Chapter II

METHODS OF PROJECTING ACTIVITY RATES

The methods are of two main types. In the first the future activity rates of the population are projected by extrapolating the past trends observed in these rates, either in the country under study or in countries whose economic development is similar. In the second certain techniques are employed to correlate the activity rates observed at a given date in various areas of a country or in countries at different levels of development with certain measures which typify economic development, such as the anticipated rate of industrialization. Furthermore, recent advances in the techniques for demographic calculations have enabled matrix methods to be used to produce a model projection of the economically active population in which the number of factors that can be allowed for in estimating the future activity rates of the population is substantially increased.

The calculations made by both these methods have their limitations. Neither the extrapolation nor the correlation method is capable of giving consistently satisfactory results, particularly in the projection of the activity rates of the youngest and oldest age groups.

A. Extrapolation methods

The first part of this section describes a number of extrapolation techniques and their application to the statistics of the economically active population of Japan. Thereafter the extrapolation method developed by the International Labour Office is described.

1. *Various extrapolation methods; application to Japan*

(a) *Direct extrapolation by age group and by sex*

Future economic activity rates may be computed by linear extrapolation of observed trends. This method is the simplest and the most frequently employed, but it is unsatisfactory for a number of reasons, the most important of which is that it can yield impossible results. For instance, in the case of age groups whose activity rates are very high, the method may yield percentages above the maximum of 100 (for one example, see annex III). These drawbacks are not overcome by resort to geometric projection if the trend being projected is a rising one. A better method is to weight the changes in the activity rate with a coefficient that will have the effect of ruling out negative percentages or percentages above 100.[1]

This correction coefficient is obtained by dividing the product of the rates of activity and inactivity at the beginning and end of the past period. The extrapolation formula is as follows:

$$a^x_{t_1} = a^x_t \; \frac{100 \pm \Delta^x_{t,t_1}}{100}$$

In this equation, a^x represents the activity rate of persons of age x; Δ^x_t, t_1 the recorded percentage increase — or decrease — in the activity rate of persons of age x in the base period, which is deemed also to apply to the projection period; t the beginning of the projection period or the end of the base period; and t_1 the end of the projection period. It should be noted that the projection period and the base period are here assumed to be of the same duration. If this is not the case, the mean rate of change for a period of the same duration must be computed. When the adjusted coefficient is applied, the percentage increase — or decrease — may be computed in two ways, using one of the following equations:[2]

$$\text{Equation } (a)\text{: } \Delta^x_{t,t_1} = \left(\frac{a^x_t}{a^x_{t_0}}\right) \left(\frac{a^x_t \times u^x_t}{a^x_{t_0} \times u^x_{t_0}}\right)$$

$$\text{Equation } (b)\text{: } \Delta^x_{t,t_1} = 1 + \left(\frac{a^x_t}{a^x_{t_0}} - 1\right) \left(\frac{a^x_t \times u^x_t}{a^x_{t_0} \times u^x_{t_0}}\right),$$

in which u^x represents the inactivity rate of persons of age x and t_0 the beginning of the base period. In (*a*) the ratio of the total increase — or decrease — is multiplied by the adjusted coefficient, whereas in (*b*) only the increase itself — or the decrease — is thus modified. Examples of projections in which these extrapolations were used are given in annexes I and II and brief illustrations of the computations themselves in tables 3 and 4.

(b) *Indirect extrapolation, by age group and by sex*

Sometimes, instead of extrapolating activity rates, it may be advantageous to use inactivity rates. This method is particularly recommended when activity rates are assumed to be increasing gradually,[3] as in the

[1] John D. Durand, *The Labour Force in the United States 1890-1960* (New York, Social Science Research Council, 1948), pp. 238 and 239. The weight adopted by the author has the property of retarding the rate of increase in the activity rates when these are very high and of accelerating their increase when they are low.

[2] In these equations $t_0 < t < t_1$.

[3] G. Bancroft, *The American Labor Force: Its Growth and Changing Composition*, (New York, Wiley, 1958), p. 176.

TABLE 3

Japan: activity rates of the male economically active population for 1965, estimated by direct extrapolation of 1955 and 1960 data for each age group, with the correction coefficient (equation (*a*))

(x)	a^x_{1955}	a^x_{1960}	$\frac{a^x_{1960}}{a^x_{1955}}$	$a^x_{1960} \times u^x_{1960}$	$a^x_{1955} \times u^x_{1955}$	$\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}$	$\left(\frac{a^x_{1960}}{a^x_{1955}}\right)$	$\left(\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}\right)$	a^x_{1965}
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7) = (5) : (6)*		*(8) = (4) × (7)*	*(9) = (3) × [100 ± (8)] : 100*
15-19	52.1	50.7 × 0.9731	50.7 × 49.3 = 2,449.51	52.1 × 47.9 = 2,495.59	0.98153		0.9551	51.2
20-24	85.2	86.8 × 1.0188	86.8 × 13.2 = 1,145.76	85.2 × 14.8 = 1,260.96	0.90864		0.9257	87.6
25-29	93.7	96.0 × 1.0245	96.0 × 4.0 = 384.0	93.7 × 6.3 = 590.31	0.65050		0.6664	96.6
30-34	95.1	97.1 × 1.0210	97.1 × 2.9 = 281.59	95.1 × 4.9 = 465.99	0.60428		0.6170	97.7
35-39	95.8	97.2 × 1.0146	97.2 × 2.8 = 272.16	95.8 × 4.2 = 402.36	0.67640		0.6863	97.9
40-44	95.9	97.1 × 1.0125	97.1 × 2.9 = 281.59	95.9 × 4.1 = 393.79	0.71617		0.7251	97.8
45-49	95.5	96.7 × 1.0126	96.7 × 3.3 = 319.11	95.5 × 4.5 = 429.75	0.74254		0.7519	97.4
50-54	93.8	95.1 × 1.0139	95.1 × 4.9 = 465.99	93.8 × 6.2 = 581.56	0.80127		0.8124	95.9
55-59	88.7	89.5 × 1.0090	89.5 × 10.5 = 939.75	88.7 × 11.3 = 1,002.31	0.93755		0.9460	90.3
60-64	81.5	81.9 × 1.0049	81.9 × 18.1 = 1,482.39	81.5 × 18.5 = 1,507.75	0.98318		0.9880	82.7
65 + ...	55.7	54.5 × 0.9785	54.5 × 45.5 = 2,479.75	55.7 × 44.3 = 2,467.51	1.00496		0.9834	54.0

TABLE 4

Japan: activity rates of the male economically active population for 1965, estimated by direct extrapolation of 1955 and 1960 data for each age group, with the correction coefficient (equation (*b*))

(x)	a^x_{1955}	a^x_{1960}	$\frac{a^x_{1960}}{a^x_{1955}}$	$\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}$	$\frac{a^x_{1960}}{a^x_{1955}} - 1$	$\left(\frac{a^x_{1960}}{a^x_{1955}} - 1\right) \left(\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}\right)$	$1 + \left(\frac{a^x_{1960}}{a^x_{1955}} - 1\right) \left(\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}\right)$	a^x_{1965}
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6) = (4) — 1*	*(7) = (6) × (5)*	*(8) = 1 + (7)*	*(9) = (3) × (8)*
15-19	52.1	50.7	0.9731	0.98153	−0.0269	−0.0264	0.9736	49.3
20-24	85.2	86.8	1.0188	0.90864	0.0188	0.0171	1.0171	88.3
25-29	93.7	96.0	1.0245	0.65050	0.0245	0.0159	1.0159	97.5
30-34	95.1	97.1	1.0210	0.60428	0.0210	0.0127	1.0127	98.3
35-39	95.8	97.2	1.0146	0.67640	0.0146	0.0099	1.0099	98.2
40-44	95.9	97.1	1.0125	0.71617	0.0125	0.0090	1.0090	98.0
45-49	95.5	96.7	1.0126	0.74254	0.0126	0.0094	1.0094	97.6
50-54	93.8	95.1	1.0139	0.80127	0.0139	0.0111	1.0111	96.2
55-59	88.7	89.5	1.0090	0.93755	0.0090	0.0084	1.0084	90.3
60-64	81.5	81.9	1.0049	0.98318	0.0049	0.0048	1.0048	82.3
65 + ...	55.7	54.5	0.9785	1.00496	−0.0215	−0.0216	0.9784	53.3

case of the female labour force. The following formula may be used:

$a_t^x = 100 - (u_t^x \cdot \alpha)$, in which $\alpha = u_t^x / u_{t_0}^x$, or the ratio of the inactivity rates of persons of age x in the periods t and t_0. Here again, the correction coefficient may be used in the same way as in the methods described earlier. An illustration is given in table 5. Examples of future activity rates extrapolated by this method, but without using the correction coefficient, are given in annex V; the correction coefficient is applied in the examples in annex IV.

(c) *Extrapolation by cohort*

This method works on the assumption that the pattern of change in the activity rates of a given age group — or cohort — during the projection period will be the same as that observed for a cohort in the same age group in an earlier period. This method is of particular value for categories of the economically active population whose activity rates show marked variations over a period of time. It is thus very useful for projecting the activity rates of the female population. The formula for extrapolation by cohort is:

$$a_{t_1}^x = a_t^x \times \frac{a_t^x}{a_{t_0}^{x_0}}$$

in which x_0 represents the age group preceding age group x, and $a_t^x / a_{t_0}^{x_0}$ the ratio of the activity rates by cohort. In order to determine the activity rates of women who will be in the 25-29 age group in 1970, the pattern of change observed in the activity rates for 1960-1965 of women who were in the 20-24 age group in 1960 is applied to the activity rates of women in the 20-24 age group in 1965 (see table 6).

TABLE 5

Japan: activity rates of the male economically active population for 1965, estimated by indirect extrapolation of 1955 and 1960 data for each age group, with and without the correction coefficient

(x)	u_{1955}^x	u_{1960}^x	$\gamma = \frac{u_{1960}^x}{u_{1955}^x}$	$u_{1960}^x \times \gamma$	a_{1965}^x [a]	$\frac{a_{1960}^x \times u_{1960}^x}{a_{1955}^x \times u_{1955}^x}$	$(u_{1960}^x \times \gamma) \left(\frac{a_{1960}^x \times u_{1960}^x}{a_{1955}^x \times u_{1955}^x}\right)$	a_{1965}^x [b]
(1)	*(2)*	*(3)*	*(4) = (3) : (2)*	*(5) = (4) × (3)*	*(6) = 100 — (5)*	*(7)*	*(8) = (5) × (7)*	*(9) = 100 — (8)*
15-19 . . .	47.9	49.3	1.0292	50.7	49.3	0.98153	49.8	50.2
20-24 . . .	14.8	13.2	0.8918	11.8	88.2	0.90864	10.7	89.3
25-29 . . .	6.3	4.0	0.6349	2.5	97.5	0.65050	1.6	98.4
30-34 . . .	4.9	2.9	0.5918	1.7	98.3	0.60428	1.0	99.0
35-39 . . .	4.2	2.8	0.6666	1.9	98.1	0.67640	1.3	98.7
40-44 . . .	4.1	2.9	0.7073	2.1	97.9	0.71617	1.5	98.5
45-49 . . .	4.5	3.3	0.7333	2.4	97.6	0.74254	1.8	98.2
50-54 . . .	6.2	4.9	0.7903	3.9	96.1	0.80127	3.1	96.9
55-59 . . .	11.2	10.5	0.9292	9.8	90.2	0.93755	9.2	90.8
60-64 . . .	18.5	18.1	0.9783	17.7	82.3	0.98318	17.4	82.6
65 years and over . . .	44.3	45.5	1.0270	46.7	53.3	1.00496	46.9	53.1

(x)	$\gamma - 1$	$(\gamma - 1) \left(\frac{a_{1960}^x \times u_{1960}^x}{a_{1955}^x \times u_{1955}^x}\right)$	$(\gamma - 1) \left(\frac{a_{1960}^x \times u_{1960}^x}{a_{1955}^x \times u_{1955}^x}\right) + 1$	$u_{1960}^x \left[(\gamma - 1) \left(\frac{a_{1960}^x \times u_{1960}^x}{a_{1955}^x \times u_{1955}^x}\right) + 1\right]$	a_{1965}^x [c]
(1)	*(10) = (4) — 1*	*(11) = (10) × (7)*	*(12) = (11) + 1*	*(13) = (3) × (12)*	*(14) = 100 — (13)*
15-19 . . .	0.0292	0.0287	1.0287	50.7	49.3
20-24 . . .	−0.1081	−0.0982	0.9018	11.9	88.1
25-29 . . .	−0.3651	−0.2375	0.7625	3.1	96.9
30-34 . . .	−0.4082	−0.2467	0.7533	2.2	97.8
35-39 . . .	−0.3333	−0.2254	0.7746	2.2	97.8
40-44 . . .	−0.2927	−0.2096	0.7904	2.3	97.7
45-49 . . .	−0.2667	−0.1980	0.8020	2.6	97.4
50-54 . . .	−0.2097	−0.1680	0.8320	4.1	95.9
55-59 . . .	−0.0708	−0.0664	0.9336	9.8	90.2
60-64 . . .	−0.0216	−0.0212	0.9788	17.7	82.3
65 + ...	0.0270	0.0272	1.0272	46.7	53.3

[a] Correction coefficient not applied.

[b] Correction coefficient (equation (*a*)) applied.

[c] Correction coefficient (equation (*b*)) applied.

