Chapter VI

EXAMPLES OF ESTIMATES BASED ON RECORDS OF POPULATION GROWTH AND DISTRIBUTION BY AGE

A. Estimation of mortality and of the birth rate from census survival rates

An application of the method of estimating mortality and fertility described in section A.2 of chapter I is illustrated below using information taken from Turkish statistics, specifically from the national censuses of 1935 and 1945. Estimation could have been based on the censuses of 1935 and 1940, but censuses at five-year intervals are rare, and the ten-year interval was chosen as more typical.

Required basic data. Distribution of the population by five-year age groups as recorded in two successive censuses taken several—preferably five or ten—years apart. Columns 2 and 3 of table 15 show the female population in Turkey in 1935 and 1945, classified by age. Ideally the population should be closed to migration during the intercensal period and the two censuses should refer to the same geographical area.

Preliminary adjustment of data. The ideal requirements as stated in the preceding paragraph are seldom perfectly satisfied. Deviations from these requirements result in biases in the final estimates or cause computational inconveniences. Preliminary adjustments of the basic data can eliminate or reduce either of these effects. The possibility of making such adjustments when they are needed, and their specific nature may differ from case to case: the decision of the analyst concerning the procedures to be followed should be influenced in each instance by the extent of the deviation from the ideal requirements and by the amount and quality of the information that is available for making corrections in the basic data. The following four types of adjustments may be both commonly needed and feasible:

Table 15. Female population of Turkey by age in 1935 and 1945 (thousands) and census survival rates for five-year cohorts

	Population reported by census		*Adjusted population, 1935*		*Adjusted population, 1945*	
Age interval (1)	*10,20, 1935* (2)	*10.21, 1945* (3)	*For "age unknowns" (col. 2 × 1.0044)* (4)	*For migration and boundary changes (col. 4 × 1.0204)* (5)	*For "age unknown" (col. 3 × 1.0014)* (6)	*Ten-year survival rates for each cohort*[a] (7)
0-4	1,297	1,185	1,303	1,329	1,187	—
5-9	1,128	1,242	1,133	1,156	1,244	—
10-14	746.0	1,074	749.2	764.2	1,076	.8096
15-19	485.9	931.5	488.0	498.0	932.8	.8069
20-24	640.2	691.7	643.0	656.1	692.7	.9060
25-29	721.3	619.1	724.4	739.3	620.0	1.2450
30-34	642.1	699.7	644.9	658.1	700.7	1.0680
35-39	509.6	578.4	511.8	522.3	579.2	.7834
40-44	473.9	558.0	476.0	485.7	558.8	.8491
45-49	314.9	378.5	316.2	322.7	379.0	.7256
50-54	384.4	434.1	386.1	394.0	434.7	.8950
55-59	195.2	219.4	196.1	200.1	219.7	.6808
60-64	297.4	349.2	298.7	304.8	349.7	.8876
65-69	105.0	124.6	105.4	107.6	124.8	.6237
70-74	126.1	133.0	126.6	129.2	133.2	.4370
75 and over	118.0	112.3	118.5	120.9	112.5	—
Unknown	35.81	13.11	—	—	—	—
Total	8,221	9,344	8,221	8,388	9,344	—

[a] Ratio of number of persons in each age interval in 1945 (from Column 6) to number ten years younger in 1935 (from Column 5).

(1) *Adjustment for persons not classified by age.* Unless the number of persons not classified by age is very small, or their proportion in the total population is very nearly the same in both censuses, "unknowns" with respect to age should be distributed in a fashion that leaves the distribution of the total population for the given sex with known ages unchanged. This is performed by multiplying the population classified by age by the ratio:

$$\frac{\text{total population}}{\text{total population} - \text{population with ages unknown}}$$

Columns 4 and 6 show the female population of Turkey in 1935 and 1945 after such an adjustment has been performed. Column 4, for example, was obtained by multiplying the population in each group shown in Column 2 by the factor of 8221/(8221-36) = 1.0044.

(2) *Adjustment for boundary changes.* Other things being equal, an increase in the territorial coverage in the second census would spuriously inflate the census survival rates; territorial losses in the intercensal period would introduce an opposite bias. If the two censuses in question do not refer to the population of the same territory comparability must be insured by reckoning the population in both censuses on an identical territorial basis. If the population involved in the adjustment constitutes a substantial portion of the total population, or if its age and sex composition is very atypical, it would be highly desirable to correct the census figures by individual age and sex groups: e.g., in case of intercensal territorial gain to add to the figures of the first census the population of the territory in question (as estimated at the time of the first census) as it was actually distributed by sex and age, or to remove the population of the affected territory from the figures of the second census age group by age group. When the population involved is small, and its characteristics are not strongly deviant from those of the rest of the country, a simpler adjustment is adequate. This statement holds true in the case of Turkey for 1935-1945. The territory of that country was increased by the province of Hatai in 1939. In 1940 the total population without this province was reported as 17,613,000 and with Hatai province as 17,821,000. An adjustment for this territorial change may be performed simply by multiplying through the 1935 census figures by the factor of (17821/17613) = 1.0118. Alternatively the 1945 census figures could have been deflated by the factor of .9883.

(3) *Adjustment for migration.* The considerations governing this adjustment are the same as those underlying the adjustment for change in territorial coverage outlined in the preceding paragraph. In Turkey it was estimated that net immigration during the 1935-1945 period amounted to some 150,000 persons. Since no detailed information is available as to the sex-age composition of the migrants, and since the numbers involved are relatively small, it may be simply assumed that their demographic characteristics (their fertility, mortality, age and sex composition) were the same as those of the rest of the population and the adjustment may be performed by multiplying through the 1935 figures by the factor of 1.0085, i.e., by the ratio of the mid-period (1940) population plus the net migratory balance to the mid-period population. Since much of the migration in question involved movements of whole families it is unlikely that the age and sex composition of the migrants was highly atypical; therefore the remaining bias due to migration after this adjustment is undoubtedly small.[1]

Since statistics on migrants by age and sex are seldom adequate, and since migrants are often concentrated in certain age and sex groups it is obvious that a substantial volume of migration may strongly bias the mortality estimates obtained from census survival rates. It should be noted, however, that the availability of certain types of census tabulations may still permit the use of the census survival method by identifying population groups that are more nearly closed to migration than the total population. An example is the calculation of mortality estimates for well-specified linguistic, racial or religious groups that are not affected by migration. A possibly more commonly feasible application may be the use of tabulations of the *native* population classified by age and sex in two consecutive censuses in population where immigration is substantial but out-migration is negligible.

Column 5 of table 15 shows the 1935 population of Turkey adjusted for comparability with the 1945 figures given in column 6. Column 5 was obtained by multiplying through column 4 by 1.0204—the product of 1.0118 (adjustment for territorial coverage) and 1.0085 (adjustment for migration).

(4) *Adjustment for the length of the intercensal period.* Computational convenience and limitations of the data (e.g. lack of classifications by single-year age groups) makes it mandatory to base the calculations on two sets of population figures referring to points of time that are to a very close approximation five, ten or perhaps fifteen years apart. When this is not the case it is necessary to "move" one of the populations involved over time to establish the desired time distance between the censuses. No such need arises in the case of the 1935 and 1945 Turkish censuses that have reference dates of 20 October and 21 October, respectively. When an adjustment *is* needed on this score the simplest procedure to follow is to assume that both the age distribution and the observed intercensal growth rate has been, or will remain, unchanged during the period to be removed from, or added to, the actual intercensal time distance in order to make that distance equal the nearest integer multiple of five years. For the sake of an example assume that the two censuses were taken 8.72 years apart and that the average yearly increase during this time was $r = .022$. Under these circumstances the population registered at the time of the second census in each age group is to be multiplied by the factor of 1.0286—i.e. by $e^{.022 \times 1.28}$; 1.28 being the additional number of years that would have elapsed beyond the actual intercensal time distance had the second census been taken exactly ten years after the first one.[2]

[1] The use of the 1940 population (or the average intercensal population) in the calculation of the correction factor implies that the migration over the decade is assumed to have been approximately evenly distributed.

[2] Where r (.022) has been calculated as follows:

$$8.72\, r = \log_e \frac{P_{t+8.72}}{P_t}.$$

Computational procedure. The essence of the computations is to find a life table (from among the model tables) that, employed to project the 1935 population, produces a 1945 population most consistent with the recorded one. The computational steps are:

(1) Apply the ten-year cohort survival rates given in table I.3 of annex I to the 1935 population distributed by age (table 15, column 5) at various "levels" of mortality.[3] An unnecessarily extravagant procedure would be to project with *all* of the tabulated model tables. In practice, it is sufficient to use a range of mortality levels to produce projections that bracket the recorded numbers above age x in 1945, where x is 10, 15, ..., 50. Levels five to eleven would have sufficed in this example, but thirteen and fifteen have been employed to illustrate the level of mortality implicit in the apparent rate of survival of the older population (over sixty five in 1935). The suggested procedure is to begin with a projection using a first guess of the mortality level, and then make projections at other levels in a spirit of trial and error. Table 16 shows the projections at levels five to fifteen;

(2) The projected populations and the adjusted census populations in 1945 are cumulated (from the "top" down) to obtain figures for the number of females above age x, x = 10, 15, 20, ... etc. (see table 17);

(3) By interpolation, find what level of mortality

[3] If the two censuses were five years apart, the survival rates given in column 7 of table I.1 would be used. If the interval were fifteen years, it would be necessary to calculate values of ${}_5L_{x+15}/{}_5L_x$ in the model life tables.

