TABLE 9. UTILIZATION OF STATISTICS ON DEATHS IN BRINGING DATA ON COSTA RICAN POPULATION AGED 5 AND OVER IN 1950 FORWARD TO 1955

Sex and age of cohort in 1950 (years)	Population enumerated, 1950[a]	Same, smoothed[b]	Deaths attributed to cohort	Estimated population, 1955	Age of cohort in 1955 (years)
Males					
5– 9........	56,789	—	594	56,195	10–14
10–14........	49,734	48,814	364	48,450	15–19
15–19........	40,418	41,773	448	41,325	20–24
20–24........	37,671	36,212	478	35,734	25–29
25–29........	28,647	29,327	460	28,867	30–34
30–34........	23,851	24,297	530	23,767	35–39
35–39........	22,908	22,184	557	21,627	40–44
40–44........	18,310	18,446	574	17,872	45–49
45–49........	14,140	14,568	704	13,864	50–54
50–54........	12,313	11,579	764	10,815	55–59
55–59........	7,889	8,747	859	7,888	60–64
60–64........	7,667	6,965	990	5,975	65–69
65–69........	4,716	5,088	1,031	4,057	70–74
70–74........	3,331	3,180	980	2,200	75–79
75–79........	1,860	—	763	1,097	80–84
80–84........	1,073	—	1,046	746	85 and over
85 and over...	719	—			
Females					
5– 9........	55,367	—	502	54,865	10–14
10–14........	48,555	48,611	310	48,301	15–19
15–19........	43,826	44,010	429	43,581	20–24
20–24........	39,386	38,679	492	38,187	25–29
25–29........	30,491	30,595	470	30,125	30–34
30–34........	23,705	24,830	541	24,289	35–39
35–39........	23,930	22,622	598	22,024	40–44
40–44........	18,074	18,548	536	18,012	45–49
45–49........	13,966	14,226	564	13,662	50–54
50–54........	11,853	11,274	645	10,629	55–59
55–59........	7,827	8,518	770	7,748	60–64
60–64........	7,248	6,649	880	5,769	65–69
65–69........	4,420	4,782	960	3,822	70–74
70–74........	3,227	3,039	904	2,135	75–79
75–79........	1,768	—	737	1,031	80–84
80–84........ / 85 and over...	1,147 / 840	—	1,280	707	85 and over

[a] Exclusive of individuals whose ages were not reported.

[b] By smoothing formula presented in p. 11. Smoothing appears desirable owing to irregularities probably due to inaccurate age reporting, such as the apparent excess of women aged 35–39 over women aged 30–34.

III. SOME CONCEPTS OF ABRIDGED LIFE TABLES AND LIFE-TABLE CONSTRUCTION

131. The core of the methods of population projections described here consists in the use of a system of model life tables to calculate the numbers of persons in each cohort of a population who will survive during successive time intervals in the future. The derivation of these model tables is briefly described in the appendix, where the survival ratios, needed for the population projections, as well as other functions of the life tables are tabulated.

132. Though the actual population projection may be carried out entirely by the tabulated survival ratios, it is first necessary to establish which of these ratios are appropriate to the given case, i.e., which ratios express most nearly the current and expected future mortality conditions. Survival ratios are not given directly in official statistics but must be calculated or otherwise derived by reference to the statistical information which exists. In almost every case, at least one step in the

general procedure of life-table construction is involved, despite all simplifications of the procedure. Some knowledge of the relevant mortality functions and of their interrelations will therefore usually be needed. The purpose of this chapter is to establish the concepts, in simplified form, which are needed to describe the relationship of survival ratios to other statistics relating to mortality.

133. Part A introduces and illustrates the main life-table functions. In part B, consideration is given to those interrelations of the several functions which have to be taken into account in particular situations. Only "abridged" life tables, i.e., tables with functions by five-year groups rather than for each year of age, are considered. The emphasis in the presentation is on simplicity and expediency, abstraction being made from the more rigorous mathematical techniques used in actuarial science.

A. The chief functions comprised in a life table

134. The starting point for the calculation of life-table values is usually the computation of death rates for the various age groups. From these rates other functions are derived, and from the latter functions survival ratios are derived, expressing the proportion of persons, among those who survive to a given age, who live on and attain the next age level.