TABLE 6

Japan: estimated activity rates of the female labour force for 1965, based on extrapolation by cohort of 1955 and 1960 data for each age group with and without the correction coefficient [a]

(x)	$\frac{a^x_{1960}}{a^x_{1955}}$	a^x_{1960}	a^x_{1965}	a^x_{1965}	a^x_{1965}	$\frac{a^x_{1960}}{a^x_{1955}} - 1$	$\left(\frac{a^x_{1960}}{a^x_{1955}} - 1\right)\left(\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}\right)$	$1 + \left[\left(\frac{a^x_{1960}}{a^x_{1955}} - 1\right)\left(\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}\right)\right]$	a^x_{1965}
(1)	(2)	(3)	(4) = (3) × (2)	(5)	(6)	(7)	(8)	(9) = (8) + 1	(10) = (3) × (9)
15-19	—	—	—	—	—	—	—	—	—
20-24	1.416	48.9	69.2	62.9	59.9	0.416	0.404	1.404	68.7
25-29	0.752	68.4	51.4	33.4	57.8	−0.248	−0.248	0.752	51.4
30-34	0.984	50.1	49.3	29.8	49.3	−0.016	−0.016	0.984	49.3
35-39	1.108	50.7	56.2	38.0	55.8	0.108	0.107	1.107	56.1
40-44	1.088	54.4	59.2	42.4	58.2	0.088	0.087	1.087	59.1
45-49	1.022	57.0	58.3	43.3	58.0	0.022	0.022	1.022	58.3
50-54	0.958	56.5	54.1	43.3	54.4	−0.042	−0.042	0.958	54.1
55-59	0.907	51.9	47.1	44.2	46.8	−0.093	−0.093	0.907	47.1
60-64	0.852	45.8	39.0	35.2	37.4	−0.148	−0.148	0.852	39.0
65 years and over	0.547	39.1	21.4	21.5	15.3	−0.453	−0.458	0.542	21.2

[a] Computations:

(4) simple extrapolation by cohort.

(5) extrapolation by cohort multiplied by the correction coefficient $\left(\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}\right)$

(6) extrapolation by cohort multiplied by the cohort correction coefficient $\left(\frac{a^x_{1960} \times u^x_{1960}}{a^x_{1955} \times u^x_{1955}}\right)$

(10) extrapolation by cohort with the correction coefficient applied to the increase — or decrease — in activity rates.

(d) *Comparison of the results obtained by the various extrapolation methods*

The results of the computations vary according to the method used. Table 7 clearly illustrates that this is so, both in the case of projections made for 1965 and in some which have been made for 1985. The differences are slight as between the techniques of direct and indirect extrapolation; the difference between projections obtained by these techniques and those derived by the method of extrapolation by cohort is wider.

The merits of the correction coefficient ($a_t^x \times u_t^x / a_{t_0}^x \times u_{t_0}^x$) are especially evident in long-term projections of activity rates for specified age groups. Annex III shows how, unless the correction coefficient is used, the direct extrapolation method yields absurd results, such as activity rates of 109 per cent for the 25-29 age group in Japan for 1985. When the projection period is short, however, the likelihood of obtaining a negative percentage or a percentage above 100 is probably negligible; in such cases, it is not essential to use the correction coefficient when computing future activity rates. The illustrative projections for Japan for the period 1960-1965 can be compared with the real pattern of labour supply in the same period. The discrepancies are very slight in the case of adult male age groups whose activity rates are relatively stable; they are considerably greater for the female population as a whole and for the youngest and oldest male groups. Such discrepancies may be explained by the fact that in projecting activity rates for these groups, the necessary adjustments have to be made for probable trends in such endogenous variables as the economic structure, school enrolment rates and the social security system; the mechanical approach imposed by the use of such rigid mathematical formulae as those described above makes no allowance for such adjustments. This problem, and the solutions it calls for, will be discussed in section D of this chapter.

Finally, it is interesting to compare the projections developed for Japan for 1985 by the different methods

TABLE 7

Japan: projections of sex- and age-specific activity rates of the economically active population, 1965 and 1985; comparison of extrapolation methods

	Activity rates [a]				*Deviation from actual data*			*Activity rates* [a]				*Deviation from official projections*		
	a	*b*	*c*	*d*	*b*	*c*	*d*	*A*	*b*	*c*	*d*	*b*	*c*	*d*
	1965							*1985*						
Males														
15-19	38.6	49.3	49.3	—	10.7	10.7	—	40.0	43.7	43.7	—	37.7	37.7	—
20-24	87.1	88.3	81.1	81.4	1.2	1.0	−5.7	85.5	93.3	91.6	—	7.8	6.1	—
25-29	97.9	97.5	96.9	94.0	−0.7	−1.0	−3.9	95.9	99.7	98.3	—	3.8	2.4	—
30-34	98.6	98.3	97.8	98.1	−0.3	−0.8	−0.5	96.8	99.7	98.9	—	2.9	2.1	—
35-39	98.4	98.2	97.8	98.6	−0.2	−0.6	0.2	97.2	99.8	98.9	—	2.6	1.7	—
40-44	98.3	98.0	97.7	98.2	−0.3	−0.6	−0.1	97.2	99.5	98.8	97.8	2.3	1.6	0.6
45-49	98.0	97.6	97.4	97.7	−0.4	−0.6	−0.3	96.8	99.5	98.5	96.4	2.7	1.7	−0.4
50-54	97.2	96.2	95.9	96.4	−1.0	−1.3	−0.8	95.2	98.7	97.4	99.3	3.5	2.2	4.1
55-59	93.8	90.3	90.2	91.0	−3.5	−3.6	−2.8	88.3	93.0	92.2	97.7	4.7	3.9	9.4
60-64	85.3	82.3	82.3	82.7	−3.0	−3.0	−2.6	78.2	83.9	84.1	91.4	5.7	5.9	13.2
65 years and over	55.1	53.3	53.3	54.6	−1.8	−1.8	−0.5	39.2	48.5	48.4	59.9	9.3	9.2	20.7
Females														
15-19	37.6	49.5	49.5	—	11.9	11.9	—	40.0	51.9	51.9	—	11.9	11.9	—
20-24	69.7	70.2	70.1	68.7	0.5	0.4	−1.0	65.0	77.4	75.9	—	12.4	10.9	—
25-29	46.4	48.7	48.7	51.4	2.3	2.3	5.0	38.4	43.3	54.3	—	4.9	15.9	—
30-34	48.0	52.3	52.2	49.3	4.3	4.2	1.3	32.8	58.9	59.2	—	26.1	26.4	—
35-39	58.3	56.5	56.3	56.1	−1.8	−2.0	−2.2	36.3	65.6	62.9	—	29.3	26.6	—
40-44	62.1	58.7	58.6	59.1	−2.4	−3.5	−3.0	39.0	65.5	64.0	61.0	26.5	25.0	22.0
45-49	62.6	58.9	58.7	58.3	−3.7	−3.9	−4.3	37.2	69.1	66.5	62.3	31.9	29.3	25.1
50-54	57.3	53.3	53.3	54.1	−4.0	−4.0	−3.2	32.1	59.1	58.7	58.1	27.0	26.6	26.0
55-59	50.1	45.7	45.7	47.1	−4.4	−4.4	−3.0	25.8	45.3	45.3	54.4	19.5	19.5	28.6
60-64	39.3	39.1	39.1	39.0	−0.2	−0.2	−0.3	16.8	39.1	39.1	44.6	22.3	22.3	27.8
65 years and over	17.6	21.7	21.7	21.2	4.1	4.1	3.6	5.3	22.9	22.9	22.8	17.6	17.6	17.5

[a] Explanatory note:

a: Official estimates for 1965; see Japan, Institute of Population Problems, *Estimates of Future Labour Force Population in Japan for 1 October from 1965 to 1985, Estimated in December 1966* (Tokyo, 1967).

A: Official projection for 1985, variant C (for an explanation of the method used, see Section B of this chapter).

b: Direct extrapolation for each age group, applying the correction coefficient to the increase — or decrease — in activity rates.

c: Indirect extrapolation for each age-group, applying the correction coefficient to the increase — or decrease — in activity rates.

d: Extrapolation by cohort, applying the correction coefficient to the increase — or decrease — in activity rates.

When the correction coefficient is not applied, impossible rates are obtained, even for 1970; see simple direct extrapolation (annex III) and extrapolation by cohort (annex VII).

described here with the country's own official estimates. The method used in making these estimates is described below in the section B.1 of this chapter, which outlines methods involving correlation techniques. In it, correlations are computed and the results corrected to allow for the probable incidence of such endogenous variables as the prospects for female employment, school enrolment rates and pension systems. The differences in the case of female activity rates are substantial but are less marked in the case of the male rates, where, even if they are significant in the youngest and oldest age groups, they are slight for the adult age groups and probably arise from the fact that the extrapolation techniques used for the long-term projections yield activity rates of very nearly 100 per cent in 1985. This figure is, of course, a practical impossibility since there will always be a small percentage of adult males who are unable to work.

These comparisons show that the results obtained by different projection techniques must be analysed carefully, particularly when they fail to take account of the potential effect of new economic or social factors in future years (see section D of this chapter).

(e) *An additional method: ratios of the activity rates of contiguous age groups*

Another method of extrapolation which has been used to estimate activity rates is to calculate the ratios of the activity rates of contiguous age groups. This extrapolation is based on the concept that there is a relationship between the activity rates of different age groups which can be expressed in the form of a ratio. On the curve of the sex-specific activity rates the age group whose activity rate is relatively the most stable is selected: for males this is generally an age group near the midpoint of the curve, for females, the age group selected as central will depend on the structure of economic activity peculiar to the country in question. Starting with this age group, the ratios of the activity rates of pairs of contiguous age groups are calculated. For example, if the 40-44 male age group is the one selected, the ratio for the 35-39 age group will be (35-39)/(40-44); for the 45-49 age groups (45-49)/(40-44), for the age 50-54 age group (50-54)/(45-49), and so on. The activity rates projection for Japan for 1960-1985 provides an example (see table 8 and annex VIII).

The first step is to calculate the ratios of contiguous (adjacent) age groups and to determine whether they have increased or decreased in an earlier period (1955-1960). An assumption is then made as to future changes in the activity rate of the age group selected as the central group. In the case of Japan, the age groups selected were the 40-44 group for males and the 20-24 group for females. For these age groups activity rates derived by indirect extrapolation were used: the rates for 1985 were 98.8 for males and 75.2 for females (see annex VIII). Next, the contiguous ratios for each age group for the target year are computed. These ratios are generally abstracted from models which applied in the earlier period and, when possible, are also adjusted to embody assumptions concerning socio-economic and demographic variables which might cause changes in the proportions of the economically active population. For the sake of simplicity, it is assumed in table 8 (column 8) that the relative increase — or decrease — in the ratios of contiguous age groups for the projection period 1960-1985 would be the same as that observed during 1955-1960.[4] The ratio for each age group at the beginning of the projection period is then multiplied by the total observed increase — or decrease — in order to project the ratios of the activity rates of contiguous age groups to the target year of 1985. These ratios and the activity rates of the central age group form the basis for the calculation of activity rates for specific age groups for the target year. A linear extrapolation is used to determine activity rates at times between the initial year and the final year of the projection period. This method has the same drawbacks as the extrapolation methods described earlier.

2. *Methods of projection by extrapolation developed by the International Labour Office*

The ILO has prepared sex-age-specific activity rate projections for the period 1960-1980 for twenty-three regions of the world [5] and some of their component countries. These projections were calculated on the basis of trends reported between 1950 and 1960 for the following age groups: 0-14, 15-19, 20-24, 25-44, 45-54, 55-64, and 65 and over. Separate projections were made for the 1960-1970 and 1970-1980 periods. These projections can be used by countries wishing to draw up activity rate forecasts for those years.[6]

An account follows of the methods used by the ILO to calculate future regional activity rates and the way in which these regional forecasts can be used to compute future activity rates for a given country.

(a) *The ILO method for regional projections*

Three estimation procedures were applied concurrently to determine activity rates for a given region.

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[4] Various assumptions can be used, depending on the specific situation. For example, one reasonable assumption would take the form of multiplying the increase — or decrease — of the ratio for each age group by the number of times by which the projection period is greater than the time interval of the base period (see Economic Commission for Latin America, *Población y mano de obra en Panamá 1950-1985*, (preliminary version) (E/CN.12/789, 1 August 1966). The projection contained in annex VIII, in which the projection period 1960-1985 is divided into two parts — 1960-1975 and 1975-1985 — provides another example. Changes for the period 1960-1975 are assumed to be the same as those in 1955-1960, and changes for the period 1975-1985 the same as those in 1960-1975.

[5] The twenty-three regions are identical with the United Nations classification system. See United Nations *Demographic Yearbook 1967* (United Nations publication, Sales No.: 68.XIII.1), p. 18. The twenty-fourth region — Polynesia and Micronesia — has been excluded for lack of proper data.

[6] For a more detailed description of these methods see: Ypsilantis, op. cit.; also "Methods of Estimation and Projection of Labour Force Sex-Age-Specific Activity Rates", document prepared by the International Labour Office for an inter-agency meeting of experts on demographic projections (New York, POP/SC/WP/6, December 1968); and J. N. Ypsilantis, "Projections of Manpower Supply", discussion paper submitted to a meeting of the International Statistical Institute on manpower projections, held at London in September 1969.

