Chapter I

THE CONCEPT OF A STABLE POPULATION

A. Malthusian populations

The concept of a stable population was first introduced into demography by Alfred J. Lotka [1] as a particular case of a Malthusian population. Lotka went on to demonstrate that a stable population could also be considered as a limit state towards which populations with unchanging mortality and fertility tended, and it is primarily this second aspect which has received attention from demographers. The distinction between the two different approaches is not very clear in Lotka's work, however, and it seemed worth while to define it more precisely at the beginning of the present work.

A Malthusian population is a population whose mortality and sex-age structure are constant. It is important to note that these characteristics are not assumed to be known. The only assumption is that they are constant. Various properties of Malthusian populations follow from this assumption.

(1) As the age structure and mortality are constant, it follows that the age distribution of deaths is also constant.

(2) If a represents the age, $C(a)$ the age structure for both sexes together, and b the crude birth rate, we have:

$$C(0) = b$$

The crude birth rate is therefore constant.

(3) As both the mortality and the age structure are constant, it follows that the crude death rate d is also constant.

(4) The rate of natural variation $r = b - d$ is therefore also constant.[2] Thus for the total numbers of population, of births and of deaths at time t, we have the following expressions in which A is a constant equal to the total number of the population at the initial condition:

$$N(t) = Ae^{rt} = N(0)e^{rt}$$
$$B(t) = bN(0)e^{rt}$$
$$D(t) = dN(0)e^{rt}$$

(5) If $p(a)$ is the survivorship function for both sexes, the number of persons of age a at time t is:

$$B(t-a)p(a) = Abe^{rt}e^{-ra}p(a)$$

They also number:

$$N(t)C(a) = Ae^{rt}C(a)$$

so that we have:

$$C(a) = be^{-ra}p(a)$$

(6) If ω represents the upper limit age of life, as $C(a)$ is a distribution, it follows from the definition:

$$\int_0^\omega C(a)da = 1$$

which is written:

$$b\int_0^\omega e^{-ra}p(a)da = 1$$

or

$$b = \frac{1}{\int_0^\omega e^{-ra}p(a)da}$$

(7) It was stated above that in a Malthusian population the sex distribution is constant. In fact, however, it is sufficient to assume that the masculinity at birth is constant. If this masculinity—the ratio of male births to female births—is represented by m, the expressions for female and male births will be respectively:

$$B_f(t) = \frac{B(t)}{1+m} = \frac{bN(0)}{1+m}e^{rt}$$
$$B_m(t) = \frac{mB(t)}{1+m} = \frac{mbN(0)}{1+m}e^{rt}$$

If $p_f(a)$ is the female survivorship function, assumed to be invariable, then the number of females of age a is:

$$B_f(t-a)p_f(a) = \frac{bN(0)}{1+m}e^{rt}e^{-ra}p_f(a)$$

and the age distribution of the female population is written:

$$C_f(a) = \frac{\frac{bN(0)}{1+m}e^{rt}e^{-ra}p_f(a)}{\frac{bN(0)}{1+m}e^{rt}\int_0^\omega e^{-ra}p_f(a)da} = \frac{e^{-ra}p_f(a)}{\int_0^\omega e^{-ra}p_f(a)da}$$

Similarly, the age distribution of the male population will be:

$$C_m(a) = \frac{e^{-ra}p_m(a)}{\int_0^\omega e^{-ra}p_m(a)da}$$

where $p_m(a)$ represents the male survivorship function, assumed to be also invariable.

(8) In particular, the two expressions for the crude female and male birth rates will be as follows:

$$C_f(0) = b_f = \frac{1}{\int_0^\omega e^{-ra}p_f(a)da}$$

$$C_m(0) = b_m = \frac{1}{\int_0^\omega e^{-ra}p_m(a)da}$$

[1] Alfred J. Lotka, *Théorie analytique des associations biologiques, deuxième partie* (Paris, Hermann, 1939), 149 pages.

[2] This is why Lotka called these populations "Malthusian populations", since Malthus dealt with populations increasing by geometric progression.

There is a very simple approximate relationship between the three rates b_f, b_m and b. It may be noted that:

$$\frac{1}{b_f} + \frac{m}{b_m} = \int_0^\omega e^{-ra}[p_f(a) + mp_m(a)]da = \int_0^\omega e^{-ra}p(a)da = \frac{1}{b}$$

In practice, b_f and b_m are always close to one another, while m varies little from 1, so that we have:

$$b \# \frac{b_f + b_m}{2}$$

It also follows that:

$$d \# \frac{d_f + d_m}{2}$$

(9) If only the female sex is considered and $\varphi(a,t)$ is taken to represent the female fertility rate for women of age a, computed from the girls at time t, we obviously have:

$$b_f = \int_u^v C_f(a)\varphi(a,t)da = b_f\int_u^v e^{-ra}p_f(a)\varphi(a,t)da$$

where u and v represent the limits of the reproductive period. Finally, we can write:

$$\int_u^v e^{-ra}p_f(a)\varphi(a,t)da = 1 \qquad (1)$$

Such are the main properties of a Malthusian population.[3]

B. Malthusian populations with known mortality (stable Malthusian populations)

Lotka goes on to consider the sub-sets H(r) obtained from the set of Malthusian populations thus defined by assuming certain fixed values for the mortality functions. Every survivorship function $p_0(a)$ has a corresponding sub-set $H_0(r)$, and all the populations of the sub-set can be obtained by successively associating with $p_0(a)$ all the possible values of r.[4]

It is within such a sub-set $H_0(r)$ that Lotka defines in the following manner what he means by a stable population. If, in addition to the mortality functions, values are also assumed for the fertility function—i.e., if it is assumed that $\varphi(a,t) = \varphi_0(a)$ is known and is independent of time, then expression (1) becomes an equation in r. It has only one real root: $r = \varrho$. The population of the sub-set H(r) which corresponds to this value ϱ is called a stable Malthusian population or, more simply, a stable population. To be more exact, it is *the* stable population corresponding to laws of mortality and fertility which are *assumed to be known and invariable*. In fact, since equation (1) in r has only one real root,[5] there is only one stable population corresponding to the given laws of mortality and fertility, and we can therefore speak of the stable population which corresponds to those laws.

[3] These properties will be dealt with again in chapter II, one by one, and various methods of formulating them will be described.

[4] Mathematically speaking, r can vary from $-\infty$ to $+\infty$. In human populations, however, the actual variations of r are much narrower. We shall revert to this question later.

[5] We shall revert to the question of the existence of this one real root in later chapters.