Table 10. Derivation of specific death rates (m_x) from statistics of deaths and population by sex and age, Luxembourg, 1946–1949

Sex and age in years (x)	*Deaths* 1946	1947	1948	1949	Average, 1946–1949	*Enumerated population 31 December 1947*	*Average age-specific death rate, 1946–1949 (1,000 m_x)*
Males							
0	188	145	142	116	148	2,082	71.1
1– 4	27	25	19	23	24	7,640	3.1
5– 9	18	19	8	9	14	9,714	1.4
10–14	15	10	7	13	11	9,947	1.1
15–19	20	20	20	21	20	12,437	1.6
20–24	39	24	23	24	28	11,843	2.4
25–29	34	29	25	13	25	10,057	2.5
30–34	34	30	33	31	32	10,143	3.2
35–39	39	61	50	47	49	12,227	4.0
40–44	61	75	64	79	70	12,226	5.7
45–49	113	98	112	101	106	11,071	9.6
50–54	101	99	108	102	102	8,996	11.3
55–59	114	144	151	141	138	7,430	18.6
60–64	173	160	190	189	178	6,470	27.5
65–69	198	246	252	228	231	5,432	42.5
70–74	224	250	259	292	256	3,928	65.2
75–79	204	217	220	264	226	2,185	103.4
80–84	150	166	153	187	164	946	173.4
85 and over	75	80	77	103	84	322	260.9
Females							
0	139	113	103	78	108	1,946	55.5
1– 4	27	34	19	22	26	7,347	3.5
5– 9	12	9	5	6	8	9,466	0.8
10–14	11	10	13	10	11	9,568	1.1
15–19	23	17	16	13	17	11,378	1.5
20–24	23	22	20	21	22	11,468	1.9
25–29	31	27	25	18	25	10,327	2.4
30–34	37	28	16	16	24	10,049	2.4
35–39	35	36	27	27	31	11,925	2.6
40–44	57	49	36	36	44	11,956	3.7
45–49	68	62	64	63	64	11,300	5.7
50–54	76	74	88	86	81	9,548	8.5
55–59	100	99	103	101	101	7,808	12.9
60–64	144	153	147	125	142	7,048	20.1
65–69	197	191	166	205	190	6,011	31.6
70–74	228	248	233	275	246	4,473	55.0
75–79	235	224	201	254	228	2,563	89.0
80–84	174	176	165	218	183	1,218	150.2
85 and over	142	130	96	151	130	497	261.6

135. Some life tables have been constructed by inverting this sequence of calculations. Where adequate statistics on deaths by age were not available, survival ratios have been inferred from statistics of the population by sex and age according to two successive census enumerations; and by working backward from these ratios, other life-table functions, including the age-specific death rate, has been obtained.

136. The functions in this sequence of computations to be considered first are: (1) m_x, i.e., the death rate for persons of a given age, x; (2) q_x, the probability of dying within a given age interval; (3) l_x, the number of survivors to a specified age from an assumed initial number of births; (4) L_x, the numbers of years lived collectively by those survivors within the given age interval; and (5) P_x, the survival ratio from one age interval to the next-higher age interval. Other functions which will be taken up subsequently are: (6) d_x, the number of deaths in the given age interval; (7) p_x, the probability of surviving during the age interval; (8) T_x, the collective number of years yet to be lived by the survivors to a given age; and (9) $°e_x$, the expectation of life of an individual of given age.

137. In all these symbols, the suffix "x" denotes age. Since we are dealing with abridged life tables, by five-year groups of ages, it denotes either the lower limit of an age group or the entire age group, depending on the nature of the function.[27]

1. *Functions in the basic sequence for computation of survival ratios*

138. Table 10 illustrates the derivation of age-specific death rates from statistics for the Grand-Duchy of Luxembourg for the years 1946 to 1949. A census was taken at the end of 1947, a date which is central for the 1946-1949 period. Dividing the average annual number of deaths in each sex-age group by the population of each group at the middle of the period, and multiplying by 1,000, the first function of the sequence, namely the age-specific death rates, 1,000 m_x, is obtained. Owing to sharp differences in mortality immediately after birth and in subsequent years of age, it is of some importance to compute separately m_0, i.e., the rate for the first year of life, and m_{1-4}, i.e., the rate for the following four years. Beginning with age 5, the rates are computed by five-year groups.

139. Averaging the numbers of deaths during four years, in this instance, is desirable both because of the small numbers of deaths in some age groups and because of year-to-year fluctuations in the factors of mortality. Where ages at death are frequently misreported, fluctuations can be smoothed out by the same formula suggested previously for smoothing a population age distribution. In the case of deaths, however, the formula cannot be applied to the first two age groups because the characteristic shape of the distribution would produce a major distortion.

140. When the m_x-values have been obtained, the remaining functions can be calculated as will presently be explained. The results of the calculations are shown for Luxembourg males, in Table 11. A brief explanation for each column is given below.

[27] Notations such as ${}_nq_x$, ${}_nL_x$, etc., are in actuarial usage, where the suffix "n" refers to the width of the age group. Thus, for a five-year group, n becomes 5, while for single-year ages it equals 1. This qualification is important for mathematical demonstrations, but need not complicate the presentation of the subject in this chapter.