TABLE 8

Japan: projections of sex- and age-specific activity rates of the economically active population (1960-1985) by the method of activity rate ratios of contiguous age groups [a]

Sex and age	Activity rates: Observed 1955	Activity rates: Observed 1960	Activity rates: Projected 1985	Ratios of contiguous age groups 1955	Ratios of contiguous age groups 1960	Ratios of contiguous age groups 1985	Increase (or decrease) in the ratios 1960/1955 = 1985/1960	Increase (decrease) 1960-1985	Increase (decrease) 5 years	Projected activity rates 1965	Projected activity rates 1970	Projected activity rates 1975	Projected activity rates 1980	Projected activity rates 1985
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8) = (6) : (5)	(9) = (4) — (3)	(10) = (9) : (5)	(11)	(12)	(13)	(14)	(15) = (4)
Males														
15-19	52.1	50.7	49.5	0.61150	0.58410	0.55793	0.955192	−1.2	−0.24	50.5	50.2	50.0	49.7	49.5
20-24	85.2	86.8	88.8	0.90928	0.90416	0.89907	0.994369	2.0	0.40	87.2	87.6	88.0	88.4	88.8
25-29	93.7	96.0	98.8	0.98527	0.98867	0.99208	1.003450	2.8	0.56	96.6	97.1	97.7	98.3	98.8
30-34	95.1	97.1	99.6	0.99269	0.99897	1.00529	1.006326	2.5	0.50	97.6	98.1	98.6	99.1	99.6
35-39	95.8	97.2	99.1	0.99895	1.00102	1.00309	1.002072	1.9	0.38	97.6	98.0	98.3	98.7	99.1
40-44	95.9	97.1	98.8	—	—	—	—	1.7	0,34	97.4	97.8	98.1	98.7	98.8
45-49	95.5	96.7	98.4	0.99582	0.99588	0.99594	1.000060	1.7	0.34	97.0	97.4	97.7	98.1	98.4
50-54	93.8	95.1	96.9	0.98219	0.98345	0.98471	1.001282	1.8	0.36	95.5	95.8	96.2	96.5	96.9
55-59	88.7	89.5	90.8	0.94562	0.94111	0.93662	0.995230	1.3	0.26	89.8	90.0	90.3	90.5	90.8
60-64	81.5	81.9	82.8	0.91882	0.91508	0.91135	0.995929	0.9	0.18	82.1	82.3	82.4	82.6	82.8
65 and over	55.7	54.5	53.6	0.68343	0.66544	0.64792	0.973676	−1.1	−0.22	54.3	54.1	54.0	53.8	53.6
Females														
15-19	48.3	48.9	53.5	0.72522	0.71491	0.70475	0.985783	4.6	0.92	49.8	50.7	51.7	52.6	53.5
20-24	66.6	68.4	75.9	—	—	—	—	7.5	1.50	69.9	71.4	72.9	74.4	75.9
25-29	51.5	50.1	52.7	0.77327	0.73245	0.69378	0.947211	2.6	0.52	50.6	51.1	51.7	52.2	52.7
30-34	49.1	50.7	56.6	0.95339	1.01197	1.07415	1.061443	5.9	1.18	51.9	53.1	54.2	55.4	56.6
35-39	52.4	54.4	61.1	1.06720	1.07297	1.07877	1.005406	6.7	1.36	55.7	57.1	58.4	59.8	61.1
40-44	55.3	57.0	63.6	1.05534	1.04779	1.04029	0.992845	6.6	1.32	58.3	59.6	61.0	62.3	63.6
45-49	54.2	56.5	63.8	0.98010	0.99122	1.00247	1.011345	7.3	1.46	58.0	59.4	60.9	62.3	63.8
50-54	50.5	51.9	57.8	0.93173	0.91858	0.90562	0.985886	5.9	1.18	53.1	54.3	55.4	56.6	57.8
55-59	45.9	45.8	49.5	0.90891	0.88246	0.85678	0.970899	3.7	0.74	46.5	47.3	48.0	48.8	49.5
60-64	39.1	39.1	42.4	0.85185	0.85371	0.85557	1.002183	3.3	0.66	39.8	40.4	41.1	41.7	42.4
65 and over	21.1	61.4	23.5	0.53964	0.54731	0.55509	1.014213	2.1	0.42	21.8	22.2	22.7	23.1	23.5

[a] Computations:

(4) = (4) 40-44 age group × (7) 35-39 age group
(4) 35-39 age group — (7) 30-34 age group

(5) = (2) 35-39 age group : (2) 40-44 age group
(2) 30-34 age group : (2) 35-39 age group
etc.

(6) = (3) 35-39 age group : (3) 40-44 age group
(3) 30-34 age group : (3) 35-39 age group

(7) = (6) × (8)
(11) = (3) + (10)
(12) = (11) + (10)
(13) = (12) + (10)
(14) = (13) + (10)

(i) The first estimate is made by multiplying the activity rates of the region in 1960 by the weighted average of the changes in the activity rates of the component countries of the region in 1950-1960. The application of this procedure to Temperate South America is illustrated in table 9.

The sex-age-specific activity rates for the region in 1970 are projected on the basis of the trend thus obtained. An example of the calculation is given in table 10 (columns 3 and 4). This method assumes the continuance in the future of the trends observed in 1950-1960 in all the component countries of the region;

(ii) The second estimate utilizes the patterns of change in activity rates observed in 1950-1960 in other regions of the world which had reached, in 1960, almost identical activity rates and similar levels of economic development. The changes in the average activity rates of these other regions are calculated and then used to project the activity rates of the region concerned. This second approach is illustrated in table 11, which sets forth the computed average change in activity rates in Japan, Southern Europe and Southern Africa in 1950-1960.[7] This pattern is then applied to the average observed activity rates for 1960 for Temperate South America. This procedure yields estimated activity rates for that region in 1970 (for results, see table 10, columns 5, 6 and 7);

[7] Decennial patterns of change in the activity rates of the twenty-three regions of the world for 1950-1960, 1960-1970 and 1970-1980 can be calculated from regional tables. See J. N. Ypsilantis, "World and Regional Estimates and Projections of Labour Force", *Sectoral Aspects of Projections for the World Economy; First Interregional Seminar on Long-term Economic Projections; Volume III: Discussion Papers*, Elsinore, 14-27 August 1966 (United Nations publication, Sales No.: E.69.II.C.3).

TABLE 9

Temperate South America: estimates of regional patterns of change in male and female age-specific activity rates according to the weighted patterns observed during 1950-1960

Sex and age groups	*Total*	*Argentina*	*Chile*	*Uruguay*	*Paraguay*
Males					
Proportion of labour force in each age group					
0-14	100.00	54.32	22.99	8.87	13.82
15-19	100.00	63.51	22.43	6.90	7.16
20-24	100.00	62.52	24.56	7.46	5.46
25-44	100.00	66.98	20.78	8.29	3.95
45-54	100.00	69.48	18.16	9.13	3.23
55-64	100.00	66.35	20.66	9.11	3.88
65 and over	100.00	64.12	22.09	7.27	6.52
Pattern of change in activity rates 1950-1960					
0-14	120.68	121.62	84.75	121.17	176.47
15-19	95.22	96.13	91.53	96.85	97.14
20-24	99.51	99.36	99.77	101.00	98.26
25-44	100.12	100.26	99.79	100.11	99.49
45-54	96.28	95.78	96.24	99.13	99.10
55-64	82.08	78.98	89.42	81.41	97.73
65 and over	71.74	71.98	68.68	56.88	96.27
Females					
Proportion of labour force in each age group					
0-14	100.00	59.47	19.50	9.75	11.28
15-19	100.00	67.89	20.28	6.76	5.07
20-24	100.00	65.25	21.99	8.05	4.71
25-44	100.00	63.45	21.33	10.55	4.67
45-54	100.00	63.35	21.21	10.77	4.67
55-64	100.00	59.79	24.39	10.27	5.55
65 and over	100.00	56.75	27.36	8.13	7.76
Pattern of change in activity rates 1950-1960					
0-14	75.93	70.21	61.68	116.13	96.05
15-19	105.14	113.86	79.34	99.40	99.30
20-24	98.26	99.62	90.69	104.36	112.00
25-44	108.48	116.09	82.39	114.65	110.36
45-54	87.63	87.82	75.91	104.55	99.40
55-64	78.66	79.75	68.50	84.63	100.60
65 and over	80.23	95.95	54.18	52.20	86.53

TABLE 10

Temperate South America: projections of sex-age-specific activity rates for 1960-1970

Age group	*Regional rates in 1960*	*Ratio changes of countries (1950-1960)* [a]	*First estimate of activity rates*	*Ratio changes of selected regions* [b]	*Estimate of activity rates based on ratio changes of selected regions*	*Second estimate of activity rates* [b]	*Third estimate of activity rates* [c]
(1)	*(2)*	*(3)*	*(4) = (2) × (3)*	*(5)*	*(6) = (2) × (5)*	*(7) = [(4) + (6)] : (2)*	*(8)*
Males							
0-14	2.42	120.68	2.92	59.68	1.44	2.18	2.10
15-19	69.84	95.22	66.50	85.91	60.00	63.25	63.24
20-24	91.03	99.51	90.58	97.24	88.52	89.55	89.54
25-44	97.25	100.12	97.37	99.99	97.24	97.31	97.75
45-54	91.52	96.23	88.12	99.11	90.71	89.42	89.51
55-64	69.10	82.08	56.72	97.16	67.14	61.93	62.21
65 and over . . .	41.03	71.74	29.43	83.55	34.28	31.86	31.82
Females							
0-14	0.96	75.93	0.68	61.36	0.59	0.64	0.63
15-19	30.38	105.14	31.94	101.85	30.94	31.43	31.41
20-24	37.40	98.26	36.75	112.95	42.24	39.47	39.47
25-44	25.33	108.48	27.48	109.80	27.81	27.63	27.73
45-54	18.93	87.63	16.59	111.27	21.06	18.81	16.92
55-64	12.18	78.66	9.58	108.86	13.26	11.41	11.39
65 and over . . .	5.89	80.23	4.73	92.05	5.42	5.06	4.98

[a] Weighted average (see table 9).

[b] Japan, Southern Europe and Southern Africa for males; Southern Europe for females (see table 11).

[c] Aggregate of projections of component countries (see tables 13 and 14).

TABLE 11

Pattern of change in male and female age-specific activity rates in selected regions: 1950-1960

Age group	*Males*				*Females*			
	Total [a]	*Japan*	*Southern Europe*	*Southern Africa*	*Total*	*Japan*	*Southern Europe*	*Southern Africa*
			Regional composition of the labour force in 1960					
0-14	100.00	6.58	82.58	10.84	100.00	5.57	86.78	7.65
15-19	100.00	39.20	50.89	9.91	100.00	53.27	39.55	7.18
20-24	100.00	42.95	48.46	8.59	100.00	56.97	37.30	5.73
25-44	100.00	41.17	51.87	6.96	100.00	57.90	37.81	4.29
45-54	100.00	37.76	56.70	5.54	100.00	58.12	38.64	3.24
55-64	100.00	39.99	54.97	5.04	100.00	58.13	39.15	2.72
65 and over . .	100.00	42.00	53.31	4.69	100.00	57.07	41.21	1.72
			Patterns of changes observed during 1950-1960					
0-14	59.68	133.33	55.08	50.00	n.a. [b]	n.a.	61.36	n.a.
15-19	85.91	81.41	88.67	89.56	n.a.	n.a.	101.85	n.a.
20-24	97.24	97.13	97.00	99.05	n.a.	n.a.	112.95	n.a.
25-44	99.99	100.82	99.42	99.34	n.a.	n.a.	109.80	n.a.
45-54	99.11	100.84	97.93	99.30	n.a.	n.a.	111.27	n.a.
55-64	97.16	102.09	93.55	97.34	n.a.	n.a.	108.86	n.a.
65 and over . .	83.55	97.35	73.27	76.73	n.a.	n.a.	92.05	n.a.

[a] The total pattern of change in activity rates 1950-1960 is the weighted average of the regional changes.

[b] n.a. = not applicable.

(iii) A third estimate of the activity rates of the region is obtained by computing the average activity rates for 1970 from the individual weighted rates for each of the countries for that year. These rates are shown in tables 12, 13 and 14 and projections for the region as a whole for 1970 in column 8 of table 10.

In general, the same procedure is applied to determine the regional sex-age-specific activity rates for 1980: the 1960 and 1970 figures for activity rates and economically active population in each of the component countries of a region provide the basic data for all three estimation methods described earlier. In the second estimate regional patterns of change for two periods, 1950-1960 and 1960-1970, are taken into account. The regional activity rates for 1965 and 1975 are derived by first fitting curves to the 1960 basic data and to the 1970 and 1980 projected sex-age-specific activity rates for each of the component countries of the region and then aggregating the results to obtain a regional average.

(b) *National projections*

Projections of national sex-age-specific activity rates for 1960-1970 are produced by linking national activity rates to regional patterns of change in activity rates. The regional patterns of change for 1950-1960 are used as models for patterns of change in the level of sex-age-specific activity rates over a ten-year period. The selection of regional models to be used for projecting the activity rates of a given country is made primarily on the basis of: similarity of the country's level of economic development to that of the models at the base date — 1960; similarity of the sex-age-specific activity rates at the base date — 1960; and similarity in the trends of these levels during 1950-1960. The national values so computed are plotted on a graph depicting the twenty-three regions. The nearest region, or cluster of regions, to the country's position on the graph are selected as the most plausible models. Their plausibility is further tested by projecting the national activity rate for 1950-1960 and comparing the findings with the known data for 1960. Other things being equal, the geographical region within which the country falls is usually selected for linkage and projection of the national sex-age-specific activity rates; the figure opposite illustrates the application of the procedure to the countries of temperate South America.