Before studying the second definition of a stable population, i.e., the concept of a stable population as the limit state of a process of demographic evolution in which mortality and fertility remain unchanged, it is worth pausing to consider the principles on which the concept of the first type of stable population is based.

A stable population is a specific population from among a sub-set of Malthusian populations selected from the wider field of all possible Malthusian populations. The sub-set is selected on the basis of *knowledge* of the mortality function. It would have been possible to select another sub-set $F_0(r)$ if the sex-age distribution functions had been assumed to be known. The method adopted by Lotka therefore ignores a whole aspect of Malthusian populations, i.e., all the sub-sets F(r) of Malthusian populations with known age distribution. We shall revert later to the consideration of these sub-sets F(r), the study of which has many practical applications.

Let us now return to the sub-sets H(r) of Malthusian populations with known mortality function, as considered by Lotka. The stable population has been defined by the supplementary assumption that the fertility function is also known. Thus, the stable population has been defined as a Malthusian population with known mortality and fertility. It is obvious, however, that in a sub-set $H_0(r)$ there are other ways to determine a particular population.

Generally speaking, it is sufficient to assume as given any index or function leading to an equation in r with a finite number of real roots. Let us consider the following examples:[6]

(*a*) If we assume, for example, a birth rate $b = b_0$, we have the equation:

$$b_0\int_0^\omega e^{-ra}p_0(a)da = 1$$

As equation (1), referred to above, has only one real root, there is therefore one, and only one, Malthusian population with a given mortality and a given crude birth rate.

(*b*) If we assume a death rate $d = d_0$, we arrive, since we have $b = d_0 + r$, at the following equation in r:

$$(d_0 + r)\int_0^\omega e^{-ra}p_0(a)da = 1$$

It can be shown that this equation has no solution, one solution or two solutions. Thus, there is not always a Malthusian population with the given mortality functions and the given crude death rate, and when one exists another generally exists also.

(*c*) It is also possible to assume a value for the age distribution at a given age h. This leads to the following equation in r:

$$C_0(h)\int_0^\omega e^{-ra}p_0(a)da = e^{-rh}p_0(h)$$

(*d*) We arrive at a similar equation if we assume the age distribution of deaths $d_0(h)$ for a given age h. This gives us the following equation in r:

$$d_0(h)\int_0^\omega e^{-ra}p_0(a)q_0(a)da = e^{-rh}p_0(h)q_0(h)$$

where $q(a)$ is the probability of dying at age a.

[6] These examples will be dealt with again in detail in later chapters.

All these problems are similar in principle to that considered by Lotka in arriving at his definition of a stable population. However, the two conditions imposed by Lotka—knowledge of the mortality and the fertility—are obviously in a different category from the two conditions corresponding to each of the other problems. They are mathematically independent and express the fundamental characteristics of a population, namely, its mortality and its fertility. This is why the case of a stable population is of much greater interest than the other particular Malthusian populations described above. This remark is all the more important when the concept of a stable population is envisaged as the limit state of a process of demographic evolution in which the mortality and fertility remain unchanged. This is the case which will be discussed next.

C. Concept of a stable population considered as a limit

The two assumptions of constant age distribution and constant mortality will define all Malthusian populations.

The third assumption, that the mortality is known, enables us to define sub-set $H_0(r)$.

Finally, the fourth assumption that the female fertility function is known, enables us to define from sub-set $H_0(r)$ the stable population corresponding to the known laws of mortality and fertility.

What happens if we suppose that the first assumption is not made, i.e., if we do not assume that the age distribution is constant? In other words, what happens if the mortality and fertility of a population remain unchanged? Let us state immediately that such a population follows a path which brings it closer and closer to the structure of the stable population corresponding to the given laws of mortality and fertility.

The demonstration of this property is one of the classic problems of the theory of "renewable resources", the solution of which calls for difficult mathematical developments if the demonstration is to be completely accurate, although a rough proof of the results can be achieved by relatively simple methods, which are described in chapter III. It is also possible to examine empirically, on the basis of actual data, what happens in a population whose female mortality and fertility remain unchanged. The principle underlying such an examination is that of computing population projections and seeing how they evolve.

D. Study of the limit population on the basis of actual cases

Attempts are made here to provide examples which are the results of projections. The starting point is the population of Eastern Germany [7] in 1957.[8] This population was selected in order to have as a starting point a very distorted age distribution.

The life table, which is assumed to be invariable, is the level 80 table of the series of model life tables published in the *Manual on Methods for Population Projections by Sex and Age*. This table corresponds to an expectation of life at birth for both sexes of 60.4 years.

As stated in the introduction, it will be necessary in this work to use two other series of model life tables which are respectively above and below the series of model life tables given in the *Manual* on methods for population projections. The series of tables given in the *Manual* will be referred to as "intermediate model life tables".

The age distribution of the female fertility rates, which are assumed to be invariable, is as follows: [9]

TABLE I.1

Age group (years)	*Distribution*
15-19	100
20-24	273
25-29	263
30-34	188
35-39	121
40-44	55
ALL AGES	1 000

The total of these rates, which is the same as the gross reproduction rate, is assumed to equal 1.50.[10] Masculinity at birth is $m = 1.05$.

As in the case of mortality, we shall later need to use other patterns of fertility with an age distribution on either side of the distribution adopted for the computation of the projections. This latter distribution will be referred to as the "intermediate model fertility distribution".

Once these conditions have been set, a projection is computed by the traditional method described in the *Manual on Methods for Population Projections by Sex and Age*.[11]

Some explanations are called for regarding the selection of the model life table and the gross reproduction rate. When a projection is computed for a population with constant mortality and fertility, often troublesome discontinuity is being introduced if rates of mortality and fertility significantly differ from those actually observed in the initial population. It was, therefore, necessary to take a mortality and a fertility which were not too far removed from the mortality and fertility actually observed in Eastern Germany in recent years.

The same mortality and fertility will be used to compute another projection based on an estimate of the population of Thailand in 1955 and it will also be necessary that the mortality and fertility adopted should not be too far removed from the mortality and fertility observed in Thailand in recent years.

It should be stated immediately, as regards fertility, that it is impossible to select a gross reproduction rate which is close both to that of Eastern Germany and to that of Thailand, since the level of fertility in Thailand is almost three times as high as in Eastern Germany. If we adopt 1.50 as the gross reproduction rate, this conforms satisfactorily with the path along which fertility is developing in Eastern Germany, but it does not rid the

[7] The designations employed and the presentation of the material in this publication do not imply the expression of any opinion whatsoever on the part of the Secretariat of the United Nations concerning the legal status of any country or territory or of its authorities, or concerning the delimitation of its frontiers.