TABLE 16. THE FEMALE POPULATION OF TURKEY 1945 (THOUSANDS) AS PROJECTED FROM THE ADJUSTED 1935 CENSUS POPULATION WITH VARIOUS "WEST" MODEL LIFE TABLES

	Projected population, 1945					
Age interval	*Level 5*	*Level 7*	*Level 9*	*Level 11*	*Level 13*	*Level 15*
10-14	1,094	1,137	1,173	1,204	1,232	1,260
15-19	1,054	1,072	1,087	1,100	1,112	1,123
20-24	793.7	700.9	712.0	721.7	730.3	738.2
25-29	437.7	447.5	456.1	463.6	470.3	476.2
30-34	565.3	579.9	592.6	603.8	613.8	622.5
35-39	625.5	643.5	659.2	673.2	685.6	696.3
40-44	547.5	564.5	579.5	592.8	604.5	614.7
45-49	427.9	441.8	454.0	464.8	474.4	482.6
50-54	387.2	400.8	412.7	423.2	432.6	440.4
55-59	243.3	253.4	262.4	270.3	277.4	283.2
60-64	269.5	284.3	297.1	308.7	319.2	327.6
65-69	117.6	126.2	134.0	141.0	147.3	152.4
70-74	143.9	158.0	170.8	182.5	193.1	201.4
75 and over	72.23	81.59	90.62	98.76	106.9	113.8

TABLE 17. THE FEMALE POPULATION OF TURKEY 1945 (THOUSANDS) AT AGE x AND OVER, ADJUSTED CENSUS FIGURES, AND PROJECTED WITH VARIOUS "WEST" MODEL LIFE TABLES

	Population x and over in 1945						
		Projected at various mortality levels					
Age x	*Adjusted census figures*	*Level 5*	*Level 7*	*Level 9*	*Level 11*	*Level 13*	*Level 15*
10	6,914	6,779	6,891	7,081	7,248	7,399	7,532
15	5,838	5,685	5,754	5,908	6,044	6,167	6,272
20	4,905	4,631	4,682	4,821	4,944	5,055	5,149
25	4,212	3,838	3,981	4,109	4,223	4,325	4,411
30	3,592	3,400	3,534	3,653	3,759	3,855	3,935
35	2,892	2,835	2,954	3,060	3,155	3,241	3,312
40	2,312	2,209	2,311	2,401	2,482	2,555	2,616
45	1,754	1,662	1,746	1,822	1,889	1,951	2,001
50	1,375	1,234	1,304	1,368	1,424	1,477	1,519
55	939.9	864.5	903.5	954.9	1,001	1,044	1,078
60	720.2	603.2	650.1	692.5	731.0	766.5	795.2
65	370.5	333.7	365.8	395.4	422.3	447.3	467.6
70	245.7	216.1	239.6	261.4	281.3	300.0	315.2
75	112.5	72.23	81.59	90.62	98.76	106.9	113.8

produces a projected population x and over exactly matching the census population in 1945, $x = 10$, 15, 20 etc. For example, the population over ten in 1945 was 6,914,000, the projected populations based on mortality levels seven and nine are 6,891,000 and 7,081,000. The level that would duplicate the 1945 census figure is 7.24. The mortality levels and corresponding values of ${}^0e_0$, ${}^0e_5$ and l_2 are given in table 18;

(4) Select the median level among the first nine in column 2 of table 18, or level 7.98, as the best single estimate of the level of mortality among Turkish females, 1935-1945;

TABLE 18. INDICES OF MORTALITY IN "WEST" MODEL LIFE TABLES CORRESPONDING TO CENSUS SURVIVAL RATES FOR TURKEY (1935-1945) FROM AGE x AND OVER TO AGE $x+10$ AND OVER

Age x (1)	*Level of mortality* (2)	${}^0e_0$ (3)	${}^0e_5$ (4)	l_2 (5)
0	7.24	35.62	46.98	.7332
5	8.09	37.73	48.32	.7523
10	10.37	43.41	51.86	.8002
15	10.80	44.50	52.53	.8090
20	7.98	37.44	48.13	.7497
25	5.96	32.39	44.90	.7010
30	7.02	35.05	46.62	.7282
35	7.21	35.53	46.92	.7324
40	9.24	40.61	50.13	.7776
45	8.42	38.54	48.83	.7595
50	10.44	43.61	51.98	.8018
55	7.31	35.79	47.09	.7348
60	7.56	36.40	47.47	.7403
65	13.81	54.06	58.17	.8772

(5) An estimate of the average intercensal crude death rate for females can be obtained by calculating the life table mortality rates corresponding to level 7.98 (shown in column 2 of table 19)[4] and multiplying these rates with the average intercensal population (or the estimated mid-period – 1940 – population) given in column 3 of the same table. The average population is calculated as the mean of the reported 1935 and 1945 populations, after adjustment for ages reported as unknown (columns 4 and 6 in table 15, respectively). The result of this operation is the average yearly number of deaths by age in the intercensal period, shown in column 4 of table 19. The ratio of the average yearly number of all deaths and the average intercensal population gives the estimated average crude death rate, d. In this example $d = (231.1/8783) = .0263$;

(6) An estimate of the increase of the female population in Turkey from 1935 to 1945 is provided by the ratio of the total populations in these years after adjustments for migration and changing territorial coverage (columns 5 and 6 in table 15), i.e., by the ratio of $(9344/8388) =$ 1.1140. The implied annual rate of natural increase is

$$r = \frac{\log_e 1.1140}{10} = .0108;$$

TABLE 19. CALCULATION OF THE AVERAGE FEMALE CRUDE DEATH IN TURKEY IN THE PERIOD 1935-1945 CORRESPONDING TO THE MEDIAN LEVEL OF MORTALITY IMPLIED BY CENSUS SURVIVAL RATES

Age x (1)	*Death rates per thousand at age x in median life table level 7.98* (2)	*Mean population 1935-1945 (thousands) (from cols. 4 and 6 in table 15* (3)	*Average annual deaths at age x (thousands) (col.2 × col.3)* (4)
0-4	79.57	1,245	99.06
5-9	7.68	1,189	9.132
10-14	5.97	912.6	5.448
15-19	7.92	710.4	5.626
20-24	9.99	667.8	6.671
25-29	11.25	672.2	7.562
30-34	12.75	672.8	8.578
35-39	14.10	545.5	7.692
40-44	15.33	517.4	7.932
45-49	16.93	347.6	5.885
50-54	22.27	410.4	9.140
55-59	29.00	207.9	6.029
60-64	43.16	324.2	13.99
65-69	59.93	115.1	6.898
70-74	89.77	129.9	11.66
75 and over	171.47	115.5	19.80
TOTAL	26.31[a]	8,783	231.1

[a] (Sum of col. 4)/(sum of col. 3).

(7) An estimate of the average annual female birth rate is obtained as the sum of the estimated death rate and rate of natural increase already calculated:

$$b = .0263 + .0108 = .0371;$$

(8) An estimate of the male birth rate may be obtained as the product of the female birth rate, the sex ratio at birth, and the ratio of the average intercensal female population to the male population. Assuming that the sex ratio at birth was 1.05, and estimating the mid-period male population (analogously to the estimation of the female population) as 8,692,000, we have

$$b(\text{males}) = .0371 \times 1.05 \times \frac{8783}{8692} = .0393 .$$

The average intercensal increase of the male population was .0153, as estimated from census figures adjusted for migration and boundary changes. This gives a male death rate of $.0393 - .0153 = .0240$.

Comments. In calculating the crude death rate the age specific death rates from the estimated life table are weighted by an age distribution that is obviously distorted by misreporting of age. Thus the resulting distribution of deaths is also erratic and its detailed features should not be accepted as a valid description of that distribution. The effect of age distortions on the calculated total number of deaths can be expected to be much smaller since the errors to a large extent are compensating ones. Never-

[4] The death rate for age 0-4 may be obtained from table I.1 as $(l_0 - l_5)/({}_1L_0 + {}_4L_1)$. The death rate for the population aged 75 and over is calculated as l_{75}/T_{75}.

theless the analyst should consider the potential bias due to this source. In particular if the proportion under age five is under-reported (either because of omission of young children or overestimation of their ages), the resulting estimate of d (and, given the intercensal r, the estimate of b) will be downward biased. In the given example, however, age-misreporting does not appear to have affected the estimate of d, once a life table has been obtained. When weighting of the m_x values of that table has been done by an intercensal population adjusted for age-mis-reporting (by means of a procedure not discussed in this *Manual*) the resulting d was .0264, instead of .0263 obtained above.

The estimate of childhood mortality is derived in this method not from the basic data themselves, but is a simple extrapolation from the estimated adult mortality (or mortality over age five) *via* the "West" model life tables. If the pattern of mortality characterizing this family of life tables is not valid for Turkey, the estimated childhood mortality, hence the derived d and b are accordingly biased. This point is discussed in section A.1.*a* of chapter IV. If, for example, the "South" family of model life tables derived from the experience of other Mediterranean countries more nearly approximates the (unknown) true pattern of Turkish mortality, the death and birth rates may be as much as .006 higher than the estimates given above. Apart from the argument of geographical, and to some extent cultural, closeness to countries known to be characterized by "South" mortality, there exists some evidence from recent surveys that the age pattern of Turkish mortality is indeed more "southern" than "western".

