Plenary Session VII: Introduction to GAMS -- A Simple CGE Model

Martín Cicowiez
CEDLAS-UNLP

Presentation for Second Training Workshop of the Project "Assessing Development Strategies to Achieve the MDGs in Asia", Manila, March 24-27, 2009

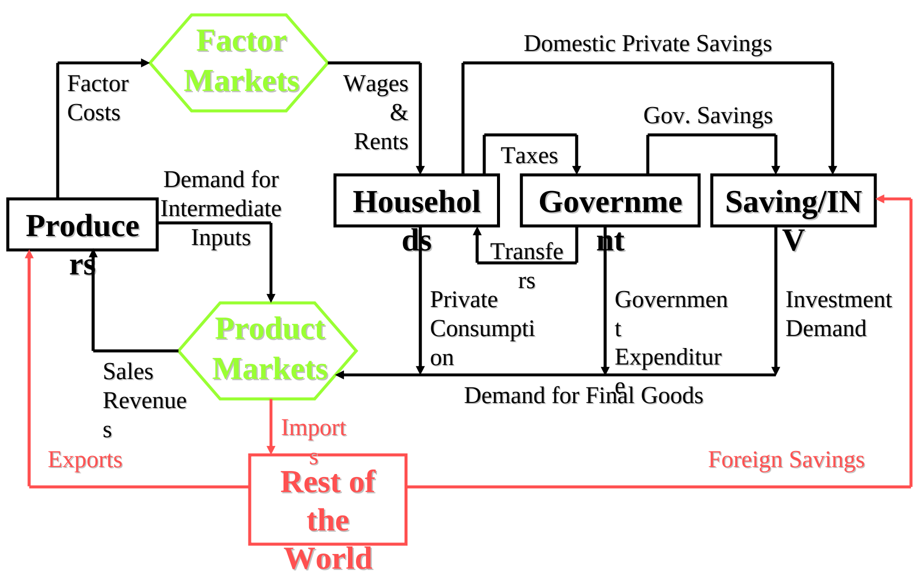
Outline

- An introduction to CGE modeling
- A simple CGE model – mathematical statement
  - (sets, SAM, variables, equations, closure rule for factor markets)
- The dataset in Excel
- A simple CGE model – GAMS version
  - (sets, parameters, calibration, variables, equations, model solution)

The CGE Methodology

- A general equilibrium model captures all the interactions between the components of an economy
  - direct and indirect effects
  - assure consistency
  - quantitative results (i.e., not only sign)
- The economic agents behave according to the principles of microeconomic optimization.
  - respond to changes in relative prices
- A CGE model is solved numerically – system of nonlinear equations.
  - functional forms are specified (LF,CD,CES)
- The macro equilibrium can be achieved in different ways (government, savings-investment, external sector) – macro closure rule.

Stylized Model Structure



Fuente: Lofgren, Robinson, El-Said (2003)

The CGE Methodology – cont.

- The data requirements used to construct a CGE model are small when compared to the number of model parameters -- calibration.
- A SAM (i.e., a picture of the economy) is used to infer the value of model parameters.
- The SAM is combined with elasticities
  - own estimations, literature review, estimations for similar countries, guesstimates.

Applications of the CGE Methodology

- tax reforms
- trade liberalization
- change in world prices
- economic growth, dynamic model
- changes in public expenditure
  - consumption of services, transfers, among others
  - MDG achievement -- MAMS

The Social Accounting Matrix

- Social Accounting Matrices (SAMs) are a key part of the database for CGE models.
- A SAM is a consistent and complete data system that captures the interdependence that exists within a socio-economic system.
  - it includes both the I/O and national accounts and institutional accounts in a consistent framework
  - shows the relationship between the functional and personal income distribution
- First SAM developed by Nobel Laureate Richard Stone for the UK in 1962; work on developing countries took off in the 1970s.

Steps in CGE Modeling

1. define the issue to be studied
2. construct a consistent mathematical model
3. data collection – construct the benchmark that will be used for calibration (see below)
4. code the model, usually using GAMS
5. replicate the benchmark -- consistency
6. conduct policy experiments
7. analysis of results – compare the counterfactual solution with the benchmark

A Simple CGE Model: Dimensions

- s productive sectors (activities/commodities)
  - 2 in the example
- f primary factors of production
  - 2 in the example
- h households
  - 1 in the example

A Simple CGE Model: Characteristics

- The government collects indirect and direct taxes and makes transfers to the households.
  - does not consume
- There is perfect competition in markets for goods and factors.