[8] *Demographic Yearbook, 1958* (United Nations publication, Sales No.: 58.XIII.1), table 5, p. 132.

[9] We shall revert to the reasons for selecting this particular age distribution of the fertility rates later.

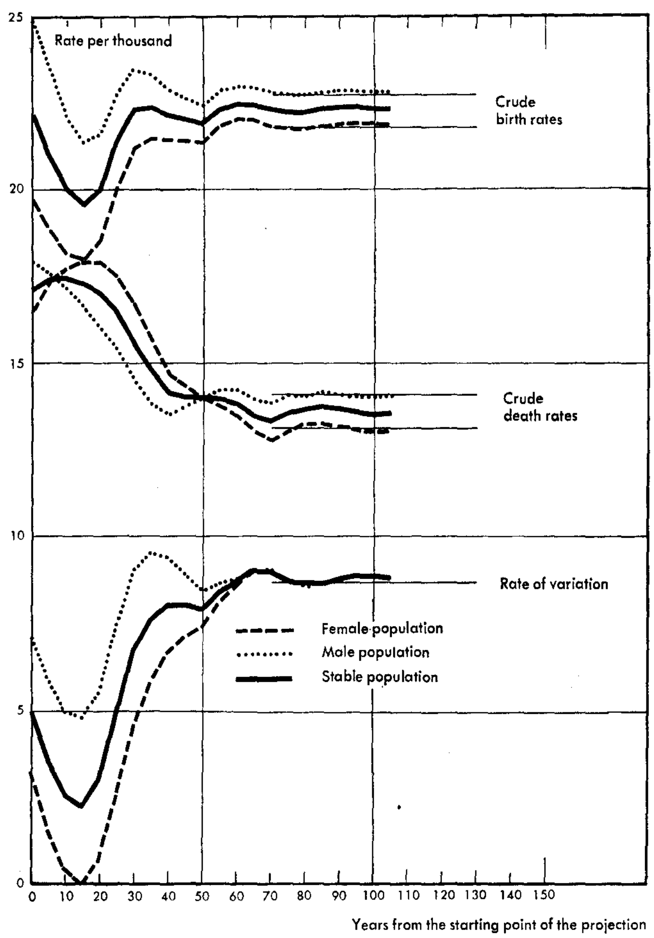
[10] Such a gross reproduction rate corresponds to a medium fertility which would lead, at the mortality rate assumed for the computations, to a stable population undergoing moderate increase.

[11] United Nations publication, Sales No.: 56.XIII.3.

projection based on the figures for Thailand of the discontinuity connected with the fertility assumption.[12] Where mortality is concerned, the level 80 of the intermediate model life table is approximately midway between the levels of mortality in Thailand and in Eastern Germany, and consequently it satisfies the requirements stated above.

These points having been made clear, we begin by commenting on the results obtained in the projection for the female population.

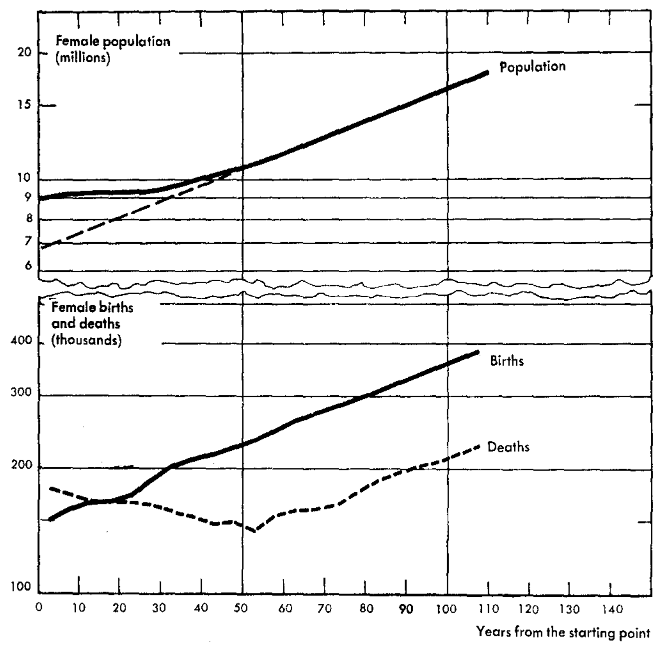
At the outset, the crude female birth and death rates show substantial fluctuations. These fluctuations diminish with time, however, and the rates finally stettle down at constant levels. The rate of natural increase of the female population, which is the difference between these two rates, follows a similar development (see graph I.1).



Graph I.1. Projections computed on the basis of the population of Eastern Germany in 1957. Variations in the crude birth rate, crude death rate and crude rate of natural increase

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50

The absolute figures for female deaths, female births and the total female population vary irregularly at the beginning but tend after a time to increase at a constant rate which is the same for births, deaths and total population. On a semi-logarithmic graph (with time indicated on the horizontal axis in a metric scale and the absolute numbers on the vertical axis in a logarithmic scale), the curves of the variation of births, deaths and total population will become, after a time, parallel straight lines whose slopes are equal to the stabilized rate of increase (see graph I.2). These straight lines will be referred to in this paper as stabilization lines.



Graph I.2. Projections computed on the basis of the population of Eastern Germany in 1957. Variations in the absolute numbers of persons in the female population, of female births and of female deaths

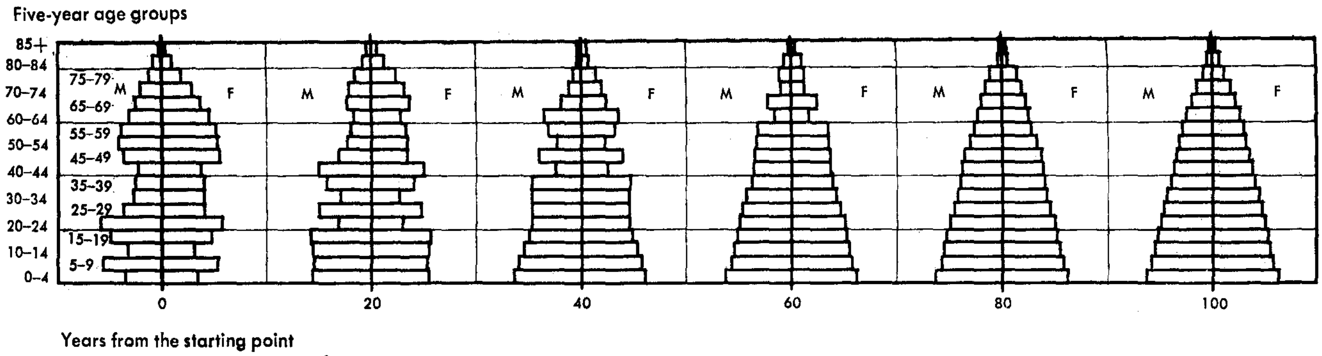
Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50.

