VIII. MAKING EMPLOYMENT PROJECTIONS USING INVERSE COBB-DOUGLAS PRODUCTION FUNCTIONS

A. Introduction

This chapter presents a method for projecting employment by industry, using employment functions (box 26) which are the inverse of Cobb-Douglas production functions (described in annex I), or their transformations.1/ These functions relate employment to value added, the capital stock and time, by industry.2/ Like the labour productivity technique described in chapter VI and the employment-value added function method described in chapter VII, this method can be used to project employment by industry as well as employment aggregates and indicators of the structure and growth of employment.

The method dispenses with a number of the explicit or implicit assumptions used by the simpler methods described in earlier chapters. Furthermore, unlike those techniques, this method explicitly allows for possible changes in the relative growth rates of various inputs (e.g., substitution of capital for labour).3/

Unlike the simpler techniques, the method is also capable of making separate estimates of the effects of input growth on employment and the effects of technical change. This makes it possible to make long-run employment projections which are more accurate than those prepared by the simpler methods, particularly if different inputs are expected to grow at different rates.

Deciding whether employment functions derived from Cobb-Douglas production functions are more suited than simpler methods for preparing long-term employment projection will involve a number of considerations. Among these considerations are: whether the Cobb-Douglas production function correctly specifies the technological relationships existing in various industries; whether adequate data are available to estimate parameters of the industry-specific Cobb-Douglas production functions (or their transformations); and whether the parameters of those functions can be correctly estimated from suitable data. In view of the importance of these considerations, they will be briefly reviewed below.

In deciding whether the Cobb-Douglas production function is the correct specification, it is important to recognize that it is only one, although the most popular, form of production function. Although this particular production function has a number of desirable properties, there may be no *a priori* reasons for choosing it over other types of production functions as a basis for making employment projections. This specific form may not be the most appropriate to use in any given situation. If the Cobb-Douglas function is a misspecification of the actual relationship between output and production inputs, the method may yield misleading projections of employment.

Box 26

Glossary

Average capital-output ratio
The capital stock of a firm, industry or economy over a time period, divided by the output produced during that period.

Employment function
A mathematical expression describing a relationship between employment and other variables, typically a measure of output and other inputs, in which employment is the dependent variable and the other variables are independent. The inverse of a Cobb-Douglas production function in which employment is the dependent variable would be an employment function.

Cobb-Douglas production function
A mathematical expression describing a relationship between a measure of output and two or more inputs (such as employed labour and capital). The function is multiplicative in the natural numbers and linear when transformed into logarithms.

Human capital
Productive investments embodied in human persons. These include skills, abilities, ideals, health etc., that result from expenditures on education, on-the-job-training and medical care.

Incremental capital-output ratio
The increase in the capital stock of a firm, industry or economy over a time period, divided by the increase in output over that period.

Simultaneous equation bias
A bias arising in statistical estimation when the dependent variable has a causal effect on the independent variables, rather than vice versa.

The method might not be readily applicable in many developing countries because reliable data needed to estimate parameters of the Cobb-Douglas production functions are not widely available. This is particularly true if the functions are to be estimated on the basis of time series data, which may be essential whenever estimates of the rate of technical change are needed.

At present, the data necessary for applying this method at the rural-urban level are not available in most developing countries.

In any given country, time series data on production and inputs may be unavailable, not sufficiently reliable or could refer to too short a time period. Cross sections including relevant information at several points in time could be even scarcer than sufficiently long time series. However, some time series data, or at least cross sections for a few different years, are essential in order to estimate the rate of technical change. Even if data of the right type are available, it may not be possible to obtain robust estimates of the relevant parameters owing to the lack of variability in the economic conditions from one year to the next or from one firm to another.

Although simple methods are available for estimating the Cobb-Douglas production function, there is an extensive econometric literature that casts doubt on the quality and utility of the estimates obtained by such methods. In particular, questions have been raised about whether the functions so estimated are really production functions at all, or merely some mixture of the production function and the profit maximizing conditions. In addition, the estimates of the parameters of the production functions obtained by ordinary least squares may be subject to a <u>simultaneous equations bias.</u> That is, changes in the independent variable may be caused by changes in the dependent variable.

There may be no such thing as an aggregate production function, that is, a technical production function at the level of industry or the entire economy. Most approaches to aggregation assume that the underlying production functions from which the aggregate production function of an industry or the economy is to be derived are additive. (An example of an additive production function is a linear production function.) Cobb-Douglas functions, however, are not additive. Thus if various segments of the industry, such as the modern or the traditional segments are growing at different rates, the estimated Cobb-Douglas production function for the industry will tend to yield poor results when used as a basis for projecting employment. That is, the coefficients of the estimated production function would take on different values depending on the relative rates of growth of the modern and traditional sectors. This is called aggregation bias.

If the <u>human capital</u> composition of the labour force or the age composition of the physical capital stock is expected to change significantly in the future (which may often be the case in developing countries), the traditional specification of the Cobb-Douglas production function can be expected to provide a poor basis for projecting employment. This is so since neither labour nor capital is homogeneous, though they are traditionally entered as homogeneous inputs in a Cobb-Douglas production function and, by implication, into any of its transformations.

This amounts to a misspecification since employees differ from one another in the amount of human capital they embody, or machines of different vintages differ from one another in the type of technology they embody.

Hence, to avoid misspecification, labour should be disaggregated by type, or a weighted index of labour inputs should be constructed and used as an input. Similarly, the capital stock need to be subdivided by vintage (or date of production), or a weighted index of capital of different ages should be prepared and used as the input.

Even if it is possible to specify, estimate and use a Cobb-Douglas production function correctly, little may be gained by using this method rather than simpler techniques to project employment, particularly if the objective is medium-term (i.e., five-year) projections. Since capital-output ratios are unlikely to change much in the medium-term, relatively simple projection techniques should serve almost as well as those involving estimated production functions. Hence production functions may perhaps be best used for longer-term projections.

B. The technique

1. Overview

This overview lists inputs required by the method based on the inverse Cobb-Douglas production functions and indicates the types of results it can generate. The overview also outlines the computational steps involved in making an employment projection with this method.

(a) Inputs

To project employment with this method the following inputs are required:

(i) Projected levels of value added, by industry;
(ii) Projected levels of the capital stock, by industry;
(iii) Estimates of the coefficients of employment functions which are the inverse of Cobb-Douglas production functions (or their transformations), by industry.4/

If, in addition to employment, shortages and/or surpluses in the labour market are to be projected, the inputs should also include:

(iv) Projected total labour force;
(v) Projected non-civilian employment.

The inputs are listed in box 27.

Since this method is described as a procedure for making quinquennial projections, the projected levels of value added and the capital stock would be for dates five years apart, starting with the initial year of the plan. Projected total labour force and projected non-civilian employment would be for the same dates. Given the appropriate annual inputs, however, the method could be used for making annual projections.

Box 27

Inputs for projecting employment using inverse Cobb-Douglas production functions or their transformations

1. <u>Value added by industry</u>

2. <u>Capital stock by industry</u>

3. <u>Coefficients of the employment functions</u>

   Coefficients of inverse Cobb-Douglas production functions or their transformations, by industry

4. <u>Total labour force</u> (if projection of labour market balances is desired)

5. <u>Non-civilian employment</u> (if projection of labour market balances is desired)

(b) <u>Outputs</u>

The outputs which this method can generate include:

(i) Levels of employment by industry;

(ii) Various employment aggregates, such as total employment and the growth in total employment;

(iii) Indicators of the structure of employment, such as proportions of total employment found in each sector, (e.g. primary, secondary and tertiary);

(iv) Rates of change in employment, including that of total employment or employment by sector.

If the inputs include projected total labour force and projected non-civilian employment, the outputs could also include:

(v) Absolute and relative levels of excess supply of labour and/or excess demand of labour.

The types of outputs that the technique can generate as part of the projection, are shown in box 28. They would be obtained for dates five years apart or for the intervening projection intervals.

(c) <u>Computational steps</u>

Levels of employment, by industry, for a given date can be projected by evaluating employment functions that are the inverses of Cobb-Douglas production functions or their transformations, using as inputs projected levels of value added and the capital stock. The procedure also permits

Box 28

Types of outputs of employment projections using inverse Cobb-Douglas production functions or their transformations

1. Employment by industry

2. Employment aggregates

   Total employment and employment by sector (e.g. primary, secondary and tertiary)

   Growth in total employment and employment by sector

3. Indicators of the structure of employment

   Proportions of employment, by sector

4. Rates of growth of employment

   Rates of growth in total employment and employment by sector

6. Labour market balances

   Absolute and relative levels of excess supply of and/or excess demand for labour

deriving for each projection date, total employment and employment in major sectors, such as primary, secondary and tertiary, along with other date-specific indicators. Also, the projection provides an estimate of the absolute growth and growth rates in employment for the intervening intervals. If a labour force projection is available, the projected total employment and the projected labour force size can be compared to calculate the surplus or shortage of labour.

2. Description

This section will first describe a particular type of Cobb-Douglas production function and then introduce employment functions that are the inverses of those Cobb-Douglas production functions and their transformations. Then it will show how to project levels of employment by industry and explain steps to derive other results. A summary of those steps is shown in box 29.

Box 29

Computational steps to project employment at the national level using inverse Cobb-Douglas production functions or their transformations

The steps used to project employment at the national level over a five-year projection interval are:

(1) Derive levels of employment or logarithms of those levels, by industry, at the end of the interval by evaluating empirically estimated industry-specific inverse Cobb-Douglas production functions or their log-linear transformations. In the process, use the assumed levels of value added, the capital stock and the suitable value of the time variable. If the logarithms of employment levels are computed, take antilogarithms of the results obtained in order to reach the levels of employment.

(2) Calculate various employment aggregates, such as total employment and the increase in total employment.

(3) Derive indicators of the employment structure, such as proportions of employment by sector.

(4) Obtain rates of growth of employment, such as the rate of growth of total employment.

(5) If the labour force projection is available, derive labour market balances by calculating the absolute and percentage levels of excess supply of or excess demand for labour.

(a) Cobb-Douglas production functions

Employment functions by industry can be obtained by inverting alternative specifications of industry-specific Cobb-Douglas production functions and, if so desired, by finding transformations of those inverse production functions. This section will describe industry-specific Cobb-Douglas production functions, which may be most suitable as the basis for deriving employment functions to be used in developing countries. These Cobb-Douglas production functions, in which the rate of technical change is considered independent of the growth of other inputs, are as follows:

$$VA(i,t') = z(i) \cdot CAP(i,t')^{a(i)} \cdot EM(i,t')^{b(i)} \cdot e^{[r(i)\ \cdot\ t']}; \qquad (1)$$

$$i = 1,\ldots,I,$$

where:

t'	is the calendar year,
$i = 1,\ldots,I$	are the industries of the nation's economy,
I	is the number of industries,
$VA(i,t')$	is the value added in industry i in year t',
$CAP(i,t')$	is the capital stock in industry i in year t',
$EM(i,t')$	is the labour employed in industry i in year t',
$z(i)$	is the intercept parameter for industry i,
$a(i)$	is the elasticity parameter relating value added to the capital stock for industry i,
$b(i)$	is the elasticity parameter relating value added to labour for industry i,
$r(i)$	is the constant rate of technical change for industry i, and
e	is the base of the natural logarithms.5/

The form of the Cobb-Douglas production functions indicated in equation (1) is one of several alternative forms (annex I). It has been selected here for the two basic reasons: first, this specification allows for technical change, which may be very important from the point of view of countries undergoing rapid economic growth based on the use of new technologies; and second, the data requirements for this specification are fewer than those of some alternative specifications.

