3 MAMS: An economy-wide model for analysis of MDG country strategies—an application to Latin America and the Caribbean [1]

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3.1 Introduction

This chapter documents MAMS (*Maquette* for MDG Simulations), the underlying methodological framework of this multi-country study for Latin America and the Caribbean (LAC), which is used to address the three development strategy questions posed in Chapter 1. MAMS is a dynamic Computable General Equilibrium (CGE) model designed to analyze strategies for achieving the Millennium Development Goals (MDGs) and, more broadly, policies for medium- and long-run growth and poverty reduction in developing countries.[2] The model is sufficiently flexible to address the key processes for MDG achievement and other development strategies in a wide range of countries, linking to country databases that capture country characteristics and may vary widely in terms of disaggregation. This chapter emphasizes model features relevant to the LAC context.

An economy-wide approach is typically needed in the analysis of development strategies whenever the outcomes of interest (for example, an MDG target) are influenced significantly not only by the direct effects of single policies, but also by indirect effects and policy interactions that feed back into the processes that determine these outcomes. For example, an increase in MDG-related government spending may have very different effects on MDG indicators depending on whether the spending increase is accompanied by an increase in foreign borrowing or domestic taxes.

Various approaches have been used both to plan or monitor progress toward achieving the MDGs and to evaluate the additional (or total) public resources needed to meet them. Clemens *et al.* (2004) and Reddy and Heuty (2004) survey a large number of studies that forecast

[1] Forthcoming as chapter 3 of a volume on *Public Policies for Human Development. Feasible Financing Strategies for Achieving the MDGs in Latin America and the Caribbean*, edited by Marco V. Sánchez, Rob Vos, Enrique Ganuza, Hans Lofgren, and Carolina Díaz-Bonilla. The volume reflects outcomes of a joint project of UNDP, UN/DESA, UN/ECLAC and the World Bank.
[2] A complementary, less comprehensive presentation of MAMS is found in Bourguignon *et al.* (2008). Lofgren (2008) presents a simplified mathematical statement.

and cost MDGs. As emphasized by Vandemoortele and Roy (2004), however, data availability and simplifying analytical assumptions severely affect the quality of quantitative estimates of all these studies.

Four major sets of limitations affect studies on MDG achievement. First, many sector studies fail to properly account for the interdependencies that exist among different MDGs and among policies designed to reach them. Second, MDG-related policies interact with the rest of the economy by altering prices, demands, and supplies of commodities and factors (including different types of labour). Third, inter-temporal equilibrium consistency is seldom checked. Financing needs, debt accumulation, and the inter-temporal sustainability of fiscal policies need to be integrated in a complete study on strategies to achieve the MDGs. Finally, as stressed by Devarajan *et al.* (2002), the policy and institutional environment is as important a component of success in achieving the MDGs as the availability of public resources or financial assistance.

Keeping these potential limitations in mind, we briefly report on some recent MDG studies and approaches:[3] (1) The UNDP Human Development Report (2005, pp. 39-48) covers most MDGs, projecting trends for individual countries. However, policies and linkages between MDGs are not considered, so this approach is not designed, and cannot be used, for the analysis of MDG strategies. (2) The SimSIP (Simulations for Social Indicators and Poverty) tool, developed by Wodon and co-authors (see Christiaensen *et al*., 2002) has a target setting module that is useful for assessing the feasibility of achieving different targets. On the other hand, the fiscal sustainability component of their method is weak, reflecting the fact that a set of independent tools cannot capture interdependencies between GDP growth, different MDG targets, program costs (including wage changes), and alternative financing approaches. (3) The different publications of the UN Millennium Project report represent a more detailed sector approach (see, for example, United Nations Millennium Project, 2005), but this approach—while rich in detail—has typically ignored or simplified the synergies across the MDGs and has not been designed to consider increasing marginal costs and the interactions with the broader economy. (4) Recognizing the need for an economy-wide perspective, Agénor *et al*. (2005) combine a macro model with an MDG module in a framework that requires relatively little data and draws on econometrically estimated parameters (a key strength of their approach). On the other hand, the macro model is highly aggregated: it has only one production sector and it does

[3] For more details, see Bourguignon *et al.* (2008).

not include intermediate inputs, factor markets, or factor wages (rents). These considerations limit its ability to analyze key aspects of MDG strategies such as how the direct exchange rate and labour market repercussions of scaled-up government programs differ depending on whether the program emphasizes, for example, education or infrastructure. Also, its high level of government and labour market aggregation makes it less informative for fiscal analysis.

The links between growth, service delivery, MDG achievements and financing outlined above demonstrate that a more sophisticated and coherent framework is needed. The analysis must consider macroeconomic factors and trade-offs between objectives. For example, increases in foreign aid (borrowing or grants, although the latter is less common in the LAC region) leads to concerns over the possibility of "Dutch disease," characterized by real exchange rate appreciation and a structural erosion of the capacity to produce tradables (for exports or domestic market), a capacity that may be needed in the future. At the same time, the content of the MDG strategy (such as whether the expansion in demand is geared toward imports or non-tradables) has a decisive impact on the magnitude of any Dutch Disease effects. On the other hand, if an MDG strategy is financed via increases in taxes or domestic borrowing, then private sector growth, investment, and consumption are all likely to suffer, negatively impacting poverty reduction and, because of indirect effects, the achievement of other MDGs (which are influenced by household incomes and consumption). A related critical issue is the pace at which large programs should be scaled up. Rapid initial expansion may drive up costs more quickly and could be more expensive in real present value terms. On the other hand, given time lags, especially in education, expanding investment too slowly may make it impossible to achieve the MDGs by 2015. By allowing the performance of policy experiments that consider these links, MAMS helps analysts and policymakers to think about the issues in a systematic manner, which helps inform the policy dialogue.

3.2 MAMS at a glance

The starting point for MAMS is the static, standard CGE model developed at the International Food Policy Research Institute (IFPRI) (Lofgren *et al.*, 2002). MAMS is significantly extended in two key respects: the inclusion of (recursive) dynamics (that is, a time dimension) and the addition of an MDG module that endogenizes MDG and education outcomes.[4] Other extensions include the endogenization of factor productivity (which depends, in the basic specification, on economic openness and government capital stocks) and the tracking of assets (liabilities) of the different institutions (factor endowments, domestic government debts, and foreign debts).

A key premise of MAMS is that it is designed to link government spending and MDG outcomes in a dynamic way, permitting several outside influences. First, it permits the returns to scale of government spending to vary with the level of service delivery. At low levels, increasing returns may prevail as network and learning effects and synergies are predominant. At high levels of service delivery, government spending may suffer from decreasing returns to scale. To exemplify, water supply, health care, and education can be relatively easily provided in densely populated areas, but become increasingly expensive as coverage expands to remote areas. Also, when mortality rates are low it becomes increasingly difficult to reduce these rates further. Similarly, if completion rates in education are already high, it is difficult to ensure that the last percentages of children complete the program.

Second, MAMS permits the effectiveness of government spending to depend on many variables. For example, spending on education may become more effective if health conditions improve (reducing absenteeism at schools), if public infrastructure improves (facilitating access to schools), if income levels rise (making parents less inclined to keep children at home or in the labour market) or if skill premiums increase (triggering a greater incentive to finish formal education). In general terms, this means that spending on services becomes more effective if demand conditions for those services are more favourable.

Third, MAMS considers that the costs of service delivery may change with macroeconomic conditions. The services are often skill intensive and in many cases also capital intensive. The more intense the MDG effort, the stronger the impact on costs as skilled labour becomes scarcer and financial conditions become tighter. From a general budgetary perspective the impacts on costs are even larger, because changes in macroeconomic conditions do not only affect MDG-related spending, but also other, non-MDG-related government spending. The relative competitiveness of different parts of the private sector is also affected.

[4] The starting point for the MDG module is the MDG model in Bourguignon *et al.* (2004). All of these models are coded and solved in the General Algebraic Modeling System (GAMS).

The first two aspects above (changing returns to scale and the impact of demand variables) are captured in the MDG functions introduced in MAMS (explained in detail in the MDG section of this chapter). The third aspect (macroeconomic interactions) is captured as the MDG functions are incorporated in a dynamic economy-wide general equilibrium framework that also includes detailed fiscal accounts (explained in more detailed terms in the core model section).

Mathematically, MAMS is divided into two modules—a core CGE module and an MDG module—both of which are integrated in a simultaneous system of linear and non-linear equations. For each time period, the core CGE module gives a comprehensive and consistent account of decisions and related payments involving production (activities producing outputs using factors and intermediate inputs), consumption (by households and the government), investment (private and government), trade (both domestic and foreign), taxation, transfers between institutions (households, government, and the rest of the world), and the distribution of factor incomes to institutions (reflecting endowments). This module also considers the constraints under which the economy operates (the budget constraints of institutions and producers; macro balances; and market constraints for factors and commodities). Lastly, in addition to these standard features of a static CGE model, the core CGE module in MAMS also updates selected parameters (including factor supplies, population, and factor productivity) on the basis of exogenous trends and past endogenous variables.

The MDG module captures the processes that determine MDG achievement in the human development area, most importantly the provision of services in the areas of education, health, and water and sanitation. The size and skill composition of the labour force is endogenized, in large measure depending on the evolution of education. The MDG module has feedback effects into the rest of the economy, primarily via the labour market.

In the model, growth depends on the accumulation of production factors (labour at different educational levels, private capital, and other factors such as land, if present) and changes in factor productivity, which is influenced by the accumulation of government capital stocks and openness to foreign trade. The structure is recursive: the decisions of economic agents depend on the past and the present, not the future; in other words, the model does not consider forward-looking behaviour.

Poverty and inequality analysis, as in other CGE models, can be performed in several ways. The simplest but least desirable method uses an elasticity calculation for poverty given changes in per-capita household consumption. Representative-household or survey-based microsimulation approaches are preferable. The former assume fixed distributions of income or consumption within each household group, providing welfare estimations directly from the CGE model results. The latter type of approach does not need to recur to the rather stringent assumption of fixed within-group income distributions. It can be either top-down, feeding CGE simulation results to a household model, or integrated, with the household model built directly into MAMS. For the purposes of this study, a survey-based, top-down microsimulation approach was used for the poverty and inequality analysis, as explained in Chapter 2, Appendix A2.1.

The disaggregation of MAMS is data-driven and flexible in most areas: subject to computer memory constraints, there is no upper limit on the number of primary factors, households, production activities, and commodities. The government is disaggregated by function to include sectors for education (by cycle or level), health (in some applications further disaggregated by type of service or technology), water and sanitation, and other public infrastructure. For the purposes of the country studies and to ensure that MDG achievement in education has explicit dynamic feedback effects on labour supply, the labour force is disaggregated by educational achievement into three types: those who have completed tertiary, completed secondary, or less than completed secondary. Further disaggregation of labour categories is possible in MAMS.

The applicability of the model to specific policy issues depends in large part on the degree of disaggregation. For example, the analysis of issues related to poverty requires a relatively detailed breakdown of household income sources (from factor endowments and the production activities in which they are employed). Similarly, it is likely preferable to disaggregate non-government production into multiple sectors and commodities (that is, services), as it will provide more specific results of the sectoral employment and income effects of an MDG strategy, pursued on its own or in conjunction with other policies, such as trade reform.

3.3 Detailed description of MAMS

The basic accounting structure and much of the underlying data of MAMS, like other CGE models, is represented by a Social Accounting Matrix (SAM). The SAM for MAMS has some unconventional features, especially because of the required detailed specification of how different MDG-related services, provided by both public and private sectors, are produced and delivered. Before describing the behavioural assumptions and mathematical structure of MAMS, we first describe the particular features of the SAM and the key accounting identities of MAMS.

The Social Accounting Matrix

A SAM is a square matrix in which each account is represented by a row and a column. It provides a comprehensive picture of the economic transactions of an economy during a time period, almost invariably one year. Each cell shows the payment from the account of its column to the account of its row. Thus, the incomes of an account appear along its row and its expenditures along its column. For each account in the SAM, total revenue (row total) should be equal to total expenditure (column total). It should be noted that SAMs almost invariably are limited to flows; additional data or assumptions are needed to define stocks. In most CGE models (including MAMS), the SAM is used to define base-year values for the bulk of the parameters in the equations that generate the corresponding payments in the model.

Table 3.1 shows a stylized and aggregated version of a SAM designed for MAMS, while Table 3.2 shows the notation that is used.[5]

[5] The SAMs used in the country studies are invariably more disaggregated in three respects: (a) government activities, commodities and investment accounts are split by government function; (b) the interest account is split into two, one for interest on domestic government debt and one for interest on foreign debts; and (c) the tax account is split into separate accounts for direct, import, export, value-added, and other domestic indirect taxes (of course, in some of the applications, some of these tax types may not exist). In contexts outside this project, MAMS has also been implemented without the MDG-education module.

Table 3.1 Stylized Macro SAM for MAMS [1/]

	act-prv	act-gov	com-prv	com-gov	f-lab	f-capprv	hhd	gov	row	taxes	int-erest	cap-hhd	cap-gov	cap-row	inv-prv	inv-gov	dstk	total
act-prv			output															
act-gov				output														
com-prv	intmed	intmed					cons		exports						inv	inv	dstk	
com-gov								cons										
f-lab	va	va																
f-capprv	va								yrow									
hhd					va	va		trnsfr	trnsfr		intdom							
gov							trnsfr		trnsfr	taxes								
row			imports			va	trnsfr	trnsfr			introw							
taxes	taxes		taxes				taxes											
interest							introw	intdom+ introw										
cap-hhd							sav							bor				
cap-gov								sav				bor		bor				
cap-row									sav									
inv-prv												inv		inv				
inv-gov													inv					
dstk												dstk	dstk					
total																		

[1/] See Table 3.2 for the explanation of the notation.

Table 3.2 Accounts and cell entries in Stylized Macro SAM for MAMS

Account	Explanation	Cell entry	Explanation
act-prv	activity - private production	bor	borrowing
act-gov	activity - government production	cons	consumption
com-prv	commodity - private production	dstk	stock (inventory) change
com-gov	commodity - government production	exports	exports
f-lab	factor - labour	imports	imports
f-capprv	factor - private capital	intdom	interest on domestic government debt
hhd	household	intmed	intermediate inputs
gov	government	introw	interest on foreign debt
row	rest of world	inv	investment (gross fixed capital formation)
taxes	taxes - domestic and trade	output	production
interest	interest (on domestic and foreign debt)	sav	savings
cap-hhd	capital account - household	taxes	taxes (direct and indirect)
cap-gov	capital account - government	trnsfr	transfers
cap-row	capital account - rest of world	va	value added
inv-prv	investment - private capital	yrow	factor income from RoW
inv-gov	investment - government capital		
dstk	stock (inventory) change		

Starting from the top left of the SAM, the activity accounts represent the entities that carry out production, allocating sales receipts to intermediates, factors (value-added) and (indirect) taxes. The commodities are activity outputs, either exported or sold domestically, and imports. The row entries of the commodity accounts represent payments from commodity demanders. The column entries show payments to the suppliers and indirect taxes (tariffs on imports and/or a sales tax on domestic sales irrespective of whether the commodity is of foreign or domestic origin). In the country studies, the accounts for the government activity and commodity are disaggregated by function, matching the requirements for the analysis of the MDGs and the educational system. Table 3.1 shows a one-to-one mapping between activities and commodities. However, MAMS permits any activity to produce multiple commodities (for example, a dairy activity may produce the commodities cheese and milk), while any commodity may be produced by multiple activities (for example, activities for small-scale and large-scale maize production may both produce the same maize commodity).[6]

[6] The SAM (and MAMS) may also include accounts and entries representing home consumption and transactions costs associated with the commodity marketing (of imports from the border to the demander; of exports from the producer to the border; and of domestic output for domestic sales). For a more detailed discussion of the treatment of these aspects of the SAM, see Lofgren *et al.* (2002, pp. 3-7).

The row entries of the two factor accounts in the SAM, labour and private capital, indicate that they earn value-added from domestic production activities and, for private capital, income from the rest of the world (this is less common for labour since it only applies to income from abroad for workers resident in the country of the SAM). In the factor columns, value-added is distributed to the owners of the factors.[7] In the country studies, labour is invariably disaggregated by education, typically into three segments with the following achievements: completed tertiary, completed secondary but not completed tertiary, and less than completed secondary. MAMS is designed to have a single factor (and SAM account) for private capital, which we define here as capital used in activities that are not part of the functions of the general government.[8] MAMS includes one type of government capital per government activity (i.e., the activities that are part of the functions of the general government). However, typically, government capital does not earn value added and, given this, it is not represented in the SAM.

The SAM in Table 3.1 includes three types of institutions: households, the government, and the rest of world (*row*). [9] Households may be disaggregated into various types and this was done in some of the country studies for this project. Each institution has a current account (its name is a shortened version of the name of the institution) and a capital account (the current-account name of the same institution prefixed by "*cap"*) linked to investment accounts and the capital accounts of other institutions. This treatment is significantly different from the more common treatment where savings and investments are handled by a unified institutional account.

In the rows of their current accounts, the domestic institutions receive their earned shares of value added, transfers from other institutions, interest income (for households), and tax revenues (for the government), while the rest of the world receives payments for the value of goods imported by the country as well as a share of value added (profit remittances), net transfers from domestic institutions (which may be negative, for instance, reflecting workers

[7] In addition to the current entries, it is not uncommon that the government owns part of the capital stock and earns part of operating surplus.

[8] Given this assumption, it is not necessary to model how the endowments and investments of different institutions (households, government, and the rest of the world) are allocated across different private capital types (perhaps disaggregated by sector); this is a key advantage given limited knowledge of the mechanisms that determine the evolution of this distribution over time.