The projection for the male population leads to the same observations as the projection for the female population (see graph I.1). The crude male mortality and fertility rates stabilize, after a time, at a level slightly above that at which the female rates stabilize. The discrepancy between the birth rates is the same as that between the death rates, so that the male rate of variation stabilizes at the same level as the female rate of variation. In other words, at the limit the female population and the male population vary at the same rate.

Finally, the age structure of the two populations, which at first is very irregular, gradually becomes more regular and eventually acquires a very regular shape, which subsequently remains invariable (see graph I.3).

We therefore see that in this projection the population approaches a state in which the mortality and age distribution are invariable. This is an exact definition of a Malthusian population. In this Malthusian state, moreover, the mortality and fertility are known, and we have seen that such knowledge enables us to define the stable population of the sub-set $H_0(r)$ associated with the mortality used in the computation. We have therefore verified empirically the result given above: *a population whose mortality and fertility remain invariable tends to become the stable population corresponding to such levels of mortality and fertility.*

[12] This discontinuity is particularly marked in the population pyramids in graph I.8.



Graph I.3. Projections computed on the basis of the population of Eastern Germany in 1957. Variations in the age distribution of the population by five-year age groups

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50.

In the graphs, we find evidence of the following properties already given above as characteristic of Malthusian populations:

(i) The total number of population varies at a constant rate which is the same for both males and females;

(ii) Births and deaths vary at the same constant rate as the population, and consequently the crude death rates and birth rates are constant;

(iii) The crude death rates and birth rates computed on the basis of the female population, using female births and female deaths, are slightly different from the corresponding rates computed on the basis of the male population, using male births and male deaths;[13]

(iv) The age structure is invariable;

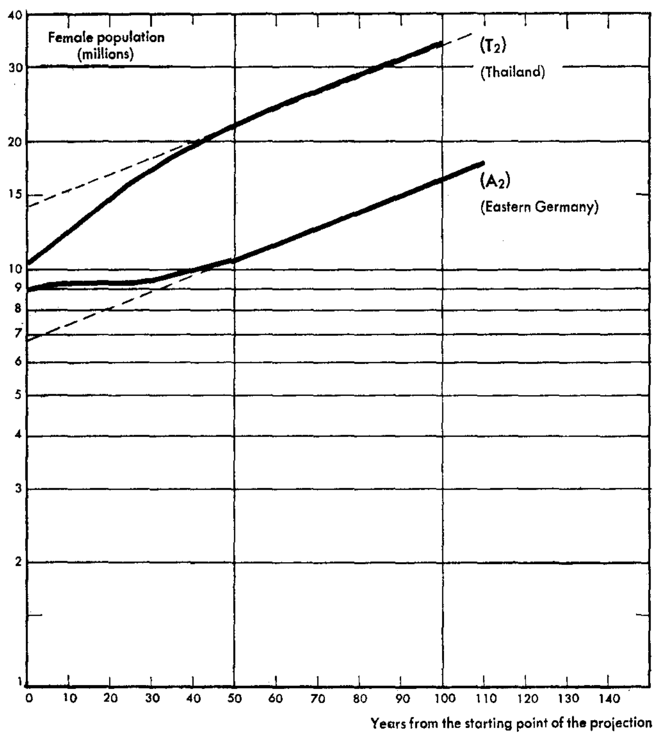
(v) There is a constant ratio between the total numbers of persons in the female and male populations.

As has been said, Lotka shows that the limit stable population is completely defined, once the laws of mortality and fertility are known. If, therefore, we compute a population projection on the basis of a population other than that of Eastern Germany in 1957, while keeping the same laws of mortality and fertility, we should arrive at a limit population identical with that described above.

Let us then repeat the preceding computations, retaining the same mortality and the same fertility, but starting from a population with an age structure different from that of the population of Eastern Germany. We have chosen for this purpose an estimate of the age structure of Thailand in 1955.[14] We find the same result as in the first computations; the demographic characteristics vary irregularly at the beginning, but become stable after a time. We note, however, that the crude birth rates, crude death rates and crude rates of increase settle down at the same levels in both projections (see graphs I.4 and I.5). The stabilized age structures are also identical in both computations (see graph I.6).

As expected, at the stable limit, the crude death rate, the crude birth rate and the crude rate of natural increase, as well as the age structure, are independent of the initial population and depend only on the laws of mortality and fertility.

In contrast, the irregular variations at the beginning and the level reached by the absolute numbers (of births, deaths, and number of persons in the population) when the rates become stabilized depend both on the initial conditions and on the laws of mortality and fertility.

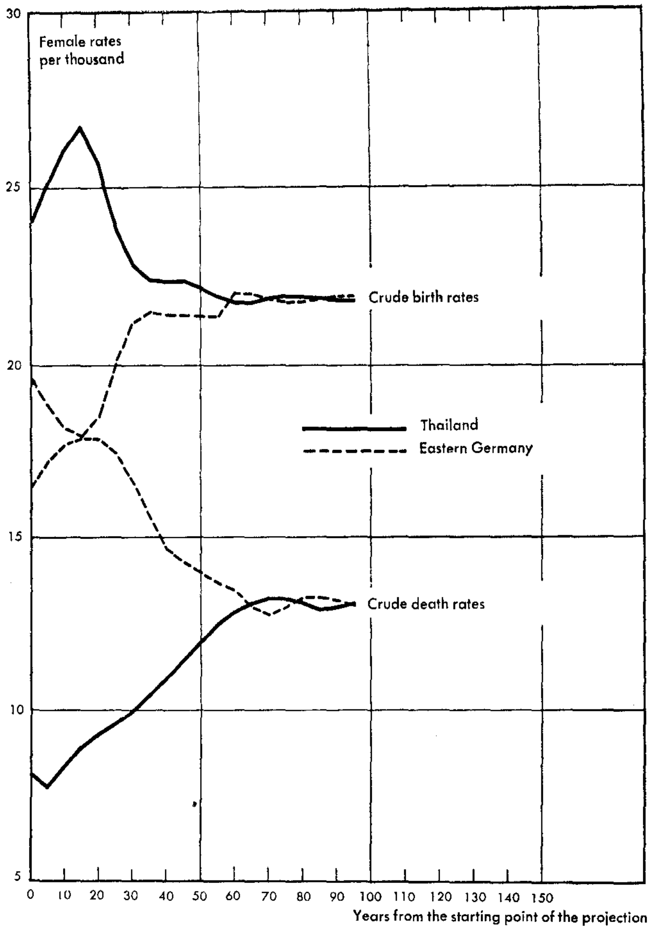


Graph I.4. Two sets of projections computed on the basis of the population of Eastern Germany in 1957 and of an estimate of the population of Thailand in 1955, respectively. Variations in the absolute number of persons in the female population

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50

[13] The male rates in the example chosen are higher than the female rates, and this characteristic is found in practice in most life tables. It is not, however, entirely impossible to perceive the contrary. This is an example of a property resulting from the fact that the mortality function is a function of human beings and as such, generally speaking, never varies greatly from the model mortality.