(b) Inverse Cobb-Douglas production functions

The production functions indicated in equation (1) can be manipulated in a variety of ways. They can be inverted to obtain employment functions, which treat employment (or labour input) as the dependent variable and consider value added, the capital stock and time as the independent variable. These employment functions, which are sometimes referred to as inverse Cobb-Douglas production functions, would be as follows:

$$EM(i,t') = a'(i) \cdot VA(i,t')^{b'(i)} \cdot CAP(i,t')^{c'(i)} \cdot e^{[d'(i)\ .\ t']}; \qquad (2)$$

$$i = 1,\ldots,I,$$

where:

$a'(i)$	is the intercept parameter for industry i,
$b'(i)$	is the elasticity parameter relating labour to value added in industry i,
$c'(c)$	is the elasticity parameter relating labour to the capital stock in industry i, and
$d'(i)$	is the parameter relating labour to the time variable in industry i.6/

The employment functions shown in equation (2) can be in turn transformed by taking the logarithms of their left and right hand sides to obtain what could be referred to as log-linear transformations of inverse Cobb-Douglas production functions. The transformed functions are as follows:

$$\ln EM(i,t') = \ln a'(i) + b'(i) \cdot \ln VA(i,t') + c'(i) \cdot \ln CAP(i,t') \qquad (3)$$

$$+ d'(i) \cdot t';$$

$$i = 1,\ldots,I,$$

where:

ln	is the natural logarithm.

The advantage of this log-linear transformation is that it can be estimated easily using the ordinary least squares (OLS) method. Moreover, once the estimates of the coefficients of the log-linear transformation are available, they can be easily used to make projections of employment.

(c) Employment by industry

Employment by industry can be projected by evaluating empirically estimated employment functions or their transformations for various dates over the projection period, using projected levels of value added and the capital stock by industry. Procedures used to evaluate the functions will depend on their functional form. Thus, if untransformed inverse Cobb-Douglas production functions were used, employment levels, by industry, for the end of the projection interval (t to t+5) would be obtained as follows:

$$EM(i,t+5) = [a'(i)]^* \cdot VA(i,t+5)^{[b'(i)]^*} \cdot CAP(i,t+5)^{[c'(i)]^*} \cdot e^{[\,[d'(i)]^* \cdot (\bar{t}'+t+5)]}; \quad i = 1,\ldots,I, \tag{4}$$

where:

t	is the year of the projection period,
$\bar{t}'$	is the calendar year designated as the initial year of the projection period,
$EM(i,t+5)$	is the labour employed in industry i at the end of the interval,
$VA(i,t+5)$	is the value added in industry i at the end of the interval,
$CAP(i,t+5)$	is the capital stock in industry i at the end of the interval,
$[a'(i)]^*$	is the estimate of the intercept coefficient of the inverse Cobb-Douglas production function for industry i,
$[b'(i)]^*$	is the estimate of the partial coefficient of the value added variable in the inverse Cobb-Douglas production function for industry i,
$[c'(i)]^*$	is the estimate of the partial coefficient of the capital stock variable in the inverse Cobb-Douglas production function for industry i, and
$[d'(i)]^*$	is the estimate of the partial coefficient of the time variable in the inverse Cobb-Douglas production function for industry i.

Estimates of the log-linear transformation of the inverse Cobb-Douglas production functions could be used to project employment. Those functions would be first used to obtain the logarithms of employment levels by industry as follows:

$$\ln EM(i,t+5) = [\ln a'(i)]^* + [b'(i)]^* \cdot \ln VA(i,t+5) + [c'(i)]^* \cdot \ln CAP(i,t+5) + [d'(i)]^* \cdot (\bar{t}'+t+5); \quad i = 1,\ldots,I, \tag{5}$$

where:

$[\ln a'(i)]^*$ is the estimate of the logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for industry i.

Once the logarithms of the labour input or employment by industry are obtained as indicated by equation (5), employment levels by industry can be obtained by taking the antilogarithms of those results:

$$EM(i,t+5) = \text{antiln}[\ \ln EM(i,t+5)\ ]; \qquad i = 1,\ldots,I, \tag{6}$$

where:

antiln is the antilogarithm of the natural logarithm.

This step completes the description of this technique for projecting employment by industry.

(d) Other results

Once the levels of employment by industry are derived for the end of a given projection interval, several useful indicators can be calculated. These indicators include employment aggregates and indicators of the employment structure as well as the rate of change in employment. (Since much of this section is very similar to sections B.2(c) of chapters VI and VII, the reader who is familiar with this material may wish to move directly to the next section.)

(i) Employment aggregates

A key aggregate that can be calculated from the projected levels of employment by industry is the level of total employment. It is also possible to obtain the levels of employment in sectors, such as the primary, secondary and tertiary sectors. Once these levels are obtained for different dates five years apart, one can further calculate increases in total and sectoral employment over the intervening five-year projection intervals.

a. Total employment

Total employment can be obtained by aggregating the levels of employment across industries. For the end of a projection interval (t to t+5) this number is:

$$EM(t+5) = \sum_{i=1}^{I} EM(i,t+5), \qquad (7)$$

where:

EM(t+5) is the total employment at the end of the interval.

b. Employment by sector

A variety of criteria can be used to aggregate industries into sectors. Thus, industries could be aggregated into primary, secondary and tertiary sectors or into agricultural, industrial and service sectors. For illustrative purposes, the primary-secondary-tertiary-sector mode of aggregation will be used. In addition, it will be assumed that the numbering of industries for which the levels of employment are being projected lists industries of the primary, secondary and tertiary sectors one after another.

i. Employment in the primary sector

Using these classifications and aggregation rules, employment in the primary sector for the end of the projection interval (t to t+5) can be obtained as:

$$EMP(t+5) = \sum_{i=1}^{Ip} EM(i,t+5), \qquad (8)$$

where:

Ip is the number of industries in the primary sector, and

EMP(t+5) is the employment in the primary sector at the end of the interval.

ii. Employment in the secondary sector

Employment in the secondary sector can be obtained as:

$$EMS(t+5) = \sum_{i=Ip+1}^{Ip+Is} EM(i,t+5), \qquad (9)$$

where:

Is	is the number of industries in the secondary sector, and
EMS(t+5)	is the employment in the secondary sector at the end of the interval.

iii. Employment in the tertiary sector

Employment in the tertiary sector can be calculated as:

$$EMT(t+5) = \sum_{i=Ip+Is+1}^{I} EM(i,t+5), \tag{10}$$

where:

EMT(t+5)	is the employment in the tertiary sector at the end of the interval.

c. Growth in total employment

The growth in total employment over the projection interval (t to t+5) equals the difference between total employment at the end and total employment at the beginning of the interval:

$$EMGR = EM(t+5) - EM(t), \tag{11}$$

where:

EMGR	is the growth in total employment during the interval.

d. Growth in employment, by sector

The increase in employment in the primary, secondary and tertiary sectors over the projection interval is respectively obtained as follows:

Growth of employment in the primary sector is calculated as:

$$EMPGR = EMP(t+5) - EMP(t), \tag{12}$$

Growth of employment in the secondary sector is calculated as:

$$EMSGR = EMS(t+5) - EMS(t), \tag{13}$$

Growth of employment in the tertiary sector is calculated as:

$$EMTGR = EMT(t+5) - EMT(t), \tag{14}$$

where:

EMPGR	is the growth of employment in the primary sector during the interval,
EMSGR	is the growth of employment in the secondary sector during the interval, and
EMTGR	is the growth of employment in the tertiary sector during the interval.

(ii) Indicators of the structure of employment

Once the various employment aggregates are obtained, it is possible to derive the proportions of employment accounted for by each sector (primary, secondary and tertiary).

a. Proportions by sector

The proportions of total employment accounted for by each sector can be obtained as follows:

The proportion of employment in the primary sector is calculated as:

$$PEMP(t+5) = EMP(t+5) \; / \; EM(t+5), \tag{15}$$

The proportion of employment in the secondary sector is calculated as:

$$PEMS(t+5) = EMS(t+5) \; / \; EM(t+5), \tag{16}$$

The proportion of employment in the tertiary sector is calculated as:

$$PEMT(t+5) = EMT(t+5) \; / \; EM(t+5), \tag{17}$$

where:

PEMP(t+5)	is the proportion of employment accounted for by the primary sector at the end of the interval,
PEMS(t+5)	is the proportion of employment accounted for by the secondary sector at the end of the interval, and
PEMT(t+5)	is the proportion of employment accounted for by the tertiary sector at the end of the interval.

(iii) Rates of growth of employment

As part of an employment projection, it is also possible to compute average annual rates of growth in employment, for the total employment and employment in major sectors.

a. Rate of growth in total employment

The average annual rate of growth of total employment for a given projection interval can be computed from the total employment at the beginning and the end of the interval. If, as part of the projection process, the planner makes the assumption that growth occurs over discrete intervals, then the percentage growth rate can be obtained using the formula for calculating a geometric growth rate:

$$GGREM = [(EM(t+5) / EM(t))^{1/5} - 1] \cdot 100, \tag{18}$$

where:

GGREM	is the average annual geometric growth rate of total employment for the interval.

Alternatively, if the planner assumes that growth is continuous, then the percentage growth rate of total employment can be calculated using the formula for calculating an exponential growth rate:

$$EGREM = [(\ln (EM(t+5) / EM(t))) / 5] \cdot 100, \tag{19}$$

where:

EGREM	is the average annual exponential growth rate of employment for the interval.

b. <u>Rates of growth in employment, by sector</u>

Assuming discrete growth, the percentage rates of growth of employment for major sectors can be obtained as follows:

Geometric growth rate for the primary sector is calculated as:

$$GGREMP = [(EMP(t+5) / EMP(t))^{1/5} - 1] \cdot 100, \tag{20}$$

Geometric growth rate for the secondary sector is calculated as:

$$GGREMS = [(EMS(t+5) / EMS(t))^{1/5} - 1] \cdot 100, \tag{21}$$

Geometric growth rate for the tertiary sector is calculated as:

$$GGREMT = [(EMT(t+5) / EMT(t))^{1/5} - 1] \cdot 100, \tag{22}$$

where:

GGREMP	is the average annual geometric growth rate of employment in the primary sector for the interval,
GGREMS	is the average annual geometric growth rate of employment in the secondary sector for the interval, and
GGREMT	is the average annual geometric growth rate of employment in the tertiary sector for the interval.

If the projections were based on the assumption of continuous growth, then the percentage rates of growth of employment by major sector would be calculated using the formula for obtaining the exponential growth rate. The calculations would be as follows:

Exponential growth rate for the primary sector is calculated as:

$$EGREMP = [(\ln (EMP(t+5) / EMP(t))) / 5] \cdot 100, \tag{23}$$

Exponential growth rate for the secondary sector is calculated as:

$$EGREMS = [(\ln (EMS(t+5) / EMS(t))) / 5] \cdot 100, \tag{24}$$

Exponential growth rate for the tertiary sector is calculated as:

$$EGREMT = [(\ln (EMT(t+5) / EMT(t))) / 5] \cdot 100, \quad (25)$$

where:

EGREMP	is the average annual exponential growth rate of employment in the primary sector for the interval,
EGREMS	is the average annual exponential growth rate of employment in the secondary sector for the interval, and
EGREMT	is the average annual exponential growth rate of employment in the tertiary sector for the interval.

(iv) Labour market balances

Once various projection results are obtained, it is possible to calculate the excess demand for labour or excess supply of labour using projections of labour force and employment as indicators of the future supply of, and demand for labour, respectively. Also, it is possible to calculate the excess demand or excess supply as a percentage of the total labour force.