THE SOCIAL ACCOUNTING MATRIX (SAM)

	s-sx	s-sy	f-lab	f-cap	t-act	t-dir	hhd	gov	total
s-sx							110		110
s-sy							110		110
f-lab	25	75							100
f-cap	75	25							100
t-act	10	10							20
t-dir							20		20
hhd			100	100				40	240
gov					20	20			40
total	110	110	100	100	20	20	240	40	

Notation in Model Mathematical Statement

- **endogenous variables**
  upper-case Latin letters
- **exogenous variables**
  lower-case Latin letters
- **parameters** – behavioral
  lower-case Latin letters or lower-case Greek letters
- **sets indices**
  lower-case Latin letters as subscripts to variables and parameters

OPTIMIZATION PROBLEMS -- PRODUCTION

The cost minimization problem consist in finding the input combination that minimizes a firm's production cost, given quantity of output. The constrained optimization problem solved by the firm can be written as

$$\min C = wl.QL + wk.QK$$

$$\text{s. a. } q = \gamma QL^{\beta_L} QK^{\beta_K}$$

where QL=labor, QK=capital, wl=wage QL, wk=wage QK, y q=production; betal, betak, and gama are parameters.

OPTIMIZATION PROBLEMS -- PRODUCTION

The lagrangean function of the cost minimization problem that the firm solves can be written as

$$\text{L} = wl.QL + wk.QK + \lambda\left(q - \gamma QL^{\beta_L} QK^{\beta_K}\right)$$

FIRST ORDER CONDITIONS (FOC)

$$\partial \text{L} / \partial QL = wl - \lambda \gamma \beta_L QL^{(\beta_L - 1)} QK^{\beta_K} = 0 \quad (1)$$

$$\partial \text{L} / \partial QK = wk - \lambda \gamma QL^{\beta_L} \beta_K QK^{(\beta_K - 1)} = 0 \quad (2)$$

$$\partial \text{L} / \partial \lambda = q - \gamma QL^{\beta_L} QK^{\beta_K} = 0 \quad (3)$$

OPTIMIZATION PROBLEMS -- PRODUCTION

Manipulating the first FOC,

$$wl - \lambda \gamma QL^{\beta_L} QK^{\beta_K} \beta_L QL^{-1} = 0$$

$$wl = \frac{\lambda q \beta_L}{QL} \qquad wl.QL = \beta_L \lambda q$$

Manipulating the second FOC,

$$wk - \lambda \gamma QL^{\beta_L} QK^{\beta_K} \beta_K QK^{-1} = 0$$

$$wk = \frac{\lambda q \beta_K}{QK} \qquad wk.QK = \beta_K \lambda q$$

OPTIMIZATION PROBLEMS -- CONSUMPTION

Given the consumer's income, yh, and prices, p1 and p2, the consumer's problem is to choose the affordable bundle that maximizes her utility. The constrained optimization problem solved by the consumer can be written as

$$\max U = QH_1^{\alpha_1} QH_2^{\alpha_2}$$

$$s.a.\ yh = p_1 QH_1 + p_2 QH_2$$

where U=Cobb-Douglas utility function, QH1 and QH2=consumption of commodity 1 and 2, and alfa1 and alfa2 are parameters.

OPTIMIZATION PROBLEMS -- CONSUMPTION

The lagrangean function of the utility maximization problem that the consumer solves can be written as

$$\text{L} = QH_1^{\alpha_1} QH_2^{\alpha_2} + \lambda\left(y - p_1 QH_1 - p_2 QH_2\right)$$

FIRST ORDER CONDITIONS (FOC)

$$\alpha_1 QH_1^{\alpha_1 - 1} QH_2^{\alpha_2} - \lambda p_1 = 0$$

$$QH_1^{\alpha_1} \alpha_2 QH_2^{\alpha_2 - 1} - \lambda p_2 = 0$$

$$yh - p_1 QH_1 - p_2 QH_2 = 0$$

OPTIMIZATION PROBLEMS -- CONSUMPTION

Manipulating the FOC,

$$\frac{\alpha_1 QH_1^{\alpha_1 - 1} QH_2^{\alpha_2}}{QH_1^{\alpha_1} \alpha_2 QH_2^{\alpha_2 - 1}} = \frac{p_1}{p_2}$$

$$\frac{\alpha_1 QH_2}{\alpha_2 QH_1} = \frac{p_1}{p_2}$$

$$p_1 QH_1 = \frac{\alpha_1}{\alpha_2} p_2 QH_2$$

$$yh = \frac{\alpha_1}{\alpha_2} p_2 QH_2 + p_2 QH_2$$

$$yh = p_2 QH_2 \left(\frac{\alpha_1}{\alpha_2} + 1 \right)$$

$$yh = p_2 QH_2 \left(\frac{\alpha_1 + \alpha_2}{\alpha_2} \right)$$

OPTIMIZATION PROBLEMS -- CONSUMPTION

$$QH_1 = \frac{\alpha_1 yh}{p_1}$$

$$QH_2 = \frac{\alpha_2 yh}{p_2}$$

ENDOGENOUS VARIABLES

- QH(s,h)
- Q(s)
- QF(f,s)
- WF(f)
- P(s)
- YH(h)
- TREV
- QFS(f)
- CPI
- YF(f)

