Probabilistic Projections of the Total Fertility Rate

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Introduction

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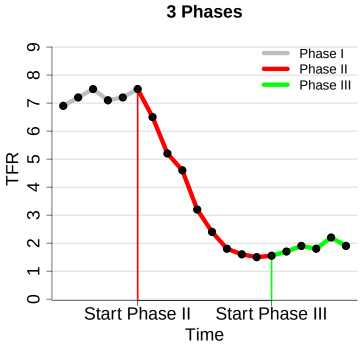

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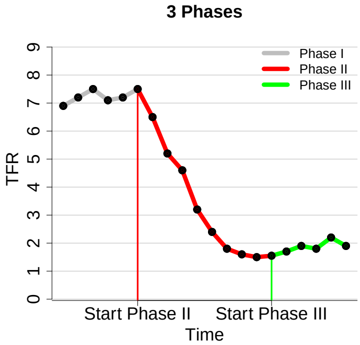

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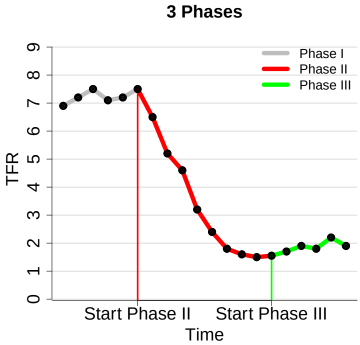

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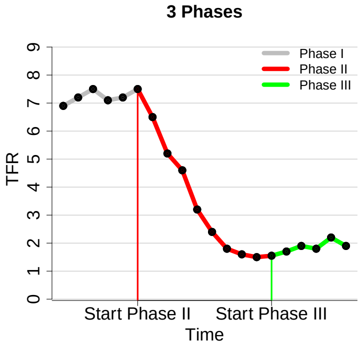

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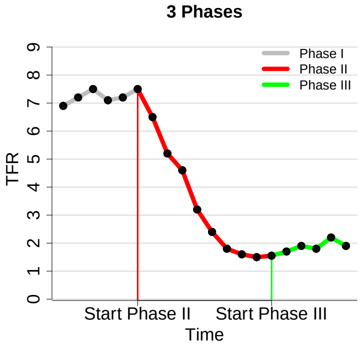

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  $\longrightarrow$ All countries are currently in phase II or III

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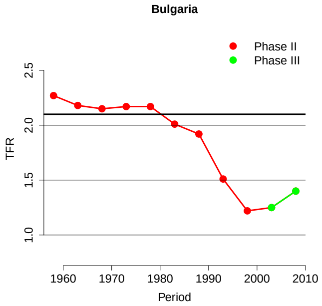

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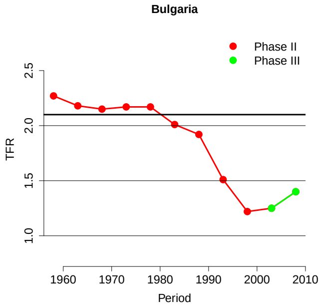

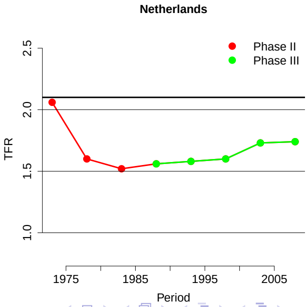

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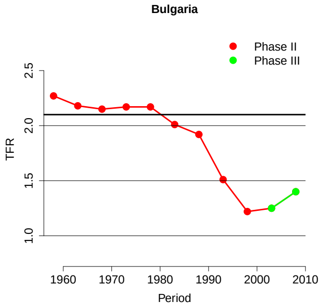
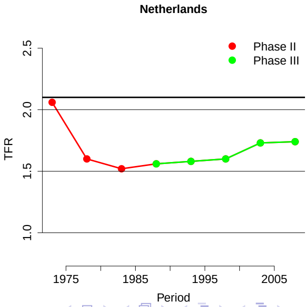
- Start of phase III before 2005-2010 observed in 20 countries
  (Singapore, Bulgaria, Czech Republic, Russian Federation, Channel Islands, Denmark, Estonia, Finland, Latvia, Norway, Sweden, United Kingdom, Italy, Spain, Belgium, France, Germany, Luxembourg, Netherlands, United States of America)





Outline

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1. Phase II: The fertility transition
2. Phase III: Post-transition low fertility
3. Results

Current UN projections for high fertility countries

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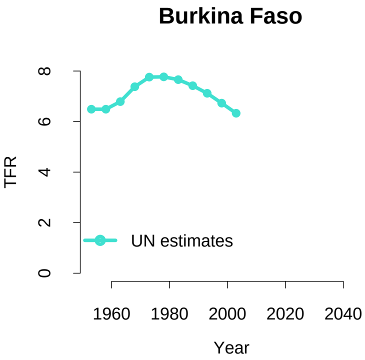