In countries where there is sizeable non-civilian employment, which may include military or internal security personnel, the projected labour force to be used in these calculations should not be the projected total labour force, which can be obtained as described in chapter V. The projected labour force that should be used is the projected civilian labour force obtained as the difference between the projected total labour force and the projected non-civilian employment, where the latter projection is an additional input.

The reason for this is related to the fact that in the projections relating to the labour market, projections of the demand for labour (or employment) will normally be those for the civilian segment of the labour market. Therefore, projections of the supply of labour (or labour force) used to compute excess supply or demand must also be those for this segment of the market.

To calculate excess supply or excess demand, therefore, the civilian labour force may first have to be calculated; this, for the end of the time interval (t to t+5), can be obtained as:

$$CLF(t+5) = LF(t+5) - NEM(t+5), \quad (26)$$

where:

CLF(t+5)	is the civilian labour force at the end of the interval,
LF(t+5)	is the total labour force at the end of the interval, and
NEM(t+5)	is the non-civilian employment at the end of the interval.

The excess supply of (or demand for) labour for the end of the interval can be obtained as the difference between the projected civilian labour force and the projected employment for that date:

$$EXL(t+5) = CLF(t+5) - EM(t+5), \tag{27}$$

where:

EXL(t+5)	is the excess supply of labour (if positive) or excess demand for labour (if negative) for the end of the interval.

The excess demand or excess supply as a percentage of the civilian labour supply (civilian labour force) can be calculated as:

$$PEXL(t+5) = [\ EXL(t+5)\ /\ CLF(t+5)\ ] \cdot 100, \tag{28}$$

where:

PEXL(t+5)	is the excess supply of labour or excess demand for labour as a percentage of the civilian labour force at the end of the interval.

C. The inputs

This section will first list the inputs used to project employment using inverse Cobb-Douglas production functions or their log-linear transformations and then describe how those inputs can be prepared.

1. Types of inputs required

The following inputs are needed to project employment using inverse Cobb-Douglas production functions or their transformations:

(i) Projected levels of value added by industry;

(ii) Projected levels of the capital stock by industry;
(iii) Estimates of the coefficients of inverse Cobb-Douglas production functions or log-linear transformations of inverse Cobb-Douglas production functions, by industry.

If projections of labour surpluses and/or shortages are also to be prepared, the inputs should include:
(iv) Projected total labour force;
(v) Projected non-civilian employment.

2. Preparation of the inputs

In order to employ the method, the user will need to have projections of value added and the capital stock, by industry. These projections, which are part of many planning exercises can be prepared by means of suitable procedures. A simple procedure for preparing value added projections is outlined in box 17 (chapter VI). A procedure for deriving historical time series of the capital stock from time series of gross investment, which can be used to make projections of the capital stock using projections of investment,is described below (box 31). Projections of the total labour force can be prepared as described in chapter V, while those of non-civilian employment can be obtained by considering likely future developments in the non-civilian sector of the economy. The user of the method must also estimate coefficients of inverse or log-linear inverse Cobb-Douglas production functions by industry. Methods for estimating these coefficients are described below.

(a) Estimates of inverse Cobb-Douglas production functions and their transformations

Estimates of the coefficients of inverse or log-linear inverse Cobb-Douglas production functions, by industry, can be obtained by a standard method of regression analysis such as OLS. Depending on the form of the function, the coefficients can be estimated from time series data, cross section data, or from a combination of the two. No matter what kind of data is used, it should include information on value added and on production inputs, labour and capital.7/

If time series data are to be used exclusively, there should be a minimum of 10 annual observations, but preferably 20 or more. Where time series are in short supply, however, they may have to be used along with cross sectional data. If cross sectional data are to be used alone, they must refer to several points in time in order to produce a suitable estimate of the rate of technical progress.

The following section describes a method which can be used to estimate parameters of the functions using time series data, by industry. In connection with this, we shall first discuss time series data required to estimate the functions. Cross section data, although not demonstrated here, are discussed in box 30.

(i) <u>Time series data</u>

Time series data on value added by industry can usually be obtained from the national accounts. Time series data on labour inputs are typically available in the form of employment data (i.e., number of employees), collected either through periodic labour force surveys of households or through periodic surveys of establishments. In the case of data obtained from establishments, coverage is often insufficient, since such surveys often exclude traditional (or informal) and very small establishments. This raises the possibility that the data on value added, which generally include the traditional sectors, and the data on labour inputs will not refer to the same units. Where the traditional sector has been omitted from the available reports on employment, it would be useful to acquire data on traditional establishments from some other source and use it as a basis for adjusting, so that total (i.e., traditional plus modern) employment can be estimated.

Box 30

Cross section data on employment, output and the capital stock

Cross section data normally come from periodic surveys of establishments, such as manufacturing surveys or farm management surveys. Such data usually include reported levels of output and reported levels of production inputs.

For establishments in non-agricultural industries the data on output normally refer to value added, while for agricultural establishments they typically refer to gross output. Given the information on gross output, levels of value added can be obtained as a difference between gross production and the value of intermediate inputs.

Establishment survey data on inputs typically include the number of employees, as a measure of labour inputs, and frequently also some estimate of the number of days or hours actually worked. Although data on actual labour inputs are conceptually more appealing for the estimation of production functions, use of the former may provide a better basis for projecting employment.

Capital stock data are typically found in one of two forms in enterprise surveys: in the form of reported book value of the firm's plant, equipment and inventory; or, particularly in the case of farm data, in the form of reported numbers and characteristics of various physical capital items (e.g. livestock structures, tractors, electric pumps). Data on physical assets can be combined into an estimate of the current value of the capital stock by assigning each item of physical capital an estimate of its market price and then adding the various items together.

Another possible problem with the available data on labour inputs is that they typically refer to the number of persons employed rather than to labour inputs *per se* (e.g., man-hours of actual work). The two variables differ to the extent that there is variation in the number of hours worked per employee, owing either to seasonal variation or to the presence of part-time workers. Although such measurement errors would result in inaccurate estimates of the true parameters of the production function, it is less important in the present context since the objective in making employment projections is typically to project the number of workers rather than the number of hours worked.

Time series data on the capital stock by industry are generally not available in the national accounts. Therefore, it is customary to use annual data on gross investment in order to estimate the value of the capital stock over time, by successively depreciating it in each year. To initiate this process, it is necessary to estimate the value of the capital stock in each industry for the initial year of the time period for which the estimation is done. Generally, this value can be estimated by assuming that the *average capital-output ratio* of the industry during the period is equal to the *incremental capital-output ratio*. Box 31 describes the principles of the procedure.

Examples of the requisite time series data on employment, value added and the capital stock, by industry, are shown in tables 127-129. These data will be used below to illustrate the estimation of the coefficients of the inverse or log-linear inverse Cobb-Douglas production functions shown in equation (2) with OLS.

(ii) Estimation procedure

If employment is to be projected using inverse Cobb-Douglas production functions, the estimates of the coefficients of those functions can be obtained as follows: take the logarithms of the inverse Cobb-Douglas production functions, shown in equation (2), and obtain the log-linear inverse Cobb-Douglas production functions indicated in equation (3). Then, to each log-linear function, add a random disturbance term to obtain:

$$\ln EM(i,t') = \ln a'(i) + b'(i) \cdot \ln VA(i,t') + c'(i) \cdot \ln CAP(i,t') + d'(i) \cdot t' + u(i,t'); \quad i = 1,\ldots,I. \qquad (29)$$

where:

$u(i,t')$	is the random disturbance term for industry i in year t'.

Box 31

The procedure to derive time series of the capital stock from time series on gross investment

The value of the capital stock over time, by industry, can be estimated by successively depreciating it in each year using a constant rate of depreciation along with annual levels of gross investment by industry. The formula most often used is the following one:

$$CAP(i,t') = CAP(i,t'-1) \cdot [\ 1 - DR(i)\ ] + INV(i,t'); \qquad (1)$$

$$i = 1,\ldots,I,$$

where:

CAP(i,t') is the capital stock in industry i in year t'.

CAP(i,t'-1) is the capital stock in industry i in year t'-1,

DR(i) is the constant annual rate of depreciation of the capital stock in industry i, and

INV(i,t') is gross investment in industry i in year t'.

However, to initiate the process of estimating the capital stock using this formula, it is necessary to have an estimate of the value of the capital stock in each industry for the initial year of the time period for which the estimation is being performed. Most of the time this value itself would have to be estimated, using the assumption that for each industry the incremental capital-output ratio equals the average capital-output ratio during this year. On the basis of this assumption, the capital stock in the initial year of the period in question, by industry, can be obtained as follows:

$$CAP(i,0) = [\ INV(i,1) \cdot VA(i,0)\ ] \ /$$

$$[\ VA(i,1) - VA(i,0) \cdot (\ 1 - DR(i))\ ], \qquad (2)$$

$$i = 1,\ldots,I,$$

(continued)

Box 31 (continued)

where:

CAP(i,0)	is the capital stock in the first (initial) year of the time period for which the time series of the capital stock by industry is being estimated,
INV(i,1)	is the gross investment in the second (next to the initial) year of the time period for which the time series of the capital stock by industry is being estimated,
VA(i,0)	is the value added in the first (initial) year of the time period for which the time series of the capital stock by industry is being estimated, and
VA(i,1)	is the value added in the second (next to the initial) year of the time period for which the time series of the capital stock by industry is being estimated.

The above formula for calculating the initial-year capital stock by industry, which is shown in equation (2), can be derived using the formula shown in equation (1) at the beginning of the box and the assumption on the equality of the average and incremental capital-output ratios.

In order to obtain estimates of the capital stock for the initial year, as indicated in equation (2), information on value added, by industry, in that year along with the information on value added and gross investment, by industry, in the subsequent year is needed. In addition, information on depreciation rates, by industry, is also needed.

Moreover, in order to use the formula indicated in equation (2), the assumption on equality of average and incremental capital-output ratios must be approximated. Unfortunately, the data used in the course of estimating the initial-year levels of the capital stock would not frequently enable one to check as to whether or not the assumption is satisfied. Under certain conditions, the data can, however, point to the fact that the assumption is violated. Thus, whenever the value of the term [VA(i,1) - VA(i,0) · (1 - DR(i))] is smaller than or equal to zero for any given industry, one can be sure that the assumption is violated for this industry.

The functions indicated in equation (29) can be estimated by OLS using time series data on employment, value added and the capital stock, such as those shown in tables 127-129. The results would include estimates of the logarithms of the intercept coefficients of the inverse Cobb-Douglas production functions, [lna'(i)]*'s, and estimates of the partial coefficients of the value added, capital stock and the time variable--[b'(i)]*'s, [c'(i)]*'s and [d'(i)]*'s.

While the estimates of the partial coefficients can be used as they are, those of the logarithms of the intercept coefficients must be transformed into estimates of the intercept coefficients themselves. This can be accomplished by taking antilogarithms of the estimates of the logarithms of the intercept coefficients:

$$[a'(i)]^* = \text{antiln}\ [\ [\ln a'(i)]^*\ ]; \qquad i = 1,\ldots,I. \tag{30}$$

If employment is to be projected using log-linear transformations of inverse Cobb-Douglas production functions, the estimates of the coefficients of those transformations can be directly obtained by estimating functions indicated in equation (29) by OLS using data such as those shown in tables 127-129. The result of this estimation would be estimates of the logarithms of the intercept coefficients of the inverse Cobb-Douglas production functions, [lna'(i)]*'s, and estimates of the partial coefficients of those functions -- [b'(i)]*'s [c'(i)]*'s and [d'(i)]*'s.

