Chapter II

METHODS OF ESTIMATION BASED ON RESPONSES TO QUESTIONS ABOUT FERTILITY AND MORTALITY

A. Estimation of fertility from reports on child-bearing in the past[1]

In many censuses and surveys there appear data on the number of children women have evei borne, tabulated by age of woman. If fertility rates have been approximately constant in the recent history of the population in question, if the reported fertility histories have not been substantially affected by migration, and if differential mortality according to the prolificacy of the woman has not had an important effect on the survival of mothers, the average number of children ever born to women past age forty-five or fifty equals total fertility. Moie precisely, the average number of children ever born per woman aged forty-five to forty-nine equals the total fertility of this cohort of women, which in turn is about the same as the total fertility of the population at the time of the census or survey, provided fertility rates in the population have been approximately constant. If the assumptions stated above are valid, it is possible to estimate the age specific fertility rate for each five-year age interval within the child-bearing span by (for example) fitting a polynomial to the reported number of children ever born by age of mother.

The uncritical use of the reported average number of children ever born as a means of estimating the fertility of a population is risky, however, because of a widespread tendency for the number of children ever born to be under-reported, especially for older women. In many censuses or demographic surveys the average number of children ever born increases too gradually with age, especially at ages above thirty or thirty-five, and indeed a common feature of many censuses is reported average numbers of children ever born that decline with age above age forty-five oı fifty. One can speculate about the causes of the apparent under-reporting of the average number of children on the part of older women. Perhaps the most important factor is that some women tend to omit children who have grown up, or who have left home. A second possibility is the inability of some illiterate respondents to report large numbers accurately. A third hypothesis is that older women tend to omit offspring who died, especially many years earlier, although there are many instances in which the proportion reported as dead rises with the age of woman in a consistent fashion. For whatever reason, the reported average number of children ever born is very frequently a downward-biased estimate of the cumulative fertility experience of women over thirty or thirty-five, and the average parity of women past age forty-five or fifty, being typically understated, would usually provide an underestimate of total fertility.

On the other hand, younger women presumably report the number of children ever born to them with much better accuracy. Such women are not asked to recall events from the remote past or to count accurately to a large number; a higher proportion of the children ever born have survived to the time of the interview and few, if any, have left the household. If age–misreporting does not cause excessive distortion, the sequence of average numbers of children ever born by age of woman duplicates closely the curve of cumulative age specific fertility rates, until an age is reached where the proportion of children ever born that are omitted by the respondents becomes significant. In other words, the early part of a curve showing the rise in the average parity of women with age should resemble closely the cumulation by age of fertility rates. On the other hand, towards the end of the child-bearing interval, as the average number of children ever born approaches total fertility, the tendency towards omission or understatement causes reported parity to fall short of cumulated fertility rates. To provide a more valid estimate, then, it would be useful to splice to the rising curve of children ever born with age at the younger ages a curve which continued to rise with age in a way reflecting more accurately the actual fertility rates above age thirty or so. If we could be sure of the approximate relationship between fertility rates at different ages in the given population, we could determine what fertility rates at the younger ages would account for the average number of children ever born reported by the younger women, and then ascribe to the older women fertility rates consistent with these rates established at the younger ages of child-bearing. For example, if all populations in less developed countries had approximately the same age pattern of fertility, and differed only according to the size of a factor expressing the level at which this age pattern of fertility operated, the level could be ascertained by looking at the average number of children ever born by younger women—say, at ages twenty to twenty-four and twenty-five to twenty-nine. It would then be possible to prepare a rough estimate of the remainder of the fertility schedule and thereby to estimate total fertility. However, no such universal age pattern of fertility exists, although it is probably true that many populations share a similar age pattern of fecundability, that is a similar age pattern

[1] W. Brass, A.J. Coale, P. Demeny, D. Heisel, F. Lorimer, A. Romaniuk, and E. van de Walle, *The Demography of Tropical Africa* (Princeton, Princeton University Press), (in press), chapter III.

of the probability of conception among women living regularly in sexual union, without practising birth control.

