the population by sex and age, while those for country B are adequate, and if it can be assumed that population trends in both countries have been

similar, the age-distribution of country B can be "pro-rated" to the population total of country A. Such a procedure is not justified, of course, unless there is firm ground for believing that the two age structures in fact resemble each other closely. An analogous procedure can be used with more assurance if sex-age data are available for a part of the country which can be presumed to be fairly representative of the whole; in that case, the statistics for the given part are "pro-rated" to an estimate of the country's total population.

115. An estimate of sex-age structure can also be arrived at if the population trends of country B are similar in some but not all respects to those of country A. An example is afforded by the population projection for Uruguay made by the United Nations staff.[21]

116. The most recent official sex-age statistics for Uruguay which could be found were those of the census of 1908, but a base estimate was required for the year 1950. Although fairly reliable vital statistics were available for most of the intervening years, it was not considered possible to construct a realistic sex-age estimate for 1950 by carrying the 1908 data forward over a period of 42 years, especially since migration—for which statistics were not adequate—had been of some importance.

117. Comparison of Uruguayan and Argentine statistics for the half-century since 1900 brought out the following facts:

(*a*) Birth rates were always appreciably lower in Uruguay than in Argentina; in fact, in most years the birth rate of Uruguay was very nearly five-sixths of that of Argentina, despite considerable changes in the rates of both countries.

(*b*) Death rates in Uruguay were at nearly all times very slightly lower than those in Argentina. The rates in both countries declined fairly regularly and about equally rapidly, those of Argentina apparently following the Uruguayan rates with a time-lag of a few years.

(*c*) International migration appeared to have affected the population of Uruguay in approximately the same proportions as that of Argentina, except that in the last decade before 1950 the net effect of migration in Uruguay seemed to have been negligible. The migration statistics were not very accurate but an error in the assumption of equal proportionate effects on the two populations would presumably introduce only a comparatively small error in the resulting population estimates.

118. In Argentina, censuses were taken in 1914 and in 1947. In view of the slight time-lag in the respective mortality rates, and the similarity of the effects of migration, the combined effects of deaths and migration in Argentina, from 1914 to 1947, were regarded as comparable to those on the population of Uruguay in a period of equal length (33 years) beginning with the census of 1908 and ending in the year 1941. Since the 33-year period is not a multiple of five years, adjustments, by interpolation between adjacent age groups, were carried out on the five-year age data of the Argentine (1914) and Uruguayan (1908) censuses so that the age groups would correspond to those of cohorts whose ages would fall within the conventional five-year groups by the end of the 33-year period.

119. Comparison of identical cohorts in the Argentine censuses of 1914 and 1947 permitted the computation of "survival-*cum*-migration ratios" (i.e., the ratio of survival, as modified by the effects of immigration and emigration) over the 33-year period for persons aged 35 years and over in 1947. "Survival-*cum*-migration ratios" were also computed, for periods of varying length, for five-year cohorts of births which occurred from 1917 to 1947, aged under 30 years in 1947. One cohort was constituted partly from small children at the 1914 census and partly from births during 1914-1917 (aged 30-34 years in 1947), and the same type of ratio was computed for this cohort. The ratios were then applied to the corresponding Uruguayan census data of 1908 and birth statistics of 1908-1941 to obtain a first estimate of the Uruguayan population, by sex and age, in 1941, which was further "pro-rated" to accord with the official estimate of total population for the same year.

120. This estimate for Uruguay in 1941 was then brought forward, first to 1946, and then to 1951, by means of Uruguayan statistics on births and on deaths by sex and age, with suitable interpolations and extrapolations for those years for which the requisite vital statistics had not yet become available. Interpolation between the resulting estimates for 1946 and 1951 yielded the required base estimate for the projection of the population from 1950 onward.

121. It is recognized that this estimating procedure may have involved errors and that high accuracy could not be claimed for the results. Nevertheless, the approximation is believed to be reasonable and not without value, no other estimate being available for any date since 1908.

122. The same degree of similarity in population trends might perhaps not be found between any other two countries, but the example cited here may be helpful in suggesting devices by which estimates can be derived in different situations.

### 2. *The method of stable populations*

123. An approximate sex-age distribution can also be constructed for a population for which a few demographic indices are known, by applying the theory of stable populations. As Lotka has demonstrated,[22] a population with constant birth rates and constant mortality rates will exhibit constant proportions of numbers in the different sex-age groups, which are determined by the birth and mortality rates. It has been further demonstrated[23] that changes in mortality, unless they are very radical, have only slight effects on this stable age structure. Hence, an estimate of the birth rate, combined with a very rough estimate of the death rate, suffices to construct an approximate age distribution if it can be assumed that the birth rate has been nearly constant for a long period. In this connexion, the model life tables described in chapter III are very helpful. The use of stable populations will be described more fully in a subsequent publication.

[21] United Nations, *The Population of South America, 1950-1980.*

[22] Alfred T. Lotka, *Théorie analytique des associations biologiques*, Paris, Hermann, 1934-1939.

[23] *Population Bulletin of the United Nations*, No. 4, December 1954. "The cause of ageing of populations: declining mortality or declining fertility?"

## E. The example of Costa Rica

124. A population projection for Costa Rica was made taking the year 1950, when the last census was taken, as a starting point.[24] Since the time when that projection was made, more recent information has become available. The latest official estimate of total population now at hand relates to mid-year 1955 and amounts to 951,093. This figure can now be taken as the basis for a new projection, with 1955 as the starting date. This recent official estimate is well in line with the results of earlier projection, according to which, assuming constant high fertility, the population would have increased from 804,900 individuals in mid-year 1950 to 945,900 in mid-year 1955. A slight revision of the earlier projection, as illustrated in chapter VII, might suffice. Nevertheless, in order to illustrate the procedures discussed in this chapter, the derivation of base estimates for 1955, which would be the starting point for a new projection, is explained here.

125. In bringing the census statistics forward from 1950 to 1955, the procedure utilizing data on births and on deaths by age might be employed. The interval is five years and the required statistics are available. A study of Costa Rican statistics,[25] however, has shown that while death registration is presumably fairly accurate, the accuracy of birth statistics is affected by frequent delays in the registration of births. The latter finding is also reflected in the series of annual recorded birth rates. The actual birth rates were probably fairly constant up to 1950, as witnessed by the following five-year averages:

| | |
|---|---|
| 1925-29 | 46.2 |
| 1930-34 | 44.6 |
| 1935-39 | 44.5 |
| 1940-44 | 43.7 |
| 1945-49 | 43.7 |
| 1950-54 | 51.1 |

For the individual years from 1950 to 1954, the following series was obtained:

| | |
|---|---|
| 1950 | 46.5 |
| 1951 | 47.6 |
| 1952 | 54.8 |
| 1953 | 53.9 |
| 1954 | 52.6 |

The series is difficult to interpret unless it is assumed that, especially in 1952 and 1953, there was a catching-up in the registration of births that had occurred in earlier years. There is little plausible reason to suppose that the actual birth rates should have fluctuated in this manner.

126. The course of the recorded crude death rates, by contrast, appears altogether plausible. The death rates declined, with increased momentum after 1940, as follows:

| | |
|---|---|
| 1925-29 | 23.2 |
| 1930-34 | 22.3 |
| 1935-39 | 20.0 |
| 1940-44 | 18.1 |
| 1945-49 | 14.0 |
| 1950 | 12.2 |
| 1951 | 11.7 |
| 1952 | 11.6 |
| 1953 | 11.7 |
| 1954 | 10.6 |

127. In the census of Costa Rica, as in the censuses of most countries, there is reason to doubt the completeness of enumeration of small children. In 1950, the Costa Rican census reported 30,261 children aged less than one year. The average annual number of births registered in 1949 and 1950[26] amounted to 35,399. If this had been the true number of births in the 12 months preceding the census, there would have had to be more than 5,000 infant deaths to result in 30,261 surviving infants, not counting the deaths of children who, by the time of the census, would have been more than 12 months old. If the latter were included, the total number of infant deaths would have had to be more than 6,000, possibly 7,000. However, on the average of 1949 and 1950, only 3,352 infant deaths were registered annually.

128. Similar rough computations can also be performed with respect to births during 1945-49, the deaths among them, and the number of children aged 1-4 in 1950. The 102,374 children of this age enumerated in the census would have been the survivors of about 129,000 births, if enumeration and registration had been exact. However, according to the statistics, only about 17,000 deaths followed those births. The conclusion is not definitive, since birth registration may have been inaccurate and since this calculation is not precise, but there is a strong indication that the enumeration of infants and small children in 1950 was quite deficient.

129. In bringing the statistics of the 1950 census forward to 1955, therefore, it is preferable not to place much reliance either on numbers of recorded births or on the number of children aged less than 5 enumerated in 1950. Instead, constant fertility in the recent period may be assumed and numbers of children aged under 5 in 1950 and those aged under 5 and 5-9 in 1955, may thus be worked out separately by the methods presented in chapter V.

130. The procedure, thus reduced to cohorts aged 5 and over in 1950, is illustrated in tables 8 and 9. The calculation is slightly inexact, since the Costa Rican census was taken on 22 May 1950 and the estimate desired is for mid-year 1955, whereas the statistics of deaths refer to the calendar years from the beginning of 1950 to the end of 1954. Most of the resulting error, however, will be eliminated when the results of the computation are "pro-rated" at a later stage. Since the data thus brought forward exclude children under 10 in 1955, "pro-rating" at this stage is not possible. Nor is it necessary, since no important errors are introduced by the possible inaccuracies in death registration or the exclusion from the calculation of almost negligible numbers of persons and deaths with undeclared ages. A definite estimate of the 1955 population by sex and age will be arrived at after use of the fertility assumption developed in chapter V.

[24] United Nations, *The Population of Central America (including Mexico), 1950-1980.*

[25] Ricardo Jiménez, "Causa de la diferencia entre el censo de población de 1950 y la estimación a esa fecha en Costa Rica", *Estadística*, vol. X, June 1952.

[26] The recorded birth rates of those two years averaged 45.0, which is plausible.

TABLE 8. DEATHS IN COSTA RICA, 1950–1954, ATTRIBUTABLE TO COHORTS OF GIVEN AGES ON 1 JANUARY 1950

| *Sex and age group (years)* | *Deaths reported in this age group* | | | | | *Total deaths, 1950–54* | *Deaths attributable to cohort initially in the given age group* [c] |
|---|---|---|---|---|---|---|---|
| | *1950* | *1951* [a] | *1952* | *1953* | *1954* [b] | | |
| *Males* | | | | | | | |
| 5– 9 | 151 | 162 | 139 | 205 | 175 | 832 | 594 |
| 10–14 | 80 | 72 | 67 | 68 | 68 | 355 | 364 |
| 15–19 | 80 | 70 | 91 | 63 | 69 | 373 | 448 |
| 20–24 | 105 | 126 | 114 | 88 | 91 | 524 | 478 |
| 25–29 | 100 | 100 | 99 | 68 | 66 | 433 | 460 |
| 30–34 | 109 | 106 | 93 | 97 | 81 | 486 | 530 |
| 35–39 | 148 | 122 | 109 | 99 | 96 | 574 | 557 |
| 40–44 | 117 | 109 | 105 | 96 | 113 | 540 | 574 |
| 45–49 | 158 | 130 | 82 | 128 | 111 | 609 | 704 |
| 50–54 | 161 | 173 | 163 | 152 | 151 | 800 | 764 |
| 55–59 | 152 | 171 | 112 | 138 | 155 | 728 | 859 |
| 60–64 | 237 | 189 | 206 | 179 | 179 | 990 | 990 |
| 65–69 | 190 | 179 | 210 | 205 | 205 | 989 | 1,031 |
| 70–74 | 235 | 212 | 224 | 209 | 193 | 1,073 | 980 |
| 75–79 | 178 | 183 | 152 | 198 | 177 | 888 | 763 |
| 80–84 | 132 | 119 | 135 | 133 | 119 | 638 | 1,046 |
| 85 and over | 155 | 136 | 135 | 159 | 142 | 727 | |
| *Females* | | | | | | | |
| 5– 9 | 148 | 158 | 133 | 137 | 156 | 732 | 502 |
| 10–14 | 63 | 56 | 51 | 49 | 52 | 271 | 310 |
| 15–19 | 83 | 73 | 68 | 62 | 64 | 350 | 429 |
| 20–24 | 107 | 129 | 88 | 88 | 96 | 508 | 492 |
| 25–29 | 107 | 107 | 107 | 76 | 79 | 476 | 470 |
| 30–34 | 104 | 102 | 97 | 91 | 70 | 464 | 541 |
| 35–39 | 160 | 131 | 99 | 126 | 102 | 618 | 598 |
| 40–44 | 123 | 115 | 118 | 106 | 115 | 577 | 536 |
| 45–49 | 114 | 93 | 94 | 103 | 91 | 495 | 564 |
| 50–54 | 127 | 137 | 125 | 122 | 122 | 633 | 645 |
| 55–59 | 126 | 142 | 122 | 137 | 130 | 657 | 770 |
| 60–64 | 204 | 163 | 182 | 171 | 163 | 883 | 880 |
| 65–69 | 174 | 163 | 185 | 194 | 163 | 879 | 960 |
| 70–74 | 220 | 198 | 208 | 228 | 187 | 1,041 | 904 |
| 75–79 | 154 | 158 | 143 | 167 | 145 | 767 | 737 |
| 80–84 | 149 | 134 | 147 | 148 | 129 | 707 | 1,280 |
| 85 and over | 172 | 151 | 195 | 218 | 190 | 926 | |

[a] Except for infants, death statistics by age for 1951 were available for both sexes only; they were estimated for each sex assuming the same sex-ratios of deaths by age as in 1950.
[b] The available statistics for 1954 provided no break-down for ages 75 and over; it was assumed that deaths occurred at ages 75–79, 80–84, and 85 and over, in the proportions of 1953.
[c] Arithmetic mean of totals of each pair of successive age groups.

TABLE 9. UTILIZATION OF STATISTICS ON DEATHS IN BRINGING DATA ON COSTA RICAN POPULATION AGED 5 AND OVER IN 1950 FORWARD TO 1955

| Sex and age of cohort in 1950 (years) | Population enumerated, 1950[a] | Same, smoothed[b] | Deaths attributed to cohort | Estimated population, 1955 | Age of cohort in 1955 (years) |
|---|---|---|---|---|---|
| *Males* | | | | | |
| 5– 9........ | 56,789 | — | 594 | 56,195 | 10–14 |
| 10–14........ | 49,734 | 48,814 | 364 | 48,450 | 15–19 |
| 15–19........ | 40,418 | 41,773 | 448 | 41,325 | 20–24 |
| 20–24........ | 37,671 | 36,212 | 478 | 35,734 | 25–29 |
| 25–29........ | 28,647 | 29,327 | 460 | 28,867 | 30–34 |
| 30–34........ | 23,851 | 24,297 | 530 | 23,767 | 35–39 |
| 35–39........ | 22,908 | 22,184 | 557 | 21,627 | 40–44 |
| 40–44........ | 18,310 | 18,446 | 574 | 17,872 | 45–49 |
| 45–49........ | 14,140 | 14,568 | 704 | 13,864 | 50–54 |
| 50–54........ | 12,313 | 11,579 | 764 | 10,815 | 55–59 |
| 55–59........ | 7,889 | 8,747 | 859 | 7,888 | 60–64 |
| 60–64........ | 7,667 | 6,965 | 990 | 5,975 | 65–69 |
| 65–69........ | 4,716 | 5,088 | 1,031 | 4,057 | 70–74 |
| 70–74........ | 3,331 | 3,180 | 980 | 2,200 | 75–79 |
| 75–79........ | 1,860 | — | 763 | 1,097 | 80–84 |
| 80–84........ | 1,073 | — | 1,046 | 746 | 85 and over |
| 85 and over... | 719 | — | | | |
| *Females* | | | | | |
| 5– 9........ | 55,367 | — | 502 | 54,865 | 10–14 |
| 10–14........ | 48,555 | 48,611 | 310 | 48,301 | 15–19 |
| 15–19........ | 43,826 | 44,010 | 429 | 43,581 | 20–24 |
| 20–24........ | 39,386 | 38,679 | 492 | 38,187 | 25–29 |
| 25–29........ | 30,491 | 30,595 | 470 | 30,125 | 30–34 |
| 30–34........ | 23,705 | 24,830 | 541 | 24,289 | 35–39 |
| 35–39........ | 23,930 | 22,622 | 598 | 22,024 | 40–44 |
| 40–44........ | 18,074 | 18,548 | 536 | 18,012 | 45–49 |
| 45–49........ | 13,966 | 14,226 | 564 | 13,662 | 50–54 |
| 50–54........ | 11,853 | 11,274 | 645 | 10,629 | 55–59 |
| 55–59........ | 7,827 | 8,518 | 770 | 7,748 | 60–64 |
| 60–64........ | 7,248 | 6,649 | 880 | 5,769 | 65–69 |
| 65–69........ | 4,420 | 4,782 | 960 | 3,822 | 70–74 |
| 70–74........ | 3,227 | 3,039 | 904 | 2,135 | 75–79 |
| 75–79........ | 1,768 | — | 737 | 1,031 | 80–84 |
| 80–84........ | 1,147 } | — | 1,280 | 707 | 85 and over |
| 85 and over... | 840 } | | | | |

[a] Exclusive of individuals whose ages were not reported.

[b] By smoothing formula presented in p. 11. Smoothing appears desirable owing to irregularities probably due to inaccurate age reporting, such as the apparent excess of women aged 35–39 over women aged 30–34.

## III. SOME CONCEPTS OF ABRIDGED LIFE TABLES AND LIFE-TABLE CONSTRUCTION

131. The core of the methods of population projections described here consists in the use of a system of model life tables to calculate the numbers of persons in each cohort of a population who will survive during successive time intervals in the future. The derivation of these model tables is briefly described in the appendix, where the survival ratios, needed for the population projections, as well as other functions of the life tables are tabulated.

132. Though the actual population projection may be carried out entirely by the tabulated survival ratios, it is first necessary to establish which of these ratios are appropriate to the given case, i.e., which ratios express most nearly the current and expected future mortality conditions. Survival ratios are not given directly in official statistics but must be calculated or otherwise derived by reference to the statistical information which exists. In almost every case, at least one step in the

general procedure of life-table construction is involved, despite all simplifications of the procedure. Some knowledge of the relevant mortality functions and of their interrelations will therefore usually be needed. The purpose of this chapter is to establish the concepts, in simplified form, which are needed to describe the relationship of survival ratios to other statistics relating to mortality.

133. Part A introduces and illustrates the main life-table functions. In part B, consideration is given to those interrelations of the several functions which have to be taken into account in particular situations. Only "abridged" life tables, i.e., tables with functions by five-year groups rather than for each year of age, are considered. The emphasis in the presentation is on simplicity and expediency, abstraction being made from the more rigorous mathematical techniques used in actuarial science.

## A. The chief functions comprised in a life table

134. The starting point for the calculation of life-table values is usually the computation of death rates for the various age groups. From these rates other functions are derived, and from the latter functions survival ratios are derived, expressing the proportion of persons, among those who survive to a given age, who live on and attain the next age level.

### Table 10. Derivation of specific death rates ($m_x$) from statistics of deaths and population by sex and age, Luxembourg, 1946–1949

| *Sex and age in years* ($x$) | *Deaths* 1946 | 1947 | 1948 | 1949 | *Average, 1946–1949* | *Enumerated population 31 December 1947* | *Average age-specific death rate, 1946–1949* (1,000 $m_x$) |
|---|---|---|---|---|---|---|---|
| *Males* | | | | | | | |
| 0 | 188 | 145 | 142 | 116 | 148 | 2,082 | 71.1 |
| 1– 4 | 27 | 25 | 19 | 23 | 24 | 7,640 | 3.1 |
| 5– 9 | 18 | 19 | 8 | 9 | 14 | 9,714 | 1.4 |
| 10–14 | 15 | 10 | 7 | 13 | 11 | 9,947 | 1.1 |
| 15–19 | 20 | 20 | 20 | 21 | 20 | 12,437 | 1.6 |
| 20–24 | 39 | 24 | 23 | 24 | 28 | 11,843 | 2.4 |
| 25–29 | 34 | 29 | 25 | 13 | 25 | 10,057 | 2.5 |
| 30–34 | 34 | 30 | 33 | 31 | 32 | 10,143 | 3.2 |
| 35–39 | 39 | 61 | 50 | 47 | 49 | 12,227 | 4.0 |
| 40–44 | 61 | 75 | 64 | 79 | 70 | 12,226 | 5.7 |
| 45–49 | 113 | 98 | 112 | 101 | 106 | 11,071 | 9.6 |
| 50–54 | 101 | 99 | 108 | 102 | 102 | 8,996 | 11.3 |
| 55–59 | 114 | 144 | 151 | 141 | 138 | 7,430 | 18.6 |
| 60–64 | 173 | 160 | 190 | 189 | 178 | 6,470 | 27.5 |
| 65–69 | 198 | 246 | 252 | 228 | 231 | 5,432 | 42.5 |
| 70–74 | 224 | 250 | 259 | 292 | 256 | 3,928 | 65.2 |
| 75–79 | 204 | 217 | 220 | 264 | 226 | 2,185 | 103.4 |
| 80–84 | 150 | 166 | 153 | 187 | 164 | 946 | 173.4 |
| 85 and over | 75 | 80 | 77 | 103 | 84 | 322 | 260.9 |
| *Females* | | | | | | | |
| 0 | 139 | 113 | 103 | 78 | 108 | 1,946 | 55.5 |
| 1– 4 | 27 | 34 | 19 | 22 | 26 | 7,347 | 3.5 |
| 5– 9 | 12 | 9 | 5 | 6 | 8 | 9,466 | 0.8 |
| 10–14 | 11 | 10 | 13 | 10 | 11 | 9,568 | 1.1 |
| 15–19 | 23 | 17 | 16 | 13 | 17 | 11,378 | 1.5 |
| 20–24 | 23 | 22 | 20 | 21 | 22 | 11,468 | 1.9 |
| 25–29 | 31 | 27 | 25 | 18 | 25 | 10,327 | 2.4 |
| 30–34 | 37 | 28 | 16 | 16 | 24 | 10,049 | 2.4 |
| 35–39 | 35 | 36 | 27 | 27 | 31 | 11,925 | 2.6 |
| 40–44 | 57 | 49 | 36 | 36 | 44 | 11,956 | 3.7 |
| 45–49 | 68 | 62 | 64 | 63 | 64 | 11,300 | 5.7 |
| 50–54 | 76 | 74 | 88 | 86 | 81 | 9,548 | 8.5 |
| 55–59 | 100 | 99 | 103 | 101 | 101 | 7,808 | 12.9 |
| 60–64 | 144 | 153 | 147 | 125 | 142 | 7,048 | 20.1 |
| 65–69 | 197 | 191 | 166 | 205 | 190 | 6,011 | 31.6 |
| 70–74 | 228 | 248 | 233 | 275 | 246 | 4,473 | 55.0 |
| 75–79 | 235 | 224 | 201 | 254 | 228 | 2,563 | 89.0 |
| 80–84 | 174 | 176 | 165 | 218 | 183 | 1,218 | 150.2 |
| 85 and over | 142 | 130 | 96 | 151 | 130 | 497 | 261.6 |

135. Some life tables have been constructed by inverting this sequence of calculations. Where adequate statistics on deaths by age were not available, survival ratios have been inferred from statistics of the population by sex and age according to two successive census enumerations; and by working backward from these ratios, other life-table functions, including the age-specific death rate, has been obtained.

136. The functions in this sequence of computations to be considered first are: (1) $m_x$, i.e., the death rate for persons of a given age, x; (2) $q_x$, the probability of dying within a given age interval; (3) $l_x$, the number of survivors to a specified age from an assumed initial number of births; (4) $L_x$, the numbers of years lived collectively by those survivors within the given age interval; and (5) $P_x$, the survival ratio from one age interval to the next-higher age interval. Other functions which will be taken up subsequently are: (6) $d_x$, the number of deaths in the given age interval; (7) $p_x$, the probability of surviving during the age interval; (8) $T_x$, the collective number of years yet to be lived by the survivors to a given age; and (9) $°e_x$, the expectation of life of an individual of given age.

137. In all these symbols, the suffix "x" denotes age. Since we are dealing with abridged life tables, by five-year groups of ages, it denotes either the lower limit of an age group or the entire age group, depending on the nature of the function.[27]

### 1. *Functions in the basic sequence for computation of survival ratios*

138. Table 10 illustrates the derivation of age-specific death rates from statistics for the Grand-Duchy of Luxembourg for the years 1946 to 1949. A census was taken at the end of 1947, a date which is central for the 1946-1949 period. Dividing the average annual number of deaths in each sex-age group by the population of each group at the middle of the period, and multiplying by 1,000, the first function of the sequence, namely the age-specific death rates, 1,000 $m_x$, is obtained. Owing to sharp differences in mortality immediately after birth and in subsequent years of age, it is of some importance to compute separately $m_0$, i.e., the rate for the first year of life, and $m_{1-4}$, i.e., the rate for the following four years. Beginning with age 5, the rates are computed by five-year groups.

139. Averaging the numbers of deaths during four years, in this instance, is desirable both because of the small numbers of deaths in some age groups and because of year-to-year fluctuations in the factors of mortality. Where ages at death are frequently misreported, fluctuations can be smoothed out by the same formula suggested previously for smoothing a population age distribution. In the case of deaths, however, the formula cannot be applied to the first two age groups because the characteristic shape of the distribution would produce a major distortion.

140. When the $m_x$-values have been obtained, the remaining functions can be calculated as will presently be explained. The results of the calculations are shown for Luxembourg males, in Table 11. A brief explanation for each column is given below.

TABLE 11. SOME FUNCTIONS OF AN ABRIDGED LIFE TABLE FOR LUXEMBOURG MALES, 1946–1949

| *Age in years* ($x$) | *Age-specific death rate* (1,000 $m_x$) | *Life-table death rate* (1,000 $q_x$) | *Survivors to exact age* ($l_x$) | *Survivors in age group* ($L_x$) | *Survival ratio* ($P_x$) ($P_b = 0.9354$) |
|---|---|---|---|---|---|
| 0.......... | 71.1 | 63.0 | 100,000 | 467,715 | 0.9862 |
| 1–4........ | 3.1 | 12.0 | 93,700 | | |
| 5–9........ | 1.4 | 7.0 | 92,576 | 461,260 | 0.9937 |
| 10–14....... | 1.1 | 5.5 | 91,928 | 458,375 | 0.9933 |
| 15–19....... | 1.6 | 8.0 | 91,422 | 455,282 | 0.9901 |
| 20–24....... | 2.4 | 11.9 | 90,691 | 450,755 | 0.9879 |
| 25–29....... | 2.5 | 12.4 | 89,611 | 445,278 | 0.9859 |
| 30–34....... | 3.2 | 15.9 | 88,500 | 438,982 | 0.9822 |
| 35–39....... | 4.0 | 19.8 | 87,093 | 431,155 | 0.9761 |
| 40–44....... | 5.7 | 28.1 | 85,369 | 420,848 | 0.9626 |
| 45–49....... | 9.6 | 47.0 | 82,970 | 405,100 | 0.9490 |
| 50–54....... | 11.3 | 55.1 | 79,070 | 384,458 | 0.9284 |
| 55–59....... | 18.6 | 89.1 | 74,713 | 356,922 | 0.8918 |
| 60–64....... | 27.5 | 129.1 | 68,056 | 318,315 | 0.8412 |
| 65–69....... | 42.5 | 192.9 | 59,270 | 267,768 | 0.7676 |
| 70–74....... | 65.2 | 281.3 | 47,837 | 205,545 | 0.6649 |
| 75–79....... | 103.4 | 410.0 | 34,381 | 136,662 | 0.5224 |
| 80–84....... | 173.4 | 592.2 | 20,284 | 71,390 | 0.3075 |
| 85 and over.. | 260.9 | 1,000.0 | 8,272 | 31,406 | |

141. The first column, x, indicates the age groups to which the functions apply. The second column, age-specific death rates, is taken from table 10. In keeping with conventions in demography, these rates are expressed per 1,000, though this is usually not done in actuarial tables. As in all life tables, this function decreases sharply from birth onward to attain a minimum near the age of 10; it then rises with increasing age, at first gradually, then more sharply.

142. The third column shows the "life-table death rate", 1,000 $q_x$. In actuarial usage, this function expresses the probability that an individual about to enter an age group will die before reaching the upper limit of that age group. Thus, of 1,000 children born, it is expected that 63.0 will die before attaining one year of age; and of 1,000 individuals aged exactly 15 years, 8.0 are expected to die before attaining the precise age of 20.

143. From the demographic point of view, a life table is regarded as a theoretical model of a population which is continuously replenished by births and depleted by deaths. In this context, the $q_x$ values can be regarded as being in the nature of death rates, though different from the specific death rates symbolized by $m_x$. Whereas $m_x$ relates the number of deaths to the persons *living within* each age group, $q_x$ relates the same number of deaths to the persons who, within a particular year, *enter* that age group. If the age group is of one year only, as in the case of age 0, $q_x$ is somewhat smaller than $m_x$ because there are more entrants into the age group during each year than persons living within it at any time,

[27] Notations such as ${}_nq_x$, ${}_nL_x$, etc., are in actuarial usage, where the suffix "n" refers to the width of the age group. Thus, for a five-year group, n becomes 5, while for single-year ages it equals 1. This qualification is important for mathematical demonstrations, but need not complicate the presentation of the subject in this chapter.

since some of those who enter die while in the group. In the case of a five-year group, $q_x$ has a value approaching five times the value of $m_x$, because there are nearly five times as many persons living in the group as enter it during any one year. For the terminal age group, 1,000 $q_x$ is 1,000 because all individuals die eventually.

144. The fourth column, survivors to the given exact age, is symbolized by $l_x$, but here the suffix "x" indicates the lower limit of each age group. As in most life tables, 100,000 births are assumed and the $l_x$ function shows how many of the 100,000 reach each age. When the tables are used for demographic purposes, the same 100,000 births are assumed to occur every year.

145. The fifth column, $L_x$, has two different meanings, depending on whether an actuarial or a demographic viewpoint is taken. From the actuarial point of view, the $L_x$-values represent the numbers of years that will be lived collectively within any one age interval by a cohort numbering 100,000 at birth and subject to the given mortality conditions. For example, if no deaths occurred up to age 5, the 100,000 children collectively would live 500,000 years up to that age. The ravages of mortality are reflected by the diminution of the $L_x$ function from one age interval to the next. From a demographic point of view, the $L_x$ function represents the age composition of a population which is constantly replenished by births at the rate of 100,000 per year and depleted by mortality at the rates represented by the $q_x$ or $m_x$ function. Owing to the assumed constant number of births, this hypothetical population, and every one of its age groups, will be of a constant size; it is therefore described as the "stationary" population corresponding to the particular life table. In this case, it would total 6,207,216 individuals.

146. The last column, $P_x$, shows the proportion of persons, among those living in the indicated age groups of the "stationary" population, who survive until they are 5 years older. This ratio attains its maximum near the age of 10 and then declines, at first gradually and then more rapidly. At the top of the column, the ratio $P_b$ is given, that is, the proportion of survivors to age groups 0-4, at the end of a five-year period, out of a cohort of births occurring at a constant rate during the five-year period. The remaining values of $P_x$ shown in the column apply to five-year age groups, with the exception of the last value, $P_{80+}$, which represents the proportion of persons, among the group aged 80 and over, who survive 5 more years, being 85 and over at the end of the period.

## 2. *Derivation of the sequence of functions from age-specific death rates*

147. The mathematical interrelations among the several functions in some instances can be expressed accurately only in terms of integral equations. Since this is not a treatise on actuarial techniques, however, the precise mathematical relations need not detain us here. The transformation of the age-specific rates, $m_x$, into life-table death rates, $q_x$, is of such a nature. The age-specific rates relate to population segments some of which have already died as a result of the mortality condition expressed by the rate itself. In addition, for the most part, they relate to five-year age groups, whereas the $q_x$ values relate to the annual number of entrants into each of these groups. There is no simple arithmetical procedure for accurate conversion of $m_x$-values into the corresponding $q_x$-values or *vice versa,* but tables have been constructed with the aid of which the transformation can be effected very rapidly.[28] If such tables cannot be readily obtained, the somewhat less accurate shortcut procedure described in part B of this chapter can be used.

148. A direct arithmetical procedure is used to obtain the numbers of survivors ($l_x$) to exact ages on the basis of the life-table death rates ($q_x$). Beginning with the assumed 100,000 births ($l_0$), $q_0$ (in this example, 63.0 per 1,000) is applied to calculate the deaths in the first year of life (6,300 in this case), and these are subtracted from the original 100,000 to compute the number of survivors at the exact age of one year. The process of multiplication and subtraction is repeated for the successive age groups until all values of $l_x$ are obtained.

149. The transformation of $l_x$ into $L_x$ is mathematically complex if a very high degree of accuracy is sought. Fortunately, a few simple arithmetical procedures are quite sufficient if only a close approximation is required; the errors involved in this simpler procedure are, on the whole, slight and for the most part negligible. One procedure can be used with respect to all age groups from 5-9 to 80-84. Separate procedures apply to the calculation of the two extreme values, $L_{0-4}$ and $L_{85+}$.

150. For age groups 5-9 to 80-84, it is quite sufficient to assume that $L_x$ equals five times the arithmetic average of $l_+$ and $l_{x+5}$.

151. Owing to the very unequal distribution of deaths over the first five years of life, this procedure would be quite unsatisfactory for the computation of $L_{0-4}$. A reasonably good approximation, however, is obtained by assuming $L_0$ (i.e., for the first year of life only) to equal $0.25\ l_0+0.75\ l_1$ (in the given example, 25,000 plus 0.75 times 93,700, which is 95,275); and by assuming that $L_{1-4}$ equals $1.9\ l_1+2.1\ l_5$ (in the present example, 1.9 times 93,700 plus 2.1 times 92,576, which is 372,440). These assumptions, taken together, lead to the formula $L_{0-4}=0.25\ l_0+2.65\ l_1+2.1\ l_5$ (and to the calculated result, in this instance, of 467,715).

152. Experience has justified the use of a very simple formula for an approximate computation of $L_{85+}$. Actually, as will be explained further on, the $L_x$-value for a terminal age group equals the $l_x$-value for the initial age of the group multiplied by the expectation of life at that age. It happens by a mere coincidence that, in most available life tables, the expectation of life at the age of 85 very nearly equals the common logarithm of $l_{85}$ when $l_{85}$ is expressed for 100,000 births. An estimate of $L_{85+}$ is therefore easily obtained by multiplying $l_{85}$ by its own logarithm. In the present example, $l_{85}$ amounts to 8,272

[28] Among the best tables of this kind are those of Reed and Merrell, which were used in the calculation of the example shown here. For the original publication of these tables, see Lowell J. Reed and Margaret Merrell, "A Short Method for Constructing an Abridged Life Table", *The American Journal of Hygiene,* Vol. 30, No. 2, September 1939. The original publication has been reproduced in full in A. J. Jaffe's *Handbook of Statistical Methods for Demographers,* United States Bureau of the Census, Washington, D.C., 1951.

and the logarithm of that figure amounts to 3.91761 which may be assumed to be the approximate expectation of life, in years, at this age. The product of these two figures is 32,406, the required value for $L_{85+}$.

153. The derivation of survival ratios ($P_x$) from survivors in age groups ($L_x$) merely requires division of successive pairs of $L_x$-values. Thus, dividing $L_{5-9}$ (461,242 in this example) by $L_{0-4}$ (467,715), we obtain 0.9862, the required figure for $P_{0-4}$. In other words, of 1,000 persons living in the age group 0-4, 986.2 are expected to be alive five years later when their ages are 5-9. The values of $P_b$ (survival from births) and $P_{80+}$ require separate mention. The first of these is computed by dividing $L_{0-4}$ by 500,000, the assumed number of births during a five-year period. The second is obtained by dividing $L_{80+}$, i.e., the sum of $L_{80-84}$ and $L_{85+}$, by $L_{85+}$.

### 3. *Other functions of the life table*

154. Table 12 shows those functions which are usually presented where a life table is published for actuarial uses. The first three columns, relating to age groups, survivors to specified ages, and life-table death rates have already been discussed. Another function, not shown here, is that of $p_x$, or the probability of survival from one specified age to another, which is simply obtained by subtracting $q_x$ from unity (or, where the rates are expressed per 1,000, $1{,}000\ p_x = 1{,}000 - 1{,}000\ q_x$).

155. Column (4) of table 12 presents the $d_x$-function, which is the number of deaths occurring within an age group, from the lower to the upper limit of age, on the assumption of 100,000 initial births. The sum of the $d_x$ values for all ages equals 100,000, representing extinction of the cohort of 100,000 births. Near age 10, the $d_x$-function attains a minimum. It then rises towards a maximum at some relatively advanced age, after which it falls off again. This function can be obtained by subtraction of successive values of $l_x$. The function can also be conceived as an age distribution of deaths in a "stationary" population.

156. Column (5), which represents the $T_x$-function, indicates the number of years that will be lived collectively, from the given age onward, by the survivors to that age from the cohort of 100,000 births. This function is obtained by cumulative addition of the $L_x$-function (column 5 in table 11), from the bottom up, i.e., beginning with $L_{85+}$ and adding $L_{80-84}$, $L_{75-79}$, and so forth.[29]

157. The last function, $°e_x$, represents the individual expectation of life. If, as in this example, 100,000 individuals at their birth are expected to live, collectively, 6,207,216 years, then the average individual can expect, at birth, to live 62.1 years. Similarly, expectations of life at other ages are obtained by dividing the corresponding $T_x$ by the corresponding $l_x$. In the first few years of life, expectation of life rises until the relatively heavy risks of dying in infancy and early childhood are past. Thereafter, with advancing age, the expectation of life declines though never by as much as the increase in age. For instance, an individual aged 60 years may expect to live another 15 years ($°e_{60}$ being 15.2), i.e., up to about the age of 75. But those who do reach 75 years of age have a further expectation of 7 years of life up to the age of 82.

## B. Practical derivations of life-table functions

158. The comparatively simple procedures so far described are adequate for the construction of an approximate abridged life table. The detailed procedures can, of course, be modified, but the general sequence of calculations should be retained.

[29] In order to obtain a separate value for $T_1$, it was also necessary to compute $L_0$ and $L_{1-4}$ by the separate formulas given on p. 23. The resulting values of $L_0$ and $L_{1-4}$ were 95,275 and 372,440, respectively.

**Table 12. Abridged life table for Luxembourg males, 1946–1949**

| (1) $x$ | (2) $l_x$ | (3) $1{,}000\ q_x$ | (4) $d_x$ | (5) $T_x$ | (6) $°e_x$ |
|---|---|---|---|---|---|
| 0 | 100,000 | 63.0 | 6,300 | 6,207,216 | 62.1 |
| 1– 4 | 93,700 | 12.0 | 1,124 | 6,111,941 | 65.2 |
| 5– 9 | 92,576 | 7.0 | 648 | 5,739,501 | 62.0 |
| 10–14 | 91,928 | 5.5 | 506 | 5,278,241 | 57.4 |
| 15–19 | 91,422 | 8.0 | 731 | 4,819,866 | 52.7 |
| 20–24 | 90,691 | 11.9 | 1,079 | 4,364,584 | 48.1 |
| 25–29 | 89,611 | 12.4 | 1,111 | 3,913,829 | 43.7 |
| 30–34 | 88,500 | 15.9 | 1,407 | 3,468,551 | 39.2 |
| 35–39 | 87,093 | 19.8 | 1,724 | 3,029,569 | 34.8 |
| 40–44 | 85,369 | 28.1 | 2,399 | 2,598,414 | 30.4 |
| 45–49 | 82,970 | 47.0 | 3,900 | 2,177,566 | 26.2 |
| 50–54 | 79,070 | 55.1 | 4,357 | 1,772,466 | 22.4 |
| 55–59 | 74,713 | 89.1 | 6,657 | 1,388,008 | 18.6 |
| 60–64 | 68,056 | 129.1 | 8,786 | 1,031,086 | 15.2 |
| 65–69 | 59,270 | 192.9 | 11,433 | 712,771 | 12.0 |
| 70–74 | 47,837 | 281.3 | 13,457 | 445,003 | 9.3 |
| 75–79 | 34,381 | 410.0 | 14,096 | 239,458 | 7.0 |
| 80–84 | 20,284 | 592.2 | 12,012 | 102,796 | 5.1 |
| 85 and over | 8,272 | 1,000.0 | 8,272 | 31,406 | 3.9 |

159. This manual provides a scheme of model life tables, in which survival ratios as well as certain other functions are tabulated. Where these model tables can be adapted to the purpose of a specific population projection, it is not necessary to go through the entire procedure of constructing a life table. Quite often, the appropriate survival ratios can be inferred from the scheme presented here, by reference to some other function for which values have been obtained from available statistics.

160. Rough calculations will often suffice as a basis for the selection of survival ratios.

### 1. *Ratio functions and cumulative functions of the life table*

161. It is useful to distinguish those functions of the life table which are in the nature of ratios from those which are the results of cumulative addition or subtraction. The first type comprises age-specific death rates ($m_x$), life-table death rates ($q_x$), and survival ratios ($P_x$). All these rates, though differently derived, are closely related to each other. If one of these functions is known, reference to a system of model life tables makes it possible to estimate immediately the approximate levels of the other two functions, without any further calculations. For instance, supposing that $m_x$ has been determined with respect to all age groups, the model life tables can be used to assess the approximate mortality levels which these particular rates represent. The survival ratios, $P_x$, corresponding to the same general mortality levels can then be located in the tables, without much further computations (see chapter IV).

162. Other functions, like the specific values of $l_x$, $L_x$ and $T_x$, do not provide an equally good indication of the corresponding values of $P_x$. For example, a given value of $l_{20}$ (survivors to the age of 20) may be the result of high mortality in infancy and low mortality in adolescence, or *vice versa.* Since the function required for population projections is $P_x$, given values of $l_x$, $L_x$ or $T_x$ have to be converted into corresponding values of $q_x$ or $P_x$ before direct reference can be made to the model life tables.

### 2. *Functions corresponding to a "general" level of mortality under average conditions*

163. The expectation of life at birth, $°e_0$, can be considered in a different category. It is both the result of cumulation of specific values ($T_0$) and a ratio (by division by $l_0$). And it is the one synthetic measure by which the "general" level of mortality can be summarized in a single figure. The model life tables present those combinations of life-table functions which, under average conditions, are likely to be found if expectation of life at birth attains a certain value. Actually, such average conditions never occur and deviations from the pattern, as implied in the model, must be expected. For example, though $°e_0$ may be 50 years, mortality rates at some ages may be higher, and at other ages lower, than indicated by the model life table for $°e_0$ equal to 50. By and large, deviations from the average pattern may be relatively small, but unusual situations occur as where adult mortality is unusually high and infant mortality low, etc.

164. If specific information on mortality conditions by age is not available and there are no special reasons to suppose that the conditions are unusual, perhaps the best estimate that can be made is that given in one of the model tables. A model table may be found, for example, which in the given instance would yield the same annual number of deaths as actually recorded. It may then be assumed that the distribution of death rates by age is the same as that of the model table.

### 3. *Derivation of required specific functions where ratio-functions are not given*

165. The model tables can be used directly to determine survival ratios for each age, if the age-specific $m_x$ or $q_x$ values are determined. Even where specific information on mortality is available, however, these ratio-functions are sometimes not determined.

166. It may be that an incomplete life table is at hand, giving only the $l_x$-values. In that case, by taking successive differences, the $d_x$-values can be obtained, and division of $d_x$ by $l_x$ yields the $q_x$ values, from which corresponding survival ratios can be inferred by means of the model tables.

167. Again, it may happen that the $L_x$-values are available. In that event, the specific survival ratios should be computed from the given $L_x$-values, instead of being inferred from the $L_x$-values in the model life tables. The same applies if the given values are those of $T_x$, from which $L_x$-values are obtained by successive subtractions.

168. It may also be that only $°e_x$-values are available. These cannot very readily be converted into other life table functions, though approximate values for some other function might be estimated by repeated trials, using some of the values tabulated in the model life tables.

169. This case is much simplified if, as sometimes happens, the two functions for which values are given in the available incomplete life table are $°e_x$ and $l_x$. $T_x$ can be calculated by multiplying $°e_x$ with $l_x$; from $T_x$, $L_x$ is obtained by successive subtractions, and $P_x$, the age-specific survival ratios, by successive divisions of $L_x$.

### 4. *Use of survival ratios computed from census statistics*

170. In paragraph 135 of part A of this chapter, it was mentioned that life tables are sometimes constructed by calculating $P_x$ values from census data. This method is advantageous where adequate statistics on deaths by age are not available, but where the age composition of the population has been determined in two successive censuses. No survival ratios can be obtained by this procedure in respect of children born during the census interval. The details of the procedure are sometimes highly complex, involving refined methods of interpolation and graduation. In the present context, however, only the survival ratios themselves are of immediate interest and there is no need to be detained with the complexities of computing other functions of the life table.