(iii) Illustrative estimation

This section will illustrate estimation of the coefficients of the inverse Cobb-Douglas production functions using the time series data presented in tables 127-129. To obtain the estimates of the coefficients of those functions one would use those data with OLS, the result of which would be the estimates presented in table 130. 8/

The results shown in table 130 appear satisfactory inasmuch as the R^2's are relatively high and the estimated coefficients have expected signs. The exceptions are a few positive coefficients of the time variable, most of which are, however, statistically insignificant. In addition, the Durbin-Watson statistics do not suggest autocorrelation.9/

If the estimates of the coefficients are not statistically significant or do not have the expected signs and alternative data are not available, it may be preferable to use the simpler projection techniques discussed in chapters VI and VII.

Table 127. Employment for the entire country, by industry: 1968-1978

(Thousands of employed persons)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
1968	1656.5	5.0	124.5	9.6	57.6	79.3	68.5	312.8
1969	1688.5	5.4	127.7	9.2	53.8	79.7	73.9	330.1
1970	1739.8	6.3	137.8	8.9	55.8	77.4	84.4	336.8
1971	1766.1	7.2	155.4	9.0	59.8	85.4	82.1	365.9
1972	1764.8	7.1	145.5	11.3	64.3	83.9	86.2	375.7
1973	1843.5	7.4	159.8	9.6	69.4	81.1	82.5	391.4
1974	1846.3	9.4	171.8	10.0	76.0	100.5	89.2	442.1
1975	1887.0	8.3	170.3	13.7	71.0	94.1	83.7	468.5
1976	1917.8	10.3	184.0	14.4	81.8	104.9	88.4	483.3
1977	1955.6	9.5	198.2	15.6	85.4	108.5	89.5	503.3
1978	1994.0	6.7	217.3	15.3	93.8	109.4	95.1	516.2

Table 128. Value added for the entire country, by industry: 1968-1978

(Millions of local currency units; constant 1968 prices)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
1968	155.2	2.2	44.6	8.2	18.4	41.2	38.1	119.2
1969	165.4	2.0	48.6	9.1	18.6	44.0	38.6	128.1
1970	172.4	2.6	52.5	9.8	19.0	44.6	41.2	139.0
1971	175.9	2.7	59.3	10.6	20.2	47.1	43.1	152.9
1972	189.3	2.5	63.6	11.6	23.1	42.6	42.4	172.2
1973	199.4	3.6	70.8	11.9	23.8	45.5	45.2	185.7
1974	202.6	3.9	74.9	12.7	22.2	46.4	44.7	207.1
1975	237.1	3.5	75.5	13.7	21.2	49.5	42.0	223.0
1976	235.2	3.8	89.6	15.4	20.7	51.8	46.4	237.5
1977	256.5	4.0	103.9	16.5	22.3	50.5	47.7	252.6
1978	260.3	2.6	118.9	18.0	23.5	50.9	50.3	266.9

Table 129. Capital stock for the entire country, by industry: 1968-1978

(Millions of local currency units; constant 1968 prices)

Year	Agri-culture	Mining	Manufac-turing	Utilities	Cons-truction	Trade	Transport	Ser-vices
1968	15.1	17.4	27.2	7.8	19.1	39.4	102.6	8.2
1969	16.1	14.1	28.8	8.5	18.3	42.1	93.5	8.6
1970	16.2	18.2	33.5	9.0	17.8	42.7	88.0	9.4
1971	16.7	17.7	34.5	9.6	17.1	40.6	85.5	9.8
1972	18.6	15.3	39.1	10.2	17.9	35.7	78.4	10.8
1973	19.2	23.5	39.2	11.1	17.0	35.3	74.4	11.4
1974	19.8	22.9	39.3	11.3	15.7	30.2	65.5	11.9
1975	25.4	21.5	42.1	11.2	13.4	32.6	58.4	12.1
1976	24.8	20.3	45.5	12.0	12.8	31.2	55.8	12.4
1977	28.2	20.8	52.5	12.9	10.9	29.3	53.1	12.8
1978	28.0	21.0	58.7	14.1	9.4	27.3	50.3	13.1

Table 130. Estimates of the coefficients of log-linear inverse Cobb-Douglas production functions, by industry: entire country a/

Industry	Intercept	Coefficients b/ Value added	Capital stock	Time variable	R-square	Durbin-Watson
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Agriculture	-14.21215	0.45694 (2.180)	-0.24581 (-2.353)	0.010 (1.939)	0.992	2.77
Mining	-37.48326	1.07969 (7.602)	-0.73550 (-3.585)	0.021 (2.624)	0.956	2.74
Manufacturing	-11.05266	0.63665 (2.133)	-0.20063 (-0.753)	0.007 (0.337)	0.977	2.24
Utilities	-112.70138	2.12628 (1.938)	-2.96373 (-3.775)	0.059 (0.695)	0.931	1.79
Construction	-66.71019	0.24704 (0.732)	-0.19959 (-0.807)	0.036 (1.640)	0.940	2.22
Trade	62.82720	1.24755 (3.128)	-0.96450 (-3.784)	-0.030 (-1.703)	0.952	2.45
Transport	124.92383	1.24437 (1.405)	-0.76079 (-0.710)	-0.062 (-0.631)	0.813	1.82
Services	57.02152	1.47648 (4.730)	-0.87672 (-4.019)	-0.029 (-1.530)	0.995	1.92

a/ Estimated by ordinary least squares (OLS).
b/ t values are shown in parentheses.

The estimates of partial coefficients (such as those presented in table 130) can be used as they are to project employment using inverse Cobb-Douglas production functions. The estimates of the logarithms of the intercept coefficients need, however, to be transformed into estimates of the intercept coefficients themselves. As illustrated in table 131, the latter estimates (column 3) can be obtained by taking antilogarithms of the estimates of the logarithms of the intercept coefficients (column 2). Thus, the estimate of the intercept coefficient for agriculture, 0.000000673, can be calculated as follows:

$$0.000000673 = \text{antiln}(-14.21214) \qquad (30)$$

where -14.21214 is the estimate of the logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for agriculture.

If estimates of the log-linear transformations of inverse Cobb-Douglas production functions are to be used to prepare employment projections, those estimates can be obtained directly by estimating functions shown in equation (29). Estimates of such functions based on the time series data presented in tables 127-129 are those shown in table 130 and discussed above.

(b) Calibration of the empirically estimated functions

After obtaining satisfactory estimates of the inverse production functions, the planner will sometimes desire to make special adjustments in the estimated coefficients. These adjustments, which are normally referred to as "calibration", are designed to make the functions better predict the levels of employment for a particular year, or group of years, of the time period to which the data used pertain, given the levels of value added and the capital stock for that year or group of years. (If left unadjusted, the functions predict the mean level of employment over the entire time period to which data refer, using the average levels of value added and the capital stock for the period).

Although the adjustments may apply to the estimates of the intercepts as well as to those of the partial coefficients, they will be most often restricted to the intercept estimates. A calibration procedure applying to intercepts of inverse and log-linear inverse Cobb-Douglas production functions is described in annex II.

D. Illustrative example of projections

The example presented below will illustrate the use of the method based on employment functions which are log-linear transformations of inverse Cobb-Douglas production functions to prepare a national projection of employment. The example will show how the relevant calculations are made for the projection interval 0-5 and will also provide complete projection results for a 20-year projection period.

Table 131. Computing estimates of intercept coefficients of inverse Cobb-Douglas production functions, by industry

Industry (1)	Intercept of log-linear inverse Cobb-Douglas production function a/ (2)	Intercept of non-linear inverse Cobb-Douglas production function b/ (3)
Agriculture	-14.21214	6.7260×10^{-7}
Mining	-37.48326	5.2629×10^{-17}
Manufacturing	-11.05265	1.5845×10^{-5}
Utilities	-112.70137	1.1335×10^{-49}
Construction	-66.71018	1.0669×10^{-29}
Trade	62.82719	1.9298×10^{27}
Transport	124.92382	1.7936×10^{54}
Services	57.02152	5.8094×10^{24}

a/ From table 130, col. 2.
b/ Antiln(Col. 2).

The calculations presented in the example will be based on the inputs contained in table 132, which shows projected levels of value added and the capital stock, by industry, for dates five years apart, starting with the initial year of the plan, which is denoted as year 0. Table 132 also shows estimates of the partial coefficients of the log-linear transformations of inverse Cobb-Douglas production functions, presented above in table 130, along with adjusted logarithms of the intercept coefficients shown in table 138 in annex II.

(a) Employment by sector

To obtain the levels of employment by industry for a given date, it is necessary to evaluate estimated log-linear transformations of inverse Cobb-Douglas production functions, by industry, using the levels of value added and the capital stock along with the appropriate value of the time variable. This would result in the logarithms of the projected levels of employment. To obtain the levels of employment themselves, it would be necessary to take antilogarithms of those results. Table 133 illustrates how the levels of employment by industry for the end of the projection interval 0-5 are calculated.10/

In particular, the logarithm of the level of employment for each industry in year 5 (column 9), is obtained as the adjusted intercept coefficient (column 2) plus the sum of three products. The first product is that of the estimate of the value added coefficient (column 3) and the logarithm of the projected level of value added (column 6). The second product is that of the estimate of the capital stock coefficient and the logarithm of the projected level of the capital stock (column 7). The third product is that of the time variable coefficient (column 5) and the value of the time variable (column 8). The level of employment in any industry (column 10) is then calculated as the antilogarithm of the logarithm of the employment level.

For example, the logarithm of the level of employment in agriculture in year 5, 7.65723, is obtained as follows:

$$7.65723 = -14.20909 + 0.45694 \cdot \ln 308.2 + (-0.24582) \cdot \ln 40.2 + 0.01015 \cdot (1980 + 5), \qquad (5)$$

where -14.20909 is the adjusted intercept coefficient for agriculture and where 0.45694 and 308.2 are, respectively, the estimate of the coefficient of the value added variable and the projected level of value added in agriculture in year 5. The estimate of the coefficient of the capital stock variable is -0.24582, and the projected level of the capital stock in this industry is 40.2. Assuming that 1980 is the initial year of the projection, the estimate of the time variable coefficient, 0.01015, is multiplied by 1985 (= 1980 + 5), the value of the time variable.