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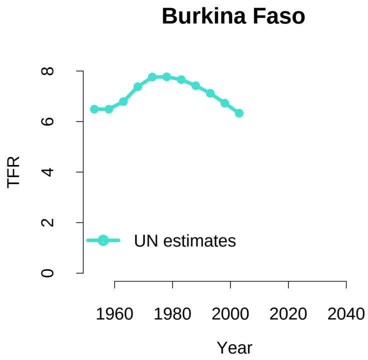

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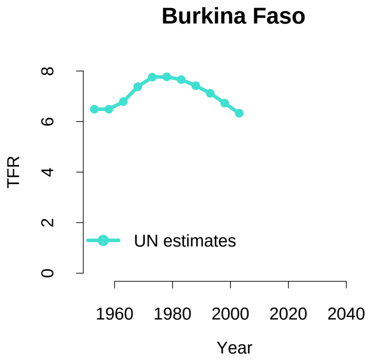

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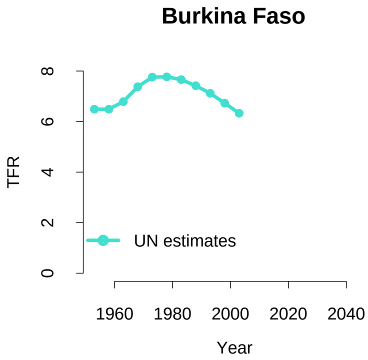

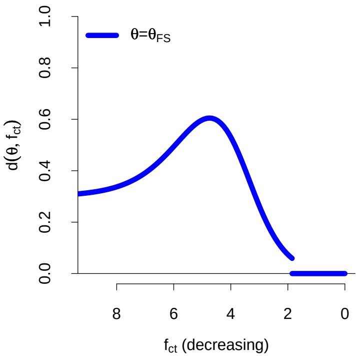

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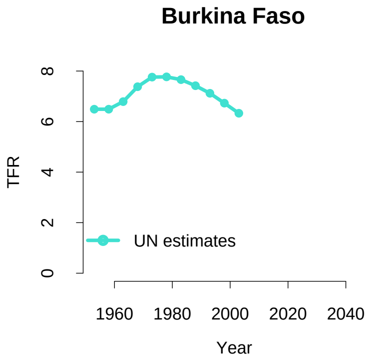

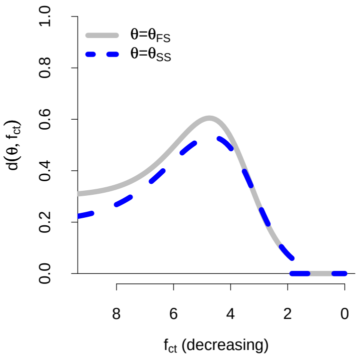

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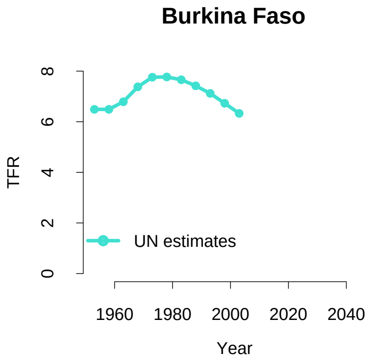

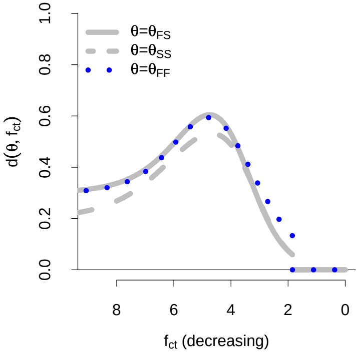

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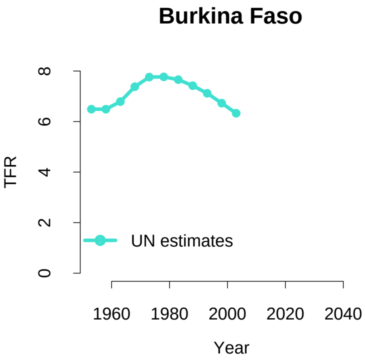