171. In computing the ratios of survival from one census to another, care must be taken that the numbers ascertained at the more recent census represent as nearly as possible the survivors of the cohorts enumerated at the previous census. Where migration has been important, something must be done to avoid distortion of the ratios by this factor. In a population mainly affected by

immigration, this is sometimes effectively done by restricting the inter-censal comparison to persons born within the country. Other factors which can vitiate the computations are faulty enumeration of some or all age groups, and mis-statement of ages. For graduation to eliminate the effects of age mis-statements, the formula presented in part B of chapter II may suffice if further adjustments are made at a later stage by reference to model life tables (as will be shown in the next chapter); however, the age mis-statements may be such that another more refined formula should be employed.

172. If the interval between the two censuses is five years, it is possible to compute directly the five-year survival ratios for five-year time periods which are required for the population projections. But additional problems arise if the census interval is longer or shorter than five years. If the interval is ten years, survival ratios for five-year age groups over ten-year periods can be calculated. A method for transforming such ratios into ratios for five-year periods is presented in the next chapter. The transformation in this case is relatively easy because the ratio for the ten-year period equals the product of ratios of two successive age groups for five-year periods. If the census interval is not a multiple of five years, the procedure becomes more complicated, but it is still possible to obtain rough estimates for survival ratios corresponding to five-year time periods.[30]

### 5. *Short-cut procedures*

173. For an approximate computation of $P_x$-values (survival ratios) from $m_x$-values (age-specific death rates), only two steps are required. Though it is suggested to find $P_x$-values from the model life tables, this short procedure may sometimes be used as an alternative approximate computation.

174. As a first step, $m_x$ may be converted into $q_x$ without recourse to detailed tabulations of the relationships between these functions.[31] This may be necessary, where such tabulations cannot be readily obtained. The approximate conversion can be made by means of factors by which $m_x$ must be multiplied to result in the required $q_x$.

175. It has been noted in paragraph 143 of part A of this chapter that $q_0$ is always somewhat smaller than $m_0$. Actually, $q_0$ is in the nature of an infant mortality rate, since infant deaths are related to births. The infant mortality rate, if known, may very well be substituted as an approximate value of $q_0$.[32] Otherwise, the following conversion table may be used:

| *1,000m₀* | *Coefficient for conversion to 1,000q₀* |
|---|---|
| 29 | 0.925 |
| 55 | 0.90 |
| 110 | 0.85 |
| 170 | 0.80 |
| 240 | 0.75 |

Other conversion factors can be interpolated for different values of $m_0$. In the example of Luxembourg, 1,000 $m_0$ for males was found to be 71.1; the corresponding value of 1,000 $q_0$ would be about 0.89 times this figure, i.e., about 63.3. This is a fairly close estimate since, according to the Reed-Merrell tables, 1,000 $q_0$ in this case is 63.0.

176. The relationship between $m_{1\text{-}4}$ and $q_{1\text{-}4}$ can similarly be summarized. As has been noted, the latter will always be somewhat less than four times the former, since this is a four-year age group. The following conversion table will suffice for rough estimates:

| *1,000m₁₋₄* | *Coefficient for conversion to 1,000q₁₋₄* |
|---|---|
| 8 | 3.8 |
| 21 | 3.6 |
| 36 | 3.4 |
| 55 | 3.2 |

For Luxembourg males, 1,000 $m_{1\text{-}4}$ was found equal to 3.1; by extrapolation from the conversion table the appropriate factor is found to be about 3.9, which gives an estimate of 12.1 for 1,000 $q_x$ — nearly the same as the figure 12.0 obtained with the use of the Reed-Merrell tables.

177. For five-year age groups, finally, $q_x$ will always be less than five times $m_x$. This difference will be slight when $m_x$ is small, but appreciable for large values of $m_x$. The following conversion table may be used, with interpolations where required:

| *1,000mₓ* | *Coefficient for conversion to 1,000qₓ* |
|---|---|
| 10 | 4.9 |
| 45 | 4.5 |
| 100 | 4 |
| 160 | 3.5 |
| 240 | 3 |

178. With these three conversion tables, values of $q_x$ for all age groups can be estimated very rapidly. Once these have been obtained, they can be rapidly transformed into corresponding values of $p_x$, i.e., the probability of surviving *each exact age interval*, since $p_x$ equals $1-q_x$. This being accomplished, approximate estimates of $P_x$, i.e., survival ratios *from one age group to the next*, can be obtained very directly, as follows.

179. For five-year age groups, $P_x$ approximates the arithmetic average of $p_x$ and $p_{x+5}$. The approximation is quite close where $p_x$ is large, i.e., where $q_x$ is small. For age groups affected by relatively high mortality, i.e., especially at advanced ages, this short-cut computation results in a more or less appreciable underestimate of $P_x$. This is a defect of the method, but not a severe one for the present purposes. All values for $P_x$, from $P_{5\text{-}9}$ to $P_{75\text{-}79}$, can thus be approximately established without any intermediate computations. The values of $P_b$ (survival from births to ages 0-4) and $P_{80+}$ (survival from ages 80 and over to ages 85 and over), and also of $P_{0\text{-}4}$, must be estimated differently. The following empirical formulae are approximately correct:

1. $P_b=0.05+p_0\ (0.53+0.42\ P_{1\text{-}4})$
2. $P_{0\text{-}4}=\dfrac{p_0 \cdot p_{1\text{-}4}\ (1+p_{5\text{-}9})}{2\ P_b}$
3. $P_{80+}=0.8\ P_{75\text{-}79}-0.1$

180. Table 13 presents the results of the computation of $q_x$ and $P_x$-values by these short-cut procedures, compared with the $q_x$ and $P_x$-values previously obtained.

[30] The procedure then involves age cohorts which do not coincide with conventional five-year groupings at both censuses, as well as the use of survival ratios for a fraction of a five-year period. With the help of some subsidiary estimates, however, the procedure can be adopted for the present purposes.

[31] Such as the Reed-Merrell tables referred to above.

[32] The infant mortality rate is not precisely $q_0$, because deaths are here related to births occurring at the same time, e.g., in the same year. Not all infants dying in one calendar year were born in the same year, and some of the infants born in a given year will die, before reaching one year of age, during the next year. Nevertheless, the difference between the infant mortality rate and $q_0$ is usually slight.

**TABLE 13. LUXEMBOURG MALES, 1946–1949. COMPUTATION OF SURVIVAL RATIOS BY SHORT-CUT AND BY LONG PROCEDURE**

| Age ($x$) | 1,000 $m_x$ | Conversion factor | *Estimates by short-cut procedure* | | | *Results of computation by long procedure* | |
|---|---|---|---|---|---|---|---|
| | | | 1,000 $q_x$ | $p_x$ | $P_x$ ($P_b = 0.9351$)[a] | $p_x$[d] | $P_x$[e] ($P_b = 0.9354$) |
| 0 | 71.1 | 0.89 | 63.3 | 0.9367 | 0.9861[b] | 0.9370 | 0.9862 |
| 1–4 | 3.1 | 3.9 | 12.1 | 0.9879 | | 0.9880 | |
| 5–9 | 1.4 | 5 | 7.0 | 0.9930 | 0.9938 | 0.9930 | 0.9937 |
| 10–14 | 1.1 | 5 | 5.5 | 0.9945 | 0.9932 | 0.9945 | 0.9933 |
| 15–19 | 1.6 | 5 | 8.0 | 0.9920 | 0.9900 | 0.9920 | 0.9901 |
| 20–24 | 2.4 | 5 | 12.0 | 0.9880 | 0.9878 | 0.9881 | 0.9879 |
| 25–29 | 2.5 | 5 | 12.5 | 0.9875 | 0.9858 | 0.9876 | 0.9859 |
| 30–34 | 3.2 | 5 | 16.0 | 0.9840 | 0.9820 | 0.9841 | 0.9822 |
| 35–39 | 4.0 | 5 | 20.0 | 0.9800 | 0.9758 | 0.9802 | 0.9761 |
| 40–44 | 5.7 | 5 | 28.5 | 0.9715 | 0.9622 | 0.9719 | 0.9626 |
| 45–49 | 9.6 | 4.9 | 47.0 | 0.9530 | 0.9488 | 0.9530 | 0.9490 |
| 50–54 | 11.3 | 4.9 | 55.4 | 0.9446 | 0.9276 | 0.9449 | 0.9284 |
| 55–59 | 18.6 | 4.8 | 89.3 | 0.9107 | 0.8908 | 0.9109 | 0.8918 |
| 60–64 | 27.5 | 4.7 | 129.2 | 0.8708 | 0.8398 | 0.8709 | 0.8412 |
| 65–69 | 42.5 | 4.5 | 191.2 | 0.8088 | 0.7642 | 0.8071 | 0.7676 |
| 70–74 | 65.2 | 4.3 | 280.4 | 0.7196 | 0.6530 | 0.7187 | 0.6649 |
| 75–79 | 103.4 | 4 | 413.6 | 0.5864 | 0.4984 | 0.5900 | 0.5224 |
| 80–84 | 173.4 | 3.4 | 589.6 | 0.4104 | 0.2987[c] | 0.4078 | 0.3075 |
| 85+ | 260.9 | ... | 1,000 | ... | | ... | |

[a] Computed by formula 1 in the accompanying text.
[b] $P_{0-4}$, computed by formula 2.
[c] $P_{80+}$, computed by formula 3.
[d] Computed from the $q_x$ values in table 11, column 3.
[e] From table 11, column 6.

# IV. ESTIMATING CURRENT LEVELS AND FUTURE TRENDS OF SURVIVAL RATIOS WITH THE USE OF MODEL LIFE TABLES

181. The main part of the procedure of a population projection by sex-age groups consists in multiplying the numbers of various cohorts living at a given time by appropriate survival ratios. The needed ratios can often be worked out individually for a particular population projection, but the procedure is greatly simplified when reference is made to a system of model life tables.

182. Tabulated values for a system of model life tables are found in the appendix, with a note explaining how they were constructed. To facilitate their use in formulating assumptions relating to the future trend of mortality, the tables have been so arranged that they can be regarded as representing successive stages in a process of declining mortality.

183. The idea of a coherent system of model life tables and of some of its uses is developed in part A of this chapter. Part B describes how, with this system, under various conditions and with varying amounts of statistical information, suitable survival ratios can be quickly obtained. The application of survival ratios to an actual population project is illustrated in part C, consideration also being given to possible variations in the systematic assumption of future changes in mortality.

## A. THE MODEL LIFE TABLES CONCEIVED AS ONE SYSTEM

184. Each model life table is designed to represent a typical combination of age-sex specific functions of mortality, or survival, corresponding to a given general level of mortality. For present purposes, the general mortality level has been determined in terms of $^{\circ}e_0$, the expectation of life at birth, for both sexes combined. Actually, the combination of mortality rates, age group by age group, in any given instance, will always differ more or less from any pattern taken as typical for the same general mortality level. It may therefore be necessary to refer to more than one model life table, and perhaps also to make interpolations between two successive tables in estimating the appropriate combination of rates for a given case.

185. A generalization is here made as to the manner in which mortality may decline, during successive five-year time periods, from the conditions of one model life table to those of the next table in the sequence. This generalization requires a rather liberal interpretation. It is not asserted that mortality will always decline in this particular way. It may decline more slowly, more rapidly, or with different rapidity for different age groups. The model assumption is merely one which is plausible under some of the more typical conditions to be found in the world today, and it can be modified as required. Apart from its uses in the estimation of future mortality trends, this model assumption also serves as the link by which the several model life tables are tied together into a coherent sequence.

### 1. *Gains in life expectation: the past record*

186. A substantial decline in rates of mortality is one of the outstanding phenomena of the modern era. This record of past achievement is relevant to considerations of the future, but is not susceptible of very accurate assessment. Adequate mortality statistics covering a sufficiently long period have not become available for a representative majority of the world's populations. In order to obtain a very rough indication of the manner in which mortality has changed in those areas for which fairly reliable statistics exist, the official life tables published in the United Nations *Demographic Yearbook* for various countries have been used. Areas were selected for which both an earlier and a more recent life table had been reproduced in the Yearbook, and these areas were grouped geographically as follows:

I. *English-speaking countries of European settlement overseas.* Europeans of the Union of South Africa, Canada, United States, Australia and New Zealand.

II. *Northern and Northwestern Europe.* Denmark, Finland, Iceland, Norway, Sweden, England and Wales, Northern Ireland and Scotland.

III. *Western and Central Europe.* Austria, Belgium, France, Germany, Netherlands.

IV. *Southern and Eastern Europe.* Bulgaria, Greece, Italy, Portugal, Spain and USSR.

V. *Latin America and Antilles.* Federal District of Brazil, Mexico, Jamaica, Trinidad and Tobago, Argentina, Chile and British Guiana.

VI. *Asia.* India, Japan, Ceylon and Cyprus.

187. For each area, two life tables of the present century were selected, namely, the earliest and the most recent, except where the area had been affected by a substantial boundary change. The central dates and the expectations of life at birth were averaged, within each group, both for the earlier and the more recent set of life tables, with results as shown below:

| *Group of countries* | *Earlier life tables* | | *More recent life tables* | | *Average gain in* $^oe_o$ *per annum* |
|---|---|---|---|---|---|
| | *Average date* | *Average* $^oe_o$ | *Average date* | *Average* $^oe_o$ | |
| I.......... | 1912 | 57 | 1949 | 68 | 0.30 |
| II.......... | 1909 | 53 | 1946 | 66 | 0.35 |
| III.......... | 1907 | 50 | 1946 | 65 | 0.39 |
| IV.......... | 1910 | 41 | 1934 | 51 | 0.42 |
| V.......... | 1917 | 38 | 1946 | 50 | 0.41 |
| VI.......... | 1914 | 40 | 1950 | 55 | 0.42 |

188. It will be noted that, with the exception of the first two groups, where expectation of life was fairly high at the outset, the average annual gain in $^oe_o$ has been more or less consistently 0.4 years. The data are too few and two unrepresentative to permit a conclusion of general validity, but they lend some support to the view that expectation of life tends to increase at a fairly uniform rate so long as a high figure has not been attained. More painstaking studies of past trends in mortality have led to a similar conclusion. Moreover, it has been shown that gains in the expectation of life have been increasingly rapid in the course of the first half of this century.[33] Available evidence, however, is not sufficiently comprehensive to permit an exact inference as to the manner in which mortality declines have accelerated in recent times, nor whether they tend to be more rapid in areas of high mortality than in those of low mortality. Estimates for the future must rely on the scanty evidence of the past, taking into consideration the special circumstances affecting the prospects of future mortality decline in specific circumstances.

[33] United Nations, *Age and Sex Patterns of Mortality* (ST/SOA/Series A, Population Studies, No. 22).

### 2. *The model assumption of future mortality decline*

189. In the past, mortality generally declined only at a slow rate where it was at a high level. As witnessed by recent experience in many parts of the world, this need no longer be so. It is now possible to cut down mortality quickly from a high level, even in poverty-stricken areas. It can, therefore, be reasonably assumed that the future mortality declines in areas where mortality is now high or moderately high will be more rapid than they were in areas where mortality conditions were similar during the first half of this century.

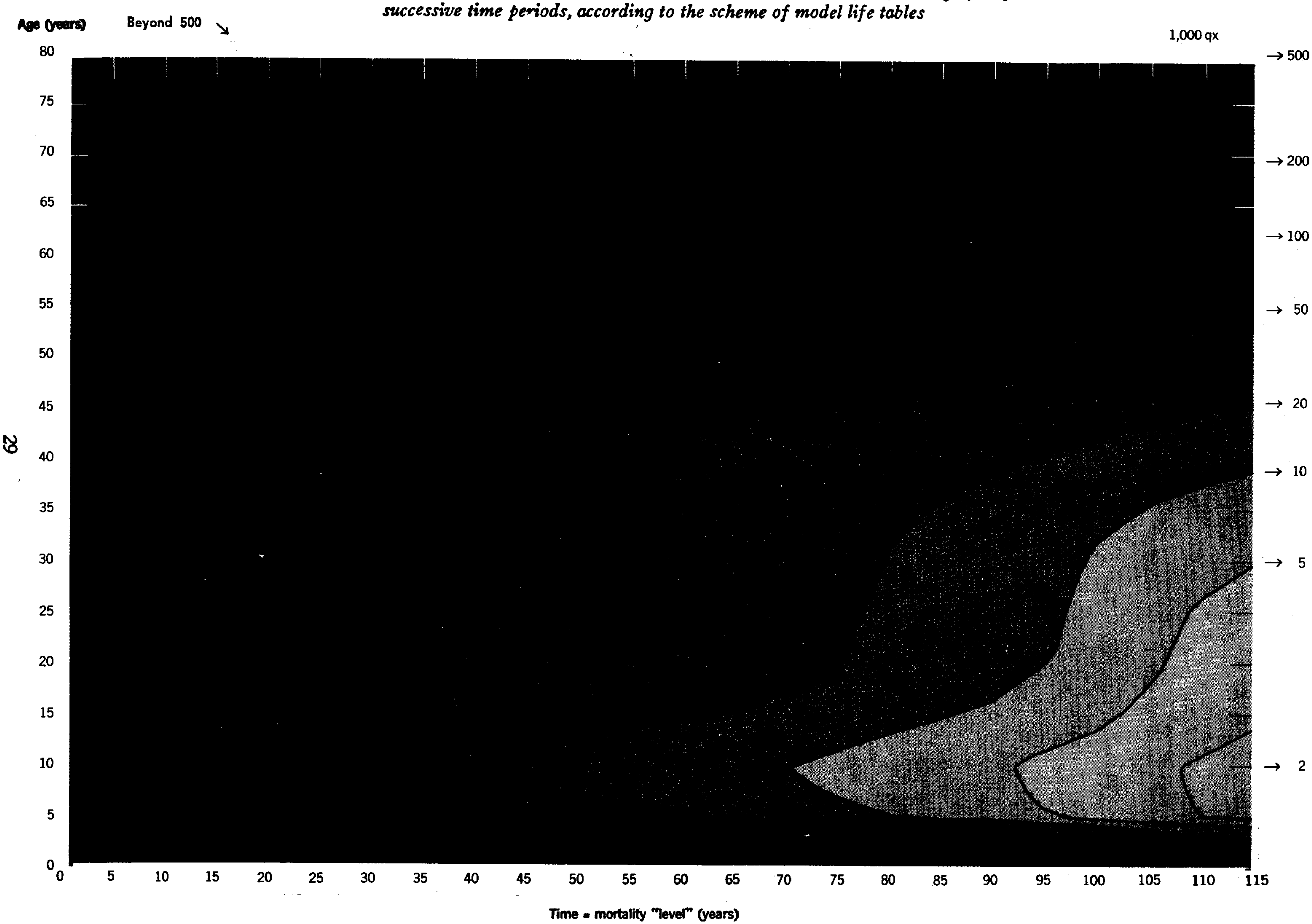
190. Important progress is still to be expected in areas where mortality is already low. Present knowledge and means for the prevention and cure of disease are still not exploited to the utmost in low-mortality countries, and new discoveries are still being made. In terms of expectation of life, however, the effects of further reductions in mortality will be less striking in the future than they were in the past. Where mortality is already very low, deaths below the age of 50 have become so infrequent that even their complete elimination—if that were possible—would result in comparatively little increase in the expectation of life. There are also some prospects for the reduction of mortality at older ages, for which death rates continue to be comparatively high, but progress in this field will probably not be rapid in the near future.

191. If a simple assumption of possible future changes in mortality under the most typical current conditions is to be made, one can envisage a uniform rate of increase in expectation of life at birth up to a certain level, beyond which any further increase will be at a slower pace. It is not suggested that such an assumption would be valid for every population, irrespective of the circumstances. Special factors affecting the prospects in each case must be taken into account.

192. The assumption proposed as a model is an annual gain of 0.5 years in expectation of life, wherever the expectation is less than 55 years. When an expectation of 55 years is attained, the model assumption implies a slight acceleration in gains[34] until the expectation approaches 65 years. Thereupon, the rate of gain slows down and becomes slight when the expectation has risen substantially higher than 70 years. The model life tables presented in the appendix have been arranged in conformity with this assumption. They are spaced at such intervals that, in conformity with the assumption, mortality would pass from the level indicated by one table to that of the next within a five-year period.

[34] The reason, as explained in the appendix, is that at this level of $^oe_o$ past observations have indicated an acceleration in the decrease of infant mortality, requiring some departure from the otherwise rather rigid scheme by which the model life tables were constructed. While, for other ages, mortality declines at a rate consistent with an annual gain of 0.5 year in $^oe_o$ the more rapid decline of infant mortality entails a somewhat more rapid rise in the expectation of life at birth.

*Diagram 1. Contour chart of life-table death rates ($1{,}000q_x$) for males in five-years age groups at successive time periods, according to the scheme of model life tables*



193. As there are 24 model life tables, their spacing by five-year time intervals implies a total time span of 115 years, during which mortality would decline from the highest to the lowest level indicated. It is not assumed that in any case mortality would indeed decline with such uniformity over so long a period. For shorter periods, however, the trends shown by the sequence are characteristic of typical situations now existing in the world.

### 3. *The model mortality surface*

194. A three-dimensional model is required for the graphic representation of changes in mortality, specific by age, in the course of time, the three dimensions being age, time, and the value of the specific rate. If such a model were carved out of a block of wood, mortality being expressed by height, time by length and age by width, the upper surface would be very uneven, as the height of the mortality rate varies both with time and with age. The shape of this "mortality surface" can be described after the fashion of a topographical map, by means of contour lines on a two-dimensional chart of time and age, the lines joining points at which the mortality rates are equal. The time-variation of life-table death rates ($1{,}000q_x$) for five-year age groups of males, according to the model system of life tables, is so represented in diagram 1. The horizontal axis measures time and the vertical axis age, while the curving lines connect points where the mortality rate is at the level indicated by the figure marked against each line. Following these curves across the chart from left to right, one sees how the range of ages subjected to relatively high mortality narrows progressively as time passes, while the range of ages having lower mortality rates is widened.

195. To return to the three-dimensional wooden model, of which the surface contours are represented by this chart, a length-wise cut through it, parallel to the time-axis, would show the trend of a certain age-specific death rate in the course of time. A cross-cut, on the other hand, would show the curve of age-specific mortality rates prevailing at a given moment of time.

196. One cut through the model which is of special interest is a cut in a diagonal direction, like the one indicated by the dotted line. Such a cut, at one-half of a right angle, forms a line along which age advances at the same rate as time. It therefore represents the mortality experience, in the course of a life span, of a cohort of persons born at the same time. The cut along the dotted line, for example, would show the experience of persons born in the year 0, 85 years old in the year 85. Since general mortality conditions are assumed to improve while the cohort grows older, its mortality experience at any age will differ from the rate which prevailed among persons who were at that age when this cohort was born. In a population projection, where each age group currently living is traced into the future, it is this type of mortality experience, relating to the same generation of individuals, which is of interest.

### 4. *Relation of specific life-table functions to the general mortality level*

197. In the model life tables, general mortality levels have been conveniently defined by $^{\circ}e_0$, the expectation of life at birth for both sexes. Moreover, a scale has been attached to the progression of $^{\circ}e_0$, in accordance with an assumption of typical current declines in mortality. The general mortality level can therefore also be designated by the number of the year in this time-scale, beginning with 0 where $^{\circ}e_0$ is 20, and ending with 115 where $^{\circ}e_0$ attains 73.9.

198. If the relationships among the mortality rates for different age groups in a given case follow a typical pattern, that is, the pattern of any one of the model life tables, then the general mortality level can be readily found if one of the age-specific rates is given. For instance, if the age-specific death rate ($1{,}000\ m_x$) for males aged 20-24 years in a given country is known to be 10.21, it is found on the corresponding line in appendix table I that this value relates to mortality level 35, where $^{\circ}e_0$ equals 37.5 years. It can then be said that this particular age-specific rate is typical of an expectation of life at birth of 37.5 years.

199. In practice, when all age-specific rates for a given country have been traced in this manner, it may be found, for instance, that the rates for young persons are generally characteristic of one mortality level, while those for older people agree more nearly with typical rates for a different mortality level. The age-specific functions of an entire life table for the given country can then be estimated by reference to the model tables for both mortality levels.

200. The values for two levels of general mortality can also be interpolated. Let it be supposed that the age-specific death rate for males aged 20-24 years is approximately 10 per 1,000. A value of exactly 10.00 is 21/89 of the way between 10.21, the value for mortality level 35, and 9.32, found at level 40, and so corresponds to a level of 36.18 for which—using the same interpolation —$^{\circ}e_0$ would be 35.59 years. Such fine interpolation, however, is not warranted; the nearest integral figure, 36 in the present instance, is sufficient.

201. The procedure is illustrated in table 14, with the age-specific mortality rates ($1{,}000\ m_x$) computed for the male population of Luxembourg in table 10. In this example, minor fluctuations in the reference to general mortality levels might be largely explained by the small numbers of deaths from which some of the specific rates were computed, but it is noticeable, in contrast with average conditions, that male mortality in Luxembourg during that period was comparatively high at older ages in view of its comparatively low level at young ages.

202. The procedure described above is useful for three purposes. In the first place, it serves as a check on the reliability of the statistics: if the general mortality "level" to which the specific observed rates are referred fluctuates sharply from one age group to the next, there is good reason to doubt the accuracy of the data. It is quite unlikely, for example, that, in any actual population, men aged 20-24 years die as if general mortality were very high while men aged 25-29 die at a rate typically observed where general mortality is very low.

203. In the second place, the peculiarities of mortality conditions appear in the general trend, age group by age group, which runs through the resulting series. Thus, in the example of table 14, it appears that the mortality at older ages corresponds to $^{\circ}e_0$ of 55-60 years only, while the mortality at younger ages is more characteristic of $^{\circ}e_0$ near 66. The situation studied here deviates, in fact, from the average pattern.

TABLE 14. REFERRAL OF AGE-SPECIFIC DEATH RATES OF LUXEMBOURG MALES, 1946–1949, TO CHARACTERISTIC GENERAL MORTALITY LEVELS

| Age (years) | 1,000 $m_x$ [a] | Values found in appendix table I | | | | Interpolated level[b] | Corresponding $e_0$[b] |
|---|---|---|---|---|---|---|---|
| | | Preceding value | | Following value | | | |
| | | Level | 1,000 $m_x$ | Level | 1,000 $m_x$ | | |
| 0........... | 71.1 | 80 | 90.18 | 85 | 70.10 | 85 | 63 |
| 1–4........ | 3.1 | 95 | 3.67 | 100 | 2.45 | 97 | 69 |
| 5–9........ | 1.4 | 90 | 1.45 | 95 | 1.15 | 91 | 66 |
| 10–14........ | 1.1 | 85 | 1.30 | 90 | 1.09 | 90 | 66 |
| 15–19........ | 1.6 | 90 | 1.81 | 95 | 1.49 | 93 | 67 |
| 20–24........ | 2.4 | 90 | 2.52 | 95 | 2.02 | 91 | 66 |
| 25–29........ | 2.5 | 90 | 2.59 | 95 | 2.08 | 91 | 66 |
| 30–34........ | 3.2 | 85 | 3.33 | 90 | 2.79 | 86 | 64 |
| 35–39........ | 4.0 | 80 | 4.50 | 85 | 3.88 | 84 | 63 |
| 40–44........ | 5.7 | 80 | 5.79 | 85 | 5.07 | 81 | 61 |
| 45–49........ | 9.6 | 70 | 9.97 | 75 | 9.07 | 72 | 56 |
| 50–54........ | 11.3 | 80 | 11.78 | 85 | 10.81 | 82 | 62 |
| 55–59........ | 18.6 | 70 | 20.00 | 75 | 18.53 | 75 | 58 |
| 60–64........ | 27.5 | 75 | 27.69 | 80 | 26.05 | 76 | 58 |
| 65–69........ | 42.5 | 70 | 44.32 | 75 | 42.02 | 74 | 57 |
| 70–74........ | 65.2 | 70 | 68.14 | 75 | 65.01 | 75 | 58 |
| 75–79........ | 103.4 | 65 | 107.64 | 70 | 103.28 | 70 | 55 |
| 80–84........ | 173.4 | 55 | 175.67 | 60 | 169.30 | 57 | 49 |
| 85+ ........ | 260.9 | 65 | 264.90 | 70 | 259.83 | 69 | 54 |

[a] Computed in table 10.

[b] Approximate, to the nearest unit.

204. Finally, as will presently be shown, this procedure makes it possible to infer the corresponding survival ratios, needed for the population projection, without a separate computation.

205. One important point, however, should be noted. This procedure is valuable in dealing with the ratio-functions of the life table, such as $m_x$, $q_x$ and $P_x$, the value of which for any age is mathematically independent of its value for other ages. The same interpretation is not impossible if the cumulated life-table functions, such as $l_x$ and $L_x$, are referred to the levels noted in the model life tables: these latter functions depend not only on mortality for the particular age, but also on mortality at other ages.

## 5. *Inference of specific $P_x$ values from general mortality levels*

206. One major advantage in using the model life tables is that most of the sequence of computations—described in chapter III—by which $P_x$-values are obtained can be dispensed with. Once the general mortality levels corresponding to specific values of either $m_x$ or $q_x$ have been obtained (as in the preceding example), the corresponding values of $P_x$ can be read off, or interpolated, in accordance with these "levels" from appendix table V.

207. One further fact, however, must still be considered: the survival ratios ($P_x$) relate to mortality conditions in two successive age groups, while death rates (whether $m_x$ or $q_x$) relate to mortality in one age group only. Theoretically, the $m_x$-values or $q_x$-values of the same population should indicate identical mortality levels.[35] The levels corresponding to age-specific survival ratios ($P_x$), on the other hand, will be intermediate between the mortality levels for the given and the next higher age group, according to $m_x$ (or $q_x$).

208. This fact is demonstrated in table 15, with the $P_x$-values which were computed by the methods of the preceding chapter (see table 11). The levels of these values were located, by interpolations like those of table 14, between the nearest values in appendix table V. These levels of the computed $P_x$-values are indeed intermediate between the levels of the $m_x$-values in the two successive age groups. Moreover, proceeding from age group to age group, the sequence of levels indicated by $P_x$ is smoother than the sequence according to $m_x$. Roughly the same effect is obtained directly by averaging levels of two successive age groups according to $m_x$. If the averaging is properly done, separate computation of $P_x$-values is no longer necessary.

209. To start with, then, only the sequence of either $m_x$ (or $q_x$) is required. Their levels are found by reference to appendix tables I, or II. These levels are then averaged for every successive pair of age groups. Using the averaged levels, the values of $P_x$ are then found in appendix table V.

210. A subtle theoretical problem relates to the exact method of averaging. If the calculation were to be carried out with great refinement and accuracy, simple averaging would introduce undesired distortions. On the

[35] Small differences in the indicated "level" result if the mode of transformation of $m_x$ into $q_x$ differs from that used in the construction of the model life tables.

TABLE 15. COMPARISON OF GENERAL MORTALITY LEVELS IN RESPECT OF SURVIVAL RATIOS FOR LUXEMBOURG MALES, 1946–1949, AS DETERMINED BY TWO DIFFERENT PROCEDURES

| Age (years) | Determination from $P_x$ values already computed: $P_x$[a] | Corresponding level[b] | Determination from $m_x$ values: Level in respect of $m_x$[c] | Same, averaged for successive age groups[d] |
|---|---|---|---|---|
| | ($P_b$=0.9354) | (88) | | (91) |
| 0 | {0.9862 | 94} | 85 | {91 |
| 1–4 | | | 97 | |
| 5–9 | 0.9937 | 90 | 91 | 90.5 |
| 10–14 | 0.9933 | 92 | 90 | 91.5 |
| 15–19 | 0.9901 | 92 | 93 | 92 |
| 20–24 | 0.9879 | 91 | 91 | 91 |
| 25–29 | 0.9859 | 89 | 91 | 88.5 |
| 30–34 | 0.9822 | 85 | 86 | 85 |
| 35–39 | 0.9761 | 82 | 84 | 82.5 |
| 40–44 | 0.9626 | 76 | 81 | 76.5 |
| 45–49 | 0.9490 | 77 | 72 | 77 |
| 50–54 | 0.9284 | 78 | 82 | 78.5 |
| 55–59 | 0.8918 | 75 | 75 | 75.5 |
| 60–64 | 0.8412 | 74 | 76 | 75 |
| 65–69 | 0.7676 | 74 | 74 | 74.5 |
| 70–74 | 0.6649 | 72 | 75 | 72.5 |
| 75–79 | 0.5224 | 66 | 70 | 65 |
| 80–84 | {0.3075 | 67} | 57 | {65 |
| 85 and over | | | 69 | |

[a] As computed in table 11.
[b] Interpolated according to values in appendix table V.
[c] Taken from table 14, where interpolation was made from values in appendix table I.
[d] Method of averaging explained in accompanying text.

other hand, if simplicity is more important than mathematical precision, the use of simple arithmetic averages of the specific levels, according to $m_x$ (or $q_x$), of two adjacent age groups can be regarded as yielding adequate results for the levels of $P_x$, from $P_{5-9}$ to $P_{70-74}$. Three of the four extreme survival ratios, $P_b$ (i.e., survival from a five-year period of births), $P_{0-4}$, $P_{75-79}$, and $P_{80+}$, relate to age groups which do not coincide with the age groups of $m_x$ (or $q_x$). For these, the averaging procedure must be varied somewhat. The procedure suggested is as follows: (*a*) assume the average of levels of $m_0$, $m_{1-4}$ and $m_{5-9}$ both for $P_b$ and $P_{0-4}$, and (*b*) assume the average of levels of $m_{75-79}$, $m_{80-84}$ and $m_{85+}$ both for $P_{75-79}$ and $P_{80+}$.

### 6. *The assumption of future trends in survival ratios*

211. The model assumption of future mortality declines is directly implied in the sequence of model life tables, according to which survival ratios pass, at the end of five years, from the level of one model life table to that of the next. They can be referred to accordingly, adding 5, in respect of every age group, to the number of the mortality level for every five-year interval.

212. In table 15, the general levels of $P_x$ have been established, age group by age group, for Luxembourg males in 1946-1949. If mortality is assumed to decline, from that period onward, in accordance with the model assumption, the future development is shown in table 16. The mortality level has been raised by 5 years in every successive period and the corresponding values of $P_x$ were inferred, by interpolation where necessary, from appendix table V. Actually, the work can be considerably shortened by rounding the mortality levels to the nearest multiple of 5. No interpolations are then necessary, while the loss of accuracy is only slight.

TABLE 16. SURVIVAL RATIOS OF LUXEMBOURG MALES IN SUCCESSIVE FUTURE PERIODS IN ACCORDANCE WITH THE MODEL ASSUMPTION OF MORTALITY DECLINES

| Age (years) | 1946–49 Level | 1951–54 Level | 1951–54 $P_x$ | 1956–59 Level | 1956–59 $P_x$ | 1961–64 Level | 1961–64 $P_x$ |
|---|---|---|---|---|---|---|---|
| ($P_b$) | (91) | (96) | (0.9600) | (101) | (0.9691) | (106) | (0.9753) |
| 0–4 | 91 | 96 | 0.9875 | 101 | 0.9914 | 106 | 0.9940 |
| 5–9 | 90.5 | 95.5 | 0.9950 | 100.5 | 0.9964 | 105.5 | 0.9975 |
| 10–14 | 91.5 | 96.5 | 0.9945 | 101.5 | 0.9959 | 106.5 | 0.9971 |
| 15–19 | 92 | 97 | 0.9923 | 102 | 0.9942 | 107 | 0.9958 |
| 20–24 | 91 | 96 | 0.9903 | 101 | 0.9928 | 106 | 0.9947 |
| 25–29 | 88.5 | 93.5 | 0.9884 | 98.5 | 0.9908 | 103.5 | 0.9929 |
| 30–34 | 85 | 90 | 0.9849 | 95 | 0.9874 | 100 | 0.9897 |
| 35–39 | 82.5 | 87.5 | 0.9794 | 92.5 | 0.9822 | 97.5 | 0.9848 |
| 40–44 | 76.5 | 81.5 | 0.9669 | 86.5 | 0.9706 | 91.5 | 0.9737 |
| 45–49 | 77 | 82 | 0.9533 | 87 | 0.9573 | 92 | 0.9600 |
| 50–54 | 78.5 | 83.5 | 0.9341 | 88.5 | 0.9387 | 93.5 | 0.9429 |
| 55–59 | 75.5 | 80.5 | 0.8991 | 85.5 | 0.9049 | 90.5 | 0.9103 |
| 60–64 | 75 | 80 | 0.8502 | 85 | 0.8573 | 90 | 0.8639 |
| 65–69 | 74.5 | 79.5 | 0.7784 | 84.5 | 0.7872 | 89.5 | 0.7955 |
| 70–74 | 72.5 | 77.5 | 0.6762 | 82.5 | 0.6866 | 87.5 | 0.6965 |
| 75–79 | 65 | 70 | 0.5326 | 75 | 0.5449 | 80 | 0.5566 |
| 80+ | 65 | 70 | 0.3186 | 75 | 0.3272 | 80 | 0.3352 |

213. Where there is reason to expect that the future trends will follow a different course, this model assumption need not be adhered to, but can be varied in several ways, according to the circumstances of the case. Among possible modifications of the model assumption, the following might be considered:

(*a*) *Assumption of slower, or faster, mortality decline.* If there is reason to believe that mortality will decline more, or less, rapidly than would be expected under typical conditions, the mortality levels from which the survival ratios are inferred may be raised by more or less than 5 during each successive period.

214. One possible assumption, in the example of Luxembourg, might be that mortality in 1946-1949 was still affected by somewhat abnormal post-war conditions. Unusually rapid improvement might then be expected in the first five-year period following 1946-1949, after which declines in mortality might revert to the more typical pattern of the model assumption.

(*b*) *Assumption of normalization of mortality pattern.* Observed differences in the levels attributed to $P_x$ for various age groups may be ascribed to abnormal conditions adversely affecting certain age groups. In this event, it may be reasonable to assume that the mortality of the affected age groups will decrease more rapidly than that of other age groups, so that the future pattern of specific mortality conditions will conform more closely to that of a model life table.

215. In the example of Luxembourg, 1946-1949, it might be assumed that abnormal conditions existed in that period which were prejudicial to the health of older persons. Mortality rates at older ages might then be assumed to decrease more rapidly than at younger ages, until they conform to the same level.

(*c*) *Assumption of differences in the mortality experience of each cohort.* Under certain conditions, the mortality of an age group may be more strongly affected by its past experience than by the current conditions affecting the entire population. For example, a cohort exposed to certain health hazards in childhood or in a time of war may have suffered permanent damage to health as a result of which it will be subject to heavier mortality risks at any subsequent age than other cohorts. Some knowledge of the prevailing causes of death and of their incidence among different age groups is necessary to evaluate this possibility in a particular case. If it appears that a certain cohort has been affected in this way, the assumption as to the future progression from one mortality level to the next should be applied to the figures for that same cohort as it grows older, instead of the figures for a given age group at successive points of time.

216. This procedure is illustrated in table 17 with reference to the mortality estimates for Luxembourg males. It is assumed for the purpose of this illustration that the comparatively heavy mortality among older persons in 1946-1949 was attributable to an impairment of their state of health resulting from special debilitating influences at some time earlier in their lives. When the same ages are attained by more recent generations whose health was not similarly damaged, it is presumed that the mortality rates for those ages will decline more rapidly than the model assumption would allow. With respect to the youngest cohorts, not yet born at the initial date, mortality is assumed to conform to the model assumption.

### B. Estimation of survival ratios under diverse practical conditions

217. The method described in the preceding pages, namely the estimation of survival ratios by referral to characteristic mortality "levels" of the model life tables, can be applied wherever adequate statistics for the com-

TABLE 17. SURVIVAL RATIOS OF LUXEMBOURG MALES IN SUCCESSIVE FUTURE PERIODS IN ACCORDANCE WITH THE ASSUMPTION OF MORTALITY DECLINE BY COHORTS

| Age (years) | 1946–49 level | 1951–54 | | 1956–59 | | 1961–64 | |
|---|---|---|---|---|---|---|---|
| | | Level | $P_x$ | Level | $P_x$ | Level | $P_x$ |
| ($P_b$) | (91) | (96) | (0.9600) | (101) | (0.9691) | (106) | (0.9753) |
| 0–4 | 91 | 96 | 0.9875 | 101 | 0.9914 | 106 | 0.9940 |
| 5–9 | 90.5 | 96 | 0.9952 | 101 | 0.9965 | 106 | 0.9976 |
| 10–14 | 91.5 | 95.5 | 0.9942 | 101 | 0.9958 | 106 | 0.9970 |
| 15–19 | 92 | 96.5 | 0.9920 | 100.5 | 0.9937 | 106 | 0.9955 |
| 20–24 | 91 | 97 | 0.9908 | 101.5 | 0.9930 | 105.5 | 0.9945 |
| 25–29 | 88.5 | 96 | 0.9896 | 102 | 0.9924 | 106.5 | 0.9939 |
| 30–34 | 85 | 93.5 | 0.9866 | 101 | 0.9901 | 107 | 0.9922 |
| 35–39 | 82.5 | 90 | 0.9808 | 98.5 | 0.9852 | 106 | 0.9884 |
| 40–44 | 76.5 | 87.5 | 0.9712 | 95 | 0.9759 | 103.5 | 0.9804 |
| 45–49 | 77 | 81.5 | 0.9529 | 92.5 | 0.9614 | 100 | 0.9664 |
| 50–54 | 78.5 | 82 | 0.9326 | 86.5 | 0.9369 | 97.5 | 0.9461 |
| 55–59 | 75.5 | 83.5 | 0.9026 | 87 | 0.9066 | 91.5 | 0.9113 |
| 60–64 | 75 | 80.5 | 0.8509 | 88.5 | 0.8618 | 92 | 0.8664 |
| 65–69 | 74.5 | 80 | 0.7794 | 85.5 | 0.7889 | 93.5 | 0.8018 |
| 70–74 | 72.5 | 79.5 | 0.6805 | 85 | 0.6917 | 90.5 | 0.7021 |
| 75–79 | 65 | 77.5 | 0.5508 | 84.5 | 0.5664 | 90 | 0.5779 |
| (80+) | 65 | 70 | (0.3186) | 82.5 | (0.3388) | 89.5 | (0.3486) |

putation of age-specific death rates, or some elements of a life table, are available. Estimating problems, however, arise where not all of this statistical detail is available or where the statistics are inaccurate or incomplete. The results, in some instances, cannot attain the same degree of accuracy as where the statistics are complete, detailed and accurate. Nevertheless, if based on good judgement, they will usually be adequate for estimating future population.

### 1. *Estimates using data on the population and on deaths by sex and age*

218. As an example of how specific death rates affected by inaccurate age declarations can be utilized, a calculation is carried out in table 18 with the statistics of population by sex and age from the Egyptian census of 1947 and Egyptian statistics on deaths by sex and age. These figures fluctuate considerably from one age group to another, in a manner which must be attributed to inaccurate age statements. As might be expected, the resulting references of the specific death rates to general "levels" of mortality also fluctuate widely. The sequence is partly smoothed as a result of the averaging process by which the "levels" of the $P_x$-values (survival ratios) are found, but is still rather irregular. It is not to be expected that mortality in Egypt should conform precisely to the pattern of any one model. But it is reasonable to expect, if the data were accurate, that deviations from the model patterns would not be abrupt, or irregular, from one age group to another.

219. In order to correct the series of "levels" to which the $P_x$ values for the different age groups should relate, it is desirable to have some information about the manner in which ages of the population and of decedents were mis-stated, and about the completeness of enumeration and death registration at certain particular ages. Without such knowledge, one might suspect both an under-enumeration of infants and an under-registration of infant deaths; the mortality rate for infants, as well as for small children, might be either too high or too low depending on which of these deficiencies is greater. Likewise, one might presume a tendency to exaggerate old age, both in the census and in death reports. If the overstatement of old age is greater in the death returns than in the census, the recorded death rates for old persons will tend to be exaggerated. In fact, for ages 85 and over, an implausibly high death rate is noted. On the other hand, if the age at death is often much overstated, too few deaths are likely to be reported at younger ages, say, between 50 and 75 years. Similar reasoning may be applicable when contrasting the mortality "levels" found for males and females of the same ages. Death reporting for females may perhaps be less complete than for males.