Table 132. Inputs for projecting employment, by industry: entire country

Industry	Year 0	5	10	15	20
	(Millions of local currency units)				
			Value added		
Agriculture	273.1	308.2	347.8	392.5	443.1
Mining	2.9	4.0	5.5	7.6	10.4
Manufacturing	140.4	212.7	322.5	489.3	742.7
Utilities	21.2	31.9	48.0	72.5	109.4
Construction	25.6	31.5	38.9	48.0	59.3
Trade	59.0	85.1	122.9	177.6	256.7
Transport	56.0	73.0	95.3	124.4	162.5
Services	312.5	464.3	691.2	1031.2	1541.4
			Capital stock		
Agriculture	31.0	40.2	52.0	67.4	87.2
Mining	22.9	28.4	35.2	43.7	54.2
Manufacturing	70.9	113.7	182.2	292.1	468.4
Utilities	16.1	22.6	31.8	44.5	62.5
Construction	10.4	13.5	17.5	22.6	29.3
Trade	29.9	37.7	47.4	59.6	75.0
Transport	60.8	97.4	156.1	250.3	401.3
Services	14.3	17.7	22.0	27.2	33.8

Estimates of the coefficients of log-linear inverse Cobb-Douglas production functions

	Adjusted intercept	Value added	Capital stock	Time variable
Agriculture	-14.20909	0.45694	-0.246	0.010
Mining	-37.46840	1.07969	-0.736	0.021
Manufacturing	-11.04406	0.63665	-0.201	0.007
Utilities	-112.66345	2.12628	-2.964	0.059
Construction	-66.69979	0.24704	-0.200	0.036
Trade	62.82550	1.24755	-0.965	-0.030
Transport	124.91470	1.24437	-0.761	-0.062
Services	57.01587	1.47648	-0.877	-0.029

Table 133. Deriving employment, by industry: entire country, year 5

Industry	Estimates of coefficients of log-linear inverse Cobb-Douglas production functions a/				In year 5				
	Adjusted intercept	Value added	Capital stock	Time variable	Value added a/	Capital stock a/	Value of time variable b/	Logarithm of projected employment c/	Projected employment d/ (thousands of persons)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Agriculture	-14.20909	0.45694	-0.24581	0.010	308.2	40.2	1985	7.657	2115.9
Mining	-37.46840	1.07969	-0.73550	0.021	4.0	28.4	1985	2.298	10.0
Manufacturing	-11.04406	0.63665	-0.20063	0.007	212.7	113.7	1985	5.669	289.8
Utilities	-112.66345	2.12628	-2.96373	0.059	31.9	22.6	1985	2.954	19.2
Construction	-66.69979	0.24704	-0.19959	0.036	31.5	13.5	1985	4.793	120.6
Trade	62.82550	1.24755	-0.96450	-0.030	85.1	37.7	1985	4.815	123.3
Transport	124.91470	1.24437	-0.76079	-0.062	73.0	97.4	1985	4.084	59.4
Services	57.01587	1.47648	-0.87672	-0.029	464.3	17.7	1985	6.599	734.2

a/ From table 132.
b/ Based on assumption that 1980 is the initial year of the projection period.
c/ (Col. 2) + (Col. 3) . (ln(Col. 6)) + (Col. 4) . (ln(Col. 7) + (Col. 5) . (Col. 8).
d/ Antiln(Col. 9).

Given the logarithm of the level of employment in agriculture in year 5, the level of employment itself, 2,115.9, is obtained by taking the antilogarithm of this result:

$$2{,}115.9 = \text{antiln}\ (7.65723). \tag{6}$$

Performing the calculations illustrated for the end of the interval 0-5 for each five-year interval of the entire projection period produces the projected levels of employment by industry for the entire period. The projected levels for the 20-year projection interval are shown in table 134.

(b) Other results

Other results that are useful in planning can be obtained as part of a projection at the national level. These include various employment aggregates, indicators of the structure of employment and the rates of growth of employment.11/

(i) Employment aggregates

The employment aggregates, which can be derived from the projection by industry, include total employment and employment in various sectors at dates five years apart. They also include increases in total employment and employment by sector over the intervening projection intervals.

a. Total employment

Total employment at the end of a given projection interval is obtained by aggregating the projected levels of employment by industry. Total employment in year 5, 3,472.3, is computed by adding the projected levels of employment by industry. Total employment is shown in table 135 for the entire 20-year projection period. The increase in total employment over this period is indicated in figure XXVIII.

b. Employment by sector

Employment in the primary, secondary and the tertiary sectors can be obtained by aggregating employment projected for various industries, using appropriate aggregation rules. For illustrative purposes, it will be assumed that the primary sector consists of agriculture and mining, the secondary sector of manufacturing, utilities and construction, and the tertiary sector of trade, transportation and services.

Table 134. Projected employment, by industry: entire country

(Thousands of persons)

Industry	Year 0	5	10	15	20
Agriculture	2027.8	2115.9	2207.9	2303.9	2404.3
Mining	7.5	10.0	13.2	17.5	23.3
Manufacturing	235.9	289.8	356.2	437.9	538.6
Utilities	16.3	19.2	22.6	26.7	31.7
Construction	100.8	120.6	144.3	172.8	206.8
Trade	113.2	123.3	134.4	146.5	159.7
Transport	83.1	59.4	42.4	30.3	21.6
Services	570.5	734.2	947.8	1227.3	1593.6

Table 135. Employment aggregates, structure and rates of growth: entire country

	Year				
	0	5	10	15	20
Employment aggregates (thousands of persons)					
Levels of employment					
Total	3155.1	3472.3	3868.8	4362.9	4979.6
Primary	2035.3	2125.8	2221.1	2321.5	2427.6
Secondary	353.0	429.6	523.2	637.4	777.0
Tertiary	766.8	916.8	1124.6	1404.0	1774.9
Growth in employment					
Total	317.2	396.5	494.1	616.6	
Primary	90.5	95.2	100.4	106.1	
Secondary	76.6	93.5	114.3	139.6	
Tertiary	150.0	207.7	279.5	370.9	
Indicators of employment structure					
Proportions of total employment by sector					
Primary	0.65	0.61	0.57	0.53	0.49
Secondary	0.11	0.12	0.14	0.15	0.16
Tertiary	0.24	0.26	0.29	0.32	0.36
Rates of growth of employment (percentage)					
Total	1.93	2.19	2.43	2.68	
Primary	0.87	0.88	0.89	0.90	
Secondary	4.01	4.02	4.03	4.04	
Tertiary	3.64	4.17	4.54	4.80	

Figure XXVIII. Total employment

i. Employment in the primary sector

Employment in the primary sector in year 5, 2,125.8, is obtained as:

$$2{,}125.8 = 2{,}115.9 + 10.0, \qquad (8)$$

where 2,115.9 and 10.0 are, respectively, projected levels of employment in agriculture and mining.

ii. Employment in the secondary sector

Employment in the secondary sector in year 5, 429.6, is obtained as:

$$429.6 = 289.8 + 19.2 + 120.6, \qquad (9)$$

where 289.8, 19.2 and 120.6 are, respectively, projected levels of employment in manufacturing, utilities and construction.

iii. Employment in the tertiary sector

Employment in the tertiary sector in year 5, 916.8, is obtained as:

$$916.8 = 123.3 + 59.4 + 734.2, \qquad (10)$$

where 123.3, 59.4 and 734.2 are, respectively, projected levels of employment in trade, transportation and services.

Employment by sector obtained for different dates over the projection period is presented in figure XXIX.

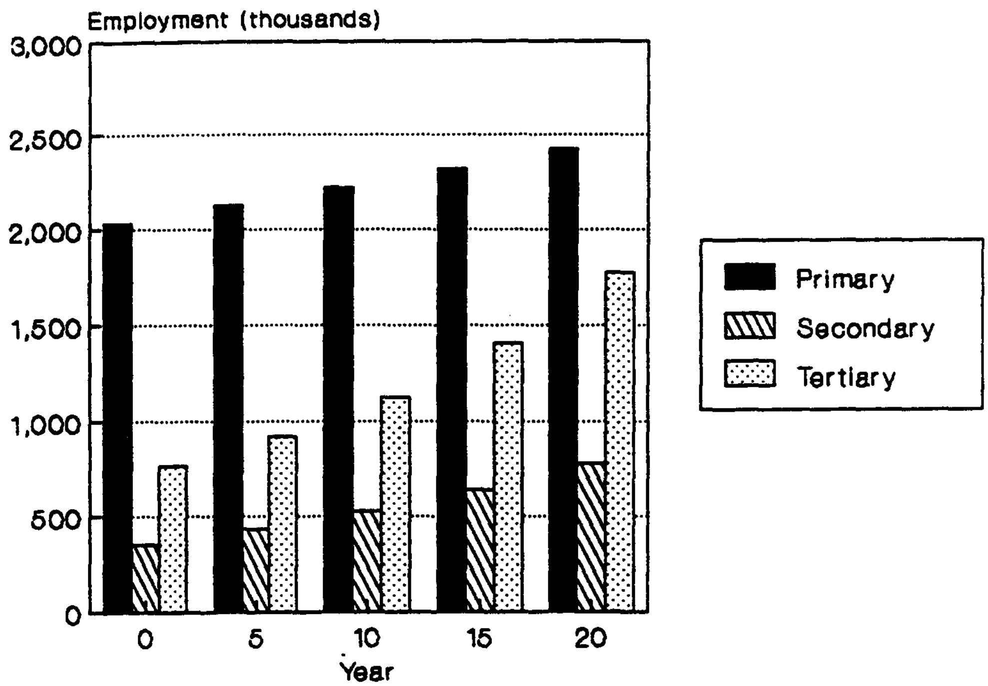
c. Growth in total employment

The growth in total employment over a given projection interval equals the difference between total employment at the end of the interval and total employment at its beginning. For the interval 0-5, the growth in total employment, 317.2, is obtained as:

$$317.2 = 3{,}472.3 - 3{,}155.1, \qquad (11)$$

where 3,155.1 and 3,472.3 are, respectively, total employment at the beginning and the end of the interval (shown in columns corresponding to years 0 and 5 respectively).

Figure XXIX. Employment: primary, secondary and tertiary sectors



d. Growth in employment, by sector

The increase in employment over the interval 0-5 in various sectors is obtained as follows:

Growth of employment in the primary sector, 90.5, is:

$$90.5 = 2{,}125.8 - 2{,}035.3, \qquad (12)$$

where 2,035.3 and 2,125.8 are, respectively, total employment in the primary sector in years 0 and 5;

Growth of employment in the secondary sector, 76.6, is:

$$76.6 = 429.6 - 353.0, \qquad (13)$$

where 353.0 and 429.6 are the levels of employment in the secondary sector in years 0 and 5; and

Growth of employment in the tertiary sector, 150.0, is:

$$150.0 = 916.8 - 766.8, \qquad (14)$$

where 766.8 and 916.8 are total employment in the tertiary sector in years 0 and 5.

(ii) Indicators of the structure of employment

Indicators of the structure of employment that can be calculated as part of an employment projection include proportions of total employment found in each sector.

a. Proportions by sector

For the end of the interval 0-5, these proportions are obtained as follows:

The proportion of employment in the primary sector, 0.61, is:

$$0.61 = 2{,}125.8 \;/\; 3{,}472.3, \qquad (15)$$

where 2,125.8 and 3,472.3 are, respectively, employment in the primary sector and the total employment;

The proportion of employment in the secondary sector, 0.12, is:

$$0.12 = 429.6 \ / \ 3{,}472.3, \tag{16}$$

where 429.6 is employment in the secondary sector;

The proportion of employment in the tertiary sector, 0.26, is:

$$0.26 = 916.8 \ / \ 3{,}472.3, \tag{17}$$

where 916.8 is employment in the tertiary sector.

