Projecting Age-Sex-Specific Mortality: A Comparison of the Modified Lee-Carter and Pattern of Mortality Decline Methods

Danan Gu, François Pelletier, and Cheryl Sawyer | PAA 2018
United Nations | Department of Social and Economic Affairs | Population Division

Objectives

• Compare the performance of the modified Lee-Carter (LC) method and the pattern of mortality decline (PMD) method in projecting age-sex-specific mortality for 155 countries

• Provide a rationale for the choice of methods to project age-specific death rates for different groups of countries in the World Population Prospects (WPP).

Methods

1. ***Pattern of mortality decline (PMD)***

$$\ln mx(t_2) = \ln m_x(t_1) - k(t_{12})\rho_x(t_{12})$$

- $m_x(t_1)$ and $m_x(t_2)$: age-specific death rate at age x at time t_1 and t_2, corresponding to life expectancy at birth $e_0(t_1)$ and $e_0(t_1)$.
- $\rho_x(t_{12})$: age-specific pattern of mortality decline from time t_1 to time t_2, $\sum \rho_x(t_{12}) = 1$.
- $k(t_{12})$: is a parameter governing the level of mortality decline over the time.

-- Andreev, Gu, and Gerland (2013)

2. ***Lee-Carter (LC) model and its variants***

$$\ln m_x(t) = a_x + b_x k(t) + \varepsilon_x(t) \sim N(0, \sigma_x^2)$$

- $m_x(t)$: age-specific death rate at age x at time t.
- a_x: baseline age pattern of mortality.
- b_x: average rate of change in age-specific death rate for a unit change in $k(t)$, $\sum b_x = 1$.
- $k(t)$: index of the overall level of mortality at time t, $\sum_{t=1}^{T} k(t) = 0.$ (T, # of empirical data points)

-- Lee and Carter (1992); Ševčíková et al.(2016).

2.1 Modified LC (MLC)

- Gender coherent [b_x(males)= b_x(females)]
- Shift in age pattern in mortality improvement [from young to older ages]

-- Li and Lee (2005); Li, Lee, and Gerland (2013)

- Three variants of age pattern (a_x) in MLC:
  - ❖ last data point without smooth (**MLC1**)
  - ❖ last data point with smooth (**MLC2**)
  - ❖ average age pattern over time (**MLC3**)

3. ***Country groups***

- HMD [HMDa (13) + HMDb (25)]
- Non-HMD countries (117)

4. ***Criteria of comparisons***

- Occurrence of sex **crossovers** in m_x
- Occurrence of **jumps** in $m_x$over time

Figure 1. Illustration of sex crossovers in m_x (left) and jumps in m_x over time (right)

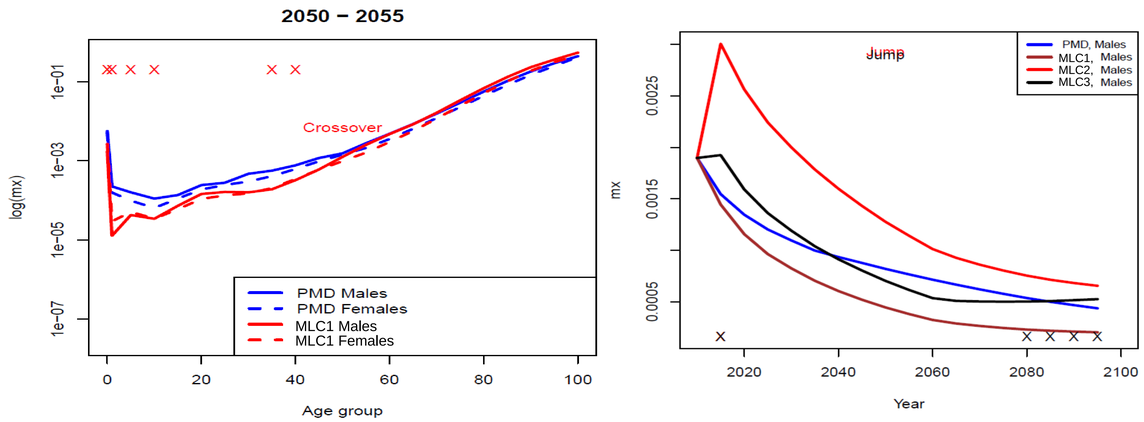

Figure 2. Occurrence rates of sex crossovers in m_x (left two) and jumps in m_x over time (right two) by method and country group

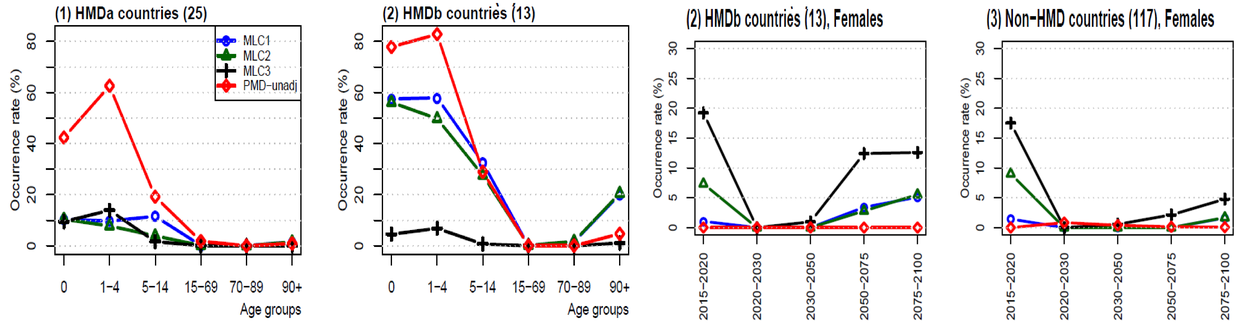

Figure 3. Comparisons between occurrence rate of sex crossovers and males' occurrence rate of jumps in m_x by projection method and country group