The above remarks suggest that the crude birth rate just derived (.0382 for the population as a whole) is lower than the actual level. It should be noted however that during the period in question the actual level of the birth rate itself must have been appreciably lower than its "normal" level. There are two reasons supporting this assumption. First, wartime conditions, such as extensive mobilization in the early 1940s probably have depressed fertility. Second, the relative size of the cohorts in the prime child-bearing ages was much below "normal" during the period because of depressed fertility and unusually high mortality due to the Balkan wars and to the first World War and its troubled aftermath in Turkey.

B. Estimation of fertility and mortality by stable population analysis

The method of deriving estimates of fertility and mortality from records of the age distribution and from information on the rate of growth under conditions when the population may be considered approximately stable is discussed in section B of chapter I. In the present section applications of this method are illustrated by three examples based on data collected in censuses in England and Wales (1871), India (1911) and Brazil (1950). These censuses exemplify three different situations with respect to the quality of the basic data, in particular with respect to the quality of data concerning age.

1. *England and Wales*, 1871

In section B of chapter I, it was shown that a stable population based on the 1871-1881 English life table and on the rate of natural increase during the same period matches very closely the age distribution as actually recorded in the census of 1881. Conversely, an index of the recorded age distribution and the rate of growth were shown to define a model stable population the parameters of which provide an excellent approximation of various demographic characteristics of the population, such as the birth rate or the expectation of life at age zero. However since the values of these parameters were known from direct statistical observations there was little justification of applying stable methods of estimation apart from proving the power of the technique under conditions when age reporting is highly reliable. The mechanics of the application of this method are illustrated in the following paragraphs also by using English data, but under somewhat less artificial circumstances. Notably stable estimates of population parameters for the period preceding 1871 will be derived from the age distribution as reported in the 1871 census and from the rate of growth between 1861 and 1871. No official life table has been prepared for the sixteen-year period preceding 1871, and the registration of births during that time is known to be slightly more defective than in the 1870s–birth statistics became virtually complete only after legislation in 1874 placed the responsibility for registering births upon the parents.[5]

Conditions for applying the method. Whether alternative methods of estimation are available or not, stable estimation should be attempted only if a case for the existence of stability with respect to the relevant demographic conditions can be established. Preferably such a case should rest on direct evidence, in particular on the constancy of the age distribution and of the rate of population growth. Examination of the distribution by age in the decennial censuses from 1841 through 1871 provides a confirmation of approximately stable conditions in England and Wales during that period (and attests to the good quality of age reported) although masculinity ratios between ages 30-44 in 1871 are noticeably smaller than the ones reported in earlier censuses. This appears to reflect the effect of excess male outmigration and suggests that the male population in this case is a less satisfactory basis for stable estimates.[6] As to the rate of growth, the slight fluctuations in the intercensal rates of increase during the three decades preceding 1871 that show no trends are reassuring but, again, cannot be taken at face value since the population was subject to net outmigration during the period. Explicit consideration of the effect of migration is clearly necessary. The average

[5] Cf. D.V. Glass, "A Note on the Under-Registration of Births in Britain in the Nineteenth Century", *Population Studies*, vol. V, No. 1 (July 1951), pp. 70-88.

[6] No illustration of these points is offered here. For a convenient source see the historical series in General Register Office, *Census 1961, England and Wales, Age, Marital Condition and General Tables* (London, 1964), pp. 30-32.

rates of intercensal growth before and after correction for net outmigration are given in table 20.[7]

Table 20 shows that there was little change in the natural rate of growth over the thirty-year period prior to 1871 and the male and female rates were reasonably close to each other. (Perfect stability would imply identical growth rates for the two sexes). The effect of the correction for migration on the female growth rate is moderate, but is much less so for the male population. On the basis of the preceding observations estimation of fertility will be derived from the female age distribution and growth rate only.

TABLE 20. AVERAGE ANNUAL RATE OF INCREASE BY DECADES BETWEEN 1841 AND 1871 FOR EACH SEX CALCULATED FROM CENSUS FIGURES AS REPORTED ("INTERCENSAL RATE OF GROWTH") AND AFTER CORRECTION FOR MIGRATION ("NATURAL RATE OF GROWTH"), ENGLAND AND WALES

Period	Males: Intercensal rate of growth	Males: Natural rate of growth	Females: Intercensal rate of growth	Females: Natural rate of growth
1841-1851	.0124	.0131	.0120	.0121
1851-1861	.0108	.0140	.0118	.0128
1861-1871	.0124	.0138	.0125	.0131

TABLE 21. FEMALE POPULATION BY AGE, ENGLAND AND WALES, 1871[a]

Age	Population (thousands)	Population (percentage)
0-4	1,534.8	13.17
5-9	1,355.6	11.64
10-14	1,203.5	10.33
15-19	1,095.7	9.40
20-24	1,052.8	9.04
25-29	937.3	8.04
30-34	813.7	6.98
35-39	700.5	6.01
40-44	639.7	5.49
45 and over	2,319.7	19.90
TOTAL	11,653.3	100.00

[a] *Source*: See foot-note 1 to the present chapter.

Required basic data, Apart from numerical evidence necessary to establish the case for the applicability of the stable method, and, in the present instance, to make a correction for migration, the required basic data are a five-year distribution of the population by sex in one census, and a count of the total population by sex at an earlier point in time to provide a rate of growth. The latter information was given in table 20. Table 21 gives the female age distribution in England and Wales up to age forty-five. It is not suggested to go beyond that age for purposes of stable estimation.

Computational procedure. (1) Obtain values of $C(x)$: proportions up to age $x(x, 5, 10, ..., 45)$ from table 21. These cumulated proportions after rounding are shown in col. 2 of table 22.

(2) From table II in annex II find the parameters of the female stable populations characterized by the $C(x)$ values on the one hand, and by the rate of natural increase for the decade preceding the census (.0131), on the other hand. This operation may be conveniently executed in the following steps:

(*a*) Given the value of $C(5)$ as reported, select two stable populations each having the required growth rate (i.e., by interpolating between stable populations tabulated for $r = .010$ and $r = .015$ to get the female growth rate of .0131) and one of the levels of mortality for which model stable populations are given in table II (i.e., levels 1, 3, ..., 23). Specifically the mortality levels should be so chosen (by means of a rough process of trial and error) that the $C(5)$ values in the resulting two model populations just bracket the $C(5)$ value in question, i.e., the ogive at age five in one of them should be just higher, and in the other just lower, than the reported value. The proper levels in this instance are levels nine and eleven. Columns 3.*a* and 3.*b* show the values of $C(5)$ in these stable populations and also the parameters (such as the birth rate) for which estimates are sought;[8]

(*b*) Repeat the above procedure for other values of $C(x)$, i.e., for $x = 10, 15, ..., 45$, using additional columns if any of the reported $C(x)$s are not bracketed by the ogives of stable populations previously calculated. In the given example no such need arises since none of the $C(x)$ values imply a mortality level higher than level eleven, or lower than level nine, given the growth rate of .0131;

(*c*) For each value of x find the interpolation factors that would be necessary to obtain the reported $C(x)$–shown in col. 2–from the corresponding values in cols. 3.*a* and 3.*b*. For example, the reported proportion up to age 10 is .248 which may be expressed as a weighted average of the figures for $C(10)$ in cols. 3.*a* and 3.*b*; specifically as $.27 \times .256 + .73 \times .245$ (cf. annex VI). Applying the same interpolation factors to other population parameters calculated for the stable populations in cols. 3.*a* and 3.*b*, such as the birth rate, one obtains parameters of the stable population defined by the given r and the observed $C(x)$. For example the birth rate corresponding to $r = .0131$ and $C(10) = .248$ is $.27 \times .0365 + .73 \times .0329 = .0339$. The results of these calculations are given in cols 4.*a* through 4.*e*. Note that one of the parameters calculated is the population death rate. Once the birth rate is calculated the death rate may of course be obtained by simply subtracting the specified growth rate from the birth rate.

[7] The corrections are based on estimates of migration prepared by Glass (*op. cit.*, pp. 85-86, "method c"). The estimated intercensal net balance in the decade preceding a given census was added to the census population and then the intercensal increase was calculated using this population and the uncorrected population a decade earlier.

[8] One of the parameters calculated is the gross reproduction rate associated with $\bar{m} = 32.1$ (the basis for this particular value of $\bar{m}$ is discussed later). Since table II contains only values of GRR for $\bar{m} = 27$, 29, 31 and 33, it is necessary first to calculate two GRR values that bracket the GRR with the correct value of $\bar{m}$, and to obtain this latter quantity by an additional step of interpolation.