(iii) Rates of growth of employment

The rates of growth of employment can be calculated for total employment and for employment in each sector.

a. Rate of growth in total employment

If growth in employment is assumed to occur over discrete intervals, the average annual growth rate of total employment for a given interval is obtained using the geometric growth rate formula. For the projection interval 0-5, this annual growth rate, 1.93 per cent (table 134), is obtained as follows:

$$1.93 = [\ (3{,}472.3 \ / \ 3{,}155.1)^{1/5} - 1\ ] \cdot 100, \tag{18}$$

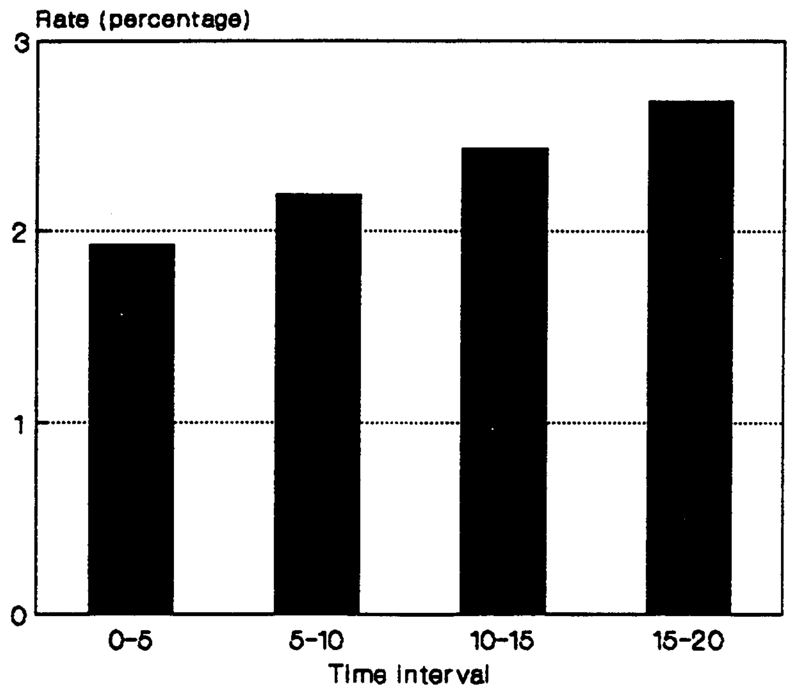
where 3,155.1 and 3,472.3 are the levels of total employment in years 0 and 5, respectively, and 5 is the length of the interval.

Rates of growth of total employment over the 20-year projection period which were computed using the geometric rate formula are shown in figure XXX.

If it is assumed that growth in employment is continuous, the average annual growth rate of total employment for a given interval is obtained by substituting the same data as above in the exponential growth rate formula. For the projection interval 0-5, this annual growth rate, 1.92 per cent, is obtained as follows:

$$1.92 = [\ \ln (3{,}472.3 \ / \ 3{,}155.1) \ / \ 5\ ] \cdot 100, \tag{19}$$

Figure XXX. Rate of growth in total employment



b. Rates of growth in employment, by sector

Assuming discrete growth, the rates of increase in employment, by sector, for the interval 0-5 are calculated as:

The annual rate of growth of employment in the primary sector, 0.87 per cent, is obtained as:

$$0.87 = [\ (2{,}125.8\ /\ 2{,}035.3)^{1/5} - 1\ ] \cdot 100, \qquad (20)$$

where 2,035.3 and 2,125.8 are the levels of employment in the primary sector in years 0 and 5, respectively;

The annual rate of growth of employment in the secondary sector, 4.01 per cent, is obtained as follows:

$$4.01 = [\ (429.6\ /\ 353.0)^{1/5} - 1\ ] \cdot 100, \qquad (21)$$

where 353.0 and 429.6 are the levels of employment in the secondary sector in years 0 and 5, respectively;

The annual rate of growth of employment in the tertiary sector, 3.64 per cent, is obtained as follows:

$$3.64 = [\ (916.8\ /\ 766.8)^{1/5} - 1\ ] \cdot 100, \qquad (22)$$

where 766.8 and 916.8 are the levels of employment in the tertiary sector in years 0 and 5, respectively.

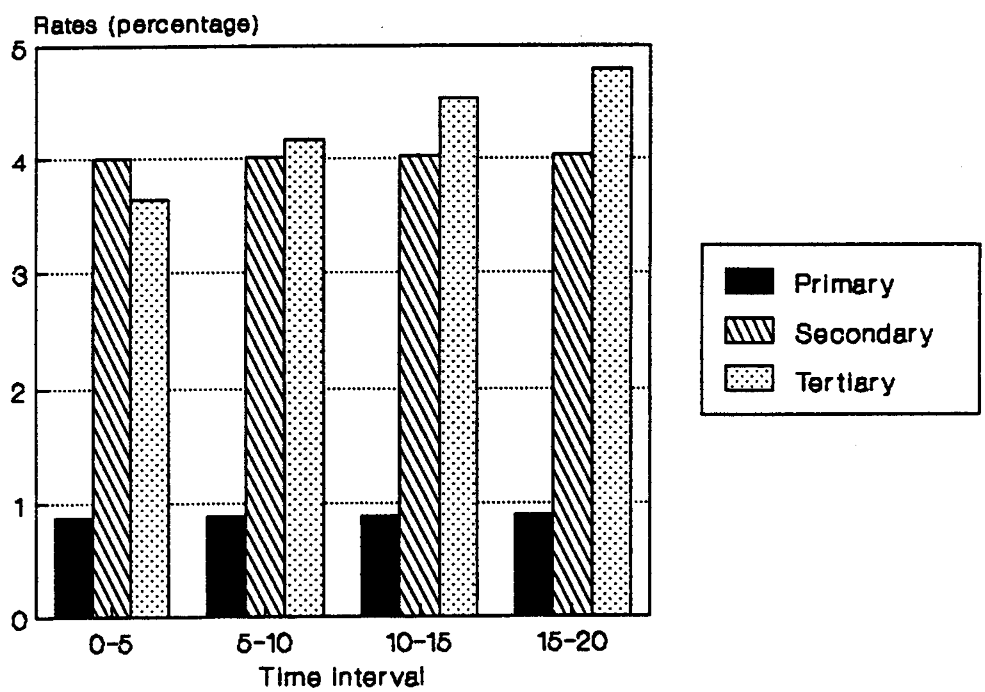
Rates of growth of employment in primary, secondary and tertiary sectors over the 20-year projection interval are shown in figure XXXI.

If continuous growth is assumed, rates of employment by sector would be calculated using the exponential growth rate formula. The calculations would be analogous to that indicated by equation (19) for total employment.

(iv) Labour market balances

If labour force projections are available for the same years as the employment projection, it is possible to calculate the levels of excess demand for or excess supply of labour. Also, it is possible to calculate the excess demand for or excess supply of labour as a percentage of the level of labour supply.

Figure XXXI. Rates of growth of employment: primary, secondary and tertiary sectors



The calculations are based on the projected total labour force, diminished where necessary by the size of non-civilian employment, and the projected total employment as indicators of the labour supply and the labour demand.

In order to illustrate these calculations, we shall use projections of the total labour force and the total employment (shown, respectively, in tables 37 and 135) along with the illustrative projections of non-civilian employment, which are shown in table 136. These calculations are illustrated for the end of the projection interval 0-5 in table 137.

The civilian labour force in year 5, 3,615.4, can be calculated as follows:

$$3{,}615.4 = 3{,}651.9 - 36.5, \tag{26}$$

where 3,651.9 and 36.5 are the projected total labour force and the projected non-civilian employment for year 5, shown in columns 2 and 3. The calculated level of civilian labour force for year 5 is shown in column 4.

The excess supply of labour for the same date, 143.1, is calculated as follows:

$$143.1 = 3{,}615.4 - 3{,}472.3, \tag{27}$$

where 3,615.4 is the civilian labour force and 3,472.3 is the total employment shown in columns 4 and 5, respectively.

The excess supply expressed as a percentage of the civilian labour force in year 5, 3.96 per cent, is calculated as follows:

$$3.96 = (143.1 \,/\, 3{,}615.4) \cdot 100. \tag{28}$$

This percentage is shown in column 7.

E. Summary

This chapter has described the method which uses empirically estimated inverse Cobb-Douglas production functions or their log-linear transformations, by industry, to make employment projections for the entire country. As part of the description of the method, the procedure utilising these functions to make projections by industry was presented. In addition, the types of inputs required by the method were described and the preparation of the inputs was discussed. Lastly, an example of a projection was described, including the various outputs that can be generated. A complete listing of the outputs that the method is capable of producing is shown in box 32.

Table 136. Projected non-civilian employment: entire country

(Thousands of persons)

Year (1)	Non-civilian employment (2)
0	32.5
5	36.5
10	41.5
15	47.4
20	53.8

Table 137. Labour market balances: entire country

Year	Total labour force a/	Non-civilian employment b/	Civilian labour force c/	Total employment d/	Excess supply/ demand e/	Excess supply/ demand f/
	(thousands of persons)					(percentage of civilian labour force)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	3251.5	32.5	3219.0	3155.1	63.9	1.98
5	3651.9	36.5	3615.4	3472.3	143.1	3.96
10	4147.8	41.5	4106.3	3868.8	237.5	5.78
15	4735.4	47.4	4688.0	4362.9	325.1	6.93
20	5377.4	53.8	5323.6	4979.6	344.0	6.46

a/ From table 37, "Labour force (Total)".
b/ From table 136.
c/ (Col. 2) - (Col. 3).
d/ From table 135, "Levels of employment (Total)".
e/ (Col. 4) - (Col. 5).
f/ ((Col. 6)/(Col. 4)) . (100).

Box 32

Outputs of the method for making employment projections using inverse Cobb-Douglas production functions or their transformations

1. Employment by industry

2. Employment aggregates

Levels of employment:

Total

Primary sector
Secondary sector
Tertiary sector

Growth in employment:

Total

Primary sector
Secondary sector
Tertiary sector

3. Indicators of the structure of employment

Proportions of employment by sector:

Primary sector
Secondary sector
Tertiary sector

4. Rates of growth of employment

Total

Primary sector
Secondary sector
Tertiary sector

5. Labour market balances

Excess supply of or excess demand for labour

Percentage excess supply of or excess demand for labour

F. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

i = 1,...,I	are the industries of the nation's economy
t	is the year of the projection period
t'	is the calendar year
$\bar{t}$'	is the calendar year designated as the initial year of the projection

(b) List of variables

CAP(i,t')	is the capital stock in industry i in year t'
CAP(i,t+5)	is the capital stock in industry i at the end of the interval
CLF(t+5)	is the civilian labour force at the end of the interval
DR(i)	is the constant annual rate of depreciation of the capital stock in industry i
EGREM	is the average annual exponential growth rate of employment for the interval
EGREMP	is the average annual exponential growth rate of employment in the primary sector for the interval
EGREMS	is the average annual exponential growth rate of employment in the secondary sector for the interval
EGREMT	is the average annual exponential growth rate of employment in the tertiary sector for the interval
EM(i,t')	is the labour employed in industry i in year t'
EM(i,t+5)	is the labour employed in industry i at the end of the interval
EM(t+5)	is the total employment at the end of the interval
EMGR	is the growth in total employment during the interval

EMP(t+5)	is the employment in the primary sector at the end of the interval
EMPGR	is the growth of employment in the primary sector during the interval
EMS(t+5)	is the employment in the secondary sector at the end of the interval
EMSGR	is the growth of employment in the secondary sector during the interval
EMT(t+5)	is the employment in the tertiary sector at the end of the interval
EMTGR	is the growth of employment in the tertiary sector during the interval
EXL(t+5)	is the excess supply of labour (if positive) or excess demand for labour (if negative) for the end of the interval
GGREM	is the average annual geometric growth rate of total employment for the interval
GGREMP	is the average annual geometric growth rate of employment in the primary sector for the interval
GGREMS	is the average annual geometric growth rate of employment in the secondary sector for the interval
GGREMT	is the average annual geometric growth rate of employment in the tertiary sector for the interval
INV(i,t')	is gross investment in industry i in year t'
LF(t+5)	is the labour force size at the end of the interval
NEM(t+5)	is the non-civilian employment at the end of the interval
PEMP(t+5)	is the proportion of employment accounted for by the primary sector at the end of the interval
PEMS(t+5)	is the proportion of employment accounted for by the secondary sector at the end of the interval
PEMT(t+5)	is the proportion of employment accounted for by the tertiary sector at the end of the interval

PEXL(t+5) is the excess supply of labour or excess demand for labour as a percentage of the total labour force at the end of the interval

VA(i,t') is the value added in industry i in year t'