TABLE 18. ESTIMATION LEVELS FOR SURVIVAL RATIOS FROM AGE-SPECIFIC DEATH RATES ACCORDING TO STATISTICS OF EGYPT, 1947

| Age in years (x) | Males | | | | Females | | | |
|---|---|---|---|---|---|---|---|---|
| | | Mortality level for | | | | Mortality level for | | |
| | 1,000 $m_x$ [a] | $m_x$ | P Computed | P Adjusted | 1,000 $m_x$ [a] | $m_x$ | $P_x$ Computed | $P_x$ Adjusted |
| | | | 34 [b] | 35 [c] | | | 38 [b] | 40 [c] |
| 0 | 219.2 | 43 | 34 [d] | 35 | 197.3 | 41 | 38 [d] | 45 |
| 1–4 | 52.3 | 14 | | | 47.2 | 19 | | |
| 5–9 | 5.9 | 45 | 40 | 40 | 4.7 | 54 | 54 | 50 |
| 10–14 | 5.0 | 35 | 43 | 40 | 3.5 | 54 | 61 | 55 |
| 15–19 | 5.2 | 51 | 52.5 | 45 | 3.5 | 68 | 70.5 | 55 |
| 20–24 | 7.1 | 54 | 50.5 | 45 | 4.1 | 73 | 69.5 | 55 |
| 25–29 | 8.6 | 47 | 43.5 | 45 | 5.5 | 66 | 59.5 | 55 |
| 30–34 | 10.8 | 40 | 40 | 45 | 8.2 | 53 | 54.5 | 55 |
| 35–39 | 12.4 | 40 | 41.5 | 45 | 8.1 | 56 | 54 | 55 |
| 40–44 | 14.2 | 43 | 47 | 45 | 9.9 | 52 | 59 | 60 |
| 45–49 | 15.2 | 51 | 50.5 | 50 | 8.7 | 66 | 64 | 60 |
| 50–54 | 20.3 | 50 | 57 | 50 | 12.7 | 62 | 74.5 | 60 |
| 55–59 | 22.1 | 64 | 66 | 50 | 11.0 | 87 | 86 | 60 |
| 60–64 | 30.3 | 68 | 63.5 | 50 | 18.0 | 85 | 82 | 60 |
| 65–69 | 50.2 | 59 | 62.5 | 50 | 31.5 | 79 | 85.5 | 60 |
| 70–74 | 70.7 | 66 | 53.5 | 50 | 45.7 | 92 | 77 | 60 |
| 75–79 | 134.2 | 41 | 45 [e] | 50 | 100.2 | 62 | 66 [e] | 60 |
| 80–84 | 183.6 | 49 | 45 [g] | 50 | 141.7 | 70 | 66 [g] | 60 |
| 85+ | 664.8 | [f] | | | 723.0 | [f] | | |

[a] Taken from United Nations *Demographic Yearbook, 1952.*

[b] Level for $P_b$; taken as average of levels for $m_0$, $m_{1-4}$ and $m_{5-9}$.

[c] Adjusted level for $P_b$.

[d] Level for $P_{0-4}$; taken as average of levels for $m_0$, $m_{1-4}$ and $m_{5-9}$.

[e] Level for $P_{75-79}$; taken as average of levels for $m_{75-79}$ and $m_{80-84}$.

[f] Specific death rate evidently too high, being outside the range of the model life tables.

[g] Level for $P_{80+}$; taken as average of levels for $m_{75-79}$ and $m_{80-84}$.

220. Though this type of reasoning does not permit any very definite conclusions, it does offer some guidance for adjusting the calculated mortality levels in order to make them more plausible. There seems to be some indication that the level advances from childhood to mature adult ages. In other words, an impression is gained that child mortality is rather high in Egypt in relation to the somewhat lower relative mortality levels in later years of life. The "adjusted" series of levels, for estimating the values of $P_x$, is clearly arbitrary and it cannot be claimed that it is accurate, but it probably represents an improvement over the clearly inaccurate original data. The improvement may be sufficient for the purpose of a population projection. The irregularities in the series of "levels" calculated from moderately inaccurate statistics are often not as great as in the present example. The series may then be smoothed by a moving average or some simple formula.

221. A somewhat different problem arises where the series of mortality levels obtained directly from the data is fairly smooth, with the exception of a few age groups. The statistics of Thailand are a case in point. In that country, an infant mortality rate of 79.8 per 1,000 was registered in 1947. This rate appears very low indeed in relation to the death rates of adults, though the latter rates show no marked inconsistencies among themselves. As appears from the calculations shown in table 19, the mortality levels for ages 10 to 55 correspond fairly well to a typical pattern. There appears to be little need to revise them, except perhaps by some slight smoothing with a moving average. Yet, the rates for early childhood and advanced ages appear markedly too low. They can hardly be regarded as consistent with the generally observed pattern. One possible explanation is that, while death registration is fairly accurate at most ages, it is incomplete for infants, small children, and old persons. This hypothesis has been used in substituting apparently more consistent mortality levels for the latter age groups, while accepting those which were calculated for the age range from 10 to 55 years. This hypothesis, which has been adopted in the absence of any specific knowledge of the functioning of death registration in the country, gives results that appear to be at least consistent and plausible.

222. The two examples given do not exhaust the types of difficulties which arise when an effort is made to estimate the mortality levels for various age groups from faulty data. Familiarity with the statistical procedures of the country, and with the kinds of errors to which the data are subject, is evidently important for a realistic adjustment. But even where such knowledge is quite limited, reasonable adjustments can be expected

TABLE 19. ESTIMATION OF LEVELS FOR SURVIVAL RATIOS FROM AGE-SPECIFIC DEATH RATES ACCORDING TO STATISTICS OF THAILAND, 1947

| Age in years (x) | *Males* | | | | *Females* | | | |
|---|---|---|---|---|---|---|---|---|
| | 1,000 $m_x$ [a] | *Mortality level for* $m_x$ | *Mortality level for* $P_x$ *Computed* | *Mortality level for* $P_x$ *Substituted* | 1,000 $m_x$ [a] | *Mortality level for* $m_x$ | *Mortality level for* $P_x$ *Computed* | *Mortality level for* $P_x$ *Substituted* |
| | | | 60 [b] | 45 [c] | | | 62 [b] | 45 [c] |
| 0............ | 57.1 | 88 | 60 [d] | 45 | 50.4 | 87 | 62 [d] | 45 |
| 1– 4......... | 18.1 | 57 | | | 16.2 | 60 | | |
| 5– 9......... | 7.7 | 34 | 35 | 45 | 7.1 | 39 | 43 | 45 |
| 10–14......... | 4.8 | 36 | 42.5 | 45 | 4.2 | 47 | 52.5 | 50 |
| 15–19......... | 5.4 | 49 | 48.5 | 45 | 4.7 | 58 | 56.5 | 55 |
| 20–24......... | 7.3 | 48 | 45.5 | 45 | 6.8 | 55 | 51 | 50 |
| 25–29......... | 9.3 | 43 | 43 | 45 | 9.0 | 47 | 47 | 45 |
| 30–34......... | 10.1 | 43 | 40.5 | 40 | 9.7 | 47 | 45.5 | 45 |
| 35–39......... | 13.1 | 38 | 39 | 40 | 11.2 | 44 | 43.5 | 45 |
| 40–44......... | 15.0 | 40 | 41 | 40 | 12.6 | 43 | 43 | 45 |
| 45–49......... | 18.8 | 42 | 42.5 | 40 | 13.3 | 43 | 46.5 | 45 |
| 50–54......... | 23.4 | 43 | 47.5 | 45 | 16.2 | 50 | 51 | 50 |
| 55–59......... | 26.9 | 52 | 53.5 | 45 | 21.0 | 52 | 55 | 50 |
| 60–64......... | 36.1 | 55 | 58.5 | 45 | 27.7 | 58 | 61.5 | 50 |
| 65–69......... | 48.6 | 62 | 61.5 | 45 | 38.3 | 65 | 66 | 50 |
| 70–74......... | 74.4 | 61 | ... | 45 | 59.8 | 67 | ... | 50 |
| 75–79......... | 123.1 [e] | .. | ... | 45 | 106.3 [f] | .. | ... | 50 |
| 80+ ......... | | | | 45 | | | | 50 |

[a] Data from United Nations *Demographic Yearbook, 1952*.

[b] Level for $P_b$; taken as average of levels for $m_0$, $m_{1-4}$ and $m_{5-9}$.

[c] Substituted level for $P_b$.

[d] Level for $P_{0-4}$; taken as average of levels for $m_0$, $m_{1-4}$ and $m_{5-9}$.

[e] Death rate for ages "75 and over"; rate is evidently too low: if this were the value of $m_{75-79}$, the corresponding level would be 50; the value of $m_{75+}$, for the same level, would be 157.7 per 1,000.

[f] Death rate for ages "75 and over"; rate is evidently too low: if this were the value of $m_{75-79}$, the corresponding level would be 56; the value of $m_{75+}$, for the same level, would be considerably greater.

to give a closer approximation to reality than unadjusted, faulty series.

223. Another problem arises where the registration of deaths is generally deficient for all age groups. In such cases, the age distribution of registered deaths may very well be approximately the same as the age distribution of all deaths, and the levels of mortality shown by the data may be consistent between age groups, but these levels are nevertheless too low.

224. Where there is good reason to believe that a substantial number of deaths escape registration, a coefficient has to be found by which to multiply the reported number of deaths in each age group in order to estimate the true total at all ages. In some countries, studies have been made of the degree of completeness of death registration. Where there is no country-wide estimate of the degree of completeness, it may be possible to arrive at a rough estimate by comparing the data for certain parts of the country, where the registration is known to be more complete, with those for the remaining areas.

225. Where death registration is confined to a particular portion of the country, such as a "registration area", it can sometimes be assumed that mortality conditions in the remainder of the country are roughly similar. Rates computed for the registration area can then be applied to the country as a whole. Sometimes, however, the registration area, or the area of presumably most accurate registration, represents comparatively advanced communities, where death rates may very well be lower than elsewhere in the country. It may then be appropriate to suppose that the same mortality conditions will be attained in the country as a whole after a certain number of years. Survival ratios computed for the area where registration is most accurate can then be used as the estimated ratios for the entire population as of a certain future date.[86]

226. Where only a relatively small portion of the deaths are registered, it is often not possible to relate these deaths to any precise segment of the population. It may still be possible to utilize the registration figures to represent the distribution by ages of decedents, but no significant death rates can be computed directly. In such a situation, it is still possible to assess the approximate mortality level under which the observed distribution of deaths by age would be expected. The estimating methods required in this connexion depend on the use of theoretical population models and are somewhat complex. They will be discussed in another publication.

## 2. *Estimates using detailed data on population and summary data on deaths*

227. The case considered here is one where statistics are available on the sex-age composition of the population but not of deaths. This is the case if only the total numbers of deaths are recorded, and also if the crude death rate or some other mortality measure, such as the expectation of life at birth, has been estimated. If the expectation of life is known or has been estimated, the problem is simple: mortality may then be assumed to conform to the model life table for which $^{o}e_0$ is the same. In the absence of more specific information, this is perhaps the best estimate that can be made.

228. Given the numbers of the population in each sex-age group and the total number of deaths, a model life table can be found, by trial and error, which would result in the same number of deaths. The age-specific death rates ($m_x$) for one of the models in appendix table I are multiplied with numbers of the population in the same sex-age groups and the results added; this process is repeated with other model life tables until the one is found which yields most nearly the given number of deaths.

229. The same procedure is also applicable where statistics of deaths are available by very broad age groups only. For example, the statistics of deaths for British Guiana in 1946 were tabulated in these age groups only: 0, 1-4, 5-14, 15-44, 45-79, and 80 and over, though a more detailed age distribution of the population is available from the census of that year. According to tabulated data, 618 deaths occurred among males aged 15-44. With the census figures and the specific death rates of several model life tables, the following results are obtained:

| *Age (years)* | *Male population* | *Mortality level 50* 1,000 $m_x$ | *Deaths* | *Mortality level 55* 1,000 $m_x$ | *Deaths* |
|---|---|---|---|---|---|
| 15–19........ | 17,770 | 5.32 | 95 | 4.79 | 85 |
| 20–24........ | 15,658 | 7.67 | 120 | 6.93 | 109 |
| 25–29........ | 12,864 | 8.04 | 103 | 7.22 | 93 |
| 30–34........ | 13,165 | 8.64 | 114 | 7.70 | 101 |
| 35–39........ | 11,080 | 9.77 | 108 | 8.67 | 96 |
| 40–44........ | 10,861 | 11.96 | 130 | 10.66 | 116 |
| TOTAL 15–44 | — | — | 670 | — | 600 |

It appears, then, that for males aged 15-44 the mortality level was somewhat less than 55.[87] Similar computations can be carried out for other sex-age groups.

## 3. *Estimates using detailed population statistics only*

230. The level of mortality can also be estimated by means of detailed population statistics only, in the absence of any statistics relating to deaths. Two cases may be distinguished: (*a*) where population statistics by sex and age are available for one date only, and (*b*) where such statistics have been obtained for at least two dates.

231. The first of these cases again requires the use of methods that depend on hypothetical model populations. This subject is to be dealt with in another publication. Since the effect of mortality on the age structure of population is comparatively slight, this method is rarely sufficient for a reliable estimate of the mortality level. Under certain conditions, however, it may be the only method available. In the second case, where census statistics on the population by sex and age at two different dates are available, mortality levels can often be esti-

[86] This type of assumption was made in the United Nations population projection for Guatemala. A life table was found only for one province of the country, which contains the capital city. Application of this table to the population of the whole country indicated that mortality in this province was comparatively low and that, in conformity with general mortality assumption, the same level of mortality might be attained by the country's total population about 5 years later. The life table was then used in the projection by shifting its time-reference 5 years into the future.

[87] This conclusion evidently depends on the supposition that both the statistics of population and of deaths are accurate.

mated with more confidence. An example is presented in table 20 with the census statistics for Iceland of December 1940 and December 1950. Actually, statistics of deaths in Iceland are available, but they are disregarded here in order to illustrate the method. It is assumed for the purpose of the illustration that the effects of migration to and from Iceland during the period in question were negligible and that the census statistics are entirely accurate. In countries lacking death statistics, where this method would be appropriate, the quality of census statistics is likely, in general, to be inferior to that of the Icelandic data, but moderately accurate census data are sufficient to give useful results. It will be seen that the results for Iceland are affected in some instances by the chance fluctuations of small numbers; this complication would not occur in the case of a country with a sizable population.

232. The first step is to obtain ten-year survival ratios for the various five-year age groups, by dividing the numbers of males and females of each age group in 1950 by the corresponding figures for the same cohorts, ten years younger, in 1940. When this has been done, it is necessary to obtain the values corresponding to $P_b$, that is, the ratios of survivors to ages 5-9 and 0-4 in 1950, among births during 1941-1945 and 1946-1950, respectively. In a country where adequate statistics of deaths are lacking, there will ordinarily be no adequate statistics of births which could be related to the census figures for the purpose of computing these ratios. It may be possible, however, to estimate the numbers of births by such methods as are to be discussed in chapter V. For the present example, the numbers of births registered in 1941-1945 and 1946-1950 (15,459 and 18,935, respectively) are introduced into the computations. The registered numbers are available for each sex separately, but here it is assumed that the sex ratio of births is 105 males per 100 females, in order to illustrate the application of an assumption which can validly be used where birth registration statistics are lacking but estimates of total numbers of births are available. The estimated $P_b$ values will be only slightly affected by any error in the assumed sex ratio of births.

233. The ten-year survival ratios so computed from the data for Iceland are shown in columns 6 and 7 of table 20. The ratio obtained for females aged 5-9 in 1940 and 15-19 in 1950 is obviously too high, even exceeding unity, which is manifestly impossible. On the other hand, the ratios for males aged 15-29 in 1940 and females aged 15-24 in 1940 appear to be too low. These peculiarities may be due to emigration and, especially in the case of females, to mis-statement of ages. The latter presumption is confirmed by the impossible survival ratio of females from ages 5-9; too many females may have been reported in the 15-19 year age group at both censuses, with the result that computed survival *to* these ages results in an excessive figure, while computed survival *from* these ages yields a figure that is too low. For the most advanced ages, the computations could not be carried out in the absence of data on numbers of persons aged 85-89, 90-94, and 95 and over, in 1950.

234. The next step is to find the general levels of mortality which correspond to the ten-year survival ratios. Now, a ten-year survival ratio for a five-year group is equivalent to the product of five-year survival ratios for two successive five-year age groups. Hence, the desired levels can be found by locating, in appendix table V, those pairs of successive five-year ratios ($P_x$-functions in the series of model life tables) which, when multiplied together, produce most nearly each of the computed ten-year ratios. The levels so found in this example are tabulated in columns 8 and 9 of table 20.

235. The simplest way of estimating the appropriate levels for five-year survival ratios from the levels obtained for the ten-year ratios is to average each successive pair of the latter levels. This procedure incidentally has the effect of "smoothing" the series. However, in this instance, critical examination of the results obtained in this manner showed that some of them were not acceptable. Those which were clearly inconsistent with other values in the series were replaced by figures which would make a consistent series. The results are tabulated in columns 10 and 11 of table 20.

236. The procedure which has been illustrated can also be applied where census age statistics are much less accurate than those of Iceland, where the time interval between censuses is not an exact multiple of five years, and where migration has been of some importance. Space does not permit a detailed discussion of all the techniques which might be applied to deal with such difficulties. Where age declarations are quite inaccurate, it is advisable to graduate the age statistics at each census with a rather refined formula before computing the ratios. Where the census interval is not a multiple of five years, the statistics of one of the censuses can be carried forward, by methods described in chapter II, to a date which is convenient for comparison with the other census; alternatively, single-year age data may be combined in five-year groups other than those terminating in 0-4 and 5-9 and further adjustments may be made in the computed survival ratios to obtain values corresponding to a multiple of five years. Where immigration has been of some importance but emigration has been negligible, survival ratios may be computed from census tabulations limited to the population born in the country, if such tabulations are available. Alternatively, ratios may be computed from statistics for the whole population and then adjusted to eliminate the estimated effects of migration.[38]

### C. Derivation of future survival ratios from statistics for Costa Rica

237. The illustrative computations relating to the population of Costa Rica presented in chapter II resulted in a preliminary estimate of the sex-age composition as of mid-year 1955 (table 9). The estimate is subject to adjustment for the apparently incomplete enumeration of children in the 1950 census. The method of this adjustment will be explained in chapter V, after methods of estimating fertility have been considered. One of the requirements for such an adjustment, however, is an estimate of mortality levels. In the following pages, the method of estimating these levels and the corresponding survival ratios is explained.

[38] If it is desired to make projections on the assumption that migration will continue in the future to exert the same influence as it had during the period between the two census dates, the survival ratios affected by immigration or emigration as derived from the census data may be used without adjustment. See chapter VII.

TABLE 20. ESTIMATION OF MORTALITY LEVELS FROM CENSUS STATISTICS BY SEX AND AGE FOR ICELAND, 1940–1950

| Age in 1940 (years) | Population, 1940 (and births, 1941–50) | | Age in 1950 (years) | Population, 1950 | | 10-year survival ratio | | Approximate level of 10-year ratio | | Estimated level of 5-year survival ratio | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Males | Females | | Males | Females | Males | Females | Males | Females | Males | Females |
| Unborn.......... | (9,698) | (9,237) | 0– 4 | 9,466 | 8,813 | (0 9761)[a] | (0.9541)[a] | (105)[a] | (90)[a] | (95)[ab] | (95)[ab] |
| | (7,918) | (7,541) | 5– 9 | 7,431 | 7,167 | (0.9385)[c] | (0.9504)[c] | (95)[c] | (95)[c] | (95) | (95) |
| 0– 4........... | 5,922 | 5,725 | 10–14 | 5,851 | 5,656 | 0.9880 | 0.9879 | 100 | 100 | 95 | 95[b] |
| 5– 9........... | 6,231 | 5,931 | 15–19 | 6,159 | 5,969 | 0.9884 | 1.0064 | 90 | [d] | 95 | 95[b] |
| 10–14........... | 6,317 | 6,127 | 20–24 | 6,229 | 6,058 | 0.9861 | 0.9887 | 95 | 95 | 90 | 90[b] |
| 15–19........... | 5,934 | 5,690 | 25–29 | 5,727 | 5,416 | 0.9651 | 0.9518 | 80 | 65 | 85[b] | 85[b] |
| 20–24........... | 5,313 | 5,097 | 30–34 | 5,109 | 4,873 | 0.9616 | 0.9561 | 80 | 70 | 85[b] | 85[b] |
| 25–29........... | 4,773 | 4,677 | 35–39 | 4,599 | 4,558 | 0.9635 | 0.9746 | 80 | 90 | 85 | 90 |
| 30–34........... | 4,458 | 4,178 | 40–44 | 4,276 | 4,057 | 0.9592 | 0.9710 | 85 | 90 | 90 | 90 |
| 35–39........... | 4,060 | 3,868 | 45–49 | 3,870 | 3,749 | 0.9532 | 0.9692 | 90 | 95 | 95 | 90 |
| 40–44........... | 3,677 | 3,670 | 50–54 | 3,453 | 3,482 | 0.9391 | 0.9488 | 95 | 90 | 100 | 90 |
| 45–49........... | 3,380 | 3,519 | 55–59 | 3,120 | 3,278 | 0.9231 | 0.9315 | 105 | 90 | 100 | 95 |
| 50–54........... | 2,574 | 2,694 | 60–64 | 2,223 | 2,423 | 0.8636 | 0.8994 | 95 | 95 | 105 | 100 |
| 55–59........... | 2,016 | 2,287 | 65–69 | 1,674 | 1,962 | 0.8304 | 0.8579 | 110 | 100 | 110 | 105 |
| 60–64........... | 1,765 | 2,087 | 70–74 | 1,297 | 1,645 | 0.7348 | 0.7882 | 110 | 110 | 110[b] | 110 |
| 65–69........... | 1,400 | 1,790 | 75–79 | 917 | 1,215 | 0.6550 | 0.6788 | [e] | 115 | 110[b] | 110 |
| 70–74........... | 1,107 | 1,486 | 80–84 | 506 | 759 | 0.4571 | 0.5108 | 110 | 110 | 110[b] | 110[b] |
| 75–79 | | | 85–89 | | | | | [f] | [f] | 110[b] | 110[b] |
| 80–84 ......... | 1,378 | 2,307[g] | 90–94 | 300 | 593 | (0.2177) | (0.2570) | [f] | [f] | 110[b] | 110[b] |
| 85+ | | | 95+ | | | | | [f] | [f] | 100[b] | 110[b] |

[a] Five-year survival ratio ($P_b$) from births in 1946–50 to ages 0–4 in 1950.

[b] Estimate arbitrarily inserted in order to obtain a consistent series.

[c] Ten-year survival ratio from births in 1941–45 to ages 5–9 in 1950.

[d] Computed survival ratio impossibly high.

[e] Computed survival ratio probably too high, and outside range of model tables.

[f] Level cannot be readily computed from the data.

238. Since the publication of the United Nations population projections for Central America,[39] an official life table for Costa Rica, relating to the 1949-1951 period, has become available and some of the relevant functions have been published in the United Nations *Demographic Yearbook, 1954,* though not separately for each sex. The $l_x$-function, as tabulated for both sexes combined, is used as a starting point for the present calculations.

239. Differences between successive values of $l_x$ result in specific values of $d_x$ and division of each $d_x$ by the corresponding $l_x$ results in $q_x$, the life-table mortality rate, as shown in table 21. The values of $q_x$, however, are for both sexes combined; they cannot be referred readily to the $q_x$-values for the model life tables (in appendix table II), which are given for each sex separately. As a rather rough working assumption, it can be supposed that each specific level of $q_x$, for the two sexes combined, approximates the average of the corresponding levels for the two sexes. The interpolations in the series of model life-table values, therefore, have to be made twice, once in respect of males and once in respect of females, and the average of the results is then taken as the required "level" in respect of each age group. It is not possible, on this basis, to find whether the position of males or of females in respect of mortality is more favourable, and it will have to be assumed that, at each age, male and female mortality rates correspond to the same level. The corresponding levels in respect of $P_x$ are then found by means of the averaging process described in paragraph 210 of this chapter.

240. The sequence of "levels" of $P_x$, found in this manner, shows no obvious signs of inconsistency. Death risks at the ages between 5 and 30 years in Costa Rica may well be low in comparison with the mortality level implied by the rates for earliest childhood and age groups over 40. The only abrupt change in the mortality levels ascribed to $P_x$-values is between ages 0-4 and 5-9, namely from 67.5 to 77.5. In part, this jump reflects the relatively high value of $q_{1-4}$, which corresponds to mortality level 61.5. It is unlikely that the registration of deaths of small children was excessive, but it is possible that some of the infants who died were reported as one year old. In the absence of any knowledge on this subject, the $P_x$-levels for these age groups are accepted as computed.

241. The next point to consider is the assumption of mortality decline. There is no doubt that mortality has declined substantially in Costa Rica during recent years (see the death rates shown in part D of chapter II), and it is highly probable that the decline will continue.

[39] *The Population of Central America (including Mexico), 1950-1980* (ST/SOA/Ser.A, No. 16).

TABLE 21. ESTIMATION OF MORTALITY LEVELS FOR COSTA RICA, 1949–1951, FROM LIFE-TABLE MORTALITY RATES ($q_x$) FOR BOTH SEXES

| Age in years ($x$) | $l_x$ (both sexes) | $d_x$ (both sexes) | 1,000 $q_x$ (both sexes) | Mortality levels: $q_x$ (males) | $q_x$ (females) | $q_x$ (both sexes) | $P_x$ |
|---|---|---|---|---|---|---|---|
| | | | | | | | 67.5[a] |
| 0 | 100,000 | 9,706 | 97.1 | 76 | 71 | 73.0 } | 67.5 |
| 1–4 | 90,294 | 5,447 | 60.3 | 62 | 61 | 61.5 } | |
| 5–9 | 84,847 | 1,155 | 13.6 | 72 | 72 | 72 | 77.5 |
| 10–14 | 83,692 | 566 | 6.8 | 84 | 82 | 83 | 84.5 |
| 15–19 | 83,126 | 791 | 9.5 | 88 | 84 | 86 | 84 |
| 20–24 | 82,335 | 1,287 | 15.6 | 84 | 80 | 82 | 80 |
| 25–29 | 81,048 | 1,505 | 18.6 | 80 | 77 | 78.5 | 76.5 |
| 30–34 | 79,543 | 1,781 | 22.4 | 75 | 74 | 74.5 | 72.5 |
| 35–39 | 77,762 | 2,166 | 27.9 | 72 | 69 | 70.5 | 70 |
| 40–44 | 75,596 | 2,557 | 33.8 | 73 | 67 | 70 | 69 |
| 45–49 | 73,039 | 3,314 | 45.4 | 74 | 63 | 68.5 | 69 |
| 50–54 | 69,725 | 4,296 | 61.6 | 76 | 62 | 69 | 66 |
| 55–59 | 65,429 | 6,123 | 93.6 | 71 | 55 | 63 | 64 |
| 60–64 | 59,306 | 7,814 | 131.8 | 74 | 57 | 65.5 | 66 |
| 65–69 | 51,492 | 9,745 | 189.3 | 75 | 59 | 67 | 66.5 |
| 70–74 | 41,747 | 11,867 | 284.3 | 73 | 59 | 66 | 67 |
| 75–79 | 29,880 | 11,940 | 399.6 | 74 | 63 | 68.5 | 71 |
| 80–84 | 17,940 | 9,349 | 521.1 | 86 | 71 | 78.5 } | 71 |
| 85+ | 8,591 | 8,591 | 1,000.0 | .. | .. | ... } | |

[a] Estimated level for $P_b$.

Efforts are being made to improve public sanitation, to combat diseases, and to educate the population in matters of hygiene. The model assumption, implying gains of about 2½ years in life expectation every five years, is accepted here as appropriate to the conditions, although a study of health problems, medical and sanitary resources, and existing plans for the development of health programmes in the future might lead to a modification of this assumption.

242. In view of deviations of the age-specific mortality indices from average patterns, the further question arises whether the calculations relevant to future mortality should be carried out in terms of constant age groups, or in respect of cohorts which advance in age as time progresses. A study of conditions in the country might help to resolve this question also, by showing whether the relatively low mortality of the younger age groups appears to be the consequence of their having received better education or nutrition, for example, than the older generations. If so, the future changes in survival levels may better be estimated in relation to cohorts than to fixed age groups.

243. For the present purpose, the simpler procedure of estimates according to age groups is adopted. The alternative of mortality estimates by cohorts seems at least equally plausible, but it would involve complications.[40] If mortality declines were estimated in relation to cohorts rather than age groups, the result would be more rapid mortality decline in the middle adult ages and probably also among children. The assumption of mortality decline in relation to fixed age groups, therefore, can be regarded as a rather conservative one.

244. The latter assumption is worked out in table 22. The mortality level for each age group is raised by 7.5 points from 1949-1951 to 1955-1960, and by 5 points for each succeeding period. The corresponding values of $P_x$ are determined by reference to appendix table V, with interpolations wherever required. Actually, this degree of refinement is hardly necessary. Mortality levels rounded to the nearest multiple of 5 and $P_x$-values derived directly from the appropriate tabulated figures would give sufficiently accurate results.

[40] First, for the period from 1949-1951 to 1955-1960, a 7½-year time interval must be allowed for, i.e., mortality levels have to be raised by 7.5 points, an assumption which does not fit well with the five-year age groups of cohorts. This problem could be resolved by shifting the mortality levels—diagonally for cohorts—first by five years, then by another five years, and taking the average of the two results. Second, lower levels, implying higher mortality, are indicated for the age groups under ten years in 1950 than for those from 10 to 25 years. Unless it is assumed that the cohorts under ten in 1950 do not share the same health advantages as the next older cohorts, the cohort method requires some assumption for a particularly rapid decline of their mortality during the next five years. Furthermore, some basis is required for estimating the initial mortality rates, at ages under ten years, of the cohorts yet to be born. It is difficult to make realistic assumptions in such matters without a full examination of the relevant conditions in the country.

TABLE 22. ESTIMATION OF FUTURE SURVIVAL RATIOS ($P_x$) FOR COSTA RICA, IN ACCORDANCE WITH THE MODEL ASSUMPTION OF MORTALITY DECLINE BY FIXED AGE GROUPS

| *Age in years* (*x*) | *Mortality "levels"* (*either sex*) | | | | | *(Survival ratios)* *Males* | | | | *Females* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *1949/51* | *1955/60* | *1960/65* | *1965/70* | *1970/75* | *1955/60* | *1960/65* | *1965/70* | *1970/75* | *1955/60* | *1960/65* | *1965/70* | *1970/75* |
| ($P_b$) .......... | (67.5) | (75.0) | (80.0) | (85.0) | (90.0) | (0.8877) | (0.9070) | (0.9262) | (0.9438) | (0.9036) | (0.9208) | (0.9380) | (0.9535) |
| 0– 4 ......... | 67.5 | 75.0 | 80.0 | 85.0 | 90.0 | 0.9648 | 0.9708 | 0.9765 | 0.9818 | 0.9669 | 0.9731 | 0.9791 | 0.9844 |
| 5– 9 ......... | 77.5 | 85 | 90 | 95 | 100 | 0.9924 | 0.9937 | 0.9949 | 0.9963 | 0.9932 | 0.9948 | 0.9962 | 0.9972 |
| 10–14 ......... | 84.5 | 92 | 97 | 102 | 107 | 0.9933 | 0.9947 | 0.9960 | 0.9972 | 0.9947 | 0.9960 | 0.9971 | 0.9978 |
| 15–19 ......... | 84 | 91.5 | 96.5 | 101.5 | 106.5 | 0.9899 | 0.9920 | 0.9940 | 0.9957 | 0.9920 | 0.9940 | 0.9956 | 0.9968 |
| 20–24 ......... | 80 | 87.5 | 92.5 | 97.5 | 102.5 | 0.9860 | 0.9886 | 0.9911 | 0.9934 | 0.9882 | 0.9908 | 0.9930 | 0.9948 |
| 25–29 ......... | 76.5 | 84 | 89 | 94 | 99 | 0.9834 | 0.9861 | 0.9886 | 0.9911 | 0.9852 | 0.9879 | 0.9904 | 0.9925 |
| 30–34 ......... | 72.5 | 80 | 85 | 90 | 95 | 0.9792 | 0.9822 | 0.9849 | 0.9874 | 0.9811 | 0.9842 | 0.9868 | 0.9893 |
| 35–39 ......... | 70 | 77.5 | 82.5 | 87.5 | 92.5 | 0.9728 | 0.9762 | 0.9794 | 0.9822 | 0.9766 | 0.9798 | 0.9828 | 0.9854 |
| 40–44 ......... | 69 | 76.5 | 81.5 | 86.5 | 91.5 | 0.9629 | 0.9669 | 0.9700 | 0.9737 | 0.9702 | 0.9737 | 0.9768 | 0.9796 |
| 45–49 ......... | 69 | 76.5 | 81.5 | 86.5 | 91.5 | 0.9483 | 0.9529 | 0.9569 | 0.9607 | 0.9602 | 0.9642 | 0.9678 | 0.9709 |
| 50–54 ......... | 66 | 73.5 | 78.5 | 83.5 | 88.5 | 0.9232 | 0.9289 | 0.9341 | 0.9887 | 0.9416 | 0.9467 | 0.9515 | 0.9558 |
| 55–59 ......... | 64 | 71.5 | 76.5 | 81.5 | 86.5 | 0.8869 | 0.8939 | 0.9003 | 0.9060 | 0.9125 | 0.9194 | 0.9256 | 0.9314 |
| 60–64 ......... | 66 | 73.5 | 78.5 | 83.5 | 88.5 | 0.8398 | 0.8478 | 0.8552 | 0.8619 | 0.8716 | 0.8800 | 0.8880 | 0.8951 |
| 65–69 ......... | 66.5 | 74 | 79.0 | 84 | 89 | 0.7678 | 0.7775 | 0.7864 | 0.7947 | 0.8037 | 0.8141 | 0.8240 | 0.8328 |
| 70–74 ......... | 67 | 74.5 | 79.5 | 84.5 | 89.5 | 0.6691 | 0.6805 | 0.6907 | 0.7003 | 0.7060 | 0.7185 | 0.7304 | 0.7411 |
| 75–79 ......... | 71 | 78.5 | 83.5 | 88.5 | 93.5 | 0.5531 | 0.5642 | 0.5748 | 0.5848 | 0.5914 | 0.6048 | 0.6171 | 0.6288 |
| 80+ .......... | 71 | 78.5 | 83.5 | 88.5 | 93.5 | (0.3328) | (0.3404) | (0.3473) | (0.3537) | (0.3552) | (0.3629) | (0.3699) | (0.3762) |

# V. ESTIMATION OF CURRENT LEVEL AND FUTURE TREND OF FERTILITY

## A. The measurement of fertility

245. In addition to the calculations concerning future numbers of survivors from various sex-age groups of the present population, a "component" projection requires estimates of the numbers of children to be born during successive future periods. These estimates are derived from the estimated numbers of surviving females in the childbearing ages at each future date, with assumptions as to their fertility.

### 1. *The relevance of alternative fertility measures for population projections*

246. Where statistics of birth by age of mother are available, calculations with respect to fertility for population projections are usually made in terms of specific birth rates for various age groups of the female population. These rates are calculated for a past period and either assumed to remain constant in the future or projected by means of suitable assumptions as to their expected trends; the future rates are then multiplied by the estimated numbers of female survivors in the corresponding age groups to obtain the estimated numbers of births. However, a current measure of age-specific birth rates cannot be obtained from the vital statistics of most countries. We shall therefore have to be concerned here mainly with the use of cruder measures.

247. The simplest measure of fertility is the crude birth rate, that is, the numbers of births annually per 1,000 of the total population. The use of this measure for estimating the future numbers of births, however, fails to take into account the results of the survivorship computations with respect to any expected future changes in the proportion of the population that consists of women in the childbearing ages. It is better for this purpose to measure fertility in terms of the annual number of births per 1,000 women of childbearing ages. The calculation of this measure requires a choice of upper and lower limits for the childbearing ages. In most circumstances, it is appropriate to let the childbearing group be represented by the female population 15-44 years old. Some births occur to girls under 15 years of age and some to women over 45, but it is not expedient to include all the ages at which any woman may bear children, for the estimated future numbers of births might then be unduly influenced by variations in the numbers of women at the extremes of the potentially fertile period, who would in any case produce very few children. Narrower limits may be more suitable in some cases; for example, where women rarely marry before the age of 18 or 19 years, better results may be obtained by taking the female population 20 to 44 years of age to represent the childbearing group.

248. Although estimates of future births based on projected rates per 1,000 women of childbearing ages take full account of changes in the size of this segment of the population, they do not take account of any changes which may be expected in the age distribution of the women within this group. Such changes may be important because fertility varies greatly with the age of women; it is always much lower, for example, in the case of women 35 to 44 years of age than of women 20 to 29. The only way to take changes in the age distribution fully into account is to make separate estimates of the future fertility rates for women in each age group, but where this cannot be done for lack of adequate data, an approximate compensation for the effects of changing age distribution can be introduced by making the computations in terms of a modified birth rate.

249. The United Nations projections were made by means of such a modified rate, called the "sex-age adjusted birth rate". Its derivation is explained below, and the discussion in the subsequent sections of this chapter is based, for the most part, on the assumption that this or some similar measure of fertility is to be employed. It is better, however, to make the calculations in terms of age-specific birth rates where the necessary data are available, and for those countries which have very detailed statistics, still more elaborate measures of fertility measures may be utilized. At this point it is relevant to consider very briefly some alternative measures and the situations in which it may be advantageous to use them if possible.

250. Given the necessary data, marital fertility rates may be calculated for married women of various ages and illegitimate birth rates for unmarried women, and these may be applied to estimates of future female population classified by marital status within each age group. Further classifications may be introduced, where the data permit, to show the distribution of married women by duration of marriage and number of children already born to them, and to measure their fertility in relation to these variables. Such measures are appropriate where substantial changes have occurred, or are expected to occur, in marriage rates, age at marriage, or the timing of births.

251. Changes in the timing of marriages and births—usually in response to changing economic conditions—may have cumulative effects on age-specific fertility rates. As an example, we may consider the effects of a tendency of women to marry at younger ages than previously. At first, the number of marriages will increase as the marriage rates for the youngest age groups rise while the older women continue to marry as before, but eventually, once the new pattern has become prevalent, the number of women marrying at older ages falls off, most women already being married before they attain those ages, and so the annual number of marriages returns to normal with a changed distribution by age at marriage. This "staggering" effect of a change in marriage habits will affect the birth rate, producing temporary variations in the number of births and their distribution by birth order, as well as a long-range effect due to the relationship between fertility and age at marriage. Soon after the increase of marriages, there is likely to follow a temporary increase in the number of first births; after a time, the number of second births will rise, etc. These and other possible effects of changes in demographic factors must be kept in view when deciding what fertility measure it is most appropriate to employ in a given situation.

## 2. *The "sex-age adjusted birth rate"*

252. The "sex-age adjusted birth rate"[41] used for the United Nations projections is defined as the number of births per 1,000 of a weighted aggregate of numbers of women in the various five-year age groups from 15 to 44. A standard set of weights is used in computing this aggregate, the weights being roughly proportional to the typical relative fertility rates of the various age groups, as determined by examination of the available statistics of births by age of mother for various countries.

253. The use of a standard system of weights for this purpose is justified by the observation that, even under greatly differing conditions of fertility, the relative levels of age-specific rates for women in the age groups from 15-19 to 40-44 are not very different. The absolute levels of these rates, of course, are higher where fertility is high than where it is low, yet on the whole the percentages of all births occurring to women in a given age group are rather similar.[42] An illustration is given by the comparison of age-specific fertility rates for France in 1936 and Formosa in 1950, shown in table 23.

TABLE 23. ABSOLUTE AND RELATIVE AGE-SPECIFIC FERTILITY RATES FOR WOMEN 14 TO 44 YEARS OF AGE: FRANCE, 1936 AND FORMOSA, 1950

| *Age of women (years)* | *Live births per 1,000 women* | | *Relative rate (total, 15–44 = 100)* | |
|---|---|---|---|---|
| | *France, 1936* | *Formosa, 1950* | *France, 1936* | *Formosa, 1950* |
| Total, 15–44[a] | 406.6 | 1,170.5 | 100.0 | 100.0 |
| 15–19 | 27.2 | 60.9 | 6.7 | 5.2 |
| 20–24 | 120.6 | 245.0 | 29.7 | 20.9 |
| 25–29 | 119.2 | 295.8 | 29.3 | 25.2 |
| 30–34 | 79.3 | 266.9 | 19.5 | 22.8 |
| 35–39 | 44.6 | 191.5 | 11.0 | 16.4 |
| 40–44 | 15.7 | 110.9 | 3.9 | 9.5 |

[a] Total of age-specific rates, excluding those of females aged less than 15 or 45 and over.

254. The relative rates are quite similar, although Formosan fertility in 1950 was almost three times that of France in 1936.

255. Relative rates have been computed for 52 countries, utilizing all the data on births by age of mother available in the *Demographic Yearbook*.[43] Averages were obtained for each country for all the years for which the data were given. These averages were averaged again for a group of 15 countries having comparatively high fertility, a group of 37 countries with relatively low fertility, and for all 52 countries combined. The variability in each of the relative rates among the 52 countries was also examined and the mean deviation from the average was computed. The results are presented in table 24. It will be noted that the variability of the relative rates is most important in the age groups 15-19 and 40-44 and is quite small in the age range from 20 to 39 years.

TABLE 24. RELATIVE AGE-SPECIFIC FERTILITY RATES OF WOMEN, AVERAGED FOR GROUPS OF COUNTRIES, AND WEIGHTS SELECTED FOR FERTILITY MEASURE

| *Age of women (years)* | *Average of relative fertility rates* | | | *Mean deviation from average (total, 52 countries)* | *Weights for computation of adjusted birth rates*[a] |
|---|---|---|---|---|---|
| | *Total, 52 countries* | *15 countries of high fertility* | *37 countries of low fertility* | | |
| Total, 15–44 | 100.0 | 100.0 | 100.0 | ... | ... |
| 15–19[b] | 6.3 | 9.3 | 5.1 | ±2.7 | 1 |
| 20–24 | 25.3 | 25.1 | 25.4 | ±3.5 | 7 |
| 25–29 | 27.6 | 25.5 | 28.5 | ±2.1 | 7 |
| 30–34 | 21.1 | 19.6 | 21.7 | ±2.1 | 6 |
| 35–39 | 13.4 | 13.7 | 13.2 | ±2.1 | 4 |
| 40–44[c] | 6.3 | 6.9 | 6.0 | ±2.2 | 1 |

[a] The method of selection of weights is explained in the accompanying text.
[b] Including an allowance for births to mothers below age 15.
[c] Including an allowance for births to mothers aged 45 and over.

256. The weights shown in the last column of table 24 are those used in computing the sex-age adjusted birth rates for the United Nations projections. Since it was considered convenient that these synthetic rates should be of the same order of magnitude as crude birth rates per 1,000 of the total population, the weights were so chosen, not only that they would be roughly proportionate to the averages of the age-specific birth rates,[44] but also that the sum of their products with the corresponding numbers of women in the various age groups would ordinarily be of the same order of magnitude as the total population. The computation of the weighted aggregate of women and the sex-age adjusted birth rate is illustrated in table 25 with data for Cyprus. In this example, the sum of products of the weights and the numbers of women in the various age groups is very nearly the same as the total population, which was officially estimated to be 484,714 at mid-year 1950. If the weighted sum of women happens to equal the total population, the adjusted birth rate will equal the crude birth rate; it will exceed the crude rate if the weighted sum is smaller, and it will be smaller than the crude rate if the weighted sum is larger, than the total population. In the case of Cyprus, the crude birth rate was 29.95 per 1,000.

[41] In the two reports on future population estimates previously cited, this measure was referred to as a "standardized birth rate". This term is now avoided since it might create confusion where birth rates are standardized for different purposes by different procedures.

[42] Considerable derivations from the normal pattern may occur in unusual situations where, in response to a severe economic crisis, or as a result of war, births are postponed for several years, but compensated again when normal conditions have been restored.

[43] United Nations, *Demographic Yearbook, 1954,* table 11. The few births occurring before age 15 were included in the 15-19 group, and the few births after age 45 in the 40-44 group.

[44] The weights are not strictly proportional to the average relative rates for all countries but, in view of the mean deviations, they are not inconsistent. They would imply the following set of age-specific relative rates: 3.8 for women aged 15-19; 26.9 for women 20-24; 26.9 for women 25-29; 23.1 for women 30-34; 15.4 for women 35-39; and 3.8 for women 40-44. Because of variability in relative rates at ages 15-19 and 40-44, it was deemed advisable to select somewhat lower weights for these two age groups than suggested by the series of observed averages. The exact weights suggested by that series would be 1.6; 6.6; 7.2; 5.5; 3.5; and 1.6. Experience shows that some modification of weights has only a slight effect on the weighted result (total, or average).

TABLE 25. COMPUTATION OF "SEX-AGE ADJUSTED BIRTH RATE" FOR CYPRUS, 1950

| Age (years) | Female population, 1950[a] | Weight | Product | Registered births, 1950 | "Sex-age adjusted birth rate" |
|---|---|---|---|---|---|
| 15-19 | 22,438 | 1 | 22,438 | ... | ... |
| 20-24 | 21,465 | 7 | 150,255 | ... | ... |
| 25-29 | 18,964 | 7 | 132,748 | ... | ... |
| 30-34 | 16,896 | 6 | 101,376 | ... | ... |
| 35-39 | 15,555 | 4 | 62,220 | ... | ... |
| 40-44 | 14,286 | 1 | 14,286 | ... | ... |
| TOTAL | ... | ... | 483,323 | 14,517 | 30.04 |

[a] Data taken from the *Demographic Yearbook, 1952.*

257. To examine variations in the ratio of weighted sum of women to total population, computations have been carried out with respect to the 62 populations for which sufficient statistics by sex and age were presented in the 1954 issue of the *Demographic Yearbook*. For 25 of the 62 populations, the weighted sum of women differed from the total population by no more than 5 per cent; for 29, it differed by at least 5 but no more than 10 per cent; differences greater than 10 per cent were found in 8 unusual cases.[45] The average of the 62 computed ratios of weighted sum of women to total population was found to be 0.983.