TABLE 11. SOME FUNCTIONS OF AN ABRIDGED LIFE TABLE FOR LUXEMBOURG MALES, 1946–1949

Age in years (x)	*Age-specific death rate* (1,000 m_x)	*Life-table death rate* (1,000 q_x)	*Survivors to exact age* (l_x)	*Survivors in age group* (L_x)	*Survival ratio* (P_x) ($P_b = 0.9354$)
0..........	71.1	63.0	100,000	467,715	0.9862
1–4........	3.1	12.0	93,700		
5–9........	1.4	7.0	92,576	461,260	0.9937
10–14.......	1.1	5.5	91,928	458,375	0.9933
15–19.......	1.6	8.0	91,422	455,282	0.9901
20–24.......	2.4	11.9	90,691	450,755	0.9879
25–29.......	2.5	12.4	89,611	445,278	0.9859
30–34.......	3.2	15.9	88,500	438,982	0.9822
35–39.......	4.0	19.8	87,093	431,155	0.9761
40–44.......	5.7	28.1	85,369	420,848	0.9626
45–49.......	9.6	47.0	82,970	405,100	0.9490
50–54.......	11.3	55.1	79,070	384,458	0.9284
55–59.......	18.6	89.1	74,713	356,922	0.8918
60–64.......	27.5	129.1	68,056	318,315	0.8412
65–69.......	42.5	192.9	59,270	267,768	0.7676
70–74.......	65.2	281.3	47,837	205,545	0.6649
75–79.......	103.4	410.0	34,381	136,662	0.5224
80–84.......	173.4	592.2	20,284	71,390	0.3075
85 and over..	260.9	1,000.0	8,272	31,406	

141. The first column, x, indicates the age groups to which the functions apply. The second column, age-specific death rates, is taken from table 10. In keeping with conventions in demography, these rates are expressed per 1,000, though this is usually not done in actuarial tables. As in all life tables, this function decreases sharply from birth onward to attain a minimum near the age of 10; it then rises with increasing age, at first gradually, then more sharply.

142. The third column shows the "life-table death rate", 1,000 q_x. In actuarial usage, this function expresses the probability that an individual about to enter an age group will die before reaching the upper limit of that age group. Thus, of 1,000 children born, it is expected that 63.0 will die before attaining one year of age; and of 1,000 individuals aged exactly 15 years, 8.0 are expected to die before attaining the precise age of 20.

143. From the demographic point of view, a life table is regarded as a theoretical model of a population which is continuously replenished by births and depleted by deaths. In this context, the q_x values can be regarded as being in the nature of death rates, though different from the specific death rates symbolized by m_x. Whereas m_x relates the number of deaths to the persons *living within* each age group, q_x relates the same number of deaths to the persons who, within a particular year, *enter* that age group. If the age group is of one year only, as in the case of age 0, q_x is somewhat smaller than m_x because there are more entrants into the age group during each year than persons living within it at any time,

since some of those who enter die while in the group. In the case of a five-year group, q_x has a value approaching five times the value of m_x, because there are nearly five times as many persons living in the group as enter it during any one year. For the terminal age group, 1,000 q_x is 1,000 because all individuals die eventually.

144. The fourth column, survivors to the given exact age, is symbolized by l_x, but here the suffix "x" indicates the lower limit of each age group. As in most life tables, 100,000 births are assumed and the l_x function shows how many of the 100,000 reach each age. When the tables are used for demographic purposes, the same 100,000 births are assumed to occur every year.

145. The fifth column, L_x, has two different meanings, depending on whether an actuarial or a demographic viewpoint is taken. From the actuarial point of view, the L_x-values represent the numbers of years that will be lived collectively within any one age interval by a cohort numbering 100,000 at birth and subject to the given mortality conditions. For example, if no deaths occurred up to age 5, the 100,000 children collectively would live 500,000 years up to that age. The ravages of mortality are reflected by the diminution of the L_x function from one age interval to the next. From a demographic point of view, the L_x function represents the age composition of a population which is constantly replenished by births at the rate of 100,000 per year and depleted by mortality at the rates represented by the q_x or m_x function. Owing to the assumed constant number of births, this hypothetical population, and every one of its age groups, will be of a constant size; it is therefore described as the "stationary" population corresponding to the particular life table. In this case, it would total 6,207,216 individuals.