[9] MAMS does not separate enterprises from other domestic institutions. In the SAM, these would have been linked to factors (enterprises receive factor incomes, reflecting their ownership of non-labour factors), "other" institutions (direct tax payments and transfers reflecting institutional ownership of the enterprise) and enterprise capital accounts (which spend on investments). In the country databases, these "other" institutions (primarily households) directly receive the factor transfers while assuming the savings and direct tax payments that otherwise would have been done by the enterprises. Other SAM payments are not affected.

remittances received by the country), and interest payments on foreign debt (*introw*). Along their columns, the outlays of the institutions are allocated to commodity purchases (consumption for the household and the government; and the exports of the SAM country for the rest of the world), direct taxes (for the household), interest payments (for indebted institutions), and savings. Some of the country studies also include an additional institution carrying out the functions of an NGO—receiving transfers from other institutions (typically the government and/or the rest of the world) and using these resources to purchase services related to health and/or education. The tax account (which in MAMS applications is disaggregated according to type of taxation) passes on its receipts from activities, commodities, and households (along the row) to the government (along the column).[10]

The account for interest payments (in applications disaggregated into accounts for domestic and foreign interest) passes on payment from the (net) borrowers to the (net) lenders. Note that the SAM (and MAMS) only captures interest payments (and related debts) of domestic institutions to the rest of the world and of the government to households. It does not capture interest payments and debts of linking domestic non-government institutions. In their rows, the capital accounts of the institutions record their financing sources, consisting of own savings and net borrowing from selected other institutions (for the government from the rest of the world and the household; for the household, from the rest of the world). The outlays of the institutional capital accounts include payments for fixed investments (*inv*) and changes in inventories (*dstk*) and net lending to other institutions (the counterpart of net borrowing). The payments from the capital account of the rest of the world to the private investment account refer to foreign direct investment (FDI). This structure makes it possible for MAMS to capture, in a simple way, the structure of institutional assets (different types of capital and financial claims) and liabilities (financial debt) and how the evolution of this structure differs under alternative scenarios. Other things being equal, if the database has multiple households, those with more rapid income growth will likely also have more rapid savings growth, acquiring increasing shares of private capital and government debt.

Like most other CGE models, MAMS is a "real" model in which inflation does not matter (only relative prices matter). Implicitly, in the SAM, the current account of the monetary

[10] The SAM and MAMS may also include direct taxes levied on factor incomes (represented by payments from factor accounts to the tax account).

sector is merged with service activities and commodities while its capital account is merged with the government capital account. Given this, in the merged government capital account, the cells for net government borrowing from other institutions are made up of multiple items. The cell for net borrowing by the government from the household is the sum of (a) net direct borrowing by government from household (net sales of government bonds on which the government pays interest); and (b) net increases in the claims of the household sector on the monetary sector (the differences between changes in broad money holdings and monetary sector credit to the household). In MAMS (but not in the SAM), the two items in this cell are treated separately, making it possible to consider the fact that (a) gives rise to interest payments and a debt whereas (b) is a grant to the government, providing it with "seignorage" (as the one who spends this new money first). The second cell, which shows net borrowing by government from the rest of the world, is the difference between (a) net direct borrowing by the government from the rest of the world; and (b) the increase in foreign exchange reserves. In MAMS, these two items are not treated separately.[11] While this treatment remains simple, it captures the important fact that the government, by means of money creation, appropriates part of private savings. Given the fact that the model does not consider effects of and private sector responses to high general inflation, MAMS should not be used for scenarios under which the resources obtained via the monetary sector are so large that inflation would accelerate. The assessment of what is a prudent upper limit for this type of borrowing should draw on expertise on the macroeconomics of each country; a few percent of GDP is often a reasonable figure.

Structure of MAMS

In the below, we discuss the mathematical statement of MAMS: firstly for the core CGE module and subsequently for the MDG module. Frequently, we will refer to Tables A3.1 and A3.2 (notation and equations for the core CGE module) and Tables A3.3 and A3.4 (notation and equations for the MDG Module), which are found in Appendix A3.1. The following notational conventions apply in Appendix A3.1 and various parts of the main text: upper case Latin letters are used for variables; exogenous variables have a bar on top, endogenous variables do not. Parameters have Greek or lower-case Latin letters. Subscripts refer to set indices. A "*0*"

[11] To verify these statements and for more details, see Agénor (2004, pp. 11-22), Rao and Nallari (2001, pp. 25-32, 176 and 168), and Barth and Hemphill (2000, pp. 71-74 and 101-106).

superscript is used to refer to base-year variable values**.** Otherwise, superscripts are exponents (i.e., not part of the name of the variable or parameter). In the presence of the "*0*" superscript, the time subscript (*t*) has been suppressed. The fact that an item is a variable and not a parameter indicates that, at least under certain model assumptions, its value is endogenous. In Tables A3.2 and A3.4, the domain column, which follows the column with the equations, is an important part of the mathematical statement—it indicates the set elements to which each equation applies.[12]

The core CGE module

As shown in Table A3.2, the core CGE module is divided into blocks covering prices; production and trade; domestic institutions; investments; system constraints and macro variables; and stock updating and productivity.[13] This section will describe in more technical terms the equations in each of these blocks.

Price block

The price block (equations 1-11) defines prices that can be expressed as functions of other endogenous variables (as opposed to being free variables that perform market-clearing functions). Among these prices, it is worth noting that transactions costs (the cost of moving the commodity between the border and the demanders or suppliers, or between domestic demanders and suppliers) are accounted for in the definitions of demander (domestic-currency) import prices, supplier (domestic-currency) export prices, and demander prices for domestic output sold domestically (equations 1, 2, and 4).

Whereas the transformation of output between exports and domestic sales typically is imperfect, the model also allows for the special cases of outputs exclusively produced for foreign markets (no domestic sales; see below discussion of equation 22) and of perfect transformability with zero exports as one possible outcome. Perfect transformability is useful for commodities that are relatively homogeneous; with only small differences depending on whether the demander is domestic or foreign (like grains). This case is covered by equation 3, which has three components: (a) the constraint that domestic supplier prices are larger than or equal to

[12] For example, in Table A3.2, the domain column of equation 1 shows that this equation does not apply to all commodities; it is limited to commodities with imports.
[13] Apart from the fact that variables are time indexed, most of the core CGE module is similar to the IFPRI standard, static CGE model described in Lofgren *et al.* (2002).

export prices in local currency units (LCU); (b) the constraint that exports are larger than or equal to zero (i.e., zero is a possible outcome); and (c) a complementary-slackness relationship according to which at least one of (a) and (b) has to hold as a strict equality—domestic supplier prices only exceed export prices if exports are zero or, if exports are above zero, then the two prices are equal. In terms of economics, this means that the export price is a floor price and that producers prefer to sell at the highest price that is offered. If the domestic price is above the export price, then nothing is exported. If, in the absence of exports, the price would have fallen below the export price, then exports will be positive, preventing a decline below the export price.[14]

Various aggregative prices—for composite supplies, for produced commodities, and value-added—are derived from relationships that define total revenue or costs as the sum of disaggregated receipts or payments (equations 5-7 and 9). The price of the aggregative intermediate commodity for any activity depends on its commodity composition and the prices of the commodities involved (equation 8). The model is homogeneous of degree zero in prices, with the CPI serving as the model *numéraire* (equation 10). Alternatively, the price index for non-tradables may serve as *numéraire* (equation 11).[15]

Production and trade block

This block (equations 12-27) includes the first-order conditions for profit-maximizing production and transformation decisions as well as cost-minimizing domestic demand decisions. Given available technology and market prices (taken as given in a perfectly competitive setting), producers maximize profits.[16] The technology is defined by a nested, two-level structure. At the top, output is a Leontief aggregation of real value-added and a real aggregate intermediate

[14] In GAMS, a model formulated as an MCP (mixed-complementarity program) can handle a combination of equations that are (a) strict equalities; and (b) inequalities linked to variables with lower limits in a mixed-complementarity relationship.

[15] The GAMS code permits the user to choose either the CPI or the price index for non-tradables as *numéraire*. As long as the model is homogeneous of degree zero in prices, this choice has no impact on the equilibrium values of real variables. This homogeneity condition is not met under macro closures with fixed savings or domestic borrowing for the government. In these cases, it is implicitly assumed that the fixed variables are indexed to the *numéraire*.

[16] Some of the country applications include a private regulated sector (typically a utility) for which behaviour deviates from the assumption of profit-maximizing output and input demand (including capital use) given market prices and rents. Each regulated activity has its own capital stock -- otherwise, there is only one private capital stock, which is mobile across private activities. For regulated activities, output prices, investment and capital use are exogenous; production is demand-driven at fixed output prices. Their capital stocks earn an endogenous, residual share of value-added which most likely deviates from the market rent; other factors earn market wages.

(equations 12-13).[17] At the bottom, these are linked to a Constant Elasticity of Substitution (CES) aggregation of primary factors (a value-added function) and a Leontief aggregation of intermediate inputs (equations 14-16). Given that the national accounts rarely attribute value-added to government capital, the CES value-added functions for government production do not include capital factors. Typically, government value-added is limited to labour.[18]

Each activity produces one or more outputs with fixed yield coefficients (equation 17). Any commodity may be produced and marketed by more than one activity. A CES approach, assuming profit-maximizing producer behaviour, is used to aggregate market sales of any commodity from different activities (equations 18-19). Production is transformed into exports and domestic sales on the basis of a CET (Constant Elasticity of Transformation) function. The profit-maximizing, optimal ratio between the quantities of exports and domestic sales is positively related to the ratio between the corresponding supply prices (equations 20-21). A less complex relationship applies to production without exports or without domestic sales (equation 22). Government and private social services are typically non-traded, i.e. they have no exports and all of the supply is from domestic producers. For any exported commodity, two alternatives are possible for export demand: (a) exogenous prices in foreign currency units (FCU) combined with an infinitely elastic demand; or (b) price-sensitive export demands (defined by constant-elasticity functions) with the FCU prices determined by domestic conditions and the exchange rate (equation 23 applies to the constant-elasticity case.) Given that, in the equations, *PWE* (the export world price) has a bar on top, we assume that (b) does not apply in this specific case (and that the set *CED* is empty).

Domestic demanders are assumed to minimize the cost of imperfectly substitutable imports and commodities from domestic production according to an *Armington* (CES aggregation) function (equations 24-25). For commodities with only one supply source, the supply from this source equals the composite supply (equation 26). The transactions (trade and transport) demand for any service commodity is the sum of demands arising from domestic sales,

[17] MAMS also permits the alternative of a CES aggregation of the real aggregates of value-added and intermediates. The choice does not tend to have a major impact on results. Most country studies used the Leonfief alternative.

[18] Nevertheless, the model accounts for the fact that government capital stocks indeed are needed in government activities by imposing investments derived from a Leontief-relationship between government activity levels and related capital stocks, with the stocks being defined on the basis of initial stocks, investment and depreciation (see equation 45). In the exceptional cases when the SAM indicates that government capital earns value-added, this value-added is a fixed share of the total value-added of the activity (in effect equivalent to a tax on value-added), not related to any market rent.

exports, and imports, each of which is the product of the quantity traded and a fixed input coefficient (showing the quantity of the service commodity per unit of trade; equation 27).

In the country studies, government service sectors invariably produce single outputs and have fixed coefficients for intermediate inputs and capital (typically without any value-added payment; also see discussion in the section "Investment block"). Given this, "profit-maximization" merely involves some flexibility in terms of the composition of their labour employment as they supply the quantities that are demanded.

Domestic institution block

This block (equations 28-44) accounts for the receipts and expenditures of all domestic institutions, both government and non-government (households) as well as current, non-trade payment flows to and from the rest of the world; i.e., factor incomes and transfers. When they represent inflows of foreign currency, these payments tend to be fixed (in FCU). The equations are structured to accommodate databases with any number of households, one government, and one entity representing the rest of the world. The payments in this block are highly interrelated since institutions often are both at the receiving and paying ends. Transfers between any two institutions may flow in both directions; however, if so, the analyst may often find it more convenient to net these in the initial model SAM.

Turning to the equations, factor incomes are defined as a function of domestic wages (which may vary across activities) and employment levels, augmented by factor incomes from the rest of the world (equation 28) and allocated across different institutions (domestic and foreign) in value shares that depend on factor endowment shares (equations 29-30). Domestic non-government institutions: (i) earn net interest incomes, defined as the difference between net interest earnings from loans to the government and net interest payments to the rest of the world on foreign debt (equation 31); (ii) transfer fixed shares of their incomes (net of direct taxes and savings) to other institutions (domestic or foreign) (equation 32); (iii) earn total gross incomes defined as the sum of factor incomes, net interest incomes, and transfers from other institutions, where the treatment of the latter differs depending on the nature of the sending institution (government, the rest of the world, or another domestic non-government institution) and the receiving institution (household or non-household) (equation 33); (iv) pay direct taxes according to rates that are fixed unless adjusted as part of the government closure rule (equation 34; note that all right-hand-side terms are exogenous); and (v) save out of incomes net of direct taxes according to marginal (and average) rates that are endogenous, depending on changes in per-capita incomes if the elasticity of savings with respect to per-capita income is different from unity (equations 35-36). Alternatively, for any given institution, the savings and/or direct tax rate may be adjusted as part of the savings-investment and government closure rules. If direct tax rates are adjusted as part of the government closure rule, either they are scaled up/down "efficiently" by a factor (*TINSADJ*) or uniformly adjusted for selected institutions (through *DTINS*). As suggested by the absence of a bar above *DTINS*, this mathematical statement assumes that changes in direct tax payments via adjustments in *DTINS* clear the government budget. This closure was the default in the country studies. The savings rates can be adjusted through similar alternative mechanisms (through *MPSADJ* or *DMPS*) as part of the savings-investment rule.

For households, incomes net of direct taxes, savings, and transfers to other institutions (defined in equation 37) are allocated across different commodities according to demand functions belonging to a Linear Expenditure System (LES), defined in per-capita form with separate equations for demands from the market and from own-production (equations 38-39). If the database explicitly considers transactions costs, then market demands include these whereas demands for own production do not.

For the remaining domestic institution, the government, current incomes come from taxes (which are disaggregated into a wide range of categories), factor endowments (the government may own non-labour factors), and transfers from other domestic institutions and the rest of the world (equation 40). The (re)current expenditures of the government are divided into consumption, transfers to domestic institutions (CPI-indexed) and the rest of the world (fixed in FCU), and interest payments on domestic and foreign debt (equations 41). For each period except the first, real government consumption, disaggregated by commodity (excluding consumption for infrastructure), is defined as the level in the previous year times a growth factor that consists of multiple terms (equation 42). In the mathematical statement, the right-hand side terms are all exogenous or lagged; in simulations with other rules for determining government consumption (including simulations targeting MDGs), one of the exogenous terms is endogenous.[19] Real government consumption of infrastructure services, also for each period except the first, is defined as the quantity of government consumption per unit of the government infrastructure capital stock times the real endowment of that capital stock by the government; i.e., the size of the capital stock determines consumption (which may represent maintenance, administration, etc.) (equation 43). Finally, government savings is simply the difference between current revenues and current expenditures (equation 44).

Investment block

This block (equations 45-53) covers the determination of government and private investment (including FDI) and how these are financed.

Government investment demand by capital stock (*DKGOV*) is defined in equation 45, which consists of three parts.[20] Different treatments are applied to service capital (used in the production of government services) and infrastructure capital (which requires government support services) (equation 45a). For service capital, growth in service production is the driving force; investment demand is determined as the difference between (i) the anticipated capital demand next year (assuming that production growth will be the same as last year and using a fixed capital-input coefficient) and (ii) the capital stock that would remain if no investments were made.[21] For infrastructure capital, government investment demand is determined as the difference between (i) an exogenous growth term times the infrastructure capital stock in *t* (similar to equation 42) and (ii) the capital stock that would remain if no investments were made.[22] A non-negativity constraint is also imposed for government investment (equation 45b). A complementary-slackness condition (equation 45c) imposes that (i) if *DKGOV* is positive, then

[19] $QGGRW_t$ (a term for government consumption growth that is not commodity-specific) is flexed if the absorption share of total government consumption is fixed. $QGGRWC_{c,t}$ (a term for government consumption growth that is commodity-specific) is flexed when some target influenced by this specific government service (*c*) is fixed for year t. For the most straightforward case, $qg01_{c,c',t}$, a parameter for mapping one *c* to another, is 1 when *c* = *c'* and zero otherwise. If the analyst wants one or more kinds of government consumption to grow in tandem with another, more than one *c* may have a value of 1 for any given *c'*. In either case, each *c* is linked to only one *c'*.

[20] Among these, only part *(a)* is an explicit equation in the GAMS code. The non-negativity constraint on *DKGOV* is handled via a lower limit on this variable. The complementary-slackness condition is imposed by associating the first equation *(a)* to the *DKGOV* variable in the GAMS model definition.

[21] In GAMS, the treatment is more general, giving the user the option to assume that the rate of expected output growth is the same as the rate of simulated output growth during the last 1, 2, or 3 years.

[22] For public infrastructure, actual *QG* (government service level) is determined by the current capital stock (see equation 43). In equation 45, the exogenous growth variable $QGGRWC_{c,t}$ (which is defined over *c*, where the relevant *c* may be public infrastructure services) is mapped to the capital stock *f* associated with *c* and drives the expansion in the capital stock.

equation 45a must hold as an equality; and (ii) if the right-hand side of equation 45a is negative, then *DKGOV* will be zero and equation 45a will hold as an inequality. This treatment is used to avoid a negative investment value (*DKGOV* < 0) in the exceptional case of an anticipated production *decline* that is larger than the depreciation rate. Equation 46 transfers the value of *DKGOV* to investment by institution, *DKINS* (for the government), a variable that is used elsewhere in the model to represent investment across all capital stocks and institutions.

The prices of new capital stocks (disaggregated by type) depend on their composition and market prices (equation 47). The resulting fixed government investment value (defined on the basis of the price and quantity information generated in the preceding equations) is financed by some combination of government savings (net of spending on stock or inventory changes), sales of government bonds (i.e., new interest-bearing borrowing), borrowing via the monetary sector, foreign borrowing, and foreign capital grants (which is separate from current government transfers from the rest of the world) (equation 48). Returning to the equations, government bond sales and borrowing via the monetary system are allocated across households on the basis of their savings shares (equations 49-50).[23]

Equation 48 concludes the series of equations that summarize the government budget (see also equations 40-41 and 44). The choice of mechanism for clearing the budget (the government closure rule) is an important part of the simulations in the country studies of this project. As noted above, changes in the variable *DTINS* (see equation 34) adjust direct tax payments sufficiently to clear the budget. The other terms in the expressions for government receipts and outlays are exogenous or determined via other mechanisms. Under the other three closures that are used, direct tax rates are exogenous while one of the following variables is endogenized to clear the budget: government bond sales ($GBORTOT_t$), government borrowing from the rest of the world ($FBOR_{gov,t}$), or government grants from the rest of the world ($FGRANT_{gov,t}$).