258. Computations have also been carried out with respect to the annual estimates of the Swedish population by sex and age, the results of which are presented in table 26. Sweden's total population increased constantly from 1938 to 1954, but the weighted sum of women stopped increasing in 1942 and declined subsequently. The weighted sum was at first greater than the total population, but after 1946 it was smaller. Measured by crude birth rates, the trend of Swedish fertility shows a peak in 1944-1945, reached by a rapid rise of the rate after 1941, and followed by an almost equally rapid decline; crude birth rates in 1951-1954 were no higher than those in 1938-1941. In terms of the sex-age adjusted birth rate, the rise in fertility from 1941 to 1944 was steeper, and the subsequent fall, from 1945 to 1951, much slower. In 1951-1954 the sex-age adjusted birth rate stood at a level intermediate between that of 1938-1941 and 1944-1945. In other words, fertility in 1951-1954 was actually higher than in 1938-1941, even though the crude birth rate was no higher, owing to the effects of a changed sex-age structure. The trends of the crude and sex-age adjusted birth rates for Sweden are also illustrated in figure 1.

259. It is emphasized that sex-age adjusted birth rates are only a simple device for eliminating the effects of changes in sex-age structure in a measurement of the fertility trend and an estimation of future births. They continue to be affected by such demographic factors

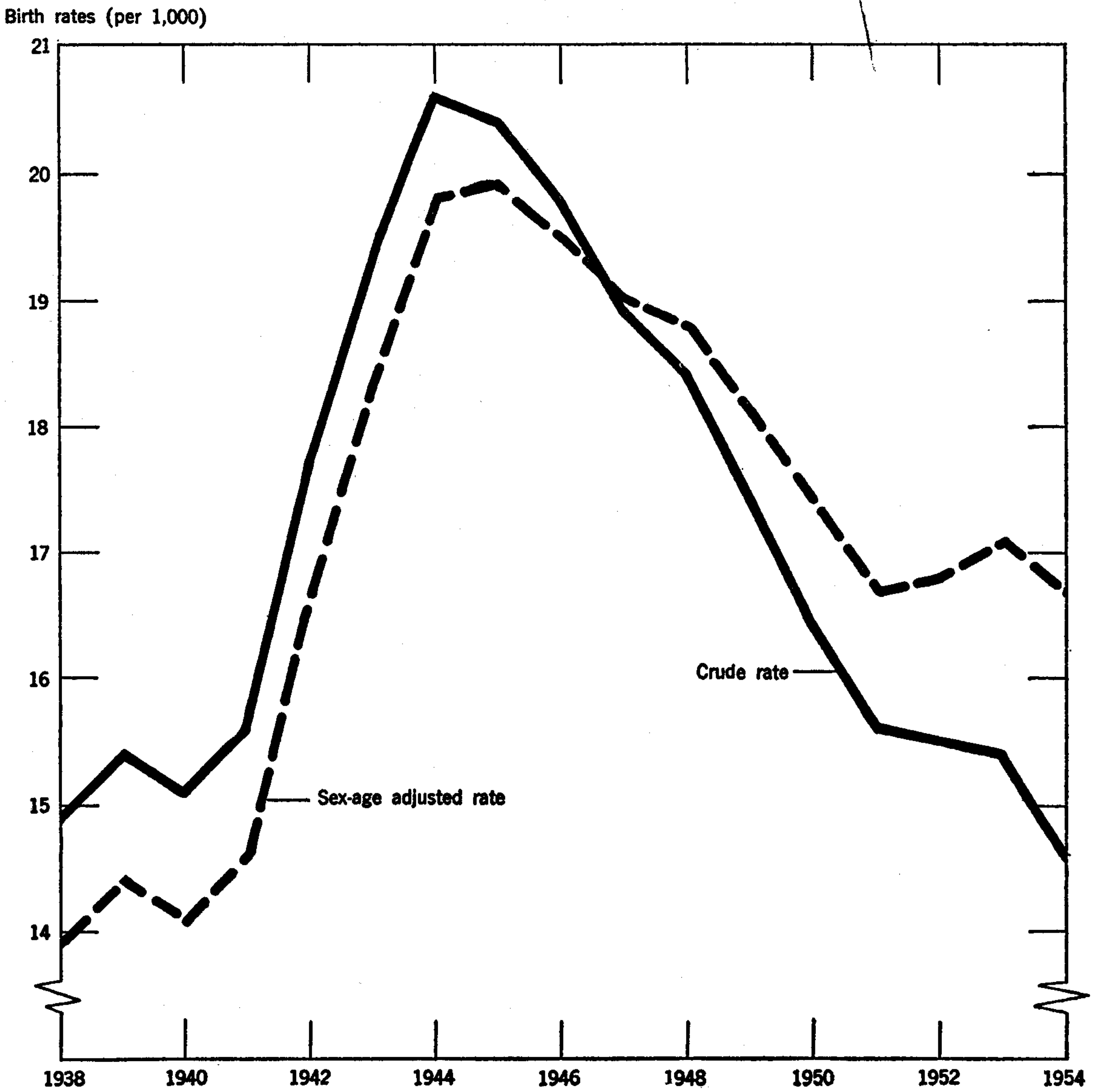
[45] These cases and the ratios of the weighted sums to the total population were: French Equatorial Africa, non-indigenous population (census 1951): 1.171; French West Africa, non-indigenous population (census 1951): 1.130; Northern Rhodesia, European population (census 1951): 1.125; Spanish Guinea, white population of the territory and indigenous population of Fernando Po (census 1950): 0.882; Haiti (census 1950): 1.111; France (estimate 1953): 0.896; West Berlin (estimate 1953): 0.803; and Gibraltar (census 1951): 1.149; The unusual sex-age structures of European settlements in African territories and Gibraltar are not surprising. The high ratio noted in the case of Haiti is perhaps largely a result of very incomplete child enumeration at the Haitian census. Low birth rates in France in the 1920's and 1930's and, in addition to past low birth rates, the selective effects of migration in West Berlin, are responsible for the fact that the age structure of the latter two populations is particularly conducive to a low crude birth rate.

TABLE 26. TREND OF SWEDISH FERTILITY, 1938–1954, AS MEASURED BY CRUDE AND BY SEX-AGE ADJUSTED BIRTH RATES

| Year | Total population (P) (thousands) | Weighted sum of women (W) (thousands) | Ratio of (W) to (P) | Number of births | Birth rate: Crude | Birth rate: Sex-age adjusted |
|---|---|---|---|---|---|---|
| 1938 | 6,297 | 6,776 | 1.076 | 93,946 | 14.9 | 13.9 |
| 1939 | 6,326 | 6,782 | 1.072 | 97,380 | 15.4 | 14.4 |
| 1940 | 6,356 | 6,812 | 1.072 | 95,778 | 15.1 | 14.1 |
| 1941 | 6,389 | 6,852 | 1.072 | 99,727 | 15.6 | 14.6 |
| 1942 | 6,432 | 6,857 | 1.066 | 113,961 | 17.7 | 16.6 |
| 1943 | 6,491 | 6,844 | 1.054 | 125,392 | 19.3 | 18.3 |
| 1944 | 6,560 | 6,827 | 1.041 | 134,991 | 20.6 | 19.8 |
| 1945 | 6,636 | 6,804 | 1.025 | 135,373 | 20.4 | 19.9 |
| 1946 | 6,719 | 6,783 | 1.010 | 132,597 | 19.7 | 19.5 |
| 1947 | 6,803 | 6,768 | 0.995 | 128,779 | 18.9 | 19.0 |
| 1948 | 6,883 | 6,736 | 0.979 | 126,683 | 18.4 | 18.8 |
| 1949 | 6,956 | 6,690 | 0.962 | 121,272 | 17.4 | 18.1 |
| 1950 | 7,017 | 6,637 | 0.946 | 115,414 | 16.4 | 17.4 |
| 1951 | 7,071 | 6,595 | 0.933 | 110,168 | 15.6 | 16.7 |
| 1952 | 7,125 | 6,545 | 0.919 | 110,088 [b] | 15.5 | 16.8 |
| 1953 | 7,192 | 6,446 | 0.896 | 110,105 [b] | 15.3 | 17.1 |
| 1954 | 7,213 | ... | (0.875) [a] | 105,396 [b] | 14.6 | 16.7 |

[a] Estimated by extrapolation of the time series.
[b] Provisional figures.

*Figure I. Fertility trend of Sweden, 1938-1954, as described by crude and by sex-age adjusted birth rates*




as the proportion of married persons in the population, duration of marriage, and number of children born previously per woman, as well as by important non-demographic factors. Such factors may or may not require to be taken into account in a particular situation. Where important, they should be considered and, provided the requisite data exist, a more pertinent fertility measure than the one derived here should be employed. The description of more elaborate fertility measurements, however, is beyond the scope of the present manual.

### B. Estimation of current or recent level of fertility

260. The determination of the current level of a birth rate raises a problem of interpretation. Birth rates, like other time series, may vary in accordance with trend and cyclical movements, but are also affected by short-period fluctuations. The position of the rate on its trend, at a given moment of time, is determined after abstraction of short-term and cyclical fluctuations. Short-term variations are usually eliminated by taking a five-year average around a central date. But if cyclical variations or changes due to some unusual circumstances have occurred in a five-year period, some other method may have to be used. Interpretation of the trend itself will be considered further on in this chapter, in connexion with assumptions of future trend.

#### 1. *Estimates based on presumably reliable statistics of births*

261. Unless there are good reasons to do otherwise, it will usually be convenient and advantageous to begin by establishing the current level of the sex-age adjusted birth rate, as explained in the preceding pages. An example of the computation of this rate was given above, utilizing the statistics of Cyprus for the year 1950, but for the purpose of a population projection, the rate for a single year does not provide a firm enough basis. In

TABLE 27. COMPUTATION OF SEX-AGE ADJUSTED BIRTH RATE FOR ICELAND, 1946–1950, FROM DATA OF CENSUSES OF DECEMBER 1940 AND DECEMBER 1950

| *Age in years* | *Female population* 1940 | 1950 | 1946–50[a] | *Weight* | *Product* | *Estimated annual number of births 1946–50* | *Sex-age adjusted birth rate* |
|---|---|---|---|---|---|---|---|
| 15–19.......... | 5,690 | 5,969 | 5,899 | 1 | 5,899 | ... | ... |
| 20–24.......... | 5,097 | 6,058 | 5,818 | 7 | 40,726 | ... | ... |
| 25–29.......... | 4,677 | 5,416 | 5,231 | 7 | 36,617 | ... | ... |
| 30–34.......... | 4,178 | 4,873 | 4.699 | 6 | 28,194 | ... | ... |
| 35–39.......... | 3,868 | 4,558 | 4,386 | 4 | 17,544 | ... | ... |
| 40–44.......... | 3,670 | 4,057 | 3,960 | 1 | 3,960 | ... | ... |
| TOTAL | ... | ... | ... | | 132,940 | 3,801 | 28.59 |

[a] The mid-point of the 1946–50 period is 7½ years after the 1940 census and 2½ years before the 1950 census. Interpolation is made in the ratio of three to one.

the years from 1948 to 1952, the following numbers of births were reported in Cyprus: 15,078, 13,234, 14,517, 14,403, and 13,358, or an annual average of 14,118. It is justifiable to assume in all but the most unusual situations that the weighted sum of women for the average of a five-year period is very nearly equal to that at the central date of the period. The average of 14,118 births is therefore divided by the weighted sum of 469,037 women in mid-1950, and the sex-age adjusted birth rate is found to be 29.21 for 1948-1952.

### 2. *Estimates by reverse survival of one cohort of children*

262. Where satisfactory statistics of births are not available, or where there are grounds to suspect incompleteness of birth registration, the recent level of fertility can be estimated from the number of children enumerated in a recent census, by calculating the probable number of births from which this number of children survived. This is accomplished by a reversal of the procedure of survival computations used for a population projection; the given number of children is divided by the appropriate survival ratio ($P_x$) to obtain the estimated number of births.

263. For example, it is assumed that the numbers of women of various ages and the number of children 0-4 years old in Iceland in 1940 and 1950, as given in table 20, are known to be accurate figures but that no statistics of births are available. The mortality levels have already been estimated, and the values of $P_b$ (survival ratio from birth to ages 0-4) referred to level 95. It is desired to estimate the sex-age adjusted birth rate for 1946-1950. The mean numbers of women of child-bearing ages in the 1946-1950 period can be estimated by interpolation, in the ratio of three to one, from the figures for 1940 and 1950. It remains to estimate the number of births.

264. In 1950 there were 9,466 males and 8,813 females aged 0-4. The survival ratios ($P_b$) at level 95, according to appendix table V, are 0.9580 for males and 0.9660 for females. By division, it is estimated that 9,881 males and 9,123 females, or a total of 19,004 children, were born during 1946-1950;[46] that is, an annual average of 3,801 births. The calculation of the adjusted birth rate is then carried out as shown in table 27.

[46] Actually, according to the official statistics, 18,935 births occurred during that period.

265. The estimated sex-age adjusted birth rate of 28.59 per 1,000, obtained by this method, may be compared with the average crude birth rate shown by the birth registration statistics for those five years, which amounted to 27.6 per 1,000, as related to a total population of 137,000 in mid-year 1948. The difference between the two rates is mainly due to the fact that the population structure slightly favoured a low birth rate.

### 3. *Estimates by reverse survival of women and two cohorts of children*

266. The conditions assumed for the preceding example are not often realized. Where statistics of the population are accurate, births and deaths are usually also recorded with at least a fair degree of accuracy, so that this procedure is unnecessary. The procedure is

TABLE 28. ESTIMATE OF NUMBERS OF BIRTHS IN THAILAND, 1942–47 AND 1937–42, BY "REVERSE SURVIVAL" OF TWO COHORTS OF CHILDREN ENUMERATED IN THE 1947 CENSUS

| | *Males* | *Females* |
|---|---|---|
| A. *Cohort born 23 May 1942 to 22 May 1947 (aged 0–4 in 1947)* | | |
| 1. Survivors enumerated 23 May 1947.. | 1,328,574 | 1,315,780 |
| 2. Mortality level for $P_b$, 1942–47...... | 45 | 45 |
| 3. Estimated survival ratio ($P_b$), 1942–47 | 0.7950 | 0.8135 |
| 4. Estimated births, 1942–47 (item 1 divided by item 3).............. | 1,671,162 | 1,617,431 |
| B. *Cohort born 22 May 1937 to 23 May 1942 (aged 5–9 in 1947)* | | |
| 5. Survivors enumerated 23 May 1947.. | 1,250,120 | 1,220,829 |
| 6. Mortality level for $P_{0-4}$, 1942–47..... | 45 | 45 |
| 7. Estimated survival ratio ($P_{0-4}$), 1942–47............................ | 0.9198 | 0.9209 |
| 8. Mortality level for $P_b$, 1937–42...... | 40 | 40 |
| 9. Estimated survival ratio ($P_b$), 1937–42 | 0.7789 | 0.7969 |
| 10. Estimated births, 1937–42 (item 5 divided by the product of items 7 and 9)......................... | 1,744,925 | 1,663,560 |

usually required where there are no reliable birth records and where the population data themselves are subject to appreciable error, especially as regards the enumeration of children 0-4. But, as has been argued in chapter II, enumeration of the 5-9 year age group is usually at least tolerably accurate, and probably more accurate than that of any other age group. The procedure of "reverse survival" must then be applied over a ten-year period, to estimate the number of births of which the enumerated children aged 5-9 years are the survivors.

267. This procedure is illustrated in tables 28, 29, and 30, by means of the statistics of Thailand in 1947 referred to in table 19. The mortality levels which were assessed for 1947 are assumed to apply also to the 1942-1947 period. In 1937-1942, mortality is assumed to have been higher; the levels are accordingly reduced by five points, as shown in lines 6 and 8 in table 28, and in columns 3 and 6 of table 29.

TABLE 29. ESTIMATE OF FEMALE POPULATION 15–44 YEARS OF AGE IN THAILAND, 1942 AND 1937, BY "REVERSE SURVIVAL" OF COHORTS ENUMERATED IN THE CENSUS OF 1947

| Age in years | Female population enumerated, May 1947 | Survivorship, 1942–47 Mortality level | Survivorship, 1942–47 Survival ratio ($P_x$) | Estimated female population, May 1942[a] | Survivorship, 1937–42 Mortality level | Survivorship, 1937–42 Survival ratio ($P_x$) | Estimated female population, May 1937[b] |
|---|---|---|---|---|---|---|---|
| 15–19.......... | 979,613 | 55 | 0.9709 | 816,115 | 50 | 0.9671 | 687,122 |
| 20–24.......... | 792,366 | 50 | 0.9606 | 664,516 | 45 | 0.9555 | 627,550 |
| 25–29.......... | 638,334 | 45 | 0.9520 | 599,624 | 40 | 0.9456 | 547,007 |
| 30–34.......... | 570,842 | 45 | 0.9490 | 517,250 | 40 | 0.9419 | 460,528 |
| 35–39.......... | 490,870 | 45 | 0.9448 | 433,771 | 40 | 0.9372 | 391,382 |
| 40–44.......... | 409,827 | 45 | 0.9369 | 366,803 | 40 | 0.9290 | 301,956 |
| 45–49.......... | 343,658 | 45 | 0.9229 | 280,517 | | | |
| 50–54.......... | 258,889 | | | | | | |

[a] Calculated by dividing the $P_x$ value for the same age group into the enumerated female population, 1947, for the next older group.

[b] Calculated by dividing the $P_x$ value for the same age group into the estimated female population, 1942, for the next older group.

268. The age groups of 1947 which must be considered comprise children of either sex aged less than 10 years and women aged 15 to 54 years. Numbers of the population living five years previously, when they were 5 years younger, are computed by dividing numbers living at the base date with the appropriate survival ratios. It is to be noted that the survival ratios are those of the preceding age group, since it is from those ages that the individuals enumerated at the base date survived. The number of children born in the preceding five-year period is similarly estimated by division with the survival ratio for births, $P_b$. These procedures are used twice. From the first set of computations, estimates are obtained of births during 1942-1947, children aged 0-4 years in 1942, and women 15-49 in 1942. Second, survival ratios for the 1937-1942 period are applied to these results, to obtain estimates of births in 1937-1942 and of women aged 15-44 in 1937.

269. By averaging the numbers of women of child-bearing ages in 1947, 1942 and 1937, mean numbers of women of those ages are obtained for each of the two five-year periods (table 30). Multiplying these numbers with the weights and adding the products, and dividing the sum into one fifth of the estimated numbers of births for the two periods, one obtains the sex-age adjusted birth rates of 47.99 in 1937-1942 and 41.54 per 1,000 in 1942-1947. The latter rate, however, is probably under-estimated, since it is probable that children 0-4 were incompletely enumerated in the 1947 census. It is not unlikely that fertility in 1942-1947 was fully as high as in 1937-1942.

270. On the assumption that the birth rates in the two periods were actually the same, the extent of under-enumeration of children 0-4 years old at the census can be estimated by taking the ratio of the sex-age adjusted birth rate as computed for 1937-1942 to that for 1942-1947. This ratio is 1.1553, which implies that there were about 115.53 children for every 100 enumerated. By applying this ratio to the enumerated numbers of children

TABLE 30. CALCULATION OF SEX-AGE ADJUSTED BIRTH RATES FOR THAILAND, 1932–37 AND 1942–47, ON THE BASIS OF ESTIMATES BY "INVERSE SURVIVAL" OF WOMEN AND TWO COHORTS OF CHILDREN ENUMERATED IN THE 1947 CENSUS

| Age in years | Weight | Estimated mean female population[a] 1937–42 | Estimated mean female population[a] 1942–47 | Estimated annual number of births[b] 1937–42 | Estimated annual number of births[b] 1942–47 | Sex-age adjusted birth rate 1937–42 | Sex-age adjusted birth rate 1942–47 |
|---|---|---|---|---|---|---|---|
| 15–19.... | 1 | 751,618 | 897,864 | | | | |
| 20–24.... | 7 | 646,033 | 728,441 | | | | |
| 25–29.... | 7 | 573,316 | 618,979 | | | | |
| 30–34.... | 6 | 488,889 | 544,046 | | | | |
| 35–39.... | 4 | 412,576 | 462,321 | | | | |
| 40–44.... | 1 | 334,380 | 388,315 | | | | |
| TOTAL | | | | 681,697 | 657,819 | 47.99 | 41.54 |
| WEIGHTED SUM | | 14,342,845 | 15,661,035 | | | | |

[a] Calculated by averaging, respectively, estimates for 1937 and 1942, and estimate for 1942 and 1947 census figure, for the same age group, as shown in table 29.

[b] Estimates from table 28, summed for the two sexes, and divided by 5 to obtain average annual numbers of births.

0-4 in 1947, it is estimated that there were 1,534,902 male and 1,520,121 female children of those ages, and that about 400,000 were omitted from the census count. If this is a correct estimate, the total population of Thailand was greater by at least 400,000 than the census figure. However, in view of the wide currency of the census figures, it is undesirable to substitute a different estimate of total population as of the date of the census, lest the users of the projections be misled by a comparison with the census figure. It is preferable, after substituting the revised figures for children aged 0-4 years, to "pro-rate" the entire age-distribution for each sex as of 1947 so that the total will agree with that of the census.

#### 4. *Estimates by the method of stable populations*

271. Given a few statistics only, it is possible to estimate various features of a population with the use of "stable population" models. Owing to the rather close relationship of fertility to population age structure, fertility is one of the population characteristics which can be estimated comparatively well in such a fashion. The use of "stable" populations for estimating purposes will be discussed in another publication.

### C. Estimation of probable future fertility trends

272. Judicious selection of realistic and plausible assumptions as to future fertility trends cannot be tied to any hard and fast rule, since no two situations are exactly alike in all respects. The assumptions must be based on reasonable judgement, derived from past observations and experience in apparently similar situations, taking all relevant conditions into account so far as possible. Since knowledge of the factors which influence human fertility is incomplete, the consideration of factors likely to be relevant is a matter of opinion.

273. The following discussion is intended to show how the results of a study of the past record of fertility trends in various parts of the world can be brought to bear on the formulation of assumptions concerning the future tendencies to be expected in a given situation. The examples of assumptions presented here are mainly illustrative and do not necessarily represent the most realistic assumptions that could be made in the situations mentioned.

274. The problem of forecasting fertility is especially important, both because of the substantial impact of this factor on the future growth and composition of the population, and because the trend is much more obscure than that of mortality. There is an almost universal tendency of mortality to decline because of the constant efforts to reduce the frequency of premature deaths, and consequently, some basic generalizations as to the probable future progress in longevity can be made with considerable assurance. Fertility, on the other hand, tends neither towards its possible maximum nor its possible minimum, but rather towards a level which represents a balance of conditions that affect it both in a positive and in a negative sense. This balance is an expression of the culture of a particular population; it varies from one population to another, and within the same population in the course of time. It is not possible to estimate very assuredly the extent to which this "normal" level is likely to be shifted as economic and social conditions, cultural values, and aspirations change. The historical observation of population trends under a great variety of conditions can, nevertheless, offer some suggestions as to what may be expected in a particular situation.

275. In a given situation, once the recent level and trend of fertility have been measured or estimated in the most relevant terms, three steps are required for a future projection. First, from these measurements, the nature of the past trend needs to be interpreted. Next, this trend should be compared with trends observed elsewhere or on previous occasions. Finally, by analogy with other observed situations, inferences should be made as to a plausible course of future development.

#### 1. *Interpretation of past trend*

276. Interpretation of the past trend in fertility requires a series of relevant figures covering a sufficiently long period of the past. Historical series of fertility data are lacking for very many countries, but in many of those countries fertility is high and has probably not changed very greatly for a long period of time. Whether this can reasonably be assumed to be true in a particular instance can be tested, if data on age composition of the population are available, by comparing these data with the age structure of a stable population. If fertility has been approximately constant, even though mortality may have changed, the population should resemble one of the stable populations tabulated in the appendix. Any significant deviation from the pattern of a stable population, which cannot be otherwise accounted for, implies that fertility has varied in the past and may provide the basis for an inference concerning the changes which have taken place.

277. Constancy of past crude birth rates is generally a sufficient indication that sex-age adjusted rates, if they could be computed, would also be fairly constant. It should be noted, however, that the accuracy of the birth statistics must be examined before a sound evaluation of the trend can be made. Progressive improvement in the completeness of birth registration has the effect of raising the birth rate even though fertility remains unchanged. Where the accuracy of the vital statistics is in doubt, it is advisable, if only for a rough check, to estimate the sex-age adjusted birth rate for one or more past dates by means of census data and the "reverse survival" method.

278. A regular rise or decline in fertility over a long period is more accurately portrayed by sex-age adjusted birth rates than by crude birth rates because the change of fertility affects the proportion of the population at childbearing ages in such a way as to retard somewhat the rise or fall of the crude birth rate. It is not necessary, however, to compute the sex-age adjusted birth rate for every year of a long past period; in fact, this is not possible where statistics on sex-age structure of the population are not available annually. Average rates may be computed around the dates of several censuses. If the intervals between census dates are too long, the ratio of total population to the weighted sum of women can then be interpolated for intermediate dates and crude birth rates for these dates can be adjusted by dividing them by this ratio.

279. Special problems are encountered where past changes in fertility have been irregular. Year-to-year fluctuations in birth rates do not complicate interpreta-

tion of the trend, since they are almost entirely eliminated by five-year averages. A difficulty arises in the case of "cyclical" changes in fertility over periods of perhaps ten or fifteen years. Such variations may occur in response to changes in economic circumstances or social attitudes, or as a result of war and subsequent demobilization. They usually involve changes in the timing of marriages and births, which may have cumulative effects on the birth rate. Indications that this has been happening may be found in statistics on marriages by age, and on births by age of mother, duration of marriage, and order of birth. A "cyclical" change may be followed by a new and different secular trend. This problem may also affect the interpretation of the current level of fertility; if a "cyclical" variation is in progress, the current rate is not representative of a long-term trend. Where there are indications of a "cyclical" variation, any effort to extrapolate trends of fertility into the future must be undertaken with special caution, preferably with the help of refined fertility measures.

### 2. *The history of fertility trends in different parts of the world as a guide to assumptions regarding future trends in particular cases*

280. Certain generalizations can be made concerning the history of fertility changes in different parts of the world, which are often helpful in arriving at plausible assumptions concerning the future trend of fertility in certain types of situations. It is useful for this purpose to distinguish situations of high fertility (birth rates of 40 per 1,000 or more), moderately high fertility (birth rates of 30 to 40), moderately low fertility (birth rates of 20 to 30), and low fertility (birth rates of less than 20).

281. High fertility has normally been marked by a certain constancy. In countries of very high fertility, fluctuations of the birth rate have been observed which could be attributed to the temporary effects of epidemic diseases or other factors, but the trend over long periods has generally been either level or slightly upward or downward. Where there has been a considerable amount of sterility due to disease, a slight upward trend may occur, as a result of improving conditions of health, and it is possible that a progressive improvement of economic conditions may have a similar effect in some circumstances. A gradual decline from very high fertility levels may be produced by changing attitudes and practices with respect to the age at marriage and the number of children that parents desire. Where such changes have been observed in the past, the new attitudes and practices have been adopted first by small groups of the population and gradually diffused among wider and wider elements, so that the general level of fertility declined only gradually at first but with growing momentum when substantially lower levels were reached. There have been few, if any, examples in modern times of a rapid decline of fertility from a very high level.

282. Moderately high fertility, with birth rates between 30 and 40 per 1,000, has often proved to be only a phase in the transition from very high to lower fertility. Sometimes it has been the result of very high fertility being maintained by one segment of the population while another segment was adopting low-fertility attitudes and practices. However, some populations have exhibited fertility at a constant, moderately high level over long periods of time. This may be the result of such factors as late marriage, unstable marital unions, polygamy, frequent and prolonged separation of couples, sexual taboos, or unusually long breast-feeding of infants. In such circumstances, there is the possibility of a substantial increase of fertility resulting from the breakdown of the customs or removal of the other conditions which have been responsible for keeping it at a moderately low level. On the other hand, there is also the possibility of a transition to lower fertility, due to the adoption over a period of time, of changed attitudes and practices relating to marriage and procreation. The situation of moderately high fertility is therefore likely, in general, to be somewhat less stable than that of very high fertility.

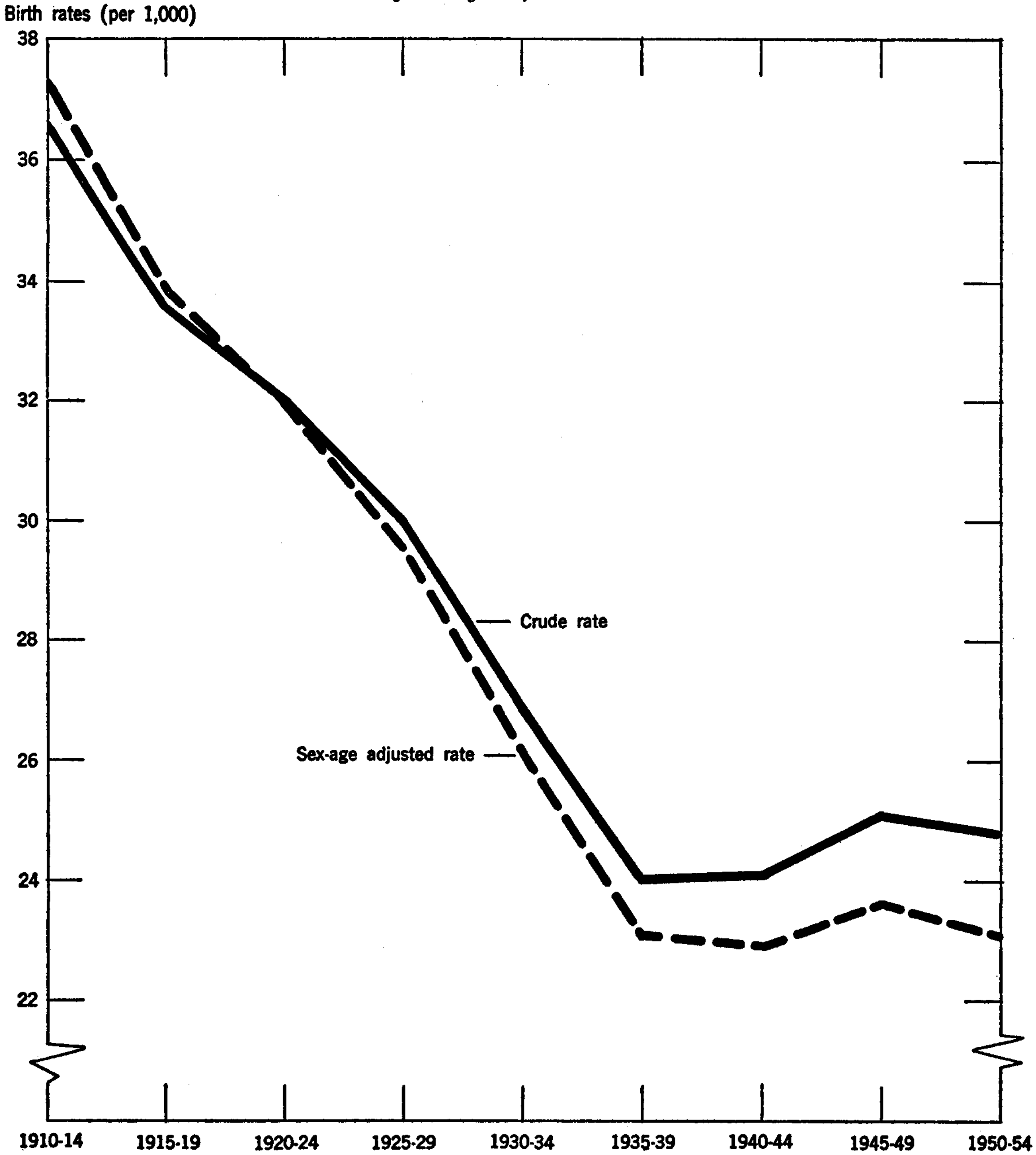
283. Moderately low fertility (birth rates of 20 to 30) has been observed in many European countries and countries of essentially European culture overseas, during recent times. In some cases, this may have been only a stage in the transition to low fertility, but in other cases it appears that moderately low fertility has been maintained over a considerable period of time, though often with considerable fluctuations. The interpretation of the long-term trends for those countries where fertility has been at such levels during the last twenty years is complicated by important cyclical variations. It seems that the fertility of a population in which most families have accepted the principle of regulating births and have adopted relatively efficient methods of doing so, is subject to relatively strong influences from cyclical changes in economic conditions and other circumstances.

284. Perhaps the clearest example of a fertility having settled at a moderately low level, after a long, steady decline, is afforded by the case of Argentina. Unlike most other populations of European origin, that of Argentina has been relatively little affected by the repercussions of war and economic depression upon the birth rate. The birth rate of Argentina declined continuously until 1933, but since then has been remarkably steady on a moderately low level (see figure II). Birth rates at a similar level have also been recorded since 1948 in other countries of low population density, high urbanization, and advanced industrialization, including Canada, the United States, Australia and New Zealand, but in these cases there were large fluctuations during the 1930's and 1940's, so that it is difficult to say whether the current rates should be considered as conforming to a long-term trend.

285. In the case of those countries where fertility has only recently declined from a moderately high to a moderately low level (for example, Japan, some of the Eastern European countries, and apparently the Soviet Union), it remains to be seen whether the trend will continue until fertility is stabilized at a considerably lower level, or whether the present moderately low level will persist. There are no cases on record of reliable statistics showing a sustained rise of fertility to moderately high or very high levels, once a moderately low level has been reached.

286. Low fertility (birth rates below 20) was observed in many countries in the European cultural sphere during the early 1930's, but subsequent developments have shown that the birth rates of that period were abnormally low, representing a depression below the

*Figure II. Fertility trend of Argentina, 1910-1954, as described by crude and by sex-age adjusted birth rates*




long-term fertility trend due to temporary postponement of marriages and births. In many of these same countries, an abnormal increase of births was registered during the late 1940's, followed by a drop to low fertility levels once more. The cyclical variation is quite apparent in the figures for Sweden (see figure I, chapter V), which was not a belligerent in World War II. It is not so clear in the figures for other countries where the effects of the war and subsequent demobilization apparently modified the course of the birth rates. The historical series of rates for this group of countries does not afford any definite basis for determining whether the current low fertility level should be accepted as conforming to the long-term trend or regarded as a temporary depression below it. A similar uncertainty attaches to the interpretation of the figures for countries like Italy and Greece, where fertility has reached a low level during the last few years only. The low rates in these cases might represent the effect of a cyclical variation superimposed upon the long-term downward trend.

287. The generalizations presented above evidently do not provide a sufficient basis for any formulae from which suitable assumptions as to future fertility trends in particular countries could be mechanically derived. They can only serve as a background against which each situation must be examined with due regard to all its relevant features. With these reservations, the following summary is offered.

288. Where fertility is very high with no clear indication of any incipient decline, it may well remain equally high for a long period in the future, and the possibility

of a moderate rise may also have to be considered. A decline may commence either in the near or more distant future; if it occurs, it is likely to be slow at first but to continue, perhaps with growing momentum, for a long period.

289. Similar considerations apply where fertility has been constant at a moderately high level, except that if a rise occurs it can be more substantial, whereas a decline, if it sets in, may perhaps be more rapid in the initial stages, inasmuch as the same factors which previously held fertility at a moderate level may now contribute to its decline.

290. Where fertility has definitely entered a decline but is still at least moderately high, it is plausible that the decline will continue over a long period, at least until a moderately low level is reached. The downward trend may, however, be accelerated if it is now slow, or retarded if it is now rapid.

291. A condition of moderately low fertility may either be stable or a prelude to lower fertility; the latter possibility is perhaps somewhat more likely if the moderately low level has only recently been achieved as a result of a long-continued decline, than if it has persisted for some time. It should be realized, however, that moderately low birth rates at a given time may be only the product of cyclical variations in a situation where distinctly lower fertility would be in accord with the long-term trend.

292. Very low fertility may likewise be a result of a cyclical depression of fertility below the long-term trend; in that event, a rise in the near future is evidently plausible. Although there is no very clear historical precedent for a sustained increase of fertility from a stable, very low position, this possibility requires consideration in areas where fertility has remained very low for a considerable period of time. The possibility of a further decrease also cannot be excluded, especially where the present low level is the result of a long-continued decline in the past. However, in such a situation a question can reasonably be raised as to how low fertility may reasonably be expected to fall in the future.

293. The fertility assumptions for the population projections made by the United Nations staff were formulated, in general, by interpreting considerations such as those stated above in the light of whatever information was at hand bearing on the special features of the case in question. In each case, the assumptions chosen were formulated in relatively simple terms; for example, where declining fertility was envisaged, the assumption of a constant reduction of the sex-age adjusted birth rate during each successive five-year period unless and until a certain reasonable minimum was reached. To map out any complex course of changes through which fertility will be presumed to pass in the future is seldom reasonable in view of the uncertainty which is inevitably attached to any assumption, even with respect to the general direction and tempo of change.

### 3. *The use of alternative assumptions as to future fertility*

294. In recognition of the fact that several rather diverse possibilities for the future often appear to be almost equally plausible, the United Nations projections were carried out for each country on three different fertility assumptions, corresponding to high, medium, and low rates of future population growth. Such a procedure is more appropriate with respect to fertility than to mortality or, as will be shown in chapter IX, to migration.

295. A population projection made on the basis of one fertility assumption only may produce an unwarranted impression of confidence in its results. Furthermore, it is less useful, as a guide in the formulation of social and economic policies and programmes, than one in which a reasonable range of future expectations is comprised.

296. Even though it is not possible to calculate the probability that any given assumption will be realized, the choice of alternatives should be guided by some consideration of their apparent likelihood. If one particular fertility trend seems more probable than others, this may be taken as a "medium" assumption. The calculations may then be carried through according to this assumption and others, considered somewhat less likely, which would yield higher and lower estimates for the future. It is useful to choose such a range of possibilities that the future course of events appears more likely to fall within than without that range. Such was the guiding principle of the choice of "high", "medium" and "low" fertility assumptions for the United Nations projections.

## D. The example of Costa Rica: estimate of fertility level, assumptions as to future fertility trends

### 1. *Estimation of the current fertility level*

297. The record of crude birth rates obtained from the statistics of registered births in Costa Rica is confused by the fact that many births are registered only after a delay which may amount to a considerable number of years. Before 1948, the statistics were tabulated only according to the year of registration, regardless of the year in which the birth was reported to have occurred. For 1948 and subsequent years, two series of official birth rates have been published, one relating to the total number of births, per 1,000 population, registered in the given year and the other to the births reported as having occurred in that year. The latter series is officially estimated to represent only 75 to 80 per cent of the actual numbers of births occurring each year. The series are shown in table 31.

298. The rates according to year of registration for the period 1921 to 1950 (with the exception of the figure for 1947, which is apparently inflated by an unusually large number of late registrations) imply more or less constant fertility at a very high level. The rates according to year of birth for 1949 and 1950 imply a similar level when the official estimate of a 20 or 25 per cent deficiency of registration is taken into account.

299. The much higher level of the rates by year of registration for 1952, 1953, and 1954 is evidently due mainly to an increase in the number of delayed registrations. However, some increase of fertility after 1950 is suggested by the rates on the basis of year of occurrence of the birth. The rising trend of these rates might be only

TABLE 31. OFFICIAL SERIES OF CRUDE BIRTH RATES PER 1,000 POPULATION BASED ON TABULATION OF BIRTHS BY YEAR OF REGISTRATION AND BY YEAR OF BIRTH, FOR COSTA RICA, 1921–1954

| *Year* | *Rate according to year of registration*[a] | *Year* | *Rate according to year of registration*[b] | *Rate according to year of birth*[a] |
|---|---|---|---|---|
| 1921–24..... | 43.4 | 1948 | 44.5[c] | 32.8 |
| 1925–29..... | 46.6 | 1949 | 44.2[c] | 36.0 |
| 1930–34..... | 45.7 | 1950 | 46.5 | 37.4 |
| 1935–39..... | 45.0 | 1951 | 47.6 | 38.1 |
| 1940–44..... | 44.9 | 1952 | 54.8 | 39.7 |
| 1945........ | 46.8 | 1953 | 53.9 | 39.7 |
| 1946........ | 45.0 | 1954 | 52.6 | 41.3 |
| 1947........ | 57.0 | | | |

[a] Figures corresponding until 1934 and for 1946 and 1947 to the series presented in the United Nations *Demographic Yearbook, 1955*. Divergent series, based on earlier official reports, are presented in previous issues of the *Yearbook*. From 1935 to 1945 figures taken from *Anuario Estadístico 1951–52 de Costa Rica*. These figures are comparable to the others.

[b] Figures presented in the United Nations *Demographic Yearbook, 1954*.

[c] Corrected figures.

a reflection of increasing completeness or promptness of registration, or it might be due to an increase in the number or the fertility of marriages, caused in turn by improved economic conditions or other factors. The recorded crude marriage rates give some support to the latter interpretation:

| *Years* | *Annual average number of marriages per 1,000 population*[a] |
|---|---|
| 1921–24................................ | 6.2 |
| 1925–29................................ | 7.6 |
| 1930–34................................ | 6.1 |
| 1935–39................................ | 6.6 |
| 1940–44................................ | 6.0 |
| 1945–49................................ | 6.5 |
| 1950–54................................ | 7.8 |

[a] United Nations *Demographic Yearbook*, 1953 and 1955.

The statistics of marriages, however, may also be affected by under-registration. Furthermore, there are many consensual unions in Costa Rica, and an increase in the number of marriages therefore does not necessarily imply a corresponding increase in the whole number of unions being formed.[47] In short, the evidence of a possible increase in marriages or fertility after 1950 is inconclusive.

300. An estimate of the sex-age adjusted birth rate for the period 1948 to 1952 can be made from the 1950 census data on female population by age groups and the average annual number of births during that period according to the series of registration statistics by year of birth, plus an allowance for the births which escaped registration. The calculations are shown in table 32.

TABLE 32. CALCULATION OF ESTIMATED AVERAGE SEX-AGE ADJUSTED BIRTH RATE FOR COSTA RICA, 1948–1952, BASED ON BIRTH REGISTRATION STATISTICS

| *Age in years* | *Female population, 1950*[a] | *Weight* | *Product* | *Estimated average annual number of births, 1948–52*[b] | *Sex-age adjusted birth rate* |
|---|---|---|---|---|---|
| 15–19.... | 43,826 | 1 | 43,826 | ... | ... |
| 20–24.... | 39,386 | 7 | 275,702 | ... | ... |
| 25–29.... | 30,491 | 7 | 213,437 | ... | ... |
| 30–34.... | 23,705 | 6 | 142,230 | ... | ... |
| 35–39.... | 23,930 | 4 | 95,720 | ... | ... |
| 40–44.... | 18,074 | 1 | 18,074 | ... | ... |
| TOTAL | ... | ... | 788,989 | 37,000–39,500 | 46.9–50.6 |

[a] Unadjusted results of the 1950 census.

[b] Average of registered births according to year of occurrence, as given in United Nations *Demographic Yearbook, 1955*, plus estimates of unregistered births according to official statement that 75 to 80 per cent of births occurring each year are registered.

301. By means of the estimates of female population of childbearing ages in 1955 derived in chapter II (table 9), together with the 1950 census figures, the sex-age

[47] In the case of Costa Rica, it is not possible to use the census statistics as an additional source of information on possible changes in the numbers of marital unions. The tabulations of the 1950 census include data on marital status but those of the preceding census, taken in 1927, do not.

TABLE 33. CALCULATION OF ESTIMATED AVERAGE SEX-AGE ADJUSTED BIRTH RATE FOR COSTA RICA, 1950–1955, BASED ON BIRTH REGISTRATION STATISTICS

| | *Female population* | | | | | |
|---|---|---|---|---|---|---|
| *Age in years* | *Enumerated, 1950*[a] | *Estimated, 1955*[b] | *Mean, 1950–55* | *Weight* | *Estimated average annual number of births, 1950–55*[c] | *Sex-age adjusted birth rate* |
| 15–19.............. | 44,010 | 48,301 | 46,156 | 1 | ... | ... |
| 20–24.............. | 38,679 | 43,581 | 41,130 | 7 | ... | ... |
| 25–29.............. | 30,595 | 38,187 | 34,391 | 7 | ... | ... |
| 30–34.............. | 24,830 | 30,125 | 27,478 | 6 | ... | ... |
| 35–39.............. | 22,622 | 24,289 | 23,456 | 4 | ... | ... |
| 40–44.............. | 18,548 | 22,024 | 20,286 | 1 | ... | ... |
| TOTAL | ... | ... | ... | ... | 42,000–44,800 | 49.2–52.5 |
| WEIGHTED SUM | ... | ... | 853,781 | | | |

[a] "Smoothed" census figures, from table 9.

[b] From table 9.

[c] Average of registered births according to year of occurrence as given in United Nations *Demographic Yearbook, 1955*, plus estimates of unregistered births according to official statement that 75 to 80 per cent of births occurring each year are registered.