146. The last column, P_x, shows the proportion of persons, among those living in the indicated age groups of the "stationary" population, who survive until they are 5 years older. This ratio attains its maximum near the age of 10 and then declines, at first gradually and then more rapidly. At the top of the column, the ratio P_b is given, that is, the proportion of survivors to age groups 0-4, at the end of a five-year period, out of a cohort of births occurring at a constant rate during the five-year period. The remaining values of P_x shown in the column apply to five-year age groups, with the exception of the last value, P_{80+}, which represents the proportion of persons, among the group aged 80 and over, who survive 5 more years, being 85 and over at the end of the period.

2. *Derivation of the sequence of functions from age-specific death rates*

147. The mathematical interrelations among the several functions in some instances can be expressed accurately only in terms of integral equations. Since this is not a treatise on actuarial techniques, however, the precise mathematical relations need not detain us here. The transformation of the age-specific rates, m_x, into life-table death rates, q_x, is of such a nature. The age-specific rates relate to population segments some of which have already died as a result of the mortality condition expressed by the rate itself. In addition, for the most part, they relate to five-year age groups, whereas the q_x values relate to the annual number of entrants into each of these groups. There is no simple arithmetical procedure for accurate conversion of m_x-values into the corresponding q_x-values or *vice versa*, but tables have been constructed with the aid of which the transformation can be effected very rapidly.[28] If such tables cannot be readily obtained, the somewhat less accurate short-cut procedure described in part B of this chapter can be used.

148. A direct arithmetical procedure is used to obtain the numbers of survivors (l_x) to exact ages on the basis of the life-table death rates (q_x). Beginning with the assumed 100,000 births (l_0), q_0 (in this example, 63.0 per 1,000) is applied to calculate the deaths in the first year of life (6,300 in this case), and these are subtracted from the original 100,000 to compute the number of survivors at the exact age of one year. The process of multiplication and subtraction is repeated for the successive age groups until all values of l_x are obtained.

149. The transformation of l_x into L_x is mathematically complex if a very high degree of accuracy is sought. Fortunately, a few simple arithmetical procedures are quite sufficient if only a close approximation is required; the errors involved in this simpler procedure are, on the whole, slight and for the most part negligible. One procedure can be used with respect to all age groups from 5-9 to 80-84. Separate procedures apply to the calculation of the two extreme values, L_{0-4} and L_{85+}.

150. For age groups 5-9 to 80-84, it is quite sufficient to assume that L_x equals five times the arithmetic average of l_+ and l_{x+5}.

151. Owing to the very unequal distribution of deaths over the first five years of life, this procedure would be quite unsatisfactory for the computation of L_{0-4}. A reasonably good approximation, however, is obtained by assuming L_0 (i.e., for the first year of life only) to equal 0.25 l_0+0.75 l_1 (in the given example, 25,000 plus 0.75 times 93,700, which is 95,275); and by assuming that L_{1-4} equals 1.9 l_1+2.1 l_5 (in the present example, 1.9 times 93,700 plus 2.1 times 92,576, which is 372,440). These assumptions, taken together, lead to the formula $L_{0-4}=0.25\ l_0+2.65\ l_1+2.1\ l_5$ (and to the calculated result, in this instance, of 467,715).

152. Experience has justified the use of a very simple formula for an approximate computation of L_{85+}. Actually, as will be explained further on, the L_x-value for a terminal age group equals the l_x-value for the initial age of the group multiplied by the expectation of life at that age. It happens by a mere coincidence that, in most available life tables, the expectation of life at the age of 85 very nearly equals the common logarithm of l_{85} when l_{85} is expressed for 100,000 births. An estimate of L_{85+} is therefore easily obtained by multiplying l_{85} by its own logarithm. In the present example, l_{85} amounts to 8,272

[28] Among the best tables of this kind are those of Reed and Merrell, which were used in the calculation of the example shown here. For the original publication of these tables, see Lowell J. Reed and Margaret Merrell, "A Short Method for Constructing an Abridged Life Table", *The American Journal of Hygiene,* Vol. 30, No. 2, September 1939. The original publication has been reproduced in full in A. J. Jaffe's *Handbook of Statistical Methods for Demographers,* United States Bureau of the Census, Washington, D.C., 1951.

and the logarithm of that figure amounts to 3.91761 which may be assumed to be the approximate expectation of life, in years, at this age. The product of these two figures is 32,406, the required value for L_{85+}.

153. The derivation of survival ratios (P_x) from survivors in age groups (L_x) merely requires division of successive pairs of L_x-values. Thus, dividing L_{5-9} (461,242 in this example) by L_{0-4} (467,715), we obtain 0.9862, the required figure for P_{0-4}. In other words, of 1,000 persons living in the age group 0-4, 986.2 are expected to be alive five years later when their ages are 5-9. The values of P_b (survival from births) and P_{80+} require separate mention. The first of these is computed by dividing L_{0-4} by 500,000, the assumed number of births during a five-year period. The second is obtained by dividing L_{80+}, i.e., the sum of L_{80-84} and L_{85+}, by L_{85+}.