The projection of activity rates in a country is obtained by linking the national data to the corresponding sex-age-specific regional data. The formula used for the projection is:

$$\frac{R_{t_1}}{R_t} \times n_t = n_{t_1}$$

in which R represented the regional rates (model), n the national rates (country), t the beginning of the projection period (1960) and t_1 the end of the projection period (1970). The national activity rates for a given sex-age group at the beginning — 1960 and at the end — 1970 — of the projection period are plotted on a graph together with the activity rates of the model (region) for the same sex-age group; the national activity rates are

TABLE 12

Chile: projections of activity rates, 1960-1980, calculated by the extrapolation method of the International Labour Office

Age group	Regional[a] activity rates			Activity rates in Chile				
	Observed (1960)	Projected (1980)	Ratio: increase or decrease	Observed (1960)	Projected (1980)	Interpolated (1965)	Interpolated (1970)	Interpolated (1975)
(1)	(2)	(3)	(4) = (3) : (2)	(5)	(6) = (5) (4)	(7)	(8)	(9)
Males								
0-14	2.5	1.5	60.0	2.3	1.2	2.0	1.8	1.5
15-19	70.4	65.0	92.3	61.7	56.9	60.5	59.3	58.1
20-24	92.2	86.0	93.3	91.6	85.5	90.1	88.5	87.0
25-44	97.5	97.0	99.5	96.9	96.9	96.9	96.9	96.9
45-54	91.7	93.0	101.5	90.0	92.1	91.2	91.5	91.8
55-64	67.4	65.0	96.4	80.6	77.8	79.9	79.2	78.5
65 and over	40.4	30.0	74.3	51.4	38.2	48.1	44.8	41.5
Females								
0-14	1.0	0.7	70.0	0.7	0.4	0.6	0.5	0.4
15-19	30.1	30.0	99.7	23.5	24.7	23.8	24.1	24.4
20-24	37.8	40.0	105.8	32.4	35.5	33.2	33.9	34.7
25-44	25.8	30.0	116.3	24.4	28.8	25.5	26.6	27.7
45-54	19.5	23.0	117.9	20.4	23.8	21.2	22.1	23.0
55-64	12.4	12.0	96.8	15.3	14.7	15.1	15.0	14.9
65 and over	5.1	4.0	78.4	7.9	6.0	7.4	7.0	6.5

[a] Temperate South America: Argentina, Chile, Paraguay and Uruguay.

TABLE 13

Temperate South America: projections of population, labour force and activity rates, by age group to 1970 (mid-year)

MALES

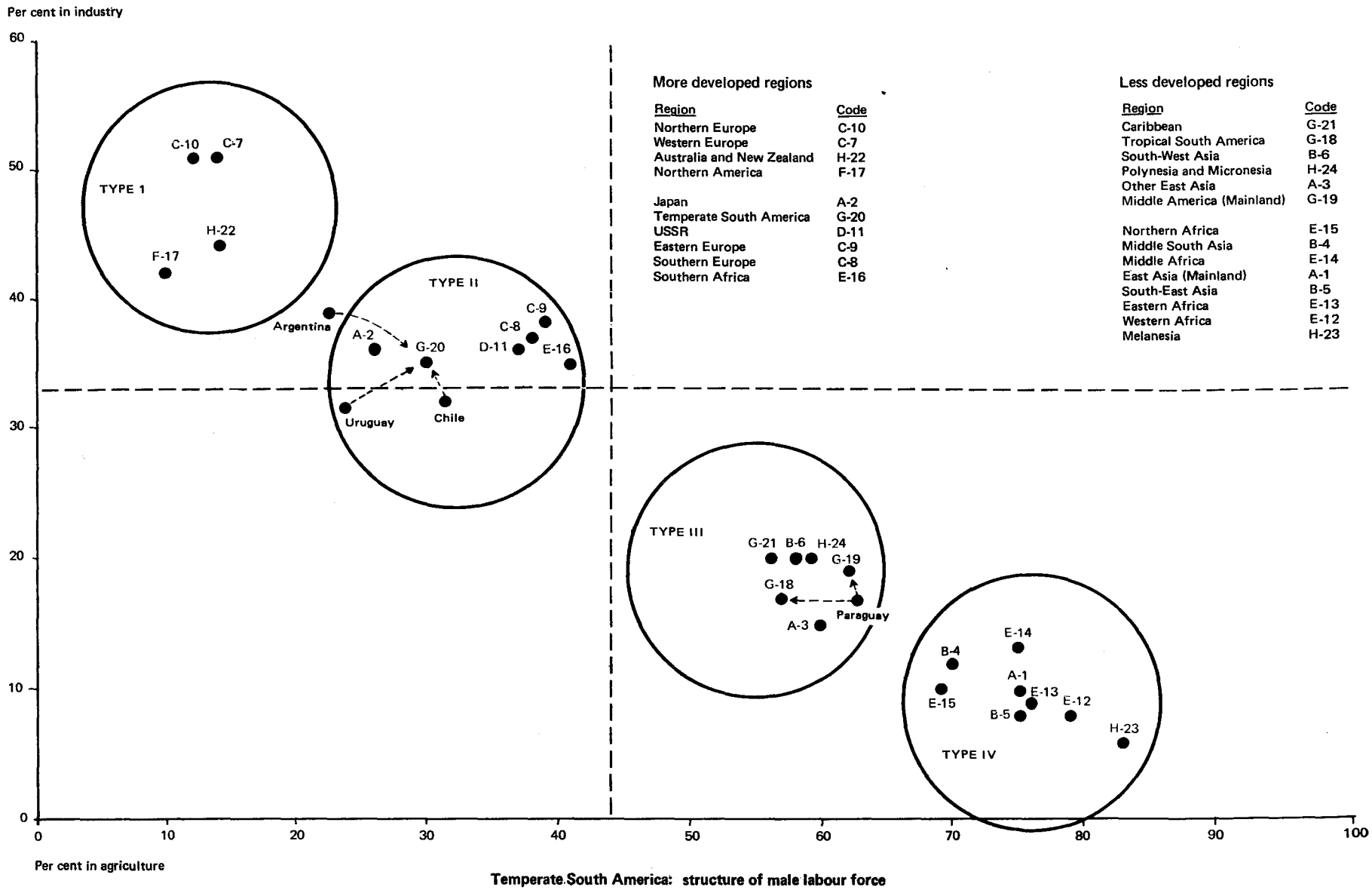
Area	Source	Total	Age group 0-14	15-19	20-24	25-44	45-54	55-64	65 and over
Population (thousands)									
Argentina	1970	12,137.4	3,510.5	1,107.9	1,016.7	3,387.5	1,313.3	973.0	828.5
Chile	1970	4,921.0	2,000.0	508.0	423.0	1,171.0	370.0	250.0	199.0
Paraguay	1970	1,153.3	519.0	130.1	108.5	250.3	68.4	42.5	34.0
Uruguay	1970	1,438.0	415.0	119.0	108.0	397.0	163.0	127.0	109.0
TOTAL	1970	19,649.7	6,444.5	1,865.0	1,656.2	5,205.8	1,914.7	1,392.5	1,170.5
Labour force (thousands)									
Argentina	1970	6,982.2	71.3	723.7	902.6	3,298.7	1,170.0	569.4	246.5
Chile	1970	2,448.2	36.0	283.7	381.0	1,159.0	328.5	180.6	79.4
Paraguay	1970	584.2	15.7	96.7	100.4	244.4	66.3	39.2	21.5
Uruguay	1970	825.4	12.4	75.3	99.0	386.8	149.0	77.8	25.1
TOTAL	1970	10,840.0	135.4	1,179.4	1,483.0	5,088.9	1,713.8	867.0	372.5
Activity rates									
Argentina	1970	57.53	2.03	65.32	88.78	97.38	89.09	58.52	29.75
Chile	1970	49.75	1.80	55.85	90.07	96.94	88.78	72.23	39.92
Paraguay	1970	50.65	3.02	74.34	92.54	97.66	96.99	92.19	63.37
Uruguay	1970	57.40	2.99	63.29	91.70	97.43	91.40	61.28	23.02
TOTAL	1970	55.17	2.10	63.24	89.54	97.75	89.51	62.26	31.82

TABLE 14

Temperate South America: projections of population, labour force and activity rates, by age group to 1970 (mid-year)

FEMALES

Area	Source	Total	Age group 0-14	15-19	20-24	25-44	45-54	55-64	65 and over
Population (thousands)									
Argentina	1970	11,912.5	3,372.9	1,068.5	976.9	3,210.7	1,321.4	1,038.3	923.8
Chile	1970	5,048.0	1,958.0	497.0	430.0	1,215.0	413.0	283.0	252.0
Paraguay	1970	1,168.7	507.5	128.0	104.4	259.0	75.5	51.0	43.3
Uruguay	1970	1,448.0	400.0	117.0	108.0	400.0	166.0	127.0	130.0
TOTAL	1970	19,577.2	6,238.4	1,810.5	1,619.3	5,084.7	1,975.9	1,499.3	1,349.1
Labour force (thousands)									
Argentina	1970	2,045.9	22.6	374.2	409.0	864.3	231.0	104.1	41.1
Chile	1970	742.1	8.8	121.0	147.7	323.1	83.9	40.7	17.1
Paraguay	1970	197.4	5.3	37.4	36.6	84.5	19.9	9.8	3.9
Uruguay	1970	283.9	3.9	36.0	45.8	138.3	39.1	16.1	4.8
TOTAL	1970	3,269.3	40.6	568.6	639.1	1,410.2	373.9	170.7	67.2
Activity rates									
Argentina	1970	17.17	0.65	35.02	41.87	26.92	17.48	10.03	4.48
Chile	1970	14.70	0.44	24.35	34.25	26.59	20.32	14.38	6.79
Paraguay	1970	16.89	1.04	29.21	35.08	32.63	26.41	19.16	8.97
Uruguay	1970	19.61	0.95	30.79	42.43	34.57	23.54	12.68	3.67
TOTAL	1970	16.70	0.63	31.41	39.47	27.73	18.92	11.39	4.98



Region	Code
Northern Europe	C-10
Western Europe	C-7
Australia and New Zealand	H-22
Northern America	F-17
Japan	A-2
Temperate South America	G-20
USSR	D-11
Eastern Europe	C-9
Southern Europe	C-8
Southern Africa	E-16

Region	Code
Caribbean	G-21
Tropical South America	G-18
South-West Asia	B-6
Polynesia and Micronesia	H-24
Other East Asia	A-3
Middle America (Mainland)	G-19
Northern Africa	E-15
Middle South Asia	B-4
Middle Africa	E-14
East Asia (Mainland)	A-1
South-East Asia	B-5
Eastern Africa	E-13
Western Africa	E-12
Melanesia	H-23

Temperate South America: structure of male labour force

then derived by fitting to the national data a curve similar to that yielded by the model. The result of this extrapolation in the case of Chile is shown in table 12.

In certain cases, particularly when computing activity rates for the female population, the national data may be extrapolated on the basis of more than one regional model. Female activity rates for Chile, for example, were projected by linkage to the projected trends of two regions, a national curve being plotted at one fourth of the distance between "temperate South America" and "tropical South America". A similar procedure was followed when the levels of activity rates and economic development were more closely paralleled by those of another region than by those observed in the geographical region in which the country was situated. Again, activity rates computed by the above procedure may be further adjusted if adequate data, such as data on urban and rural population, intercensal surveys, or information on economic development changes, educational policy and future pension policy are available.

The projected national sex-age-specific activity rates for 1970 obtained by the above procedures are then applied to the population projections for the countries concerned in order to derive the number of economically active persons in each sex-age group. The tabulations for temperate South America and its component countries are shown in tables 13 and 14; experience in using data for the twenty-three geographical regions as models has shown their usefulness.[8] There is evidence, however, that a reclassification of individual countries into a certain number of model groups — irrespective of their geographical situation — on the basis of the similarity of the demographic, economic and other relevant factors which affect their activity rates would provide more refined results than those obtained with the present United Nations classification method.

B. Correlation methods

Some of the extrapolation methods described above can be used if statistical data are available which record the pattern of sex-age-specific activity rates observed in the past and from which fairly consistent trends emerge. Naturally, even in such cases it may be desirable to introduce adjustments for those age groups whose activity rates are liable to be affected by specific developments which are known or can be anticipated, such as changes in the compulsory education age range. In many developing countries, however, the proportions of economically active persons of both sexes in each age group cannot be projected into the future on the basis of trends observed in the past, owing to the absence or inadequacy of comparable historical data. Another possible approach is to employ regression analysis. Its use requires that consideration first be given to whether the data for the regions — or zones or districts — of the country indicate a close relationship between the activity rates and certain characteristics of economic and social development, or whether the estimate of future activity rates can be compared with the data of another country or other countries: the advantage of the former method is that it is based solely on the data for the country concerned; the second method can be used where the other proves unsuitable.

Methods of this kind have been used for the manpower projections recently prepared for certain countries by the United Nations and the specialized agencies, and by other research organizations and institutes. Some of them are described below.