The age pattern of fecundability is not identical in all populations primarily because of differences in the incidence of secondary sterility, or in the average age at which childbearing ceases. But even if differences in the age pattern of fecundability could be ignored, it would still not be possible to ignore differences by age in exposure to the risk of pregnancy. The major source of such differences is in the age pattern of the establishment of regular sexual unions—through marriage or other socially sanctioned institutions. The average age at first marriage for females in populations not commonly practising contraception varies from less than fifteen in India before the second World War to over twenty-five in many parts of Europe during the nineteenth and early twentieth centuries. In societies where consensual unions and formal marriages are both common, the age of entry into the former is typically much younger than into the latter, and consensual unions do not ordinarily involve as regular exposure to intercourse as marriage. There are similar though quantitatively less important differences in the dissolution of sexual unions. The incidence of widowhood depends upon the level of mortality, and the frequency of divorce upon law and custom. Societies also differ with regard to remarriage of the widowed and divorced.

To adjust the reported numbers of children ever born for a characteristic omission on the part of older women it is therefore necessary to have reliable evidence of the particular age pattern of fertility in the population in question. There are two potential sources of direct information on child-bearing rates by age of mother. The first is birth registration, and the second responses to a survey or census question on births during the year preceding the survey, either tabulated by age of woman. Of course if either of these sources were known to be accurate in the coverage of births the resultant data could be used directly to construct fertility measures. But even when the fertility rates derived from these sources are not accurate, they may be approximately correct in form. In other words there is the possibility that even if the number of births registered or reported understates or overstates the true number, the degree of understatement or overstatement is not age selective. This possibility is a plausible one with regard to the events reported by respondents for the preceding year, but is much less so for registered births. Incomplete registration cannot ordinarily be used to indicate the age pattern of fertility because there is no reason for supposing that the population covered by registration is representative of the whole population with respect to the fertility schedule. If birth registration is associated with literacy, for example, there may well be a tendency for the births occurring to younger women to be more completely registered than those occurring to older women. Other likely sources of bias include differential completeness of registration in regions that are not uniform with respect to the age pattern of fertility—in urban areas as compared to rural, for example.

The other potential source of information about the age pattern of fertility is survey data on births occurring during some preceding period, typically a year, before a census or demographic survey. The number of births reported in response to such questions has not proven accurate. Experience seems to indicate that the source of the inaccuracy is not a systematic tendency for women to fail to report births that have occurred or to exaggerate the number of these births but rather the difficulty that respondents have in identifying properly the length of the interval for which births should be reported. In some surveys there is a net tendency for women to report births that occurred in a period that is less than a year and in other surveys to report on the average events that occurred during more than the preceding year. The factors causing this reference period error seem likely to depend on general cultural conditions, the circumstances of the particular survey, including the wording of questions, the instructions to enumerators and the like. There seems no reason to expect an association between errors in reference period and the age of respondents. Moreover, the population covered by questions on births during the preceding interval is the same as that covered by questions about children ever born so that inconsistencies caused by differences in coverage or in forms of age-misreporting are avoided—differences that may exist between a population included in a register and a population covered by a survey.

The necessary questions on births during the preceding year on the one hand and on children ever born on the other, both tabulated by age of woman, have been included in a number of demographic surveys in Africa. William Brass has designed for use primarily in Africa a method of fertility estimation that accepts as essentially correct the *pattern* of fertility rates by age indicated by the births reported as occurring during the preceding year, and that accepts as an essentially correct indication of the *level* of fertility the average number of children reported as ever born by younger women. The method requires the estimation of the average value of cumulative fertility by age over the same age intervals (usually 15 to 19, 20 to 24, 25 to 29 etc.) for which the average number of children ever born is reported. It is then assumed that the source of the difference between the estimated average value of cumulated fertility at the younger ages (such as 20 to 24 or 25 to 29) and the average number of children ever born at these ages is an erroneous perception of the reference period by the respondents. The multiplier that would be needed to bring cumulative fertility at the younger ages in line with the reported average number of children ever born is determined, and the reported numbers of births at all ages are multiplied by this factor.