TABLE 22. DERIVATION OF STABLE POPULATION ESTIMATES OF FERTILITY AND MORTALITY BASED ON A REPORTED AGE DISTRIBUTION AND THE RATE OF GROWTH. ENGLAND AND WALES, FEMALES, 1871

Age x	C(x) (proportion up to age x)	Values of C(x) and various parameters in female stable populations with r = .0131		Values of various parameters in female stable populations with C(x) as shown in col. 2 and with r = .0131				
		Level 9	Level 11	Birth rate	Death rate	Level of mortality	$^0e_0$	GRR ($\bar{m}$ = 32.1)
(1)	(2)	(3.a)	(3.b)	(4.a)	(4.b)	(4.c)	(4.d)	(4.e)
5	.132	.139	.131	.0334	.0202	10.8	44.4	2.37
10	.248	.256	.245	.0339	.0208	10.5	43.6	2.41
15	.351	.363	.349	.0334	.0203	10.7	44.3	2.38
20	.445	.461	.444	.0331	.0200	10.9	44.7	2.35
25	.536	.548	.530	.0341	.0210	10.3	43.3	2.42
30	.616	.626	.607	.0346	.0215	10.1	42.6	2.46
35	.686	.695	.677	.0347	.0216	10.0	42.5	2.46
40	.746	.756	.739	.0344	.0213	10.2	42.9	2.44
45	.801	.810	.794	.0345	.0214	10.1	42.8	2.45
Birth rate		.0365	.0329					
Death rate		.0234	.0198					
$^0e_0$		40.0	45.0					
GRR ($\bar{m}$ = 31)		2.52	2.28					
GRR ($\bar{m}$ = 33)		2.65	2.39					
GRR ($\bar{m}$ = 32.1)		2.59	2.34					

(3) Ideally each combination of C(x) and r for a given sex should define the same stable population: the parameters of this model then could be accepted as valid for the actual population of that sex. In practice however a more or less tightly clustered series of stable populations are determined by the various pairs of C(x) and r. The procedure of selecting a single best estimate (or selecting estimates located within a narrower range than the range of all obtained stable estimates) depends on the nature of identifiable errors in the data, especially with respect to age misreporting. In the given example the consistency of the estimates shown in cols 4.*a* through 4.*e* is gratifyingly high (see figure XX for a graphical representation of the series of birth rates obtained). This finding tends to confirm the original assumption of stability and the good quality of age reporting. Given these circumstances, and lacking information that would single out some reported C(x) values as particularly reliable, or relatively defective, the selection of the stable population with an ogive median among those considered provides the best available choice. To find this population, rank the estimated nine birth rates according to their absolute values, and select the intermediate (the fifth largest) in this series. In the given example the median stable population is the one associated with the reported C(25), giving the following estimates for the female population: birth rate = .0341, death rate = .0210, expectation of life at birth = 43.3 years, and gross reproduction rate = 2.42.

(4) Estimates for the male population and for the population as a whole may be obtained from the parameters calculated for the females plus the knowledge of the sex ratio at birth (male births/female births) and the sex ratio (males/females) in the total population. The ratio of registered male births to female births in the five years preceding 1871 was 1.041. The number of males enumerated in 1871 was 11,058.9 thousands. (The female population was given in table 21.) The male birth rate then is calculated as

$$\text{female birth rate} \times \frac{\text{sex ratio at birth}}{\text{sex ratio of the population}} = .0341 \times \frac{1.041}{.949} = .0374 .$$

The total birth rate can be obtained as a population-weighted average of the rates calculated separately for the two sexes or directly from the female birth rate as

$$\text{female birth rate} \times \frac{\text{female population}}{\text{total population}} \times (1+\text{sex ratio at birth}) = .0341 \times .513 \times 2.041 = .0357 .$$

(5) By subtracting the appropriate rate of natural increase from the estimated male birth rate and total birth rate death rates for males (.0374 − .0138 = .0236) and for the total population (.0357 − .0134 = .0223) are obtained.

(6) An estimate of the male expectation of life at birth (and/or any desired life table parameter) is obtained from table II by finding the level of mortality in the male model stable population having the estimated male death rate (.0236) and the male rate of natural increase (.0138). The level is 10.0, implying an $^0e_0$ of 39.7 years. Since mortality in England is known to be well described by the model "West" life tables, the level of mortality in this instance may be estimated also by simply assuming that the

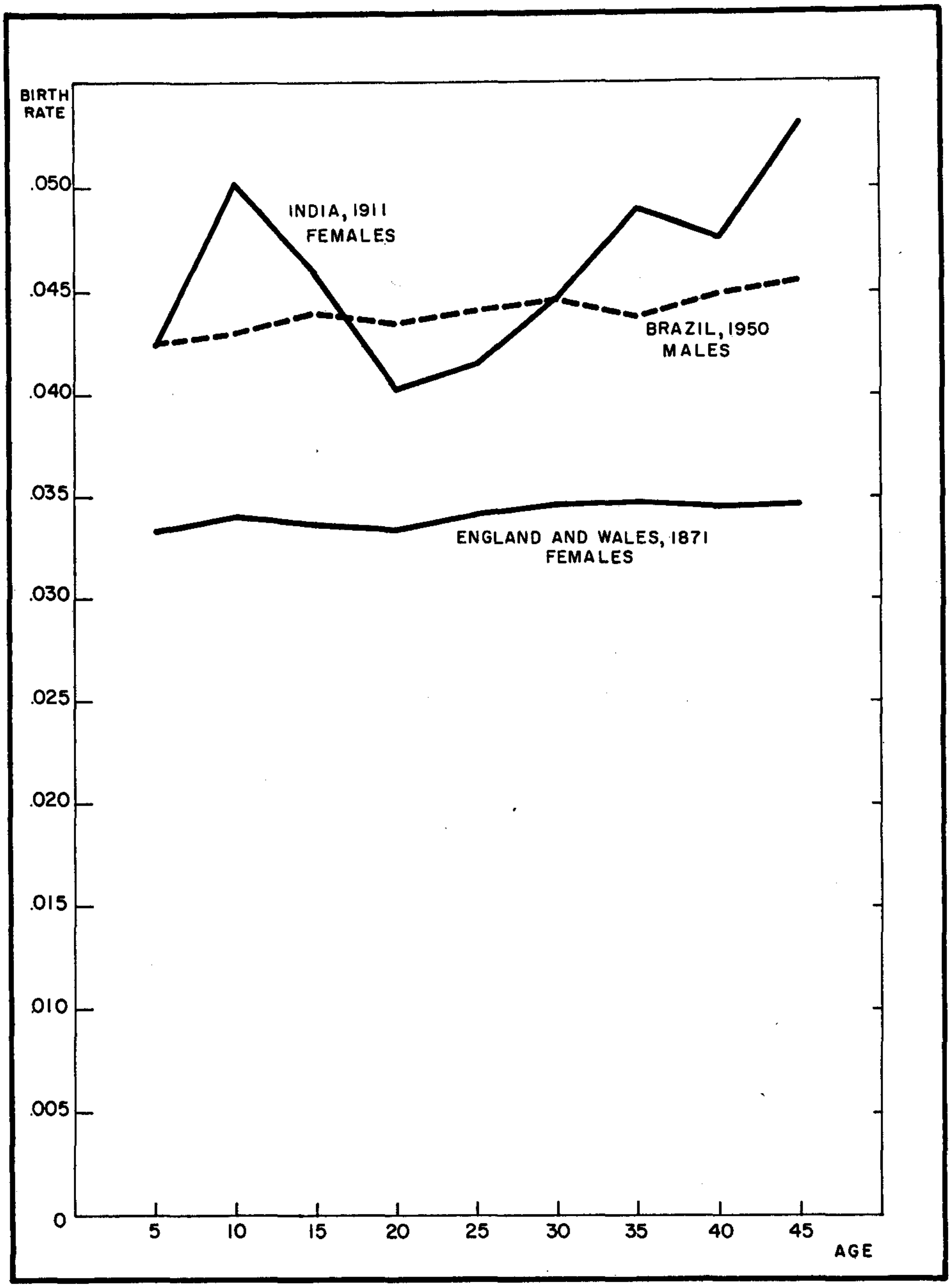

Figure XX. Stable population estimates of the birth rate derived from reported proportions up to age *x*—C(*x*)—in censuses of England and Wales, India and Brazil for the year and sex as indicated and from the average rate of natural increase for the same sex during the ten-year period preceding each census

relation of male mortality is the same as in the tables shown in annex I, i.e., that the level of male mortality is 10.3, as is for the females, hence that the male ${}^{0}e_0$ is 39.9.

(7) Given the sex ratio at birth an estimate of total fertility can be obtained from the estimated GRR ($\bar{m}$ = 32.1): TF = 2.42 × 2.041 = 4.94 children per woman.

Estimation of $\bar{m}$. The calculation of estimates of the gross reproduction rate as described above presupposes the existence of an estimate of the mean age in the schedule of the age specific fertility rates ($\bar{m}$) prevailing in the given population. Such rates are not available for England and Wales for the period in question. Section B.5 of chapter I describes two methods for indirect estimation of $\bar{m}$; due to lack of data on children ever born only the first of these methods—based on the reported proportions married—may be applied here. An illustration of the computation is given in table 23. The standard age pattern of marital fertility (applicable for populations whose birth control practices are negligible) shown in col. 3 is taken from table 1. Note that the absolute magnitudes of the hypothetical age specific fertility rates calculated in col. 4 have no practical significance. What is relevant for the problem at hand is merely the age pattern of these imputed rates: the mean age of the fertility schedule is computed as the average of the central ages in each age interval (col. 5) weighted by the entries in col. 4.