VA(i,t+5) is the value added in industry i at the end of the interval

(c) List of special symbols

a'(i) is the intercept parameter for industry i

a(i) is the elasticity parameter relating value added to the capital stock for industry i

antiln is the antilogarithm of the natural logarithm

b'(i) is the elasticity parameter relating labour to value added for industry i

b(i) is the elasticity parameter relating value added to labour for industry i

c'(i) is the elasticity parameter relating labour to the capital stock for industry i

d'(i) is the parameter relating labour to the time variable for industry i

e is the base of the natural logarithm

I is the number of industries

Ip is the number of industries in the primary sector

Is is the number of industries in the secondary sector

ln is the natural logarithm

r(i) is the constant rate of technical change for industry i

u(i,t') is the random disturbance term for industry i in year t'

z(i) is the intercept parameter for industry i

[a'(i)]*	is the estimate of the intercept coefficient of the inverse Cobb-Douglas production function for industry i
[b'(i)]*	is the estimate of the partial coefficient of the value added variable in the inverse Cobb-Douglas production function for industry i
[c'(i)]*	is the estimate of the partial coefficient of the capital stock variable in the inverse Cobb-Douglas production function for industry i
[d'(i)]*	is the estimate of the partial coefficient of the time variable in the inverse Cobb-Douglas production function for industry i
[lna'(i)]*	is the estimate of the logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for industry i

2. Equations

A. Description

(a) Cobb-Douglas production functions

$$VA(i,t') = z(i) \cdot CAP(i,t')^{a(i)} \cdot EM(i,t')^{b(i)} \cdot e^{[r(i)\ .\ t']}; \quad i = 1,\ldots,I \tag{1}$$

(b) Inverse Cobb-Douglas production functions

$$EM(i,t') = a'(i) \cdot VA(i,t')^{b'(i)} \cdot CAP(i,t')^{c'(i)} \cdot e^{[d'(i)\ .\ t']}; \quad i = 1,\ldots,I, \tag{2}$$

$$\begin{aligned} lnEM(i,t') = {} & lna'(i) + b'(i) \cdot lnVA(i,t') + c'(i) \cdot lnCAP(i,t') \\ & + d'(i) \cdot t'; \\ & i = 1,\ldots,I, \end{aligned} \tag{3}$$

(c) Employment by industry

$$EM(i,t+5) = [a'(i)]^* \cdot VA(i,t+5)^{[b'(i)]^*} \cdot CAP(i,t+5)^{[c'(i)]^*} \cdot \tag{4}$$

$$e[\ [d'(i)]^* \cdot (\bar{t}'+t+5)];$$

$$i = 1,\ldots,I,$$

$$\ln EM(i,t+5) = [\ln a'(i)]^* + [b'(i)]^* \cdot \ln VA(i,t+5) \tag{5}$$

$$+ [c'(i)]^* \cdot \ln CAP(i,t+5) + [d'(i)]^* \cdot (\bar{t}'+t+5);$$

$$i = 1,\ldots,I,$$

$$EM(i,t+5) = \text{antiln}[\ \ln EM(i,t+5)\ ]; \tag{6}$$

$$i = 1,\ldots,I,$$

(d) Other results

(i) Employment aggregates

a. Total employment

$$EM(t+5) = \sum_{i=1}^{I} EM(i,t+5) \tag{7}$$

b. Employment by sector

i. Employment in the primary sector

$$EMP(t+5) = \sum_{i=1}^{Ip} EM(i,t+5) \tag{8}$$

ii. Employment in the secondary sector

$$EMS(t+5) = \sum_{i=Ip+1}^{Ip+Is} EM(i,t+5) \tag{9}$$

iii. Employment in the tertiary sector

$$EMT(t+5) = \sum_{i=Ip+Is+1}^{I} EM(i,t+5) \quad (10)$$

c. Growth in total employment

$$EMGR = EM(t+5) - EM(t) \quad (11)$$

d. Growth in employment, by sector

$$EMPGR = EMP(t+5) - EMP(t) \quad (12)$$

$$EMSGR = EMS(t+5) - EMS(t) \quad (13)$$

$$EMTGR = EMT(t+5) - EMT(t) \quad (14)$$

(ii) Indicators of the structure of employment

a. Proportions by sector

$$PEMP(t+5) = EMP(t+5) / EM(t+5) \quad (15)$$

$$PEMS(t+5) = EMS(t+5) / EM(t+5) \quad (16)$$

$$PEMT(t+5) = EMT(t+5) / EM(t+5) \quad (17)$$

(iii) Rates of growth of employment

a. Rate of growth in total employment

$$GGREM = [(EM(t+5) / EM(t))^{1/5} - 1] \cdot 100, \quad (18)$$

$$EGREM = [(\ln (EM(t+5) / EM(t))) / 5] \cdot 100, \quad (19)$$

b. Rates of growth in employment by sector

$$GGREMP = [(EMP(t+5) / EMP(t))^{1/5} - 1] \cdot 100, \quad (20)$$

$$GGREMS = [(EMS(t+5) / EMS(t))^{1/5} - 1] \cdot 100, \quad (21)$$

$$GGREMT = [(EMT(t+5) / EMT(t))^{1/5} - 1] \cdot 100, \quad (22)$$

$$EGREMP = [(\ln (EMP(t+5) / EMP(t))) / 5] \cdot 100, \quad (23)$$

$$EGREMS = [(\ln (EMS(t+5) / EMS(t))) / 5] \cdot 100, \quad (24)$$

$$EGREMT = [(\ln (EMT(t+5) / EMT(t))) / 5] \cdot 100, \quad (25)$$

(iv) Labour market balances

$$CLF(t+5) = LF(t+5) - NEM(t+5), \quad (26)$$

$$EXL(t+5) = CLF(t+5) - EM(t+5), \quad (27)$$

$$PEXL(t+5) = [EXL(t+5) / CLF(t+5)] \cdot 100 \quad (28)$$

B. The inputs

1. Types of inputs required

2. Preparation of the inputs

(a) Estimates of inverse Cobb-Douglas production functions and their transformations

(i) Time series data

(ii) Estimation procedure

$$\ln EM(i,t') = \ln a'(i) + b'(i) \cdot \ln VA(i,t') + c'(i) \cdot \ln CAP(i,t') \tag{29}$$
$$+ d'(i) \cdot t' + u(i,t');$$
$$i = 1,\ldots,I,$$

$$[a'(i)]^* = \text{antiln}\ [\ [\ln a'(i)]^*\ ]; \tag{30}$$
$$i = 1,\ldots,I$$

$$a'(i) = z(i)^{-1/b(i)},$$

$$b'(i) = 1/b(i),$$

$$c'(i) = -a(i)/b(i),$$

and

$$d'(i) = -r(i)/b(i).$$

Hence, the parameters of the employment function indicated in equation (2) could be derived directly from the parameters of the Cobb-Douglas production function shown in equation (1).

Notes

1/ Annex I describes various forms of the Cobb-Douglas production function and their properties.

2/ Throughout the chapter, "value added" and "capital stock" will refer, respectively, to value added and the capital stock measured in constant prices.

3/ However, it assumes that the elasticity of substitution between production factors is equal to 1, which may not necessarily be the case in any given industry.

4/ This description of the method assumes that the only two relevant inputs of production are labour and capital.

5/ The functions shown in equation (1) were obtained by adding the industry dimension to the Cobb-Douglas production function shown in equation (2) in annex I.

6/ The parameters of employment functions indicated in equation (2) are related to the parameters of the Cobb-Douglas production functions shown in equation (1) as follows:

7/ Value added is most often used as the measure of output in production function analysis, although it is fairly common to use gross output as the dependent variable in agricultural production functions (in which case, purchased inputs, such as seed and fertilizer, are treated as inputs).

8/ To obtain the results shown in table 130, time series data presented in table 127 through 129 were subject to a logarithmic transformation. Values assigned to the time variable were 1968 through 1978.

9/ For the discussion of autocorrelation and the use of Durbin Watson statistics, see chapter VII, section C.

10/ The log-linear transformations of inverse Cobb-Douglas production functions used in this example were estimated, among other things, from the time series on employment shown in table 127, which are expressed in units of 1,000 employed persons. Therefore, the levels of employment in this illustrative example will be given in thousands of employed persons.

11/ Much of this section is similar to section D.1(c) in chapters VI or chapter VII. The reader who is familiar with that material may wish to move directly to the next section.

Annex I

THE COBB-DOUGLAS PRODUCTION FUNCTION

The Cobb-Douglas production function, which is by far the most widely used form of production function, has been applied in various planning exercises, especially in a wide range of planning models.a/ The function, which was formulated by Cobb and Douglas in an attempt to explain the relative constancy of the shares of capital and labour in national income, is widely used in theories of income distribution, production and economic growth.

The Cobb-Douglas function may assume a number of different specifications, several of which will be introduced and discussed below. Irrespective of the specification, however, the function has a number of properties. First, it embodies the assumption that inputs can be fairly freely substituted for one another, although not as freely as with some other functions. Secondly, it has the property that if one or more inputs is increased, the productivity of the other inputs always increases. Third, the function is homogeneous, in that when all inputs are simultaneously increased, output always increases by a constant proportion--not necessarily equal to 1 --of the increase in inputs.

Simpler specifications of the Cobb-Douglas function assume that production factors (labour and capital) are homogeneous or they do not allow for change in technology. More complex specifications treat some or all factors of production as composite production inputs or allow for technical change. This annex will introduce various specifications of the Cobb-Douglas production function, starting with the simplest. The calendar year will be used as unit observation.

A. Function without technical change

The simplest Cobb-Douglas production function, which assumes homogeneous inputs and no technical change, can be represented in the following general form:

$$VA(t') = z \cdot CAP(t')^{a} \cdot EM(t')^{b}, \tag{1}$$

where:

t'	is the calendar year,
$VA(t')$	is the value added in year t',b/
$CAP(t')$	is the measure of the capital stock in year t',

EM(t')	is the measure of labour employed in year t',
z	is the intercept parameter,
a	is the elasticity parameter (box 33) relating to the capital stock, and
b	is the elasticity parameter relating to labour.

If the elasticity parameter of an input is less than one, there will be diminishing returns to that input. The sum of the two elasticity parameters, a + b, provides a measure of returns to scale. The returns to scale can be either increasing, constant, or decreasing, depending on whether the sum, a + b, is greater than, equal to, or less than unity. In this connection, if one is willing to assume constant returns to scale (an assumption often made in practice), the number of parameters to be estimated is reduced by 1, since in this case, b = 1 - a.

B. Functions with technical change

The Cobb-Douglas production function indicated in equation (1) is suitable for estimating production relationships on the basis of cross-section data. However, over time, the production function should reflect the process of technical change. This can be done in one of several ways, the most common of which is to assume that the technical change is "disembodied"; that is, it affects the function's intercept alone (Solow, 1957). The following specification is typical of this approach:

$$VA(t') = z \cdot CAP(t')^{a} \cdot EM(t')^{b} \cdot e^{(r\ t')}, \tag{2}$$

where:

r	is the constant rate of disembodied technical change, and
e	is the base of the natural logarithm.

Another approach to specifying technical change in the production function is to assume that it affects the elasticity parameters, a and b. The following specification provides an example of this approach:

$$VA(t') = z \cdot CAP(t')^{a(t')} \cdot EM(t')^{b(t')}, \tag{3}$$

Box 33

Glossary

<u>Diminishing returns</u>
A situation where if one factor of production is increased by small, constant amounts, all other factor quantities being held constant, then after some point the resulting increases in output become smaller and smaller.

<u>Disembodied technical change</u>
A type of technical change that influences output through shifts in the level of the production function rather than via factors of production.