EQUATIONS

$$QF_{fs}WF_f = \delta_{fs}P_s(1 - ta_s)Q_s$$

$$Q_s = \phi_s \prod_f QF_{fs}^{\delta_{fs}}$$

$$YF_f = \sum_s WF_f QF_{fs}$$

$$YH_h = \sum_f shry_{hf} YF_f + shrt_h TREV$$

EQUATIONS

$$QH_{sh}P_s = \alpha_{sh} YH_h (1 - ty_h)$$

$$CPI = \sum_s cwts_s P_s$$

$$TREV = \sum_s ta_s P_s Q_s + \sum_h ty_h YH_h$$

$$QFS_f = \sum_s QF_{fs}$$

$$Q_s = \sum_h QH_{sh}$$

NUMBER OF VARIABLES AND EQUATIONS

QUANTITY OF VARIABLES
2 s + 3 f + h + (f x s) + (s x h) + 2

QUANTITY OF EQUATIONS
2 s + 2 f + h + (f x s) + (s x h) + 2

DIFFERENCE
f

Number of Variables and Equations – cont.

- Typically, a General Equilibrium model only determines relative prices; select a numeraire.
- By virtue of Walras Law, one equation can be dropped from the model
  - in our case, we keep only one commodity market

Closure Rule Factor Markets

- Capital
  - WF(fcap) flexible
  - QFS(fcap) fix
- Labor
  - option 1
    - WF(flab) flexible
    - QFS(flab) fix
  - option 2
    - WF(flab) fix
    - QFS(flab) flexible

CALIBRATION

$$ta_{s-sx} = \frac{10}{110} = 0.0909$$

$$QF_{fs} WF_f = \delta_{fs} P_s (1 - ta_s) Q_s$$

$$\delta_{fs} = \frac{QF^0_{fs} WF^0_f}{P^0_s (1 - ta_s) Q^0_s}$$

$$WF^0_f = P^0_s = 1$$

CALIBRATION

$$\delta_{lab,sx} = \frac{25}{(1-0.09)110} = 0.25 \qquad \delta_{cap,sx} = \frac{75}{(1-0.09)110} = 0.75$$

$$Q_s = \phi_s \prod_f QF_{fs}^{\delta_{fs}}$$

$$\phi_s = \frac{Q_s^0}{\prod_f \left(QF_{fs}^0\right)^{\delta_{fs}}}$$

$$\phi_{sx} = \frac{110}{25^{0.25}75^{0.75}} = 1.93$$

Calibration – cont.

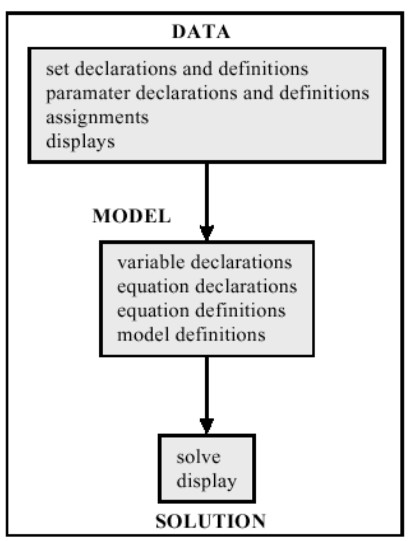
- When using other functional forms, we need to give a value to the free parameters; specifically, the elasticity of substitution in the case of CES function
  - conduct sensitivity analysis

$$\beta_L = \frac{wlQL^{\frac{1}{\sigma}}}{\left(wrQK^{\frac{1}{\sigma}} + wlQL^{\frac{1}{\sigma}}\right)} \qquad \beta_K = \frac{wrQK^{\frac{1}{\sigma}}}{\left(wrQK^{\frac{1}{\sigma}} + wlQL^{\frac{1}{\sigma}}\right)}$$

The GAMS Code

- The GAMS (General Algebraic Modeling System) software can solve non-linear equations system (for example, a CGE model).
  - can be downloaded for free (demo version – limited model size) from <*www.gams.com*>
- A GAMS file is a text file with .gms extension
  - can be edited and run using the GAMSIDE.
  - no case sensitive
  - each sentence finishes with ;
  - can read datasets from Excel
- See Brooke, Kendrick, Meeraus, and Raman (2008). GAMS: A User Guide. GAMS Corporation.