The basic principle underlying the Brass method of fertility estimation is simple enough, and the computations are complicated only by the difficulty of estimating the average value of cumulated fertility for the same age intervals for which average children ever born are given[2]. If approximate age specific fertility rates based on reported births by age of mother are tabulated only in five-year age intervals, cumulative fertility can be calculated directly only at the boundaries of these age intervals. Thus five times the age specific fertility rate of women 15 to 19 gives the cumulative fertility to age 20; this value plus five

[2] W. Brass, *et al.* loc. cit.

times the fertility rate of women 20 to 24 gives the cumulative fertility to age 25, and so on. It is necessary therefore to estimate from cumulative fertility to ages 20, 25, 30 etc., what would be the average value of cumulative fertility in the five-year age groups for which average parity (or average number of children ever born) is reported. If age specific fertility rates were constant within each five-year age interval the average value of cumulative fertility would be closely approximated by simple linear interpolation to the midpoint of each age interval. In fact, the typical pattern of fertility rates (increasing as they do from the earliest age of child-bearing to a peak usually in the late twenties) creates a curve of cumulative fertility that is not linear and that therefore requires a more complicated form of estimation than simple linear interpolation. Brass has calculated variable interpolation factors to be applied to the readily calculated values of cumulated fertility to the boundaries of the age intervals. The selection of appropriate interpolating factors is determined by how rapidly the reported fertility rates increase from the first to the second age group of women, since it is the steepness of the rise in fertility that determines the curvature in cumulative fertility with age over the early portion of the cumulative fertility function. Tables in annex IV present the interpolation factors—table IV.1 to be used when age specific fertility rates are available for the five-year age groups bounded by exact ages 14.5 and 19.5, 19.5 and 24.5 etc.;[3] and table IV.2 to be used when such rates are available for the conventional five-year age groups. In chapter VII there is a fully worked out example of the estimation of fertility by this method.

There are many censuses and surveys that have included a question on children ever born, with the responses tabulated by age of woman, but that have not included a question on births in the preceding year, so that there is no possibility of using the method just described. As stated earlier, it is a mistake to assume that different populations have the same age pattern of fertility—even populations not employing contraception or abortion. However, the form of the fertility schedules in populations that employ little birth control differs primarily in the way in which fertility rises from the first ages of childbearing to the ages where fertility is a maximum, and relatively much less in the way fertility declines after the peak is reached. This greater relative variability in the early part of fertility schedules results from the fact that the rise of fertility with age is strongly affected by customs and institutions governing the establishment of sexual unions—strongly affected, that is to say, by the age pattern of nuptiality in societies where formal marriage is a principal determinant of cohabitation. In other words, when birth control is not widely practised, the shape of the early part of the fertility schedule is dominated by the age pattern of entry into regular sexual relations rather than by the age schedule of fecundability; and in contrast the decline of fertility with age—when birth control is not practised—is generally governed more by declining fecundability than by customs and institutions. The decline of fecundability with age in different populations is likely to follow a roughly similar pattern while no such similarity is found with respect to the age of entry into cohabitation. These considerations suggest the hypothesis that the ratio of the average parity of women at the end of child-bearing to the average parity of a younger group (say women 25-29) is closely related to the relative parity of women early and late in their twenties. The reasoning behind the hypothesis is as follows: (*a*) if the average parity of women 25-29 is an unusually large multiple of the average parity of women 20-24, child-bearing does not begin early in this population since the high ratio implies that fertility is unusually high at ages 22.5 to 27.5 compared to before 22.5. It follows that in this population an unusually large fraction of total fertility occurs in the later years of child-bearing, and the ratio of final average parity to the average at ages 25 to 29 is therefore unusually large; (*b*) on the other hand, an unusually low ratio of parity at 25-29 to parity at 20-24 indicates that high rates of child-bearing began early, that an unusually small fraction of total fertility occurs in the later years of child-bearing, and that the ratio of final average parity to the average at 25-29 is unusually low.