[14] *The Population of South-East Asia (including Ceylon and China (Taiwan))*, 1950-1980 (United Nations publication, Sales No.: 59.XIII.2).



Graph I.5. Two sets of projections computed on the basis of the population of Eastern Germany in 1955 and of an estimate of the population of Thailand in 1955, respectively. Variations in the crude female birth rate and the crude female death rate

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50

It is to be noted, first of all, that the levels reached in the stabilization phase vary considerably, according to the initial age structure.

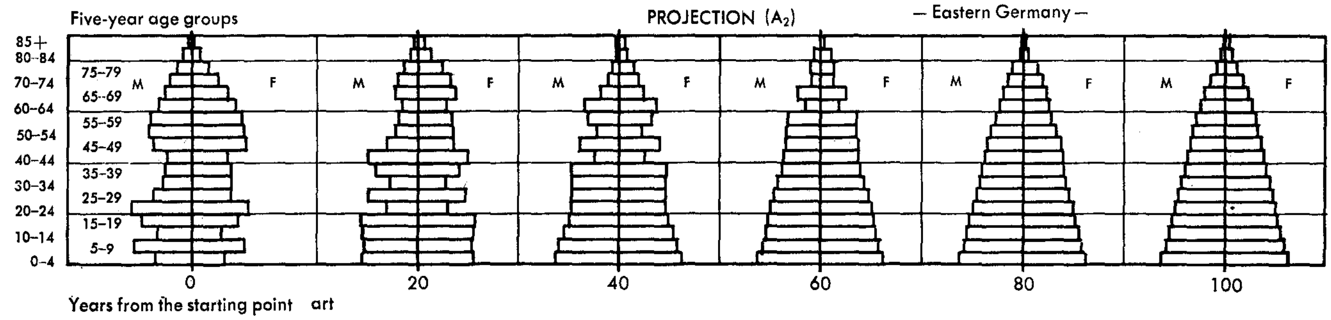
One hundred years after the starting point, the female population is increasing each year by 8.7 per thousand, whether the age structure of Eastern Germany or that of Thailand is taken as the basis; when an initial population of one million women is assumed, however, the figure at the end of 100 years is 1,977,376 if the age structure of Eastern Germany is taken as a basis, but 3,275,000 if the age structure of Thailand is used.

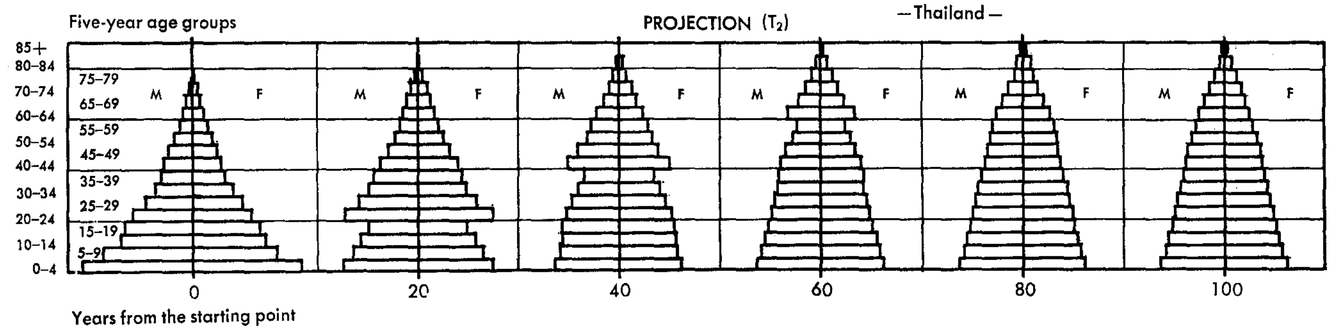
The irregular oscillations observed at the beginning do not follow any general laws. The diversity of the initial age structures exerts its full effect in this phase of the computations. In contrast, certain regular patterns are to be observed in the levels reached by the absolute numbers at the period when stabilization is taking place.

In order to establish what these regular patterns are, let us repeat the preceding computations, retaining the same law of mortality, but taking different fertility rates in succession.

Two other projections (A_1) and (A_3) have been computed on the basis of the population of Eastern Germany by associating the same mortality as in projection (A_2) with two other levels of fertility, one corresponding to a gross reproduction rate of 0.75 [15] (projection A_1), the other corresponding to a gross reproduction rate of 1.17 (projection A_3). This latter level of fertility is such that the stable level corresponds to an unchanging number of persons in the population; the population is then said to be stationary. In both cases, intermediate model distributions of the age-specific fertility rates have been adopted.

[15] Such a rate corresponds to a very low fertility leading to a declining stable population.





Graph I.6. Two sets of projections computed on the basis of the population of Eastern Germany in 1957 and of an estimate of the population of Thailand in 1955, respectively. Variations in the age distribution of the population by five-year age groups

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated with a constant intermediate model fertility corresponding to a gross reproduction rate of 1.50.

Finally, similar projections (T_1) and (T_3) have been computed on the basis of the population of Thailand in 1955. Thus, we now have, for each of the two initial populations, two sets of three projections (A) and (T), which correspond to each other two by two in accordance with the assumptions regarding mortality and fertility.

Graphs I.7 and I.8 show how the age structure varies in these several projections.

At year zero, the population pyramids of each set are the same in all three projections. They correspond to the initial populations, which are the same in all three projections. The pyramid for set (A) is very different from that for set (T), however, as the initial populations chosen are, of course, very different.