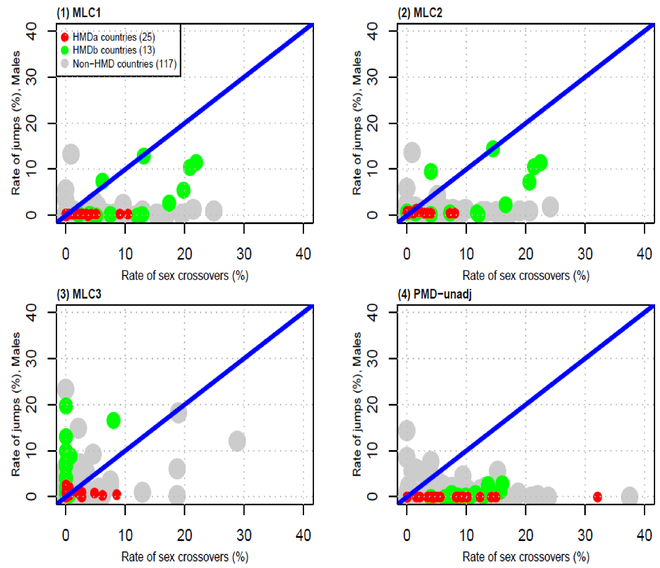

Table 1. Average occurrence rates of sex crossovers in m_x (left) and jumps in m_x over time for males (right) by method and country group

	MLC 1	MLC2	MLC3	PMD-unadj	PMD-WPP
	(1)	(2)	(3)	(4)	(5)
Country classification by quality of data					
All countries (155)	3.9	3.4	1.5	6.8	---
All HMD countries (38)	5.3	4.6	1.0	8.5	---
HMDa countries (25)	2.2	1.4	1.2	7.5	---
HMDb countries (13)	11.2	10.6	0.7	10.6	---
Non-HMD countries (117)	3.5	3.0	1.7	6.2	---
Country classification according to the approach used in the *2017 Revision*					
PMD-WPP (130)	4.3	3.7	1.6	6.7	**3.2**
MLC1 (8)	**1.4**	1.2	1.4	4.4	---
MLC2 (5)	1.2	**0.3**	2.8	7.2	---
MLC3 (12)	3.2	2.0	**0.4**	9.6	---

	MLC 1	MLC2	MLC3	PMD-unadj	PMD-WPP
	(1)	(2)	(3)	(4)	(5)
Country classification for HMD vs. non-HMD					
All countries (155)	0.7	1.2	2.5	1.0	---
All HMD countries (38)	1.5	2.0	3.1	0.3	---
HMDa countries (25)	0.2	0.7	0.8	0.0	---
HMDb countries (13)	4.0	4.6	7.3	0.9	---
Non-HMD countries (117)	0.5	0.9	2.4	1.2	---
Country classification according to the approach used in the *2017 Revision*					
PMD-WPP (130)	0.8	1.3	2.9	1.1	**1.2**
MLC1 (8)	**0.2**	0.6	1.0	0.0	---
MLC2 (5)	0.3	**0.7**	0.8	0.0	---
MLC3 (12)	0.2	0.6	**0.7**	0.0	---

Conclusions

- MLC3 produced fewer sex crossovers, but more jumps.
- MLC1 produced fewer jumps yet more sex crossovers.
- MLC works quite well for HMDa countries.
- Sex crossovers by PMD-unadj could be avoided by adjusting sex ratio in m_x.
- PMD is preferred for HMDb and non-HMD countries.

Recommendations

- **For MLC**: More flexible base year age pattern of m_x
- **For PMD**: Sex-coherent age pattern of mortality improvement should be used.
- **For MLC and PMD**: For HMDb and Non-HMD countries, base-year mortality age pattern may incorporate regional or subregional patterns.
- Refine the female-male gap in e0 in BHM projection to avoid sex gaps in e0 that are too narrow.

References

1. Andreev, K. Gu, D., Gerland, P. (2013). Age Patterns of Mortality Improvement by Level of Life Expectancy at Birth with Applications to Mortality Projections. Paper presented at the Annual Meeting of the Population Association of America, New Orleans, LA. http://paa2013.princeton.edu/papers/132554.

2. Gu, D., Pelletier F. and Sawyer, C. (2017). Projecting Age-sex-specific Mortality: A Comparison of the Modified Lee-Carter and Pattern of Mortality Decline Methods, UN Population Division, Technical Paper No. 6. New York: United Nations. https://esa.un.org/unpd/wpp/Publications/Files/WPP2017_TechnicalPaperNo6.pdf

3. Lee, R.D. and Carter, L. R. (1992). Modeling and forecasting US mortality. Journal of the American Statistical Association, 87(419): pp. 659–671.

4. Li, N. and Lee, R.D. (2005). Coherent mortality forecasts for a group of populations: An extension of the Lee–Carter method. Demography, 42(3): pp. 575–594.

5. _____, Lee, R., & Gerland, P. (2013). Extending the Lee-Carter method to model the rotation of age patterns of mortality decline for long-term projections. Demography, 50(6): pp. 2037–2051.

6. Ševčíková H., Li, N., Kantorová, V., Gerland, P., and Raftery, A.E. (2016). Age-specific mortality and fertility rates for probabilistic population projections. In R. Schoen (eds.). Dynamic Demographic Analysis: pp. 285-310. Cham, Switzerland: Springer.