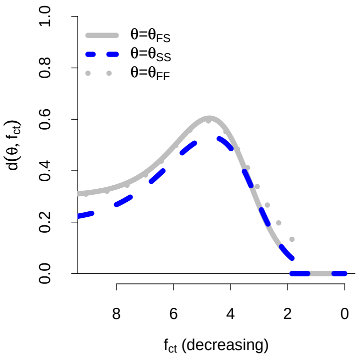

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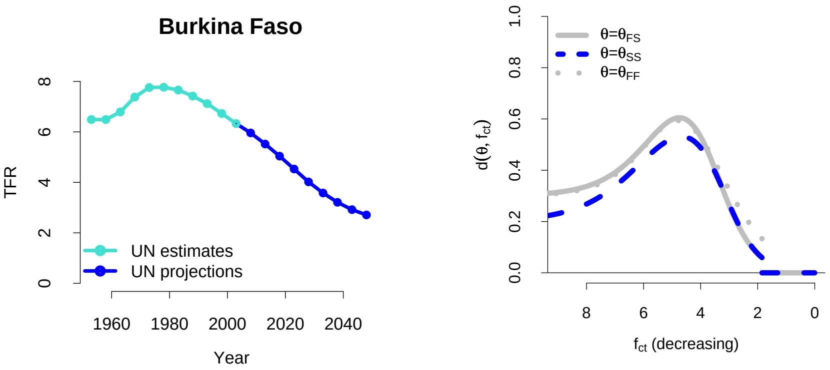

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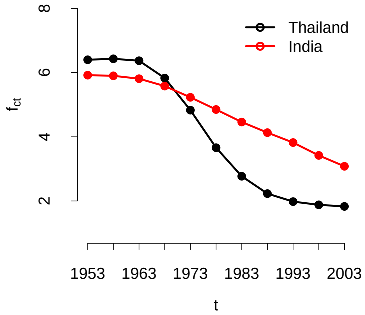

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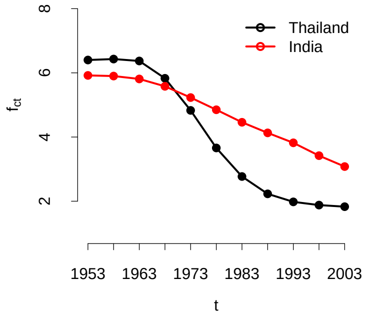

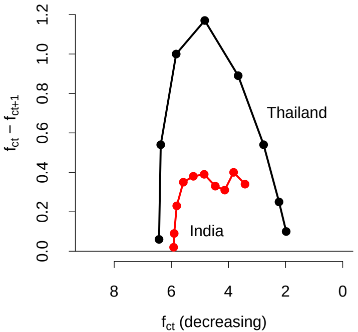

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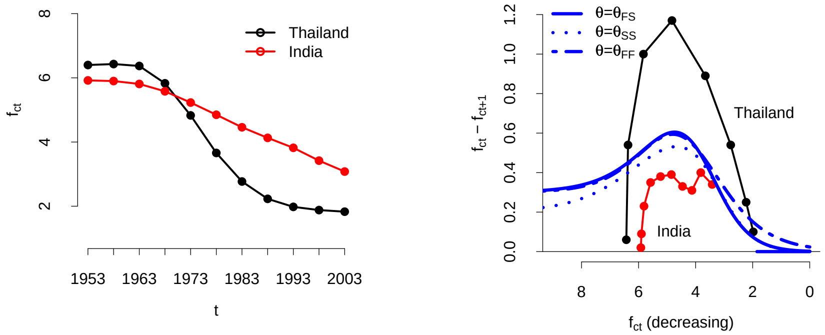

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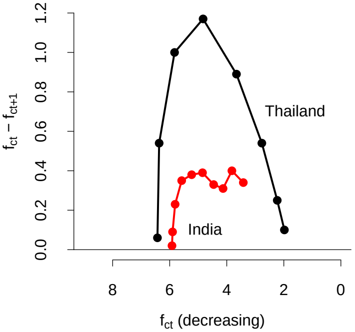

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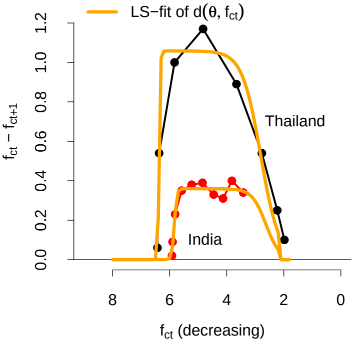

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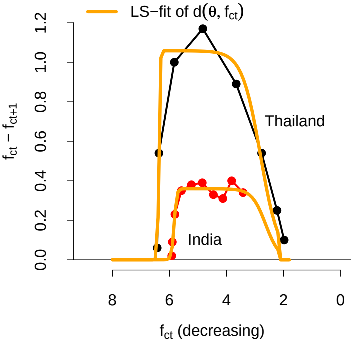

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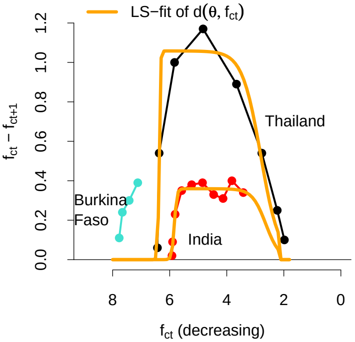