1. *Method based on the existence of correlations between the activity rates of the population and specified characteristics of economic development in various regions of a country*

The different stages of economic development obtaining in individual geographical regions of a particular country at a given time gives some idea of the dynamic and changing effect of the economic development process on the activity rates of various population groups. For this reason the future trend of the activity rate of the country as a whole is assumed to follow a pattern similar to that of the activity rates of the various regions of the country, measured from the least developed to the most developed.

(a) *Example of projections of labour supply for Central America, Panama and Mexico, 1950-1980* [9]

The basic assumption is that there is a close relationship between a country's level of industrialization and the sex-age-specific activity rates of its population. Since the development of industry is unequal in different regions of the country, the regional activity rates are necessarily different. In order to measure this correlation quantitatively, means must be found to measure the level of industrialization. It was accepted that the rate of industrialization is correctly represented by the proportion of the economically active population engaged in non-agricultural occupations. Accordingly the relationship between that rate and the sex-specific activity rates was analysed (see table 15). The correlation coefficient thus obtained is significant, the correlation being positive for females and negative for males.

The rates of industrialization which should be attained in 1980 were then selected for each country. Sex-specific activity rates for 1980 were computed from the results obtained by the analyses set out above. These activity rates were then distributed by age, assuming the continuance of the 1950 ratio of the

[8] Inverse activity rate projections carried out with a view to "predicting" 1950 rates from 1960 data show that the average error with this method is less than 2 per cent for a decennial projection of patterns of change in the economically active population, as compared with 10 per cent or more when activity rates are assumed to be constant.

[9] *Human Resources of Central America, Panama and Mexico, 1950-1980, in relation to some aspects of Economic Development* (United Nations publication, Sales No.: 60.XIII.1).

TABLE 15

Correlation and regression coefficients for economically active proportion of the population, by sex, in relation to level of industrialization in selected Central American countries, Panama and Mexico (1950)

Country and sex [a]	*Correlation coefficient*	*Regression equations* [b] $y = a - bx$ $y^1 = a^1 + b^1x$
Costa Rica		
Males	−0.71	$y = 95{,}188 - 0.100\,x$
Females	+0.96	$y^1 = 4{,}310 + 0.251\,x$
El Salvador		
Males	−0.74	$y = 87{,}650 - 0.093\,x$
Females	+0.92	$y^1 = 3{,}276 + 0.356\,x$
Guatemala		
Males	−0.46	$y = 80{,}220 - 0.70\,x$
Females	+0.76	$y^1 = 4{,}812 + 0.182\,x$
Nicaragua		
Males	−0.88	$y = 99{,}410 - 0.138\,x$
Females	+0.72	$y^1 = 6{,}680 + 0.234\,x$
Panama		
Males	−0.88	$y = 83{,}592 - 0.101\,x$
Females	+0.88	$y^1 = 8{,}397 + 0.226\,x$
Mexico		
Males	−0.86	$y = 93{,}434 - 0.139\,x$
Females	+0.76	$y^1 = 4{,}010 + 0.189\,x$

SOURCE: *Human Resources of Central America, Panama and Mexico, 1950-1980, in relation to some Aspects of Economic Development* (United Nations publication, Sales No.: 60.XIII.1).

[a] In Costa Rica and Mexico the economically active percentage of the population is the male or female population of 12 years and over; in El Salvador and Panama, of 10 years and over; in Nicaragua, of 14 years and over; and in Guatemala, of 7 years and over.

[b] Explanation of symbols: y = percentage of economically active males; y^1 = percentage of economically active females; and x = percentage of population engaged in non-agricultural occupations. The data for these percentages were computed by provinces or departments in the specified Central American countries, of Panama, and by states in Mexico, and were taken from the 1950 population censuses of the respective countries.

activity rates of each age group to the activity rates for the total population of each sex. Next, adjustments were then made for young people in the 10-14 and 15-19 age groups, and for persons aged 65 and over, to take account of the probable decline, in relation to the first estimate, in the future activity rates for those age groups. For the other age groups, however, the results of the computation were regarded as valid.

A number of comments on the value of this method are called for:

(i) The rate of industrialization cannot be clearly distinguished from the activity rate of the population. Auto-correlation is not excluded;

(ii) The use of the square of the correlation coefficient (r^2) to indicate the percentage of the variance explained by the correlation is not very satisfactory except in a limited number of cases, namely, females in Costa Rica and El Salvador ($r^2 = 0.92$ and 0.85 respectively); for males in Nicaragua and females in Panama r^2 is only 0.78. For other population groups studied results range from 0.5 to 0.6. Lastly, for males in Guatemala r^2 is only 0.21. Hence the explanatory value of the correlations is average or weak;

(iii) The parameter b or b', according to sex, is indicative, in the regression equation shown in table 15, of the intensity of the action of the explanatory variable on the explained variable. This parameter is moderately significant for females but it is nearly always very weak for males.

(b) *Projection of the economically active population of Japan, 1965-1985* [10]

A generally similar method was applied in the case of Japan, but there the analysis was more precise, with greater age group detail. The first step was a relatively detailed analysis of activity rate trends observed in the past. This analysis showed that industrialization, like increases in *per capita* income and other indicators of rising levels of living, tended to be associated with decreasing rates of economic activity. Social and institutional changes which may contribute to changes in activity rates were also taken into consideration and account was taken of the possible effect of aggregate labour demand in relation to a given target to be attained at the end of the projection period. After this comprehensive study of the trends observed in the past and of possible future determinants of manpower, regression analyses were used. The percentages of non-agricultural workers, i.e. the numbers employed in the secondary and tertiary sectors, were regarded as independent variables, and the sex-age-specific activity rates as the dependent variables. The regression equations were based on 1960 data for the forty-six administrative districts (prefectures).

It was anticipated that during the projection period, 1965-1985, the proportion of the labour force engaged in agriculture would diminish by about 60 per cent, falling from 32.5 per cent in 1965 to 13 per cent in 1985, and the estimates of activity rates for 1985 were made on the basis of this fundamental assumption. The values for the intermediate years were obtained by linear interpolation. However, in order to show the possible range of variations in future activity rates, three variants were prepared: variant A, based on constant activity rates at the levels recorded in the 1965 census; variant B, calculated by averaging variants A and C; and variant C, based on the assumption described above. The last variant was regarded as being the most plausible. It is interesting to note that it assumed a substantial reduction in activity rates, especially among females (see table 16).

2. *Correlation applied to observed differences between countries*

When it is not considered desirable to make projections by the methods described above or to use extrapolation methods because the trends observed in the past do not give an accurate picture of the evolution of the labour force, a substitute method can be used: it is based on the relationship between activity rates

[10] Japan, Institute of Population Problems, op. cit.

TABLE 16

Japan: past and projected activity rates, 1930-1985

Sex and age group	Activity rates					Deviation from 1960 rates		
	1930 [a]	1955 [b]	1960	1965 [b]	1985	1930	1965	1985
Males								
Total	*89.0*	*82.6*	*85.0*	*83.4*	*83.5*	*4.0*	*−1.6*	*−2.5*
15-19	78.4	54.3	50.7	38.6	40.0	27.7	−12.1	−10.7
20-24	91.8	88.1	86.8	87.1	85.5	5.0	0.3	−1.3
25-29	96.7	96.2	96.0	97.9	95.9	0.7	1.9	−0.1
30-34	98.0	97.0	97.1	98.6	65.8	0.9	1.5	−0.3
35-39	98.1	97.3	97.2	98.4	97.2	0.9	1.2	0.0
40-44	97.9	97.4	97.1	98.3	97.2	0.8	1.2	0.1
45-49	97.0	97.0	96.7	98.0	96.8	0.3	1.3	0.1
50-54	95.3	95.5	95.1	97.2	95.2	0.2	2.1	0.1
55-59	91.9	91.1	89.5	93.8	88.4	2.4	4.3	−1.1
60-64	85.3	82.4	81.9	85.3	78.2	3.4	3.4	−3.7
65 and over	63.0	56.4	54.5	55.1	39.2	8.5	0.6	−15.3
Females								
Total	*48.8*	*48.1*	*50.9*	*49.8*	*32.5*	*−2.1*	*−1.1*	*−18.4*
15-19	61.7	50.1	48.9	37.6	40.0	12.8	−11.3	−8.9
20-24	53.7	68.2	68.4	69.7	65.0	−14.7	1.3	−3.4
25-29	46.5	51.8	50.1	46.4	38.4	−3.6	−3.7	−11.7
30-34	48.7	49.6	50.7	48.0	32.8	−2.0	−2.7	−17.9
35-39	51.9	53.4	54.5	58.3	36.3	−2.6	−3.8	−18.2
40-44	53.7	55.5	57.1	62.1	39.0	−3.4	5.0	−18.1
45-49	53.5	54.4	56.1	62.6	37.2	−2.6	6.5	−18.9
50-54	50.7	51.3	51.9	57.3	32.1	−1.2	5.4	−19.8
55-59	45.0	45.7	45.8	50.1	25.8	−0.8	4.3	−20.0
60-64	35.3	38.4	39.1	39.3	16.8	−3.8	0.2	−22.3
65 and over	18.5	20.6	21.4	17.6	5.3	−2.9	−3.8	−16.1

SOURCE: Japan, Institute of Population Problems, *Estimates of Future Labour Force Population in Japan for October 1 from 1965 to 1985, Estimated in December 1966* (Tokyo, 1967), pp. 4 and 17.

[a] Based on the "gainful worker" concept.

[b] Based on a 1 per cent sample.

and certain economic and social variables which are assumed to have an effect on future rates of economic activity, particularly among young people and those in the older age groups. In order to prepare manpower projections for the Philippines,[11] for example, data from some thirty countries were examined with a view to measuring by correlation coefficients the association between:

(*a*) The age-specific activity rates for males in the youngest and oldest age groups, and the percentages of the male labour force engaged in agriculture. The results were as follows:

Males 10-14 years	+0.64
Males 15-19 years	+0.62
Males 55-64 years	+0.86
Males 65 years and over	+0.82

(*b*) The age-specific activity rates for males in these age groups and the percentage of the population 10 years of age and over who were literate. The results were as follows:

Males 10-14 years	−0.77
Males 15-19 years	−0.46
Males 55-64 years	−0.70
Males 65 years and over	−0.79

(*c*) The male and female activity rates for the youngest age groups and the percentage of persons in those age groups who were attending school. The results were as follows:

Males 10-14 years	−0.87
Males 15-19 years	−0.94
Females 10-14 years	−0.54

(*d*) The activity rates of young persons and the combined factors of school attendance and degree of industrialization (as measured by the percentage of active males in agriculture). The results were as follows:

Males 10-14 years	+0.89
Males 15-19 years	+0.94
Females 10-14 years	+0.63

Thus negative correlations were found between school attendance and activity rates in the young age groups, and also between literacy and the sex-age-specific

[11] *Population Growth and Manpower in the Philippines*, a joint study by the United Nations and the Government of the Philippines, 1960 (United Nations publication, Sales No.: 61.XIII.2), pp. 19-23 and 54-66.

activity rates. On the other hand, fairly high positive correlations were found between the importance of agriculture as a field of employment and activity rates for all the age groups tested. The multiple correlation coefficients between the activity rates of young persons and school attendance combined with the degree of industrialization were very significant for boys and adolescents of the two age groups 10-14 and 15-19, and were fairly high for girls aged 10 to 14. The regression equations used to estimate future activity rates were therefore based on the association with prospective changes in the percentage of the labour force engaged in agriculture in the case of males in the age groups 55-64 and 65 and over, and on the percentages attending school and engaged in agriculture, in the case of males in the 10-14 and 15-19 age groups and females aged 10-14.[12] The regression equations are given below:

Males 10-14 years . .	$x_1 = 42.20 + 156\,x_2 - 0.488\,x_3$
Males 15-19 years . . .	$x_1 = 89.68 + 0.026\,x_2 - 0.812\,x_3$
Females 10-14 years . .	$x_1 = 6.34 + 0.072\,x_2 - 0.071\,x_3$
Males 55-64 years . . .	$Y = 79.75 + 0.178\,x$
Males 65 years and over	$Y = 28.33 + 0.684\,x$

In these equations x_1 represents the percentage economically active in the given age group; x_2 the percentage of economically active males engaged in agriculture; x_3 the percentage in the given age group attending school; Y the percentage economically active in the given age group; and x the percentage of economically active males engaged in agriculture.

The results obtained were then adjusted as follows:

(*a*) For males under 20 years (10-14 and 15-19 years) and for females under 15 years, the activity rates were projected by means of a multiple regression equation, bearing in mind the probable future trend of school attendance and industrialization. The percentages of young people attending school were projected to the target year on the basis of trends observed in the past. Since a high level of school attendance (among young people 10-14 years) already obtained at the beginning of the projection period, it was assumed that the rate of increase would slacken. For adolescents, the 15-19 year group, it was assumed that the past rate of increase would be maintained;

(*b*) For males aged 20 to 24, it was assumed that the proportion of those attending school at the beginning of the projection period would increase in the future at the same rate as that of the 18-19 age group in the previous two decades. That assumption was made because of the lack of international data on school attendance for the 20-24 age group and because of the absence of a close association between the activity rate and the degree of industrialization;

(*c*) For males aged 25 to 55, it was assumed that the activity rates would remain constant until the end of the projection period, owing to the relative stability of the activity rates of males of these age groups, as shown by the national census data and the data of other countries;

(*d*) For females over 15 years, no close correlation was found between their activity rates and industrialization or school attendance. Consequently, it was assumed that the activity rates of females of all ages over 15 would remain constant until the target year, in both urban and rural sectors;

(*e*) The industrialization trend is measured by the percentage of the labour force engaged in agriculture, and this was assumed to fall slightly — from 68.8 to 67.3 per cent — during the first five years of the projection period (1957-1962), which in fact represents a very moderate rate of industrialization. For the latter part of the projection period, a slightly more rapid rate of industrialization was adopted: it was assumed that the percentage of the labour force engaged in agriculture would fall to 65.3 per cent in 1967, to 62.8 per cent in 1972 and 59.7 per cent in 1977. The assumptions used in this projection were arbitrary and a different assumption might have been adopted for nearly every part of the projection period, with somewhat different results.