TABLE. 23 CALCULATION OF $\bar{m}$ FROM REPORTED PROPORTIONS MARRIED AND FROM THE STANDARD AGE PATTERN OF MARITAL FERTILITY RATES, ENGLAND AND WALES, 1871

Age interval (1)	*Proportion of married females* (2)	*Standard marital fertility rates* (3)	*Hypothetical fertility rates (col. 2 × col. 3)* (4)	*Median age* (5)	*Col. 4 × col. 5* (6)
15-19	.032	1.178[a]	.0377	17.5	.66
20-24	.343	1.000	.3430	22.5	7.72
25-29	.624	.935	.5834	27.5	16.04
30-34	.735	.853	.6270	32.5	20.38
35-39	.766	.685	.5247	37.5	19.68
40-44	.758	.349	.2645	42.5	11.24
45-49	.740	.051	.0377	47.5	1.79
			2.4180		77.51

Hence, $\bar{m} = \frac{77.51}{2{,}418} = 32.1$

[a] 1.2 −.7 (proportion married at age 15−19) = 1.2 −.7 × .032 = 1.178.

Implicit in the above procedure is the assumption that all births occur in marriage. However the proportion of illegitimate births in England at the time amounted to some 6 per cent of all births. It would be possible to extend the calculation just outlined to obtain a hypothetical age pattern of fertility that takes into account illegitimate births as well, e.g., by assuming that illegitimate fertility rates by age of mother had the same pattern as was recorded in Sweden in the 1860s. Calculations show that the $\bar{m}$ resulting from such assumptions would differ little from the value obtained in table 23–it would be less by not more than .2 year.

Comments. Some of the estimates obtained above may be compared with data from vital registration or other estimates. For example, with respect to the birth rate for the total population we have:

Stable estimate	.0357
Vital registration	
1866-1870	.0351
1861-1865	.0350
Vital registration corrected for under-registration of births[9]	
1866-1870	.0357
1861-1865	.0358

Official life tables are available for the periods 1838-1854 and 1871-1880, but not for the sixteen years preceding the census for which the stable estimates derived above may be considered as relevant. The stable life table estimates do, however, suggest a plausible trend of mortality change when compared with the official life tables mentioned. In terms of $^0e_0$ the following comparison can be made:

	Life table for England and Wales, 1838-1854	*Stable estimate from 1871 census*	*Life table for England and Wales, 1871-1880*
Females ...	41.85	43.3	44.62
Males	39.91	39.9	41.35

According to vital registration for the sixteen-year period from 1855 to 1870 the average death rate for the total population was .0223. This figure is identical to the stable population estimate of the same quantity derived from the age distribution in the census of 1871.

2. *India, 1911*

Another illustration of the technique of obtaining vital rates by the stable method based on a census record of the age distribution and an intercensal growth rate is offered in this section using Indian statistics, notably $C(x)$ from the census of 1911 and r for the period 1901-1911. In contrast to the previous example, information on age distribution and population growth constitutes virtually the only valid basis for establishing vital rates for India relating to the period in question. The usefulness of that information is fortunately greatly enhanced by the applicability of stable population analysis. The argument supporting the assumption of stability rests mainly on two considerations. First, the series of decennial censuses in India up to 1911 shows a remarkable degree of stability. This point is sufficiently well-illustrated in figure XI. To be sure, the detailed shape of the reported age distributions is far from what would result from sustained past constancy of vital rates. But the fact that the marked peculiarities of the age distribution are reproduced census after census at the same *age* (as opposed to the same *cohort*) conclusively proves that the explanation for these peculiarities lies in an essentially unchanged pattern of age–misreporting rather than in violent past deviations from stable levels of fertility and mortality.

Second, the series of intercensal growth rates preceding 1911 lack any detectable trend away from the horizontal; rather ups are followed by downs in regular succession.[10]

Constancy of age distribution and fluctuating growth rates are consistent with indirect or qualitative knowledge on the main features of the demographic situation in pre-1911 India. Such features are frequent short-term changes in mortality conditions but the absence of lasting improvement or deterioration in the chances of dying; a sustained high level of fertility explained by the lack of contraceptive practices and by quasi-universal and early marriage; and, finally, the essential closedness of the population with respect to external migration.

Stable conditions notwithstanding, no high precision can be expected from estimates derived by stable analysis in the Indian case, primarily because of the defects in age–reporting mentioned earlier. Nevertheless, identification of systematic deviations from the expected stable distributions, as discussed in detail in section B.*c* of chapter I, permits an interpretation of even seemingly inconsistent series of stable estimates, thus considerably reducing the apparent range of uncertainty. As was

[9] Glass, *op. cit.*, p. 85.

[10] See Davis, *op. cit.*, pp. 27-28 and 85.

TABLE 24. DERIVATION OF STABLE POPULATION ESTIMATES OF FERTILITY AND MORTALITY BASED ON A REPORTED AGE DISTRIBUTION AND THE RATE OF GROWTH. INDIA, 1911, FEMALES

Age x (1)	C(x) (proportion up to age x) India, 1911, females (2)	Values of C(x) and various parameters in female stable populations with r = .0073 and levels of mortality as indicated: Level 1 (3.a)	Level 3 (3.b)	Level 5 (3.c)	Level 7 (3.d)	Values of various parameters in female stable populations with C(x) as shown in col. 2 and with r = .0073: Birth rate (4.a)	Death rate (4.b)	Level of mortality (4.c)	$^0e_0$ (4.d)	GRR ($\bar{m}$ = 28.2) (4.e)
5	.141		.151	.139		.0421	.0348	4.7	29.2	2.68
10	.276	.296	.272			.0503	.0430	2.7	24.2	3.19
15	.375		.383	.360		.0457	.0384	3.7	26.8	2.91
20	.455			.457	.435	.0403	.0330	5.2	30.4	2.57
25	.548		.573	.546		.0413	.0340	4.9	29.6	2.63
30	.640		.653	.625		.0449	.0376	3.9	27.3	2.85
35	.725	.754	.724			.0487	.0414	2.8	24.8	3.09
40	.782		.784	.759		.0478	.0405	3.2	25.4	3.03
45	.847	.861	.837			.0531	.0458	2.2	22.9	3.38
Birth rate		.0597	.0484	.0408	.0353					
Death rate		.0524	.0411	.0335	.0280					
$^0e_0$		20.0	25.0	30.0	35.0					
GRR ($\bar{m}$ = 27)......		3.67	2.98	2.53	2.21					
GRR ($\bar{m}$ = 29)		3.88	3.13	2.64	2.30					
GRR ($\bar{m}$ = 28.2) ...		3.80	3.07	2.60	2.26					

shown in chapter I—see in particular figures VIII, IX and X—Indian age distributions are characterized by what has been described in this *Manual* as the "African-South Asian" pattern. Rules for analysing such distributions were set forth in section B.4 in chapter I. According to these rules it is preferable to use only the female population as a basis for estimation as far as the reported age distribution is concerned. Table 24 shows the derivation of various population parameters from the 1911 female age distribution in India (col. 2) and from the 1901-1911 average female growth rate (r = .0073).[11] The computational procedures underlying this table are exactly analogous to those used and explained in connexion with table 22 above. Note that the intermediate step of calculating parameters for stable populations with the proper growth rate but with approximate ("bracketing") levels of mortality requires in this instance the use of more than two mortality levels (see cols. 3.*a* through 3.*d*) owing to the fact the reported age distribution is less consistently close to one single stable distribution than was the case in the previous example.

The mean age of the fertility schedule ($\bar{m}$ = 28.2) used in the computation was estimated from the standard marital fertility schedule shown in table 1 and from the proportions of married females in India, 1911. The latter, for five-year age groups, were as follows :[12]

15-19	*20-24*	*25-29*	*30-34*	*35-39*	*40-44*	*45-49*
.818	.902	.882	.814	.742	.597	.522

[11] Both the age distribution and the growth rate are for the current (post-partition) territory of India. They were calculated by Mr. S. B. Mukherjee in his "A Study of the Vital Rates in India and West Bengal" (unpublished manuscript, Princeton, 1965) which he kindly made available to the authors.

[12] See the preceding foot-note for the source of these data.

The procedure of calculation was the same as that discussed in connexion with table 23 above.

Inspection of the sequence of the derived birth rates (column 4.*a* in table 24), also reproduced in figure XX, shows the same characteristic pattern as was found from direct comparisons of the reported population with model stable populations (cf. figures VIII and IX). This suggests that such comparisons are not necessarily required for the identification of the general character of age-reporting errors. Once a case for applying the stable method has been established the analysis may proceed directly to the calculation of birth rates and other parameters implied by the various pairs of C(x) and r. A judgement on the pattern of age-misreporting, hence on the rules of estimation to be applied, then can be based on the results of this calculation, in particular on the sequence of the birth rates obtained for x = 5, 10, ..., 45. In the present case the rules call for the acceptance of the parameter values associated with C(35) as the best single estimates. Given the female rates, parameter values for the male population and for the population as a whole are to be obtained in the same fashion as was shown in the preceding example, i.e., by using the available information on average intercensal growth (in this case .0082 for males and .0077 for the total population), on the sex ratio of the population (1.037), and assuming—in the absence of information to the contrary—that the sex ratio at birth was 1.05. The male expectation of life and/or other life table indices are determined by finding the level of mortality in the male stable population having the reported male growth rate and the death rate as derived earlier. Some of the stable estimates resulting from these calculations are summarized in table 25.