Each alternative closure has specific macroeconomic repercussions. Increases in government bond sales reduces the amount of financing that is available for private investment (cf. equation 51) while increases foreign grants or foreign borrowing tend to permit more rapid

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[23] The savings shares are adjusted by a distortion term ($gbdist_i$) that reflects deviations between household shares of government borrowing and savings. Implicitly, the burden of monetary system borrowing is felt by other agents since it extracts real purchasing power from them by reducing the value of the old money that they hold. In the absence of an explicit treatment of money in this model, this burden is here allocated across households on the basis of their savings shares.

growth in GDP and private final demand (consumption and investment). Reliance on foreign resources also tends to bring about real exchange rate appreciation, slower export growth, and more rapid growth in imports and production for domestic markets. The strength of these effects depends on the growth impact of the expansion in government spending as well as on whether the new spending has high or low import shares. If the country later needs to reverse the switch toward production of non-tradables (for example, because of a decline in foreign grants in the future), and its structure is rigid, it may end up suffering from "Dutch Disease." Expansion in foreign borrowing is less favourable than grants since it drives up the foreign debt (which, in the absence of debt relief, eventually has to be repaid) and related interest payments (more or less burdensome depending on loan conditions). The alternative of raising direct taxes tends to be less favourable to growth in GDP and private final demand than reliance on foreign resources. However, given that most of the cut in household disposable income is born by consumption as opposed to savings and investment, the direct tax alternative is more favourable than domestic government borrowing for long-run growth in GDP and private final demand.

Equation 51 defines the fixed investment values for non-government institutions—all terms do not apply to each institution—as own savings, net of spending on stock (inventory) changes and lending to the government, and augmented by borrowing, capital grants and FDI from the rest of the world. For the latter, the fixed investment value is simply the value of FDI (fixed in FCU) times the exchange rate. (The FDI term is invariably fixed at zero for domestic institutions.) Implicitly, equation 51 shows the rule that the country studies used for ensuring that total savings and total investment are equal: given that government and households savings, government investment, and FDI all are determined by other rules, the clearing variable is private household investment ($INVVAL_{h,t}$).

For each non-government institution, real investment in different capital stocks (investment by destination) is determined by its total fixed investment values, the prices of capital goods, and exogenous value shares by capital stock; the value share is unity if the database only specifies a single private capital type (equation 52).[24]

[24] Typically, the model will only have one private capital stock, i.e. the value of the share parameter is unity for this capital type. If the model has more than one private capital stock, the allocation between the different stocks may be endogenized, possibly deviating from the base-level allocation in response to changes in relative profit rates, a relationship that would need to be specified in one or more additional equations.

The final equation in this block defines total investment demand by commodity source (often referred to as investment by origin). It is defined on the basis of real gross fixed capital formation (both private and government; investment by destination) and the capital composition parameter (equation 53).

Other system constraints: foreign exchange, factors, and commodities

In the preceding, we discussed alternative mechanisms for clearing two of the macro constraints of the model, the government budget and the savings-investment balance. The current block (equations 54-58) includes the remaining system constraints: the balance of payments and the markets for factors and commodities.

The balance of payments (or foreign exchange constraint) (equation 54) imposes equality between foreign exchange uses (spending on imports, factor incomes and transfers to the rest of the world, and interest payments on foreign debts) and sources (export revenues, transfers, factor incomes, borrowing, capital grants, and FDI).[25] In the country studies, the (real) exchange rate (EXR_t) clears this balance. For example, other things being equal, depreciation (an increase in EXR_t) will remove a deficit by raising supplies for export relative to supplies for domestic sales while reducing domestic use of imports relative to domestic use of domestic output.

The market constraint for factors (equation 55), which applies to all factors except government capital, states that total demand for any factor (the left-hand side) equals the total endowment times the employment rate (one minus the unemployment rate). This is straightforward if the unemployment rate is exogenous—if so, in any time period, the economy-wide wage variable ($WF_{f,t}$) will clear the market by influencing the quantities demanded.

[25] Implicitly, an additional system constraint, the savings-investment balance, also holds: by channelling domestic savings and the terms that make up foreign savings to investment, the model equations assure that total savings and total investment are equal.

[*Figure 3.1* Labour market adjustment with endogenous unemployment]

Figure 3.1 shows the functioning of factor markets with endogenous unemployment. The supply curve is upward-sloping, reflecting that, ceteris paribus, workers request higher wages as the labour market gets tighter. When the market reaches a state of "full employment", that is when the minimum unemployment rate is reached (which here is set at 5 percent but varies across the country studies), the supply curve turns vertical. When the factor market is below full employment, the unemployment rate ($UERAT_{f,t}$) is the clearing variable; at full employment, the economy-wide wage ($WF_{f,t}$) clears the market. Unemployment should be seen as broadly defined, representing the degree of underutilization of the factor (and the potential for increased utilization), due to a combination of full or partial unemployment (i.e., also considering underemployment).

Equations 56-57 specify our treatment of the labour market. Workers have a reservation (minimum) wage ($WFRES_{f,t}$) below which they will not work (equation 56). It is defined as a function of the economy-wide wage in the base year, and the ratios between current and base-year values for the (un)employment rate, household consumption per capita (as indicator of real living standards), and the CPI. The ratio terms are raised to elasticities that determine their importance (an elasticity of zero implies that a term has no importance). Equation 57 consists of three parts: (a) the constraint that the economy-wide wage for each factor cannot fall below the endogenous reservation wage; (b) the constraint that the unemployment rate cannot fall below an exogenous minimum (*ueratmin*);[26] and (c) a complementary slackness condition, which states that either (a) or (b) but not both are slack (non-binding). In other words: if the unemployment rate is above its minimum, then the wage must be at the reservation level; if the wage is above the reservation level, then the unemployment rate must be at its minimum.

Note that, at the activity level, the wage paid is the product of $WF_{f,t}$ and $WFDIST_{f,a,t}$ (cf. equations 15 and 28). $WFDIST_{f,a,t}$, a distortion (or differential) term that typically is exogenous, reflects relative wage differences across activities. In some cases it may be desirable to impose an exogenous time path for the employment of specific factors in selected activities (drawing on

[26] The level of the base-year unemployment rate relative to the minimum unemployment rate indicates the potential for employment growth over and above the growth rate of the labour stock.

other pieces of information, for example data on the expected evolution of sectors based on the exploitation of natural resources). For the factor-activity-time combination in question, the analyst only has to flex the wage distortion variable ($WFDIST_{f,a,t}$) and fix the employment variable ($QF_{f,a,t}$). Such an assumption can coexist with factor markets with or without endogenous unemployment.

For each composite commodity, the supply is set equal to the sum of demands (equation 58). As noted earlier, composite supplies stem from two sources, imports and domestic supplies to domestic markets (cf. equation 24); for each commodity with both sources, demand is allocated between them on the basis of relative prices. The market-clearing variables are the quantity (*QM*) for imports and, for domestic output, the price (*PDS* for suppliers and *PDD* for demanders, with a wedge between the two in the presence of transactions costs).

Asset stock updating and productivity block

The equations in this block update institutional stocks of assets and liabilities, and TFP by activity (equations 59-66). Except for equations defining arguments for the definition of TFP, all equations in this block include lagged relationships. They do not apply to the first year, for which the values of the variables defined in this block are fixed.

In most country studies, MAMS has a single representative household and this is also the implicit assumption in this mathematical statement.[27] For capital, the stock of any institution (household, government, and rest of world) is defined as the sum of its previous-period stock (adjusted for depreciation), new investments, and exogenous adjustments (which may reflect the impact of natural disasters or institutional changes, removing parts of the capital stock from production) (equation 59). The evolution of labour endowments in defined in equation 81. For other factors (for example agricultural land), the growth in institutional endowments ($QFINS_{i,f,t}$) is exogenous. Except for the absence of depreciation, the relationships that hold for foreign debt (equations 60) and government bonds (equations 61) are identical to those used for capital. For

[27] In applications with multiple households, it was necessary to specify how the population in each household evolves over time. Our general principle was that the household types that exist in the base-year (characterized by patterns for generation and spending of incomes) continue to exist but grow at different rates depending on the types of labour that they control. The non-labour endowments of each household type grow at the same rate as its population, scaled upwards or downwards to ensure that the total for these endowments across all household respect economy-wide constraints.

foreign debt, the treatment is potentially more complex since the model allows for the possibility of non-paid interest (which is added to the debt) and debt relief.

This block includes further a set of equations used to define total factor productivity (TFP) for each activity. To simplify the algebra, equations 62-63 define real GDP at market prices and the real trade-to-GDP ratio.

In equation 64, the TFP of each activity (a variable that appears in equation 14, the CES value-added function) is defined as the product of a trend term, changes due to capital accumulation, and changes due to variations in economic openness (defined by the real trade-to-GDP ratio). The effects of capital accumulation and changes in openness depend on the values of exogenous elasticities—if they are set at zero, the effect is zero and then only the trend term matters. In the definition of the trend term (equation 65), the first of the trend growth terms, $\alpha vag_{a,t}$ is invariably exogenous. The second term, *CALTFPG* is endogenized when a certain GDP level is targeted (used in the base simulations of the country studies). In this context, the parameter *tfp01* has been used to control relative TFP growth rates across activities (with values ranging between zero and unity). However, apart from the base simulation, all right-hand terms are either exogenous or lagged while GDP is endogenous.[28] The trade-to-GDP ratio, the most common indicator of economic openness (in terms of outcome, not policy stance) is defined in real terms (to avoid the impact of nominal changes, for example due to exchange rate depreciation) and with a potential lag to avoid unrealistically large immediate productivity effects of changes in openness): in any time period, the numerator in the last term of equation 64 is a weighted average of current and past trade-to-GDP ratios. The parameter for the length of the lag is part of the country-specific database. The final equation in this block, equation 66, defines real GDP at factor cost; it is flexible unless *CALTFPG* is flexible.

The fact that the elasticity parameters in equation 64 are disaggregated (by activity for trade and by activity and function for capital) make it possible to specify different channels and magnitudes for the productivity effects of trade and different types of capital stocks. In the

[28] When developing the model base run, *CALTFPG* may be endogenous or exogenous. If it is endogenous, real GDP (at factor cost) should be fixed (growing exogenously over time). If so, the analyst should review the resulting economy-wide growth in TFP as well as efficiency growth in different activities ($ALPHAVA_{a,t}$) and, if needed, adjust the targeted real GDP levels. On the other hand, if *CALTFPG* is exogenous (and real GDP endogenous), the analyst should monitor overall GDP growth and, if needed, adjust either *CALTFPG* or αvag. The estimates of initial capital stocks and depreciation rates may also have to be revisited. For non-base runs, the determinants of trend TFP growth (*ALPHAVA2)* are typically fixed, while real GDP growth is determined by growth in factor employment and endogenous TFP changes.

country studies, the productivity effects of capital are limited to infrastructure capital.[29] Depending on the degree of disaggregation of these capital stocks and activities, the productivity effects can be more or less finely targeted. For example, if irrigation and road capital stocks are singled out, these could meaningfully be linked to agriculture (especially crop activities) and transportation services, respectively; other sectors would only be influenced indirectly by these productivity changes. On the other hand, if infrastructure capital is a single capital stock, the selection of targeted sectors would have to be more general (implicitly reflecting some assumed composition of this broader spending type).

The MDG module

The MDG module (equations 67-83) specifies the mechanisms that determine the values for the indicators related to the different MDGs and educational behaviour as well as the size and disaggregation (by educational achievement) of the labour force. The rest of the economy, which was presented in the preceding sections, influences the evolution of the MDGs and the educational sector through variables related to household consumption, the provision of different types of MDG-related services, labour wages, and capital stocks in infrastructure. In its turn, the MDG module influences the rest of the economy through its impact on the size and composition of the labour force. In addition, the evolution of one set of MDGs can influence other MDGs. The notation and the equations of the MDG module are respectively presented in Tables A3.3 and A3.4.

MAMS focuses on the MDGs that typically are most costly and have the greatest interactions with the rest of the economy: universal primary school completion (MDG 2; measured by the net primary completion rate), reduced under-five and maternal mortality rates (MDGs 4 and 5), and increased access to improved water sources and basic sanitation (part of MDG 7). The poverty MDG (MDG 1) is simulated in the microsimulation model (see Chapter 2, Appendix A2.1); it is not targeted given the absence of tools (in MAMS and in most real-world, developing-country contexts) that policymakers could use to fine-tune poverty outcomes.[30]

[29] It would also be possible to include the effects of improved health on the productivity of labour (with these effects disaggregated by labour type and activity). These were not considered in the country studies for lack of estimates of the quantitative relationship.
[30] Implicitly, when MDGs 4 and 5 are achieved, the expansion in health services and other determinants may be sufficient to achieve MDG 6 (to halt and reverse the spread of HIV/AIDS, malaria and other diseases). MDG 3

As explained in the introductory section of this chapter, MDG outcomes depend on government and private sector provision of MDG-related services as well as demand conditions for those services. Table 3.3 lists the determinants that were included in a typical country study. In most cases they were identified on the basis of sector studies underpinned by econometric analysis and subject to the constraints of an economy-wide model like MAMS (including the fact that it is difficult to include finely disaggregated actions, like increasing coverage of certain types of vaccinations).[31] Beyond per-capita real service delivery (either public or a combination of public and private as is the case in most of the LAC region), the determinants include other MDGs (for example, better access to water and sanitation may improve health outcomes—MDGs 4 and 5), as well as public infrastructure, per-capita household consumption, and wage incentives (through the ratio of labour wages of different educational levels). Other potential determinants (like the impact of education on health) were not included given that their effects tend to make themselves felt only over time periods longer than the ones covered by the country simulations.

Table 3.3 Determinants of non-poverty MDGs

MDG	Service delivery	Household consumption per capita	Wage incentives	Public infra-structure	Other MDGs
2 Primary education	x	x	x	x	4
4. Under-five mortality	x	x		x	7a, 7b
5. Maternal mortality	x	x		x	7a, 7b
7a. Access to safe water	x	x		x	
7b. Access to basic sanitation	x	x		x	

In the equations of this module, the treatment of the education MDG (2) is separate from the treatment for the remaining MDGs (4, 5, 7a, 7b) since, rather than targeting MDG 2 directly,

(elimination of gender disparity in education and empowering women) was not addressed due to data issues. However, note that, if MDG 2 is achieved, gender equality is achieved in primary education.

[31] The country studies of this volume show that the relationships between the determinants and the non-poverty MDGs in the MAMS model hold from a statistical point of view for a number of Latin American and Caribbean countries. Kamaly (2006) provides examples of the literature on health and education whose findings, although sometimes contradictory, show broad support also in sub-Saharan Africa for the inclusion of the determinants referred to in Table 3.3.

the model defines (and may target) specific educational behavioural outcomes that jointly determine the value for MDG 2.

The first three equations define arguments that enter both education and MDG functions. Equations 67-68 define aggregate human development (HD) services (which include both MDG and education services). For each service type, equation 67 separates demand into two aggregates, government and non-government, according to who is paying for the service. Typically, services paid for by the government (non-government) are also supplied by a government (non-government) activity, but this is not necessarily the case. Equation 68 generates an economy-wide aggregate (which below is fed into the determination of MDG and education outcomes), permitting two alternative assumptions: services paid for by government and non-government are perfect substitutes (simply summed) or imperfect substitutes (according to a CES function). Equation 69 defines average real household consumption per capita (*QHPC*) as total household consumption (both marketed and home commodities) at base-year prices divided by total population.

The educational component consists of equations 70-79. It is disaggregated by cycle (with three cycles as a typical level of disaggregation). For each cycle, educational quality (*EDUQUAL*) is defined as the ratio between real services per student (aggregated services divided by total enrolment) in the current year and in the base year; i.e., in the base-year, educational quality is indexed to one (equation 70). Within any cycle, the model endogenizes the following aspects of student behaviour (or outcomes):

- the shares of the enrolled that pass their current grade, drop out, or repeat the grade next year (referred to as *pass*, *dropout*, and *rep*). The sum of these shares is unity—i.e., during the school year, a student must either pass, drop out, or become a repeater (this applies to each grade and for each cycle as a whole). Note that the term “pass” throughout this chapter and the model refers both to students who successfully complete a grade and continue to a higher grade within the cycle, and to students who successfully finish the last year of a given education cycle (and thus graduate);
- the shares among the passers from their current grade (*pass*) who graduate from their current cycle (*grdcyc*) or continue to a higher grade within this cycle (*contcyc*). In terms of shares: *grdcyc* + *contcyc* = *pass*;

- the shares among cycle graduates who exit the school system (*grdexit*) or continue to next cycle (*grdcont*). The sum of these shares is also unity. For graduates from the last cycle, the share of those who exit is unity; and
- the share of the cohort of the 1st year in primary school that enters school (*g1entry*).

Drawing on the above information, we can define the number of enrolled students by cycle and year. Equation 71 defines the number of "old" enrolled students in any cycle (i.e., those who were enrolled in the same cycle last year) as the sum of those who: (i) continue within the cycle after successful completion of an earlier grade; and (ii) repeat the grade they were in last year. The number of "new" enrolled students is defined in equation 72 as the sum of: (i) cohort entrants (only for the 1st grade of the primary cycle); (ii) other, non-cohort entrants entering any cycle in the educational system; and (iii) graduates from the relevant earlier cycle last year who chose to continue.[32] The total number of enrolled students in a cycle is the sum of old and new students (equation 73).