TABLE 34. ESTIMATES OF MORTALITY LEVELS AND SURVIVAL RATIOS FOR CHILDREN AND WOMEN IN COSTA RICA, 1945–50 AND 1940–45

| *Sex and age (x)* | *Mortality level, both sexes* | | | *Survival ratio ($P_x$)* | |
|---|---|---|---|---|---|
| | *1949–51* | *1945–50* | *1940–45* | *1945–50* | *1940–45* |
| Values for $P_b$ | | | | | |
| Males | 67.5 | 65 | 60 | 0.8557 | 0.8406 |
| Females | | | | 0.8739 | 0.8594 |
| Values for $P_{0-4}$ | | | | | |
| Males | 67.5 | 65 | ... | 0.9518 | ... |
| Females | | | | 0.9537 | ... |
| Values for $P_x$, females | | | | | |
| 15–19 | 84 | 81.5 | 76.5 | 0.9873 | 0.9846 |
| 20–24 | 80 | 77.5 | 72.5 | 0.9823 | 0.9789 |
| 25–29 | 76.5 | 74 | 69 | 0.9785 | 0.9748 |
| 30–34 | 72.5 | 70 | 65 | 0.9735 | 0.9695 |
| 35–39 | 70 | 67.5 | 62.5 | 0.9690 | 0.9646 |
| 40–44 | 69 | 66.5 | 61.5 | 0.9628 | 0.9581 |
| 45–49 | 69 | 66.5 | ... | 0.9518 | ... |

adjusted birth rate can also be estimated for the period 1950-1955, using the registered number of births for the years 1950-1954 inclusive, with the same proportional allowance for under-registration. This calculation is presented in table 33.

302. For comparison with these estimates, sex-age adjusted birth rates can be estimated for the period 1945-1950 and 1940-1945, by "reverse survival" of the cohorts of children aged 0-4 and 5-9 and of women 20 to 54 years of age, as enumerated in the 1950 census. The mortality levels for 1945-1950 and 1940-1945 can be deduced from those evaluated for 1949-1951 and future periods in chapter IV, table 22. There is evidence that mortality declined substantially during the 1940's; hence the assumption of a change of 5 points in the mortality level every 5 years, which was used for estimating the future survival ratios, may also be applied to the period 1940-1950. The derivation of the estimated mortality levels and the corresponding values of $P_x$ from appendix table V, for the sex-age groups involved in the computations, is shown in table 34. Tables 35, 36 and 37 show the computations of the estimated sex-age adjusted birth rates, by the same methods illustrated with the data for Thailand in tables 28, 31 and 30.

TABLE 35. ESTIMATE OF NUMBERS OF BIRTHS IN COSTA RICA, 1945–50 AND 1940–45, BY "REVERSE SURVIVAL" OF TWO COHORTS OF CHILDREN ENUMERATED IN THE 1950 CENSUS

| | *Males* | *Females* |
|---|---|---|
| A. *Cohort born 23 May 1945 to 22 May 1950 (aged 0–4 in 1950)* | | |
| 1. Survivors enumerated, 22 May 1950 | 67,481 | 65,154 |
| 2. Estimated survival ratio ($P_b$), 1945–1950 | 0.8557 | 0.8739 |
| 3. Estimated births, 1945–1950 | 78,861 | 74,555 |
| B. *Cohort born 23 May 1940 to 22 May 1945 (aged 5–9 in 1950)* | | |
| 4. Survivors enumerated, 22 May 1950 | 56,789 | 55,367 |
| 5. Estimated survival ratio ($P_{0-4}$), 1945–1950 | 0.9518 | 0.9537 |
| 6. Estimated survival ratio ($P_b$), 1940–1945 | 0.8406 | 0.8594 |
| 7. Estimated births, 1940–1945 | 70,979 | 67,553 |

TABLE 36. ESTIMATE OF FEMALE POPULATION OF COSTA RICA, 1945 AND 1940, BY "REVERSE SURVIVAL" OF COHORTS ENUMERATED IN THE CENSUS OF 1950

| *Age in years* | *Female population enumerated, May 1950* | *Survival ratio ($P_x$), 1945–50* | *Estimated female population, May 1945* | *Survival ratio ($P_x$), 1940–45* | *Estimated female population, May 1940* |
|---|---|---|---|---|---|
| 15–19 | 43,826 | 0.9873 | 39,893 | 0.9846 | 31,525 |
| 20–24 | 39,386 | 0.9823 | 31,040 | 0.9789 | 24,748 |
| 25–29 | 30,491 | 0.9785 | 24,226 | 0.9748 | 25,216 |
| 30–34 | 23,705 | 0.9735 | 24,581 | 0.9695 | 19,239 |
| 35–39 | 23,930 | 0.9690 | 18,652 | 0.9646 | 15,038 |
| 40–44 | 18,074 | 0.9628 | 14,506 | 0.9581 | 12,998 |
| 45–49 | 13,966 | 0.9518 | 12,453 | | |
| 50–54 | 11,853 | | | | |

303. We now have the following estimates of the sex-age adjusted birth rate:

49.2-52.5 for 1950-1955, according to the official vital statistics and the official estimate of 75 to 80 per cent completeness of registration;

46.9-50.6 on the same basis for 1948-1952;

42.25 for 1945-1950 according to the "reverse survival" estimates;

44.93 for 1940-1945 according to the "reverse survival" estimates.

304. The difference between the estimates based on the vital statistics for 1948-1952 and the estimate from the "reverse survival" computations for 1945-1950 could be reconciled with an hypothesis of unchanging fertility, on the assumption that only 83.5 to 90 per cent of the children aged 0-4 in 1950 were enumerated in the census. The assumption of at least a 10 per cent deficiency in the enumeration of this age group is not inconsistent with the general experience of census-taking in various countries with social conditions like those of Costa Rica.

305. The differences between the estimate by "reverse survival" for 1940-1945 and the estimates based on vital statistics for later periods imply either that fertility

TABLE 37. CALCULATION OF SEX-AGE ADJUSTED BIRTH RATES FOR COSTA RICA, 1940–45 AND 1945–50, ON THE BASIS OF ESTIMATES BY "REVERSE SURVIVAL" OF WOMEN AND TWO COHORTS OF CHILDREN ENUMERATED IN THE 1950 CENSUS

| | | *Estimated mean female population* | | *Estimated annual number of births* | | *Sex-age adjusted birth rate* | |
|---|---|---|---|---|---|---|---|
| *Age in years* | *Weight* | *1940–45* | *1945–50* | *1940–45* | *1945–50* | *1940–45* | *1945–50* |
| 15–19.......... | 1 | 41,860 | 35,709 | | | | |
| 20–24.......... | 7 | 35,213 | 27,894 | | | | |
| 25–29.......... | 7 | 27,358 | 24,721 | | | | |
| 30–34.......... | 6 | 24,143 | 21,910 | | | | |
| 35–39.......... | 4 | 21,291 | 16,845 | | | | |
| 40–44.......... | 1 | 16,290 | 13,752 | | | | |
| TOTAL ... | | ... | ... | 27,706 | 30,683 | 44.93 | 42.25 |
| WEIGHTED SUM ... | | 726,169 | 616,606 | | | | |

increased after 1940-1945 or that the official estimate of the degree of completeness of birth registration is under-stated. To be sure, the estimated rate for 1940-1945 might be somewhat too low because of incomplete enumeration of children aged 5-9 in the census or an excessively high estimate of the survival rates applicable to this cohort in 1940-1945 and 1945-1949, but errors from these sources are not likely to be very large. If fertility was actually constant throughout the period 1940 to 1955 at approximately the level indicated by the "reverse survival" estimate for 1940-1945, it follows that the proportion of births registered in 1948-1952 was about 84 per cent, and in 1950-1955 about 88 per cent, instead of the 75 to 80 per cent estimated by the Costa Rican authorities. Such proportions of completeness are not implausible, since the true proportion is presumably not exactly known to the authorities. On the other hand, a moderate increase of fertility between 1940-1945 and 1950-1955, sufficient to bring the sex-age adjusted rate near the lower end, at least, of the range of estimates based on the adjusted vital statistics, is also possible.

306. The United Nations projections for various countries where vital statistics are imperfect were based, as a rule, on estimates of current fertility levels derived from "reverse survival" computations applied to census data, rather than estimates from the vital statistics, unless there were definite reasons for preferring the latter basis. In the case of Costa Rica, the sex-age adjusted rate of 44.93 estimated by "reverse survival" for 1940-1945 is therefore accepted as representing the current fertility level for the sake of consistency with this general procedure, although the lower limits of the estimates derived from vital statistics appear equally plausible. The sex-age adjusted birth rate of 44.93 for 1940-1945 is presumed to have prevailed without any major change for some time in the past.

## 2. *Estimation of numbers of children aged 0-4 and 5-9 in 1955*

307. A preliminary estimate of the sex-age distribution of the Costa Rican population in 1955 is shown in table 9. This estimate is incomplete because it does not take account of incomplete enumeration of the age group 0-4 in the 1950 census, nor of births during 1950-1955.

308. A minimum estimate of the deficiency in enumeration of children 0-4 in 1950, on the assumption of a constant sex-age adjusted birth rate of 44.93 per 1,000 in 1940-1945 and 1945-1950 is represented by the difference between this rate and the figure of 42.25 obtained by "reverse-survival" of the cohort aged 0-4 in 1950—that is, a difference of 5.7 per cent of the higher figure. A larger percentage of omissions from the enumeration would be obtained by accepting the birth rate derived from vital statistics. The statistics do not suggest that completeness of enumeration differed markedly as between boys and girls. New estimates for numbers of each sex aged 0-4 years in 1950 are obtained by dividing the numbers enumerated by 0.943; that is, 71,560 males and 69,092 females aged 0-4 instead of 67,481 males and 65,154 females, as enumerated.

309. Assuming a constant sex-age adjusted birth rate of 44.93, births during 1950-1955 are estimated by multiplying this rate with the weighted sum of the mean estimates of female population aged 15-44 in the period 1950-1955, as shown in table 33. The resulting estimate of 38,360 births annually corresponds to a total of 191,800 during the period from mid-year 1950 to mid-year 1955, which exceeds the registered total for the years 1950-1954, inclusive, according to the year-of-occurrence series, by 23,871, implying under-registration to the extent of approximately 12.5 per cent during this period.

TABLE 38. ESTIMATION OF NUMBERS OF SURVIVORS AGED 0–4 AND 5–9 IN COSTA RICA, 1955

| | *Males* | *Females* |
|---|---|---|
| A. *Cohort born 1950–1955 (aged 0–4 in 1955)* | | |
| 1. Estimated births, 1950–1955........... | 98,239 | 93,561 |
| 2. Mortality level for $P_b$, 1950–1955....... | 70 | 70 |
| 3. Estimated survival ratio ($P_b$), 1950–1955.. | 0.8703 | 0.8882 |
| 4. Estimated survivors, 1955.............. | 85,497 | 83,101 |
| B. *Cohort born 1945–1950 (aged 5–9 in 1955)* | *Males* | *Females* |
| 5. Estimated survivors, 1950.............. | 71,560 | 69,092 |
| 6. Mortality level for ($P_{0-4}$), 1950–1955..... | 70 | 70 |
| 7. Estimated survival ratio ($P_{0-4}$), 1950–1955 | 0.9584 | 0.9607 |
| 8. Estimated survivors, 1955.............. | 68,583 | 66,377 |

310. On the assumption of 105 male per 100 female births, it is estimated that there were 98,239 male and 93,561 female births during the period 1950-1955. The numbers of survivors aged 0-4 and 5-9 in 1955 are then computed as shown in table 38.

311. The estimates of the Costa Rican population by sex and age groups for 1955 now remain to be brought into conformity with the official estimate of total population for that year. With the new estimates for age groups 0-4 and 5-9 derived in table 38, the provisional estimates made, up to this point, add to a total of 948,924, whereas the official estimate of total population, as of mid-year 1955, amounts to 951,093. The figures for the various sex-age groups are pro-rated to the latter total as shown in table 39.

TABLE 39. FINAL ESTIMATE OF POPULATION OF COSTA RICA, MID-YEAR 1955, BY SEX AND AGE, AS BASIS FOR THE POPULATION PROJECTION

| *Age in years* | *Provisional estimate* [a] *Males* | *Provisional estimate* [a] *Females* | *Final estimate* [b] *Males* | *Final estimate* [b] *Females* |
|---|---|---|---|---|
| All ages | 474,559 | 474,365 | 475,644 | 475,449 |
| 0-4 | 85,497 | 83,101 | 85,692 | 83,291 |
| 5-9 | 68,583 | 66,377 | 68,740 | 66,529 |
| 10-14 | 56,195 | 54,865 | 56,323 | 54,990 |
| 15-19 | 48,450 | 48,301 | 48,561 | 48,411 |
| 20-24 | 41,325 | 43,581 | 41,419 | 43,681 |
| 25-29 | 35,734 | 38,187 | 35,816 | 38,274 |
| 30-34 | 28,867 | 30,125 | 28,933 | 30,194 |
| 35-39 | 23,767 | 24,289 | 23,821 | 24,345 |
| 40-44 | 21,627 | 22,024 | 21,676 | 22,074 |
| 45-49 | 17,872 | 18,012 | 17,913 | 18,053 |
| 50-54 | 13,864 | 13,662 | 13,896 | 13,693 |
| 55-59 | 10,815 | 10,629 | 10,840 | 10,653 |
| 60-64 | 7,888 | 7,748 | 7,906 | 7,766 |
| 65-69 | 5,975 | 5,769 | 5,989 | 5,782 |
| 70-74 | 4,057 | 3,822 | 4,066 | 3,831 |
| 75-79 | 2,200 | 2,135 | 2,205 | 2,140 |
| 80-84 | 1,097 | 1,031 | 1,100 | 1,033 |
| 85 and over | 746 | 707 | 748 | 709 |

[a] Ages 10 and over according to table 9; ages 0-4 and 5-9 according to table 38.
[b] Pro-rated to total of 951,093.

### 3. *Assumption as to future fertility trend*

312. For the projection of the population of Costa Rica, beginning with the year 1950, which was included in the United Nations report, *The Population of Central America (including Mexico), 1950-1980*, the following assumptions as to fertility were made:

"High": Calculated rate, 1940-1945, remaining constant;

"Medium": Rate decreasing after 1950 by 5 per cent each quinquennium;

"Low": Rate decreasing after 1950 by 10 per cent each quinquennium.

These assumptions were formulated largely with a view to achieving comparability of results for the entire group of countries composing the region.

313. For a projection beginning in 1955 and taking into account the newly available statistics of registered births by year of occurrence for 1950-1954, it appears advisable to include an assumption of higher fertility in view of the upward trend of the recorded birth rates in the last years. As a "high" assumption therefore the future maintenance of an age-sex adjusted birth rate of about 50 per 1,000 is assumed, corresponding to the general level of the rates calculated from adjusted vital statistics for the period since 1948.

314. As a "medium" assumption, it is assumed that the rate of 44.93 per 1,000 calculated for 1940-1945 by reverse survival of age group 5-9 in the 1950 census, will be maintained from 1955 to 1975. Birth rates have been remarkably constant over a long period of the past. Contacts with areas of lower fertility are comparatively slight. Severe population pressure, forcing a decrease in fertility, will not necessarily arise in the near future.

315. Nevertheless, the possibility of a fertility decrease should not be disregarded. Social changes connected with urbanization, industrialization, and increased education may tend to modify attitudes towards the raising of families in this direction. A gradual decline, such as by 5 per cent of the last previous value in every five-year period, is possible. This is treated as the "low" assumption.

316. It will be understood that this assessment is highly tentative. Greater familiarity with conditions in the country might provide a basis for more realistic assumptions. The present assumptions result in the following future sex-age adjusted birth rates:

| | *1955-60* | *1960-65* | *1965-70* | *1970-75* |
|---|---|---|---|---|
| "High" | 50.00 | 50.00 | 50.00 | 50.00 |
| "Medium" | 44.93 | 44.93 | 44.93 | 44.93 |
| "Low" | 43.81 [a] | 41.62 [b] | 39.54 [b] | 37.56 [b] |

[a] 2½ per cent decline from 44.93, assumed up to 1960.
[b] 5 per cent decline from value for last previous quinquennium.

## VI. COMPUTATION OF THE POPULATION PROJECTION: THE EXAMPLE OF COSTA RICA

317. The computations for the population projection begin with the base estimate of population by sex-age groups for the starting date and proceed by successive multiplication of each cohort by the appropriate survival ratios, survivors of future births being added in. These calculations can be conveniently carried out with worksheets of the type illustrated with the figures for Costa Rica in tables 40 and 41. The first worksheet consists of alternating columns for the numbers of individuals in the various sex-age groups at each date and the survival ratios for the interval between that date and the next. The other worksheet, illustrated by table 41, is used to compute the future numbers of births, on the given assumptions as to future trends of the sex-age adjusted birth rate, from the estimated future numbers of women of childbearing ages.

318. The method of calculation is explained below with the Costa Rican figures as an illustration, under each of the three chosen hypotheses with respect to future fertility.

## Table 40. Calculation of survivors of the Costa Rican population, 1955–1975

### *Males and females*

| *Sex and age (x) in years* | *Estimated population, 1955* | *Survival ratio ($P_x$), 1955–60* | *Estimated survivors, 1960* | *Survival ratio ($P_x$), 1960–65* | *Estimated survivors, 1965* | *Survival ratio ($P_x$), 1965–70* | *Estimated survivors, 1970* | *Survival ratio ($P_x$), 1970–75* | *Estimated survivors, 1975* |
|---|---|---|---|---|---|---|---|---|---|
| *Males* | | ($P_b$=0.8877) | | ($P_b$=0.9070) | | ($P_b$=0.9262) | | ($P_b$=0.9438) | |
| 0– 4 | 85,692 | 0.9648 | 101,335 | 0.9708 | 118,676 | 0.9765 | 139,437 | 0.9818 | 166,266 |
| 5– 9 | 68,740 | 0.9924 | 82,676 | 0.9937 | 98,376 | 0.9949 | 115,887 | 0.9963 | 136,899 |
| 10–14 | 56,323 | 0.9933 | 68,218 | 0.9947 | 82,155 | 0.9960 | 97,874 | 0.9972 | 115,458 |
| 15–19 | 48,561 | 0.9899 | 55,946 | 0.9920 | 67,856 | 0.9940 | 81,826 | 0.9957 | 97,600 |
| 20–24 | 41,419 | 0.9860 | 48,071 | 0.9886 | 55,498 | 0.9911 | 67,449 | 0.9934 | 81,474 |
| 25–29 | 35,816 | 0.9834 | 40,839 | 0.9861 | 47,523 | 0.9886 | 55,004 | 0.9911 | 67,004 |
| 30–34 | 28,933 | 0.9792 | 35,221 | 0.9822 | 40,271 | 0.9849 | 46,891 | 0.9874 | 54,514 |
| 35–39 | 23,821 | 0.9728 | 28,331 | 0.9762 | 34,594 | 0.9794 | 39,663 | 0.9822 | 46,389 |
| 40–44 | 21,676 | 0.9629 | 23,173 | 0.9669 | 27,657 | 0.9700 | 33,881 | 0.9737 | 38,957 |
| 45–49 | 17,913 | 0.9483 | 20,872 | 0.9529 | 22,406 | 0.9569 | 26,827 | 0.9607 | 32,900 |
| 50–54 | 13,896 | 0.9232 | 16,987 | 0.9289 | 19,889 | 0.9341 | 21,440 | 0.9387 | 25,773 |
| 55–59 | 10,840 | 0.8869 | 12,829 | 0.8939 | 15,779 | 0.9003 | 18,578 | 0.9060 | 20,126 |
| 60–64 | 7,906 | 0.8398 | 9,614 | 0.8478 | 11,468 | 0.8552 | 14,206 | 0.8619 | 16,832 |
| 65–69 | 5,989 | 0.7678 | 6,639 | 0.7775 | 8,151 | 0.7864 | 9,807 | 0.7947 | 12,244 |
| 70–74 | 4,066 | 0.6697 | 4,598 | 0.6805 | 5,162 | 0.6907 | 6,410 | 0.7003 | 7,794 |
| 75–79 | 2,205 | 0.5531 | 2,723 | 0.5642 | 3,129 | 0.5748 | 3,565 | 0.5848 | 4,489 |
| 80–84 | 1,100 | | 1,220 | | 1,536 | | 1,799 | | 2,085 |
| 85+ | 748 | 0.3328 | 615 | 0.3404 | 625 | 0.3473 | 751 | 0.3537 | 902 |
| Born before 1955 | 475,644 | | 458,572 | | 443,699 | | 428,187 | | 411,573 |
| Born after 1955 | — | | 101,335 | | 217,052 | | 353,198 | | 516,223 |
| Total males | 475,644 | | 559,907 | | 660,751 | | 781,385 | | 927,796 |
| *Females* | | ($P_b$=0.9036) | | ($P_b$=0.9208) | | ($P_b$=0.9380) | | ($P_b$=0.9535) | |
| 0– 4 | 83,291 | 0.9669 | 98,246 | 0.9731 | 114,754 | 0.9791 | 134,500 | 0.9844 | 159,990 |
| 5– 9 | 66,529 | 0.9932 | 80,534 | 0.9948 | 95,603 | 0.9962 | 112,356 | 0.9972 | 132,402 |
| 10–14 | 54,990 | 0.9947 | 66,077 | 0.9960 | 80,115 | 0.9971 | 95,240 | 0.9978 | 112,041 |
| 15–19 | 48,411 | 0.9920 | 54,699 | 0.9940 | 65,813 | 0.9956 | 79,883 | 0.9968 | 95,030 |
| 20–24 | 43,681 | 0.9882 | 48,024 | 0.9908 | 54,371 | 0.8930 | 65,523 | 0.9948 | 79,627 |
| 25–29 | 38,274 | 0.9852 | 43,166 | 0.9879 | 47,582 | 0.9904 | 53,990 | 0.9925 | 65,182 |
| 30–34 | 30,194 | 0.9811 | 37,708 | 0.9842 | 42,644 | 0.9868 | 47,125 | 0.9893 | 53,585 |
| 35–39 | 24,345 | 0.9766 | 29,623 | 0.9798 | 37,112 | 0.9828 | 42,081 | 0.9854 | 46,621 |
| 40–44 | 22,074 | 0.9702 | 23,775 | 0.9737 | 29,025 | 0.9678 | 36,474 | 0.9796 | 41,467 |
| 45–49 | 18,053 | 0.9602 | 21,416 | 0.9642 | 23,150 | 0.9678 | 28,352 | 0.9700 | 35,730 |
| 50–54 | 13,693 | 0.9416 | 17,334 | 0.8467 | 20,649 | 0.9515 | 22,405 | 0.9558 | 27,501 |
| 55–59 | 10,653 | 0.9125 | 12,893 | 0.9194 | 16,410 | 0.9256 | 19,648 | 0.9314 | 21,415 |
| 60–64 | 7,766 | 0.8716 | 9,721 | 0.8800 | 11,854 | 0.8880 | 15,189 | 0.8951 | 18,300 |
| 65–69 | 5,782 | 0.8037 | 6,769 | 0.8141 | 8,554 | 0.8240 | 10,526 | 0.8328 | 13,596 |
| 70–74 | 3,831 | 0.7060 | 4,647 | 0.7185 | 5,511 | 0.7304 | 7,048 | 0.7411 | 8,766 |
| 75–79 | 2,140 | 0.5914 | 2,705 | 0.6048 | 3,339 | 0.6171 | 4,025 | 0.6288 | 5,223 |
| 80–84 | 1,033 | | 1,266 | | 1,636 | | 2,060 | | 2,531 |
| 85 + | 709 | 0.3552 | 619 | 0.3629 | 684 | 0.3699 | 858 | 0.3762 | 1,098 |
| Born before 1955 | 475,449 | | 460,976 | | 448,449 | | 435,187 | | 420,642 |
| Born after 1955 | — | | 98,246 | | 210,357 | | 342,096 | | 499,463 |
| Total females | 475,449 | | 559,222 | | 658,806 | | 777,283 | | 920,105 |
| Total, both sexes | 951,093 | | 1,119,129 | | 1,319,557 | | 1,558,668 | | 1,847,901 |

TABLE 41. CALCULATION OF ESTIMATED NUMBERS OF BIRTHS IN COSTA RICA, 1955–75, UNDER "MEDIUM" FERTILITY ASSUMPTION

| | *1955* | *1960* | *1965* | *1970* | *1975* | *Weight* |
|---|---|---|---|---|---|---|
| *Estimated female population* | | | | | | |
| Age in years: | | | | | | |
| 15–19 | 48,411 | 54,699 | 65,813 | 79,883 | 95,030 | 1 |
| 20–24 | 43,681 | 48,024 | 54,371 | 65,523 | 79,627 | 7 |
| 25–29 | 38,274 | 43,166 | 47,582 | 53,990 | 65,182 | 7 |
| 30–34 | 30,194 | 37,708 | 42,644 | 47,125 | 53,585 | 6 |
| 35–39 | 24,345 | 29,623 | 37,112 | 42,081 | 46,621 | 4 |
| 40–44 | 22,074 | 23,775 | 29,025 | 36,474 | 41,467 | 1 |
| WEIGHTED SUM | 922,714 | 1,061,544 | 1,212,821 | 1,404,022 | 1,658,154 | |

| | | | | |
|---|---|---|---|---|
| MEAN WEIGHTED SUM (INTERVENING PERIODS) | 992,129 | 1,137,182 | 1,308,422 | 1,531,088 |
| *Sex-age adjusted birth rate* | | | | |
| Assumed annual rate | 44.93 | 44.93 | 44.93 | 44.93 |
| Quinquennial rate[a] | | | | |
| Males | 115.06 | 115.06 | 115.06 | 115.06 |
| Female | 109.59 | 109.59 | 109.59 | 109.59 |
| *Estimated number of births* | | | | |
| Male | 114,154 | 130,844 | 150,547 | 176,167 |
| Female | 108,727 | 124,624 | 143,390 | 167,792 |

[a] Calculated on the assumption of 105 male per 100 female births.

A. ESTIMATES ASSUMING "MEDIUM" FERTILITY

319. The calculations for the "medium" fertility assumption are shown in tables 40 and 41. The base population estimates by sex and age for mid-year 1955, from table 39, are entered in the first column of table 40. Survival ratios for future time-periods, from table 22, are entered in the appropriate columns. The results obtained by means of these figures alone are those which appear below the solid line, for each sex, in table 40. They refer to individuals born before mid-year 1955. They are obtained by multiplying each number with its survival ratio and entering the product in the column

TABLE 42. CALCULATION OF ESTIMATED NUMBERS OF BIRTHS IN COSTA RICA, 1955–75, UNDER "HIGH" AND "LOW" FERTILITY ASSUMPTIONS

| | *1955–60* | *1960–65* | *1965–70* | *1970–75* |
|---|---|---|---|---|
| *"High" assumption* | | | | |
| Mean weighted sum of female population 15–44 | 992,129 | 1,137,182 | 1,308,422 | 1,536,448 |
| Sex-age adjusted birth rate: | | | | |
| Assumed annual rate | 50.00 | 50.00 | 50.00 | 50.00 |
| Quinquennial rate:[a] | | | | |
| Male | 128.05 | 128.05 | 128.05 | 128.05 |
| Female | 121.95 | 121.95 | 121.95 | 121.95 |
| Estimated number of births: | | | | |
| Male | 127,042 | 145,616 | 167,543 | 196,742 |
| Female | 120,990 | 138,679 | 159,562 | 187,370 |
| *"Low" assumption* | | | | |
| Mean weighted sum of female population 15–44 | 992,129 | 1,137,182 | 1,308,422 | 1,529,900 |
| Sex-age adjusted birth rate; | | | | |
| Assumed annual rate | 43.81 | 41.62 | 39.54 | 37.56 |
| Quinquennial rate,[a] | | | | |
| Male | 112.20 | 106.59 | 101.26 | 96.19 |
| Female | 106.85 | 101.51 | 96.44 | 91.61 |
| Estimated number of births: | | | | |
| Male | 111,317 | 121,212 | 132,491 | 147,161 |
| Female | 106,009 | 115,435 | 126,184 | 140,154 |

[a] Calculated on the assumption of 105 male per 100 female births.

for the next date, on the line for the next-higher age group.

320. For example, initially there are 85,692 males aged 0-4 years. Multiplication of this figure by the survival ratio $P_{0-4}$ for 1955-1960 (0.9648) results in the estimate of 82,676 male survivors aged 5-9 years in 1960. Multiplication of the latter figure by the survival ratio $P_{5-9}$ for 1960-1965 (0.9937) yields 82,155 males aged 10-14 years in 1965, and so forth.

321. In table 41, the numbers of women in age groups 15-19 to 40-44 are copied from table 40. At the stage of calculations described so far, all of these age groups are available except for women aged 15-19 years in 1975, who are themselves the survivors of future births. At this stage, therefore, weighted sums for each date can be derived only up to 1970. Averaging these, mean weighted sums are derived for the periods 1955-1960, 1960-1965, and 1965-1970. The annual sex-age adjusted birth rate, for the "medium" assumption, is 44.93 per 1,000, or 224.65 in a five-year period. If there are 105 male per 100 female births, the rates for five-year periods are 115.06 male births to 109.59 female births per 1,000 of the weighted sum. Multiplying the mean weighted sums of women for each period by these rates yields the estimates of numbers of births of each sex up to 1970.

322. These estimated numbers of births are multiplied by the survival ratios ($P_b$) for the corresponding periods and the resulting estimate of survivors aged 0-4 years at the end of each period is entered in table

TABLE 43. CALCULATION OF ESTIMATED NUMBERS OF SURVIVORS IN 1960–1975 FROM BIRTHS IN 1955–1975, FOR COSTA RICA, ACCORDING TO "HIGH" AND "LOW" FERTILITY ASSUMPTIONS

| *Age in years* | *Estimated survivors, 1960* | *Survival ratio ($P_x$), 1960–65* | *Estimated survivors, 1965* | *Survival ratio ($P_x$), 1965–70* | *Estimated survivors, 1970* | *Survival ratio ($P_x$), 1970–75* | *Estimated survivors, 1975* |
|---|---|---|---|---|---|---|---|
| ***High assumption*** | | | | | | | |
| *Males* | | | | | | | |
| 0– 4 | 112,775 | 0.9708 | 132,074 | 0.9765 | 155,178 | 0.9818 | 185,685 |
| 5– 9 | | | 109,482 | 0.9949 | 128,970 | 0.9963 | 152,354 |
| 10–14 | | | | | 108,924 | 0.9972 | 128,493 |
| 15–19 | | | | | | | 108,619 |
| Born before 1955 | 458,572 | | 443,699 | | 428,187 | | 411,573 |
| Born after 1955 | 112,775 | | 241,556 | | 393,072 | | 575,151 |
| TOTAL MALES | 571,347 | | 685,255 | | 821,259 | | 986,724 |
| *Females* | | | | | | | |
| 0– 4 | 109,327 | 0.9731 | 127,696 | 0.9791 | 149,669 | 0.9844 | 178,657 |
| 5– 9 | | | 106,386 | 0.9962 | 125,027 | 0.9972 | 147,334 |
| 10–14 | | | | | 105,982 | 0.9978 | 124,677 |
| 15–19 | | | | | | | 105,749 |
| Born before 1955 | 460,976 | | 448,449 | | 435,187 | | 420,642 |
| Born after 1955 | 109,327 | | 234,082 | | 380,678 | | 556,417 |
| TOTAL FEMALES | 570,303 | | 682,531 | | 815,865 | | 977,059 |
| TOTAL, BOTH SEXES | 1,141,650 | | 1,367,786 | | 1,637,124 | | 1,963,783 |
| ***Low assumption*** | | | | | | | |
| *Males* | | | | | | | |
| 0– 4 | 98,816 | 0.9708 | 109,939 | 0.9765 | 122,713 | 0.9818 | 138,891 |
| 5– 9 | | | 95,931 | 0.9949 | 107,355 | 0.9963 | 120,480 |
| 10–14 | | | | | 95,442 | 0.9972 | 106,958 |
| 15–19 | | | | | | | 95,175 |
| Born before 1955 | 458,572 | | 443,699 | | 428,187 | | 411,573 |
| Born after 1955 | 98,816 | | 205,870 | | 325,510 | | 461,504 |
| TOTAL MALES | 557,388 | | 649,569 | | 753,697 | | 873,077 |
| *Females* | | | | | | | |
| 0– 4 | 95,790 | 0.9731 | 106,293 | 0.9791 | 118,361 | 0.9844 | 133,637 |
| 5– 9 | | | 93,213 | 0.9962 | 104,071 | 0.9972 | 116,515 |
| 10–14 | | | | | 92,859 | 0.9978 | 103,780 |
| 15–19 | | | | | | | 92,655 |
| Born before 1955 | 460,976 | | 448,449 | | 435,187 | | 420,642 |
| Born after 1955 | 95,790 | | 199,506 | | 315,291 | | 446,587 |
| TOTAL FEMALES | 556,766 | | 647,955 | | 750,478 | | 867,229 |
| TOTAL, BOTH SEXES | 1,114,154 | | 1,297,524 | | 1,504,175 | | 1,740,306 |

40. For example, the estimate of 114,154 male births in 1955-1960 is multiplied by $P_b$ for 1955-1960 (0.8877) to obtain the estimated number of males aged 0-4 in 1960 (101,335). By applying the appropriate survival ratios for successive time periods, the estimates of survivors for all age groups above the solid line in table 40 can now be computed, except the group 0-4 years old in 1975.

323. The estimate of female survivors aged 15-19 years in 1975 so obtained is now entered in table 41, and the calculations of births are completed for the 1970-1975 period, so that the estimates of survivors aged 0-4 in 1975 can be entered in table 40, completing the projection.

### B. Estimates on alternative fertility assumptions

324. In addition to the "medium" fertility assumption, projections are desired, in accordance with the last section of chapter V, on the "high" assumption of a constant sex-age adjusted birth rate of 50 per 1,000 and on the "low" assumption of a continuous decline from the rate of 44.93 per 1,000. These alternative assumptions affect only the numbers of survivors from births occurring after 1955.

325. The alternative estimates of births are obtained by a modification of table 41, as shown in table 42. The weighted sums of women of childbearing age remain unchanged whatever fertility is assumed, up to the period 1970-1975, when new weighted sums must be calculated to take account of the larger or smaller estimates of female births in 1955-1960, 15-19 years old in 1975.

326. The estimated future number of births from table 42 are multiplied by the $P_b$ ratios and the results inserted in table 43 in order to compute the corresponding numbers of survivors at ages 0-4 in 1960, 0-9 in 1965, 0-14 in 1970, and 0-19 in 1975. Table 43 represents a substitute for that segment of table 40 which is above the solid line.

327. The survivors from the population born before 1955, obtained in table 40, are unaffected by the alternative assumptions of future fertility. Thus, the future total population figures under each of the two alternative assumptions are obtained by addition of survivors born before 1955 to the alternate estimates of survivors from future births. On the "medium" assumption, the projection yields a total population of about 1,850,000 by 1975. The results of the "high" and "low" assumptions imply that the 1975 figure might exceed 1,900,000 or fall somewhat below 1,800,000. It seems unlikely, unless unforeseeable events should occur, that the population would be either more than, say, 1,950,000 or less than 1,750,000.

# VII. MIGRATION

328. Many population projections are computed with reference only to the factors of fertility and mortality and without regard to any changes which may be brought about by immigration or emigration. Such projections are adequate where there is little reason for expecting migration to play an important part during the period covered by the projection. International migration has in fact had relatively little influence on the trends of population in the majority of countries during recent times, and in most cases it may be plausible to assume that its effect will continue to be minor. However, where such an assumption does not appear to be realistic, it is important so far as possible to introduce the factor of migration into the calculations. It should be considered that where the volume of migration seems unlikely to be great enough to have much influence on the size of the future population, it may nevertheless have a disproportionately large effect on the numbers in certain sex-age groups, owing to the peculiar sex-age composition of the migrant population.

329. As a rule, the future course of migration is more difficult to predict with any assurance than that of either fertility or mortality. Migration is very sensitive to changing economic conditions, not only within the country for which projections are to be made, but also in other countries from which it may draw immigrants or to which it may send emigrants. Changing legal and administrative regulations and changing public sentiments may also have a great effect. Furthermore, in very many countries it is not even possible to get a satisfactory measure of the past trends of immigration and emigration. The statistics on this subject are often very faulty, and their coverage and definitions are often not adequate for the purpose of measuring those inward and outward movements which have a lasting influence on the population of the country.[48] For these reasons, calculations relating to the possible effects of migration are commonly carried out separately from those relating to births and deaths, and the results are presented separately so that different estimates can readily be derived on various assumptions as to future amounts of immigration and emigration. It is in this manner that the factor of migration was treated in the population projections prepared by the United Nations staff.

### A. The model of recent overseas movements of European origin

330. The United Nations projections for Latin American countries were accompanied by a model which could be used to estimate the effects of any given annual amounts of immigration and emigration upon the population figures for various sex-age groups, computed without regard to migration, for various countries in the region.[49] This model is reproduced in tables 47 to 50, inclusive. It is based on the statistics of postwar immigration and emigration for Argentina, and is relevant to recent migration from Europe to countries overseas and to the return movements of previous migrants,

[48] United Nations, *Problems of Migration Statistics*, Population Studies, ST/SOA/Ser. A., No. 5.

[49] *The population of South America, 1950-1980*, Annex C.

or of the descendants of earlier migrant generations. The model obtained from these data cannot be applied without reservations to international movements occurring under different conditions, but other models, suited to various circumstances, can be constructed by the same methods.

331. The model presents the effects of immigration at the rate of 100,000 a year and emigration at 25,000 a year, leaving an annual net immigration of 75,000. This ratio of immigrants to emigrants corresponds roughly to recent Argentine experience. Emigration from Argentina, as from most countries which have received large numbers of European immigrants over a long period of time, now consists mainly of former immigrants returning to their native countries.

332. The calculations relating to the effects of the immigration and of the emigration are presented separately in this model, so that they can be adapted to any desired assumption as to the relative numbers of immigrants and emigrants. Separate treatment of immigrants and emigrants is necessary because of differences in their sex-age composition. Young adults, particularly young men, are most numerous among the immigrants. Where emigrants are mostly former immigrants returning to their countries of origin, somewhat older ages are more heavily represented among them because of time-lags between immigration and return migration. The sex-age distributions of immigrants and emigrants were derived, by means of interpolation, from Argentinian statistics for the years 1948-1952.[50] These data were found comparable with similar statistics for Brazil, Canada and Australia and are probably fairly representative of the structure of recent trans-oceanic movements originating in Europe.

B. Projection of the migrant population

333. The statistics of migrants are classified by their ages at the moment of arrival in (or departure from) the country of immigration. At the end of a five-year period, most of those migrants who entered (or left) the country toward the end of the period would still be in the same five-year age group, while most of those who arrived (or departed) at the beginning of the period would be in the next older five-year group. In order to calculate their effect on the number of persons within each age group at the end of the period, it is necessary to estimate the distribution of all migrants arriving (or departing) throughout a period of five years by age either at the beginning or the end of the period. An estimate of this age distribution at the beginning is obtained by averaging numbers of migrants of given ages at the moment when the movement occurs with numbers of migrants aged five years less, and multiplying the results by five. This process leaves a residual figure for children born during the five-year period under consideration (born abroad in the case of immigrants, born in the country *and emigrated* during the period, in the case of emigrants.)

334. During the five-year period in which their movement took place, migrants (whether immigrants or emigrants) can be presumed to have resided in the country, on an average, for 2½ years. Their risk of death, from the moment of their arrival (or departure) to the end of the period, is therefore roughly equal to one-half of the risk of death of a cohort of corresponding age during a five-year period. Hence, in computing the model sex-age groups of migrants were projected for the five-year period of their movement by the equivalent of one-half the mortality, i.e., by $\frac{1}{2}(1+P_x)$, of the non-migrant population. For subsequent periods, the numbers of survivors of the migrants were computed by applying the same survival ratios used for the non-migrant population.

335. The survival ratios used were those constructed for the projection of the population of Argentina in the period 1960-65.[51] Argentinian mortality is fairly low and comparable with that of the European countries where most of the migrants originated. European immigrants to Argentina or other countries are likely to establish themselves, for the most part, in an environment in which similar rates of mortality can be maintained, even though these rates may be lower than those experienced by the native population in some countries of immigration.

336. In order to preserve the flexibility intended for this model, constant mortality at the projected 1960-1965 level was assumed throughout a 30-year period. This was done at the expense of a slight inconsistency with population projections in which mortality is assumed to be declining. The device, however, has the advantage that the calculation of the demographic effect of, say, 100,000 annual immigrants during 1950-1955 upon the population in the year 1955 or 1960 is identical with that of 100,000 annual immigrants during 1965-1970 on the population of 1970 or 1975. The model is thereby rendered independent of time-reference and can be utilized in various combinations.

337. A special problem attaches to the estimation of births to the migrants. Since migration requires a certain period of preparation and a period of adjustment after the move, and often involves separation of husbands and wives for more or less long periods, the fertility of migrants is likely to be relatively low before the move and for some time afterward. Furthermore, migrants are not representative of the general population; couples with large families are less likely to migrate than single persons and childless couples.

338. It has already been mentioned that a residual figure of children born prior to arrival (or departure) of the migrants is obtained in the process of adjusting the age data of migrants to a given time-reference. On the average, these children would have been born during one-half of a five-year period. The ratio of the number of these children to the number of women of child-bearing ages among the migrants yields an estimate of the sex-age adjusted birth rate of the migrants during a period up to five years prior to their migration. In the case of immigrants to Argentina, fertility so estimated was found to be abnormally low and, in the case of emigrants, even lower.

[50] The interpolated figures are those of Pan, Chia-lin, in "Effects of recent and possible future migration on the population of Argentina, Brazil, Italy and India" (paper submitted at the World Population Conference, Rome, 31 Aug.-10 Sept., 1954), United Nations, E/Conf.13/325.

[51] Argentinian mortality was projected in accordance with an earlier scheme of model life tables on the basis of official vital statistics for the years 1946-1948. See United Nations, *The Population of South America 1950-1980,* ST/SOA/Ser. A. Population Studies No. 21. New York, 1955, p. 41.

339. After an initial period of adjustment, the fertility of migrants may possibly surpass that of the non-migrant population, as the single migrants marry and begin to raise children while migrant couples who had previously postponed births make up for the postponement. On the other hand, it is also possible that the migrants' fertility will remain low even after a long period of adjustment. In the absence of specific information, a wide variety of hypotheses can be made.

340. The fertility assumption selected for the present purpose, though lacking any very substantial basis in facts, appears realistic enough without being too complicated. It was assumed that during a period of 2½ years after the time of migration the fertility of the migrants would rise from the estimated low level prior to migration to equal that of the non-migrant population of Argentina;[52] thereafter, it would remain at that level. Since the migrants during a five-year period arrive at various times throughout the period, the fertility of the entire migrant groups—in periods preceding and following their movements as well as the period of movement itself—was estimated by averaging the results for migrants moving near the beginning and near the end of the period.

## C. Calculation of the cumulative and net effects of migration

341. By the methods described so far, it is possible to calculate separately the effects of an annual movement of 100,000 immigrants and 25,000 emigrants (chiefly "return migrants"), which continues during one five-year period only. The future effects of such a movement are projected by survival ratios and sex-age adjusted birth rates. If the movements were to occur only during 1950-1955 and then to stop abruptly, the effects would be those shown in tables 47 and 48, for immigrants and emigrants, respectively. The net effect, obtained by subtracting that of emigration from that of immigration, is indicated in table 49.

342. Actual conditions in a particular case will require various modifications of these assumptions. If the assumed annual numbers of immigrants and emigrants are different from 100,000 and 25,000, respectively, it is only necessary to multiply the figures by appropriate coefficients. For example, assuming an annual immigration of 200,000 persons and an annual emigration of 50,000 persons, one would multiply each figure in tables 47, 48, and 49 by two. If 100,000 annual immigrants and 50,000 annual emigrants are assumed, table 49 cannot be utilized directly but the required figures are obtained by subtracting double the figures of table 48 from the figures of table 47. In similar fashion, any other combination of assumptions as regards the volume of immigration and emigration can be applied to tables 47 and 48.

[52] The fertility of the non-migrants was taken as the sex-age adjusted birth rate estimated on the basis of official statistics for the years 1948-1952 (see *The Population of South America, 1950-1980*, p. 41) and assumed to remain constant on the "medium" assumption for the Argentinian population. The Argentine birth rate is comparable to that of other overseas countries of European immigration and to the fertility levels which prevailed recently in parts of Southern Europe where many of the migrants originate. It has, moreover, been practically constant for a period of more than twenty years (see also chapter V, part C).

343. The time-reference can be varied quite freely. One may assume that no migration occurs until a certain year in the future, when a five-year period of movements begins. The effects will be those presented in the tables, except that they will occur at later dates.

344. A movement lasting for five years only, however, is not likely to occur except under special conditions, such as the transfer of a group of refugees. Normal migratory movements usually continue over much longer periods, although the annual volume of migration varies in the course of time.

345. The purpose of table 50 is to show the effects of immigration and emigration of the given annual volume which continues throughout a thirty-year period. Though such a constant volume of migration is hardly to be expected, the model affords a simple means of evaluating the approximate effect of a movement varying from one period to another about an average of the magnitude indicated. The figures are obtained by successive addition, column after column, of the numbers shown on each line of table 49. Thus, the figures in the 1955 column of table 49 give the results of migration during the five years immediately preceding the given date; the figures in the 1960 column show the effect of a movement which occurred five to ten years prior to the given date; the sums of these two sets of figures which appear in the 1960 column of table 50 represent the cumulative effect of a movement which continued uniformly during the ten-year period preceding the given date.