Naturally all figures in table 25 are to be regarded as rough approximations. Yet, on the basis of knowledge

TABLE 25. STABLE POPULATION PARAMETERS FOR INDIA, 1911, DERIVED FROM THE FEMALE AGE DISTRIBUTION, THE FEMALE AND MALE GROWTH RATES, AND THE SEX RATIO OF THE POPULATION AS REPORTED; AND BY ASSUMING THAT THE SEX RATIO AT BIRTH IS 1.05

	Females	*Males*	*Total population*
Birth rate	.0487	.0493	.0490
Death rate	.0414	.0411	.0413
Level of mortality	2.8	4.0	—
$^{0}e_0$	24.8	25.3	—
GRR ($\bar{m}$ = 28.2)	3.09	—	—
Total fertility	6.33	—	—

of the pattern of age–misreporting it is possible to assert with a high degree of certainty that the female birth rate was higher than .046—the estimate associated with C(15). Furthermore, the value derived from C(35) is strongly supported by the only slightly higher estimate (.050) implied by C(10) and *r*. Most likely this latter figure is, or is close to, what may be considered a fair upper estimate of the birth rate. These statements are qualified by the fact that there is no direct evidence confirming the validity of the "West" pattern of mortality in the Indian case. Use of alternative stable population families in the above calculations would have typically resulted in higher estimates of the birth and death rates, hence in higher estimates of total fertility and lower expectation of life. As to the relation of male and female mortalities, the strong masculinity of the population—demonstrated by the other Indian censuses as well—conclusively indicates that this particular relation incorporated in the model life tables is not duplicated in India.

3. *Brazil, 1950*

Using age distribution data from the Brazilian census of 1950 jointly with the rate of growth between 1940 and 1950 to derive stable estimates exemplifies the application of the stable method under conditions when age–reporting is typically "Latin American" in its characteristics. The applicability of the method in this instance is supported by somewhat less satisfactory evidence than in the two preceding examples because of the lack of an extended series of previous censuses of reasonably good quality. Nevertheless the case for assuming stability is convincing. It is based on the close similarity of the 1940 and 1950 age distributions; on high and essentially identical levels of fertility as indicated by 1940 and 1950 census reports on children ever born; on the relative unimportance of international migration; and on the little or no change in mortality prior to 1950 as evidenced by reports on proportions of children surviving in the 1950 and 1940 censuses.

The Latin American character of the pattern of age–misreporting is revealed by comparing the actual age distributions to model stable distributions, or, more directly, by calculating estimates of the birth rate for males and females from C(*x*) and *r*. As a result the basic stable analysis is to be limited to the male population. Table 26 shows the parameters implied by the male age distribution (column 2) and the male growth rate (r = .0232). The computations underlying this table were explained above in connexion with table 22 (cf. also table 24). Naturally, the *male* model stable populations of annex II were utilized in this case.

TABLE 26. STABLE POPULATION ESTIMATES OF FERTILITY AND MORTALITY BASED ON THE AGE DISTRIBUTION OF THE MALE POPULATION OF BRAZIL AS REPORTED IN THE CENSUS OF 1950 AND ON r = .0232, THE ANNUAL RATE OF GROWTH OF THAT POPULATION IN THE 1940-1950 INTERCENSAL PERIOD

Age x	*C(x) (proportion up to age x) Brazil, 1950, males*	*Values of various parameters in male stable populations with C(x) as indicated in column 2 and with r = .0232*			
		Birth rate	*Death rate*	*Level of mortality*	$^{0}e_0$
(1)	*(2)*	*(3.a)*	*(3.b)*	*(3.c)*	*(3.d)*
5	.164	.0422	.0190	12.1	45.0
10	.302	.0430	.0198	11.7	43.9
15	.424	.0438	.0206	11.3	42.9
20	.527	.0436	.0204	11.4	43.2
25	.619	.0441	.0209	11.1	42.4
30	.698	.0447	.0215	10.9	41.8
35	.760	.0436	.0204	11.4	43.2
40	.819	.0447	.0215	10.9	41.8
45	.867	.0456	.0224	10.5	40.9

The series of male birth rates and other parameters given in table 26 are located within rather narrow limits. The median in the series, which is the best single estimate, is associated with C(15). Assuming a sex ratio at birth of 1.05, and considering that the sex ratio of the population (as reported by the census) was .9933, the female birth rate is obtained by multiplying the male birth rate by the ratio .9933/1.05. The birth rate for the whole population is obtained as

$$\frac{\text{male birth rate}}{\text{sex ratio at birth}} \times \frac{\text{male population}}{\text{total population}} \times (1+\text{sex ratio at birth}).$$

Death rates are calculated by subtracting the rates of average intercensal growth (.0238 for females and .0235 for the total population) from the estimated birth rates.

The value of $\bar{m}$ necessary to obtain an estimate of the (female) gross reproduction rate can be estimated by both methods suggested in chapter I, section B.5. However the method of applying a standard marital fertility schedule to the proportions married among females 15-49 (which in this case yields an estimated $\bar{m}$ of 28.8 years) is open to the objection, serious in the case of Brazil, of ignoring the fertility of women reported as single but living in *de facto* unions. Thus the estimate obtained from the relation $\bar{m} = 2.25\ P_3/P_2 + 23.95$ is to be preferred. The value of P_3/P_2—the ratio of children ever born per woman 25-29 and 20-24— was 2.289 according to the 1950 census. Hence we have $\bar{m}$ = 29.1. The female gross reproduction rate, as well as the female expectation of life and other parameters, then can be obtained by reading these values in a female stable population determined by any two of the parameters previously calculated, such as the female death rate and the female rate of growth. Table 27 summarizes the main results.

TABLE 27. STABLE POPULATION PARAMETERS FOR BRAZIL, 1950, DERIVED FROM THE MALE AGE DISTRIBUTION, THE MALE AND FEMALE GROWTH RATES, AND THE SEX RATIO OF THE POPULATION AS REPORTED; AND BY ASSUMING THAT THE SEX RATIO AT BIRTH IS 1.05

	Males	*Females*	*Total population*
Birth rate	.0438	.0414	.0426
Death rate	.0206	.0176	.0191
Level of mortality	11.3	12.0	—
$^0e_0$	42.9	47.5	—
GRR ($\bar{m}$ = 29.1)	—	2.83	—
Total fertility	—	5.80	—

As was the case in the previous example no vital statistics are available with which these estimates could be confronted. In view of the high consistency of the values implied by the various C(x)s, the major uncertainty with respect to the goodness of the estimates originates, once again, in the choice of the mortality pattern underlying the model stable populations utilized: the "West" pattern— and in particular the early childhood mortality implied by a given adult mortality in that pattern— may or may not be a close approximation of Brazil's actual experience.[13] The application of the census survival method (cf. the discussion in section A in this chapter and in chapter 1) for the Brazilian male population yields an $^0e_0$ value of 42.4 that appears to confirm the validity of the mortality estimate shown in table 27 (hence, given the stable age distribution, the validity of the birth rate estimate). But this is not pertinent to the problem stated above, since both methods have essentially the same weakness in estimating childhood mortality. Unlike in the case of India, however, census information on child survival rates in Brazil supplies a basis for a direct estimation of child mortality thus permitting a check on, and improvement of, the estimates shown in table 27. This topic will be taken up in chapters VII and VIII below.

C. ESTIMATION OF FERTILITY AND MORTALITY BY STABLE POPULATION ANALYSIS WHEN THE POPULATION IS QUASI-STABLE

The examples given in the previous section demonstrated the technical details of extracting population estimates from information on age distribution and growth under conditions of approximate constancy of fertility and mortality. When mortality has been declining, but other requisites of stability obtain— a situation often encountered in the contemporary world— stable analysis is still frequently attempted, the practice being defended by the argument that the age distribution in such so-called quasi-stable populations is always close to that of a population which is stable in the strict sense, and is characterized by the current fertility and mortality of the population in which mortality has been declining. However, such estimates contain a bias which, depending on the duration and speed of the change in mortality, may be substantial. Section C of chapter I describes a method by which such a bias can be eliminated or at least considerably lessened by using information on the nature of the mortality decline. The method is illustrated below by the example of two populations; that of India in 1961 and of Mexico in 1960.

1. *India, 1961*

The assumption of essentially stable demographic conditions, on the basis of which estimates for the India of 1911 were derived above, is less defensible after the census of 1921. While there are no signs that would indicate a change in fertility, the growth of the population has been accelerating since the 1920s, undoubtedly reflecting a more or less steady decline of mortality from the high plateau of the 1881-1921 period. Under such conditions stable estimates should be adjusted to take care of the effects of that decline.

Computational procedure. (1) Table 28 shows stable estimates of the birth rate and the gross reproduction rate that are calculated in exactly the same manner explained in connexion with tables 22 and 24 above. The inputs in this instance are the 1961 female age distribution,[14] and

TABLE 28. STABLE POPULATION ESTIMATES OF THE BIRTH RATE (b) AND OF THE GROSS REPRODUCTION RATE (GRR) BASED ON THE AGE DISTRIBUTION OF THE FEMALE POPULATION OF INDIA AS REPORTED IN THE CENSUS OF 1961 AND ON THE ANNUAL RATE OF NATURAL INCREASE OF THAT POPULATION IN THE 1951-1961 INTERCENSAL PERIOD (r = .0189)

Age x (1)	C(x) (*proportion up to age* x) *India, 1961, females* (2)	*Values of* b *and of GGR in female stable populations with* C(x) *as in col. 2 and with* r = .0189	
		Birth rate (3.a)	GRR ($\bar{m}$ = 28.8) (3.b)
5	.154	.0396	2.64
10	.303	.0476	3.18
15	.412	.0447	2.98
20	.493	.0393	2.62
25	.583	.0397	2.64
30	.668	.0415	2.76
35	.738	.0425	2.83
40	.794	.0422	2.81

[13] An at least qualitatively identifiable source of bias in these calculations also arises from the fact that no allowance was made for external migration in reckoning the growth rate. The rate of *natural* increase may have been perhaps .0002 smaller than the average intercensal rate of growth. If so, the birth rates are underestimated by roughly the same amount, while the error introduced into the death rates (also underestimated) is about twice as large.