[*Figure 3.2* Logistic function for education]

Equations 74-78 model the share variables that identify different aspects of student behaviour. For each cycle, a logistic function (equation 74) defines *SHREDU*, the shares for 1st year in-cohort entry, for graduates from the current grade, and for graduates who decide to continue to next cycle (i.e. *g1entry*, *pass*, and *grdcont*, the elements of the set *BLOG*). The logistic form was selected since it makes it possible to impose extreme (for education it is a maximum of one) values for the function and to incorporate extraneous information about elasticities and conditions under which target values are achieved. Another advantage is that it allows for segments of increasing and decreasing marginal returns to improvements in the determinants of educational behaviour. The only endogenous variable in the logistic function (*ZEDU*), is defined in a constant-elasticity (CE) function (equation 75) as determined by: (i) educational quality; (ii) wage incentives, defined as relative wage gains from continued schooling (i.e., the relative wage gain that students can achieve if they complete a cycle that is sufficiently high to enable them to climb to the next higher level in the labour market); (iii) the

[32] This category includes non-cohort entrants to the 1st primary year of primary school (who may represent a significant number during a transitional period of primary school expansion). It may also include immigrants from other countries.

under-five mortality rate (a proxy for the health status of the school population); (iv) the size of the infrastructure capital stock; and (v) household consumption per capita. Figure 3.2 illustrates the logistic functional form for education. The observed base-year value for *SHREDU* is generated at the base-year value for *ZEDU*. The parameters of the function have to be defined such that the maximum share is one, the base-year elasticities of *SHREDU* with respect to each determinants of *ZEDU* is replicated and, under values for the determinants of *ZEDU* identified in the database, a target level for *SHREDU* is realized. In terms of the algebra, the parameters in equations 74-75 are selected as follows:

- the parameter $extedu$ shows the extreme (maximum) value (here unity) to which the behaviour share should converge as the value of the intermediate variable approaches infinity;
- the parameter αedu is calibrated so that, under base-year conditions, the behavioural share replicates the base-year value;
- the parameters βedu and φedu are calibrated so that the two equations: (i) replicate the base-year elasticities of the behavioural share (*SHREDU*) with respect to the arguments of the CE function; and (ii) achieve a behavioural target (e.g. a share very close to one for *g1entry*, the share of the relevant age cohort that enters first grade) under a set of values for the arguments of the CE function that have been identified by other studies; and
- the value of the parameter γedu determines how the base-year point on the logistic function is positioned relative to the inflection point (where the curve switches from increasing to decreasing marginal returns as the determinants of educational behaviour improve).

Equation 75 is calibrated so that, in the base year (under base-year conditions), $ZEDU_{b,c,t} = SHREDU^{0}_{b,c}$. (Note that the left-hand term enters the denominator of the second term in equation 74.)

Drawing on the shares defined in the preceding equations, the shares for repeaters, dropouts, and cycle graduates exiting from the school system (*rep*, *dropout*, and *grdexit*; elements in the set *BRES*) are defined residually (equation 76). The formulation considers the fact that, as noted above, selected shares have to sum to unity. If more than one variable in *BRES* has to be adjusted in relation to one or more elements in *BLOG* (as is the case for the adjustment of shares for repeaters and dropouts in response to changes in the share of graduates), then all adjusted variables are scaled up or down by the same factor.[33] The share of graduates from a cycle (*grdcyc*) is defined as the share for the total number of passers in the cycle (*pass*) divided by the number of years in the cycle times an adjustment term (since students may not be evenly distributed across grades) whereas the residual share is assigned to graduates within a cycle (*contcyc*) (equations 77-78).

We use the net completion rate as our MDG 2 indicator. It is defined as the product of the relevant 1st-year primary school entry rate (*g1entry*) and the passing rates (*pass*) over time for the cohort that graduate from primary school in the current year (equation 79).[34]

The labour force participation rate is defined as the labour force (*QFINS*) divided by the population in labour force age that is not enrolled in secondary and tertiary cycles (equation 80).[35] Institutional labour endowments (*QFINS* for labour) are defined as the sum of the following components (equation 81): (i) remaining labour from the preceding year; (ii) new labour force entrants among students who exited from the school system in the previous year (with separate terms for non-tertiary graduates, tertiary graduates, and dropouts); and (iii) new labour force entrants from the non-student population who reach the age at which they, to the extent that they seek work, become part of the labour force. Depending on their highest completed grade, the new labour force entrants are allocated to a specific labour category.

The treatment underlying MDGs 4, 5, 7a and 7b is similar but less complex. For these, a logistic function directly defines the MDG indicators as a function of an intermediate variable that is defined in a related CE function (equations 82-83). The values for the parameters $extmdg$, αmdg, βmdg, and φmdg are defined following the same principles as the corresponding parameters in the logistic and CE functions for education. The arguments of the CE function are similar except for that the relevant service supply is expressed in per-capita form (not per enrolled student).

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[33] The equation is formulated so that it works for cases with one or more than one term in any of the sums over related shares (defined by the mappings *MBB* and *MBB2*) in either of the sets *BRES* and *BLOG*.

[34] In other words, in order for 100% of the cohort to complete the primary cycle on time, it is necessary that all of them enter at the time of their first year and then that all manage to pass each year (i.e., successfully complete each grade) up to the final year of the cycle. Given that we do not generate separate pass rates for students in the relevant cohort (as opposed to students outside this cohort), we assume that the rates for in-cohort students are identical to the over-all rates for students in the cycle.

[35] It is assumed that, as an acceptable approximation, students in secondary and above are in labour force age. If not, this definition should be adjusted.

3.4 Overview of MAMS data needs and sources

The data needs of the core CGE module of MAMS are similar to those of other CGE models. Additional data (for the SAM and other parameters) is needed primarily for the MDG module but also to capture some other extensions, mostly related to the treatment of the government.

In an earlier section, we presented the structure of a SAM for the full (MDG) version of MAMS, as applied in the country studies. The following aspects of this SAM give rise to data requirements that go beyond what is needed for most SAMs for CGE models:[36] (i) government consumption and investment spending must be disaggregated into functions that correspond to policy tools for addressing the relevant MDGs and providing education at the three major levels (primary, secondary and tertiary); (ii) labour must be disaggregated by level of education in a manner that matches the educational system; and (iii) the SAM must include accounts for foreign and domestic interest payments. In addition, if they are important, it is also preferable to single out separate private activities and commodities in the MDG and education area; for example, the private sector may account for significant shares of the total supply of education, especially at higher levels. In other respects, the SAM is very similar to standard practice.

Other data and sources

Other than the SAM, MAMS requires data that in part coincides with those of many other models, most importantly elasticities to capture substitutability between factors in production, transformability of output between exports and domestic sales, substitutability between imports and domestic commodities in domestic demand, and responses in household consumption to income changes.

The other data requirements, specific to MAMS, are mostly due to its extensions to include MDG indicators and their determinants, an extended education module, and relatively detailed government accounts. The links between labour, education, and population necessitate consistent base-year data on employment (by activity and labour type), unemployment (by

[36] A SAM for the core (non-MDG) version of MAMS deviates from this in that may have a very aggregate treatment of sectors (activities and commodities), factors, and institutions. The minimum degree of detail is two sectors (private and government); two factors (labour and private capital), three institutions (government, household, and rest of world), two investment accounts (for private and government capital).

labour type), and enrolment (by level or cycle).[37] The model also requires projections for the total population and the population of three age groups: the cohort of entrants to primary school, the cohort of entrants to labour-force age, and the broader population group in working age.

Both for education (for each of the three levels) and four of the non-education MDGs in MAMS (labelled 4, 5, 7a, and 7b), it is necessary to provide data for base-year outcomes: in education, cycle-specific rates for entry, pass, repetition and dropout; outside education, the rates that define other MDG indicators (e.g. the under-five mortality rate). For these non-education MDGs, information on the situation in 1990 is needed to define the targets for 2015. To model how these outcomes change over time, two pieces of information are needed: (a) base-year elasticities (linking outcomes to determinants), preferably estimated using logit or probit models; and (b) a path for the evolution of these determinants that makes it possible to reach a set of future values (typically targets for 2015). This information is used to calibrate the functions so that they replicate base-year outcomes and elasticities, reach each MDG under specified conditions, and have the upper limits that were specified exogenously.

Finally, also for non-labour factors, it is necessary to define base-year stocks. For private capital, these may be defined on the basis of base-year data for rents, profit rates, and depreciation rates.[38] For each type of government capital, the base-year stock is defined on the basis of historical data on service growth, investments, depreciation rates, and the assumption that the capital stock over time has grown at the same rate as real services. For other factors such as agricultural land and natural resources, base-year stocks can typically be defined so that base-year rents are normalized to unity; data on the future stock growth is also needed—as opposed to labour and capital, growth for these factors is exogenous.

The construction of this database requires the analysts to consult existing SAMs and input-output tables, other standard databases (both country-specific and those of international organizations, covering national accounts, government budgets, and the balance of payments), surveys (of households, labour, and health conditions), and relevant research on trade, production, consumption, and human development, including available MDG strategies and other analyses of the determinants of MDG outcomes. Sector-focused MDG studies (in health,

[37] For the 18 country studies, the base year falls within the period 2000-2005, i.e. approximately at the mid-point between the starting year on the basis of which many MDG targets are defined (1990) and the target year (2015).
[38] The following formula is used to define the base-year private capital stock: *qfcap* = *samrent*/(*netprfrat* + *deprrat*) where *qfcap* = the stock; *samrent* = total VA to private capital in SAM; *netprfrat* = the net profit rate (in decimal form); *deprrat* = the depreciation rate (also in decimal form).

education, water and sanitation, and public infrastructure), public expenditure reviews and other types of country-level economic studies are often valuable sources. In the country studies for this project, it was typically necessary to complement available studies with new survey-based research to better understand the determination of MDG and education outcomes.[39]

As shown, MAMS does not replace other forms of sectoral research in human development; on the contrary, it draws extensively on and stimulates such research. Without sector studies that provide a strong empirical basis, the analysis of MDG strategies in an economy-wide framework (whether MAMS or any other) loses much of its power.

3.5 Concluding remarks

This chapter has documented MAMS, a dynamic CGE model that constitutes the central methodological framework of this study and, together with a microsimulation model, and complemented with sector studies, is used to answer the three development strategy questions posed in Chapter 1. MAMS is designed to analyze strategies for achieving selected MDGs and, more broadly, policies for medium- and long-run growth and poverty reduction in developing countries. The development of MAMS is and has been driven by a strongly felt need for an economy-wide approach to development strategy analysis that considers the different effects of government interventions, not only because of their resource requirement but also through their impact on human development, including the size and educational characteristics of the labour force, and public infrastructure. MAMS is sufficiently flexible to address the key processes for MDG achievement and other development strategies in a wide range of countries. The requirement is to link the model to country databases that capture country characteristics; the databases may vary widely in terms of disaggregation. The development of such a database, undertaken for the country studies in this volume, requires a considerable research effort. At the same time several aspects of MAMS were further developed to better accommodate many of the specificities of LAC countries.

In the country studies of this volume, MAMS is first used to generate a benchmark, business-as-usual scenario for a period that typically starts around 2000 and ends in 2015; as indicated by its name, the aim of this scenario is to represent a plausible projection into the

[39] The micro-simulation approach to poverty and inequality analysis that was followed in the different country studies requires access to a recent household survey.

future, drawing on recent trends. As a second step, MAMS is used to simulate a set of alternative scenarios under which growth in government MDG services is adjusted endogenously to achieve selected MDGs under alternative financing policies. The scenarios target different combinations of MDGs 2 (universal completion primary school), 4 (reduced under-five mortality rates), 5 (reduced maternal mortality rates), and 7 (improved access rates for water and/or sanitation). The alternative sources for the additional financing are grant aid, foreign borrowing, domestic bonds, and direct taxes.

As the analysis in Chapter 2 made clear, the results vary systematically depending on the targets, the financing mechanism, the initial starting point (how far is the country from achieving the MDG), and overall macroeconomic conditions (including GDP growth under the base or business-as-usual scenario). The country analysis shows that reliance on foreign financing tends to appreciate the real exchange rate, increase import growth and reduce export growth. Real exchange rate appreciation is more significant if the spending increase primarily is directed toward non-tradables (the case for most scenarios except for those that target the water and sanitation MDGs, which tend to be relatively more investment and import-intensive). Among the two foreign financing alternatives, grant aid differs in that it does not leave the country with a debt that requires interest payment and may be the source of sustainability problems. This form of financing of the MDG strategy does not adversely affect private consumption or private investment as is the case in taxation or domestic-borrowing strategies of increased MDG-related spending. Under the domestic financing alternatives, government expansion leaves fewer resources in the hands of households and the private sector. When a government expansion is financed via direct taxes, for example, household disposable income declines, with a stronger reduction (in value terms) for private consumption than private savings and private investment. The government will likely have to spend more to make up for a reduction in private MDG-related spending, rendering this alternative more expensive (cf. Chapter 2). Other things being equal, GDP growth may suffer slightly. Poverty reduction is compromised in the short run while the medium-run results up to 2015 are very heterogeneous.

The effects of the simulations on the labour market are also systematic. For the scenarios that target MDG 2, labour force growth declines in the least educated segment and overall but expands for the more educated segments; these effects are particularly strong in countries that start out with the lowest educational achievements. Accordingly, wages grow more strongly while unemployment declines for the least educated. However, this tendency is mitigated given the fact that MDG strategies also tend to generate more rapid demand growth for more educated labour. Overall, the gains in productivity and average wages from having a more educated labour force are considerable, both for the nation and for the workers who climb the educational ladder.

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Appendix A3.1: Mathematical statement of MAMS

Table A3.1 Sets, parameters, and variables for core CGE modules of MAMS model

SETS			
Symbol	Explanation	Symbol	Explanation
$a \in A$	activities	$f, f' \in F$	factors
$a \in ACES\ (\subset A)$	activities with CES function between Value Added and Intermediate inputs	$f \in FCAP(\subset F)$	capital factors
$a \in ALEO\ (\subset A)$	activities with Leontief function between value added and intermediate inputs	$f \in FCAPGOV(\subset FCAP)$	government capital factors
$c \in C$	commodities	$f \in FEXOG(\subset F)$	factors with exogenous growth rates
$c \in CD(\subset C)$	commodities with domestic sales of domestic output	$f \in FLABN(\subset F)$	non-labour factors
$c \in CDN(\subset C)$	commodities not in *CD*	$f \in FUEND(\subset F)$	factors with endogenous unemployment
$c \in CE(\subset C)$	exported commodities	$h \in H(\subset INSDNG)$	households (incl. NGOs)
$c \in CEN(\subset C)$	commodities not in *CE*	$i \in INS$	institutions (domestic and rest of world)
$c \in CECETN(\subset C)$	exported commodities without CET function	$i \in INSD(\subset INS)$	domestic institutions
$c \in CINF(\subset C)$	infrastructure commodity	$i \in INSDNG(\subset INSD)$	domestic non-government institutions
$c \in CM(\subset C)$	imported commodities	$i \in INSNG(\subset INS)$	non-government institutions
$c \in CMN(\subset C)$	commodities not in *CM*	$(f, a) \in MFA$	mapping showing that disaggregated factor *f* is used in activity *a*
$c \in CT(\subset C)$	transaction service commodities	$t \in T$	time periods

PARAMETERS—LATIN LETTERS			
$capcomp_{c,f}$	quantity of commodity *c* per unit of new capital *f*	$pwse_{c,t}$	world price for export substitutes (FCU)
$cwts_c$	weight of commodity *c* in the CPI	$qdst_{c,i,t}$	quantity of stock (inventory) change

$depr_f$	depreciation rate for factor *f*	$\overline{qe}_{c,t}$	export demand for *c* if *pwe* = *pwse* (world price for substitutes)
$dintrat_{i,t}$	interest rate on government bonds for domestic institution *i*	$qfhhtot_{f,t}$	total household stock of exogenous, non-labour factors
$dwts_c$	domestic sales price weights	$qfinsadj_{i,f,t}$	exogenous factor stock adjustment
$fdebtrelief_{i,t}$	foreign debt relief for domestic institution *i*	$qfpc_{i,f,t}$	per-capita quantity of exogenous-supply factor *f* by institution *i* and year *t*
$fdi_{i,t}$	foreign direct investment by institution *i* (rest of world) (FCU)	$rqgadj_{c,c',t}$	parameter linking government consumption growth across commodities
$fintrat_{i,t}$	interest rate on foreign debt for domestic institution *i* (paid)	$shii_{i,i'}$	share of net income of *i*' to *i* (*i*' ∈ INSDNG)
$fintratdue_{i,t}$	interest rate on foreign debt for domestic institution *i* (due)	$ta_{a,t}$	tax rate for activity *a*
$fprd_{f,a,t}$	productivity of factor *f* in activity *a*	$te_{c,t}$	export tax rate
$gbdist_i$	distortion factor for government borrowing from institution *i*	$tf_{f,t}$	direct tax rate for factor *f*
$gfcfshr_{f,i,t}$	share of gross fixed capital formation for institution *i* in capital factor *f*	$tfp01_{a,t}$	0-1 parameter for activities with endogenous TFP growth
$ica_{c,a}$	quantity of *c* as intermediate input per unit of aggregate intermediate in activity *a*	$tfpelasqg_{a,f,t}$	elasticity of TFP for activity *a* with respect to government capital stock *f*
$icd_{c,c',t}$	trade input of *c* per unit of commodity *c*' produced & sold domestically	$tfpelastrd_a$	elasticity of TFP for *a* with respect to GDP trade share
$ice_{c,c',t}$	trade input of *c* per unit of commodity *c*' exported	$tfptrdwt_{t,t'}$	weight of period *t*' in tfp-trade link in *t*
$icm_{c,c',t}$	trade input of *c* per unit of commodity *c*' imported	$tgap_{t,t'}$	gap between *t* and *t*' (years used for calculation of expected growth rate for *QA*)
$ifa_{f,a}$	quantity of capital f per unit of government activity *a*	$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$igf_{c,f,t}$	quantity of gov consumption per unit of gov infrastructure capital stock f	$tinsbar_{i,t}$	exogenous component in direct tax rate for domestic institution *i*
$inta_a$	quantity of aggregate intermediate input per unit of activity *a*	$tm_{c,t}$	import tariff rate
iva_a	quantity of value-added per unit of activity *a*	$tq_{c,t}$	rate of sales tax