Suppose that the average number of children ever born (average parity) to women 15-19 is designated P_1, to women 20-24 P_2, and so on, until P_7 designates the average parity of women 45-49, Suppose the average parity of women reaching age 50—assumed to be the upper limit of child-bearing—is designated TF (for total fertility). Our hypothesis is, then, that TF/P_3 is closely related to P_3/P_2. The usefulness of this hypothetical relationship, should it prove valid, is that it provides a possible method of estimating total fertility when older women under-report the number of children they have borne, and younger women report parity accurately.

The hypothesis obviously cannot be tested by examining the relationships among average numbers of children ever born reported by women over 50, 25-29 and 20-24 in a number of populations that do not practise birth control, because it is the very inaccuracy of these reports that leads us to consider the expected relationship. The test employed is based on the cumulation of age specific fertility rates in a number of apparently reliable fertility schedules (based on virtually complete birth registration) to construct a set of necessarily consistent average parity schedules. Thus the average number of children ever born was calculated at ages 20-24, 25-29 and on reaching age 50 in a group of women subject to each of the given fertility schedules. By *constructing* the average parties in this way, we insured that there would be no distortion from faulty reporting, or changing fertility. The ratios of TF/P_3 and of P_3/P_2 are plotted in figure XVI. The relationship is gratifyingly close, and is well represented by simple equality of the two fractions, or by the formula $TF = (P_3)^2/P_2$.

This formula provides an estimate of total fertility under the following conditions:

(1) Fertility at ages 15-29 has been constant in the recent past;

(2) The age pattern of fertility conforms to the typical age relationships found in populations practising little

[3] When age specific fertility rates are based on births reported in the preceding year, the mothers were approximately one half-year younger when the births occurred than at the time of the survey. Therefore births reported by women between exact ages 15 and 20 serve as a basis for estimating fertility rates for women 14.5 to 19.5. This slight displacement of age must be allowed for in calculating cumulative fertility.