346. The assumption of a constant and uniform movement, implied in table 50 can be varied. For example, if the annual volume of both immigration and return migration is expected to increase (or diminish) in the course of time, the figures of table 49 can be cumulated after they have been multiplied by various constants assumed to be appropriate for the various time periods. If the trend of immigration is expected to differ from that of emigration, the cumulative effects of immigration and emigration will first have to be calculated separately from tables 47 and 48 and the net cumulative effect obtained by subtracting the cumulative results of emigration from those of immigration.

## D. Application of the model to a population projection

347. The figures shown in table 50 or some appropriate multiple or modification of them, can be added directly to a population projection made on the basis of assumptions with respect to fertility and mortality. This separate treatment of the migration factor is appropriate, both because of the great uncertainty of predictions concerning migration and because of the interest which attaches to the effects of migration as distinct from natural increase. Of all the assumptions, that pertaining to migration is mostly likely to require early revision. Revisions can be readily made by adding different multiples of the migration model.

348. It is well to emphasize that, unless the movements are unusually large, the demographic effects of international migration will be surprisingly small. Under the assumptions stated in the report on future populations of South America, Argentina's population would in-

**TABLE 44. MIGRATION MODEL BASED ON STATISTICS OF BULGARIA, 1935–38: COMPUTATION OF DISTRIBUTION OF IMMIGRANTS AND EMIGRANTS DURING A FIVE-YEAR PERIOD BY SEX AND AGE AT THE BEGINNING OF THE PERIOD**

*Emigrants and immigrants*

| *Age in years* | *Distribution of recorded migrants by age at time of migration*[a] | | *Recorded distribution pro-rated to 20,000 emigrants and 2,000 immigrants* | | *Estimated distribution of 20,000 annual immigrants and 2,000 annual emigrants during a 5-year period by age at beginning of period* | |
|---|---|---|---|---|---|---|
| | *Males* | *Females* | *Males* | *Females* | *Males* | *Females* |
| *Emigrants* | | | | | | |
| TOTAL | 55,271 | 36,523 | 12,044 | 7,958 | 60,221 [b] | 39,794 [b] |
| 0– 4........ | 5,504 | 5,153 | 1,199 | 1,123 | 5,825 | 5,425 |
| 5– 9........ | 5,192 | 4,805 | 1,131 | 1,047 | 5,795 | 4,940 |
| 10–14........ | 5,447 | 4,263 | 1,187 | 929 | 5,255 | 3,798 |
| 15–19........ | 4,200 | 2,707 | 915 | 590 | 4,608 | 3,078 |
| 20–24........ | 4,258 | 2,941 | 928 | 641 | 6,088 | 3,348 |
| 25–29........ | 6,918 | 3,205 | 1,507 | 698 | 7,022 | 3,318 |
| 30–34........ | 5,977 | 2,886 | 1,302 | 629 | 5,922 | 2,868 |
| 35–39........ | 4,895 | 2,377 | 1,067 | 518 | 4,692 | 2,250 |
| 40–44........ | 3,718 | 1,752 | 810 | 382 | 3,605 | 1,762 |
| 45–49........ | 2,900 | 1,482 | 632 | 323 | 2,778 | 1,488 |
| 50–54........ | 2,200 [c] | 1,250 [c] | 479 | 272 | 2,070 | 1,280 |
| 55–59........ | 1,600 [c] | 1,100 [c] | 349 | 240 | 1,472 | 1,090 |
| 60–64........ | 1,100 [c] | 900 [c] | 240 | 196 | 982 | 898 |
| 65–69........ | 700 [c] | 750 [c] | 153 | 163 | 600 | 680 |
| 70–74........ | 400 [c] | 500 [c] | 87 | 109 | 328 | 435 |
| 75–79........ | 200 [c] | 300 [c] | 44 | 65 | 138 | 228 |
| 80–84........ | 50 [c] | 120 [c] | 11 | 26 | 35 | 82 |
| 85 and over.. | 12 [c] | 32 [c] | 3 | 7 | 8 | 18 |
| *Immigrants* | | | | | | |
| TOTAL | 4,391 | 1,996 | 1,377 | 625 | 6,881 [d] | 3,125 [d] |
| 0– 4........ | 67 | 75 | 21 | 23 | 182 | 158 |
| 5– 9........ | 165 | 127 | 52 | 40 | 312 | 222 |
| 10–14........ | 233 | 156 | 73 | 49 | 440 | 312 |
| 15–19........ | 328 | 242 | 103 | 76 | 492 | 505 |
| 20–24........ | 299 | 403 | 94 | 126 | 732 | 572 |
| 25–29........ | 635 | 329 | 199 | 103 | 982 | 422 |
| 30–34........ | 618 | 211 | 194 | 66 | 922 | 298 |
| 35–39........ | 558 | 169 | 175 | 53 | 808 | 215 |
| 40–44........ | 473 | 105 | 148 | 33 | 650 | 132 |
| 45–49........ | 357 | 64 | 112 | 20 | 475 | 85 |
| 50–54........ | 250 [c] | 45 [c] | 78 | 14 | 335 | 58 |
| 55–59........ | 180 [c] | 30 [c] | 56 | 9 | 235 | 38 |
| 60–64........ | 120 [c] | 20 [c] | 38 | 6 | 142 | 25 |
| 65–69........ | 60 [c] | 12 [c] | 19 | 4 | 70 | 15 |
| 70–74........ | 30 [c] | 5 [c] | 9 | 2 | 35 | 8 |
| 75–79........ | 15 [c] | 2 [c] | 5 | 1 | 15 | 2 |
| 80–84........ | 2 [c] | 1 [c] | 1 | 0 | 2 | 0 |
| 85 and over.. | 1 [c] | 0 [c] | 0 | 0 | 0 | 0 |

[a] Sums of figures for the four years 1935–38 from United Nations, *Sex and Age of International Migrants: Statistics for 1918–1947* (ST/SOA/Series A/No. 11), pp. 156–157.

[b] Rough estimates. Data for five-year age groups above age 50 were not given in the source.

[c] Totals for the two sexes include residual estimates of 2,998 male and 2,808 female emigrants born during the five-year period.

[d] Totals for the two sexes include residual estimates of 52 male and 58 female immigrants born during the five-year period.

crease from about 17 million in 1950 to about 26 million in 1980 without migration (medium fertility assumption). Continuous migration on the scale of the model, i.e., 100,000 annual immigrants and 25,000 annual emigrants, would raise the population total in 1980 by about 3 million, but it is doubtful whether such a volume of migration would be maintained throughout the period. The rejuvenating effect of this movement on Argentina's population would be very slight.

349. The case is entirely different with internal migration. Population growth and structure in parts of a country, such as its cities, provinces and districts, are often very substantially affected as a result of internal movements. Projections of populations for segments of a country, by the cohort-survival method, cannot be made realistically unless the effects of internal movements can be included in the calculations. The requisite statistics, however, are not available in a majority of situations.

### E. Alternative models for the projection of migration

350. The model based on data for Argentina that is presented in tables 47 to 50 may be applied, with suitable modifications, in other situations where migration can be presumed to be of a similar type. If important movements of a different type are anticipated, a different model will be required, based on the most pertinent statistics that can be found. For illustrative purposes, an example is given below of the construction of such a model on the basis of statistics for Bulgaria in the period 1935-1938. These data are not necessarily relevant to any existing situation. They have been chosen merely for an illustration of methods.

### F. Sex-age distribution of migrants

351. In Bulgaria during 1935-1938, 91,794 emigrants and 6,387 immigrants of known ages were recorded.[53] Most of these movements occurred between Bulgaria and other countries of Europe. A model of annual migration of the order of 20,000 emigrants and 2,000 immigrants can be derived from these figures.

352. The derivation of the age distribution of migrants at the beginning of a five-year period is shown in table 44. The Bulgarian statistics do not show age groupings above the age of 50; hence, five-year age groups were estimated very roughly in accordance with the characteristic patterns found in other countries possessing sufficiently detailed statistics. The numbers were pro-rated to 20,000 immigrants and 2,000 emigrants, as assumed in the model. The cohorts were estimated by averaging successive age groups and multiplying by five. This procedure yields a residual estimate of the number of migrant children, aged 0-4 years at the time of migration but not yet born at the beginning of the period, which will be used in estimating the fertility of the migrants.

### G. Mortality

353. For the sake of this example, it is assumed that the migrants are subject to the mortality of model life table 60, where the expectation of life at birth is of the order of 50 years. To preserve flexibility of the model, mortality is assumed to be constant. For periods subsequent to the five-year period of movement, the survival ratios for entire five-year periods apply. For the period during which the movements occur, the survival ratios are determined by the formula $\frac{1}{2}(1 + P_x)$, except for the survival ratios of migrant children born during the period. Mortality of children is greatest immediately after birth, but children do not necessarily migrate very shortly after birth. The survival ratio for the migrant children born during the period is therefore estimated at $\frac{1}{3}(1 + 2\,P_b)$. The survival ratios are tabulated in table 45.

[53] United Nations. *Sex and Age of International Migrants: Statistics for 1918-1947*. ST/SOA/Ser.A/ No. 11. New York, 1953, pp. 156-157.

Table 45. Model based on statistics of Bulgaria, 1935–38: survival ratios of migrants for the period when migration occurs and for subsequent periods

| Age in years X | Survival ratio ($P_x$) for period of migration: Males | Females | Survival ratio ($P_x$) for subsequent periods: Males | Females |
|---|---|---|---|---|
| | ($P_b$=0.8937) | ($P_b$=0.9063) | ($P_b$=0.8406) | ($P_b$=0.8594) |
| 0–4 | 0.9722 | 0.9732 | 0.9445 | 0.9463 |
| 5–9 | 0.9917 | 0.9914 | 0.9834 | 0.9828 |
| 10–14 | 0.9914 | 0.9910 | 0.9827 | 0.9819 |
| 15–19 | 0.9870 | 0.9872 | 0.9741 | 0.9745 |
| 20–24 | 0.9844 | 0.9847 | 0.9688 | 0.9694 |
| 25–29 | 0.9836 | 0.9837 | 0.9673 | 0.9674 |
| 30–34 | 0.9822 | 0.9827 | 0.9644 | 0.9654 |
| 35–39 | 0.9790 | 0.9811 | 0.9580 | 0.9622 |
| 40–44 | 0.9734 | 0.9778 | 0.9467 | 0.9556 |
| 45–49 | 0.9647 | 0.9718 | 0.9294 | 0.9437 |
| 50–54 | 0.9522 | 0.9626 | 0.9045 | 0.9252 |
| 55–59 | 0.9339 | 0.9474 | 0.8678 | 0.8949 |
| 60–64 | 0.9070 | 0.9230 | 0.8141 | 0.8459 |
| 65–69 | 0.8684 | 0.8855 | 0.7367 | 0.7710 |
| 70–74 | 0.8167 | 0.8331 | 0.6334 | 0.6662 |
| 75–79 | 0.7524 | 0.7684 | 0.5049 | 0.5369 |
| (80 and over) | (0.6494) | (0.6606) | (0.2988) | (0.3211) |

### H. Fertility

354. The fertility of migrants prior to their movement is not necessarily similar to their fertility after migration; but nevertheless the birth performance of women prior to migration is worth examining. If it is assumed that children aged less than five years always migrate with their mothers, the relation of the number of children of this age to women of childbearing ages among the recorded migrants provides an estimate of fertility. Dividing the recorded numbers of migrant children, after pro-rating, by the survival ratios, $P_b$, for the five-year period, and again by five, we obtain annual estimates of 285.3 male and 261.3 female births to emigrants, i.e., a total of 546.6 births; and 5.0 male and 5.4 female or 10.4 births of both sexes to immigrants. The corresponding mean weighted sum of women is calculated by averaging the numbers of women of each age group 15-44 with the numbers of women aged five years more and weighting the results. The calculation is shown in table 46.

TABLE 46. MIGRATION MODEL BASED ON STATISTICS OF BULGARIA, 1935–38: COMPUTATION OF SEX-AGE ADJUSTED BIRTH RATES OF MIGRANTS DURING THE FIVE-YEAR PERIOD PRECEDING MIGRATION

| *Age in years* | *Female migrants*[a] | | | *Weight* | *Estimated annual number of births* | *Sex-age adjusted birth rate* |
|---|---|---|---|---|---|---|
| | *Given ages* | *5 years older* | *Mean* | | | |
| *Emigrants* | | | | | | |
| 15–19 | 590 | 641 | 616 | 1 | | |
| 20–24 | 641 | 698 | 670 | 7 | | |
| 25–29 | 698 | 629 | 664 | 7 | | |
| 30–34 | 629 | 518 | 574 | 6 | | |
| 35–39 | 518 | 382 | 450 | 4 | | |
| 40–44 | 382 | 323 | 352 | 1 | | |
| TOTAL | ... | ... | ... | ... | 546.6 | 35.15 |
| WEIGHTED SUM | ... | ... | 15,550 | | | |
| *Immigrants* | | | | | | |
| 15–19 | 76 | 126 | 101 | 1 | | |
| 20–24 | 126 | 103 | 114 | 7 | | |
| 25–29 | 103 | 66 | 84 | 7 | | |
| 30–34 | 66 | 53 | 60 | 6 | | |
| 35–39 | 53 | 33 | 43 | 4 | | |
| 40–44 | 33 | 20 | 26 | 1 | | |
| TOTAL | ... | ... | ... | ... | 10.4 | 5.09 |
| WEIGHTED SUM | ... | ... | 2,045 | | | |

[a] From pro-rated distribution by age at time of migration, shown in table 44.

355. For emigrant women, the resulting estimate of fertility could plausibly represent normal conditions, but the estimated fertility of immigrant women, prior to migration, is extraordinarily low, possibly because many of them were single women. For the purpose of this illustrative example, it is assumed that the fertility of emigrant women remains constant but that the fertility of immigrant women remains at the estimated low level only until the end of the five-year period of migration; then it rises to equal that of emigrant women at the end of the next five-year period. The sex-age adjusted birth rate of immigrants, then, would amount to 5.09 per 1,000 during the period of migration, 20.12 per 1,000 in the next five-year period, and 35.15 per 1,000 in subsequent periods.

### I. CALCULATION OF DEMOGRAPHIC EFFECTS

The reductions in various sex-age groups of the population at successive future dates which result from the assumed annual emigration of 20,000 persons during an initial five-year period are now computed by projecting the emigrant population shown in the last two columns of table 44. The additions resulting from the annual immigration of 2,000 are likewise computed by projecting the immigrant population in the last two columns of the table. The details of the computation are omitted here, since they follow precisely the pattern for a population projection shown in the worksheets of chapter VI. When the resulting model of the effects of emigration and immigration during a single five-year period has been obtained, it is used to calculate the model of cumulative effects of migration continuing at the same rate in future periods, in the manner explained above with reference to the model based on the statistics of Argentina.

TABLE 47. MIGRATION MODEL BASED ON STATISTICS OF ARGENTINA, 1948–52: DEMOGRAPHIC GAIN RESULTING FROM 100,000 ANNUAL IMMIGRANTS ARRIVING DURING 1950–55

| *Sex and age in years* | *Arrivals in any one year* | *Addition to the population* | | | | | |
|---|---|---|---|---|---|---|---|
| | | *1955* | *1960* | *1965* | *1970* | *1975* | *1980* |
| TOTAL, BOTH SEXES. | 100,000 | 523,400 | 569,400 | 609,100 | 638,100 | 659,600 | 681,900 |
| MALES, TOTAL | 58,600 | 304,100 | 326,400 | 345,000 | 357,300 | 365,800 | 373,700 |
| 0–4 | 3,800 | 23,700 | 30,900 | 29,700 | 26,100 | 25,600 | 28,700 |
| 5–9 | 3,200 | 17,400 | 23,400 | 30,500 | 29,300 | 25,800 | 25,300 |
| 10–14 | 3,700 | 17,400 | 17,300 | 23,200 | 30,400 | 29,200 | 25,700 |
| 15–19 | 5,300 | 22,400 | 17,300 | 17,200 | 23,100 | 30,200 | 29,000 |
| 20–24 | 7,900 | 32,700 | 22,200 | 17,100 | 17,100 | 22,900 | 29,900 |
| 25–29 | 9,200 | 42,400 | 32,300 | 22,000 | 16,900 | 16,800 | 22,600 |
| 30–34 | 7,600 | 41,700 | 41,800 | 31,900 | 21,700 | 16,700 | 16,600 |
| 35–39 | 5,900 | 33,500 | 41,000 | 41,100 | 31,300 | 21,300 | 16,400 |
| 40–44 | 4,400 | 25,500 | 32,800 | 40,100 | 40,200 | 30,600 | 20,800 |
| 45–49 | 3,100 | 18,400 | 24,600 | 31,600 | 38,700 | 38,800 | 29,600 |
| 50–54 | 1,900 | 12,200 | 17,500 | 23,400 | 30,100 | 36,800 | 36,900 |
| 55–59 | 1,200 | 7,500 | 11,400 | 16,300 | 21,800 | 28,000 | 34,300 |
| 60–64 | 700 | 4,500 | 6,700 | 10,200 | 14,600 | 19,600 | 25,200 |
| 65–69 | 400 | 2,600 | 3,800 | 5,700 | 8,600 | 12,400 | 16,600 |
| 70–74 | 200 | 1,400 | 2,000 | 3,000 | 4,400 | 6,700 | 9,600 |
| 75–79 | 100 | 600 | 900 | 1,400 | 2,000 | 2,900 | 4,500 |
| 80–84 | — | 200 | 300 | 500 | 700 | 1,000 | 1,600 |
| 85+ | — | — | 100 | 200 | 300 | 400 | 600 |
| FEMALES, TOTAL | 41,400 | 219,300 | 243,000 | 264,100 | 280,800 | 293,800 | 308,200 |
| 0–4 | 3,300 | 22,700 | 29,700 | 28,500 | 25,100 | 24,600 | 27,500 |
| 5–9 | 3,000 | 15,500 | 22,500 | 29,300 | 28,200 | 24,800 | 24,300 |
| 10–14 | 3,100 | 15,200 | 15,400 | 22,400 | 29,200 | 28,100 | 24,700 |
| 15–19 | 3,700 | 17,000 | 15,100 | 15,400 | 22,200 | 29,100 | 27,900 |
| 20–24 | 4,800 | 21,300 | 16,900 | 15,000 | 15,200 | 22,000 | 28,800 |
| 25–29 | 5,600 | 26,000 | 21,000 | 16,700 | 14,800 | 15,000 | 21,800 |
| 30–34 | 4,600 | 25,600 | 25,700 | 20,700 | 16,500 | 14,600 | 14,800 |
| 35–39 | 3,500 | 20,300 | 25,200 | 25,300 | 20,500 | 16,200 | 14,400 |
| 40–44 | 2,800 | 15,700 | 20,000 | 24,800 | 24,900 | 20,100 | 16,000 |
| 45–49 | 2,200 | 12,400 | 15,400 | 19,500 | 24,200 | 24,300 | 19,600 |
| 50–54 | 1,700 | 9,500 | 12,000 | 14,800 | 18,800 | 23,400 | 23,500 |
| 55–59 | 1,200 | 6,900 | 9,000 | 11,400 | 14,100 | 17,900 | 22,200 |
| 60–64 | 800 | 4,800 | 6,400 | 8,300 | 10,500 | 13,000 | 16,500 |
| 65–69 | 500 | 3,200 | 4,200 | 5,600 | 7,300 | 9,200 | 11,500 |
| 70–74 | 300 | 1,900 | 2,600 | 3,400 | 4,600 | 6,000 | 7,500 |
| 75–79 | 200 | 1,000 | 1,400 | 1,800 | 2,400 | 3,300 | 4,300 |
| 80–84 | — | 300 | 600 | 800 | 1,100 | 1,400 | 1,900 |
| 85+ | — | — | 100 | 300 | 500 | 700 | 900 |

TABLE 48. MIGRATION MODEL BASED ON STATISTICS OF ARGENTINA, 1948–52: DEMOGRAPHIC LOSS RESULTING FROM 25,000 ANNUAL EMIGRANTS DURING 1950–1955

| *Sex and age in years* | *Departures in any one year* | *Subtraction from population* | | | | | |
|---|---|---|---|---|---|---|---|
| | | *1955* | *1960* | *1965* | *1970* | *1975* | *1980* |
| TOTAL, BOTH SEXES | 25,000 | 127,020 | 126,800 | 123,700 | 117,800 | 110,700 | 104,200 |
| MALES, TOTAL | 16,800 | 84,500 | 82,800 | 79,400 | 74,400 | 68,600 | 63,100 |
| 0–4 | 200 | 3,200 | 3,800 | 3,200 | 2,500 | 2,500 | 3,100 |
| 5–9 | 200 | 1,100 | 3,200 | 3,800 | 3,100 | 2,500 | 2,400 |
| 10–14 | 300 | 1,300 | 1,100 | 3,200 | 3,700 | 3,100 | 2,400 |
| 15–19 | 500 | 1,900 | 1,300 | 1,100 | 3,100 | 3,700 | 3,100 |
| 20–24 | 1,100 | 3,800 | 1,900 | 1,300 | 1,100 | 3,100 | 3,700 |
| 25–29 | 1,700 | 6,900 | 3,800 | 1,900 | 1,200 | 1,100 | 3,100 |
| 30–34 | 1,900 | 8,900 | 6,800 | 3,700 | 1,800 | 1,200 | 1,100 |
| 35–39 | 2,000 | 9,600 | 8,800 | 6,700 | 3,700 | 1,800 | 1,200 |
| 40–44 | 2,100 | 10,100 | 9,400 | 8,600 | 6,500 | 3,600 | 1,800 |
| 45–49 | 2,000 | 10,100 | 9,700 | 9,100 | 8,300 | 6,300 | 3,500 |
| 50–54 | 1,600 | 9,000 | 9,600 | 9,300 | 8,600 | 7,900 | 6,000 |
| 55–59 | 1,200 | 6,900 | 8,400 | 9,000 | 8,600 | 8,000 | 7,300 |
| 60–64 | 900 | 5,000 | 6,200 | 7,500 | 8,100 | 7,700 | 7,200 |
| 65–69 | 600 | 3,400 | 4,200 | 5,300 | 6,400 | 6,800 | 6,600 |
| 70–74 | 300 | 2,000 | 2,700 | 3,300 | 4,100 | 4,900 | 5,300 |
| 75–79 | 100 | 900 | 1,400 | 1,800 | 2,200 | 2,700 | 3,300 |
| 80–84 | — | 300 | 500 | 700 | 900 | 1,200 | 1,400 |
| 85+ | — | — | 100 | 200 | 400 | 500 | 700 |
| FEMALES, TOTAL | 8,200 | 42,700 | 44,000 | 44,300 | 43,400 | 42,100 | 41,100 |
| 0–4 | 200 | 3,100 | 3,700 | 3,000 | 2,400 | 2,400 | 3,000 |
| 5–9 | 200 | 1,100 | 3,100 | 3,600 | 3,000 | 2,400 | 2,300 |
| 10–14 | 300 | 1,200 | 1,100 | 3,000 | 3,600 | 3,000 | 2,400 |
| 15–19 | 300 | 1,500 | 1,200 | 1,100 | 3,000 | 3,600 | 3,000 |
| 20–24 | 500 | 2,200 | 1,500 | 1,200 | 1,100 | 3,000 | 3,500 |
| 25–29 | 700 | 3,100 | 2,100 | 1,500 | 1,200 | 1,100 | 3,000 |
| 30–34 | 800 | 3,700 | 3,000 | 2,100 | 1,500 | 1,200 | 1,100 |
| 35–39 | 800 | 4,000 | 3,600 | 3,000 | 2,100 | 1,500 | 1,200 |
| 40–44 | 900 | 4,300 | 3,900 | 3,600 | 2,900 | 2,100 | 1,400 |
| 45–49 | 900 | 4,400 | 4,200 | 3,800 | 3,500 | 2,900 | 2,000 |
| 50–54 | 800 | 4,100 | 4,300 | 4,000 | 3,700 | 3,400 | 2,800 |
| 55–59 | 600 | 3,500 | 3,900 | 4,100 | 3,800 | 3,500 | 3,200 |
| 60–64 | 500 | 2,700 | 3,200 | 3,600 | 3,800 | 3,500 | 3,200 |
| 65–69 | 300 | 1,900 | 2,900 | 2,800 | 3,200 | 3,300 | 3,100 |
| 70–74 | 200 | 1,200 | 1,500 | 2,000 | 2,300 | 2,600 | 2,700 |
| 75–79 | 100 | 600 | 800 | 1,100 | 1,400 | 1,600 | 1,900 |
| 80–84 | — | 200 | 300 | 500 | 600 | 800 | 1,000 |
| 85+ | — | — | 100 | 200 | 300 | 400 | 500 |

TABLE 49. MIGRATION MODEL BASED ON STATISTICS OF ARGENTINA, 1948–52: NET DEMOGRAPHIC EFFECT OF 100,000 ANNUAL IMMIGRANTS AND 25,000 ANNUAL EMIGRANTS DURING 1950–55

| *Sex and age in years* | *Balance in any one year* | *1955* | *1960* | *1965* | *1970* | *1975* | *1980* |
|---|---|---|---|---|---|---|---|
| TOTAL, BOTH SEXES | 75,000 | 396,200 | 442,600 | 485,400 | 520,300 | 548,900 | 577,700 |
| MALES, TOTAL | 41,800 | 219,600 | 243,600 | 265,600 | 282,900 | 297,200 | 310,600 |
| 0–4 | 3,600 | 20,500 | 27,100 | 26,500 | 23,600 | 23,100 | 25,600 |
| 5–9 | 3,000 | 16,300 | 20,200 | 26,700 | 26,200 | 23,300 | 22,900 |
| 10–14 | 3,400 | 16,100 | 16,200 | 20,000 | 26,700 | 26,100 | 23,300 |
| 15–19 | 4,800 | 20,500 | 16,000 | 16,100 | 20,000 | 26,500 | 25,900 |
| 20–24 | 6,800 | 28,900 | 20,300 | 15,800 | 16,000 | 19,800 | 26,200 |
| 25–29 | 7,500 | 35,500 | 28,500 | 20,100 | 15,700 | 15,700 | 19,500 |
| 30–34 | 5,700 | 32,800 | 35,000 | 28,200 | 19,900 | 15,500 | 15,500 |
| 35–39 | 3,900 | 23,900 | 32,200 | 34,400 | 27,600 | 19,500 | 15,200 |
| 40–44 | 2,300 | 15,400 | 23,400 | 31,500 | 33,700 | 27,000 | 19,000 |
| 45–49 | 1,100 | 8,300 | 14,900 | 22,500 | 30,400 | 32,500 | 26,100 |
| 50–54 | 300 | 3,200 | 7,900 | 14,100 | 21,500 | 28,900 | 30,900 |
| 55–59 | — | 600 | 3,000 | 7,300 | 13,200 | 20,000 | 27,000 |
| 60–64 | 200 | 500 | 500 | 2,700 | 6,500 | 11,900 | 18,000 |
| 65–69 | 200 | 800 | 400 | 400 | 2,200 | 5,600 | 10,000 |
| 70–74 | 100 | 600 | 700 | 300 | 300 | 1,800 | 4,300 |
| 75–79 | — | 300 | 500 | 400 | 200 | 200 | 1,200 |
| 80–84 | — | 100 | 200 | 200 | 200 | 200 | 200 |
| 85+ | — | — | — | — | 100 | 100 | 100 |
| FEMALES, TOTAL | 33,200 | 176,600 | 199,000 | 219,800 | 237,400 | 251,700 | 267,100 |
| 0–4 | 3,100 | 19,600 | 26,000 | 25,500 | 22,700 | 22,200 | 24,500 |
| 5–9 | 2,800 | 14,400 | 19,400 | 25,700 | 25,200 | 22,400 | 22,000 |
| 10–14 | 2,800 | 14,000 | 14,300 | 19,400 | 25,600 | 25,100 | 22,300 |
| 15–19 | 3,400 | 15,500 | 13,900 | 14,300 | 19,200 | 25,500 | 24,900 |
| 20–24 | 4,300 | 19,100 | 15,400 | 13,800 | 14,100 | 19,000 | 25,300 |
| 25–29 | 4,900 | 22,900 | 18,900 | 15,200 | 13,600 | 13,900 | 18,800 |
| 30–34 | 3,800 | 21,900 | 22,700 | 18,600 | 15,000 | 13,400 | 13,700 |
| 35–39 | 2,700 | 16,300 | 21,600 | 22,300 | 18,400 | 14,700 | 13,200 |
| 40–44 | 1,900 | 11,400 | 16,100 | 21,200 | 22,000 | 18,000 | 14,600 |
| 45–49 | 1,300 | 8,000 | 11,200 | 15,700 | 20,700 | 21,400 | 17,600 |
| 50–54 | 900 | 5,400 | 7,700 | 10,800 | 15,100 | 20,000 | 20,700 |
| 55–59 | 600 | 3,400 | 5,100 | 7,300 | 10,300 | 14,400 | 19,000 |
| 60–64 | 300 | 2,100 | 3,200 | 4,700 | 6,700 | 9,500 | 13,300 |
| 65–69 | 200 | 1,300 | 1,800 | 2,800 | 4,100 | 5,900 | 8,400 |
| 70–74 | 100 | 700 | 1,100 | 1,400 | 2,300 | 3,400 | 4,800 |
| 75–79 | 100 | 400 | 600 | 700 | 1,000 | 1,700 | 2,400 |
| 80–84 | — | 100 | 300 | 300 | 500 | 600 | 900 |
| 85+ | — | — | — | 100 | 200 | 300 | 400 |

TABLE 50. MIGRATION MODEL BASED ON STATISTICS OF ARGENTINA, 1948–52: NET DEMOGRAPHIC EFFECT OF 100,000 ANNUAL IMMIGRANTS AND 25,000 ANNUAL EMIGRANTS ARRIVING AND DEPARTING THROUGHOUT THE PERIOD 1950–1980

| *Sex and age in years* | *Balance in any one year* | *1955* | *1960* | *1965* | *1970* | *1975* | *1980* |
|---|---|---|---|---|---|---|---|
| TOTAL, BOTH SEXES | 75,000 | 396,200 | 838,800 | 1,324,200 | 1,844,500 | 2,393,400 | 2,971,100 |
| MALES, TOTAL | 41,800 | 219,600 | 463,200 | 728,800 | 1,011,700 | 1,308,900 | 1,619,500 |
| 0–4 | 3,600 | 20,500 | 47,600 | 74,100 | 97,700 | 120.800 | 146,400 |
| 5–9 | 3,000 | 16,300 | 36,500 | 63,200 | 89,400 | 112,700 | 135,600 |
| 10–14 | 3,400 | 16,100 | 32,300 | 52,300 | 79,000 | 105,100 | 128,400 |
| 15–19 | 4,800 | 20,500 | 36,500 | 52,600 | 72,600 | 99,100 | 125,000 |
| 20–24 | 6,800 | 28,900 | 49,200 | 65,000 | 81,000 | 100,800 | 127,000 |
| 25–29 | 7,500 | 35,500 | 64,000 | 84,100 | 99,800 | 115,500 | 135,000 |
| 30–34 | 5,700 | 32,800 | 67,800 | 96,000 | 115,900 | 131,400 | 146,900 |
| 35–39 | 3,900 | 23,900 | 56,100 | 90,500 | 118,100 | 137,600 | 152,800 |
| 40–44 | 2,300 | 15,400 | 38,800 | 70,300 | 104,000 | 131,000 | 150,000 |
| 45–49 | 1,100 | 8,300 | 23,200 | 45,700 | 76,100 | 108,600 | 134,700 |
| 50–54 | 300 | 3,200 | 11,100 | 25,200 | 46,700 | 75,600 | 106,500 |
| 55–59 | — | 600 | 3,600 | 10,900 | 24,100 | 44,100 | 71,100 |
| 60–64 | 200 | 500 | — | 2,700 | 9,200 | 21,100 | 39,100 |
| 65–69 | 200 | 800 | 1,200 | 800 | 1,400 | 7,000 | 17,000 |
| 70–74 | 100 | 600 | 1,300 | 1,600 | 1,300 | 500 | 4,800 |
| 75–79 | — | 300 | 800 | 1,200 | 1,400 | 1,200 | — |
| 80–84 | — | 100 | 300 | 500 | 700 | 900 | 700 |
| 85+ | — | — | — | — | 100 | 200 | 300 |
| FEMALES, TOTAL | 33,200 | 176,600 | 375,600 | 595,400 | 832,800 | 1,084,500 | 1,351,600 |
| 0–4 | 3,100 | 19,600 | 45,600 | 71,100 | 93,800 | 116,000 | 140,500 |
| 5–9 | 2,800 | 14,400 | 33,800 | 59,500 | 84,700 | 107,100 | 129,100 |
| 10–14 | 2,800 | 14,000 | 28,300 | 47,700 | 73,300 | 98,400 | 120,700 |
| 15–19 | 3,400 | 15,500 | 29,400 | 43,700 | 62,900 | 88,400 | 113,300 |
| 20–24 | 4,300 | 19,100 | 34,500 | 48,300 | 62,400 | 81,400 | 106,700 |
| 25–29 | 4,900 | 22,900 | 41,800 | 57,000 | 70,600 | 84,500 | 103,300 |
| 30–34 | 3,800 | 21,900 | 44,600 | 63,200 | 78,200 | 91,600 | 105,300 |
| 35–39 | 2,700 | 16,300 | 37,900 | 60,200 | 78,600 | 93,300 | 106,500 |
| 40–44 | 1,900 | 11,400 | 27,500 | 48,700 | 70,700 | 88,700 | 103,300 |
| 45–49 | 1,300 | 8,000 | 19,200 | 34,900 | 55,600 | 77,000 | 94,600 |
| 50–54 | 900 | 5,400 | 13,100 | 23,900 | 39,000 | 59,000 | 79,700 |
| 55–59 | 600 | 3,400 | 8,500 | 15,800 | 26,100 | 40,500 | 59,500 |
| 60–64 | 300 | 2,100 | 5,300 | 10,000 | 16,700 | 26,200 | 39,500 |
| 65–69 | 200 | 1,300 | 3,100 | 5,900 | 10,000 | 15,900 | 24,300 |
| 70–74 | 100 | 700 | 1,800 | 3,200 | 5,500 | 8,900 | 13,700 |
| 75–79 | 100 | 400 | 1,000 | 1,700 | 2,700 | 4,400 | 6,800 |
| 80–84 | — | 100 | 400 | 700 | 1,200 | 1,800 | 2,700 |
| 85+ | — | — | — | 100 | 300 | 600 | 1,000 |

# VIII. ADDITIONAL ESTIMATES DERIVED FROM POPULATION PROJECTIONS

## A. POPULATION CHARACTERISTICS

356. Population projections by sex and age form a basis for the estimation of a variety of features of the future population, such as the size of the school population, the numbers of economically active persons, families, voters, persons eligible for old-age pensions, etc.

357. One method of deriving such estimates is to apply to the projected figures for each of the age groups, or sex-age groups, of the population an estimate of the proportion belonging to the category in question. The size of the economically active population can be estimated by applying to each sex-age group an estimated percentage of persons engaged in economic activities. The estimated number of married men (or women) can be obtained by applying to each age group the estimated future per-

centage, who will be in the married state, and the number of families may then be calculated by applying a certain estimated ratio to the number of married couples.

358. Another method, usually requiring more refined statistics, but capable of producing better results, is the use of an inflow-outflow model. For each sex-age group at a given time there is a specific rate of entry into, or departure from, a given category, corresponding to age-specific death rates or marriage rates. Thus, beginning from a given size of the married population, future numbers of married couples can be estimated by applying to each age group the specific marriage rates and the corresponding rates of deaths, divorces, separations and annulments, to the extent that this is practicable. Analogous methods can be used to estimate future numbers of students enrolled in school and, with some limitations, the future size of the labour force.

### B. Geographic distribution

359. Much interest attaches also to foreseeable changes in the geographic distribution of the population. Future population estimates, especially by sex-and-age groups, for small areas, towns or districts are beset with many difficulties, as explained in chapter I. Yet rough estimates of total future population of various regions of the country, or urban and rural areas, can often be conveniently derived from future population estimates for the entire country. Reasonable assumptions as to future changes in the pattern of population distribution can sometimes be founded on observations of past changes. Again, two types of methods are available.

360. The simpler method is the "ratio method", where it is assumed that the ratio of population in each segment of the country to the country's total population will change at the same rate as it did in the past or at some rate estimated on other grounds. Supposing that a certain province contained 25 per cent of the population of the entire country in 1920 and 28 per cent in 1950, one may infer that it will contain about 29 per cent in 1960, 30 per cent in 1970, etc., unless there is reason to assume otherwise. This method, however, cannot always be used without some modification if some patently absurd results are to be avoided. Good judgment is indispensable.

361. The "apportionment method" implies the assumption that each of a country's divisions will continue to get the same share of the country's population increase which it received in the past, or a share changing according to some assumed pattern. Supposing that a country's population, in a given past period, increased by one million while that of a particular province increased by 100,000, it is assumed, for this method, that this province will also have one-tenth of the entire country's population growth in any future period. This method, like the previous one, can lead to improbable results unless modifications are judiciously introduced wherever necessary.

### C. Interpolations

362. The methods of projection described in this manual yield future estimates by sex and age for future dates five years apart. Estimates may be required with respect to other dates, intermediate between those of the projection. These can be derived by interpolation of the projected results, using the proportions of the time-interval. Such interpolations are on the whole adequate except with respect to age groups born at a time when there was a marked fluctuation in the birth rate. With regard to such groups, modifications may be required, with the help of numbers of births in the corresponding years of the past. The method of time-interpolation and the adjustment for particular age groups with irregular numbers are discussed in chapter II.

363. The present methods yield estimates only for five-year groups of ages, whereas estimates for certain single years of age, or age groups not delimited by multiples of five, are often needed for specific purposes. Ages of compulsory school attendance, of eligibility for voting, or for military service are cases in point. The corresponding estimates can be derived from the population projections by means of interpolation.

**Table 51. The Sprague Multipliers**

| | $N_1$ | $N_2$ | $N_3$ | $N_4$ | $N_5$ |
|---|---|---|---|---|---|
| | | | First end-panel | | |
| $n_1$ | +0.3616 | −0.2768 | +0.1488 | −0.0336 | — |
| $n_2$ | +0.2640 | −0.0960 | +0.0400 | −0.0080 | — |
| $n_3$ | +0.1840 | +0.0400 | −0.0320 | +0.0080 | — |
| $n_4$ | +0.1200 | +0.1360 | −0.0720 | +0.0160 | — |
| $n_5$ | +0.0704 | +0.1968 | −0.0848 | +0.0176 | — |
| | | | First next-to-end panel | | |
| $n_1$ | +0.0336 | +0.2272 | −0.0752 | +0.0144 | — |
| $n_2$ | +0.0080 | +0.2320 | −0.0480 | +0.0080 | — |
| $n_3$ | −0.0080 | +0.2160 | −0.0080 | +0.0000 | — |
| $n_4$ | −0.0160 | +0.1840 | +0.0400 | −0.0080 | — |
| $n_5$ | −0.0176 | +0.1408 | +0.0912 | −0.0144 | — |
| | | | Mid-panel | | |
| $n_1$ | −0.0128 | +0.0848 | +0.1504 | −0.0240 | +0.0016 |
| $n_2$ | −0.0016 | +0.0144 | +0.2224 | −0.0416 | +0.0064 |
| $n_3$ | +0.0064 | −0.0336 | +0.2544 | −0.0336 | +0.0064 |
| $n_4$ | +0.0064 | −0.0416 | +0.2224 | +0.0144 | −0.0016 |
| $n_5$ | +0.0016 | −0.0240 | +0.1504 | +0.0848 | −0.0128 |
| | | | Last next-to-end panel | | |
| $n_1$ | −0.0144 | +0.0912 | +0.1408 | −0.0176 | — |
| $n_2$ | −0.0080 | +0.0400 | +0.1840 | −0.0160 | — |
| $n_3$ | +0.0000 | −0.0080 | +0.2160 | −0.0080 | — |
| $n_4$ | +0.0080 | −0.0480 | +0.2320 | +0.0080 | — |
| $n_5$ | +0.0144 | −0.0752 | +0.2272 | +0.0336 | — |
| | | | Last end-panel | | |
| $n_1$ | +0.0176 | −0.0848 | +0.1968 | +0.0704 | — |
| $n_2$ | +0.0160 | −0.0720 | +0.1360 | +0.1200 | — |
| $n_3$ | +0.0080 | −0.0320 | +0.0400 | +0.1840 | — |
| $n_4$ | −0.0080 | +0.0400 | −0.0960 | +0.2640 | — |
| $n_5$ | −0.0336 | +0.1488 | −0.2768 | +0.3616 | — |

364. A simple method yielding generally adequate results is based on a graduation formula devised by Sprague, from which certain coefficients were deduced by Glover.[54] These coefficients, or "Sprague multipliers",

[54] The method is described in A. J. Jaffe's *Handbook of Statistical Methods for Demographers* (United States, Bureau of the Census, Washington, 1951), pp. 94-96. Reference is made to Glover's derivation from the Sprague formula as put forth by J. W. Glover in U. S. Bureau of the Census, *United States Life Tables*, Washington, 1921, pp. 344 and 345.

are used for the derivation of numbers in single-year ages within a given five-year age group from numbers in this and certain adjacent five-year age groups.

365. For all five-year age groups except the first two and the last two this can be done with a constant set of multipliers namely, those of the "mid-panel", as laid out in table 51. The numbers in the given five-year age groups is designated by $N_3$, those in the preceding two groups by $N_1$ and $N_2$, and those in the following groups by $N_4$ and $N_5$. The numbers in the single-year age groups within $N_3$ are designated by $n_1$, $n_2$, $n_3$, $n_4$ and $n_5$. To obtain the estimated number for a single-year age group, numbers in the several five-year groups are multiplied by the coefficients shown in the corresponding line of the mid-panel, and the results are added. For example, $N_1$ is estimated as $- 0.0128 N_1 + 0.0848 N_2 + 0.1504 N_3 - 0.0240 N_4 + 0.0016 N_5$; etc.

366. This cannot be done with respect to the first two and last two of the five-year groups, i.e., if ages are grouped by five years up to age 85, age groups 0-4, 5-9, 75-79 and 80-84. In these cases, the first two and last two of the "end panels" apply, respectively. For example, to estimate the number aged 0, i.e., $n_1$ where $N_1$ is the number aged 0-4, it is necessary to add 0.3616 times the number aged 0-4, −0.2768 times the number aged 5-9, 0.1488 times the number aged 10-14, and −0.0336 times the number aged 15-19. To estimate the number aged 5, i.e., $n_1$ where $N_2$ is the number aged 5-9, one must add 0.0336 times the number aged 0-4, 0.2272 times the number aged 5-9, −0.0752 times the number aged 10-14, and 0.0144 times the number aged 15-19.

367. In table 52, numbers of males aged 0-14 years in 1975, according to the projection for Costa Rica (see table 40) are interpolated, by single years of age, by means of each of the first three panels shown in table 51. It should be noted that, for the purpose of computations of the first three sets of single-year age groups, the same five-year groups are designated by $N_1$, $N_2$, $N_3$ and $N_4$, respectively. If the computation is continued for single-year ages 15-19, $N_1$ of the mid-panel will refer to ages 5-9, $N_2$ to ages 10-14, and so forth.

TABLE 52. INTERPOLATION OF NUMBERS OF MALES AGED 0–14 YEARS, BY SINGLE YEARS OF AGE, FOR COSTA RICA, 1955, ACCORDING TO "MEDIUM" FERTILITY ASSUMPTION, BY MEANS OF THE "SPRAGUE MULTIPLIERS"

| Age (years) | $N_1$ (Ages 0–4) | $N_2$ (Ages 5–9) | $N_3$ (Ages 10–14) | $N_4$ (Ages 15–19) | $N_5$ (Ages 20–24) | Sum |
|---|---|---|---|---|---|---|
| | | | PROJECTED NUMBERS | | | |
| | 166,266 | 136,899 | 115,458 | 97,600 | 81,474 | |
| | | | FIRST END-PANEL | | | |
| 0 | 60,122 | −37,894 | 17,180 | −3,279 | | 36,129 |
| 1 | 43,894 | −13,142 | 4,618 | −781 | | 34,589 |
| 2 | 30,593 | 5,476 | −3,695 | +781 | | 33,155 |
| 3 | 19,952 | 18,618 | −8,313 | 1,562 | | 31,819 |
| 4 | 11,705 | 26,942 | −9,791 | 1,718 | | 30,574 |
| | | | FIRST NEXT-TO-END PANEL | | | |
| 5 | 5,587 | 31,103 | −8,682 | 1,405 | | 29,413 |
| 6 | 1,330 | 31,761 | −5,542 | 781 | | 28,330 |
| 7 | −1,330 | 29,570 | −924 | 0 | | 27,316 |
| 8 | −2,660 | 25,189 | 4,618 | −781 | | 26,366 |
| 9 | −2,926 | 19,275 | 10,530 | −1,405 | | 25,474 |
| | | | MID-PANEL | | | |
| 10 | −2,128 | 11,609 | 17,365 | −2,342 | 130 | 24,634 |
| 11 | −266 | 1,971 | 25,678 | −4,060 | 521 | 23,844 |
| 12 | 1,064 | −4,600 | 29,373 | −3,279 | 521 | 23,079 |
| 13 | 1,064 | −5,695 | 25,678 | 1,405 | −130 | 22,322 |
| 14 | 266 | −3,286 | 17,365 | 8,276 | −1,043 | 21,578 |

## IX. REVISION OF POPULATION PROJECTIONS

368. It is only to be expected that actual future population trends will differ from those anticipated in a projection. The projection will therefore have to be reconsidered whenever new information is received. The new information may relate to the present size of the population, its composition by sex and age, or the current rates of mortality, fertility, or migration. If the differences between the actual trends and those which were forecast are not great, the projection may still represent reasonable expectations for the more distant future. But considerable departures from the expected trends may occur only a few years after the projection was made.