$mps01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates	$trnsfr_{i,i',t}$	Exogenous transfer from institution *i'* to institution *i*
$mpsbar_{i,t}$	Exogenous component in savings rate for domestic institution *i*	$trnsfr_{f,i',t}$	Exogenous transfer from institution *i'* to factor *f*
$poptot_t$	total population by year	$trnsfrpc_{i,i',t}$	per-capita transfers from institution *i'* to household institution *i*
$pwm_{c,t}$	import world price of *c* (FCU)	$tva_{a,t}$	rate of value-added tax for activity *a*

PARAMETERS—GREEK LETTERS			
αac_c	shift parameter for domestic commodity aggregation function	$\delta va_{f,a}$	CES value-added function share parameter for factor *f* in activity *a*
$\alpha vag_{a,t}$	exogenous component of efficiency (TFP) for activity *a*	$\gamma h_{a,c,h}$	per capita household subsistence consumption of home commodity *c* from activity *a*
αq_c	*Armington* function shift parameter	$\gamma m_{c,h}$	per capita household subsistence cons of marketed commodity *c*
αt_c	CET function shift parameter	ρac_c	domestic commodity aggregation function exponent
$\beta h_{a,c,h}$	marginal share of household consumption on home commodity *c* from activity *a*	ρq_c	*Armington* function exponent
$\beta m_{c,h}$	marginal share of household consumption spending on marketed commodity *c*	ρsav_i	elasticity of savings rate with respect to per-capita income for institution (household) *h*
δac_a	share parameter for domestic commodity aggregation function	ρt_c	CET function exponent
δq_c	*Armington* function share parameter	ρva_a	CES value-added function exponent
δt_c	CET function share parameter	$\theta_{a,c}$	yield of output *c* per unit of activity *a*

VARIABLES			
$ALPHAVA_{a,t}$	efficiency parameter in the CES value-added function	$PVA_{a,t}$	value-added price (factor income per unit of activity)
$ALPHAVA2_{a,t}$	endogenous TFP trend term by *a*	$PVAAVG_t$	average value-added price
$CALTFPG_t$	calibration factor for TFP growth	$PWE_{c,t}$	export world price of *c* (FCU)

CPI_t	consumer price index	$PX_{c,t}$	aggregate producer price for commodity
$GBORMS_{i,t}$	implicit government Central Bank borrowing (deficit monetization) from institution *i*	$PXAC_{a,c,t}$	price of commodity *c* from activity *a*
$GBORMSTOT_t$	total government Central Bank borrowing (deficit monetization)	$QD_{c,t}$	quantity sold domestically of domestically produced *c*
$GBOR_{i,t}$	change in holding of government bonds for domestic institution *i*	$QF_{f,a,t}$	quantity demanded of factor *f* by activity *a*
$GBORTOT_t$	total change in holding of government bonds	$QFINS_{i,f,t}$	real endowment of factor *f* for institution *i*
$DKGOV_{f,t}$	gross government investment in *f*	$QG_{c,t}$	quantity of government consumption of commodity *c*
$DKINS_{i,f,t}$	gross change in capital stock (investment in) *f* for institution *i*	$QH_{c,h,t}$	quantity consumed by household *h* of marketed commodity *c*
$DMPS_t$	uniform point change in savings rate of selected domestic institutions	$QHA_{a,c,h,t}$	quantity consumed of home commodity *c* from act *a* by hhd *h*
DPI_t	producer price index for non-traded output	$QINTA_{a,t}$	quantity of aggregate intermediate input used by activity *a*
$DTINS_t$	uniform point change in direct tax rate of selected domestic institutions	$QINT_{c,a,t}$	quantity of commodity *c* as intermediate input to activity *a*
EG_t	government expenditures	$QINV_{c,t}$	quantity of investment demand for commodity *c*
$EH_{h,t}$	consumption spending for household	$QM_{c,t}$	quantity of imports of commodity *c*
EXR_t	exchange rate (LCU per unit of FCU)	$QQ_{c,t}$	quantity of goods supplied to domestic market (composite supply)
$FBOR_{i,t}$	foreign borrowing for domestic institution *i*	$QT_{c,t}$	quantity of trade and transport demand for commodity *c*
$FDEBT_{i,t}$	foreign debt for domestic inst *i*	$QVA_{a,t}$	quantity of (aggregate) value-added
$FGRANT_{i,t}$	foreign grants to domestic institution *i* (FCU)	$QX_{c,t}$	aggregated quantity of domestic output of commodity
$GDEBT_{i,t}$	endowment of government bonds for *i*	$QXAC_{a,c,t}$	quantity of output of commodity *c* from activity *a*
$GDPREAL_t$	real GDP at market prices	$QGGRW_t$	real government consumption growth for all *c* in *t* relative to *t-1*
$GDPREALFC_t$	real GDP at factor cost	$QGGRWC_{c,t}$	real government consumption growth of *c* in *t* relative to *t-1*
$GSAV_t$	government savings	$SHIF_{i,f,t}$	share of institution *i* in income of factor *f*

$INSSAV_{i,t}$	savings of domestic non-government institution *i*	$TINS_{i,t}$	direct tax rate for domestic non-government institution *i*
$INVVAL_{i,t}$	investment value for institution *i*	$TINSADJ_{t}$	direct tax scaling factor
$MPS_{i,t}$	marginal propensity to save for domestic non-gov institution *i*	$TRDGDP_{t}$	foreign trade as share of GDP
$MPSADJ_{t}$	savings rate scaling factor	$TRII_{i,i',t}$	transfers from institution *i*' to *i* (both in the set INSDNG)
$PA_{a,t}$	activity price (unit gross revenue)	$WF_{f,t}$	average price of factor
$PDD_{c,t}$	demand price for commodity *c* produced & sold domestically	$WFDIST_{f,a,t}$	wage distortion factor for factor *f* in activity *a*
$PDS_{c,t}$	supply price for commodity *c* produced & sold domestically	$WFRES_{f,t}$	reservation wage for factor *f*
$PE_{c,t}$	export price (domestic currency)	$YF_{f,t}$	income of factor *f*
$PINTA_{a,t}$	aggregate intermediate input price for activity *a*	YG_{t}	government revenue
$PK_{f,t}$	price of new capital stock *f*	$YI_{i,t}$	income of domestic non-government institution
$PM_{c,t}$	import price (domestic currency)	$YIF_{i,f,t}$	income to domestic institution *i* from factor *f*
$POP_{i,t}$	population by household	$YIINT_{i,t}$	interest payment on government bonds to institution *i*
$PQ_{c,t}$	composite commodity price		

Table A3.2 Equations for the core CGE module of MAMS model

#	Equation	Domain	Description
Price Block			
(1)	$PM_{c,t} = pwm_{c,t} \cdot (1+tm_{c,t}) \cdot EXR_t + \sum_{c' \in C}(PQ_{c',t} \cdot icm_{c',c,t})$ $\begin{bmatrix} import\ price \\ (LCU) \end{bmatrix} = \begin{bmatrix} import\ price \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} tariff \\ adjustment \end{bmatrix} \cdot \begin{bmatrix} exchange\ rate \\ (LCU\ per\ FCU) \end{bmatrix} + \begin{bmatrix} transaction \\ costs \end{bmatrix}$	$c \in CM$ $t \in T$	Import price
(2)	$PE_{c,t} = \overline{PWE}_{c,t} \cdot (1-te_{c,t}) \cdot EXR_t - \sum_{c' \in C}(PQ_{c',t} \cdot ice_{c',c,t})$ $\begin{bmatrix} export\ price \\ (LCU) \end{bmatrix} = \begin{bmatrix} export\ price \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} tariff \\ adjustment \end{bmatrix} \cdot \begin{bmatrix} exchange\ rate \\ (LCU\ per\ FCU) \end{bmatrix} - \begin{bmatrix} transaction \\ costs \end{bmatrix}$	$c \in CE$ $t \in T$	Export price
(3)	$(a) \quad PDS_{c,t} \geq PE_{c,t}$ $\quad (b) \quad QE_{c,t} \geq 0$ $\begin{bmatrix} domestic\ supply \\ price \end{bmatrix} \geq \begin{bmatrix} export\ price \\ (LCU) \end{bmatrix}$ $\quad \begin{bmatrix} export \\ quantity \end{bmatrix} \geq [0]$ .... $(c) \quad (PDS_{c,t} - PE_{c,t})(QE_{c,t} - 0) = 0$ $\begin{bmatrix} Complementary\ slackness\ relationship: \\ 1.\ If\ domestic\ price\ exceeds\ export\ price\ then\ export\ quantity\ is\ zero. \\ 2.\ If\ export\ quantity\ excceds\ zero,\ then\ domestic\ price\ equals\ export\ price \end{bmatrix}$	$c \in$ $(CD \cap$ $CECETN)$ $t \in T$	For non-CET exportables with domestic sales: (a) domestic floor price, (b) non-negative export quantity constraints; and (c) related complementary-slackness relationship.
(4)	$PDD_{c,t} = PDS_{c,t} + \sum_{c' \in C}(PQ_{c',t} \cdot icd_{c',c,t})$ $\begin{bmatrix} domestic\ demander \\ price \end{bmatrix} = \begin{bmatrix} domestic\ supplier \\ price \end{bmatrix} + \begin{bmatrix} transaction \\ costs \end{bmatrix}$	$c \in CD$ $t \in T$	Domestic demander price for domestic commodity
(5)	$PQ_{c,t} \cdot (1-tq_{c,t}) \cdot QQ_{c,t} = PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t}$ $\begin{bmatrix} absorption \\ (at\ demand\ prices \\ net\ of\ sales\ tax) \end{bmatrix} = \begin{bmatrix} domestic\ demander \\ price\ times \\ domestic\ sales\ quantity \end{bmatrix} + \begin{bmatrix} import\ price \\ times \\ import\ quantity \end{bmatrix}$	$c \in$ $(CD \cup CM)$ $t \in T$	Absorption
(6)	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$ $\begin{bmatrix} producer\ price \\ times\ marketed \\ output\ quantity \end{bmatrix} = \begin{bmatrix} domestic\ supplier \\ price\ times \\ domestic\ sales\ quantity \end{bmatrix} + \begin{bmatrix} export\ price \\ times \\ export\ quantity \end{bmatrix}$	$c \in$ $(CD \cup CE)$ $t \in T$	Marketed output value
(7)	$PA_{a,t} = \sum_{c \in C} PXAC_{a,c,t} \cdot \theta_{a,c}$ $\begin{bmatrix} activity \\ price \end{bmatrix} = \begin{bmatrix} producer\ prices \\ times\ yields \end{bmatrix}$	$a \in A$ $t \in T$	Activity price

(8)	$PINTA_{a,t} = \sum_{c \in C} PQ_{c,t} \cdot ica_{c,a}$ $\begin{bmatrix} aggregate \\ intermediate \\ input\ price \end{bmatrix} = \begin{bmatrix} intermediate\ input\ cost \\ per\ unit\ of\ aggregate \\ intermediate\ input \end{bmatrix}$	$a \in A$ $t \in T$	Aggregate intermediate input price
(9)	$PA_{a,t} \cdot (1 - ta_{a,t}) \cdot QA_{a,t} =$ $PVA_{a,t} \cdot QVA_{a,t} + PINTA_{a,t} \cdot QINTA_{a,t}$ $\begin{bmatrix} activity\ price \\ (net\ of\ taxes) \\ times\ activity\ level \end{bmatrix} = \begin{bmatrix} value\text{-}added \\ price\ times \\ quantity \end{bmatrix} + \begin{bmatrix} aggregate\ intermediate \\ input\ price\ times\ quantity \end{bmatrix}$	$a \in A$ $t \in T$	Activity revenue and costs
(10)	$\overline{CPI}_t = \sum_{c \in C} PQ_{c,t} \cdot cwts_c$ $[CPI] = \begin{bmatrix} prices\ times \\ weights \end{bmatrix}$	$t \in T$	Consumer price index
(11)	$DPI_t = \sum_{c \in CD} PDS_{c,t} \cdot dwts_c$ $\begin{bmatrix} price\ index\ for \\ non\text{-}tradables \end{bmatrix} = \begin{bmatrix} supplier\ price\ for\ output \\ marketed\ domestically \\ times\ weights \end{bmatrix}$	$t \in T$	Price index for non-tradables

Production and trade block

(12)	$QVA_{a,t} = iva_a \cdot QA_{a,t}$ $\begin{bmatrix} demand\ for \\ value\text{-}added \end{bmatrix} = f \begin{bmatrix} activity \\ level \end{bmatrix}$	$a \in ALEO$ $t \in T$	Demand for aggregate value-added
(13)	$QINTA_{a,t} = inta_a \cdot QA_{a,t}$ $\begin{bmatrix} demand\ for\ aggregate \\ intermediate\ input \end{bmatrix} = f \begin{bmatrix} activity \\ level \end{bmatrix}$	$a \in ALEO$ $t \in T$	Demand for aggregate intermediate input
(14)	$QVA_{a,t} = ALPHAVA_{a,t} \cdot \left(\sum_{f \in F} \delta va_{f,a} \cdot \left(fprd_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho va_a} \right)^{-\frac{1}{\rho va_a}}$ $\begin{bmatrix} quantity\ of\ aggregate \\ value\text{-}added \end{bmatrix} = CES \begin{bmatrix} factor \\ inputs \end{bmatrix}$	$a \in A$ $t \in T$	Value-added
(15)	$WF_{f,t} \cdot \overline{WFDIST}_{f,a,t} = PVA_{a,t} \cdot \left(1 - tva_{a,t}\right) \cdot QVA_{a,t}$ $\cdot \left(\sum_{f' \in F} \delta va_{f',a} \cdot \left(fprd_{f',a,t} \cdot QF_{f',a,t} \right)^{-\rho va_a} \right)^{-1} \cdot \delta va_{f,a} \cdot fprd_{f,a,t}^{-\rho va_a} \cdot QF_{f,a,t}^{-\rho va_a - 1}$ $\begin{bmatrix} marginal\ cost\ of \\ factor\ f\ in\ activity\ a \end{bmatrix} = \begin{bmatrix} marginal\ revenue\ product \\ of\ factor\ f\ in\ activity\ a \end{bmatrix}$	$a \in A$ $f \in F$ $t \in T$	Factor demand

(16)	$$QINT_{c,a,t} = ica_{c,a} \cdot QINTA_{a,t}$$ [intermediate demand for commodity c from activity a] = f [aggregate intermediate input quantity for activity a]	$c \in C$ $a \in A$ $t \in T$	Disaggregated intermediate input demand
(17)	$$QXAC_{a,c,t} + \sum_{h \in H} QHA_{a,c,h,t} = \theta_{a,c} \cdot QA_{a,t}$$ [quantity of output of commodity c from activity a] + [quantity consumed of home commodity c from activity a in all households] = [activity-specific marketed production of commodity c]	$a \in A$ $c \in C$ $t \in T$	Commodity production and allocation between market and home
(18)	$$QX_{c,t} = \alpha ac_c \cdot \left(\sum_{a \in A} \delta ac_{a,c} \cdot QXAC_{a,c,t}^{-\rho ac_c} \right)^{-\frac{1}{\rho ac_c}}$$ [aggregate marketed production of commodity c] = CES [output of commodity c from activity a]	$c \in$ $(CE \cup CD)$ $t \in T$	Output aggregation function
(19)	$$\frac{PXAC_{a,c,t}}{PX_{c,t}} = QX_{c,t} \cdot \sum_{a' \in A} \left(\delta ac_{a',c} \cdot QXAC_{a',c,t}^{-\rho ac_c} \right)^{-1} \cdot \delta ac_{a,c} \cdot QXAC_{a,c,t}^{-\rho ac_c - 1}$$ [ratio of price of commodity c from acitivty a to average output price] = f [aggregate marketed commodity output and output of commodity c from activity a]	$a \in A$ $c \in C$ $t \in T$	Ratio of prices for output aggregation function
(20)	$$QX_{c,t} = \alpha t_c \cdot \left(\delta t_c \cdot QE_{c,t}^{\rho t_c} + (1 - \delta t_c) \cdot QD_{c,t}^{\rho t_c} \right)^{\frac{1}{\rho t_c}}$$ [aggregate marketed domestic output] = CET [export quantity, domestic sales of domestic output]	$c \in (CD \cap$ $CECET)$ $t \in T$	Output transformation (CET) function
(21)	$$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}} \cdot \frac{1 - \delta t_c}{\delta t_c} \right)^{\frac{1}{\rho t_c - 1}}$$ [export-domestic supply ratio] = f [export-domestic price ratio]	$c \in (CD \cap$ $CECET)$ $t \in T$	Export-domestic supply ratio
(22)	$$QX_{c,t} = QD_{c,t} + QE_{c,t}$$ [aggregate marketed domestic output] = [domestic market sales of domestic output [for c ∈ (CD ∩ CEN)]] + [exports [for c ∈ (CE ∩ CDN)]]	$c \in$ $(CD \cap CEN) \cup$ $(CE \cap CDN) \cup$ $(CD \cap CECETN)$ $t \in T$	Output transformation for outputs without exports, exports without domestic sales, and non-CET exports with domestic sales