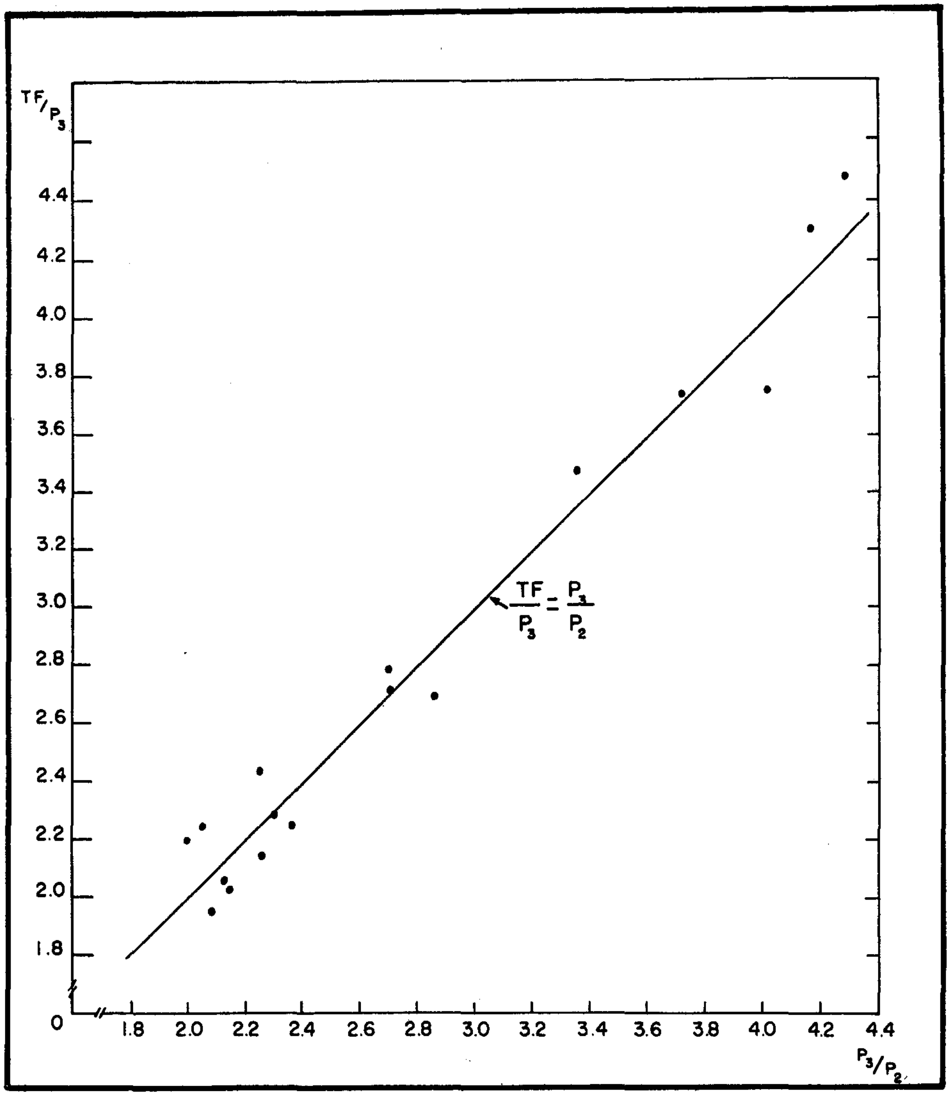

Figure XVI. Ratio of TF (total fertility) to P_3 (average parity at age 25-29) *versus* ratio of P_3 to P_2 (average parity at age 20-24) calculated from fertility schedules of populations not practising birth control

birth control, implying (*a*) that the age pattern of declining fecundability is typical; and (*b*) that widowhood, divorce, and other forms of dissolution of sexual unions do not have an unusual age incidence from age thirty to forty-five in the population in question.

The value of $(P_3)^2/P_2$ can be compared with the average parity reported by women over 50, and 45-49. Lower parity reported by women over 50 than at 45-49 is an indication of likely omission of children ever born by older women. If under these circumstances $(P_3)^2/P_2$ exceeds P_7, the estimate gains in credibility and is to be preferred to the numbers supplied by the older women. Approximate equality of P_7, the average parity reported by women over 50, and $(P_3)^2/P_2$ indicates that any of the three figures is an acceptable estimate of total fertility. But if $(P_3)^2/P_2$ is substantially *less* than P_7, or if the average parity reported by women over 50 is much *greater* than P_7, the wisest course is not to attempt an estimate of current fertility by manupulation of the data on children ever born.

B. Estimates of mortality based on proportions surviving among children ever born

In some of the censuses and demographic surveys in which women are asked to report the total number of children ever born to them, there is an additional question asking the number of surviving children. It has long been realized that the proportion surviving depends on the level of infant and child mortality, and census reports in which data on surviving children are included often contain comments that variations within the enumerated population in proportions surviving can be considered an index of differential mortality. Recently William Brass has greatly increased the usefulness of data of this sort by developing a method of translating proportions surviving and proportions dead among the children ever born to women in different age groups into conventional measures of mortality. His technique makes it possible under certain circumstances to estimate the proportion of children born who survive to age 1, 2, 3, 5, 10, 15, ..., 35 from the proportion reported as surviving among children ever born to women 15 to 19, 20 to 24, 25 to 29, ..., 60 to 64. Because a full account of this technique appears in another publication[4] there is no attempt in this *Manual* to explain the somewhat surprising correspondence between l_x values and data on child survival in women's reports of children ever born, nor to justify in detail the adjustments described below. Our discussion is restricted to a brief outline of the conditions under which the technique may be applied, and of the steps involved in constructing estimates.