C. Matrix representation of future labour force trends

Matrix computation enlarges the scope of population projection work, since it enables the combined action of different factors which may affect the evolution of the population to be assessed; a large number of variables can be introduced. Hence such procedures can be very useful in projections of the economically active population, which are influenced by a large number of factors. Nevertheless, while the method is obviously useful, its limitations must be clearly understood: the method is merely a helpful technical device for making the computations for a projection of the economically active population. The value of the computations depends, however, on the assumptions made as to the various factors which play a role in the evolution of the economically active population. Matrix computation in no way helps to define these assumptions; it simply makes it possible better to determine their effect on the numbers of the population seeking employment.

The principles of this new technique and methods for applying it will be described by reference to an example: that of the projection of labour supply in Mexico for 1965-1985.[13] This projection was made by sex and by age, a distinction being made between the urban and rural economically active populations.

In view of the constraints of the method in question, the inactive population, including children, must also

[12] For males over 55, the trend of school attendance is not applicable, so that the regression equation was limited to the association between percentages of the labour force and industrialization.

[13] This example is taken from the study by L. Tabah, "Representations matricielles de perspectives de population active", *Population*, No. 3, May-June 1968, pp. 437-476.

be estimated and distributed between the urban and rural populations. The model represents the total population and shows its evolution at all ages; it must therefore allow for future births.

Before the types of assumption needed for the functioning of the model, and its principles and operation are described, the meaning of the symbols used must be given: the number of males of age x, economically active and urban, at the beginning of the projection, will be denoted by $t^n x; m; a; u$. To represent the number of females, it is sufficient to replace m by f. To denote the inactive, i is substituted for a, and to denote the number of rural workers, r is substituted for u. The time reference is represented by the letter t placed before the letter n.

1. *Basic assumptions*

(a) *Probabilities of accession to and separation from activity or inactivity*

These probabilities, represented by the sign θ, were established on the basis of information derived from population censuses according to the following formulae, in which a_x represents the activity rate at age x for the base year:

Probability of accession to activity:

$$\theta_{i,a} = \frac{a_{x+1} - a_x}{1 - a_x}$$

Probability of separation from activity:

$$\theta_{a,i} = \frac{a_x - a_{x+1}}{a_x}$$

Probability of non-separation from inactivity:

$$\theta_{a,i} = \frac{1 - a_{x+1}}{1 - a_x}$$

Probability of non-separation from activity:

$$\theta_{a,a} = \frac{a_{x+1}}{a_x}$$

(b) *Probabilities of migration*

The probabilities of migration, represented by the symbol μ, were calculated as between urban and rural areas and by sex, but without using differential migration rates, which show whether persons were active or inactive in their areas of origin before migration. These calculations were prepared on the basis of census results.

(c) *Probabilities of survival*

The probable survival ratios were calculated in the urban and rural areas, without allowing for differential mortality depending on active and inactive status but assuming that migrants were subject to the pattern of mortality of the area form which they came. These probabilities are represented by the symbol P.

(d) *Probabilities of births*

Births during the projection period must be calculated because the method projects the evolution not only of the active population, but also of the inactive population. No differential fertility, depending on whether the women are active or inactive, was assumed. However, births must be distributed between future births to urban and rural, migrant and non-migrant women, and they must be distributed by sex. A calculation of this type, applied to urban females in a five-year projection of the Mexican population in 1960-1965, is cited here as an example.

The number of births of both sexes to urban females in the age group x to $x + 4$ during the year 1960 can be estimated by the following formula, in which fertility is represented by the sign φ:

$$(60^n\, x, x + 4; f; u) \cdot (60\, \varphi\, x, x + 4; u).$$

This is only an approximation, because it is the number of urban women of age x, $x + 4$ at 30 June 1960 which should have been multiplied by $60\, \varphi\, x, x + 4; u$. Births to urban women not emigrating during the whole projection period, in the absence of mortality, amount to:

$$(60^n\, x, x + 4; f; u) \cdot (\mu\, x, x + 4; f; u, u) \cdot (60\, \varphi\, x, x + 4; u).$$

The number of births at the end of the period, i.e. in 1964, in this group of women can be estimated by the following formula:

$$(60^n\, x, x + 4; f; u) \cdot (\mathrm{P}\, x, x + 4; f; u) \cdot (\mu\, x, x + 4; f; u, u) \cdot (65\, \varphi\, x + 5, x + 5 + 4; u)$$

This, too, is only an approximation, because it is the number of urban women who have not emigrated and who, by 30 June 1964, formed the age group $x + 4, x + 4 + 4$ which should have been multiplied by rate $60\, \varphi\, x, x + 4, x + 4 + 4$ to find the births in this group during the year 1964. Thus the average annual number of births between 1960 and 1964 can be estimated as the arithmetic mean of the last two expressions:

$$1/2\ [(60^n\, x, x + 4; f; u)\ (\mu\, x, x + 4; f; u)]\ [(60\, \varphi\, x, x + 4; u) + (65\, \varphi\, x + 5 + 4; u)(\mathrm{P}\, x, x + 4; f; u)].$$

To obtain the total number of births during the five years of the period for the same group of women, this quantity is multiplied by 5; then, to obtain male births alone, the sex ratio k is applied. To estimate the number of boys surviving on 1 January 1965, for example, it is sufficient to apply the projected male survival ratio for urban populations.

2. *Principles of the matrix*

The purpose of the matrix is to represent either entries through birth or expected future movements for each sex as between the economically inactive and active populations, and the urban and rural populations, or the expected movements within each of these groups on account of aging. In projecting these movements, the degree of detail as regards the ages of the population can be varied: in table 17, in order to simplify matters, only three age groups were used. Since there are three criteria — sex, active or inactive status, and urban or rural residence, there are eight possible initial situations, if age is not taken into con-

TABLE 17

Projection of the economically active population by sex and age with migration to and from rural and urban areas

Initial situation

Situation at end of period		Active urban males (1 2 3)	Inactive urban males (4 5 6)	Active urban females (7 8 9)	Inactive urban females (10 11 12)	Active rural males (13 14 15)	Inactive rural females (16 17 18)	Active rural females (19 20 21)	Inactive rural females (22 23 24)	Vector describing the initial situation	Sub-vectors
Active urban males	1									$0^n1;\ m;\ a;\ u$	$0^N m;\ a;\ u$
	2	$m_2^1\ M_1^1$	$m_2^4\ M_1^2$	M_1^3	M_1^4	$m_2^{13}\ M_1^5$	$m_2^{16}\ M_1^6$	M_1^7	M_1^8	$0^n2;\ m;\ a;\ u$	
	3	m_3^2	m_3^5			m_3^{14}	m_3^{17}			$0^n3;\ m;\ a;\ u$	
Inactive urban males	4			$m_4^7\ m_4^8\ m_4^9$	$m_4^{10}\ m_4^{11}\ m_4^{12}$			$m_4^{19}\ m_4^{20}\ m_4^{21}$	$m_4^{22}\ m_4^{23}\ m_4^{24}$	$0^n1;\ m;\ i;\ u$	$0^N m;\ i;\ u$
	5	$m_5^1\ M_2^1$	$m_5^4\ M_2^2$	M_2^3	M_2^4	$m_5^{13}\ M_2^5$	$m_5^{16}\ M_2^6$	M_2^7	M_2^8	$0^n2;\ m;\ i;\ u$	
	6	m_6^2	m_6^5			m_6^{14}	m_6^{17}			$0^n3;\ m;\ i;\ u$	
Active urban females	7									$0^n1;\ f;\ a;\ u$	$0^N f;\ a;\ u$
	8	M_3^1	M_3^2	$m_8^7\ M_3^3$	$m_8^{10}\ M_3^4$	M_3^5	M_3^6	$m_8^{19}\ M_3^7$	$m_8^{22}\ M_3^8$	$0^n2;\ f;\ a;\ u$	
	9			m_9^8	m_9^{11}			m_2^{20}	m_9^{23}	$0^n3;\ f;\ a;\ u$	
Inactive urban females	10			$m_{10}^7\ m_{10}^8\ m_{10}^9$	$m_{10}^{10}\ m_{10}^{11}\ m_{10}^{12}$			$m_{10}^{19}\ m_{10}^{20}\ m_{10}^{21}$	$m_{10}^{22}\ m_{10}^{23}\ m_{10}^{24}$	$0^n1;\ f;\ i;\ u$	$0^N f;\ i;\ u$
	11	M_4^1	M_4^2	$m_{11}^7\ M_4^3$	$m_{11}^{10}\ M_4^4$	M_4^5	M_4^6	$m_{11}^{19}\ M_4^7$	$m_{11}^{22}\ M_4^8$	$0^n2;\ f;\ i;\ u$	
	12			m_{12}^8	m_{12}^{11}			m_{12}^{20}	m_{12}^{23}	$0^n3;\ f;\ i;\ u$	
Active rural males	13									$0^n1;\ m;\ a;\ r$	$0^N m;\ a;\ r$
	14	$m_{14}^1\ M_5^1$	$m_{14}^4\ M_5^2$	M_5^3	M_5^4	$m_{14}^{13}\ M_5^5$	$m_{14}^{16}\ M_5^6$	M_5^7	M_5^8	$0^n2;\ m;\ a;\ r$	
	15	m_{15}^2	m_{15}^5			m_{15}^{14}	m_{15}^{17}			$0^n3;\ m;\ a;\ r$	
Inactive rural males	16			$m_{16}^7\ m_{16}^8\ m_{16}^9$	$m_{16}^{10}\ m_{16}^{11}\ m_{16}^{12}$			$m_{16}^{19}\ m_{16}^{20}\ m_{16}^{21}$	$m_{16}^{22}\ m_{16}^{23}\ m_{16}^{24}$	$0^n1;\ m;\ i;\ r$	$0^N m;\ i;\ r$
	17	$m_{17}^1\ M_6^1$	$m_{17}^4\ M_6^2$	M_6^3	M_6^4	$m_{17}^{13}\ M_6^5$	$m_{17}^{16}\ M_6^6$	M_6^7	M_6^8	$0^n2;\ m;\ i;\ r$	
	18	m_{18}^2	m_{18}^5			m_{18}^{14}	m_{18}^{17}			$0^n3;\ m;\ i;\ r$	
Active rural females	19									$0^n1;\ f;\ a;\ r$	$0^N f;\ a;\ r$
	20	M_7^1	M_7^2	$m_{20}^7\ M_7^3$	$m_{20}^{10}\ M_7^4$	M_7^5	M_7^6	$m_{20}^{19}\ M_7^7$	$m_{20}^{22}\ M_7^8$	$0^n2;\ f;\ a;\ r$	
	21			m_{21}^8	m_{21}^{11}			m_{21}^{20}	m_{21}^{23}	$0^n3;\ f;\ a;\ r$	
Inactive rural females	22			$m_{22}^7\ m_{22}^8\ m_{22}^9$	$m_{22}^{10}\ m_{22}^{11}\ m_{22}^{12}$			$m_{22}^{19}\ m_{22}^{20}\ m_{22}^{21}$	$m_{22}^{22}\ m_{22}^{23}\ m_{22}^{24}$	$0^n1;\ f;\ i;\ r$	$0^N f;\ i;\ r$
	23	M_8^1	M_8^2	$m_{23}^7\ M_8^3$	$m_{23}^{10}\ M_8^4$	M_8^5	M_8^6	$m_{23}^{19}\ M_8^7$	$m_{23}^{22}\ M_8^8$	$0^n2;\ f;\ i;\ r$	
	24			m_{24}^8	m_{24}^{11}			m_{24}^{20}	m_{24}^{23}	$0^n3;\ f;\ i;\ r$	

sideration.[14] Each initial situation is expressed by a vector (see penultimate column of the table). To complete this initial structure, the vector is pre-multiplied by a square matrix containing the same number of rows and columns as the vector, i.e. eight, since there are eight possible situations. The matrix is divided into 64 sub-matrices. Each of them can be expressed by the formula M_i^j, in which i denotes the final situation and j the initial situation. Age is not taken into account at this stage of the construction of the matrix. If, for example, three age groups are used, the number of sub-matrices becomes: $24 \times 24 = 576$.