3. *Other functions of the life table*

154. Table 12 shows those functions which are usually presented where a life table is published for actuarial uses. The first three columns, relating to age groups, survivors to specified ages, and life-table death rates have already been discussed. Another function, not shown here, is that of p_x, or the probability of survival from one specified age to another, which is simply obtained by subtracting q_x from unity (or, where the rates are expressed per 1,000, $1{,}000\ p_x = 1{,}000 - 1{,}000\ q_x$).

155. Column (4) of table 12 presents the d_x-function, which is the number of deaths occurring within an age group, from the lower to the upper limit of age, on the assumption of 100,000 initial births. The sum of the d_x values for all ages equals 100,000, representing extinction of the cohort of 100,000 births. Near age 10, the d_x-function attains a minimum. It then rises towards a maximum at some relatively advanced age, after which it falls off again. This function can be obtained by subtraction of successive values of l_x. The function can also be conceived as an age distribution of deaths in a "stationary" population.

156. Column (5), which represents the T_x-function, indicates the number of years that will be lived collectively, from the given age onward, by the survivors to that age from the cohort of 100,000 births. This function is obtained by cumulative addition of the L_x-function (column 5 in table 11), from the bottom up, i.e., beginning with L_{85+} and adding L_{80-84}, L_{75-79}, and so forth.[29]

157. The last function, $°e_x$, represents the individual expectation of life. If, as in this example, 100,000 individuals at their birth are expected to live, collectively, 6,207,216 years, then the average individual can expect, at birth, to live 62.1 years. Similarly, expectations of life at other ages are obtained by dividing the corresponding T_x by the corresponding l_x. In the first few years of life, expectation of life rises until the relatively heavy risks of dying in infancy and early childhood are past. Thereafter, with advancing age, the expectation of life declines though never by as much as the increase in age. For instance, an individual aged 60 years may expect to live another 15 years ($°e_{60}$ being 15.2), i.e., up to about the age of 75. But those who do reach 75 years of age have a further expectation of 7 years of life up to the age of 82.

B. Practical derivations of life-table functions

158. The comparatively simple procedures so far described are adequate for the construction of an approximate abridged life table. The detailed procedures can, of course, be modified, but the general sequence of calculations should be retained.

[29] In order to obtain a separate value for T_1, it was also necessary to compute L_0 and L_{1-4} by the separate formulas given on p. 23. The resulting values of L_0 and L_{1-4} were 95,275 and 372,440, respectively.

Table 12. Abridged life table for Luxembourg males, 1946–1949

(1) x	(2) l_x	(3) $1{,}000\ q_x$	(4) d_x	(5) T_x	(6) $°e_x$
0	100,000	63.0	6,300	6,207,216	62.1
1– 4	93,700	12.0	1,124	6,111,941	65.2
5– 9	92,576	7.0	648	5,739,501	62.0
10–14	91,928	5.5	506	5,278,241	57.4
15–19	91,422	8.0	731	4,819,866	52.7
20–24	90,691	11.9	1,079	4,364,584	48.1
25–29	89,611	12.4	1,111	3,913,829	43.7
30–34	88,500	15.9	1,407	3,468,551	39.2
35–39	87,093	19.8	1,724	3,029,569	34.8
40–44	85,369	28.1	2,399	2,598,414	30.4
45–49	82,970	47.0	3,900	2,177,566	26.2
50–54	79,070	55.1	4,357	1,772,466	22.4
55–59	74,713	89.1	6,657	1,388,008	18.6
60–64	68,056	129.1	8,786	1,031,086	15.2
65–69	59,270	192.9	11,433	712,771	12.0
70–74	47,837	281.3	13,457	445,003	9.3
75–79	34,381	410.0	14,096	239,458	7.0
80–84	20,284	592.2	12,012	102,796	5.1
85 and over	8,272	1,000.0	8,272	31,406	3.9

159. This manual provides a scheme of model life tables, in which survival ratios as well as certain other functions are tabulated. Where these model tables can be adapted to the purpose of a specific population projection, it is not necessary to go through the entire procedure of constructing a life table. Quite often, the appropriate survival ratios can be inferred from the scheme presented here, by reference to some other function for which values have been obtained from available statistics.