The Brass method of estimating mortality enables the analyst to construct the survival function of a life table up to early adult ages. The conditions that would make this computation accurate are:

(1) The age specific fertility schedule has been approximately constant in the recent past (at least for the younger women), and the approximate form of the schedule is known;

(2) Infant and child mortality rates have been approximately constant in recent years;

(3) There is no powerful association between age of mother and infant mortality or between death rates of mothers and of their children;

(4) Omission rates of dead children and of surviving children are about the same in the reported numbers ever born;

(5) The age pattern of mortality among infants and children conforms approximately to the model life tables.

Under such ideal conditions it has been shown that the proportion of children dying before their first birthday is not very different from the proportion dead among those ever born to women 15 to 19, the proportion dying before the second birthday not very different from the proportion dead among the children ever born to women 20 to 24, before the third birthday to the proportion dead among children ever born to women 25 to 29, before the fifth birthday to the proportion dead among children ever born to women 30 to 34, before the tenth equal to the proportion dead among children ever born to women 35 to 39 etc. These approximations are very close for a population characterized neither by a very early nor by a very late start in child-bearing. If fertility begins at a very early age the children ever born to women in each age group would be exposed to more prolonged risks of mortality, and, therefore, the proportion dead would tend to be higher for each age group of mothers than when fertility has a later start. Brass has constructed a set of adjustment factors that can be used to modify the estimates of proportions dying defore age 1, 2, 3, 5 and so on, in accordance with whether the starting point of fertility in the population in question is early or late. The index of early or late fertility is the ratio of the average number of children ever born in the first two age groups of women —P_1/P_2. In making adjustment for estimates derived from the proportion dead among the children ever born to older women, the index of early or late fertility is the mean age ($\overline{m}$) or the median age ($\overline{m}'$) of the fertility schedule. These adjustment factors are given in annex V; in table V.1 (to be used when children ever born and surviving are tabulated by the conventional five-year age groups) and in table V.2 (to be used when the census tabulations are by ten year groups of women: 15-24, 25-34 etc.). Examples of estimating mortality by this method are worked out in chapter VII.

Of course the conditions specified as ideal are seldom if ever completely fulfilled. Because of the sensitivity of the estimate of the proportion dying before age one to peculiarities or defects in the data the estimate of infant mortality directly derived by this technique does not justify much confidence. On the other hand, estimates of child mortality up to older childhood ages based on the reports by older women of still-living children and of children who have died are especially subject to reporting errors and also to the effects of the possibly different mortality levels in the more distant past. The estimates that appear to reflect the best chance of minimizing error from various sources are those for the proportion dying before age two and age three. Because of the prevalence of falling death rates in many of the less developed countries in recent years, it should be borne in mind that the life table value estimated by these procedures represents the average mortality experience over the preceding four or five years in the determination of the proportion dead before the second birthday, and in the preceding six to eight years, in the estimate of proportion dead before the third birthday.

The ideal conditions listed above were not fully realized in the population of Hungary before the censuses of 1930, 1949 1960; nor in the population of Canada just before the census of 1941. Mortality was changing, the age pattern of mortality in Hungary does not conform to the "West" pattern, and fertility was not constant. Questions in the censuses concerning children ever born and surviving were limited to married women only. Nonetheless, when estimates are made by the Brass techniques they come remarkably close to estimating the proportions dying before age two in the four or five years preceding each of the censuses, even though the range of mortality estimated is from a high of over 200 per thousand to a low of less than 70 per thousand, as shown in table 3. Columns 2 and 3 of table 3 were obtained by calculating the average level of infant mortality in the given country and time for a four and five year period before the given census and then using the relationship between ${}_1q_0$ and ${}_2q_0$ as shown in the nearest official life tables to estimate the proportion dying before age two. Column 4 is derived from the reported proportions surviving to women aged twenty to twenty-four, using the multipliers of annex table V.1, selected according to the reported P_1/P_2 ratios.