(23)	$QE_{c,t} = \overline{qe}_{c,t} \cdot \left(\frac{PWE_{c,t}}{pwse_{c,t}} \right)^{\rho_{e_c}}$ $\begin{bmatrix} \text{export} \\ \text{demand} \end{bmatrix} = f \begin{bmatrix} \text{trend export quantity, world price} \\ \text{for exports relative to world} \\ \text{price for export substitutes} \end{bmatrix}$	$c \in CED$ $t \in T$	Export demand with CE demand function
(24)	$QQ_{c,t} = \alpha q_c \cdot \left(\delta q_c \cdot QM_{c,t}^{-\rho q_c} + (1 - \delta q_c) \cdot QD_{c,t}^{-\rho q_c} \right)^{-\frac{1}{\rho q_c}}$ $\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = f \begin{bmatrix} \text{import quantity, domestic} \\ \text{use of domestic output} \end{bmatrix}$	$c \in$ $(CM \cap CD)$ $t \in T$	Composite supply (*Armington*) function
(25)	$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \cdot \frac{\delta q_c}{1 - \delta q_c} \right)^{\frac{1}{1+\rho q_c}}$ $\begin{bmatrix} \text{import-domestic} \\ \text{demand ratio} \end{bmatrix} = f \begin{bmatrix} \text{domestic-import} \\ \text{price ratio} \end{bmatrix}$	$c \in$ $(CM \cap CD)$ $t \in T$	Import-domestic demand ratio
(26)	$QQ_{c,t} = QD_{c,t} + QM_{c,t}$ $\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{domestic use of} \\ \text{marketed domestic} \\ \text{output [for} \\ c \in (CD \cap CMN)] \end{bmatrix} + \begin{bmatrix} \text{imports [for} \\ c \in (CM \cap CDN)] \end{bmatrix}$	$c \in$ $(CD \cap CMN)$ $\cup$ $(CM \cap CDN)$ $t \in T$	Composite supply for non-imported outputs and non-produced imports
(27)	$QT_{c,t} = \sum_{c' \in C'} \left(icm_{c,c',t} \cdot QM_{c',t} + ice_{c,c',t} \cdot QE_{c',t} + icd_{c,c',t} \cdot QD_{c',t} \right)$ $\begin{bmatrix} \text{trade and transport} \\ \text{demand for commodity } c \end{bmatrix} = \begin{bmatrix} \text{from imports} \end{bmatrix} + \begin{bmatrix} \text{from exports} \end{bmatrix} + \begin{bmatrix} \text{from marketed} \\ \text{domestic output} \end{bmatrix}$	$c \in CT$ $t \in T$	Demand for transaction services

Domestic institution block

(28)	$YF_{f,t} = \sum_{a \in A} WF_{f,t} \cdot \overline{WFDIST}_{f,a,t} \cdot QF_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$ $\begin{bmatrix} \text{income of} \\ \text{factor } f \end{bmatrix} = \begin{bmatrix} \text{sum of activity payments} \\ \text{(activity-specific wages} \\ \text{times employment levels)} \end{bmatrix} + \begin{bmatrix} \text{income to factor } f \\ \text{from Rest of World} \end{bmatrix}$	$f \in F$ $t \in T$	Factor income
(29)	$SHIF_{i,f,t} = \frac{QFACINS_{i,f,t}}{\sum_{i' \in INS} QFACINS_{i',f,t}}$ $\begin{bmatrix} \text{share of institution } i \text{ in} \\ \text{the income of factor } f \end{bmatrix} = \begin{bmatrix} \text{endowment of institution } i \text{ of factor } f \\ \text{divided by total endowment of factor } f \end{bmatrix}$	$i \in INS$ $f \in F$ $t \in T$	Institutional shares in factor incomes
(30)	$YIF_{i,f,t} = SHIF_{i,f,t} \cdot \left[\left(1 - tf_{f,t} \right) \cdot YF_{f,t} \right]$ $\begin{bmatrix} \text{income of} \\ \text{institution } i \\ \text{from factor } f \end{bmatrix} = \begin{bmatrix} \text{share of income} \\ \text{of factor } f \text{ to} \\ \text{institution } i \end{bmatrix} \cdot \begin{bmatrix} \text{income of factor } f \\ \text{(net of tax)} \end{bmatrix}$	$i \in INS$ $f \in F$ $t \in T$	Institutional factor incomes

(31)	$$YIINT_{i,t} = gintrat_{i,t} \cdot GDEBT_{i,t} - fintrat_{i,t} \cdot FDEBT_{i,t} \cdot EXR_t$$ $$\begin{bmatrix} net\ interest \\ income\ of \\ institution\ i \end{bmatrix} = \begin{bmatrix} interest\ earnings \\ on\ government \\ bonds \end{bmatrix} - \begin{bmatrix} interest \\ payments \\ on\ foreign\ debt \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Institutional net interest income
(32)	$$TRII_{i,i',t} = shii_{i,i'} \cdot (1 - MPS_{i',t}) \cdot (1 - TINS_{i',t}) \cdot YI_{i',t}$$ $$\begin{bmatrix} transfer\ from \\ institution\ i'\ to\ i \end{bmatrix} = \begin{bmatrix} share\ of\ net\ income \\ of\ institution\ i' \\ transfered\ to\ i \end{bmatrix} \cdot \begin{bmatrix} income\ of\ institution \\ i',\ net\ of\ savings\ and \\ direct\ taxes \end{bmatrix}$$	$i \in INS$ $i' \in INSDNG$ $t \in T$	Intra-institutional transfers
(33)	$$YI_{i,t} = \sum_{f \in F} YIF_{i,f,t} + \sum_{i' \in INSDNG'} TRII_{i,i',t} + YIINT_{i,t}$$ $$\begin{bmatrix} income\ of \\ institution\ i \end{bmatrix} = \begin{bmatrix} factor \\ income \end{bmatrix} + \begin{bmatrix} transfers\ from\ other \\ domestic\ non\text{-}government \\ institutions \end{bmatrix} + \begin{bmatrix} net \\ interest \\ income \end{bmatrix}$$ $$+ trnsfr_{i,gov,t} \cdot \overline{CPI}_t + trnsfrpc_{i,gov,t} \cdot POP_{i,t} \cdot \overline{CPI}_t$$ $$+ \begin{bmatrix} transfers\ from\ government \\ to\ non\text{-}household\ institutions \end{bmatrix} + \begin{bmatrix} transfers\ from \\ government\ to\ households \end{bmatrix}$$ $$+ trnsfr_{i,row,t} \cdot EXR_t + trnsfrpc_{i,row,t} \cdot POP_{i,t} \cdot EXR_t$$ $$+ \begin{bmatrix} transfers\ from\ Rest\ of\ World \\ to\ non\text{-}household\ institutions \end{bmatrix} + \begin{bmatrix} transfers\ from \\ Rest\ of\ World\ to\ households \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Income of domestic, non-government institutions
(34)	$$TINS_{i,t} = tinsbar_{i,t} \cdot \left(1 + \overline{TINSADJ}_t \cdot tins01_i\right) + DTINS_t \cdot tins01_i$$ $$\begin{bmatrix} direct\ tax \\ rate\ for \\ institution\ i \end{bmatrix} = \begin{bmatrix} exogenous\ rate\ adjusted \\ for\ scaling\ for \\ selected\ institutions \end{bmatrix} + \begin{bmatrix} point\ change \\ for\ selected \\ institutions \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Direct tax rates for domestic non-government institutions
(35)	$$MPS_{i,t} = mpsbar_{i,t} \cdot \left(\frac{\left(1 - TINS_{i,t}\right) \cdot YI_{i,t}}{\overline{POP}_{i,t}} \right)^{\rho_{sav_i} - 1} \cdot \left(1 + \overline{MPSADJ}_t \cdot mps01_i\right)$$ $$\begin{bmatrix} marginal \\ propensity \\ to\ save \end{bmatrix} = \begin{bmatrix} exogenous \\ term \end{bmatrix} \cdot \begin{bmatrix} adjustment\ for \\ per\text{-}capita \\ post\text{-}tax\ income \end{bmatrix} \cdot \begin{bmatrix} scaling\ adjustment \\ for\ selected \\ institutions \end{bmatrix}$$ $$+ \overline{DMPS}_t \cdot mps01_i$$ $$+ \begin{bmatrix} point\text{-}change \\ adjustment\ for \\ selected\ institutions \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Savings rates for domestic non-government institutions
(36)	$$INSSAV_{i,t} = MPS_{i,t} \cdot \left(1 - TINS_{i,t}\right) \cdot YI_{i,t}$$ $$\begin{bmatrix} savings\ for \\ institution\ i \end{bmatrix} = \begin{bmatrix} savings \\ rate\ for \\ institution\ i \end{bmatrix} \cdot \begin{bmatrix} income\ of \\ institution\ i \\ (net\ of\ direct\ taxes) \end{bmatrix}$$	$i \in INSDNG$	Savings for domestic non-government institutions

(37)	$$EH_{h,t} = \left(1 - \sum_{i \in INSDNG} shii_{i,h}\right) \cdot \left(1 - MPS_{h,t}\right) \cdot (1 - TINS_{h,t}) \cdot YI_{h,t}$$ $$\begin{bmatrix} \textit{household income} \\ \textit{disposable for} \\ \textit{consumption} \end{bmatrix} = \begin{bmatrix} \textit{household income, net of direct} \\ \textit{taxes, savings, and transfers to} \\ \textit{other non-government institutions} \end{bmatrix}$$	$h \in H$ $t \in T$	Household consumption expenditure
(38)	$$QH_{c,h,t} = \overline{POP}_{h,t} \cdot$$ $$\left(\gamma m_{c,h} + \frac{\beta m_{c,h} \cdot \left(\left[\dfrac{EH_{h,t}}{\overline{POP}_{h,t}} \right] - \sum_{c' \in C} PQ_{c',t} \cdot \gamma m_{c',h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a,c',t} \cdot \gamma h_{a,c',h} \right)}{PQ_{c,t}} \right)$$ $$\begin{bmatrix} \textit{quantity of} \\ \textit{household demand} \\ \textit{for commodity } c \end{bmatrix} = f \begin{bmatrix} \textit{household} \\ \textit{consumption} \\ \textit{spending, prices} \end{bmatrix}$$	$c \in C$ $h \in H$ $t \in T$	Household consumption demand for commodities from market
(39)	$$QHA_{a,c,h,t} = \overline{POP}_{h,t} \cdot$$ $$\left(\gamma h_{a,c,h} + \frac{\beta h_{a,c,h} \cdot \left(\left[\dfrac{EH_{h,t}}{\overline{POP}_{h,t}} \right] - \sum_{c' \in C} PQ_{c',t} \cdot \gamma m_{c',h} - \sum_{a' \in A} \sum_{c' \in C} PXAC_{a',c',t} \cdot \gamma h_{a',c',h} \right)}{PXAC_{a,c,t}} \right)$$ $$\begin{bmatrix} \textit{quantity of household demand} \\ \textit{for commodity } c \textit{ from activity } a \end{bmatrix} = f \begin{bmatrix} \textit{household consumption} \\ \textit{spending, prices} \end{bmatrix}$$	$a \in A$ $c \in C$ $h \in H$ $t \in T$	Household consumption demand for own production
(40)	$$YG_t = \sum_{i \in INSDNG} TINS_{i,t} \cdot YI_{i,t} + \sum_{f \in F} tf_{f,t} \cdot YF_{f,t} + \sum_{a \in A} ta_{a,t} \cdot PA_{a,t} \cdot QA_{a,t}$$ $$\begin{bmatrix} \textit{government} \\ \textit{revenue} \end{bmatrix} = \begin{bmatrix} \textit{direct taxes} \\ \textit{from institutions} \end{bmatrix} + \begin{bmatrix} \textit{direct taxes} \\ \textit{from factors} \end{bmatrix} + [\textit{activity tax}]$$ $$+ \sum_{a \in A} tva_{a,t} \cdot PVA_{a,t} \cdot QVA_{a,t} + \sum_{c \in CM} tm_{c,t} \cdot pwm_{c,t} \cdot QM_{c,t}$$ $$+ [\textit{value-added tax}] + [\textit{import tariffs}]$$ $$+ \sum_{c \in CE} te_{c,t} \cdot \overline{PWE}_{c,t} \cdot QE_{c,t} \cdot EXR_t + \sum_{c \in C} tq_{c,t} \cdot PQ_{c,t} \cdot QQ_{c,t}$$ $$+ [\textit{export taxes}] + [\textit{sales tax}]$$ $$+ \sum_{f \in F} YIF_{gov,f,t} + \sum_{i \in INSDNG} TRII_{gov,i,t} + trnsfr_{gov,row,t} \cdot EXR_t$$ $$+ [\textit{factor income}] + \begin{bmatrix} \textit{transfers from} \\ \textit{domestic institutions} \end{bmatrix} + [\textit{transfers from RoW}]$$	$t \in T$	Government recurrent revenue

(41)	$$EG_t = \sum_{c\in C} PQ_{c,t} \cdot QG_{c,t} + \sum_{i\in INSDNH} trnsfr_{i,gov,t} \cdot \overline{CPI}_t$$ $$\begin{bmatrix} government \\ spending \end{bmatrix} = \begin{bmatrix} government \\ consumption \end{bmatrix} + \begin{bmatrix} transfers\ to\ domestic \\ non\text{-}household\ institutions \end{bmatrix}$$ $$+\sum_{h\in H} trnsfrpc_{h,gov,t} \cdot \overline{POP}_{h,t} \cdot \overline{CPI}_t + trnsfr_{row,gov,t} \cdot EXR_t$$ $$+ \begin{bmatrix} transfers\ to\ domestic \\ households \end{bmatrix} + \begin{bmatrix} transfers\ to \\ Rest\ of\ World \end{bmatrix}$$ $$+\sum_{i\in INS} gintrat_{i,t} \cdot GDEBT_{i,t} + fintrat_{gov,t} \cdot FDEBT_{gov,t} \cdot EXR_t$$ $$+ \begin{bmatrix} interest\ payment \\ on\ domestic\ debt \end{bmatrix} + \begin{bmatrix} interest\ payment \\ on\ foreign\ debt \end{bmatrix}$$	$t \in T$	Government recurrent expenditures
(42)	$$QG_{c,t} = QG_{c,t-1}$$ $$\cdot\left(1 + \overline{QGGRW}_t + \sum_{c'\in C} qg01_{c,c',t} \cdot \overline{QGGRWC}_{c',t}\right)$$ $$\begin{bmatrix} real\ government \\ consumption \\ of\ c\ in\ t \end{bmatrix} = \begin{bmatrix} real\ government \\ consumption \\ of\ c\ in\ t-1 \end{bmatrix} \cdot \left[1 + \begin{bmatrix} adjustment\ for\ uniform \\ consumption\ growth, \\ e.g.\ absorption\ share \end{bmatrix} + \begin{bmatrix} adjustment\ for\ growth \\ specific\ to\ one\ or \\ more\ commodities \end{bmatrix}\right]$$	$c \in C$ $c \notin CINF$ $t \in T$ $t > 1$	Real government consumption (excluding infrastructure services)
(43)	$$QG_{c,t} = \sum_{\substack{i\in INS \\ f\in F}} igf_{c,f,t} \cdot QFINS_{i,f,t}$$ $$\begin{bmatrix} real\ government \\ consumption \\ of\ c\ in\ t \end{bmatrix} = \begin{bmatrix} quantity\ of\ gov\ consumption \\ per\ unit\ of\ gov\ infrastructure \\ capital\ stock\ f \end{bmatrix} \cdot \begin{bmatrix} real\ endowment\ of \\ factor\ f\ for \\ institution\ i \end{bmatrix}$$	$c \in CINF$ $t \in T$ $t > 1$	Real government consumption of infrastructure services
(44)	$$GSAV_t = YG_t - EG_t$$ $$\begin{bmatrix} government \\ savings \end{bmatrix} = \begin{bmatrix} government \\ recurrent\ revenue \end{bmatrix} - \begin{bmatrix} government \\ recurrent\ expenditures \end{bmatrix}$$	$t \in T$	Government savings
Investment block			

(45)	*(a)* $DKGOV_{f,t} \geq \sum_{a \in A \,\|(f,a) \in MFA} ifa_{f,a,t} \cdot QA_{a,t} \cdot EXP\left(\ln\left(\frac{QA_{a,t}}{QA_{a,t-1}}\right)\right)\Big\|_{f \in FCAPGOVSER}$ $\begin{bmatrix} \textit{government investment} \\ \textit{demand for capital f} \end{bmatrix} \geq \begin{bmatrix} \textit{demand for government service capital in t+1:} \\ \textit{capital coefficient times expected activity level in t+1} \end{bmatrix}$ $+\left(\left(1+\sum_{c \in C} qg01_{f,c,t} \cdot \overline{QGGRWC}_{c,t}\right) \cdot QFINS_{gov,f,t}\right)\Big\|_{f \in FCAPGOVINF}$ $+\begin{bmatrix} \textit{demand for government infrastructure capital in t + 1:} \\ \textit{growth rate times infra capital stock in t} \end{bmatrix}$ $-QFINS_{gov,f,t} \cdot (1-depr_{f,t})$ $-\begin{bmatrix} \textit{remaining capital stock (after dep-} \\ \textit{reciation) in t+1 if no investment in t} \end{bmatrix}$ .... *(b)* $DKGOV_{f,t} \geq 0$ $[\textit{government investment}] \geq [\textit{zero}]$ .... *(c)* $\left(DKGOV_{f,t} - DKGOVDEM_{f,t}\right) \cdot \left(DKGOV_{f,t} - 0\right) = 0$ where $DKGOVDEM_{f,t}$ = right-hand of part *(a)* of Equation 45 $\begin{bmatrix} \textit{Complementary slackness relationship:} \\ \textit{1. If government investment exceeds its demand then this investment level is zero.} \\ \textit{2. If the government investment level is above zero, then it equals its demand} \end{bmatrix}$	$f \in$ *FCAPGOV* $t \in T$ $t > 1$	Real government demand for investment in capital stock *f*
(46)	$DKINS_{gov,f,t} = DKGOV_{f,t}$ $\begin{bmatrix} \textit{gross investment in f of} \\ \textit{institution ins (here "ins" = gov)} \end{bmatrix} = \begin{bmatrix} \textit{gross government investment} \\ \textit{demand for capital} \end{bmatrix}$	$f \in$ *FCAPGOV* $t \in T$ $t > 1$	Real government investment in capital stock *f* (investment by destination)
(47)	$PK_{f,t} = \sum_{c \in C} capcomp_{c,f} \cdot PQ_{c,t}$ $\begin{bmatrix} \textit{price of new} \\ \textit{capital stock} \end{bmatrix} = \begin{bmatrix} \textit{total value of commodities c} \\ \textit{per unit of new capital} \end{bmatrix}$	$f \in FCAP$ $t \in T$	Price of new capital stock
(48)	$\sum_{f \in FCAPGOV} PK_{f,t} \cdot DKINS_{gov,f,t} = GSAV_t - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,gov,t} + \overline{GBORTOT}_t$ $\begin{bmatrix} \textit{government fixed} \\ \textit{investment value} \end{bmatrix} = \begin{bmatrix} \textit{government} \\ \textit{savings} \end{bmatrix} - \begin{bmatrix} \textit{spending on} \\ \textit{stock changes} \end{bmatrix} + \begin{bmatrix} \textit{total change in holdings} \\ \textit{of government bonds} \end{bmatrix}$ $+\overline{GBORMSTOT}_t + \left(\overline{FBOR}_{gov,t} + \overline{FGRANT}_{gov,t}\right) \cdot EXR_t$ $+\begin{bmatrix} \textit{Government Central Bank} \\ \textit{borrowing (deficit monetization)} \end{bmatrix} + \begin{bmatrix} \textit{foreign borrowing and} \\ \textit{foreign grants (in LCU)} \end{bmatrix}$	$t \in T$	Government investment value and financing