369. Some minor revisions are possible without extensive recalculations. For instance, if the size of the popu-

lation at the starting date is found to be larger than had been estimated, because of under-enumeration in the census, the distribution by sex and age groups is not necessarily affected, nor the estimated levels and trends of fertility and mortality. The entire projection is then readily revised by multiplying each figure with a constant, that is, the ratio of the revised to the original estimate of the population at the starting date.

370. If the actual trend of the population departs from the projection as a result of migration, where no allowance has been made for this function in the projection, an adjustment can be made by applying a model of the effects of migration. The results of the model, multiplied by a suitable constant, are simply added in. If the need for such corrections can already be foreseen at the time when the projection is made, it is well to work out such a model in advance so that corrections can be readily made as and when the need arises.

371. If the trends of fertility deviate from those assumed, revisions can also be carried out without extensive recalculation. The age groups affected are those which will be born after the base date of the forecast. If fertility is at a higher or lower level than was expected, but otherwise no change in the assumed future trend is indicated, each of the age groups of future birth cohorts may be simply multiplied with a given constant. If the trend of fertility changes in a fashion which casts doubt on the validity of the assumptions concerning the form of the future trend, that part of the forecast which pertains to survivors from future births may have to be recalculated.

372. With the passing of time, it may appear that actual trends tend to conform to one of the alternative assumptions rather than to the assumptions originally regarded as "medium", or "most likely". New alternative assumptions may then have to be devised, in accordance with the new experience.

373. The new information may be such that the projection can no longer be regarded as useful. This may happen especially if a new census or survey shows the age composition of the population at the starting date to be quite different from the estimated—though this is rather unlikely. Recalculation of the entire projection also becomes necessary if experience shows a large error in the mortality.

374. Revisions of population projections should not be made hastily, or after only a short period of new observations. Projections which are too frequently revised can only create unnecessary confusion. And observations made during a brief period may show only temporary deviations from a trend.

375. On the other hand, population projections should not remain unrevised for too long a period. In the course of time, observations will deviate more and more from any one assumption made in the original calculations, so that the unrevised forecast becomes worthless.

## *Appendix*

# THE MODEL LIFE TABLES

376. In preparing population projections for Central and South America, the United Nations staff used a system of model life tables for purposes of estimating current and future mortality levels for various sex-age groups in the different countries.[55] One shortcoming of that system of model tables was that comparative trends of life-table death rates, by sex and age ($q_x$) were estimated chiefly on the basis of the experience of a limited number of countries, most of them in Europe, in recent times. The system was therefore most reliable for a limited range of mortality levels—with expectations of life at birth between about 50 and 60 years—but less reliable for conditions of higher or lower mortality.

377. More recently, a systematic study of mortality patterns, by sex and age, has been completed by the United Nations staff, permitting the construction of model life tables on a firmer basis and with greater validity for extreme as well as intermediate mortality conditions. The derivation of this new scheme of tables has been explained in detail in another publication[56] and will only be briefly recapitulated here.

### A. Method of derivation of model life tables

378. The tables are based on the observed close correlations—in a wide selection of countries and at different periods of time—between life-table death rates ($q_x$) for pairs of adjacent age groups. For each such pair of $q_x$-values, this correlation was established in the form of a parabolic regression equation. For instance, given the value of $q_{10-14}$, the value of $q_{15-19}$ which normally go with it can be estimated with the regression equation for this pair of values. From the value of $q_{15-19}$ so obtained, the corresponding value of $q_{20-24}$ can be found by using the regression function for that pair of values, and so forth. Thus, all the values of $q_x$ consistent with any one $q_x$-value can be computed. By doing this for a sequence of forty different values of $q_0$, forty model life tables, ranging from very high to very low mortality, were established. The calculation was first carried out using life-table functions of both sexes combined. The $q_x$-values were then determined for each sex separately on the basis of observed correlations between given levels of mortality and sex differences in the life-table death rates pertaining to each age group.

### B. Adaptation of the model life tables for the purpose of population projections

379. It has been explained in chapter IV how the system of model tables for different values of $q_0$ was

[55] The method of construction of that scheme of life tables is described in *The Population of Central America (including Mexico), 1950-1980.*

[56] United Nations, *Age and Sex Pattern of Mortality* (ST/SOA/Ser.A/Population Studies, No. 22).

adapted to represent a time sequence of tables in a typical situation of declining mortality. In view of past observations, it was assumed that, where mortality is high or moderately high, favourable conditions in the future would bring about, on the average, a gain in expectation of life of about two-and-a-half years within each five-year period of time. Accordingly, the $q_x$-values were interpolated for intervals corresponding to expectations of life of 20, 22.5, 25, etc., years to obtain a sequence of model life tables which would succeed each other at intervals of five years. This time sequence should merely be regarded as "normative", since in a given situation, it may well be more realistic to expect that mortality will decline either more or less rapidly.

380. Where the expectation of life is already high, however, it cannot be presumed to go on increasing indefinitely at this pace. For high expectations of life, therefore, the sequence had to be further modified with a view to a gradual approach to minimum levels of mortality. The minimum values were based on Bourgeois-Pichat's estimate of the lowest levels of mortality attainable when all avoidable deaths have been eliminated.[57] It was assumed that, in the time series of life-table death rates for any given sex-age groups, the maximum decline during a five-year period would be one-third of the difference between the present level and the minimum estimated by Bourgeois-Pichat.[58] In this manner, a slowing down in the decline of death rates was assumed where the proportion of avoidable deaths is no longer great; that is, chiefly where the expectation of life at birth approaches or exceeds 70 years.

381. A further modification was made with respect to the decline in $q_0$, i.e., the life-table death rate of infants, for expectations of life greater than 55 years. At this point there is a discontinuity in the observed relations between the mortality of small children and the expectation of life, which the parabolic equations cannot take fully into account.[59] According to observations and consistent with what are believed to be the attainable minimum levels of infant mortality, the decline in $q_0$ for expectations of life greater than 55 years should at first be more rapid, but eventually less rapid, than that implied in the model life tables. Hence, in adapting the model life tables for the purpose of population projections, different values, more in accordance with observations and theory, were substituted for $q_0$ where expectations of life exceed 55 years. It was assumed that from that point onward the difference between the given level of $q_0$ and the minimum attainable level would be narrowed by 15 per cent in the first five-year period, 20 per cent in the next period, 25 per cent in the next, 30 per cent in the next, and thereafter at the assumed maximum rate of one-third. This assumption is consistent with the modification of the time series of mortality declines for the highest expectations of life. The maximum decline in $q_x$-values—by one-third of all the deaths which can still be avoided—is attained at successively higher and higher ages.

382. As a result of these modifications, the expectation of life ceases to increase by exactly 2½ years every five years once the expectation of 55 years has been exceeded. The rate of gain slows down appreciably when an expectation of 70 years has been attained.

### C. Computation of life-table values in the adapted scheme

383. From the $q_x$-values obtained by the interpolations and modifications mentioned above, the $l_x$-values were computed, starting with 100,000 births, by successive multiplication with $(1-q_x)$, i.e., $p_x$. The $L_x$-values were then obtained by the procedures outlined in part A of chapter III, and the $P_x$-values were derived from those. The $q_x$-values are basic to the entire scheme.

384. The derivation of $m_x$-values was somewhat more devious. Values of $d_x$ were obtained by differencing the $l_x$-values. From these, $m_x$ was computed from the relation

$$m_x = \frac{d_x}{L_x}.$$

Because of crudity of the formulae employed in deriving $L_x$ from $l_x$, the relationship between the $m_x$-values so obtained and the original $q_x$-values deviates appreciably in some instances from that which should have been expected on the basis of the Reed-Merrell tables. This inaccuracy, however, is not a serious handicap to their utilization in the contexts considered in this manual.

[57] Jean Bourgeois-Pichat, "La mortalité biologique de l'homme", *Population,* 1952, No. 3.

[58] This assumption appears to be justified by the observed slowing down in the decline of infant mortality in countries where infant mortality is already very low.

[59] *Age and sex pattern of mortality* (ST/SOA/Series A, Population Studies No. 22), p. 21 and figure 11.

## Table I. Age-specific death rates

*Mortality level (or time-*

| Sex and age ($x$) in years | Level 0 ($°e_0 = 20$) | Level 5 ($°e_0 = 22.5$) | Level 10 ($°e_0 = 25$) | Level 15 ($°e_0 = 27.5$) | Level 20 ($°e_0 = 30$) | Level 25 ($°e_0 = 32.5$) | Level 30 ($°e_0 = 35$) | Level 35 ($°e_0 = 37.5$) | Level 40 ($°e_0 = 40$) | Level 45 ($°e_0 = 42.5$) | Level 50 ($°e_0 = 45$) | Level 55 ($°e_0 = 47.5$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MALES | | | | | | | | | | | | |
| 0........ | 442.63 | 411.67 | 371.41 | 341.60 | 316.20 | 293.25 | 70.17 | 248.20 | 229.41 | 211.29 | 193.65 | 177.35 |
| 1-4....... | 77.96 | 67.11 | 58.27 | 50.98 | 44.99 | 39.73 | 35.13 | 30.97 | 27.55 | 24.31 | 21.44 | 18.85 |
| 5-9....... | 18.64 | 16.21 | 14.16 | 12.44 | 11.00 | 9.72 | 8.58 | 7.55 | 6.71 | 5.90 | 5.20 | 4.56 |
| 10-14...... | 11.45 | 10.09 | 8.92 | 7.91 | 7.05 | 6.28 | 5.59 | 4.96 | 4.43 | 3.92 | 3.47 | 3.07 |
| 15-19...... | 14.09 | 12.88 | 11.75 | 10.68 | 9.76 | 8.90 | 8.05 | 7.25 | 6.57 | 5.91 | 5.32 | 4.79 |
| 20-24...... | 17.93 | 16.66 | 15.41 | 14.21 | 13.20 | 12.23 | 11.17 | 10.21 | 9.32 | 8.47 | 7.67 | 6.93 |
| 25-29...... | 21.13 | 19.38 | 17.70 | 16.12 | 14.73 | 13.43 | 12.13 | 10.98 | 9.94 | 8.96 | 8.04 | 7.22 |
| 30-34...... | 25.43 | 22.95 | 20.66 | 18.55 | 16.74 | 15.07 | 13.48 | 12.08 | 10.84 | 9.69 | 8.64 | 7.70 |
| 35-39...... | 31.77 | 28.20 | 25.01 | 22.15 | 19.76 | 17.59 | 15.59 | 13.87 | 12.38 | 11.01 | 9.77 | 8.67 |
| 40-44...... | 40.65 | 35.72 | 31.40 | 27.62 | 24.50 | 21.71 | 19.20 | 17.06 | 15.17 | 13.45 | 11.96 | 10.66 |
| 45-49...... | 50.53 | 44.37 | 39.05 | 34.41 | 30.54 | 27.10 | 24.07 | 21.52 | 19.22 | 17.14 | 15.38 | 13.85 |
| 50-54...... | 59.69 | 53.02 | 47.17 | 42.02 | 37.74 | 33.92 | 30.43 | 27.46 | 24.82 | 22.49 | 20.36 | 18.56 |
| 55-59...... | 71.38 | 64.55 | 58.34 | 52.73 | 47.98 | 43.66 | 39.64 | 36.14 | 33.01 | 30.18 | 27.68 | 25.49 |
| 60-64...... | 84.96 | 78.49 | 72.35 | 66.59 | 61.60 | 56.97 | 52.48 | 48.48 | 44.87 | 41.57 | 38.65 | 36.12 |
| 65-69...... | 106.42 | 100.14 | 93.94 | 87.92 | 82.65 | 77.63 | 72.48 | 67.82 | 63.58 | 59.64 | 56.00 | 52.77 |
| 70-74...... | 144.12 | 137.21 | 130.28 | 123.34 | 116.91 | 110.72 | 104.51 | 98.77 | 93.31 | 88.21 | 83.43 | 79.17 |
| 75-79...... | 194.69 | 186.80 | 178.81 | 170.84 | 163.60 | 156.14 | 148.65 | 141.69 | 135.04 | 128.82 | 122.91 | 117.55 |
| 80-84...... | 274.44 | 264.86 | 254.17 | 243.76 | 234.75 | 225.25 | 215.65 | 206.60 | 198.10 | 190.09 | 182.54 | 175.67 |
| 85+....... | 511.36 | 460.12 | 418.80 | 387.42 | 363.99 | 344.56 | 327.70 | 313.93 | 302.48 | 292.58 | 284.10 | 276.97 |
| FEMALES | | | | | | | | | | | | |
| 0........ | 398.43 | 365.17 | 335.65 | 309.22 | 283.41 | 259.87 | 239.81 | 220.56 | 202.22 | 184.42 | 167.81 | 151.94 |
| 1-4....... | 79.80 | 68.66 | 59.60 | 52.13 | 45.75 | 40.16 | 35.51 | 31.29 | 27.71 | 24.33 | 21.37 | 18.69 |
| 5-9....... | 19.44 | 16.90 | 14.76 | 12.96 | 11.41 | 10.02 | 8.85 | 7.79 | 6.89 | 6.03 | 5.29 | 4.61 |
| 10-14...... | 13.10 | 11.54 | 10.19 | 9.03 | 8.02 | 7.10 | 6.31 | 5.59 | 4.98 | 4.39 | 3.87 | 3.40 |
| 15-19...... | 15.97 | 14.59 | 13.29 | 12.09 | 10.93 | 9.86 | 8.92 | 8.02 | 7.21 | 6.42 | 5.70 | 5.02 |
| 20-24...... | 19.91 | 18.50 | 17.10 | 15.77 | 14.35 | 13.02 | 11.88 | 10.85 | 9.73 | 8.67 | 7.72 | 6.83 |
| 25-29...... | 23.98 | 21.98 | 20.07 | 18.28 | 16.52 | 14.89 | 13.46 | 12.17 | 10.82 | 9.56 | 8.38 | 7.37 |
| 30-34...... | 28.30 | 25.52 | 22.96 | 20.61 | 18.40 | 16.38 | 14.64 | 13.11 | 11.56 | 10.14 | 8.91 | 7.81 |
| 35-39...... | 33.17 | 29.44 | 26.09 | 23.10 | 20.40 | 17.96 | 15.92 | 14.15 | 12.41 | 10.83 | 9.51 | 8.34 |
| 40-44...... | 37.24 | 32.75 | 28.82 | 25.36 | 22.29 | 19.54 | 17.29 | 15.36 | 13.55 | 11.90 | 10.50 | 9.27 |
| 45-49...... | 42.25 | 37.18 | 32.79 | 28.95 | 25.59 | 22.61 | 20.11 | 17.99 | 16.01 | 14.22 | 12.67 | 11.30 |
| 50-54...... | 49.17 | 43.80 | 39.06 | 34.86 | 31.03 | 27.62 | 24.82 | 22.42 | 20.10 | 18.02 | 16.30 | 14.70 |
| 55-59...... | 57.84 | 52.49 | 47.57 | 43.08 | 38.86 | 35.03 | 31.86 | 29.08 | 26.36 | 23.88 | 21.74 | 19.81 |
| 60-64...... | 72.71 | 67.28 | 62.15 | 57.29 | 52.49 | 48.07 | 44.34 | 41.00 | 37.64 | 34.56 | 31.76 | 29.18 |
| 65-69...... | 93.87 | 88.49 | 83.15 | 77.92 | 72.47 | 67.35 | 62.99 | 58.99 | 54.81 | 50.94 | 47.43 | 44.21 |
| 70-74...... | 129.46 | 123.41 | 117.29 | 111.22 | 104.86 | 98.83 | 93.42 | 88.36 | 83.13 | 78.25 | 73.71 | 69.49 |
| 75-79...... | 183.54 | 176.33 | 168.86 | 161.47 | 153.68 | 145.74 | 138.89 | 132.45 | 125.61 | 119.19 | 113.20 | 107.63 |
| 80-84...... | 261.12 | 251.93 | 242.11 | 232.48 | 222.14 | 211.79 | 202.85 | 194.50 | 185.42 | 176.94 | 169.01 | 161.57 |
| 85+....... | 456.14 | 417.09 | 386.93 | 362.32 | 341.38 | 323.96 | 310.57 | 299.33 | 288.89 | 279.84 | 272.19 | 265.53 |

[a] Equivalent values of $°e_0$ shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

(1,000 $m_x$) OF MODEL LIFE TABLES

*reference in years)* [a]

| Level 60 ($°e_0$ = 50) | Level 65 ($°e_0$ = 52.5) | Level 70 ($°e_0$ = 55) | Level 75 ($°e_0$ = 57.6) | Level 80 ($°e_0$ = 60.4) | Level 85 ($°e_0$ = 63.2) | Level 90 ($°e_0$ = 65.8) | Level 95 ($°e_0$ = 68.2) | Level 100 ($°e_0$ = 70.2) | Level 105 ($°e_0$ = 71.7) | Level 110 ($°e_0$ = 73.0) | Level 115 ($°e_0$ = 73.9) | Sex and age (x) in years |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | MALES |
| 161.16 | 145.05 | 129.59 | 110.85 | 90.18 | 70.10 | 52.51 | 39.13 | 30.35 | 24.55 | 20.73 | 18.18 | ........0 |
| 16.44 | 14.15 | 12.10 | 10.18 | 8.45 | 6.79 | 5.20 | 3.67 | 2.45 | 1.64 | 1.11 | 0.75 | .......1–4 |
| 3.98 | 3.44 | 2.96 | 2.51 | 2.12 | 1.76 | 1.45 | 1.15 | 0.81 | 0.56 | 0.39 | 0.28 | .......5–9 |
| 2.70 | 2.34 | 2.05 | 1.77 | 1.53 | 1.30 | 1.09 | 0.89 | 0.66 | 0.47 | 0.35 | 0.26 | ......10–14 |
| 4.29 | 3.78 | 3.32 | 2.90 | 2.51 | 2.14 | 1.81 | 1.49 | 1.13 | 0.80 | 0.59 | 0.44 | ......15–19 |
| 6.22 | 5.48 | 4.84 | 4.22 | 3.62 | 3.04 | 2.52 | 2.02 | 1.49 | 1.08 | 0.80 | 0.61 | ......20–24 |
| 6.45 | 5.68 | 4.97 | 4.30 | 3.70 | 3.14 | 2.59 | 2.08 | 1.57 | 1.18 | 0.92 | 0.75 | ......25–29 |
| 6.84 | 6.00 | 5.24 | 4.55 | 3.91 | 3.33 | 2.79 | 2.29 | 1.82 | 1.43 | 1.18 | 1.00 | ......30–34 |
| 7.70 | 6.77 | 5.94 | 5.19 | 4.50 | 3.88 | 3.31 | 2.79 | 2.31 | 1.90 | 1.63 | 1.45 | ......35–39 |
| 9.50 | 8.41 | 7.45 | 6.58 | 5.79 | 5.07 | 4.45 | 3.87 | 3.35 | 2.88 | 2.52 | 2.27 | ......40–44 |
| 12.49 | 11.22 | 9.97 | 9.07 | 8.16 | 7.33 | 6.61 | 5.92 | 5.32 | 4.71 | 4.17 | 3.76 | ......45–49 |
| 16.95 | 15.43 | 14.08 | 12.85 | 11.78 | 10.81 | 9.96 | 9.14 | 8.40 | 7.67 | 6.95 | 6.30 | ......50–54 |
| 23.52 | 21.65 | 20.00 | 18.53 | 17.19 | 15.98 | 14.93 | 13.99 | 13.02 | 12.08 | 11.22 | 10.37 | ......55–59 |
| 33.83 | 31.58 | 29.53 | 27.69 | 26.05 | 24.59 | 23.24 | 21.96 | 20.70 | 19.42 | 18.22 | 17.13 | ......60–64 |
| 49.82 | 46.92 | 44.32 | 42.02 | 39.85 | 37.92 | 36.13 | 34.42 | 32.74 | 31.02 | 29.46 | 28.01 | ......65–69 |
| 75.35 | 71.61 | 68.14 | 65.01 | 62.09 | 59.41 | 56.94 | 54.60 | 52.36 | 50.13 | 48.10 | 46.28 | ......70–74 |
| 112.56 | 107.64 | 103.28 | 99.39 | 95.73 | 92.40 | 89.26 | 86.32 | 83.35 | 80.36 | 77.33 | 74.56 | ......75–79 |
| 169.30 | 162.98 | 157.07 | 151.67 | 146.64 | 141.99 | 137.74 | 133.68 | 129.39 | 126.39 | 122.42 | 118.21 | ......80–84 |
| 270.72 | 264.90 | 259.83 | 255.32 | 251.20 | 247.52 | 244.26 | 241.34 | 238.71 | 236.36 | 234.24 | 232.36 | ......85+ |
| | | | | | | | | | | | | FEMALES |
| 136.41 | 121.72 | 107.62 | 91.92 | 74.54 | 57.57 | 42.65 | 31.27 | 23.79 | 18.84 | 15.57 | 13.39 | ........0 |
| 16.19 | 13.87 | 11.65 | 9.80 | 7.92 | 6.14 | 4.55 | 3.09 | 2.06 | 1.38 | 0.93 | 0.63 | .......1–4 |
| 3.99 | 3.41 | 2.90 | 2.42 | 1.97 | 1.56 | 1.18 | 0.85 | 0.60 | 0.42 | 0.29 | 0.21 | .......5–9 |
| 2.96 | 2.55 | 2.17 | 1.82 | 1.49 | 1.18 | 0.91 | 0.67 | 0.50 | 0.36 | 0.26 | 0.20 | ......10–14 |
| 4.38 | 3.79 | 3.27 | 2.77 | 2.30 | 1.86 | 1.45 | 1.10 | 0.84 | 0.60 | 0.44 | 0.33 | ......15–19 |
| 5.97 | 5.19 | 4.46 | 3.76 | 3.13 | 2.54 | 1.99 | 1.49 | 1.11 | 0.80 | 0.60 | 0.47 | ......20–24 |
| 6.45 | 5.58 | 4.80 | 4.06 | 3.38 | 2.73 | 2.19 | 1.70 | 1.29 | 0.97 | 0.76 | 0.62 | ......25–29 |
| 6.80 | 5.89 | 5.09 | 4.34 | 3.64 | 3.00 | 2.47 | 1.97 | 1.56 | 1.23 | 1.00 | 0.85 | ......30–34 |
| 7.28 | 6.31 | 5.48 | 4.70 | 4.00 | 3.37 | 2.84 | 2.35 | 1.95 | 1.60 | 1.38 | 1.22 | ......35–39 |
| 8.15 | 7.15 | 6.29 | 5.49 | 4.78 | 4.14 | 3.58 | 3.06 | 2.65 | 2.31 | 2.04 | 1.87 | ......40–44 |
| 10.06 | 8.95 | 8.00 | 7.11 | 6.33 | 5.62 | 5.02 | 4.45 | 4.00 | 3.62 | 3.28 | 3.02 | ......45–49 |
| 13.22 | 11.91 | 10.78 | 9.74 | 8.80 | 7.94 | 7.20 | 6.51 | 5.98 | 5.51 | 5.15 | 4.83 | ......50–54 |
| 18.05 | 16.48 | 15.05 | 13.71 | 12.53 | 11.45 | 10.45 | 9.52 | 8.87 | 8.33 | 7.92 | 7.62 | ......55–59 |
| 26.79 | 24.70 | 22.82 | 21.04 | 19.41 | 17.89 | 16.58 | 15.33 | 14.47 | 13.78 | 13.10 | 12.59 | ......60–64 |
| 41.19 | 38.47 | 35.91 | 33.46 | 31.26 | 29.21 | 27.41 | 25.71 | 24.47 | 23.39 | 22.33 | 21.29 | ......65–69 |
| 65.38 | 61.59 | 58.10 | 54.80 | 51.80 | 48.96 | 46.46 | 44.08 | 42.31 | 40.53 | 38.68 | 36.89 | ......70–74 |
| 102.30 | 97.39 | 92.63 | 88.03 | 83.82 | 79.85 | 76.31 | 72.96 | 70.52 | 68.24 | 65.40 | 62.47 | ......75–79 |
| 154.48 | 147.96 | 141.94 | 136.17 | 130.92 | 125.92 | 121.48 | 117.21 | 113.98 | 110.99 | 107.61 | 104.01 | ......80–84 |
| 259.56 | 254.36 | 249.76 | 245.57 | 241.69 | 238.20 | 235.22 | 232.54 | 230.61 | 228.97 | 227.41 | 225.98 | ......85+ |

## Table II. Life-table death rates[a]

*Mortality level (or time-*

| Sex and age (x) in years | Level 0 ($^{o}e_{o}$ = 20) | Level 5 ($^{o}e_{o}$ = 22.5) | Level 10 ($^{o}e_{o}$ = 25) | Level 15 ($^{o}e_{o}$ = 27.5) | Level 20 ($^{o}e_{o}$ = 30) | Level 25 ($^{o}e_{o}$ = 32.5) | Level 30 ($^{o}e_{o}$ = 35) | Level 35 ($^{o}e_{o}$ = 37.5) | Level 40 ($^{o}e_{o}$ = 40) | Level 45 ($^{o}e_{o}$ = 42.5) | Level 50 ($^{o}e_{o}$ = 45) | Level 55 ($^{o}e_{o}$ = 47.5) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MALES | | | | | | | | | | | | |
| 0........ | 332.31 | 310.55 | 290.49 | 271.93 | 255.59 | 240.38 | 224.65 | 209.25 | 195.73 | 182.39 | 169.09 | 156.53 |
| 1–4....... | 267.98 | 235.27 | 207.67 | 184.21 | 164.43 | 146.69 | 130.86 | 116.31 | 104.16 | 92.50 | 82.07 | 72.52 |
| 5–9....... | 89.06 | 77.89 | 68.40 | 60.33 | 53.53 | 47.42 | 42.01 | 37.06 | 33.00 | 29.09 | 25.65 | 22.55 |
| 10–14...... | 55.68 | 49.22 | 43.61 | 38.78 | 34.66 | 30.93 | 27.57 | 24.47 | 21.89 | 19.42 | 17.22 | 15.24 |
| 15–19...... | 68.08 | 62.41 | 57.05 | 52.03 | 47.65 | 43.52 | 39.45 | 35.60 | 32.34 | 29.14 | 26.28 | 23.68 |
| 20–24...... | 85.81 | 79.96 | 74.20 | 68.61 | 63.91 | 59.35 | 54.34 | 49.76 | 45.52 | 41.46 | 37.63 | 34.07 |
| 25–29...... | 100.33 | 92.40 | 84.74 | 77.49 | 71.05 | 64.96 | 58.89 | 53.45 | 48.50 | 43.81 | 39.40 | 35.44 |
| 30–34...... | 119.56 | 108.54 | 98.22 | 88.66 | 80.35 | 72.62 | 65.19 | 58.61 | 52.79 | 47.33 | 42.27 | 37.76 |
| 35–39...... | 147.14 | 131.73 | 117.68 | 104.93 | 94.14 | 84.24 | 75.03 | 67.03 | 60.05 | 53.58 | 47.67 | 42.45 |
| 40–44...... | 184.52 | 163.96 | 145.60 | 129.18 | 115.45 | 102.96 | 91.58 | 81.81 | 73.06 | 65.05 | 58.06 | 51.94 |
| 45–49...... | 224.27 | 199.69 | 177.89 | 158.44 | 141.87 | 126.88 | 113.54 | 102.09 | 91.67 | 82.19 | 74.04 | 66.96 |
| 50–54...... | 259.66 | 234.09 | 210.97 | 190.11 | 172.43 | 156.31 | 141.39 | 128.48 | 116.84 | 106.46 | 96.89 | 88.69 |
| 55–59...... | 302.86 | 277.92 | 254.60 | 232.95 | 214.20 | 196.83 | 180.32 | 165.73 | 152.50 | 140.30 | 129.43 | 119.81 |
| 60–64...... | 350.36 | 328.05 | 306.35 | 285.41 | 266.90 | 249.36 | 231.93 | 216.20 | 201.75 | 188.28 | 176.23 | 165.63 |
| 65–69...... | 420.21 | 400.41 | 380.42 | 360.40 | 342.45 | 325.04 | 306.80 | 289.97 | 274.30 | 259.50 | 245.62 | 233.12 |
| 70–74...... | 529.66 | 510.83 | 491.26 | 471.32 | 452.34 | 433.58 | 414.31 | 396.07 | 378.29 | 361.39 | 345.17 | 330.47 |
| 75–79...... | 654.67 | 636.83 | 618.02 | 598.55 | 580.59 | 561.54 | 541.86 | 523.16 | 504.82 | 487.20 | 470.10 | 454.28 |
| 80–84...... | 813.99 | 796.15 | 777.25 | 757.58 | 739.73 | 720.64 | 700.45 | 681.17 | 662.40 | 644.34 | 626.70 | 610.36 |
| FEMALES | | | | | | | | | | | | |
| 0........ | 306.76 | 286.66 | 268.15 | 251.01 | 233.73 | 217.48 | 203.25 | 189.25 | 175.59 | 162.01 | 149.05 | 136.40 |
| 1–4....... | 273.39 | 240.03 | 211.87 | 187.93 | 166.95 | 148.16 | 132.18 | 117.46 | 104.75 | 92.58 | 81.81 | 71.93 |
| 5–9....... | 92.69 | 81.06 | 71.16 | 62.79 | 55.45 | 48.87 | 43.29 | 38.19 | 33.85 | 29.70 | 26.08 | 22.81 |
| 10–14...... | 63.42 | 56.06 | 49.69 | 44.18 | 39.29 | 34.87 | 31.08 | 27.58 | 24.58 | 21.70 | 19.16 | 16.85 |
| 15–19...... | 76.77 | 70.38 | 64.32 | 58.68 | 53.21 | 48.11 | 43.60 | 39.32 | 35.42 | 31.61 | 28.10 | 24.79 |
| 20–24...... | 94.84 | 88.38 | 82.02 | 75.84 | 69.27 | 63.03 | 57.70 | 52.80 | 47.48 | 42.44 | 37.88 | 33.58 |
| 25–29...... | 113.13 | 104.19 | 95.57 | 87.38 | 79.35 | 71.80 | 65.10 | 59.03 | 52.67 | 46.68 | 41.02 | 36.18 |
| 30–34...... | 132.14 | 119.96 | 108.56 | 97.99 | 87.96 | 78.66 | 70.62 | 63.45 | 56.19 | 49.43 | 43.57 | 38.31 |
| 35–39...... | 153.14 | 137.11 | 122.47 | 109.21 | 97.05 | 85.94 | 76.53 | 68.32 | 60.19 | 52.70 | 46.43 | 40.86 |
| 40–44...... | 170.34 | 151.35 | 134.41 | 119.24 | 105.55 | 93.15 | 82.87 | 73.98 | 65.52 | 57.77 | 51.15 | 45.28 |
| 45–49...... | 191.05 | 170.10 | 151.54 | 134.96 | 120.27 | 106.99 | 95.75 | 86.09 | 76.96 | 68.68 | 61.40 | 54.93 |
| 50–54...... | 218.97 | 197.41 | 177.92 | 160.33 | 144.01 | 129.18 | 116.86 | 106.14 | 95.70 | 86.19 | 78.28 | 70.92 |
| 55–59...... | 252.65 | 232.02 | 212.56 | 194.48 | 177.08 | 161.05 | 147.54 | 135.54 | 123.64 | 112.68 | 103.07 | 94.38 |
| 60–64...... | 307.59 | 288.00 | 268.95 | 250.56 | 232.02 | 214.56 | 199.56 | 185.95 | 172.01 | 159.03 | 147.09 | 135 97 |
| 65–69...... | 380.19 | 362.28 | 344.19 | 326.08 | 306.78 | 288.25 | 272.07 | 257.04 | 241.02 | 225.91 | 212.00 | 199.06 |
| 70–74...... | 488.91 | 471.53 | 453.48 | 435.07 | 415.44 | 396.24 | 378.63 | 361.87 | 344.11 | 327.22 | 311.21 | 296.02 |
| 75–79...... | 629.00 | 611.86 | 593.79 | 575.08 | 555.05 | 534.15 | 515.43 | 497.53 | 477.96 | 459.17 | 441.16 | 424.03 |
| 80–84...... | 789.92 | 772.63 | 754.27 | 735.19 | 714.27 | 692.37 | 672.99 | 654.29 | 633.46 | 613.38 | 594.02 | 575.44 |

[a] Equivalent values of $^{o}e_{o}$ shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

($1{,}000q_x$) OF MODEL LIFE TABLES

*reference in years)* [a]

| Level 60 ($°e_0 = 50$) | Level 65 ($°e_0 = 52.5$) | Level 70 ($°e_0 = 55$) | Level 75 ($°e_0 = 57.6$) | Level 80 ($°e_0 = 60.4$) | Level 85 ($°e_0 = 63.2$) | Level 90 ($°e_0 = 65.8$) | Level 95 ($°e_0 = 68.2$) | Level 100 ($°e_0 = 70.2$) | Level 105 ($°e_0 = 71.7$) | Level 110 ($°e_0 = 73.0$) | Level 115 ($°e_0 = 73.9$) | Sex and age (x) in years |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | MALES |
| 143.78 | 130.82 | 118.11 | 102.34 | 84.47 | 66.60 | 50.52 | 38.01 | 29.67 | 24.11 | 20.41 | 17.94 | ........0 |
| 63.56 | 54.98 | 47.22 | 39.88 | 33.21 | 26.78 | 20.56 | 14.55 | 9.75 | 6.55 | 4.42 | 3.00 | .......1–4 |
| 19.72 | 17.03 | 14.68 | 12.48 | 10.55 | 8.75 | 7.20 | 5.73 | 4.06 | 2.80 | 1.96 | 1.40 | .......5–9 |
| 13.40 | 11.66 | 10.18 | 8.81 | 7.61 | 6.48 | 5.44 | 4.49 | 3.31 | 2.36 | 1.73 | 1.31 | ......10–14 |
| 21.22 | 18.74 | 16.49 | 14.39 | 12.45 | 10.64 | 8.99 | 7.42 | 5.64 | 4.01 | 2.93 | 2.21 | ......15–19 |
| 30.63 | 27.02 | 23.90 | 20.89 | 17.94 | 15.10 | 12.52 | 10.07 | 7.44 | 5.37 | 3.99 | 3.07 | ......20–24 |
| 31.74 | 28.00 | 24.54 | 21.25 | 18.35 | 15.59 | 12.87 | 10.34 | 7.83 | 5.90 | 4.61 | 3.75 | ......25–29 |
| 33.61 | 29.56 | 25.87 | 22.48 | 19.36 | 16.49 | 13.86 | 11.39 | 9.05 | 7.14 | 5.86 | 5.01 | ......30–34 |
| 37.75 | 33.28 | 29.28 | 25.60 | 22.26 | 19.22 | 16.43 | 13.85 | 11.50 | 9.47 | 8.12 | 7.22 | ......35–39 |
| 46.41 | 41.17 | 36.57 | 32.36 | 28.52 | 25.04 | 22.02 | 19.16 | 16.61 | 14.30 | 12.50 | 11.30 | ......40–44 |
| 60.58 | 54.57 | 48.66 | 44.36 | 40.00 | 36.00 | 32.50 | 29.15 | 26.23 | 23.30 | 20.66 | 18.61 | ......45–49 |
| 81.28 | 74.28 | 67.99 | 62.25 | 57.21 | 52.65 | 48.57 | 44.67 | 41.11 | 37.63 | 34.15 | 31.01 | ......50–54 |
| 111.05 | 102.67 | 95.26 | 88.53 | 82.41 | 76.85 | 71.96 | 67.56 | 63.06 | 58.62 | 54.58 | 50.54 | ......55–59 |
| 155.94 | 146.35 | 137.49 | 129.48 | 122.30 | 115.83 | 109.82 | 104.10 | 98.40 | 92.61 | 87.14 | 82.15 | ......60–64 |
| 221.49 | 209.98 | 199.52 | 190.11 | 181.21 | 173.20 | 165.69 | 158.46 | 151.32 | 143.92 | 137.21 | 130.87 | ......65–69 |
| 317.01 | 303.71 | 291.13 | 279.61 | 268.74 | 258.62 | 249.22 | 240.20 | 231.51 | 222.76 | 214.70 | 207.39 | ......70–74 |
| 439.18 | 424.08 | 410.44 | 398.05 | 386.23 | 375.29 | 364.86 | 354.98 | 344.87 | 334.57 | 323.99 | 314.24 | ......75–79 |
| 594.78 | 578.98 | 563.92 | 549.84 | 536.52 | 523.97 | 512.30 | 500.96 | 490.21 | 480.19 | 468.67 | 456.23 | ......80–84 |
| | | | | | | | | | | | | FEMALES |
| 123.75 | 111.54 | 99.58 | 85.99 | 70.59 | 55.19 | 41.33 | 30.55 | 23.37 | 18.58 | 15.39 | 13.26 | ........0 |
| 62.64 | 53.92 | 45.50 | 38.41 | 31.17 | 24.24 | 18.03 | 12.27 | 8.22 | 5.52 | 3.72 | 2.52 | .......1–4 |
| 19.73 | 16.88 | 14.39 | 12.03 | 9.82 | 7.77 | 5.89 | 4.23 | 3.01 | 2.08 | 1.46 | 1.05 | .......5–9 |
| 14.68 | 12.65 | 10.81 | 9.05 | 7.40 | 5.86 | 4.53 | 3.36 | 2.49 | 1.79 | 1.32 | 1.01 | ......10–14 |
| 21.65 | 18.78 | 16.22 | 13.74 | 11.43 | 9.24 | 7.25 | 5.48 | 4.17 | 2.99 | 2.20 | 1.67 | ......15–19 |
| 29.42 | 25.59 | 22.06 | 18.63 | 15.54 | 12.61 | 9.89 | 7.44 | 5.51 | 4.01 | 3.01 | 2.35 | ......20–24 |
| 31.75 | 27.51 | 23.73 | 20.09 | 16.74 | 13.55 | 10.88 | 8.47 | 6.41 | 4.83 | 3.78 | 3.08 | ......25–29 |
| 33.45 | 29.01 | 25.11 | 21.44 | 18.04 | 14.91 | 12.25 | 9.81 | 7.79 | 6.12 | 5.00 | 4.26 | ......30–34 |
| 35.73 | 31.08 | 27.03 | 23.23 | 19.81 | 16.69 | 14.11 | 11.68 | 9.70 | 7.99 | 6.85 | 6.09 | ......35–39 |
| 39.94 | 35.11 | 30.94 | 27.06 | 23.62 | 20.48 | 17.76 | 15.21 | 13.19 | 11.49 | 10.17 | 9.29 | ......40–44 |
| 49.06 | 43.80 | 39.20 | 34.93 | 31.17 | 27.71 | 24.78 | 21.99 | 19.79 | 17.94 | 16.28 | 14.99 | ......45–49 |
| 63.98 | 57.83 | 52.50 | 47.53 | 43.06 | 38.91 | 35.37 | 32.01 | 29.47 | 27.15 | 25.41 | 23.85 | ......50–54 |
| 86.36 | 79.12 | 72.54 | 66.29 | 60.76 | 55.65 | 50.91 | 46.47 | 43.37 | 40.78 | 38.84 | 37.38 | ......55–59 |
| 125.54 | 116.32 | 107.95 | 99.95 | 92.57 | 85.61 | 79.59 | 73.84 | 69.81 | 66.61 | 63.41 | 61.01 | ......60–64 |
| 186.71 | 175.46 | 164.76 | 154.38 | 144.98 | 136.09 | 128.27 | 120.77 | 115.32 | 110.49 | 105.77 | 101.05 | ......65–69 |
| 280.98 | 266.86 | 253.73 | 241.01 | 229.29 | 218.09 | 208.12 | 198.52 | 191.33 | 184.00 | 176.35 | 168.88 | ......70–74 |
| 407.31 | 391.61 | 376.08 | 360.75 | 346.50 | 332.80 | 320.43 | 308.53 | 299.75 | 291.47 | 281.04 | 270.15 | ......75–79 |
| 557.17 | 540.05 | 523.83 | 507.96 | 493.16 | 478.85 | 465.91 | 453.25 | 443.51 | 434.43 | 423.99 | 412.71 | ......80–84 |

## TABLE III. SURVIVORS TO EXACT AGES

*Mortality level or (time-*

| *Sex and age (x) in years* | *Level 0* ($^{o}e_0$ = 20) | *Level 5* ($^{o}e_0$ = 22.5) | *Level 10* ($^{o}e_0$ = 25) | *Level 15* ($^{o}e_0$ = 27.5) | *Level 20* ($^{o}e_0$ = 30) | *Level 25* ($^{o}e_0$ = 32.5) | *Level 30* ($^{o}e_0$ = 35) | *Level 35* ($^{o}e_0$ = 37.5) | *Level 40* ($^{o}e_0$ = 40) | *Level 45* ($^{o}e_0$ = 42.5) | *Level 50* ($^{o}e_0$ = 45) | *Level 55* ($^{o}e_0$ = 47.5) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MALES | | | | | | | | | | | | |
| 0......... | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 |
| 1......... | 66,769 | 68,545 | 70,951 | 72,807 | 74,441 | 75,962 | 77,535 | 79,075 | 80,427 | 81,761 | 83,091 | 84,347 |
| 5......... | 48,876 | 52,418 | 56,217 | 59,395 | 62,201 | 64,819 | 67,389 | 69,878 | 72,050 | 74,198 | 76,272 | 78,230 |
| 10......... | 44,523 | 48,335 | 52,372 | 55,812 | 58,871 | 61,745 | 64,558 | 67,288 | 69,672 | 72,040 | 74,316 | 76,466 |
| 15......... | 42,044 | 45,956 | 50,088 | 53,648 | 56,831 | 59,835 | 62,778 | 65,641 | 68,147 | 70,641 | 73,036 | 75,301 |
| 20......... | 39,182 | 43,088 | 47,230 | 50,857 | 54,123 | 57,231 | 60,301 | 63,304 | 65,943 | 68,583 | 71,117 | 73,518 |
| 25......... | 35,820 | 39,643 | 43,726 | 47,368 | 50,664 | 53,834 | 57,024 | 60,154 | 62,941 | 65,740 | 68,441 | 71,013 |
| 30......... | 32,226 | 35,980 | 40,021 | 43,697 | 47,064 | 50,337 | 53,666 | 56,939 | 59,888 | 62,860 | 65,744 | 68,496 |
| 35......... | 28,373 | 32,075 | 36,090 | 39,823 | 43,282 | 46,682 | 50,168 | 53,602 | 56,727 | 59,885 | 62,965 | 65,910 |
| 40......... | 24,198 | 27,850 | 31,843 | 35,644 | 39,207 | 42,750 | 46,404 | 50,009 | 53,321 | 56,676 | 59,963 | 63,112 |
| 45......... | 19,733 | 23,284 | 27,207 | 31,040 | 34,681 | 38,348 | 42,154 | 45,918 | 49,425 | 52,989 | 56,482 | 59,834 |
| 50......... | 15,307 | 18,634 | 22,367 | 26,122 | 29,761 | 33,482 | 37,368 | 41,230 | 44,894 | 48,634 | 52,300 | 55,828 |
| 55......... | 11,332 | 14,272 | 17,648 | 21,156 | 24,629 | 28,248 | 32,085 | 35,933 | 39,649 | 43,456 | 47,233 | 50,877 |
| 60......... | 7,900 | 10,306 | 13,155 | 16,228 | 19,353 | 22,688 | 26,299 | 29,978 | 33,603 | 37,359 | 41,120 | 44,781 |
| 65......... | 5,132 | 6,925 | 9,125 | 11,596 | 14,188 | 17,031 | 20,199 | 23,497 | 26,824 | 30,325 | 33,873 | 37,364 |
| 70......... | 2,975 | 4,152 | 5,654 | 7,417 | 9,329 | 11,495 | 14,002 | 16,684 | 19,466 | 22,456 | 25,553 | 28,654 |
| 75......... | 1,399 | 2,031 | 2,876 | 3,921 | 5,109 | 6,511 | 8,201 | 10,076 | 12,102 | 14,341 | 16,733 | 19,185 |
| 80......... | 483 | 738 | 1,099 | 1,574 | 2,143 | 2,855 | 3,757 | 4,805 | 5,993 | 7,354 | 8,867 | 10,470 |
| 85......... | 90 | 150 | 245 | 382 | 558 | 798 | 1,125 | 1,532 | 2,023 | 2,616 | 3,310 | 4,080 |
| FEMALES | | | | | | | | | | | | |
| 0......... | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 |
| 1......... | 69,324 | 71,334 | 73,185 | 74,899 | 76,627 | 78,252 | 79,675 | 81,075 | 82,441 | 83,799 | 85,095 | 86,360 |
| 5......... | 50,372 | 54,212 | 57,679 | 60,823 | 63,834 | 66,658 | 69,144 | 71,552 | 73,805 | 76,041 | 78,133 | 80,148 |
| 10......... | 45,703 | 49,818 | 53,575 | 57,004 | 60,294 | 63,400 | 66,151 | 68,819 | 71,307 | 73,783 | 76,095 | 78,320 |
| 15......... | 42,805 | 47,025 | 50,913 | 54,486 | 57,925 | 61,189 | 64,095 | 66,921 | 69,554 | 72,182 | 74,637 | 77,000 |
| 20......... | 39,519 | 43,715 | 47,638 | 51,289 | 54,843 | 58,245 | 61,300 | 64,290 | 67,090 | 69,900 | 72,540 | 75,091 |
| 25......... | 35,771 | 39,851 | 43,731 | 47,399 | 51,044 | 54,574 | 57,763 | 60,895 | 63,905 | 66,933 | 69,792 | 72,569 |
| 30......... | 31,724 | 35,699 | 39,552 | 43,257 | 46,994 | 50,656 | 54,003 | 57,300 | 60,539 | 63,809 | 66,929 | 69,943 |
| 35......... | 27,532 | 31,417 | 35,258 | 39,018 | 42,860 | 46,671 | 50,189 | 53,664 | 57,137 | 60,655 | 64,013 | 67,263 |
| 40......... | 23,316 | 27,109 | 30,940 | 34,757 | 38,700 | 42,660 | 46,348 | 49,998 | 53,698 | 57,458 | 61,041 | 64,515 |
| 45......... | 19,344 | 23,006 | 26,781 | 30,613 | 34,615 | 38,686 | 42,507 | 46,299 | 50,180 | 54,139 | 57,919 | 61,594 |
| 50......... | 15,648 | 19,093 | 22,723 | 26,481 | 30,452 | 34,547 | 38,437 | 42,313 | 46,318 | 50,421 | 54,363 | 58,211 |
| 55......... | 12,222 | 15,324 | 18,680 | 22,235 | 26,067 | 30,084 | 33,945 | 37,822 | 41,885 | 46,075 | 50,107 | 54,083 |
| 60......... | 9,134 | 11,769 | 14,709 | 17,911 | 21,451 | 25,239 | 28,937 | 32,696 | 36,706 | 40,883 | 44,942 | 48,979 |
| 65......... | 6,324 | 8,380 | 10,753 | 13,423 | 16,474 | 19,824 | 23,162 | 26,616 | 30,392 | 34,381 | 38,331 | 42,319 |
| 70......... | 3,920 | 5,344 | 7,052 | 9,046 | 11,420 | 14,110 | 16,860 | 19,775 | 23,067 | 26,614 | 30,205 | 33,895 |
| 75......... | 2,003 | 2,824 | 3,854 | 5,110 | 6,676 | 8,519 | 10,476 | 12,619 | 15,129 | 17,905 | 20,805 | 23,861 |
| 80......... | 743 | 1,096 | 1,566 | 2,171 | 2,970 | 3,969 | 5,076 | 6,341 | 7,898 | 9,684 | 11,627 | 13,743 |
| 85......... | 156 | 249 | 385 | 575 | 849 | 1,221 | 1,660 | 2,192 | 2,895 | 3,744 | 4,720 | 5,835 |