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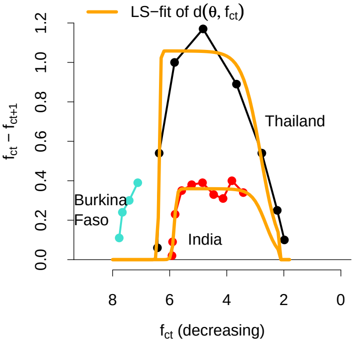

Random walk with drift:

$$f_{c,t+1} \quad = \quad f_{c,t} - d(\boldsymbol{\theta}_c, f_{c,t}) + \varepsilon_{c,t},$$

with $\begin{cases} f_{c,t} & \text{TFR for country } c\text{, 5-year period } t \\ d(\boldsymbol{\theta}_c, f_{c,t}) & \text{5-year decrement} \\ \varepsilon_{c,t} & \text{Random distortions} \end{cases}$

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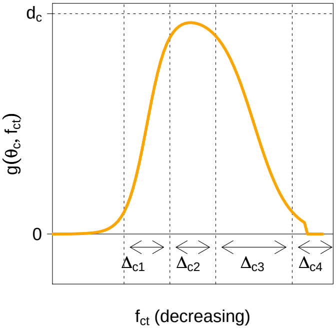

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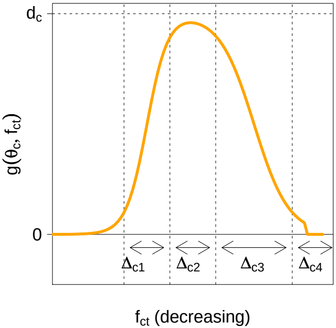

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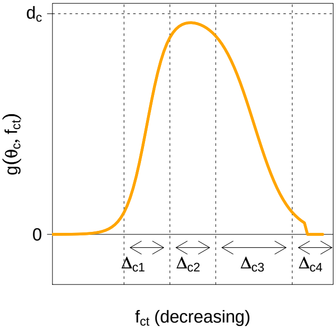

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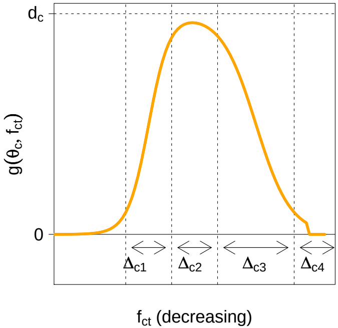

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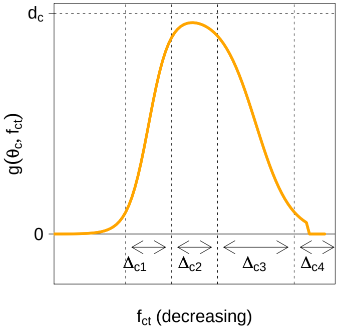

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- Estimate these parameters with a Bayesian hierarchical model

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with χ the world mean, and ψ^2 the variance of the d_c^*'s.

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  $\longrightarrow$ Many TFR trajectories

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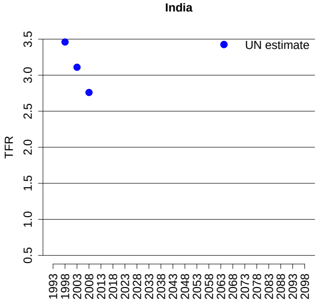

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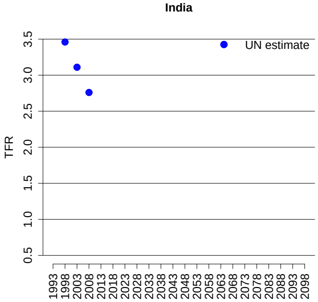

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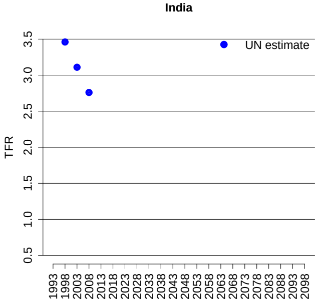

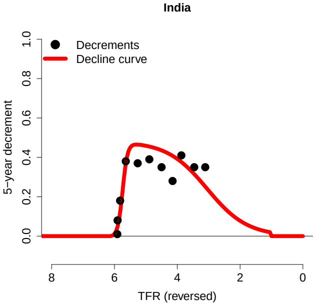

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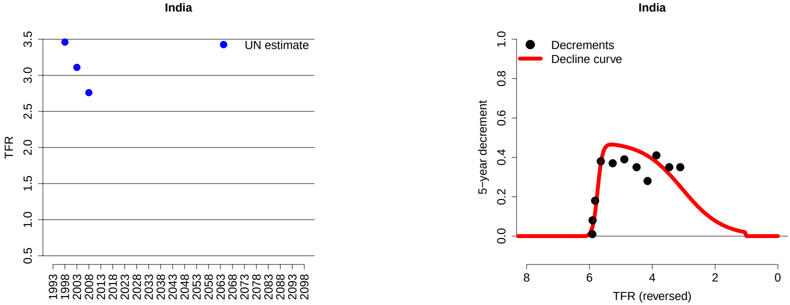