Each sub-matrix represents movement from one of the eight possible initial situations — for example, active urban males — for a given age group — say, group 1 — either to the next age group in the same situation or to a different situation (such as transfer of active urban males into the inactive urban males group). Such movements are not possible, of course, to every block in the matrix: for example, there can be no transfer of urban active females into the urban actives males group, which would literally represent a change of sex. Of the 576 theoretical possible sub-matrices of table 17, only 112 can be filled in. They fall into three groups:

(*a*) Forty-eight sub-matrices correspond to entries into the population, i.e. births. These births derive from the four female groups: urban and rural females, active and inactive. The births are distributed among four possible groups, namely, males and females, urban and rural, all necessarily being inactive, since they are children;

(*b*) Sixteen sub-matrices record simple age shifts within individual active groups: m_2^1, m_8^7, m_{14}^{13} and m_{20}^{19} for movement from age group 1 to age group 2, and m_3^2, m_9^8, m_{15}^{14} and m_{21}^{20} for movements from group 2 to group 3. In the inactive groups, sub-matrices m_5^4, m_6^5, m_{12}^{11}, m_{17}^{16}, m_{18}^{17}, m_{23}^{22} and m_{24}^{23} represent the same age shifts;

(*c*) Sixteen sub-matrices represent changes as between inactivity and activity within either urban or rural areas:

Transitions from inactivity to activity:

$$m_2^4,\ m_3^5,\ m_8^{10},\ m_9^{11},\ m_{14}^{16},\ m_{15}^{17},\ m_{20}^{22},\ m_{21}^{23};$$

Transitions from activity to inactivity:

$$m_5^1,\ m_6^2,\ m_{11}^7,\ m_{12}^8,\ m_{17}^{13},\ m_{18}^{14},\ m_{23}^{19},\ m_{24}^{20}.$$

(*d*) Thirty-two sub-matrices correspond to migrations from and to rural and urban areas. Some of these migrations are combined with changes in activity status, when migrants at the same time move out of inactive into active status or vice versa:

a. The following sub-matrices represent migrations without change in activity status: m_{14}^1, m_{15}^2, m_{17}^4, m_{18}^5, m_{20}^7, m_{21}^8, m_{23}^{10}, m_{24}^{11}, m_2^{13}, m_3^{14}, m_5^{16}, m_6^{17}, m_8^{19}, m_2^{20}, m_{11}^{22}, m_{12}^{23};

b. The remaining sub-matrices represent migration movements from and to urban and rural areas with an accompanying change in activity status: m_{17}^1, m_{18}^2, m_{14}^4, m_{15}^5, m_{23}^7, m_{24}^8, m_{20}^{10}, m_{21}^{11}, m_5^{13}, m_6^{14}, m_2^{16}, m_3^{17}, m_{11}^{19}, m_{12}^{20}, m_8^{22}, m_9^{23}.

In practice it is not very likely that some of the situations represented will occur—that active urban males, for example, will become active rural males, since the movement is generally in the reverse direction (m_{14}^1, m_{18}^2). On the other hand, the sub-matrices m_2^{13} and m_3^{14}, which record the drift of agricultural population to towns, are of very great practical importance.

3. *Computation of the components of the matrix*

The example used will again be the population of Mexico, broken down into five-year age groups and distributed by sex, urban and rural segments of the population, and economically active and inactive segments of the population. The way in which the formulae are constructed when this structure is to be moved forward in five-year stages is explained below. The meaning of the symbols used is given first:

$60^{\eta}\, x, x + 4; f; a; u$ is the number of active urban females of age $x, x + 4$ in 1960;

$60^{\eta}\, x, x + 4; f; a; r$, the number of active rural females of age $x, x + 4$ in 1960;

$60^{\eta}\, x, x + 4; f; i; u$, the number of inactive urban females of age $x, x + 4$ in 1960;

$60^{\eta}\, x, x + 4; f; i; r$, the number of inactive rural females of age $x, x + 4$ in 1960;

$60\, \varphi\, x, x + 4; u$ and $65\, \varphi\, x, x + 4; u$ are the fertility rates from age x to age $x + 4$ in urban areas in 1960 and 1965, no allowance being made, for lack of observed data, for differential fertility depending on activity status; the introduction of this factor would not, however, give rise to any difficulty;

$60\, \varphi\, x, x + 4; r$ and $65\, \varphi\, x, x + 4; r$, are the same rates for rural areas;

$\mathrm{P}x, x + 4; f; u$ and $\mathrm{P}x, x + 4; f; r$ are the probable survival ratios in urban and rural areas in 1960-1964 for females between the ages of x and $x + 4$ at 1 January 1960; here again no allowance is made for differential mortality depending on activity status, although this could be done;

$\mathrm{P}n; f; v$ and $\mathrm{P}n; f; r$ are the probable survival ratios of girls born in 1960-1964 in urban and rural areas, no allowance being made for differential mortality depending on whether the mothers are active or inactive;

$\theta\, x, x + 4; f; u, r$ represents the probabilities of migration — mortality being ignored — in 1960-1964 from urban to rural areas for females between the ages of x and $x + 4$ at 1 January 1960, no allowance being made for differential migration depending on activity status;

$\theta\, x, x + 4; f; r, u$ represents the same probabilities for migration in the opposite direction;

$\theta\, x, x + 4; f; i, a; u$ represents the probabilities for inactive urban females of age $x, x + 4$, of entering activity before $x + 5, x + 5 + 4$;

$\theta\, x, x + 4; f; i, i; u$ represents the probabilities for the same females of not leaving inactive status;

[14] If we distinguish three age groups, as is done in table 17, there are twenty-four possible starting-points.

$\theta\, x, x+4; f; i, a; r$ and $\theta\, x, x+4; f; i, i; r$ represent the foregoing two probabilities for rural females;

$\theta\, x, x+4; f; a, i; u$ represents the probabilities, for active urban females of age $x, x+4$, of leaving active status before age $x+5, x+5+4$;

$\theta\, x, x+4; f; a, a; u$ represents the probabilities for the same female group of not leaving active status;

$\theta\, x, x+4; f; a, i; r$ and $\varphi\, x, x+4; f; a, a; r$ represent the foregoing two probabilities for rural females;

k is the masculinity proportion at birth;

all these expressions are in the same form when they concern males, except that the symbol m replaces f.

Each of the components of the sub-matrices can now be calculated.

Sub-matrix M_1^1: urban active males remaining active and urban:

The components of the sub-diagonal m_2^1 and m_3^2 represent combinations of probabilities of survival in urban areas, probabilities for active urban males of not leaving active status and probabilities of not migrating, i.e.:

$$(\mathrm{P}x, x+4; m; u) \cdot (\varphi\, x, x+4; m; a, a; u) \cdot (\varphi\, x, x+4; m; u, u).$$

Sub-matrix M_1^2: inactive urban males entering activity in urban areas:

The components of the sub-diagonal m_2^4 and m_3^5 this time represent combinations of probabilities of survival in urban areas, probabilities for inactive urban males of entering activity and probabilities of not migrating from the urban sector, i.e.:

$$(\mathrm{P}x, x+4; m; u) \cdot (\varphi\, x, x+4; m; i, a; u) \cdot (\varphi\, x, x+4; m; u, u).$$

The components of the other two other sub-matrices in the first row can easily be worked out:

Sub-matrix M_1^5 : active rural males migrating but remaining active:

$$(\mathrm{P}x, x+4; m; r) \cdot (\varphi\, x, x+4; m; a, a; r) \cdot (\varphi\, x, x+4; m; r, u).$$

Sub-matrix M_1^6: inactive rural males migrating and entering activity:

$$(\mathrm{P}x, x+4; m; r) \cdot (\varphi\, x, x+4; m; i, a; r) \cdot (\varphi\, x, x+4; m; r, u).$$

Similarly, for the first two sub-matrices of the second row:

Sub-matrix M_2^1: active urban males becoming inactive but not migrating:

$$(\mathrm{P}x, x+4; m; u) \cdot (\varphi\, x, x+4; m; a, i; u) \cdot (\mu\, x, x+4; m; u, u).$$

Sub-matrix M_2^2: inactive urban males remaining inactive but not migrating:

$$(\mathrm{P}x, x+4; m; u) \cdot (\varphi\, x, x+4; m; i, i; u, u) \cdot (\mu\, x, x+4; m; u, u).$$

Sub-matrices M_2^3 and M_2^4: survivors of male births to non-migrating active or inactive urban females:

The components of these two sub-matrices incorporate births occurring during the projection interval and hence serve to replenish the population. The method of calculating these sub-matrices was described in the passages concerning basic assumptions. The formula used is:

$$(60^{\eta}\, x, x+4; f; u \cdot \frac{5\,k}{2}\, \mathrm{P}n; m; u \cdot \mu\, x, x+4; f; u, u)$$

$$(60^{\varphi}\, x, x+4; u + 65^{\varphi}\, x+5, x+5+4; u \cdot \mathrm{P}x, x+4; f; u).$$

In these expressions, the coefficients of $60^{\eta}\, x, x+4; f; u$ are the components of the first row of the sub-matrix M_2^3. The sum of the expressions for all the age groups ($x = 15, \ldots, 45$) represents the total number of surviving male children of ages 0 to 4 years as of 1 January 1965. For the purpose of this calculation it was assumed that the migration of children born during the projection period could not occur independently of that of the mothers, so that the migration of mothers was identified with the migration of children. If these two migratory movements were assumed to be independent, the probability of migration of the mothers $\mu\, x, x+4; u, u$ would have to be replaced in the foregoing expression by that of the births $\mu\, n; m; u, u$ and the formula would be:

$$(60^{\eta}\, x, x+4; f; u \cdot \frac{5\,k}{2}\, \mathrm{P}n; m; u \cdot \mu n; m; u, u)$$
$$(60^{\varphi}\, x, x+4; u + 65^{\varphi}\, x+5, x+5+4; u \cdot \mathrm{P}x, x+4; f; u).$$

Sub-matrix M_2^5: active rural males becoming inactive and migrating:

$$(\mathrm{P}x, x+4; m, r) \cdot (\varphi\, x, x+4; m; a, i; r) \cdot (\mu\, x, x+4; m; r, u).$$

Sub-matrix M_2^6: inactive rural males remaining inactive and migrating:

$$(\mathrm{P}x, x+4; m; r) \cdot (\varphi\, x, x+4; m; i, i) \cdot (\mu\, x, x+4; m; r, u).$$

Sub-matrices M_2^7 and M_2^8: survivors of male births to migrating active or inactive rural females:

The same expression is used as for M_2^3 and M_2^4, the symbol r being substituted for u in the mortality and fertility functions and for the first index u at the base of the migration function, i.e.:

$$\left(\frac{5\,k}{2} \cdot \mathrm{P}n; m; r\, x \cdot \mu\, x, x+4; f; r, u\right)$$
$$(60^{\varphi}\, x, x+4; r + 65^{\varphi}\, x+5, x+5+4; r \cdot \mathrm{P}x, x+4; f; r).$$

The reader will easily find for himself the other components of the matrix, using similar reasoning.

The final result is shown in table 18 below.

D. Limitations of the methods described in this chapter

The methods described above project future sex-age-specific activity rates mechanically on the basis of past observed trends or on the basis of correlations established with the current distribution of activity rates in different parts of a country or in different countries. They have two potential drawbacks.

The reference period used in, for example, estimates by extrapolation, is often very short — five years in the example we gave for Japan. There are definite hazards involved in projecting the development of activity rates

over a twenty-year period by extrapolating from the observed trend for such a short period, inasmuch as the projection period is much longer than the reference period. This drawback is hard to eliminate. Many countries have information on trends in activity rates for only a short period, and even in those countries which have longer series, the data for the earlier years are difficult to handle because they relate to economic and social structures which have been made obsolete by technical advances or changing customs. Adjustments must therefore be made to the activity rates for the under-20 age groups and most certainly for the 65 and over age groups.[15] One example of such adjustments has already been given for the Philippines.[16] A second example which is cited below relates to Central America, Panama and Mexico;[17] table 19 gives the principal results of that projection.

Downward adjustment for the 10-14 age group in 1980 was substantial but relatively slight for the 15-19 group. The main assumption for the 10-14 males was that by 1980 activity rates for that group for the country as a whole would be no greater than those prevailing in 1950 among the urban population. In other words, it was assumed that with the level of economic development that might be reached by 1980 the standards of school facilities and school enrolment for the 10-14 boys for the whole country would equal those for urban children in 1950. For males aged 15-19 the adjustment made some allowance for longer schooling, but not to the same degree. The activity rates for girls aged 10-14 were assumed to be approximately the same as those of 1950, since more employment opportunities for girls aged 13 and 14, resulting from increased urbanization,

[15] In some cases, however, pension schemes affect workers under the age of 65. That is true, in particular, of some of the highly developed countries.

[16] *Population Growth and Manpower in the Philippines*, a joint study by the United Nations and the Government of the Philippines, 1960, (United Nations publication, Sales No.: 61.XIII.2), pp. 42-43.

[17] *Human Resources of Central America, Panama and Mexico, 1950-1980, in relation to some Aspects of Economic Development* (United Nations publication, Sales No.: 60.XIII.1).