[a] Equivalent values of $^{o}e_0$ shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

## ($l_x$) OF MODEL LIFE TABLES

*reference in years)* [a]

| Level 60 ($°e_0$ = 50) | Level 65 ($°e_0$ = 52.5) | Level 70 ($°e_0$ = 55) | Level 75 ($°e_0$ = 57.6) | Level 80 ($°e_0$ = 60.4) | Level 85 ($°e_0$ = 63.2) | Level 90 ($°e_0$ = 65.8) | Level 95 ($°e_0$ = 68.2) | Level 100 ($°e_0$ = 70.2) | Level 105 ($°e_0$ = 71.7) | Level 110 ($°e_0$ = 73.0) | Level 115 ($°e_0$ = 73.9) | Sex and age (x) in years |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | MALES |
| 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | ..........0 |
| 85,622 | 86,918 | 88,189 | 89,766 | 91,553 | 93,340 | 94,948 | 96,199 | 97,033 | 97,589 | 97,959 | 98,206 | ..........1 |
| 80,180 | 82,139 | 84,025 | 86,186 | 88,513 | 90,840 | 92,996 | 94,799 | 96,087 | 96,950 | 97,526 | 97,911 | ..........5 |
| 78,599 | 80,740 | 82,792 | 85,110 | 87,579 | 90,045 | 92,326 | 94,256 | 95,697 | 96,679 | 97,335 | 97,774 | .........10 |
| 77,546 | 79,799 | 81,949 | 84,360 | 86,913 | 89,462 | 91,824 | 93,838 | 95,380 | 96,451 | 97,167 | 97,646 | .........15 |
| 75,900 | 78,304 | 80,598 | 83,146 | 85,831 | 88,510 | 90,999 | 93,142 | 94,842 | 96,064 | 96,882 | 97,430 | .........20 |
| 73,575 | 76,188 | 78,672 | 81,409 | 84,291 | 87,173 | 89,860 | 92,204 | 94,136 | 95,548 | 96,495 | 97,131 | .........25 |
| 71,240 | 74,055 | 76,741 | 79,679 | 82,744 | 85,814 | 88,704 | 91,251 | 93,399 | 94,984 | 96,050 | 96,767 | .........30 |
| 68,846 | 71,866 | 74,756 | 77,888 | 81,142 | 84,399 | 87,475 | 90,212 | 92,554 | 94,306 | 95,487 | 96,282 | .........35 |
| 66,247 | 69,474 | 72,567 | 75,894 | 79,336 | 82,777 | 86,038 | 88,963 | 91,490 | 93,413 | 94,712 | 95,587 | .........40 |
| 63,172 | 66,614 | 69,913 | 73,438 | 77,073 | 80,704 | 84,143 | 87,258 | 89,970 | 92,077 | 93,528 | 94,507 | .........45 |
| 59,345 | 62,979 | 66,511 | 70,180 | 73,990 | 77,799 | 81,408 | 84,714 | 87,610 | 89,932 | 91,596 | 92,748 | .........50 |
| 54,521 | 58,301 | 61,989 | 65,811 | 69,757 | 73,703 | 77,454 | 80,930 | 84,008 | 86,548 | 88,468 | 89,872 | .........55 |
| 48,466 | 52,315 | 56,084 | 59,985 | 64,008 | 68,039 | 71,880 | 75,462 | 78,710 | 81,475 | 83,639 | 85,330 | .........60 |
| 40,908 | 44,659 | 48,373 | 52,218 | 56,180 | 60,158 | 63,986 | 67,606 | 70,965 | 73,930 | 76,351 | 78,320 | .........65 |
| 31,847 | 35,282 | 38,722 | 42,291 | 46,000 | 49,739 | 53,384 | 56,893 | 60,227 | 63,290 | 65,875 | 68,070 | .........70 |
| 21,751 | 24,567 | 27,449 | 30,466 | 33,638 | 36,875 | 40,080 | 43,227 | 46,284 | 49,192 | 51,732 | 53,953 | .........75 |
| 12,198 | 14,149 | 16,183 | 18,339 | 20,646 | 23,036 | 25,456 | 27,882 | 30,322 | 32,734 | 34,971 | 36,999 | .........80 |
| 4,943 | 5,957 | 7,057 | 8,255 | 9,569 | 10,966 | 12,415 | 13,914 | 15,458 | 17,015 | 18,581 | 20,119 | .........85 |
| | | | | | | | | | | | | FEMALES |
| 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 | ..........0 |
| 87,625 | 88,846 | 90,042 | 91,401 | 92,941 | 94,481 | 95,867 | 96,945 | 97,663 | 98,142 | 98,461 | 98,674 | ..........1 |
| 82,136 | 84,055 | 85,945 | 87,890 | 90,044 | 92,191 | 94,139 | 95,755 | 96,860 | 97,600 | 98,095 | 98,425 | ..........5 |
| 80,515 | 82,636 | 84,708 | 86,833 | 89,160 | 91,475 | 93,585 | 95,350 | 96,568 | 97,397 | 97,952 | 98,322 | .........10 |
| 79,333 | 81,591 | 83,792 | 86,047 | 88,500 | 90,939 | 93,161 | 95,030 | 96,328 | 97,223 | 97,823 | 98,223 | .........15 |
| 77,615 | 80,059 | 82,433 | 84,865 | 87,488 | 90,099 | 92,486 | 94,509 | 95,926 | 96,932 | 97,608 | 98,059 | .........20 |
| 75,332 | 78,010 | 80,615 | 83,284 | 86,128 | 88,963 | 91,571 | 93,806 | 95,397 | 96,543 | 97,314 | 97,829 | .........25 |
| 72,940 | 75,864 | 78,702 | 81,611 | 84,686 | 87,758 | 90,575 | 93,011 | 94,786 | 96,077 | 96,946 | 97,528 | .........30 |
| 70,500 | 73,663 | 76,726 | 79,861 | 83,158 | 86,450 | 89,465 | 92,099 | 94,048 | 95,489 | 96,461 | 97,113 | .........35 |
| 67,981 | 71,374 | 74,652 | 78,006 | 81,511 | 85,007 | 88,203 | 91,023 | 93,136 | 94,726 | 95,800 | 96,522 | .........40 |
| 65,266 | 68,868 | 72,342 | 75,895 | 79,586 | 83,266 | 86,637 | 89,639 | 91,908 | 93,638 | 94,826 | 95,625 | .........45 |
| 62,064 | 65,852 | 69,506 | 73,244 | 77,105 | 80,959 | 84,490 | 87,668 | 90,089 | 91,958 | 93,282 | 94,192 | .........50 |
| 58,093 | 62,044 | 65,857 | 69,763 | 73,785 | 77,809 | 81,502 | 84,862 | 87,434 | 89,461 | 90,912 | 91,946 | .........55 |
| 53,076 | 57,135 | 61,080 | 65,138 | 69,302 | 73,479 | 77,353 | 80,918 | 83,642 | 85,813 | 87,381 | 88,509 | .........60 |
| 46,413 | 50,489 | 54,486 | 58,627 | 62,887 | 67,188 | 71,196 | 74,943 | 77,803 | 80,097 | 81,840 | 83,109 | .........65 |
| 37,747 | 41,630 | 45,509 | 49,576 | 53,770 | 58,044 | 62,064 | 65,892 | 68,831 | 71,247 | 73,184 | 74,711 | .........70 |
| 27,141 | 30,521 | 33,962 | 37,628 | 41,441 | 45,385 | 49,147 | 52,811 | 55,662 | 58,138 | 60,278 | 62,094 | .........75 |
| 16,086 | 18,569 | 21,190 | 24,054 | 27,082 | 30,281 | 33,399 | 36,517 | 38,977 | 41,193 | 43,337 | 45,319 | .........80 |
| 7,123 | 8,541 | 10,090 | 11,836 | 13,726 | 15,781 | 17,838 | 19,966 | 21,690 | 23,298 | 24,963 | 26,615 | .........85 |

## TABLE IV. SURVIVORS WITHIN AGE

*Mortality level (or time-*

| Sex and age (x) in years | Level 0 (°e₀ = 20) | Level 5 (°e₀ = 22.5) | Level 10 (°e₀ = 25) | Level 15 (°e₀ = 27.5) | Level 20 (°e₀ = 30) | Level 25 (°e₀ = 32.5) | Level 30 (°e₀ = 35) | Level 35 (°e₀ = 37.5) | Level 40 (°e₀ = 40) | Level 45 (°e₀ = 42.5) | Level 50 (°e₀ = 45) | Level 55 (°e₀ = 47.5) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MALES | | | | | | | | | | | | |
| $T_0$ ....... | 1,980,306 | 2,214,814 | 2,475,323 | 2,722,611 | 2,957,495 | 3,194,138 | 3,441,523 | 3,689,987 | 3,924,285 | 4,165,152 | 4,404,679 | 4,637,355 |
| 0–4....... | 304,577 | 316,722 | 331,076 | 342,668 | 352,891 | 362,419 | 371,985 | 381,293 | 389,437 | 397,482 | 405,362 | 412,803 |
| 5–9....... | 233,498 | 251,882 | 271,472 | 288,018 | 302,680 | 316,410 | 329,868 | 342,915 | 354,305 | 365,595 | 376,470 | 386,740 |
| 10–14...... | 216,418 | 235,728 | 256,150 | 273,650 | 289,255 | 303,950 | 318,340 | 332,322 | 344,548 | 356,702 | 368,380 | 379,418 |
| 15–19...... | 203,065 | 222,610 | 243,295 | 261,262 | 277,385 | 292,665 | 307,698 | 322,362 | 335,225 | 348,060 | 360,382 | 372,048 |
| 20–24...... | 187,505 | 206,828 | 227,390 | 245,562 | 261,968 | 277,662 | 293,312 | 308,645 | 322,210 | 335,808 | 348,895 | 361,328 |
| 25–29...... | 170,115 | 189,058 | 209,368 | 227,662 | 244,320 | 260,428 | 276,725 | 292,732 | 307,072 | 321,500 | 335,462 | 348,772 |
| 30–34...... | 151,498 | 170,138 | 190,278 | 208,800 | 225,865 | 242,548 | 259,585 | 276,352 | 291,538 | 306,862 | 321,772 | 336,015 |
| 35–39...... | 131,428 | 149,812 | 169,832 | 188,668 | 206,222 | 223,580 | 241,430 | 259,028 | 275,120 | 291,402 | 307,320 | 322,555 |
| 40–44...... | 109,828 | 127,835 | 147,625 | 166,710 | 184,720 | 202,745 | 221,395 | 239,818 | 256,865 | 274,162 | 291,112 | 307,365 |
| 45–49...... | 87,600 | 104,795 | 123,935 | 142,905 | 161,105 | 179,575 | 198,805 | 217,870 | 235,798 | 254,058 | 271,955 | 289,155 |
| 50–54...... | 66,598 | 82,265 | 100,038 | 118,195 | 135,975 | 154,325 | 173,632 | 192,908 | 211,358 | 230,225 | 248,832 | 266,762 |
| 55–59...... | 48,080 | 61,445 | 77,008 | 93,460 | 109,955 | 127,340 | 145,960 | 164,778 | 183,130 | 202,038 | 220,882 | 239,145 |
| 60–64...... | 32,580 | 43,078 | 55,700 | 69,560 | 83,852 | 99,298 | 116,245 | 133,688 | 151,068 | 169,210 | 187,482 | 205,362 |
| 65–69...... | 20,268 | 27,692 | 36,948 | 47,532 | 58,792 | 71,315 | 85,502 | 100,452 | 115,725 | 131,952 | 148,565 | 165,045 |
| 70–74...... | 10,935 | 15,458 | 21,325 | 28,345 | 36,095 | 45,015 | 55,508 | 66,900 | 78,920 | 91,992 | 105,715 | 119,598 |
| 75–79...... | 4,705 | 6,922 | 9,938 | 13,738 | 18,130 | 23,415 | 29,895 | 37,202 | 45,238 | 54,238 | 64,000 | 74,138 |
| 80–84...... | 1,432 | 2,220 | 3,360 | 4,890 | 6,752 | 9,132 | 12,205 | 15,842 | 20,040 | 24,925 | 30,442 | 36,375 |
| 85+....... | 176 | 326 | 585 | 986 | 1,533 | 2,316 | 3,433 | 4,880 | 6,688 | 8,941 | 11,651 | 14,731 |
| FEMALES | | | | | | | | | | | | |
| $T_0$ ....... | 2,019,694 | 2,271,979 | 2,524,846 | 2,777,294 | 3,040,235 | 3,307,683 | 3,559,943 | 3,813,632 | 4,074,257 | 4,343,703 | 4,605,783 | 4,866,027 |
| 0–4....... | 314,490 | 327,880 | 340,066 | 351,211 | 362,113 | 372,350 | 381,341 | 390,108 | 398,459 | 406,753 | 414,581 | 422,165 |
| 5–9....... | 240,188 | 260,075 | 278,135 | 294,568 | 310,320 | 325,145 | 338,238 | 350,928 | 362,780 | 374,560 | 385,570 | 396,170 |
| 10–14...... | 221,270 | 242,108 | 261,220 | 278,725 | 295,548 | 311,472 | 325,615 | 339,350 | 352,152 | 364,912 | 376,830 | 388,300 |
| 15–19...... | 205,810 | 226,850 | 246,378 | 264,438 | 281,920 | 298,585 | 313,488 | 328,028 | 341,610 | 355,205 | 367,942 | 380,228 |
| 20–24...... | 188,225 | 208,915 | 228,422 | 246,720 | 264,718 | 282,048 | 297,658 | 312,962 | 327,488 | 342,082 | 355,830 | 369,150 |
| 25–29...... | 168,738 | 188,875 | 208,208 | 226,640 | 245,095 | 263,075 | 279,415 | 295,488 | 311,110 | 326,855 | 341,802 | 356,280 |
| 30–34...... | 148,140 | 167,790 | 187,025 | 205,688 | 224,635 | 243,318 | 260,480 | 277,410 | 294,190 | 311,160 | 327,355 | 343,015 |
| 35–39...... | 127,120 | 146,315 | 165,495 | 184,438 | 203,900 | 223,328 | 241,342 | 259,155 | 277,088 | 295,282 | 312,635 | 329,445 |
| 40–44...... | 106,650 | 125,288 | 144,302 | 163,425 | 183,288 | 203,365 | 222,138 | 240,742 | 259,695 | 278,992 | 297,400 | 315,272 |
| 45–49...... | 87,480 | 105,248 | 123,760 | 142,735 | 162,668 | 183,082 | 202,360 | 221,530 | 241,245 | 261,400 | 280,705 | 299,512 |
| 50–54...... | 69,675 | 86,042 | 103,508 | 121,790 | 141,298 | 161,578 | 180,955 | 200,338 | 220,508 | 241,240 | 261,175 | 280,735 |
| 55–59...... | 53,390 | 67,732 | 83,472 | 100,365 | 118,795 | 138,308 | 157,205 | 176,295 | 196,478 | 217,395 | 237,622 | 257,655 |
| 60–64...... | 38,645 | 50,372 | 63,655 | 78,335 | 94,812 | 112,658 | 130,248 | 148,280 | 167,745 | 188,160 | 208,182 | 228,245 |
| 65–69...... | 25,610 | 34,310 | 44,512 | 56,172 | 69,735 | 84,835 | 100,055 | 115,978 | 133,648 | 152,488 | 171,340 | 190,535 |
| 70–74...... | 14,808 | 20,420 | 27,265 | 35,390 | 45,240 | 56,572 | 68,340 | 80,985 | 95,490 | 111,298 | 127,525 | 144,390 |
| 75–79...... | 6,865 | 9,800 | 13,550 | 18,202 | 24,115 | 31,220 | 38,880 | 47,400 | 57,568 | 68,972 | 81,080 | 94,010 |
| 80–84...... | 2,248 | 3,362 | 4,878 | 6,865 | 9,548 | 12,975 | 16,840 | 21,332 | 26,982 | 33,570 | 40,868 | 48,945 |
| 85+....... | 342 | 597 | 995 | 1,587 | 2,487 | 3,769 | 5,345 | 7,323 | 10,021 | 13,379 | 17,341 | 21,975 |

[a] Equivalent values of $°e_0$ shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

GROUPS ($L_x$) OF MODEL LIFE TABLES

*reference in years)* [a]

| Level 60 (°e₀ = 50) | Level 65 (°e₀ = 52.5) | Level 70 (°e₀ = 55) | Level 75 (°e₀ = 57.6) | Level 80 (°e₀ = 60.4) | Level 85 (°e₀ = 63.2) | Level 90 (°e₀ = 65.8) | Level 95 (°e₀ = 68.2) | Level 100 (°e₀ = 70.2) | Level 105 (°e₀ = 71.7) | Level 110 (°e₀ = 73.0) | Level 115 (°e₀ = 73.9) | Sex and age (x) in years |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | MALES |
| 4,872,148 | 5,117,013 | 5,356,512 | 5,613,376 | 5,882,432 | 6,153,098 | 6,411,344 | 6,647,631 | 6,855,509 | 7,027,221 | 7,160,427 | 7,264,597 | ....... $T_0$ |
| 420,276 | 427,825 | 435,153 | 443,871 | 453,493 | 463,115 | 471,904 | 479,005 | 483,920 | 487,206 | 489,396 | 490,859 | .......0–4 |
| 396,948 | 407,198 | 417,042 | 428,240 | 440,230 | 452,212 | 463,305 | 472,638 | 479,460 | 484,072 | 487,152 | 489,212 | .......5–9 |
| 390,362 | 401,348 | 411,852 | 423,675 | 436,230 | 448,768 | 460,375 | 470,235 | 477,692 | 482,825 | 486,255 | 488,550 | ......10–14 |
| 383,615 | 395,258 | 406,368 | 418,765 | 431,860 | 444,930 | 457,058 | 467,450 | 475,555 | 481,288 | 485,122 | 487,690 | ......15–19 |
| 373,688 | 386,230 | 398,175 | 411,388 | 425,305 | 439,208 | 452,148 | 463,365 | 472,445 | 479,030 | 483,442 | 486,402 | ......20–24 |
| 362,038 | 375,608 | 388,532 | 402,720 | 417,588 | 432,468 | 446,410 | 458,638 | 468,838 | 476,330 | 481,362 | 484,745 | ......25–29 |
| 350,215 | 364,802 | 378,742 | 393,918 | 409,715 | 425,532 | 440,448 | 453,658 | 464,882 | 473,225 | 478,842 | 482,622 | ......30–34 |
| 337,732 | 353,350 | 368,308 | 384,455 | 401,195 | 417,940 | 433,782 | 447,938 | 460,110 | 469,298 | 475,498 | 479,672 | ......35–39 |
| 323,548 | 340,220 | 356,200 | 373,330 | 391,022 | 408,702 | 425,452 | 440,552 | 453,650 | 463,725 | 470,600 | 475,235 | ......40–44 |
| 306,292 | 323,982 | 341,060 | 359,045 | 377,658 | 396,258 | 413,878 | 429,930 | 443,950 | 455,022 | 462,810 | 468,138 | ......45–49 |
| 284,665 | 303,200 | 321,250 | 339,978 | 359,368 | 378,755 | 397,155 | 414,110 | 429,045 | 441,200 | 450,160 | 456,550 | ......50–54 |
| 257,468 | 276,540 | 295,182 | 314,490 | 334,412 | 354,355 | 373,335 | 390,980 | 406,795 | 420,058 | 430,268 | 438,005 | ......55–59 |
| 223,435 | 242,435 | 261,142 | 280,508 | 300,470 | 320,492 | 339,665 | 357,670 | 374,188 | 388,512 | 399,975 | 409,125 | ......60–64 |
| 181,888 | 199,852 | 217,738 | 236,272 | 255,450 | 274,742 | 293,425 | 311,248 | 327,980 | 343,050 | 355,565 | 365,975 | ......65–69 |
| 133,995 | 149,622 | 165,428 | 181,892 | 199,095 | 216,535 | 233,660 | 250,300 | 266,278 | 281,205 | 294,018 | 305,058 | ......70–74 |
| 84,872 | 96,790 | 109,080 | 122,012 | 135,710 | 149,778 | 163,840 | 177,772 | 191,515 | 204,815 | 216,758 | 227,380 | ......75–79 |
| 42,852 | 50,265 | 58,100 | 66,485 | 75,538 | 85,005 | 94,678 | 104,490 | 114,450 | 124,372 | 133,880 | 142,795 | ......80–84 |
| 18,259 | 22,488 | 27,160 | 32,332 | 38,093 | 44,303 | 50,826 | 57,652 | 64,756 | 71,988 | 79,324 | 86,584 | ......85+ |
| | | | | | | | | | | | | FEMALES |
| 5,130,792 | 5,393,553 | 5,652,382 | 5,921,457 | 6,204,598 | 6,490,665 | 6,756,688 | 6,998,544 | 7,180,317 | 7,323,693 | 7,434,057 | 7,519,058 | ....... $T_0$ |
| 429,692 | 436,957 | 444,096 | 451,782 | 460,386 | 468,976 | 476,739 | 482,990 | 487,213 | 490,036 | 491,921 | 493,179 | .......0–4 |
| 406,628 | 416,728 | 426,632 | 436,808 | 448,010 | 459,165 | 469,310 | 477,762 | 483,570 | 487,492 | 490,118 | 491,868 | .......5–9 |
| 399,620 | 410,568 | 421,250 | 432,200 | 444,150 | 456,035 | 466,865 | 475,950 | 482,240 | 486,550 | 489,438 | 491,362 | ......10–14 |
| 392,370 | 404,125 | 415,562 | 427,280 | 439,970 | 452,595 | 464,118 | 473,848 | 480,635 | 485,388 | 488,578 | 490,705 | ......15–19 |
| 382,368 | 395,172 | 407,620 | 420,372 | 434,040 | 447,655 | 460,142 | 470,788 | 478,308 | 483,688 | 487,305 | 489,720 | ......20–24 |
| 370,680 | 384,685 | 398,292 | 412,238 | 427,035 | 441,802 | 455,365 | 467,042 | 475,458 | 481,550 | 485,650 | 488,392 | ......25–29 |
| 358,600 | 373,818 | 388,570 | 403,680 | 419,610 | 435,520 | 450,100 | 462,775 | 472,085 | 478,915 | 483,518 | 486,602 | ......30–34 |
| 346,202 | 362,592 | 378,445 | 394,668 | 411,672 | 428,642 | 444,170 | 457,805 | 467,960 | 475,538 | 480,652 | 484,088 | ......35–39 |
| 333,118 | 350,605 | 367,485 | 384,752 | 402,742 | 420,682 | 437,100 | 451,655 | 462,610 | 470,910 | 476,565 | 480,368 | ......40–44 |
| 318,325 | 336,800 | 354,620 | 372,848 | 391,728 | 410,562 | 427,818 | 443,268 | 454,992 | 463,990 | 470,270 | 474,542 | ......45–49 |
| 300,392 | 319,740 | 338,408 | 357,518 | 377,275 | 396,920 | 414,980 | 431,325 | 443,808 | 453,548 | 460,485 | 465,345 | ......50–54 |
| 277,922 | 297,948 | 317,342 | 337,252 | 357,718 | 378,220 | 397,138 | 414,450 | 427,690 | 438,185 | 445,732 | 451,138 | ......55–59 |
| 248,722 | 269,060 | 288,915 | 309,412 | 330,472 | 351,668 | 371,372 | 389,652 | 403,612 | 414,775 | 423,052 | 429,045 | ......60–64 |
| 210,400 | 230,298 | 249,988 | 270,508 | 291,642 | 313,080 | 333,150 | 352,088 | 366,585 | 378,360 | 387,560 | 394,550 | ......65–69 |
| 162,220 | 180,378 | 198,678 | 218,010 | 238,028 | 258,572 | 278,028 | 296,758 | 311,232 | 323,462 | 333,655 | 342,012 | ......70–74 |
| 108,068 | 122,725 | 137,880 | 154,205 | 171,308 | 189,165 | 206,365 | 223,320 | 236,598 | 248,328 | 259,038 | 268,532 | ......75–79 |
| 58,022 | 67,775 | 78,200 | 89,725 | 102,020 | 115,155 | 128,092 | 141,208 | 151,668 | 161,228 | 170,750 | 179,835 | ......80–84 |
| 27,443 | 33,579 | 40,399 | 48,199 | 56,792 | 66,251 | 75,836 | 85,860 | 94,053 | 101,750 | 109,770 | 117,775 | ......85+ |

## TABLE V. SURVIVAL RATIOS ($P_x$)

*Mortality level (or time-*

| *Sex and age (x) in years* | *Level 0* ($°e_0$ = 20) | *Level 5* ($°e_0$ = 22.5) | *Level 10* ($°e_0$ = 25) | *Level 15* ($°e_0$ = 27.5) | *Level 20* ($°e_0$ = 30) | *Level 25* ($°e_0$ = 32.5) | *Level 30* ($°e_0$ = 35) | *Level 35* ($°e_0$ = 37.5) | *Level 40* ($°e_0$ = 40) | *Level 45* ($°e_0$ = 42.5) | *Level 50* ($°e_0$ = 45) | *Level 55* ($°e_0$ = 47.5) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MALES | | | | | | | | | | | | |
| (Births).... | (0.6092) | (0.6334) | (0.6622) | (0.6853) | (0.7058) | (0.7248) | (0.7440) | (0.7626) | (0.7789) | (0.7950) | (0.8107) | (0.8256) |
| 0–4....... | 0.7666 | 0.7953 | 0.8200 | 0.8405 | 0.8577 | 0.8731 | 0.8868 | 0.8993 | 0.9098 | 0.9198 | 0.9287 | 0.9369 |
| 5–9....... | 0.9269 | 0.9359 | 0.9436 | 0.9501 | 0.9556 | 0.9606 | 0.9651 | 0.9691 | 0.9725 | 0.9757 | 0.9785 | 0.9811 |
| 10–14...... | 0.9383 | 0.9444 | 0.9498 | 0.9547 | 0.9590 | 0.9629 | 0.9666 | 0.9700 | 0.9729 | 0.9758 | 0.9783 | 0.9806 |
| 15–19...... | 0.9234 | 0.9291 | 0.9346 | 0.9399 | 0.9444 | 0.9487 | 0.9532 | 0.9574 | 0.9612 | 0.9648 | 0.9681 | 0.9712 |
| 20–24...... | 0.9073 | 0.9141 | 0.9207 | 0.9271 | 0.9326 | 0.9379 | 0.9434 | 0.9484 | 0.9530 | 0.9574 | 0.9615 | 0.9653 |
| 25–29...... | 0.8906 | 0.8999 | 0.9088 | 0.9171 | 0.9245 | 0.9313 | 0.9381 | 0.9440 | 0.9494 | 0.9545 | 0.9592 | 0.9634 |
| 30–34...... | 0.8675 | 0.8805 | 0.8925 | 0.9036 | 0.9130 | 0.9218 | 0.9301 | 0.9373 | 0.9437 | 0.9496 | 0.9551 | 0.9599 |
| 35–39...... | 0.8357 | 0.8533 | 0.8692 | 0.8836 | 0.8957 | 0.9068 | 0.9170 | 0.9258 | 0.9336 | 0.9408 | 0.9473 | 0.9529 |
| 40–44...... | 0.7976 | 0.8198 | 0.8395 | 0.8572 | 0.8722 | 0.8857 | 0.8980 | 0.9085 | 0.9180 | 0.9267 | 0.9342 | 0.9408 |
| 45–49...... | 0.7603 | 0.7850 | 0.8072 | 0.8271 | 0.8440 | 0.8594 | 0.8734 | 0.8854 | 0.8964 | 0.9062 | 0.9150 | 0.9226 |
| 50–54...... | 0.7219 | 0.7469 | 0.7698 | 0.7907 | 0.8086 | 0.8251 | 0.8406 | 0.8542 | 0.8664 | 0.8776 | 0.8877 | 0.8965 |
| 55–59...... | 0.6776 | 0.7011 | 0.7233 | 0.7443 | 0.7626 | 0.7798 | 0.7964 | 0.8113 | 0.8249 | 0.8375 | 0.8488 | 0.8587 |
| 60–64...... | 0.6221 | 0.6428 | 0.6633 | 0.6833 | 0.7011 | 0.7182 | 0.7355 | 0.7514 | 0.7660 | 0.7798 | 0.7924 | 0.8037 |
| 65–69...... | 0.5395 | 0.5582 | 0.5772 | 0.5963 | 0.6139 | 0.6312 | 0.6492 | 0.6660 | 0.6820 | 0.6972 | 0.7116 | 0.7246 |
| 70–74...... | 0.4303 | 0.4478 | 0.4660 | 0.4847 | 0.5023 | 0.5202 | 0.5386 | 0.5561 | 0.5732 | 0.5896 | 0.6054 | 0.6199 |
| 75–79...... | 0.3044 | 0.3207 | 0.3381 | 0.3559 | 0.3724 | 0.3900 | 0.4083 | 0.4258 | 0.4430 | 0.4595 | 0.4757 | 0.4906 |
| (80+)..... | (0.1095) | (0.1280) | (0.1483) | (0.1678) | (0.1850) | (0.2023) | (0.2195) | (0.2355) | (0.2502) | (0.2640) | (0.2768) | (0.2882) |
| FEMALES | | | | | | | | | | | | |
| (Births).... | (0.6290) | (0.6558) | (0.6801) | (0.7024) | (0.7242) | (0.7447) | (0.7627) | (0.7802) | (0.7969) | (0.8135) | (0.8292) | (0.8443) |
| 0–4....... | 0.7637 | 0.7932 | 0.8179 | 0.8387 | 0.8570 | 0.8732 | 0.8870 | 0.8996 | 0.9105 | 0.9209 | 0.9300 | 0.9384 |
| 5–9....... | 0.9212 | 0.9309 | 0.9392 | 0.9462 | 0.9524 | 0.9579 | 0.9627 | 0.9670 | 0.9707 | 0.9742 | 0.9773 | 0.9801 |
| 10–14...... | 0.9301 | 0.9370 | 0.9432 | 0.9487 | 0.9539 | 0.9586 | 0.9628 | 0.9666 | 0.9701 | 0.9734 | 0.9764 | 0.9792 |
| 15–19...... | 0.9146 | 0.9209 | 0.9271 | 0.9330 | 0.9390 | 0.9446 | 0.9495 | 0.9541 | 0.9587 | 0.9631 | 0.9671 | 0.9709 |
| 20–24...... | 0.8965 | 0.9041 | 0.9115 | 0.9186 | 0.9259 | 0.9327 | 0.9387 | 0.9442 | 0.9500 | 0.9555 | 0.9606 | 0.9651 |
| 25–29...... | 0.8779 | 0.8884 | 0.8983 | 0.9076 | 0.9165 | 0.9249 | 0.9322 | 0.9388 | 0.9456 | 0.9520 | 0.9577 | 0.9628 |
| 30–34...... | 0.8581 | 0.8720 | 0.8849 | 0.8967 | 0.9077 | 0.9178 | 0.9265 | 0.9342 | 0.9419 | 0.9490 | 0.9550 | 0.9604 |
| 35–39...... | 0.8390 | 0.8563 | 0.8719 | 0.8861 | 0.8989 | 0.9106 | 0.9204 | 0.9289 | 0.9372 | 0.9448 | 0.9513 | 0.9570 |
| 40–44...... | 0.8203 | 0.8400 | 0.8576 | 0.8734 | 0.8875 | 0.9003 | 0.9110 | 0.9202 | 0.9290 | 0.9369 | 0.9439 | 0.9500 |
| 45–49...... | 0.7965 | 0.8175 | 0.8364 | 0.8533 | 0.8686 | 0.8825 | 0.8942 | 0.9043 | 0.9140 | 0.9229 | 0.9304 | 0.9373 |
| 50–54...... | 0.7663 | 0.7872 | 0.8064 | 0.8241 | 0.8407 | 0.8560 | 0.8688 | 0.8800 | 0.8910 | 0.9012 | 0.9098 | 0.9178 |
| 55–59...... | 0.7238 | 0.7437 | 0.7626 | 0.7805 | 0.7981 | 0.8145 | 0.8285 | 0.8411 | 0.8538 | 0.8655 | 0.8761 | 0.8859 |
| 60–64...... | 0.6627 | 0.6811 | 0.6993 | 0.7171 | 0.7355 | 0.7530 | 0.7682 | 0.7822 | 0.7967 | 0.8104 | 0.8230 | 0.8348 |
| 65–69...... | 0.5782 | 0.5952 | 0.6125 | 0.6300 | 0.6487 | 0.6668 | 0.6830 | 0.6983 | 0.7145 | 0.7299 | 0.7443 | 0.7578 |
| 70–74...... | 0.4636 | 0.4799 | 0.4970 | 0.5143 | 0.5330 | 0.5519 | 0.5689 | 0.5853 | 0.6029 | 0.6197 | 0.6358 | 0.6511 |
| 75–79...... | 0.3275 | 0.3431 | 0.3600 | 0.3772 | 0.3959 | 0.4156 | 0.4331 | 0.4500 | 0.4687 | 0.4867 | 0.5040 | 0.5206 |
| (80+)..... | (0.1320) | (0.1508) | (0.1694) | (0.1878) | (0.2066) | (0.2251) | (0.2409) | (0.2556) | (0.2708) | (0.2850) | (0.2979) | (0.3099) |

[a] Equivalent values of $°e_0$ shown in parentheses refer to expectation of life at birth, for both sexes, in years.

**OF MODEL LIFE TABLES**

*reference in years)* [a]

| *Level 60* ($^{\circ}e_0$ = 50) | *Level 65* ($^{\circ}e_0$ = 52.5) | *Level 70* ($^{\circ}e_0$ = 55) | *Level 75* ($^{\circ}e_0$ = 57.6) | *Level 80* ($^{\circ}e_0$ = 60.4) | *Level 85* ($^{\circ}e_0$ = 63.2) | *Level 90* ($^{\circ}e_0$ = 65.8) | *Level 95* ($^{\circ}e_0$ = 68.2) | *Level 100* ($^{\circ}e_0$ = 70.2) | *Level 105* ($^{\circ}e_0$ = 71.7) | *Level 110* ($^{\circ}e_0$ = 73.0) | *Level 115* ($^{\circ}e_0$ = 73.9) | *Sex and age (x) in years* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | MALES |
| (0.8406) | (0.8557) | (0.8703) | (0.8877) | (0.9070) | (0.9262) | (0.9438) | (0.9580) | (0.9678) | (0.9744) | (0.9788) | (0.9817) | ....(Births) |
| 0.9445 | 0.9518 | 0.9584 | 0.9648 | 0.9708 | 0.9765 | 0.9818 | 0.9867 | 0.9908 | 0.9936 | 0.9954 | 0.9966 | .......0–4 |
| 0.9834 | 0.9856 | 0.9876 | 0.9893 | 0.9909 | 0.9924 | 0.9937 | 0.9949 | 0.9963 | 0.9974 | 0.9982 | 0.9986 | .......5–9 |
| 0.9827 | 0.9848 | 0.9867 | 0.9884 | 0.9900 | 0.9914 | 0.9928 | 0.9941 | 0.9955 | 0.9968 | 0.9977 | 0.9982 | ......10–14 |
| 0.9741 | 0.9772 | 0.9798 | 0.9824 | 0.9848 | 0.9871 | 0.9893 | 0.9913 | 0.9935 | 0.9953 | 0.9965 | 0.9974 | ......15–19 |
| 0.9688 | 0.9725 | 0.9758 | 0.9789 | 0.9819 | 0.9847 | 0.9873 | 0.9898 | 0.9924 | 0.9944 | 0.9957 | 0.9966 | ......20–24 |
| 0.9673 | 0.9712 | 0.9748 | 0.9781 | 0.9811 | 0.9840 | 0.9866 | 0.9891 | 0.9916 | 0.9935 | 0.9948 | 0.9956 | ......25–29 |
| 0.9644 | 0.9686 | 0.9725 | 0.9760 | 0.9792 | 0.9822 | 0.9849 | 0.9874 | 0.9897 | 0.9917 | 0.9930 | 0.9939 | ......30–34 |
| 0.9580 | 0.9628 | 0.9671 | 0.9711 | 0.9746 | 0.9779 | 0.9808 | 0.9835 | 0.9860 | 0.9881 | 0.9897 | 0.9907 | ......35–39 |
| 0.9467 | 0.9523 | 0.9575 | 0.9617 | 0.9658 | 0.9696 | 0.9728 | 0.9759 | 0.9786 | 0.9812 | 0.9835 | 0.9851 | ......40–44 |
| 0.9294 | 0.9359 | 0.9419 | 0.9469 | 0.9516 | 0.9558 | 0.9596 | 0.9632 | 0.9664 | 0.9696 | 0.9727 | 0.9752 | ......45–49 |
| 0.9045 | 0.9121 | 0.9189 | 0.9250 | 0.9306 | 0.9356 | 0.9400 | 0.9441 | 0.9481 | 0.9521 | 0.9558 | 0.9594 | ......50–54 |
| 0.8678 | 0.8767 | 0.8847 | 0.8919 | 0.8985 | 0.9044 | 0.9098 | 0.9148 | 0.9198 | 0.9249 | 0.9296 | 0.9341 | ......55–59 |
| 0.8141 | 0.8244 | 0.8338 | 0.8423 | 0.8502 | 0.8573 | 0.8639 | 0.8702 | 0.8765 | 0.8830 | 0.8890 | 0.8945 | ......60–64 |
| 0.7367 | 0.7487 | 0.7598 | 0.7698 | 0.7794 | 0.7881 | 0.7963 | 0.8042 | 0.8119 | 0.8197 | 0.8269 | 0.8335 | ......65–69 |
| 0.6334 | 0.6469 | 0.6594 | 0.6708 | 0.6816 | 0.6917 | 0.7012 | 0.7102 | 0.7192 | 0.7283 | 0.7372 | 0.7454 | ......70–74 |
| 0.5049 | 0.5193 | 0.5326 | 0.5449 | 0.5566 | 0.5675 | 0.5779 | 0.5878 | 0.5976 | 0.6072 | 0.6176 | 0.6280 | ......75–79 |
| (0.2988) | (0.3091) | (0.3186) | (0.3272) | (0.3352) | (0.3426) | (0.3493) | (0.3556) | (0.3613) | (0.3666) | (0.3721) | (0.3775) | .....(80+) |
| | | | | | | | | | | | | FEMALES |
| (0.8594) | (0.8739) | (0.8882) | (0.9036) | (0.9208) | (0.9380) | (0.9535) | (0.9660) | (0.9744) | (0.9801) | (0.9838) | (0.9864) | ....(Births) |
| 0.9463 | 0.9537 | 0.9607 | 0.9669 | 0.9731 | 0.9791 | 0.9844 | 0.9892 | 0.9925 | 0.9948 | 0.9963 | 0.9973 | .......0–4 |
| 0.9828 | 0.9852 | 0.9874 | 0.9895 | 0.9914 | 0.9932 | 0.9948 | 0.9962 | 0.9972 | 0.9981 | 0.9986 | 0.9990 | .......5–9 |
| 0.9819 | 0.9843 | 0.9865 | 0.9886 | 0.9906 | 0.9925 | 0.9941 | 0.9956 | 0.9967 | 0.9976 | 0.9982 | 0.9987 | ......10–14 |
| 0.9745 | 0.9778 | 0.9809 | 0.9838 | 0.9865 | 0.9891 | 0.9914 | 0.9935 | 0.9952 | 0.9965 | 0.9974 | 0.9980 | ......15–19 |
| 0.9694 | 0.9735 | 0.9771 | 0.9807 | 0.9839 | 0.9869 | 0.9896 | 0.9920 | 0.9940 | 0.9956 | 0.9966 | 0.9973 | ......20–24 |
| 0.9674 | 0.9718 | 0.9756 | 0.9792 | 0.9826 | 0.9858 | 0.9884 | 0.9909 | 0.9929 | 0.9945 | 0.9956 | 0.9963 | ......25–29 |
| 0.9654 | 0.9700 | 0.9739 | 0.9777 | 0.9811 | 0.9842 | 0.9868 | 0.9893 | 0.9913 | 0.9929 | 0.9941 | 0.9948 | ......30–34 |
| 0.9622 | 0.9669 | 0.9710 | 0.9749 | 0.9783 | 0.9814 | 0.9841 | 0.9866 | 0.9886 | 0.9903 | 0.9915 | 0.9923 | ......35–39 |
| 0.9556 | 0.9606 | 0.9650 | 0.9691 | 0.9727 | 0.9759 | 0.9788 | 0.9814 | 0.9835 | 0.9853 | 0.9868 | 0.9879 | ......40–44 |
| 0.9437 | 0.9493 | 0.9543 | 0.9589 | 0.9631 | 0.9668 | 0.9700 | 0.9731 | 0.9754 | 0.9775 | 0.9792 | 0.9806 | ......45–49 |
| 0.9252 | 0.9318 | 0.9377 | 0.9433 | 0.9482 | 0.9529 | 0.9570 | 0.9609 | 0.9637 | 0.9661 | 0.9680 | 0.9695 | ......50–54 |
| 0.8949 | 0.9030 | 0.9104 | 0.9175 | 0.9238 | 0.9298 | 0.9351 | 0.9402 | 0.9437 | 0.9466 | 0.9491 | 0.9510 | ......55–59 |
| 0.8459 | 0.8559 | 0.8653 | 0.8743 | 0.8825 | 0.8903 | 0.8971 | 0.9036 | 0.9083 | 0.9122 | 0.9161 | 0.9196 | ......60–64 |
| 0.7710 | 0.7832 | 0.7948 | 0.8059 | 0.8162 | 0.8259 | 0.8345 | 0.8429 | 0.8490 | 0.8549 | 0.8609 | 0.8668 | ......65–69 |
| 0.6662 | 0.6804 | 0.6940 | 0.7073 | 0.7197 | 0.7316 | 0.7422 | 0.7525 | 0.7602 | 0.7677 | 0.7764 | 0.7852 | ......70–74 |
| 0.5369 | 0.5523 | 0.5672 | 0.5819 | 0.5955 | 0.6088 | 0.6207 | 0.6323 | 0.6410 | 0.6493 | 0.6592 | 0.6697 | ......75–79 |
| (0.3211) | (0.3313) | (0.3406) | (0.3495) | (0.3576) | (0.3652) | (0.3719) | (0.3781) | (0.3828) | (0.3869) | (0.3913) | (0.3957) | .....(80+) |

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