160. Rough calculations will often suffice as a basis for the selection of survival ratios.

1. *Ratio functions and cumulative functions of the life table*

161. It is useful to distinguish those functions of the life table which are in the nature of ratios from those which are the results of cumulative addition or subtraction. The first type comprises age-specific death rates (m_x), life-table death rates (q_x), and survival ratios (P_x). All these rates, though differently derived, are closely related to each other. If one of these functions is known, reference to a system of model life tables makes it possible to estimate immediately the approximate levels of the other two functions, without any further calculations. For instance, supposing that m_x has been determined with respect to all age groups, the model life tables can be used to assess the approximate mortality levels which these particular rates represent. The survival ratios, P_x, corresponding to the same general mortality levels can then be located in the tables, without much further computations (see chapter IV).

162. Other functions, like the specific values of l_x, L_x and T_x, do not provide an equally good indication of the corresponding values of P_x. For example, a given value of l_{20} (survivors to the age of 20) may be the result of high mortality in infancy and low mortality in adolescence, or *vice versa*. Since the function required for population projections is P_x, given values of l_x, L_x or T_x have to be converted into corresponding values of q_x or P_x before direct reference can be made to the model life tables.

2. *Functions corresponding to a "general" level of mortality under average conditions*

163. The expectation of life at birth, $°e_0$, can be considered in a different category. It is both the result of cumulation of specific values (T_0) and a ratio (by division by l_0). And it is the one synthetic measure by which the "general" level of mortality can be summarized in a single figure. The model life tables present those combinations of life-table functions which, under average conditions, are likely to be found if expectation of life at birth attains a certain value. Actually, such average conditions never occur and deviations from the pattern, as implied in the model, must be expected. For example, though $°e_0$ may be 50 years, mortality rates at some ages may be higher, and at other ages lower, than indicated by the model life table for $°e_0$ equal to 50. By and large, deviations from the average pattern may be relatively small, but unusual situations occur as where adult mortality is unusually high and infant mortality low, etc.

164. If specific information on mortality conditions by age is not available and there are no special reasons to suppose that the conditions are unusual, perhaps the best estimate that can be made is that given in one of the model tables. A model table may be found, for example, which in the given instance would yield the same annual number of deaths as actually recorded. It may then be assumed that the distribution of death rates by age is the same as that of the model table.

3. *Derivation of required specific functions where ratio-functions are not given*

165. The model tables can be used directly to determine survival ratios for each age, if the age-specific m_x or q_x values are determined. Even where specific information on mortality is available, however, these ratio-functions are sometimes not determined.

166. It may be that an incomplete life table is at hand, giving only the l_x-values. In that case, by taking successive differences, the d_x-values can be obtained, and division of d_x by l_x yields the q_x values, from which corresponding survival ratios can be inferred by means of the model tables.

167. Again, it may happen that the L_x-values are available. In that event, the specific survival ratios should be computed from the given L_x-values, instead of being inferred from the L_x-values in the model life tables. The same applies if the given values are those of T_x, from which L_x-values are obtained by successive subtractions.

168. It may also be that only $°e_x$-values are available. These cannot very readily be converted into other life table functions, though approximate values for some other function might be estimated by repeated trials, using some of the values tabulated in the model life tables.

169. This case is much simplified if, as sometimes happens, the two functions for which values are given in the available incomplete life table are $°e_x$ and l_x. T_x can be calculated by multiplying $°e_x$ with l_x; from T_x, L_x is obtained by successive subtractions, and P_x, the age-specific survival ratios, by successive divisions of L_x.

4. *Use of survival ratios computed from census statistics*

170. In paragraph 135 of part A of this chapter, it was mentioned that life tables are sometimes constructed by calculating P_x values from census data. This method is advantageous where adequate statistics on deaths by age are not available, but where the age composition of the population has been determined in two successive censuses. No survival ratios can be obtained by this procedure in respect of children born during the census interval. The details of the procedure are sometimes highly complex, involving refined methods of interpolation and graduation. In the present context, however, only the survival ratios themselves are of immediate interest and there is no need to be detained with the complexities of computing other functions of the life table.

171. In computing the ratios of survival from one census to another, care must be taken that the numbers ascertained at the more recent census represent as nearly as possible the survivors of the cohorts enumerated at the previous census. Where migration has been important, something must be done to avoid distortion of the ratios by this factor. In a population mainly affected by

immigration, this is sometimes effectively done by restricting the inter-censal comparison to persons born within the country. Other factors which can vitiate the computations are faulty enumeration of some or all age groups, and mis-statement of ages. For graduation to eliminate the effects of age mis-statements, the formula presented in part B of chapter II may suffice if further adjustments are made at a later stage by reference to model life tables (as will be shown in the next chapter); however, the age mis-statements may be such that another more refined formula should be employed.