(49)	$GBOR_{i,t} = \frac{gbdist_i \cdot INSSAV_{i,t}}{\sum_{i' \in INSDNG'} gbdist_{i'} \cdot INSSAV_{i',t}} \cdot \overline{GBORTOT}_t$ $\begin{bmatrix} \textit{change in holdings of} \\ \textit{government bonds} \\ \textit{by institution i} \end{bmatrix} = \frac{\begin{bmatrix} \textit{savings by} \\ \textit{by institution i} \end{bmatrix}}{\begin{bmatrix} \textit{total institution} \\ \textit{savings value} \end{bmatrix}} \cdot \begin{bmatrix} \textit{(scaled) total change} \\ \textit{in holdings of} \\ \textit{government bonds} \end{bmatrix}$	$i \in INSDNG$ $t \in T$	Allocation of government bond borrowing across domestic non-government institutions
(50)	$GBORMS_{i,t} = \frac{gbdist_i \cdot INSSAV_{i,t}}{\sum_{i' \in INSDNG'} gbdist_{i'} \cdot INSSAV_{i',t}} \cdot \overline{GBORMSTOT}_t$ $\begin{bmatrix} \textit{Government Central Bank} \\ \textit{borrowing by institution i} \end{bmatrix} = \frac{\begin{bmatrix} \textit{savings by} \\ \textit{by institution i} \end{bmatrix}}{\begin{bmatrix} \textit{total institution} \\ \textit{savings value} \end{bmatrix}} \cdot \begin{bmatrix} \textit{(scaled) total Government} \\ \textit{Central Bank borrowing} \end{bmatrix}$	$i \in INSDNG$ $t \in T$	Allocation of the burden of Central Bank borrowing across domestic non-government institutions
(51)	$INVVAL_{i,t} = INSSAV_{i,t} - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,i,t} - GBOR_{i,t}$ $\begin{bmatrix} \textit{non-government fixed} \\ \textit{investment value} \end{bmatrix} = [\textit{savings}] - \begin{bmatrix} \textit{stock} \\ \textit{changes} \end{bmatrix} - \begin{bmatrix} \textit{change in holdings of} \\ \textit{government bonds} \end{bmatrix}$ $-GBORMS_{i,t} + \left(\overline{FBOR}_{i,t} + \overline{FGRANT}_{i,t} + fdi_{i,t}\right) \cdot EXR_t$ $-\begin{bmatrix} \textit{Government Central} \\ \textit{Bank borrowing} \end{bmatrix} + \begin{bmatrix} \textit{foreign borrowing, grants,} \\ \textit{and direct investment (in LCU)} \end{bmatrix}$	$i \in INSNG$ $t \in T$	Investment financing for non-government institutions
(52)	$PK_{f,t} \cdot DKINS_{i,f,t} = gfcfshr_{f,i,t} \cdot INVVAL_{i,t}$ $\begin{bmatrix} \textit{non-government spending} \\ \textit{on capital stock f} \end{bmatrix} = \begin{bmatrix} \textit{total fixed investment value} \\ \textit{times share for capital stock f} \end{bmatrix}$	$i \in INSNG$ $f \in FCAP$ $t \in T$	Non-government investment by capital stock (investment by destination)
(53)	$QINV_{c,t} = \sum_{f \in FCAP} \left(capcomp_{c,f} \cdot \sum_{i \in INS} DKINS_{i,f,t} \right)$ $\begin{bmatrix} \textit{real investment demand} \\ \textit{for commodity c} \end{bmatrix} = \begin{bmatrix} \textit{demand for c for each type of capital,} \\ \textit{summed over all institutions and capital types} \end{bmatrix}$	$c \in C$ $t \in T$	Total real investment demand by commodity (investment by origin or source)

Constraints for foreign exchange, factors, and commodities

(54)	$$\sum_{c\in CM} pwm_{c,t}\cdot QM_{c,t} + \frac{\sum_{f\in F} YIF_{row,f,t}}{EXR_t} + \frac{\sum_{i\in INSDNG} TRII_{row,i,t}}{EXR_t}$$ $$\begin{bmatrix} import \\ spending \end{bmatrix} + \begin{bmatrix} factor\ income \\ to\ Rest\ of\ World \end{bmatrix} + \begin{bmatrix} transfers\ from\ domestic \\ non\text{-}gov\ institutions\ to\ RoW \end{bmatrix}$$ $$+trnsfr_{row,gov,t} + \sum_{i\in INSD} fintrat_{i,t}\cdot FDEBT_{i,t}$$ $$+\begin{bmatrix} transfers\ from \\ government\ to\ RoW \end{bmatrix} + \begin{bmatrix} interest\ payment \\ on\ foreign\ debt \end{bmatrix}$$ $$= \sum_{c\in CE} \overline{PWE}_{c,t}\cdot QE_{c,t} + \sum_{i\in INSDNH} trnsfr_{i,row,t} + \sum_{h\in H} trnsfrpc_{h,row,t}\cdot \overline{POP}_{h,t}$$ $$= \begin{bmatrix} export \\ revenue \end{bmatrix} + \begin{bmatrix} transfers\ from\ RoW\ to\ domestic \\ non\text{-}household\ institutions \end{bmatrix} + \begin{bmatrix} transfers\ from\ RoW\ to \\ domestic\ households \end{bmatrix}$$ $$+\sum_{f\in F} trnsfr_{f,row,t} + \sum_{i\in INSD}\left(\overline{FBOR}_{i,t} + \overline{FGRANT}_{i,t}\right) + fdi_{row,t}$$ $$+\begin{bmatrix} factor\ income \\ from\ RoW \end{bmatrix} + \begin{bmatrix} borrowing \\ from\ RoW \end{bmatrix} + \begin{bmatrix} grants \\ from\ RoW \end{bmatrix} + \begin{bmatrix} foreign\ direct \\ investment \end{bmatrix}$$	$t\in T$	Balance of payments (in FCU)
(55)	$$\sum_{a\in A} QF_{f,a,t} = \left(1 - UERAT_{f,t}\right)\cdot \sum_{i\in INS} QFINS_{i,f,t}$$ $$\begin{bmatrix} demand\ \ for \\ market\ factor\ \ f \end{bmatrix} = \begin{bmatrix} 1\ \text{-}\ unemployment\ rate \\ (i.e.,\ employment\ rate) \end{bmatrix} \cdot \begin{bmatrix} sum\ of\ all\ institutional \\ endowments\ of\ factor\ f \end{bmatrix}$$	$f\in F$ $t\in T$	Factor markets
(56)	$$WFRES_{f,t} = WF_f^0 \cdot \left(\frac{QHPC_t}{QHPC^0}\right)^{\varphi_f^{wfqhpc}} \cdot \left(\frac{\left(1-UERAT_{f,t}\right)}{\left(1-UERAT_f^0\right)}\right)^{\varphi_f^{wferat}} \cdot \left(\frac{CPI_t}{CPI^0}\right)^{\varphi_f^{wfcpi}}$$ $$\begin{bmatrix} reservation\ wage \\ for\ factor\ f \\ in\ year\ t \end{bmatrix} = \begin{bmatrix} economy\text{-}wide\ wage \\ for\ factor\ f\ in \\ the\ base\ year \end{bmatrix} \cdot \begin{bmatrix} adjustment\ due\ to\text{: } per\text{-}capita\ household \\ consumption;\ employment\ rate;\ and \\ CPI\ (all\ relative\ to\ base\ year\ values) \end{bmatrix}$$	$f\in$ $FUEND$ $t\in T$	Reservation wage
(57)	(a) $$WF_{f,t} \geq WFRES_{f,t}$$ $$\begin{bmatrix} economy\text{-}wide \\ wage\ for\ fac\text{-} \\ tor\ f\ in\ year\ t \end{bmatrix} \geq \begin{bmatrix} reservation \\ wage\ for\ fac\text{-} \\ tor\ f\ in\ year\ t \end{bmatrix}$$ (b) $$UERAT_{f,t} \geq ueratmin_{f,t}$$ $$\begin{bmatrix} unemployment \\ rate\ for\ factor \\ f\ \ in\ year\ t \end{bmatrix} \geq \begin{bmatrix} minimum\ unem\text{-} \\ ployment\ \ rate\ for \\ factor\ f\ \ in\ year\ t \end{bmatrix}$$ .... (c) $$\left(WF_{f,t} - WFRES_{f,t}\right)\cdot\left(UERAT_{f,t} - ueratmin_{f,t}\right) = 0$$ [*Complementary slackness relationship:* *1. If wage exceeds reservation wage then unemployment rate is at its minimum.* *2. If unemployment rate excceds its minimum, then wage equals reservation wage*]	$f\in$ $FUEND$ $t\in T$	For factors with endogenous unemployment: (a) Wage and (b) unemployment constraints; and (c) related complementary-slackness relationship

(58)	$$QQ_{c\,t} = \sum_{a \in A} QINT_{c,a,t} + \sum_{h \in H} QH_{c,h,t} + QG_{c,t}$$ $$\begin{bmatrix} composite \\ supply \end{bmatrix} = \begin{bmatrix} intermediate \\ use \end{bmatrix} + \begin{bmatrix} household \\ consumption \end{bmatrix} + \begin{bmatrix} government \\ consumption \end{bmatrix}$$ $$+QINV_{c,t} + \sum_{i \in INS} qdst_{c,i,t} + QT_{c,t}$$ $$+\begin{bmatrix} fixed \\ investment \end{bmatrix} + \begin{bmatrix} stock \\ change \end{bmatrix} + \begin{bmatrix} trade\ and \\ transport \end{bmatrix}$$	$c \in C$ $t \in T$	Composite commodity markets

Asset stock updating and productivity block

(59)	$$QFINS_{i,f,t} = (1 - depr_{f,t-1}) \cdot QFINS_{i,f,t-1} + DKINS_{i,f,t-1} + qfinsadj_{i,f,t-1}$$ $$\begin{bmatrix} stock\ of\ capital \\ type\ f\ held \\ by\ institution\ i \end{bmatrix} = \begin{bmatrix} non\text{-}depreciated \\ capital\ stock \end{bmatrix} + \begin{bmatrix} fixed\ invest\text{-} \\ ment\ in\ t\text{-}1 \end{bmatrix} + \begin{bmatrix} exogenous\ adjustment \\ in\ capital\ stock \end{bmatrix}$$	$i \in INS$ $f \in FCAP$ $t \in T$ $t>1$	Capital stocks by institution
(60)	$$FDEBT_{i,t} = FDEBT_{i,t-1} + FBOR_{i,t-1}$$ $$+\left(fintratdue_{i,t-1} - fintrat_{i,t-1} \right) \cdot FDEBT_{i,t-1} - fdebtrelief_{i,t-1}$$ $$\begin{bmatrix} foreign \\ debt\ in\ t \end{bmatrix} = \begin{bmatrix} foreign \\ debt\ in\ t\text{-}1 \end{bmatrix} + \begin{bmatrix} foreign\ bor\text{-} \\ rowing\ in\ t\text{-}1 \end{bmatrix} + \begin{bmatrix} unpaid\ interest\ on \\ foreign\ debt\ in\ t\text{-}1 \end{bmatrix} - \begin{bmatrix} foreign\ debt \\ relief\ in\ t\text{-}1 \end{bmatrix}$$	$i \in INSD$ $t \in T$ $t > 1$	Foreign debt of domestic institutions
(61)	$$GDEBT_{i,t} = GDEBT_{i,t-1} + GBOR_{i,t-1}$$ $$\begin{bmatrix} stock\ of\ government \\ bond\ held\ by \\ institution\ i \end{bmatrix} = \begin{bmatrix} redistributed\ holdings\ of \\ stock\ of\ government\ bond \\ held\ by\ institution\ i\ in\ t\text{-}1 \end{bmatrix} + \begin{bmatrix} government \\ borrowing \\ from\ i\ in\ t\text{-}1 \end{bmatrix}$$	$i \in$ $INSDNG$ $t \in T$ $t > 1$	Government bond holdings of domestic institutions
(62)	$$GDPREAL_t = \sum_{c \in C} \sum_{h \in H} PQ^0_c \cdot QH_{c,h,t} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC^0_{a,c} \cdot QHA_{a,c,h,t}$$ $$\left[real\ GDP \right] = \begin{bmatrix} household\ market \\ consumption \end{bmatrix} + \begin{bmatrix} household\ own \\ production\ consumption \end{bmatrix}$$ $$+\sum_{c \in C} PQ^0_c \cdot QG_{c,t} + \sum_{c \in C} PQ^0_c \cdot QINV_{c,t} + \sum_{c \in C} \sum_{i \in INS} PQ^0_c \cdot qdst_{c,i,t}$$ $$+\begin{bmatrix} government \\ consumption \end{bmatrix} + \begin{bmatrix} fixed \\ investment \end{bmatrix} + \begin{bmatrix} stock \\ change \end{bmatrix}$$ $$+\sum_{c \in CE} EXR^0 \cdot PWE^0_c \cdot QE_{c,t} - \sum_{c \in CM} EXR^0 \cdot PWM^0_c \cdot QM_{c,t}$$ $$+\left[exports \right] - \left[imports \right]$$	$t \in T$	Real GDP at market prices
(63)	$$TRDGDP_t = \frac{\sum_{c \in CE} EXR^0 \cdot PWE^0_c \cdot QE_{c,t} + \sum_{c \in CM} EXR^0 \cdot PWM^0_c \cdot QM_{c,t}}{GDPREAL_t}$$ $$\begin{bmatrix} ratio\ of \\ trade\ to\ GDP \end{bmatrix} = \frac{\left[real\ trade \right]}{\left[real\ GDP \right]}$$	$t \in T$	Real Trade-GDP ratio

(64)	$$ALPHAVA_{a,t} = ALPHAVA2_{a,t} \cdot \prod_{f \in FCAP} \left[\frac{\sum_{i \in INS} QFINS_{i,f,t}}{\sum_{i \in INS} QFINS^{0}_{i,f}} \right]^{tfpelasqg_{a,f,t}} \cdot \left(\frac{\sum_{t' \in T} tfptrdwt_{t,t'} \cdot TRDGDP_{t'}}{TRDGDP^{0}} \right)^{tfpelastrd_{a}}$$ $$\begin{bmatrix} efficiency \\ term\ for \\ activity\ a \end{bmatrix} = \begin{bmatrix} trend \\ term\ for \\ activity\ a \end{bmatrix} \cdot \begin{bmatrix} product\ of: ratio\ of\ all \\ current\ real\ capital \\ endowment\ f\ to\ inital \\ value,\ raised \\ to\ the\ relevant\ elasticity \end{bmatrix} \cdot \begin{bmatrix} weighted\ avg.\ (over\ time) \\ of\ ratios\ of\ openness \\ to\ initial\ value,\ raised \\ to\ the\ relevant\ elasticity \end{bmatrix}$$	$a \in A$ $t \in T$ $t > 1$	Efficiency (TFP) by activity
(65)	$$ALPHAVA2_{a,t} = ALPHAVA2_{a,t-1} \cdot \left(1 + \alpha vag_{a,t} + \overline{CALTFPG}_{t} \cdot tfp01_{a,t}\right)$$ $$\begin{bmatrix} trend\ term\ for \\ activity\ a\ in\ t \end{bmatrix} = \begin{bmatrix} trend\ term\ for \\ activity\ a\ in\ t\text{-}1 \end{bmatrix} \cdot \begin{bmatrix} growth\ adjust\text{-} \\ ment\ factor \end{bmatrix}$$	$a \in A$ $t \in T$ $t > 1$	TFP trend term by activity
(66)	$$GDPREALFC_{t} = \sum_{a \in A} PVA^{0}_{a} \cdot \left(1 - tva^{0}_{a,t}\right) \cdot QVA_{a,t}$$ $$\begin{bmatrix} real\ GDP \\ at\ factor\ cost \end{bmatrix} = \begin{bmatrix} value\text{-}added \\ net\ of\ taxes \end{bmatrix}$$	$t \in T$	Real GDP at factor cost