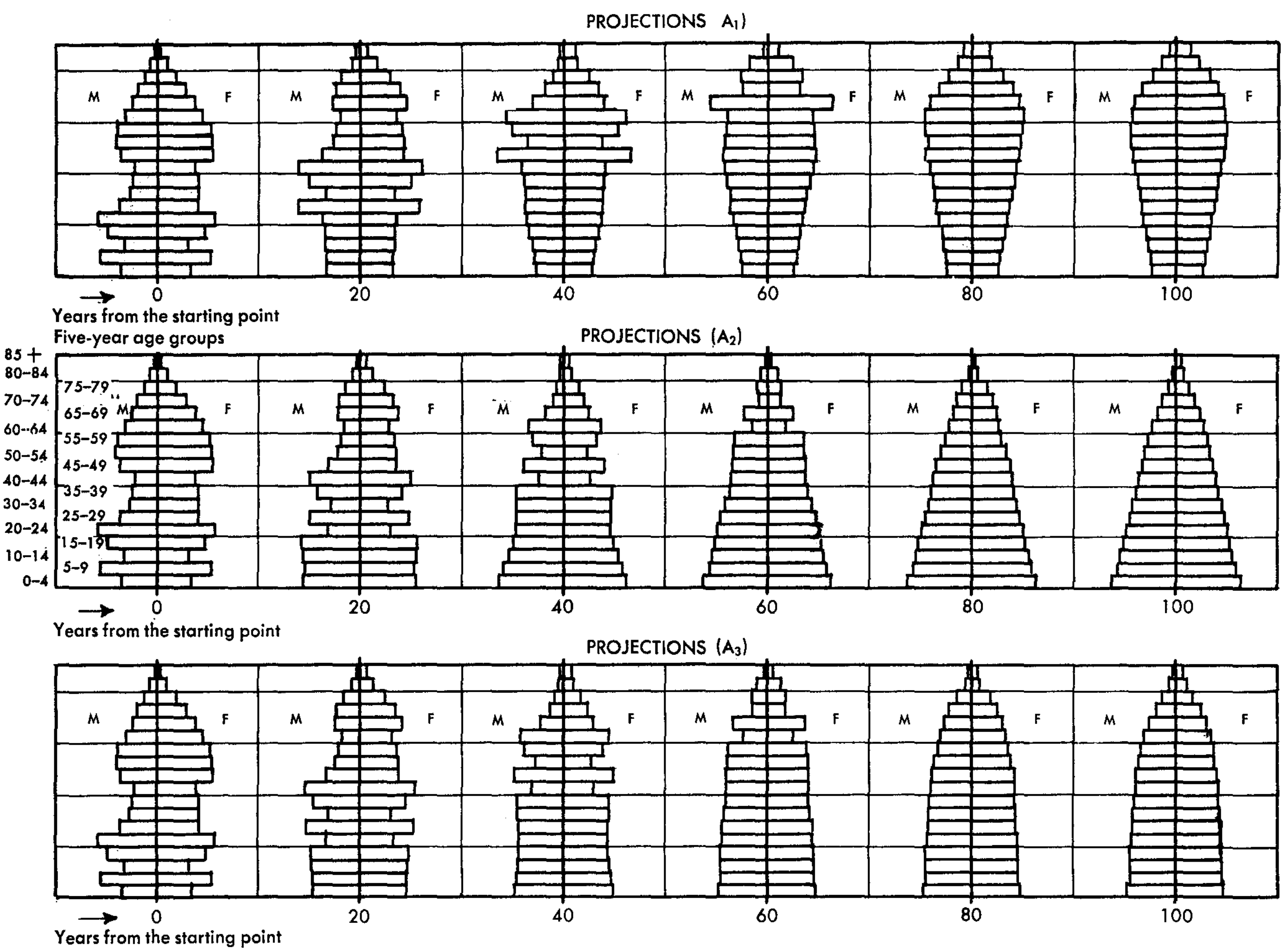
One hundred years after the starting point, the three pyramids of each of the two sets are very different from each other, because in the three projections very different fertilities were associated with the same mortality. However, the pyramids in sets (A) and (T) corresponding to the same fertility are identical, because the age structure of a stable population does not depend on the initial population.

On a semi-logarithmic graph for each projection, a stabilization straight line for total population, similar to those in graph I.4, can be drawn. Graph I.9 shows these straight lines for series (A), while graph I.10 shows them for series (T). Finally, graph I.11 shows the straight lines thus obtained for both series (A) and series (T).

When fertility varies in a continuous manner, we have a family of straight lines for each series, and it is to such a family that the three straight lines marked on graphs I.9, I.10 and I.11 belong. Each family has an envelope of which three tangents are known. The part of this envelope corresponding to the two series of projections [16] has been drawn in graph I.11. This envelope is a curve, concave to the origin, which passes through a maximum for the stationary population (points S and S′ in graph I.11).

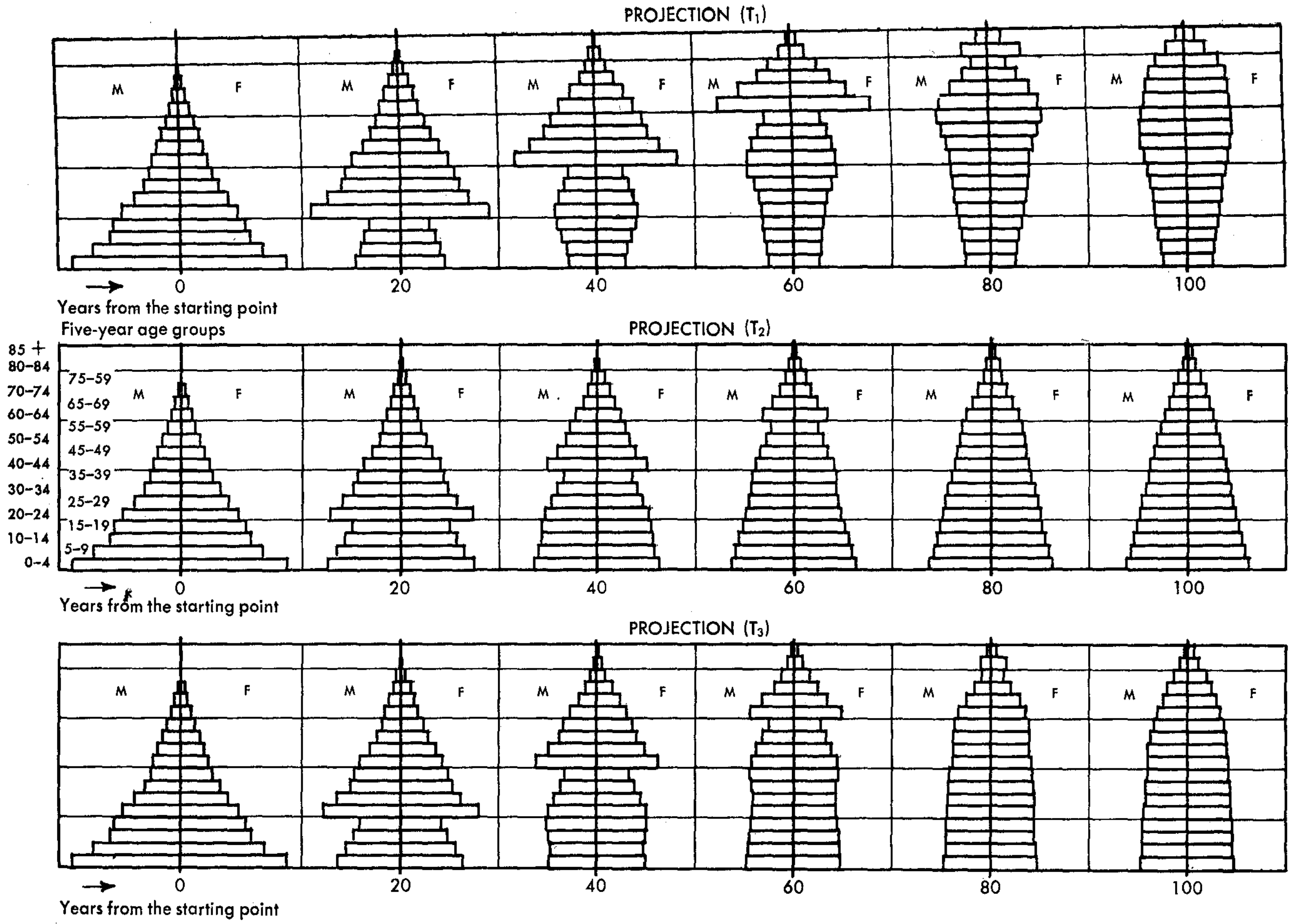
If the intrinsic rate of natural variation is small, as is often the case, the envelope is reduced in practice to its peak S. If, then, starting from different initial populaions of equal numbers—for example, female populations of one million with different age distributions as may be found in different countries—we carry out the same computations as above, we obtain for each country a peak S, and the series of these peaks forms a curve the shape of which depends only to a minor degree on the fertility and mortality used for the computation. To all