How to Organize a GAMS Code



GAMS Basic Statements

- SETS
- PARAMETERS (declaration and definition)
- read data from EXCEL
- calibration of model parameters
- VARIABLES
- EQUATIONS (declaration and definition)
- MODEL (declaration, definition, and solution)
- DISPLAY statements

The GAMS Syntax

Using mathematical notation,

$$s = \{sx, sy\}$$

$$f = \{lab, cap\}$$

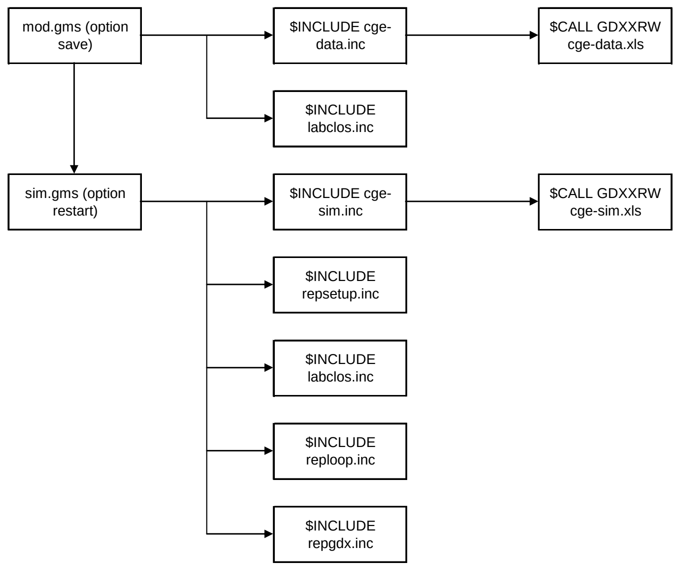
$$Q_s = \phi_s \prod_f QF_{fs}^{\delta_{fs}}$$

Using GAMS notation,

```
SET
      s /s-sx,s-sy/
      f /f-lab,f-cap/
;

Q(s) =E= phi(s)*PROD(f, QF(f,s)**delta(f,s));
```

Files Organization



The Data File

- The benchmark data for model calibration is introduced in cge-data.xls – read in cge-data.inc
  - elements in sets
    - ac, s(ac), f(ac), flab(f), fcap(f), h(ac)
  - SAM
  - labor market closure rule
    - base and default for simulations
- Now,
  - open cge-data.xls
  - run mod.gms with the option s=save\mod

SIMULATIONS

- SIMUL 1
  double the numeraire
- SIMUL 2 (tfp-sy)
  double the total factor productivity in sector s-sy
- SIMUL 3 (taxcut-sx)
  eliminate tax sector s-sx
- SIMUL 4 (taxcut-sx-unemp)
  repeat SIMUL 2 assuming the existence of unemployment with a fixed real wage

The Simulation File

- The information for each simulation is introduced in cge-sim.xls
  - name
  - shocks
  - labor market closure rule
- Now,
  - open cge-sim.xls
  - run sim.gms with the option r=save\mod s=save\sim
  - open report.gdx to see simulation results

RESULTS

variable			base	tfpy	tcutx-fe	tcutx-ue
QX	s-sx		110.0	110.0	113.9	115.3
	s-sy		110.0	220.0	106.0	109.9
QFX	f-lab	s-sx	25.0	25.0	26.8	28.1
	f-lab	s-sy	75.0	75.0	73.2	76.7
	f-cap	s-sx	75.0	75.0	76.7	76.7
	f-cap	s-sy	25.0	25.0	23.3	23.3
QFSX	f-lab		100.0	100.0	100.0	104.8
	f-cap		100.0	100.0	100.0	100.0
PX	s-sx		1.000	1.333	0.964	0.976
	s-sy		1.000	0.667	1.036	1.024
WF	f-lab		1.000	1.333	1.023	1.000
	f-cap		1.000	1.333	1.073	1.099
TREVX			40.0	53.3	30.0	30.7

Final Comments

- The same model can be used with different databases (i.e., model size is data driven); only need to change the Excel file – similar to MAMS
  - As an exercise, add one more household to the SAM and solve the model again; repeat SIMUL 3 and analyze the distributive effects of the shock

Recommended References

- Kehoe, P. J. and Kehoe, T. J. (1994). A Primer on Static Applied General Equilibrium Models. Federal Reserve Bank of Minneapolis Quarterly Review 18 (1).
- Lofgren, H. (2000, Revised 2003). Exercises in General Equilibrium Modeling Using GAMS and Key to Exercises in CGE Modeling Using GAMS. Microcomputers in Policy Research 4. IFPRI.
- Lofgren, H.; Lee Harris, R. and Robinson, S. (2002). A Standard Computable General Equilibrium (CGE) Model in GAMS. Microcomputers in Policy Research 5. IFPRI.