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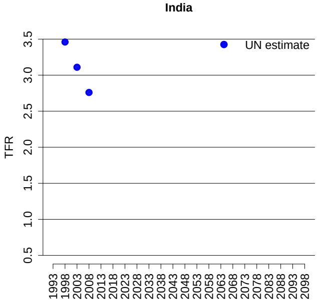

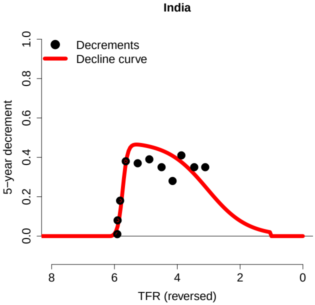

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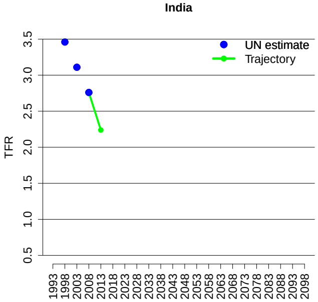

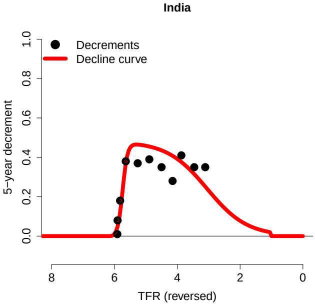

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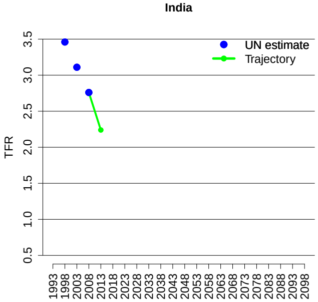

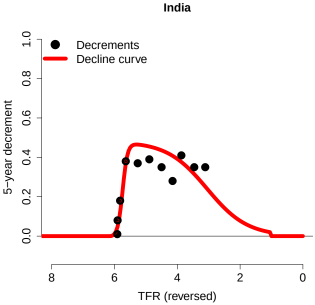

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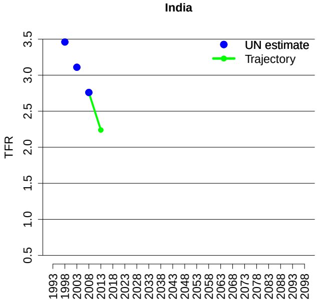

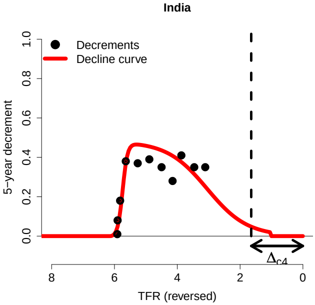

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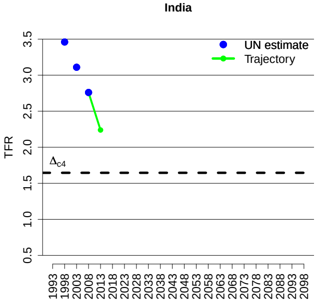

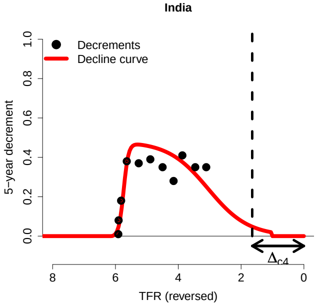

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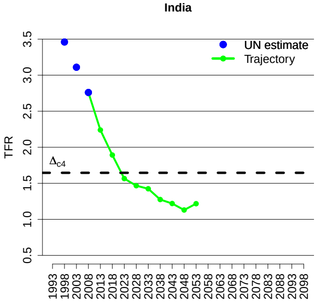

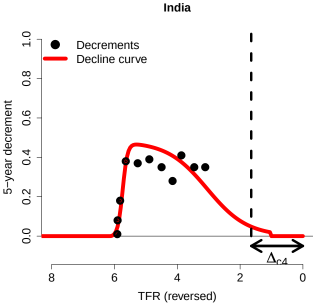

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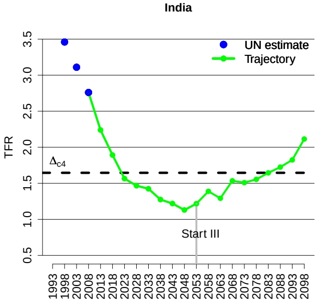