TABLE 18

Projection of the Mexican population (1960-1965): absolute numbers

	1960	*1965*	*1970*	*1975*	*1980*	*1985*
			(Thousands)			
Economically active urban males	4,151.8 (11.5) [a]	5,279.8 (12.5)	6,636.2 (13.4)	8,329.8 (14.3)	10,364.9 (15.1)	12,753.9 (15.8)
Economically inactive urban males	4,728.6 (13.1)	6,100.3 (14.4)	7,633.5 (15.4)	9,405.0 (16.1)	11,509.2 (16.8)	14,032.4 (17.4)
Economically active urban females	1,391.2 (3.9)	1,646.2 (3.9)	2,016.5 (4.1)	2,540.8 (4.2)	3,224.4 (4.7)	4,092.5 (5.1)
Economically inactive urban females	7,982.3 (22.2)	10,098.9 (23.9)	12,514.0 (25.2)	15,293.5 (26.3)	18,638.2 (27.2)	22,490.0 (27.9)
Economically active rural males	4,569.1 (12.7)	4,924.1 (11.7)	5,293.2 (10.7)	5,715.2 (9.8)	6,186.3 (9.0)	6,702.8 (8.3)
Economically inactive rural males	4,518.4 (12.6)	4,842.7 (11.5)	5,307.3 (10.7)	5,850.0 (10.0)	6,444.2 (9.4)	7,137.6 (8.9)
Economically active rural females	519.3 (1.4)	562.4 (1.3)	615.9 (1.2)	675.6 (1.2)	781.7 (1.1)	889.7 (1.1)
Economically inactive rural females	8,142.6 (22.6)	8,804.1 (20.8)	9,586.4 (19.3)	10,480.4 (18.0)	11,445.6 (16.7)	12,508.3 (15.5)
TOTAL	36,003.3 (100.0)	42,258.5 (100.0)	49,603.1 (100.0)	58,290.2 (100.0)	68,594.5 (100.0)	80,608.1 (100.0)
Urban population	18,253.9 (50.7)	23,125.2 (54.7)	28,800.2 (58.1)	35,569.0 (58.9)	43,736,7 (63.8)	53,368.9 (66.2)
Rural population	17,749.4 (49.3)	19,133.3 (45.3)	20,802.8 (41.9)	22,721.2 (41.1)	24,857.8 (36.2)	27,238.4 (33.8)

[a] The figures in brackets represent the percentage of population in each category.

TABLE 19

Central America, Panama and Mexico: activity rates (1950 and 1980)

Sex and age	1950 census data						Projection for 1980					
	El Salvador	*Costa Rica*	*Guatemala*	*Nicaragua*	*Panama*	*Mexico*	*El Salvador*	*Costa Rica*	*Guatemala*	*Nicaragua*	*Panama*	*Mexico*
Both sexes												
10 and over	49.7	49.7	48.7	47.9	50.1	46.7	51.0	48.9	48.9	48.9	48.7	46.3
Males												
10 and over	84.5	84.8	84.4	85.1	78.6	82.9	80.2	80.3	81.2	82.3	76.1	77.0
10-14	37.8	37.4	39.9	40.0	17.4	—	20.0	20.0	25.0	25.0	10.0	—
15-19	88.9	91.1	90.6	89.6	68.3	—	84.0	84.0	87.0	86.0	66.0	—
20-24	95.6	96.7	96.6	96.9	94.8	—	92.9	94.2	94.4	94.8	93.4	—
25-34	97.1	98.4	97.8	98.4	97.8	—	94.3	95.9	95.5	96.3	96.3	—
35-44	97.5	98.6	97.9	98.7	98.2	—	94.7	96.1	95.7	96.5	96.7	—
45-54	97.5	97.6	97.3	98.5	97.1	—	94.7	95.1	95.1	96.3	95.6	—
55-64	95.4	94.8	94.7	97.3	89.6	—	92.7	92.4	92.7	95.1	98.3	—
65 and over	82.7	74.0	74.1	86.3	70.3	—	75.1	70.3	74.0	80.0	63.0	—
Females												
10 and over	16.2	15.2	12.5	13.0	20.3	12.5	21.9	17.6	15.7	15.7	20.9	16.0
10-14	7.9	5.0	6.4	6.4	5.3	—	8.0	3.5	7.0	7.0	4.0	—
15-19	20.7	22.5	15.8	15.0	23.4	—	29.1	27.3	19.9	18.5	24.8	—
20-24	20.9	22.6	14.9	16.3	29.6	—	29.4	27.4	18.8	19.7	31.3	—
25-34	17.4	17.2	13.0	14.5	25.2	—	24.5	20.8	16.4	17.5	26.7	—
35-44	17.3	15.7	13.9	14.3	24.6	—	24.4	19.0	17.5	17.2	26.1	—
45-54	15.9	13.3	13.5	13.7	20.8	—	22.4	16.1	17.1	16.5	22.0	—
55-64	13.5	9.1	12.3	13.1	15.0	—	19.0	11.0	15.6	15.8	15.9	—
65 and over	10.6	5.6	8.9	8.9	8.4	—	11.8	6.8	11.9	10.7	8.6	—

SOURCE: *Human Resources of Central America, Panama and Mexico, 1950-1980, in relation to some Aspects of Economic Development* (United Nations publication, Sales No.: 60.XII.1), tables XXV, XXVI, XXVII, XXVIII, XXIX and XXX, pp. 138-140.

might be offset by increased school enrolment among girls aged 10-12. For males aged 65 and over, the activity rates resulting from the first approximation were slightly decreased to allow for a greater frequency of retirement as general standards of living rise, mainly as a result of accelerated economic development and the gradual expansion of social security schemes. For women aged 65 and over, it appeared reasonable to make a slight downward adjustment in some cases, whereas in others the first approximation results were left unadjusted.

Even in the case of adult males, methods of projection by direct extrapolation, for example, may yield absurd results. Annexes I and II show that in the case of Japan, for instance, the application of these methods gives, for the 30-34 age group, an activity rate of 99.8 or 100 per cent in 1980, depending on whether equation (*a*) or equation (*b*) is used for the corrective coefficient. In every country there will always be a small percentage of adult males who are incapable of working for health and other reasons. Every result arrived at by mechanical projection must therefore be submitted to careful scrutiny.

Activity rates derived by correlation and regression analyses should not always be taken as strictly reliable; they must be subjected to certain adjustments based on anticipated economic and social development and its effects on the labour force. Some judgements on trends in activity rates may be better than the mechanical results of a mathematical operation which takes into account only some of the many factors involved.[18] This applies particularly to the determination of the activity rates of economically active females in the youngest and oldest age groups. In such cases, in view of the uncertainties inherent in such judgements, it seems essential to formulate several assumptions concerning changes in the rates, allowing for such factors as prospects for increased school enrolment or pension schemes.

E. Measuring the relative incidence of population growth, changes in activity rates and other variables on the future size of the economically active population

A projection of the size of the economically active population, broken down by sex and age, is obtained by multiplying the number of persons in the projected population by the projected activity rates. The relative incidence of population growth and of changes in activity rates on the future size and sex-age structure of the economically active population must then be determined. Table 20 shows patterns of change in the total economically active population, by sex and by age, of the United States of America from 1965 to 1980 arising from population growth, on one hand, and changes in activity rates, on the other.

What strikes one first of all is that nearly 88 per cent of the total increase in the male economically active population and about 72 per cent of the increase in the female economically active population are attributed to population growth. In the case of males 65 years and over, the sharp projected decline in activity rates will be almost entirely offset by the increase in the numbers of the group. Changes in activity rates are much more marked in the female economically active population, especially in the age groups in the 35-64 year range. The effect of these two factors may be analysed in greater detail in order to ascertain, for instance, their proportionate contribution to changes in the activity rates of the economically active population as a whole. A simple method is utilized for this purpose. The total activity rate is taken to be:

$$a_\Sigma = \frac{A_\Sigma}{P_\Sigma} = \frac{A_m + A_f}{P_\Sigma} = \frac{A_m}{P_m} \cdot \frac{P_m}{P_\Sigma} + \frac{A_f}{P_f} \cdot \frac{P_f}{P_\Sigma} = a_m p_m + a_f p_f$$

This equation is used, in conjunction with the data in table 20, to derive the following changes in the total activity rate from the incidence of the activity rates of the male and female economically active populations and the proportion of total population they represent:

	Symbol	(t) 1965	(t_1) 1980
Economically active population (14 years and over)			
TOTAL . . .	A_Σ	78.357	101.408
Male	A_m	51.705	65.981
Female	A_f	26.653	36.427
Population (14 years and over)			
TOTAL . . .	P_Σ	138.261	173 908
Male	P_m	67.205	84.123
Female	P_f	71.056	89.785
Activity rates			
TOTAL . . .	a_Σ	56.7	58.3
Male	a_m	76.9	77.2
Female	a_f	37.5	40.6
Proportion of total population			
Male	$p_m = \frac{P_m}{P_\Sigma}$	48.6	48.4
Female	$p_f = \frac{P_f}{P_\Sigma}$	51.4	51.6

The data in table 20 are also used in the following procedure:

Change in the male activity rate:

$$p_{m_{t_1}} \left(a_{m_{t_1}} - a_{m_t}\right), \text{ or } 48.4\,(77.2 - 76.9) = 0.145;$$

Change in the female activity rate:

$$p_{f_{t_1}} \left(a_{f_{t_1}} - a_{f_t}\right) \text{ or } 51.6\,(40.6 - 37.5) = 1.599;$$

[18] S. Cooper and D. F. Johnston, "Comments on the Denberg-Strand-Dukler approach", *Industrial Relations*. vol. 5, No. 1, October 1966, p. 73.

TABLE 20

United States of America: changes in the total economically active population, by sex and age, arising from population growth and changes in activity rates, 1965 and 1980

Sex and age groups	*Total economically active population (thousands)*			*Changes in total economically active population*				*Relative incidence of changes (per cent)*	
	1965 (observed)	*(1980) (projected)*	*1980 [a] (projected)*	*Total*	*Due to population growth*	*Due to changes in activity rates*	*Totals*	*In activity rates*	*In population numbers*
(1)	*(2)*	*(3)*	*(4)*	*(5) = (3) — (2)*	*(6) = (4) — (2)*	*(7) = (3) — (4)*	*(8) = (3) : (2)*	*(9) = (3) : (4)*	*(10) = (8) — (9)*
Both sexes									
14 years and over	78,357	101,408	98,606	23,051	20,249	2,802	29.4	2.8	26.6
Males									
14 years and over	51,705	64,981	64,691	13,276	12,986	290	25.7	0.4	25.3
14-19	4,591	5,744	5,610	1,153	1,019	134	25.1	2.4	22.7
20-24	5,926	9,065	8,960	3,139	3,034	105	53.0	1.2	51.8
25-34	10,653	17,590	17,554	6,937	6,901	36	65.1	0.2	64.9
35-44	11,504	12,084	12,021	580	517	63	5.0	0.5	4.5
45-54	10,131	10,219	10,144	88	13	75	0.9	0.7	0.2
55-64	6,768	8,185	8,134	1,417	1,366	51	20.9	0.6	20.3
65 years and over	2,131	2,096	2,584	−35	453	−488	−1.6	−18.9	17.3
Females									
14 years and over	26,653	36,427	33,669	9,774	7,016	2,758	36.7	8.2	28.5
14-19	2,940	3,832	3,571	892	631	261	30.3	7.3	23.0
20-24	3,375	5,380	5,084	2,005	1,709	296	59.4	5.8	53.6
25-34	4,336	7,347	7,019	3,011	2,683	328	69.4	4.7	64.7
35-44	5,724	6,386	5,862	662	138	524	11.6	8.9	2.7
45-54	5,714	6,805	5,787	1,091	73	1,018	19.1	17.6	1.5
55-64	3,587	5,337	4,579	1,750	992	758	48.8	16.6	32.2
65 years and over	976	1,340	1,281	364	305	59	37.3	4.6	32.7

SOURCE: United States of America, Bureau of Labor Statistics, Special Labor Force Report No. 73, *Monthly Labour Review*, September 1966, p. 986.

[a] Economically active population in 1980 calculated on the basis of 1965 activity rates. As the figures given have been rounded off to the nearest whole number, the sum of the figures in the individual groups may not be equal to the totals given.

Share of the male population:

$(a_{m_t} p_{m_{t_1}} + a_{f_t} p_{f_t}) - (a_{m_t} p_{m_t} + a_{f_t} p_{f_t})$, or $(76.9 \times 48.4 + 37.5 \times 51.4) - (76.9 \times 48.6 + 37.5 \times 51.4) = -0.154$;

Share of the female population:

$(a_{m_t} p_{m_t} + a_{f_t} p_{f_{t_1}}) - (a_{m_t} p_{m_t} + a_{f_t} p_{f_t})$, or $(76.9 \times 48.6 + 37.5 \times 51.6) - (76.9 \times 48.6 + 37.5 \times 51.4) = 0.075$.

The absolute increase in the activity rate of the total economically active population in 1965-1980 is approximately 1.6 per cent; the analysis above shows that it is the product of the following changes:

	Absolute change	*Relative change*
(1) Increase in male activity rates	0.145	8.70
(2) Increase in female activity rates	1.599	96.04
(3) Decrease in the proportion of males in the total population	−0.154	−9.24
(4) Increase in the proportion of females in the total population	0.075	4.50
Total (1), (2) and (4)	1.819	109.24
Total (3)	−0.154	−9.24
Total (1), (2), (3) and (4)	1.665	100.00
Remainder	0.065	3.91

This procedure demonstrates that the increase resulting from higher female activity rates is approximately equal to the increase in the activity rates of the total economically active population. The changes in other components are minimal.

The method described above may also be utilized for projecting female activity rates by marital status. As a rule, however, there is no theoretical single measure of the separate incidence of each relevant factor on the economically active population. Changes in each factor considered occur simultaneously, so that the effect of one change is necessarily influenced by other changes.[19]

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[19] In addition to the method described here, reference may be made to the method used to project the economically active population of the United Kingdom of Great Britain and Northern Ireland, especially its female segment. See W. Beckerman and J. Sutherland, "Married women at work in 1972", *National Institute Economic Review*, No. 23, February 1963, pp. 59-60. See also W. Beckerman and associates, *The Britsh Economy in 1975* (Cambridge University Press, 1965), chap. III.