172. If the interval between the two censuses is five years, it is possible to compute directly the five-year survival ratios for five-year time periods which are required for the population projections. But additional problems arise if the census interval is longer or shorter than five years. If the interval is ten years, survival ratios for five-year age groups over ten-year periods can be calculated. A method for transforming such ratios into ratios for five-year periods is presented in the next chapter. The transformation in this case is relatively easy because the ratio for the ten-year period equals the product of ratios of two successive age groups for five-year periods. If the census interval is not a multiple of five years, the procedure becomes more complicated, but it is still possible to obtain rough estimates for survival ratios corresponding to five-year time periods.[30]

5. *Short-cut procedures*

173. For an approximate computation of P_x-values (survival ratios) from m_x-values (age-specific death rates), only two steps are required. Though it is suggested to find P_x-values from the model life tables, this short procedure may sometimes be used as an alternative approximate computation.

174. As a first step, m_x may be converted into q_x without recourse to detailed tabulations of the relationships between these functions.[31] This may be necessary, where such tabulations cannot be readily obtained. The approximate conversion can be made by means of factors by which m_x must be multiplied to result in the required q_x.

175. It has been noted in paragraph 143 of part A of this chapter that q_0 is always somewhat smaller than m_0. Actually, q_0 is in the nature of an infant mortality rate, since infant deaths are related to births. The infant mortality rate, if known, may very well be substituted as an approximate value of q_0.[32] Otherwise, the following conversion table may be used:

1,000m₀	*Coefficient for conversion to 1,000q₀*
29	0.925
55	0.90
110	0.85
170	0.80
240	0.75

[30] The procedure then involves age cohorts which do not coincide with conventional five-year groupings at both censuses, as well as the use of survival ratios for a fraction of a five-year period. With the help of some subsidiary estimates, however, the procedure can be adopted for the present purposes.

[31] Such as the Reed-Merrell tables referred to above.

[32] The infant mortality rate is not precisely q_0, because deaths are here related to births occurring at the same time, e.g., in the same year. Not all infants dying in one calendar year were born in the same year, and some of the infants born in a given year will die, before reaching one year of age, during the next year. Nevertheless, the difference between the infant mortality rate and q_0 is usually slight.

Other conversion factors can be interpolated for different values of m_0. In the example of Luxembourg, 1,000 m_0 for males was found to be 71.1; the corresponding value of 1,000 q_0 would be about 0.89 times this figure, i.e., about 63.3. This is a fairly close estimate since, according to the Reed-Merrell tables, 1,000 q_0 in this case is 63.0.

176. The relationship between $m_{1\text{-}4}$ and $q_{1\text{-}4}$ can similarly be summarized. As has been noted, the latter will always be somewhat less than four times the former, since this is a four-year age group. The following conversion table will suffice for rough estimates:

1,000m₁₋₄	*Coefficient for conversion to 1,000q₁₋₄*
8	3.8
21	3.6
36	3.4
55	3.2

For Luxembourg males, 1,000 $m_{1\text{-}4}$ was found equal to 3.1; by extrapolation from the conversion table the appropriate factor is found to be about 3.9, which gives an estimate of 12.1 for 1,000 q_x — nearly the same as the figure 12.0 obtained with the use of the Reed-Merrell tables.

177. For five-year age groups, finally, q_x will always be less than five times m_x. This difference will be slight when m_x is small, but appreciable for large values of m_x. The following conversion table may be used, with interpolations where required:

1,000mₓ	*Coefficient for conversion to 1,000qₓ*
10	4.9
45	4.5
100	4
160	3.5
240	3

178. With these three conversion tables, values of q_x for all age groups can be estimated very rapidly. Once these have been obtained, they can be rapidly transformed into corresponding values of p_x, i.e., the probability of surviving *each exact age interval*, since p_x equals $1-q_x$. This being accomplished, approximate estimates of P_x, i.e., survival ratios *from one age group to the next*, can be obtained very directly, as follows.

179. For five-year age groups, P_x approximates the arithmetic average of p_x and p_{x+5}. The approximation is quite close where p_x is large, i.e., where q_x is small. For age groups affected by relatively high mortality, i.e., especially at advanced ages, this short-cut computation results in a more or less appreciable underestimate of P_x. This is a defect of the method, but not a severe one for the present purposes. All values for P_x, from $P_{5\text{-}9}$ to $P_{75\text{-}79}$, can thus be approximately established without any intermediate computations. The values of P_b (survival from births to ages 0-4) and P_{80+} (survival from ages 80 and over to ages 85 and over), and also of $P_{0\text{-}4}$, must be estimated differently. The following empirical formulae are approximately correct:

1. $P_b = 0.05 + p_0\ (0.53 + 0.42\ P_{1\text{-}4})$
2. $P_{0\text{-}4} = \dfrac{p_0 \cdot p_{1\text{-}4}\ (1 + p_{5\text{-}9})}{2\ P_b}$
3. $P_{80+} = 0.8\ P_{75\text{-}79} - 0.1$

180. Table 13 presents the results of the computation of q_x and P_x-values by these short-cut procedures, compared with the q_x and P_x-values previously obtained.