[16] Formulae enabling the abscissae and ordinates of the points of contact of each straight line with the envelope to be calculated are to be found in annex I. See also the numerical applications in chapter III.



Graph I.7. Three sets of projections computed on the basis of the population of Eastern Germany in 1957. Variations in the age distribution of the population by five-year age groups

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 (projection A_1), 1.50 (projection A_2) and 1.17 (projection A_3).



Graph I.8. Three sets of projections computed on the basis of an estimate of the population of Thailand in 1955. Variations in the age distribution of the population by five-year age groups

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 projection (T_1), 1.50 (projection T_2) and 1.17 (projection T_3).

intents and purposes, the shape of this curve depends only on the initial age distributions of the populations, and it therefore enables those age distributions to be compared from the point of view of their growth potential.[17] See annex I for more details on how to construct and use these curves.

Such are the main properties of stable populations considered as the limit of a process of demographic evolution in which mortality and fertility remain unchanged. Obviously, the verification of these properties in some particular case is not sufficient to establish their validity, and a mathematical demonstration is necessary. This will be given in chapter III in a non-rigorous form used by Lotka. Already, one may note the usefulness of the concept of the limit stable population in demographic analysis.

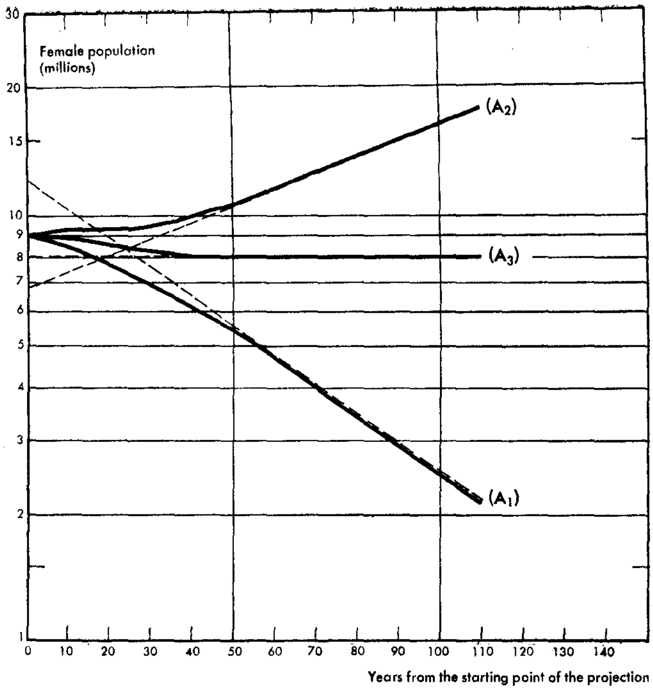
If at a given moment the mortality and fertility of an actual population are stabilized at the levels attained at that moment, we set in motion a process of demographic evolution which approaches the stable population corresponding to the mortality and fertility of that moment.

This stable population is, in a way, the development of the conditions of mortality and fertility of the moment. It is determined entirely by the unique knowledge of the laws of fertility and mortality. It does not depend particularly on the age distribution of the moment. This age distribution makes itself felt at the beginning of the stabilization process, but with the passage of time its influence progressively diminishes, and at the limit it disappears completely. All the characteristics of this stable population are therefore actually characteristics of the moment. They describe what would happen if the mortality and fertility of the moment remained for ever unchanged from the level they had attained at that moment. They are by definition referred to as the intrinsic characteristics. We speak of the intrinsic crude birth rate, the intrinsic crude death rate and the intrinsic rate of natural variation. We must never forget, however, that these are characteristics of the moment, although they are defined as being part of a process of development.