[14] Source: *Census of India, 1961 Census, Age Tables,* p. 54.

the rate of female natural increase for 1951-1961, ($r = .0189$) that was obtained by adjusting the intercensal rate for changed territorial coverage and for net immigration. The mean age of the fertility schedule was estimated from an imputed age specific fertility schedule in the same manner as shown in table 23. The proportions married among females 15-49 that were used in the calculation are as follows (1961 census data):

15-19	20-24	25-29	30-34	35-39	40-44	45-49
.696	.918	.942	.915	.871	.777	.698

The resulting $\bar{m}$ equals 28.8 years.

(2) The preliminary stable estimates of b and GRR (columns 3.a and 3.b in table 28) are to be adjusted for the distorting effects of changing mortality using the adjustments listed in table III.1. Since the preliminary estimates are based on $C(x)$ and the average rate of growth during the ten years preceding the time to which $C(x)$ refers, the appropriate section of that table is its "Part (a)". To extract the correct adjustment factors from the tabulated figures it is first necessary to estimate values of two indices; namely t, the approximate length of time (in years) for which the decline of mortality has been proceeding; and k, a parameter that describes the speed of the decline.

(3) The value of t may be estimated in this instance as 40 years, the time that has elapsed between 1921 – the date up to which growth rates for India showed a regular sequence of ups and downs, and after which acceleration of growth was uninterrupted – and 1961, the date of the latest census.

(4) The parameter k can be derived from the average rate of acceleration of the growth rate itself using the empirical relation $k = 17.8 \times \Delta r/\Delta t$. The absolute increase in the growth rate can be obtained by subtracting from the 1951-1961 rate of increase (which may be thought of as referring to the year 1956), the level of growth that prevailed, on the average, up to 1921. The latter may be estimated from the ratio of 1921 all-India population to the same population in 1881. This ratio is 1.1877, therefore, $r_{1921} = (\log_e 1.1877)/40 = .00430$. The value of $\Delta r/\Delta t$ is then

$$\frac{.0189 - .0043}{1956 - 1921} = \frac{.0146}{35} = .000417\,;$$

hence $k = .000417 \times 17.8 = .0074$.

(5) Column 3 in table 29 shows the adjustments as taken directly from table III.1, which is tabulated for $k = .01$. For other values of k it is necessary to scale these fractions up or down in the same proportion that the actual value of k bears to .01–i.e., in this instance by $.0074/.01 = .74$. This is shown in column 4.

(6) Column 4 thus contains proportions to be added to or subtracted from the preliminary estimates. It is convenient to transform these adjustments into multipliers by adding 1 to each entry (see column 5). Column 6 gives the products of these multipliers and the preliminary stable estimates, i.e., the adjusted (quasi-stable) estimates of the female birth rate and the gross reproduction rate.

(7) Selection of a single estimate for b and GRR among those associated with the various $C(x)$ values should be carried out in the same fashion as was explained and illustrated for "pure" stable estimates in section B of this

TABLE 29. ESTIMATION OF THE BIRTH RATE AND OF THE GROSS REPRODUCTION RATE FOR THE FEMALE POPULATION OF INDIA, 1961, BY ADJUSTMENT OF PRELIMINARY STABLE ESTIMATES OF THESE PARAMETERS (AS CALCULATED IN TABLE 28) FOR THE EFFECTS OF DECLINING MORTALITY

Age x (1)	*Stable population estimates derived from C(x) and 10-year intercensal r* (2)	*Adjustments from table III.1, part* (a) t = 40, k = .01 (3)	*Adjustments for* t = 40, k = .0074 (*col. 3×.74*) (4)	*Adjustment factors* (*1+col. 4*) (5)	*Adjusted estimates* (*col. 2×col. 5*) (6)
		Birth rate			
5	.0396	−.043	−.032	.968	.0383
10	.0476	−.032	−.024	.976	.0465
15	.0447	−.004	−.003	.997	.0446
20	.0393	.026	.019	1.019	.0400
25	.0397	.051	.034	1.034	.0410
30	.0415	.073	.054	1.054	.0437
35	.0425	.092	.068	1.068	.0454
40	.0422	.114	.084	1.084	.0457
		Gross reproduction rate ($\bar{m}$ = 28.8)			
5	2.64	−.010	−.007	.993	2.62
10	3.18	.001	.001	1.001	3.18
15	2.98	.031	.023	1.023	3.05
20	2.62	.062	.046	1.046	2.74
25	2.64	.088	.065	1.065	2.81
30	2.76	.111	.082	1.082	2.99
35	2.83	.130	.096	1.096	3.10
40	2.81	.153	.113	1.113	3.13

chapter and of chapter I. In the present instance the estimates derived from C(35) are to be preferred to the rest. The male birth rate and the birth rate for the total population are calculated by assuming a sex ratio at birth of 1.05 and by accepting the reported masculinity ratio of the population (1.062).

(8) Death rates (for the two sexes and for the total population) are obtained by subtracting the rates of natural increase (adjusted intercensal growth rates) from the appropriate birth rate estimates. The expectation of life or any other index of mortality is determined by reading the level of mortality in the *stable* populations (one for the males, one for the females) determined by the vital rates calculated earlier. No adjustment of such estimates for quasi-stability is warranted, or indeed desirable. The principal parameter values derived by following the above steps of calculation are exhibited in table 30.

TABLE 30. ESTIMATES OF VARIOUS POPULATION PARAMETERS FOR INDIA, 1961, OBTAINED BY ADJUSTING STABLE ESTIMATES OF THESE PARAMETERS FOR THE EFFECTS OF DECLINING MORTALITY

	Females	*Males*	*Total population*
Birth rate	.0454	.0449	.0451
Death rate	.0265	.0254	.0259
Level of mortality	7.7	9.1	—
$^0e_0$	36.8	37.5	—
GRR ($\bar{m}$ = 28.8)	3.10	—	—
Total fertility	6.36	—	—

Comments. Application of the stable method to the population of India in 1961 without adjustment for declining mortality results in a series of birth rate estimates unlike those typically produced by age distributions subject to the African-South Asian pattern of age-misreporting. When quasi-stability of the underlying demographic conditions is allowed for, however, the pattern familiar from the analysis of the 1911 Indian age distribution is fully re-established. The apparent range of uncertainty–apart from any error in the observed r, or in the assumption concerning the pattern of mortality–as to the actual level of the (female) birth rate is remarkably small; what may be tentatively considered maximum and minimum estimates (those associated with C(10) and C(15), respectively) differ only by about .002 (.0465 *versus* .0446). Comparisons with the estimates derived for 1911 (cf. table 25) show virtually identical gross reproduction rates but appreciably reduced birth rates in 1961. The changed relationship of these two indices is of course a necessary consequence of destabilization. It may be noted that the actual birth rate of the quasi-stable (1961) population is below the intrinsic rate: maintenance of the estimated mortality level and of the GRR would result eventually in a higher birth rate than the one shown in table 30.

2. *Mexico, 1960*

Any attempt to estimate Mexican fertility and mortality exclusively from census data is bound to be a highly artificial enterprise since the country has a vital registration system of long tradition and its statistics on births and deaths for the past two or three decades at least, are considered virtually complete. Also the assumption of constant fertility underlying both the stable and the quasi-stable methods discussed here is apparently valid only as a rough approximation. Apart from the violent demographic disturbance caused by the Mexican revolution in the second decade of the century (the consequences of which are now less visible than they were in earlier censuses), shifts in the age distribution and trends in population growth in recent decades reflect the influence of a slight but not negligible increase in Mexican fertility that subtly reinforces, and is superimposed on, the dominating effect on changes in those variables exerted by the very rapid decline of mortality since the mid-1930's. There is no practical way to separate such effects in stable population analysis. A straightforward application of stable methods (including the method of correcting for the presumed effects of declining mortality) for the analysis of Mexican data then can be expected to reveal some inconsistencies. The existence of direct information on vital rates, not used in the stable estimates, offers the advantage of making explicit the nature of such inconsistencies, and should also reveal other biases involved in stable analysis that may commonly occur in other applications, yet that are ordinarily not possible to nail down in the absence of independent evidence.