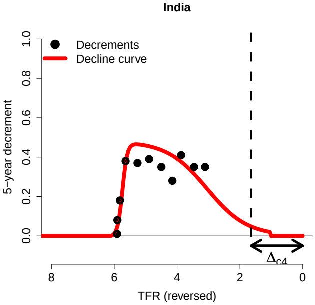

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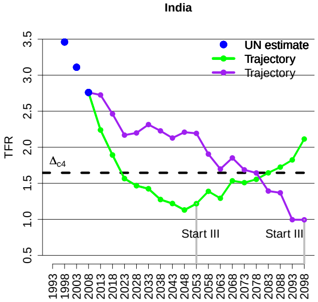

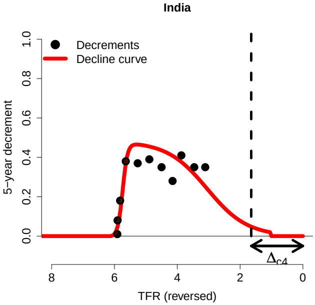

Outline

1. Phase II: The fertility transition

2. Phase III: Post-transition low fertility

3. Results

Phase III: What happens post-fertility-transition?

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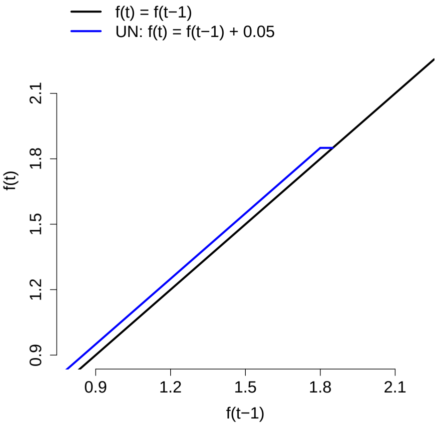

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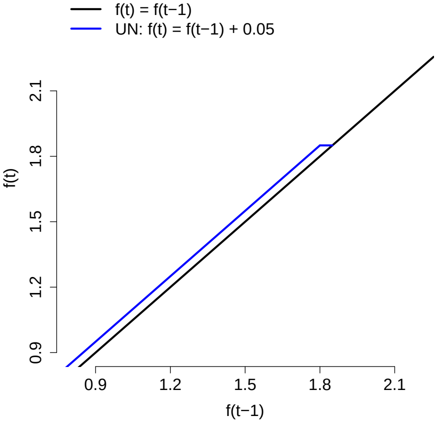

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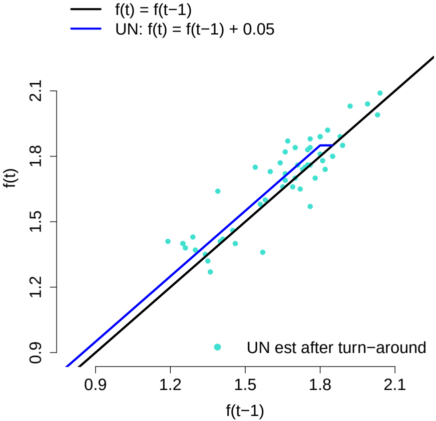

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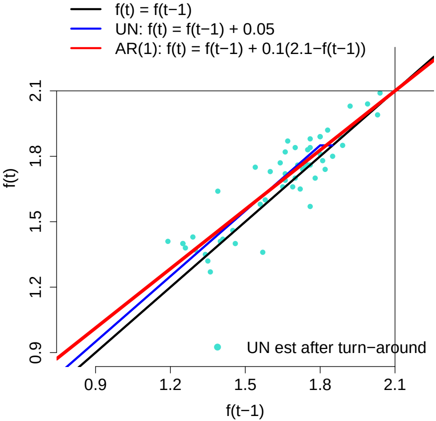

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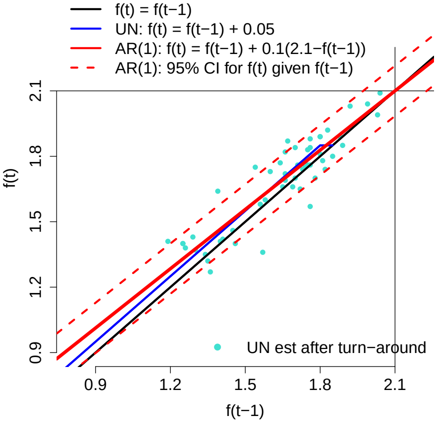