TABLE 13. LUXEMBOURG MALES, 1946–1949. COMPUTATION OF SURVIVAL RATIOS BY SHORT-CUT AND BY LONG PROCEDURE

Age (x)	1,000 m_x	Conversion factor	Estimates by short-cut procedure			Results of computation by long procedure	
			1,000 q_x	p_x	P_x (P_b=0.9351)[a]	p_x[d]	P_x[e] (P_b=0.9354)
0	71.1	0.89	63.3	0.9367	0.9861[b]	0.9370	0.9862
1–4	3.1	3.9	12.1	0.9879		0.9880	
5–9	1.4	5	7.0	0.9930	0.9938	0.9930	0.9937
10–14	1.1	5	5.5	0.9945	0.9932	0.9945	0.9933
15–19	1.6	5	8.0	0.9920	0.9900	0.9920	0.9901
20–24	2.4	5	12.0	0.9880	0.9878	0.9881	0.9879
25–29	2.5	5	12.5	0.9875	0.9858	0.9876	0.9859
30–34	3.2	5	16.0	0.9840	0.9820	0.9841	0.9822
35–39	4.0	5	20.0	0.9800	0.9758	0.9802	0.9761
40–44	5.7	5	28.5	0.9715	0.9622	0.9719	0.9626
45–49	9.6	4.9	47.0	0.9530	0.9488	0.9530	0.9490
50–54	11.3	4.9	55.4	0.9446	0.9276	0.9449	0.9284
55–59	18.6	4.8	89.3	0.9107	0.8908	0.9109	0.8918
60–64	27.5	4.7	129.2	0.8708	0.8398	0.8709	0.8412
65–69	42.5	4.5	191.2	0.8088	0.7642	0.8071	0.7676
70–74	65.2	4.3	280.4	0.7196	0.6530	0.7187	0.6649
75–79	103.4	4	413.6	0.5864	0.4984	0.5900	0.5224
80–84	173.4	3.4	589.6	0.4104	0.2987[c]	0.4078	0.3075
85+	260.9	...	1,000	...		...	

[a] Computed by formula 1 in the accompanying text.
[b] P_{0-4}, computed by formula 2.
[c] P_{80+}, computed by formula 3.
[d] Computed from the q_x values in table 11, column 3.
[e] From table 11, column 6.

IV. ESTIMATING CURRENT LEVELS AND FUTURE TRENDS OF SURVIVAL RATIOS WITH THE USE OF MODEL LIFE TABLES

181. The main part of the procedure of a population projection by sex-age groups consists in multiplying the numbers of various cohorts living at a given time by appropriate survival ratios. The needed ratios can often be worked out individually for a particular population projection, but the procedure is greatly simplified when reference is made to a system of model life tables.

182. Tabulated values for a system of model life tables are found in the appendix, with a note explaining how they were constructed. To facilitate their use in formulating assumptions relating to the future trend of mortality, the tables have been so arranged that they can be regarded as representing successive stages in a process of declining mortality.

183. The idea of a coherent system of model life tables and of some of its uses is developed in part A of this chapter. Part B describes how, with this system, under various conditions and with varying amounts of statistical information, suitable survival ratios can be quickly obtained. The application of survival ratios to an actual population project is illustrated in part C, consideration also being given to possible variations in the systematic assumption of future changes in mortality.

A. THE MODEL LIFE TABLES CONCEIVED AS ONE SYSTEM

184. Each model life table is designed to represent a typical combination of age-sex specific functions of mortality, or survival, corresponding to a given general level of mortality. For present purposes, the general mortality level has been determined in terms of $°e_o$, the expectation of life at birth, for both sexes combined. Actually, the combination of mortality rates, age group by age group, in any given instance, will always differ more or less from any pattern taken as typical for the same general mortality level. It may therefore be necessary to refer to more than one model life table, and perhaps also to make interpolations between two successive tables in estimating the appropriate combination of rates for a given case.

185. A generalization is here made as to the manner in which mortality may decline, during successive five-year time periods, from the conditions of one model life table to those of the next table in the sequence. This generalization requires a rather liberal interpretation. It is not asserted that mortality will always decline in this particular way. It may decline more slowly, more rapidly, or with different rapidity for different age groups. The model assumption is merely one which is plausible under some of the more typical conditions to be found in the world today, and it can be modified as required. Apart from its uses in the estimation of future mortality trends, this model assumption also serves as the link by which the several model life tables are tied together into a coherent sequence.