Not surprisingly the tests proposed earlier for detecting errors in age-reporting when applied to Mexican census data reveal the existence of a "Latin American" pattern. Hence the male age distribution is accepted as the main basis for stable estimates. Table 31 sets forth the elements of the calculation leading to a series of estimates of the male birth rate, adjusted for the effects of declining mortality. The basic data on which these calculations are built are the male age distribution (column 2) and the average male growth rate for the period of 1950-1960 (.0316). The stable estimates implied by these variables (column 3) are adjusted for quasi-stability in a manner analogous to the computations shown earlier in table 29. The parameters k and t used in the adjustment process were obtained by comparing the male growth rate for the 1930-1940 intercensal period (.0181) with that for 1950-1960. Specifically, $\Delta r/\Delta t = .0316-.0181/20 = .0135/20 = .000675$; and $k = \Delta r/\Delta t \times 17.8 = .0120$. Some acceleration of the growth rate increase probably occurred before the 1930s, although very little of such acceleration is suggested by the difference between the r's for 1930-1940 and 1920-1930, or by registered death rates in the 1920s and early 1930s. Naturally such comparisons may be somewhat misleading owing to possible changes in census coverage and improvement in vital registration. On the other hand an examination of the shifts in the age distribution of deaths, a statistic not necessarily affected by moderate omission rates, indicates fluctuating mortality before the mid-1930s followed by a clear sustained upturn. In any event it is evident that far the greatest proportion of the improvement in mortality took place in the twenty-five year period preceding the census of 1960. Consequently the value of t was taken as 25 years.

From column 7 of table 31 the median of the birth rates is selected as the single most acceptable estimate among

TABLE 31. ESTIMATION OF THE BIRTH RATE FOR THE MALE POPULATION OF MEXICO, 1960, BY ADJUSTMENT OF PRELIMINARY STABLE ESTIMATES OF THAT PARAMETER (DERIVED FROM REPORTED C (x) FOR 1960 AND THE INTERCENSAL GROWTH RATE—r = .0316—FOR 1950-1960) FOR THE EFFECTS OF DECLINING MORTALITY

Age x (1)	C(x) (*proportion up to age* x), *Mexico, 1960, males* (2)	*Birth rates in male stable populations; with* C(x) *as in col. 2 and with* r = .0316 (3)	*Adjustments from table III.1 part* (a) t = 25; k = .01 (4)	*Adjustments for* t = 25, k = .012 (*col. 4×1.2*) (5)	*Adjustment factors* (*1+col. 5*) (6)	*Adjusted estimates of the birth rate* (*col. 3×col. 6*) (7)
5	.169	.0383	−.028	−.034	.966	.0370
10	.325	.0424	−.025	−.030	.970	.0411
15	.454	.0436	−.004	−.005	.995	.0434
20	.554	.0422	.021	.025	1.025	.0433
25	.635	.0405	.053	.064	1.064	.0431
30	.704	.0395	.081	.097	1.097	.0433
35	.762	.0385	.099	.119	1.119	.0431
40	.818	.0394	.106	.127	1.127	.0444

those associated with x = 10, 15, ..., 40. C(20) and C(30) are tied for the median position — as a matter of fact all birth rates implied by C(15) through C(35) are virtually indistinguishable. From the estimated male birth rate the male death rate is calculated by subtracting the growth rate. Mortality indices are then obtainable from the stable population defined by these vital rates. Rates for females and for the total population are calculated in a similar fashion, having first derived the birth rates for these groups from the male birth rate via the sex ratio at birth (1.05, assumed) and the sex ratio of the population as a whole (.995, reported).

A somewhat more roundabout process is to be followed in finding the (female) gross reproduction rate. The following steps are required here: (*a*) estimate $\bar{m}$;[15] (*b*) having selected the median *male* adjusted birth rate in column 7, say the one derived from C(20) and r, find (by the usual method, i.e., by using the sex ratios at birth and in the population) the *stable* (unadjusted) female birth rate associated with the *stable* (unadjusted) male birth rate (Column 3) implied by C(20) and r (results: male birth rate, .0422; female birth rate, .0400); (*c*) this female birth rate plus the reported female growth rate (.0290) determine a stable population: read the value of GRR with the appropriate $\bar{m}$ from this stable population (GRR ($\bar{m}$ = 28.8) = 2.75); (*d*) adjust the GRR thus obtained for quasi-stability using parameter values as in the earlier calculation (t = 25, k = .012) and selecting the adjustment factors appropriate for the gross reproduction rate and for the proper x, in this instance 20. (Adjusted GRR ($\bar{m}$ = 28.8) = 2.75 × 1.054 = 2.90. The corresponding value for total fertility is 5.95. This compares with an estimate of total fertility from number of children ever born, as P_3^2/P_2 which gives 5.99). The results of the above calculations are given in table 32.

The correction for quasi-stability that affects all the estimates shown in table 32 was based on a value of the parameter k obtained from the acceleration of the growth rate. It is interesting to check the consistency of that estimate of k with the result of an alternative method, also described in chapter I, section C, that utilizes the changing composition of deaths to measure the tempo of mortality change. The index of the age distribution employed is the proportion of deaths over sixty-five within all deaths over age five. This index for Mexico can be calculated for each year from 1936 on; it shows clear upward trend with relatively minor yearly fluctuations. To minimize the effects of the latter it is better to calculate the index for periods longer than one year. In the following illustration (which is limited to the male population) the average for 1936-1939 and 1956-1959 are used; their values are .220 and .322, respectively. This change has occurred in twenty years. If we had an estimate of the $^0e_0$ at the base period, and if we knew the change in the expectation of life at birth during these twenty years it would be possible to read a tabulated value of kt in annex table III.3, hence to estimate k. If fertility is constant, and the age distribution is quasi-stable, it is possible to obtain just such a base-period value for $^0e_0$ and a value for Δ^0e_0 using the indices of the age distribution of deaths given above. The procedure is as follows:

(1) By means of the tabulation in annex II calculate $^0e_0$ in the stable populations defined by the 1950-1960

TABLE 32. ESTIMATES OF VARIOUS POPULATION PARAMETERS FOR MEXICO, 1960, OBTAINED BY ADJUSTING STABLE ESTIMATES OF THESE PARAMETERS FOR THE EFFECTS OF DECLINING MORTALITY

	Males	*Females*	*Total population*
Birth rate	.0433	.0410	.0422
Death rate	.0117	.0120	.0119
Level of mortality	16.6	15.4	—
$^0e_0$	55.5	56.0	—
GRR ($\bar{m}$ = 28.8)	—	2.90	—
Total fertility	—	5.95	—

15 The method based on standard marital fertility is not applicable because of the prevalence of consensual marriages in Mexico. Using the reported P_3/P_2 ratio (2.141), $\bar{m}$ is calculated as 28.8 years. This is the value accepted in the following calculation. Note however, that calculated direct from birth statistics, $\bar{m}$ is appreciably higher — 29.3 years.

intercensal r, and (a) C(10) in 1960 and (b) C(15) in 1960. (The results are 57.2 and 54.9.) Calculate the average of these two figures (56.0). This gives an estimate of the terminal (end-period) ${}^0e_0$.

(2) Using the same tables calculate the index births/population 15-44 (this index of fertility is included in table II) defined by the same parameters as given in point (1) above (results: .1006 and .1036) and take their average (.1021).

(3) From annex table III.2 obtain $\Delta{}^0e_0$ as the difference between two separately calculated estimates of ${}^0e_0$, each defined by the average index of births/population 15-49 as calculated in point (2), and by the index of the distribution of death for the two dates as given above (.322 and .22). The result is: $\Delta{}^0e_0 = 56.6-40.1 = 16.5$.

(4) Calculate an estimate of the base period ${}^0e_0$ as the difference between the end period ${}^0e_0$—see point (1)—and the $\Delta{}^0e_0$—see point (3)—i.e., as $56.0-16.5 = 39.5$.

Table III.3 can now be used to get kt the value of which in this example is .1686. Hence $k = .1686/20 = .0084$. If the adjustment of the preliminary stable estimates is carried out with this value of k, the procedure is exactly the same as shown in table 31 with the exception that column 4 is multiplied by .84, instead of 1.2, to get column 5.

Comments. There is a substantial difference between the two independent estimates of k obtained above which cannot be attributed to the approximate nature of the techniques involved, or explained by biases in the reporting of the age of dead persons. A more fundamental cause of this difference is that fertility has been increasing, and part of the acceleration of growth (reflected in the first estimate of k) is not attributable to mortality decline, which is alone measured by the shift in the age distribution of deaths. Fertility increase also biases the latter measure downwards. Naturally if stable estimates are corrected only for mortality decline, but the shift in age distribution is reinforced by fertility increase also, the resulting estimates will have a downward bias. The actual value of k to be used in this instance actually should be larger than .012 (cf. foot-note 20 to chapter I).

Even if the estimates given in table 32 were obtained by a more adequate correction for quasi-stability, their values would still be affected by a more substantial bias owing to the inadequate representation of the true pattern of mortality in Mexico by the "West" model life tables which underlie the above calculations. Section A.1.b. of chapter I gives a general statement of this problem. In the present instance there is ample evidence from the life tables prepared for Mexico since 1930 that the relation of child mortality (e.g., l_5) to "adult" mortality (e.g., ${}^0e_{10}$) is much closer to the "South" pattern than it is to the "West". For Mexico this factor alone would cause the estimated birth rates to be some .004 lower than their actual value, and of course there is a corresponding distortion in the other parameter values as well. This example thus shows the basic weakness of stable (or quasi-stable) estimates derived from $C(x)$ and r: their dependence on a well-chosen model life table family. There is often no information available on the true pattern, and no basis for a good choice. This difficulty is however eliminated, and the power of stable techniques greatly increased, when censuses provide data on child survival. Examples of estimation under such circumstances are discussed in chapter VIII below.