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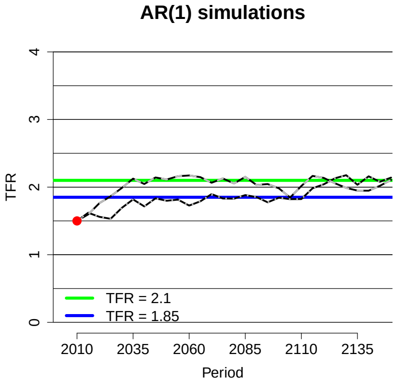

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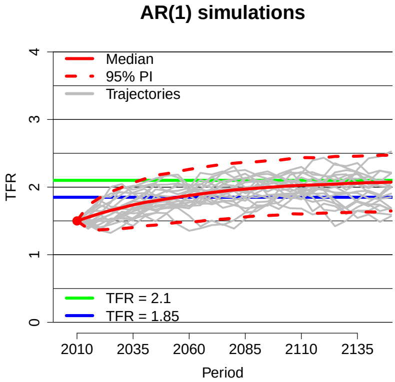

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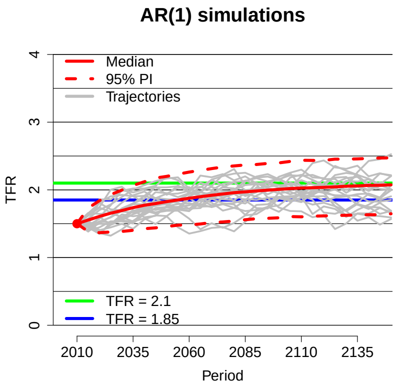

Asymptotic 95% projection interval (PI) given by [1.7,2.5]

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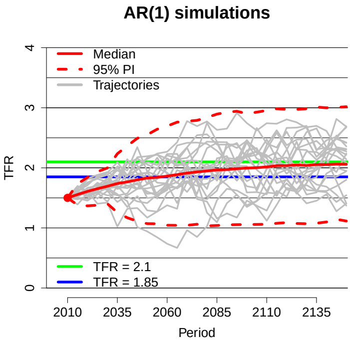

Outline

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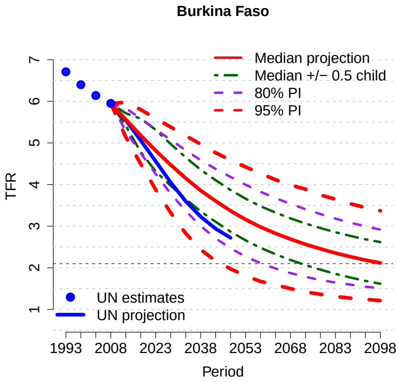

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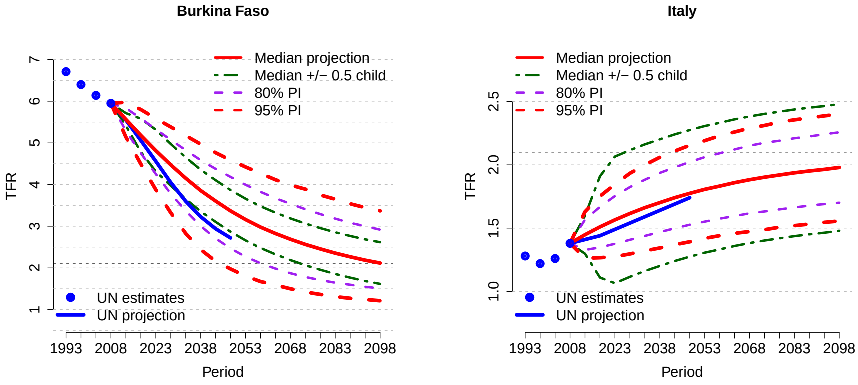

Decline curve; world level

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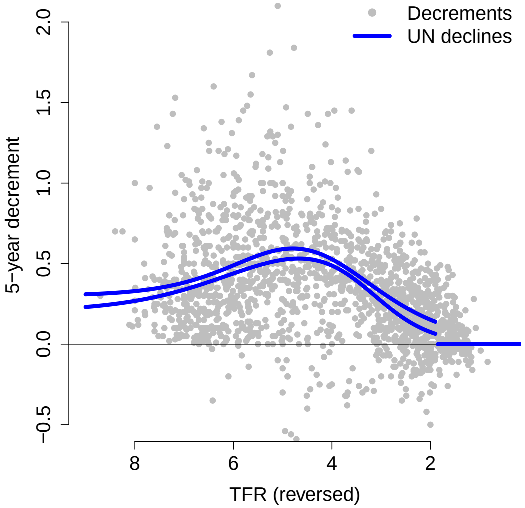