<u>Elasticity parameter</u>
A parameter indicating the extent to which value added changes during a specified period of time (usually a year) in response to a given change in the amount of labour of capital used.

<u>Vintage capital models</u>
A class of economic models in which the aggregate capital stock consists of capital of different years of production (vintages).

where:

a(t')	is the elasticity parameter relating to the capital stock in year t', and
b(t')	is the elasticity parameter relating to labour in year t'.

As indicated by equation (3), this type of approach assumes that the elasticity parameters are related to time (Brown, 1966).

An alternative treatment of technical changes is to assume that it is embodied within one or more of the inputs. Thus, for example, it has been argued that technical change cannot occur independently of investment, and that it is in fact embodied in the physical characteristics of new machines. These ideas underlie the <u>vintage capital models</u> (Solow, 1959). Another approach has been to assume that technical change is embodied in the labour force in the form of "knowledge" (Griliches, 1967; Christensen and Jorgensan, 1970). The Cobb-Douglas production function with embodied technical change is written as follows:

$$VA(t') = z \cdot CAP^*(t')^a \cdot EM^*(t')^b, \tag{4}$$

where:

CAP*(t')	is the index of capital inputs in year t', which reflects quality improvements in this input over time, and
EM*(t')	is the index of labour inputs in year t', which reflects quality improvements in this input over time.

C. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

t'	is the calendar year

(b) List of variables

CAP(t')	is the measure of the capital stock in year t'
CAP*(t')	is the index of capital inputs in year t', which reflects quality improvements in this input over time
EM(t')	is the measure of labour employed in year t'
EM*(t')	is the index of labour inputs in year t', which reflects quality improvements in this input over time
VA(t')	is the value added in year t'

(c) List of special symbols

a	is the elasticity parameter relating to the capital stock
a(t')	is the elasticity parameter relating to the capital stock in year t'
b	is the elasticity parameter relating to labour

$b(t')$	is the elasticity parameter relating to labour in year t'
e	is the base of the natural logarithm
r	is the constant rate of disembodied technical change
z	is the intercept parameter

2. Equations

(a) Function without technical change

$$VA(t') = z \cdot CAP(t')^{a} \cdot EM(t')^{b} \tag{1}$$

(b) Functions with technical change

$$VA(t') = z \cdot CAP(t')^{a} \cdot EM(t')^{b} \cdot e^{(r\, t')} \tag{2}$$

$$VA(t') = z \cdot CAP(t')^{a(t')} \cdot EM(t')^{b(t')} \tag{3}$$

$$VA(t') = z \cdot CAP^{*}(t')^{a} \cdot EM^{*}(t')^{b} \tag{4}$$

Notes

a/ The Cobb-Douglas production function has been widely estimated during the past 50 years for a variety of sectors in many different countries, and with a variety of types of data including both aggregate and firm-level data, and both time series and cross-section data. See, for example: Walters (1970); Murti and Sastry (1957); Hildebrand and Liu (1965).

b/ In a Cobb-Douglas production function, one can use other measures of output instead of value added, such as gross output.

Annex II

PROCEDURE TO CALIBRATE INVERSE COBB-DOUGLAS PRODUCTION FUNCTIONS

The planner may wish to make adjustments in the estimated transformed Cobb-Douglas production functions by industry in order to make it possible to accurately predict employment, by industry, for a given historical year or time period, using the levels of value added and the capital stock for that year or period. These adjustments, which are usually referred to as "calibration", may be necessary, for example, where the planner wishes to use them to make projections of employment originating in the year that immediately follows the time period to which the data used to estimate the functions refer, rather than in the later, initial year of the plan.

Calibrating inverse Cobb-Douglas production functions may involve adjustments in the estimates of the intercept coefficients or in the estimates of the partial coefficients or both. Adjustments in the intercepts are more straightforward than those in the partial coefficients and, therefore, calibration is often restricted to the former coefficients. This annex describes a procedure that can be used to adjust intercepts of inverse Cobb-Douglas production functions as well as intercepts of log-linear inverse Cogg-Douglas production functions. It will also illustrate the use of the procedure to adjust intercepts of the latter form of functions.

A. The procedure

The procedure to calculate adjusted intercept coefficients of the inverse Cobb-Douglas production functions makes use of estimates of the partial coefficients of those functions as well as observed levels of employment, value added and the capital stock for the selected year or time period. Also, it uses an appropriate value of the time variable which refers to the chosen year or the mid-year of the selected time period. For a selected year, adjusted intercept coefficients are obtained as follows:

$$[\ [a'(i)]^{*}\ ]' = EM(i,t+5)\ /\ [\ VA(i,t+5)^{[b'(i)]^{*}} \cdot CAP(i,t+5)^{[c'(i)]^{*}} \cdot e^{[\ [d'(i)]^{*}\ \cdot\ t'\ ]}\ ]; \quad i = 1,\dots,I, \tag{1}$$

where:

$i = 1,\dots,I$ are the industries of the nation's economy,

I	is the number of industries,
t'	is the given calendar year,
EM(i,t')	is the observed labour or employment in industry i in year t',
VA(i,t')	is the observed value added in industry i in year t',
CAP(i,t')	is the observed capital stock in industry i in year t',
[[a'(i)]*]'	is the adjusted intercept coefficient of the inverse Cobb-Douglas production function for industry i,
[b'(i)]*	is the estimate of the partial coefficient of the value added variable in the inverse Cobb-Douglas production function for industry i,
[c'(i)]*	is the estimate of the partial coefficient of the capital stock variable in the inverse Cobb-Douglas production function for industry i, and
[d'(i)]*	is the estimate of the partial coefficient of the time variable in the inverse Cobb-Douglas production function for industry i.

To adjust intercepts of log-linear inverse Cobb-Douglas production functions, one would use estimates of the partial coefficients as well as observed levels of employment, value added and the capital stock for the selected year or time period. Also, one would need to use an appropriate value of the time variable. For a selected year adjusted intercept coefficients would be obtained as follows:

$$[\,[\ln a'(i)]^*\,]' = \ln EM(i,t') - [\,[b'(i)]^* \cdot \ln VA(i,t') + [c'(i)]^* \cdot \ln CAP(i,t') + [d'(i)]^* \cdot t'\,]; \quad i = 1,\ldots,I, \tag{2}$$

where:

[[lna'(i)]*]'	is the adjusted logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for industry i.

In equations (1) and (2), t' stands for a selected year of the time period to which the data used to estimate inverse Cobb-Douglas production functions refer. In instances where the planner wishes to perform adjustments in the intercept coefficients using data for a few or several years rather then a single year, the adjustments can also be made using expressions shown in equation (1) or equation (2). In that instance, the observed levels of employment, value added and the capital stock used would be the mean levels of those variables for, say, three or five years centred on that particular year, t'.

B. Illustrative example of calibration

This example will illustrate the application of the calibration procedure by calculating adjusted intercepts of the log-linear inverse Cobb-Douglas production functions, by industry, estimated in chapter VIII (table 130). The example will use observations on employment, value added and the capital stock for 1978, which are respectively shown in tables 127-129.

Table 138 illustrates the calculation of the adjusted intercepts for the functions in question. The adjusted intercept coefficient (column 9) for any industry is obtained as the difference between the logarithm of the observed level of employment for the industry in 1978 (column 8) and the sum of three products. The first product is obtained by multiplying the estimated value added coefficient for the industry (column 2) by the logarithm of the observed level of value added for the industry in 1978 (column 5). The second product is found by multiplying the estimated capital stock coefficient (column 3) by the logarithm of the observed capital stock for the industry in 1978 (column 5). The third product is the result of multiplying the time variable coefficient for the industry (column 4) by the value of the time variable for year 1978 (column 6).

For example, the adjusted intercept in the function for agriculture, -14.20909, is obtained as follows:

$$-14.20909 = \ln(1{,}994.0) - [0.45694 \cdot \ln(260.3) + -0.24582 \cdot \ln(28.0) + 0.01015 \cdot 1978\ ], \qquad (1)$$

where 1,994.0 is the employment in agriculture in 1978, while 0.45694 and 260.3 are, respectively, the estimate of the value added coefficient in the function for agriculture and the value added in this industry in 1978 0-0.24582 and 28.0. The estimate of the capital stock coefficient in the function for agriculture and the capital stock in this industry in 1978 are -0.24582, 28.0 and 0.01015. The estimate of the time variable coefficient for agriculture in 1978 is the value of the time variable which is 0.01015.

Table 138. Computing adjusted intercept coefficients for log-linear inverse Cobb-Douglas production functions: entire country, year 1978

	Estimates of partial coefficients a/			In year 1978				
Industry	Value added	Capital stock	Time variable	Value added b/	Capital stock c/	Value of time variable	Employment d/	Adjusted intercept coefficient e/
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Agriculture	0.45695	-0.24581	0.01015	260.3	28.0	1978	1994.0	-14.20909
Mining	1.07969	-0.73550	0.02051	2.6	21.0	1978	6.7	-37.46840
Manufacturing	0.63666	-0.20063	0.00717	118.9	58.7	1978	217.3	-11.04406
Utilities	2.12628	-2.96373	0.05919	18.0	14.1	1978	15.3	-112.66345
Construction	0.24705	-0.19959	0.03584	23.5	9.4	1978	93.8	-66.69978
Trade	1.24756	-0.96450	-0.03025	50.9	27.3	1978	109.4	62.82550
Transport	1.24438	-0.76079	-0.06180	50.3	50.3	1978	95.1	124.91469
Services	1.47648	-0.87672	-0.02869	266.9	13.1	1978	516.2	57.01586

a/ From table 130.
b/ From table 128, year 1978.
c/ From table 129, year 1978.
d/ From table 127, year 1978.
e/ (ln(Col. 8)) - ((Col. 2) . (ln(Col. 5)) + (Col. 3) . (ln(Col. 6)) + (Col. 4) . (Col. 7)).

C. Notation and equations

1. Indices, variables and special symbols

(a) List of indices

i = 1,...,I	are industries of the nation's economy
t'	is the given calendar year

(b) List of variables

CAP(i,t')	is the observed capital stock in industry i in year t'
EM(i,t')	is the observed labour or employment in industry i in year t'
VA(i,t')	is the observed value added in industry i in year t'

(c) List of special symbols

[[a'(i)]*]'	is the adjusted intercept coefficient of the inverse Cobb-Douglas production function for industry i
[b'(i)]*	is the estimate of the partial coefficient of the value added variable in the inverse Cobb-Douglas production function for industry i
[c'(i)]*	is the estimate of the partial coefficient of the capital stock variable in the inverse Cobb-Douglas production function for industry i
[d'(i)]*	is the estimate of the partial coefficient of the time variable in the inverse Cobb-Douglas production function for industry i
I	is the number of industries
[[lna'(i)]*]'	is the adjusted logarithm of the intercept coefficient of the inverse Cobb-Douglas production function for industry i

2. Equations

A. The procedure

$$[\,[a'(i)]^{*}\,]' = EM(i,t+5) \,/\, [\, VA(i,t+5)^{[b'(i)]^{*}} \cdot CAP(i,t+5)^{[c'(i)]^{*}} \cdot e^{[\,[d'(i)]^{*} \cdot t'\,]}\,]; \quad i = 1,\ldots,I \tag{1}$$

$$[\,[\ln a'(i)]^{*}\,]' = \ln EM(i,t') - [\,[b'(i)]^{*} \cdot \ln VA(i,t') + [c'(i)]^{*} \cdot \ln CAP(i,t') + [d'(i)]^{*} \cdot t'\,]; \quad i = 1,\ldots,I \tag{2}$$

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