When the potential sources of bias are considered, it is evident that the estimation of child mortality by this procedure tends almost always to err on the low side, if the estimate is not accurate. The presumption of bias in this direction is based on a judgement that respondents are much more likely to omit from a summary of their

[4] See the discussion in W. Brass, *et al.*, op. cit. For an earlier exposition see W. Brass, "The Construction of Life Tables from Child Survivorship Ratios", in Union internationale, *International Population Conference, New York, 1961*, (London, 1963), vol. I, pages 294-301.

TABLE 3. PROPORTIONS DYING BEFORE AGE 2 (VALUES OF $_2q_0$) IN SPECIFIED PERIODS AS DERIVED FROM VITAL REGISTRATION DATA FOR HUNGARY AND CANADA, AND AS ESTIMATED FROM PROPORTIONS OF CHILDREN SURVIVING REPORTED IN CENSUSES OF THE SAME COUNTRIES

Census	*From vital registration data*		
	Average of 4 years preceding the census	*Average of 5 years preceding the census*	*Estimated from census survivorship rates*
Hungary			
1930	.204	.203	.203
1949	.138	.133	.137
1960	.063	.063	.067
Canada			
1941	.069	.071	.072

TABLE 4. INFANT MORTALITY RATES ($_1q_0$) ASSOCIATED WITH $_2q_0$ VALUES THAT WERE ESTIMATED FROM PROPORTIONS OF CHILDREN SURVIVING AS REPORTED IN VARIOUS CENSUSES AND AVERAGE INFANT MORTALITY RATES DURING THE FOUR YEARS PRECEDING EACH CENSUS AS REGISTERED IN VITAL STATISTICS

Country	*Year*	*Average of $_1q^0$ in the fort years preceding the census, according to vital registration data*	*Estimates of $_1q^0$ obtained from $_2q^0$ according to "West" model life tables, based on census reports*
Barbados	1946	.161	.162
British Guiana	1946	.119	.131
Brunei	1960	.097	.119
Cyprus	1960	.030	.050
Egypt	1947	.152	.212
Fiji Islands	1946	.067	.115
Fiji Islands	1956	.051	.093
North Borneo	1951	.114	.186
Peru	1940	.128[a]	.202
Sarawak	1960	.065	.114
Seychelles	1960	.049	.069
Western Samoa	1956	.038	.095
Western Samoa	1961	.023	.080
Windward Islands	1946	.112	.129
Yugoslavia	1953	.117	.133

[a] 1940 only.

experience to date children who have died than those who have survived.

Estimates of infant mortality have been obtained in a number of populations by first applying the method here described to detmine the proportion dying before the second birthday and then assuming that this proportion is related to infant mortality as in the "West" model life tables. It is remarkable in view of the expected downward bias that these estimates exceed (often by a major extent) the average level of infant mortality derived from registered births and deaths in the four years before the survey in the countries shown in table 4. This comparison indicates the widespread prevalence of under-registration of infant mortality.

The demonstrated accuracy of the Brass method of estimating infant and child mortality in Hungary and Canada, in conjunction with the fact that it appears in many of the less developed countries to provide an estimate much closer to actuality than can be obtained from registered data suggests that this method will prove a powerful and welcome addition to the techniques available to demographic analysts. Unfortunately, the method does not provide information about adult mortality.[5] To estimate the expectation of life at birth or the crude death rate from these childhood survival rates, it is necessary to make an assumption about the relationship between death rates at different ages. It is a simple mechanical procedure to select a model life table having the same proportions surviving to age two as is indicated by this technique of estimation. However, there is little basis for confidence that the relationship between mortality rates at different ages in the "West" model tables holds closely in a population in Africa, Asia or Latin America.

[5] Concerning the possibility of obtaining such information from census reports on the proportion of orphans in a population, see however Louis Henry, "Mesure indirecte de la mortalité des adultes", *Population*, 1960, No. 3, pages 457-466.