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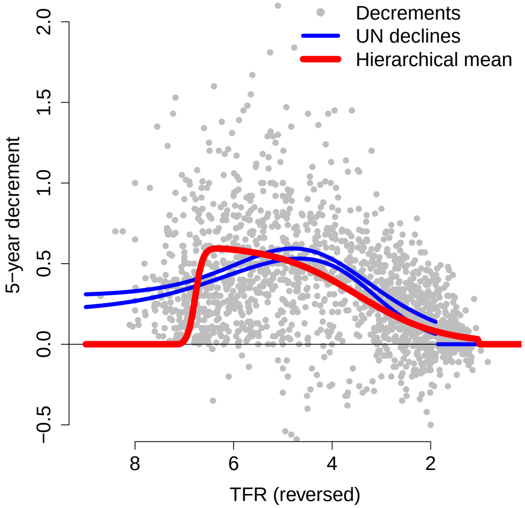

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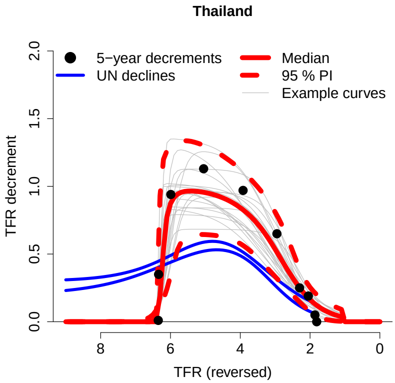

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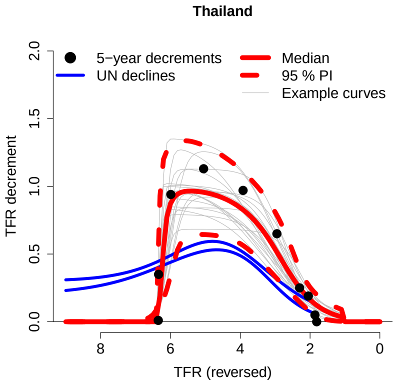

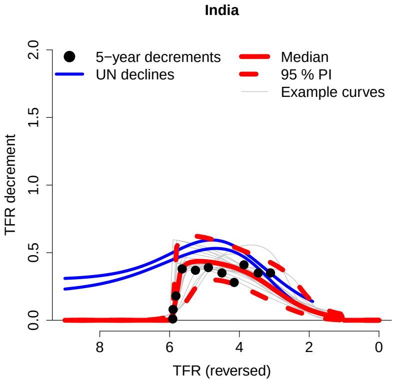

Decline curve; Burkina Faso

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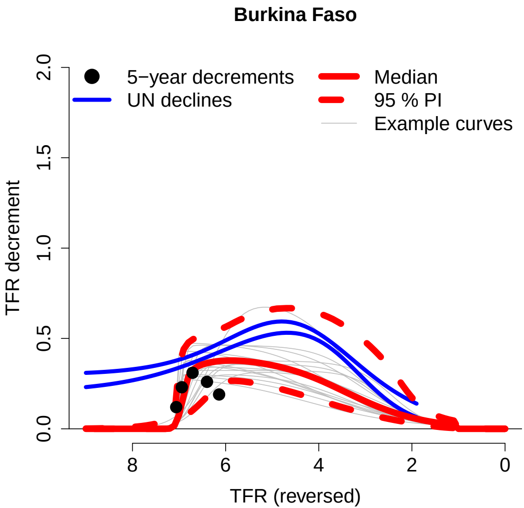

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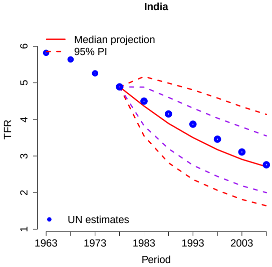

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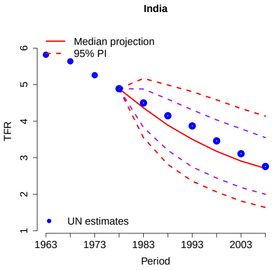

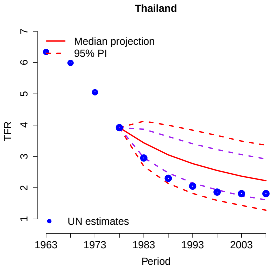

Out-of-sample model validation

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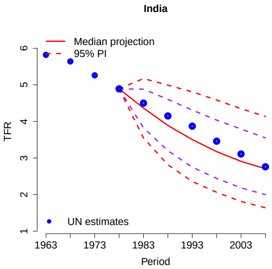

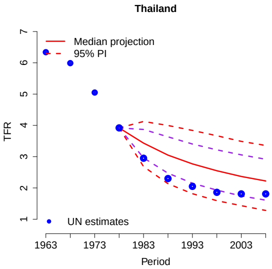

- Summary of model validation results:

Project	Above Median	Coverage 95%PI	Coverage 80%PI
from 1980	43%	91%	77%
from 1995	36%	93%	79%

Summary Bayesian TFR Projection Model

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- Results: Country-specific projections that include an uncertainty assessment