E. A generalization of the concept of a limit stable population

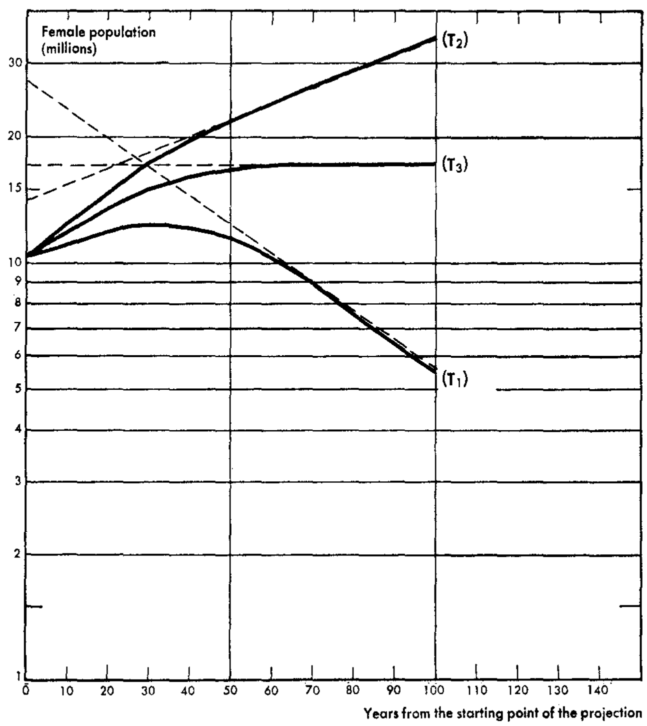
What has been done for stable populations can obviously also be done for all the particular Malthusian populations of sub-set $H_0(r)$. We may ask, for example, what happens in a population where *the mortality and the crude birth rate* remain constant, or in a population

[17] This point was the subject of a paper submitted to the Société de statistique de Paris by Mr. P. Vincent. Cf. P. Vincent, "*Potentiel d'accroissement d'une population*", *Journal de la Société de statistique de Paris* (January-February 1945), pp. 16 *et seq.* Some comments on the method developed by Mr. Vincent will be found in annex I.



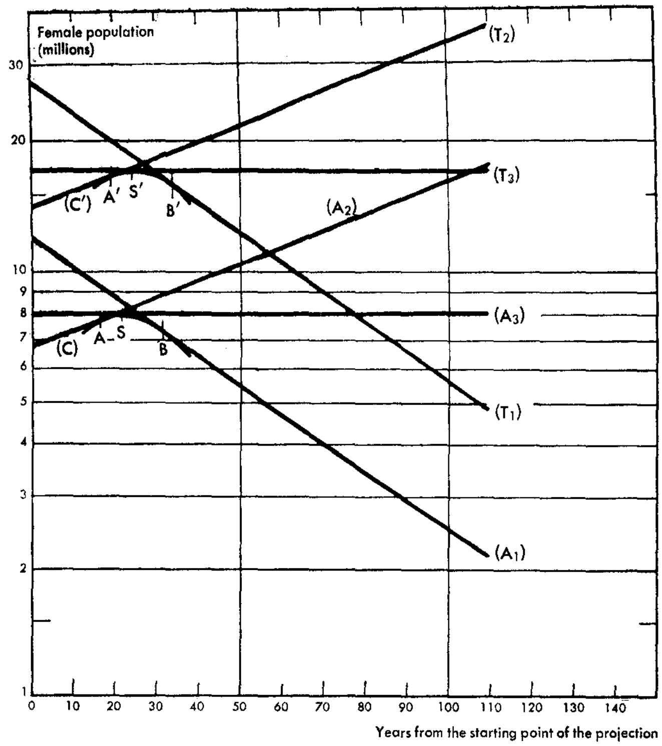
Graph I.9. Three sets of projections computed on the basis of the population of Eastern germany in 1957. Variations in the absolute number of persons in the female population

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 (projection A_1), 1.50 (projection A_2) and 1.17 (projection A_3).



Graph I.10. Three sets of projections computed on the basis of an estimate of the population of Thailand in 1955. Variations in the absolute number of persons in the female population

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 (projection T_1), 1.50 (projection T_2) and 1.17 (projection T_3)



Graph I.11. Two series of three sets of projections computed on the basis of the population of Eastern Germany in 1957 (series A) and of an estimate of the population of Thailand in 1955 (series T). Stabilization straight lines for the female population

Constant intermediate model life table of level 80, corresponding to an expectation of life at birth for both sexes of 60.4 years, associated successively with three constant intermediate model fertility corresponding to gross reproduction rates of 0.75 (projections A_1 and T_1), 1.50 (projections A_2 and T_2) and 1.17 (projections A_3 and T_3)

where *the mortality and the crude death rate* remain constant, and so forth. In each of the cases considered above we show that, in general, the population approaches the corresponding Malthusian population of the sub-set $H_0(r)$ or, when the given conditions are applicable to several Malthusian populations[18] of sub-set $H_0(r)$, towards one of the corresponding Malthusian populations.

F. Priority of the concept of a stable population

It is easy to see why, as mentioned earlier, the concept of a stable population has become quite popular in demographic analysis. The laws of mortality and fertility of the moment can be considered, quite satisfactorily, as indices of health conditions and of the reproductive behaviour of couples. The stable population thus enables an assessment of health conditions and reproductive behaviour to be made. In contrast, the laws of mortality and of the crude birth rate, for example, while still forming a good index of health conditions, give a picture of the reproductive behaviour of couples which is distorted by the age distribution of the moment. Part of this current age distribution is thus transferred into the limit Malthusian population, while the influence of the age distribution of the moment disappears from the stable

[18] This is so, for example, in the case of a population whose survivorship function and crude death rate remain invariable (see above).

population. What is stated above regarding the laws of mortality and of the crude birth rate taken together is equally true for all the other associations considered.

It is no doubt of interest from a theoretical point of view to see how the concept of a stable population fits into a wider set of Malthusian populations of different types and to show that, by the use of mathematics, it is possible to define processes similar to those leading to the limit of the notion of stable populations, but taking as the starting point conditions different from those which are at the basis of stable populations. It must be kept in mind, however, that none of these attempts can go very far, whereas the concept of stable populations has many useful applications.

G. Malthusian populations with known age distributions

In contrast, a study of the other sub-sets of Malthusian populations which were referred to at the beginning of this chapter (the sub-sets F(*r*)) leads to important results. These other sub-sets are in a sense symmetrical with the sub-sets H(*r*). Each sub-set H(*r*) is made up of all Malthusian populations for which the mortality function is known. The populations of each sub-set H(*r*) therefore satisfy the following conditions:

(i) *Constant* age distribution;

(ii) *Constant and known* mortality.

Each sub-set F(*r*) consists of Malthusian populations where age distribution is known. The populations of each sub-set F(*r*) therefore satisfy the following conditions:

(i) *Constant and known* age distribution;

(ii) *Constant* mortality.

The study of the sub-sets F(*r*) lends itself to developments similar to those described above in connexion with the sub-sets H(*r*).

Here we are breaking new ground, since Lotka and his successors did not study the sub-sets F(*r*). The main properties of the sub-sets F(*r*) are indicated in chapter IV. For the moment, we shall confine ourselves, in chapters II and III, to a further study of the properties of the populations of sub-sets H(*r*) with known mortality functions and, in particular, to giving practical ways of calculating the various characteristics of these populations.