Table A3.3 Notation for MDG module of MAMS model

SETS			
Symbol	Explanation	Symbol	Explanation
$a \in A$	activities	$i \in INSG$	government institution
$b \in B$	student behavioural characteristics ={*rep* = repeater; *dropout* = dropout; *pass* = pass; *grdcont* = continuing graduate; *grdexit* = exiting graduate; *g1entry* = entrant to grade 1; *grdcyc*= pass from last cycle-year; *contcyc* = pass within cycle}	$i \in INSNGAGG$	aggregate (domestic) non-government institution
$b \in BLOG$ $(\subset B)$	student behaviour determined by logistic function ={*pass, grdcont, g1entry*}	$b, b' \in MBB$	mapping between *b* (in *BRES*) and *b'* (in *BLOG*): ={(*rep, dropout).grd, grdexit.grdcont*}
$b \in BRES$ $(\subset B)$	student behaviour determined by residual scaling ={*rep* = repeater; *dropout* = dropout; *grdexit* = exiting graduate}	$b, b' \in MBB2$	mapping between *b* (in *BRES*) and all elements *b'* (also in *BRES)* that are related to the same element(s) in *BLOG*: ={*rep.*(*rep, dropout*)*, dropout.*(*rep, dropout*)*, grdexit.grdexit*}
$c \in C$	commodities	$c, c' \in MCE$	mapping private and public education into 1 education commodity, by cycle = {*c-edup*.(*c-edup*, *c-edupng*)} where *c-edupng* is private primary; similarly for *c-edus* and *c-edut*
$c \in CEDU$ $(\subset C)$	education services ={*c-edup* = primary; *c-edus* = secondary; *c-edut* = tertiary}; can include both private and public education	$c, c' \in MCHDC$	human development service *c* is aggregated to *c'*
$c \in CEDUT$ $(\subset C)$	tertiary education services = {*c-edut*}	$c, c' \in MCM$	mapping between aggregate (CMDG) and disaggregated MDG service commodities (CHLTH and CWTSN) = {*c-hlt*.(*c-hlt1g*, *c-hlt2g*, *c-hlt3g*, *c-hlt1ng*, *c-hlt2ng*, *c-hlt3ng*} and {*c-wtsn*.(*c-wtsn*)}
$c \in CELA$	educational cycle that corresponds to the age at which non-students would enter the labour force	$mdg \in MDG$	selected MDG indicators ={*mdg2, mdg4, mdg5, mdg7a, mdg7b*}
$c \in CHLTH$ $(\subset C)$	health services (public) ={*c-hlt1g* = low-tech; *c-hlt2g* = medium-tech; *c-hlt3g* = high-tech}; corresponding private health services labelled with "*ng*"	$mcyc(c, b, t', t)$	MDG2 in *t* is defined as the product over selected combinations of *b* and *t'* (where $t' \in T11$) = {*pass, g1entry*}
$cmdg \in CMDG$	aggregate MDG (non-education) service commodities = {*c-hlt* = aggregate health in MDG functions, not in C; *c-wtsn* = water-sanitation services}	$mdg \in MDGSTD$	MDG indicators ={*mdg4* = under-5 mortality rate; *mdg5* = maternal mortality rate; *mdg7a* = access to safe water; *mdg7b* = access to basic sanitation}

$c \in CWTSN$ $(\subset C)$	water-sanitation service commodities {*c-wtsn* = water-sanitation services}	$f, c \in MFC$	mapping indicating that students who have completed cycle *c* belong to labour type *f* ={*f-labn.(c-edup); f-labs.(c-edus); f-labt.(c-edut)*}
$eduarg \in EDUARG$	arguments in CE function for educational behaviour ={*edu-qual* = quantity of services per student; *w-prem* = semiskilled-unskilled wage ratio; *w-prem2* = skilled-semiskilled wage ratio; *mdg4* = under-five mortality rate; *fcapinf* = infrastructure capital stocks; *qhpc* = per-capita hhd consumption}	$mdgarg \in MDGARG$	arguments in CE function for MDGs ={*cmdg* = agg commodities; *mdg* = different MDGs; *fcapinf* = infrastructure capital stocks; *hhdconspc* = per-capita hhd consumption }
$f \in FEXOG$	factors with exogenous growth	$t \in T$	time periods
$f \in FLAB$	labour factors {*f-labn* = less than completed secondary education; *f-labs* = complete secondary education (without completed tertiary); *f-labt* = completed tertiary education	$t \in T11$	time periods including preceding years for MDG2 calculation
$h \in H$	households (excl. NGOs) ={*h* = the single household}		

PARAMETERS			
$\alpha edu_{b,c}$	constant in logistic function for educational behaviour	$extmdg_{mdg}$	maximum value for MDG 7a and 7b; minimum value for MDG 4 and 5
$\alpha educe_{b,c}$	constant in CE function for educational behaviour	$grdcont01_{c,c'}$	0-1 constant showing that for *c'* next cycle is *c*
αmdg_{mdg}	constant in logistic function for MDG achievement	ord_t	ordinal position of *t* in the set *T*
αmce_{mdg}	constant in CE function for intermediate MDG variable	$popg1_t$	population in age cohort entering grade 1
αhd_c	efficiency term in CES aggregation function for human development	$poplab_t$	population of labour force age
$\beta edu_{b,c}$	constant in logistic function for educational behaviour	$poplabent_t$	population in age cohort entering labour force (age at end of a model education cycle)
βlog_{mdg}	constant in logistic function for MDG achievement	$poptot_t$	total population in *t*
$\delta hd_{c,i}$	share parameter for HD CES function	$qg1entncoh_{c,t}$	number of non-cohort (non-1st-year-primary) entrants to first cycle
$\varphi edu_{b,c,eduarg}$	elasticity of behaviour *b* in cycle *c* with respect to argument *eduarg* in educational CE function	$shif^0_{i,f,t}$	share of domestic institution *i* in income of factor *f*
$\varphi m_{mdg,mdgarg}$	elasticity of *mdg* with respect to argument *mdgarg* in CE function for MDG	$shrdemot01_{c,c'}$	0-1 parameter showing that for dropouts from *c'* the highest cycle is *c*
$\gamma edu_{b,c}$	parameter in logistic function for education	$shred^0_{b,c}$	base-year share for behavioural indicator behav in cycle *c*
γmdg_{mdg}	parameter in logistic function for non-education MDGs	$shrgrdcyc_c$	share of graduates (passing students) graduating from cycle c in base-year
ρhd_c	exponent in CES aggregation function for human development	$shrlabent_{c,t}$	share of drop-outs and leavers in cycle *c* that enter the labour force

$depr_{f,t}$	depreciation rate for factor *f*	$shrlabent2_{f,t}$	share of labour type *f* of labour force entrants without education
$discrat$	discount rate	$yrcyc_{c}$	years in school cycle for each education cycle *c*
$extedu_{b,c}$	maximum share for educational behaviour *b* in cycle *c*		

VARIABLES			
$EDUQUAL_{c,t}$	educational quality in cycle *c* in year *t*	$QH_{c,h,t}$	consumption of commodity *c* in *t* by household *h*
EG_{t}	government expenditures	$QHA_{a,c,h,t}$	quantity consumed of home commodity *c* from activity *a* by household *h*
$INVVAL_{i,t}$	investment value for institution *i*	$QHPC_{t}$	Per-capita household consumption in *t*
$MDGVAL_{mdg,t}$	value for MDG indicator *mdg* in *t*	$QQ_{c,t}$	quantity of goods supplied to domestic market (composite supply)
$PQ_{c,t}$	price of commodity *c* in *t*	$QXHLTH_{mdg,t}$	government and NGO provision of aggregated health services related to health MDG
$PXAC_{a,c,t}$	price of commodity *c* from activity *a*	$SHREDU_{b,c,t}$	share of students in cycle *c* with behaviour *b* in *t*
$QENR_{c,t}$	total number of students enrolled in cycle *c* in year *t*	$WF_{f,t}$	economy-wide wage for factor *f* in *t*
$QENROLD_{c,t}$	number of old students enrolled in cycle *c* in year *t*	$ZEDU_{b,c,t}$	intermediate variable for educational outcome (defined by CE function; entering logistic function)
$QENRNEW_{c,t}$	number of new students enrolled in cycle *c* in year *t*	$ZMDG_{mdg,t}$	intermediate variable for standard MDGs (4-5-7a-7b) (defined by CE function; entering logistic function)
$QFACINS_{i,f,t}$	endowment of labour type *f* for institution *i* in *t*		

Table A3.4 Equations for MDG module of MAMS model

#	Equation	Domain	Description
(67)	$QHD_{c,i,t} = \sum_{\substack{c'\in C \\ \vert (c,c')\in MCHDC \\ \cup \vert i\in INSG}} QG_{c',t} + \sum_{\substack{c'\in C \\ \vert (c,c')\in MCHDC \\ \cup \vert i\in INSNGAGG}} \left(QQ_{c',t} - QG_{c',t}\right)$ $\begin{bmatrix} \text{demand for HD (MDG or educ)} \\ \text{service } c \text{ by aggregate demander } i \end{bmatrix} = \begin{bmatrix} \text{sum of gov and non-gov} \\ \text{demand for HD service} \end{bmatrix}$	$c \in C$ $i \in I$ $t \in T$	Separation of human development (HD services into government and non-government
(68)	$QHDAGG_{c,t} = \alpha hd_c \cdot \sum_{i\in INS} \left(\delta hd_{c,i} \cdot QHD_{c,i,t}^{-\rho hd_c}\right)^{-\frac{1}{\rho hd_c}} \Bigg\vert_{c\in CHDCES}$ $+ \sum_{i\in INS} QHD_{c,i,t} \Bigg\vert_{c\in CHDPRFSUB}$ $\begin{bmatrix} \text{aggregate demand for HD} \\ \text{(MDG or educ) service ac} \end{bmatrix} = \begin{bmatrix} \text{aggregation of HD demand as imperfect substit-} \\ \text{utes (CES) or as perfect substitutes (summed)} \end{bmatrix}$	$c \in C$ $i \in I$ $t \in T$	Aggregation of human development (HD) services (i.e., MDG and education)
(69)	$QHPC_t = \frac{\sum_{c\in C}\sum_{h\in H} PQ_c^0 \cdot QH_{c,h,t} + \sum_{a\in A}\sum_{c\in C}\sum_{h\in H} PXAC_{a,c}^0 \cdot QHA_{a,c,h,t}}{poptot_t}$ $\begin{bmatrix} \text{real household} \\ \text{conumption per capita} \end{bmatrix} = \begin{bmatrix} \text{total household consumption at base -} \\ \text{year prices divided by total population} \end{bmatrix}$	$t \in T$	Real household consumption per capita
(70)	$EDUQUAL_{c,t} = \frac{QHDAGG_{c,t}}{QENR_{c,t}} \Big/ \frac{QHDAGG_c^0}{QENR_c^0}$ $\begin{bmatrix} \text{educational quality} \\ \text{in cycle } c \text{ in year } t \end{bmatrix} = \begin{bmatrix} \text{real services per student} \\ \text{in cycle } c \text{ in t} \end{bmatrix} \div \begin{bmatrix} \text{real services per student} \\ \text{in cycle } c \text{ in base-year} \end{bmatrix}$	$c \in CEDU$ $t \in T$ $t > 1$	Educational quality
(71)	$QENROLD_{c,t} = SHREDU_{contcyc,c,t\text{-}1} \cdot QENR_{c,t-1} + SHREDU_{rep,c,t\text{-}1} \cdot QENR_{c,t-1}$ $\begin{bmatrix} \text{number old students} \\ \text{enrolled in cycle } c \text{ in } t \end{bmatrix} = \begin{bmatrix} \text{enrolled in cycle } c \text{ in } t - 1 \\ \text{who continue in } c \end{bmatrix} + \begin{bmatrix} \text{enrolled in } c \text{ in} \\ t - 1 \text{ who repeated } c \end{bmatrix}$	$c \in CEDU$ $t \in T$ $t > 1$	Enrolment old students
(72)	$QENRNEW_{c,t} = SHREDU_{g1entry,c,t\text{-}1} \cdot popg1_t + qg1entncoh_{c,t}$ $+ \sum_{c'\in C} grdcont01_{c,c'} \cdot SHREDU_{grdcont,c,t\text{-}1} \cdot SHREDU_{grdcyc,c',t\text{-}1} \cdot QENR_{c',t-1}$ $\begin{bmatrix} \text{number new students} \\ \text{enrolled in cycle } c \text{ in } t \end{bmatrix} = \begin{bmatrix} \text{(cohort) students entering} \\ \text{cycle } c \ (c = primary) \end{bmatrix}$ $+ \begin{bmatrix} \text{(non - cohort) students entering} \\ c \text{ from outside school system} \end{bmatrix} + \begin{bmatrix} \text{enrolled in preceding cycle } c' \text{ in} \\ t - 1 \text{ who graduated and entered } c \end{bmatrix}$ +	$c \in CEDU$ $t \in T$ $t > 1$	Enrolment new students
(73)	$QENR_{c,t} = QENROLD_{c,t} + QENRNEW_{c,t}$ $\begin{bmatrix} \text{total number enrolled} \\ \text{in cycle } c \text{ in } t \end{bmatrix} = \begin{bmatrix} \text{enrolled old students} \\ \text{in cycle } c \text{ in } t \end{bmatrix} + \begin{bmatrix} \text{enrolled new students} \\ \text{in cycle } c \text{ in } t \end{bmatrix}$	$c \in CEDU$ $t \in T$ $t > 1$	Total Enrolment

(74)	$$SHREDU_{b,c,t} = exted_{b,c} + \frac{\alpha edu_{b,c}}{1 + EXP\left(\gamma edu_{b,c} + \beta edu_{b,c} \cdot ZEDU_{b,c,t}\right)}$$ $$\begin{bmatrix} \text{student share with} \\ \text{behavior } b \text{ in cycle } c \end{bmatrix} = \begin{bmatrix} \text{logistic function of intermediate} \\ \text{behavior variable} \left(ZEDU_{b,c,t}\right) \end{bmatrix}$$	$b \in BLOG$ $c \in CEDU$ $t \in T$	Student behaviour (logistic function)[40]
(75)	$$ZEDU_{b,c,t} = \alpha educe_{b,c} \cdot \left(EDUQUAL_{c,t}\right)^{\varphi edu_{b,c,edu-qual}} \cdot \left(\frac{WF_{f-labs,t}}{WF_{f-labn,t}}\right)^{\varphi edu_{b,c,w-prem}} \cdot \left(\frac{WF_{f-labt,t}}{WF_{f-labs,t}}\right)^{\varphi edu_{b,c,w-prem}} \cdot MDGVAL_{mdg4,t}^{\varphi edu_{b,c,mdg4}} \cdot \prod_{f \in FCAPGOVINF} \left(\sum_{i \in INS} QFINS_{i,f,t}\right)^{\varphi edu_{b,c,f}} \cdot QHPC_t^{\varphi edu_{b,c,qhpc}}$$ $$\begin{bmatrix} \text{intermediate variable for student} \\ \text{share with behavior } b \text{ in cycle } c \end{bmatrix} = \begin{bmatrix} \text{exogenous} \\ \text{trend value} \end{bmatrix} \cdot \begin{bmatrix} \text{influence of: education quality (service per student);} \\ \text{wage premia (for } c \leq \text{secondary and } c \geq \text{tertiary, resp.);} \\ \text{student health (proxied by MDG4); level of infra-} \\ \text{structure; and per-capita household consumption} \end{bmatrix}$$	$b \in BLOG$ $c \in C$ $t \in T$	Student behaviour (CE function defining intermediate variable)[41]
(76)	$$SHREDU_{b,c,t} = \left(1 - \sum_{\substack{b' \in BLOG \\ \mid (b,b') \in MBB}} SHREDU_{b',c,t}\right) \frac{SHREDU_{b,c}^{0}}{\sum_{\substack{b' \in BRES \\ \mid (b,b') \in MBB2}} SHREDU_{b',c}^{0}}$$ $$\begin{bmatrix} \text{student share} \\ \text{with behavior} \\ b \text{ in cycle } c \end{bmatrix} = \begin{bmatrix} \text{residual value (1 less sum} \\ \text{of shares for related} \\ \text{elements in BLOG)} \end{bmatrix} \cdot \begin{bmatrix} \text{initial share of } b \text{ in} \\ \text{total shares for related} \\ \text{residual elements} \end{bmatrix}$$	$b \in BRES$ $c \in CEDU$ $t \in T$	Student behaviour (defined residually, given left-hand side of the logistic function for education).
(77)	$$SHREDU_{grdcyc,c,t} = \frac{SHREDU_{pass,c,t}}{yrcyc_c} \cdot \left(\frac{shrgrdcyc_c}{\frac{1}{yrcyc_c}}\right)^{\frac{1-SHREDU_{pass,c,t}}{1-SHREDU_{pass,c}^{0}}}$$ $$\begin{bmatrix} \text{student share that} \\ \text{graduates from} \\ \text{cycle } c \text{ in year } t \end{bmatrix} = \frac{\begin{bmatrix} \text{student share that passes} \\ \text{each grade within cycle } c \end{bmatrix}}{\begin{bmatrix} \text{number of years} \\ \text{in cycle } c \end{bmatrix}} \cdot \begin{bmatrix} \text{adjustment term : ratio between base - year} \\ \text{share of cycle graduates in total graduates} \\ \text{OVER the share of the last year in total} \\ \text{number of years in cycle } c \end{bmatrix}$$	$c \in CEDU$ $t \in T$	graduation rate by cycle (ratio cycle graduates over enrolment)

[40] The α and β parameters in the logistic functions (equations 74 and 82) have been calibrated so that (i) under base-year conditions, the left-hand side variables (showing student behaviour shares or MDG values) will replicate base-year values; and (ii) under conditions derived from supporting studies of health and education, the left-hand side variables will take on values indicative of or compatible with MDG achievement.

[41] In the computer program, equations 75 and 83 (constant-elasticity functions defining intermediate variables for educational behaviour or MDG achievement) are more complex in two respects. First, the terms that are raised to exponents, which represent elasticities, are all divided by base-year values. This formulation was preferred given our desire to simulate scenarios with changes in elasticities but without any changes in simulated base-year values for left-hand-side variables. Second, for the element *grdcont* ∈ BLOG, the decision to continue to the next education cycle depends on the values for the right-hand side variables that correspond to the next cycle.

(78)	$SHREDU_{contcyc,c,t} = SHREDU_{pass,c,t} - SHREDU_{grdcyc,c,t}$ $\begin{bmatrix} \textit{student share that} \\ \textit{continues in cycle} \\ \textit{c in year t} \end{bmatrix} = \begin{bmatrix} \textit{student share that} \\ \textit{passes each grade} \\ \textit{within cycle c} \end{bmatrix} - \begin{bmatrix} \textit{student share that} \\ \textit{graduates from} \\ \textit{cycle c in year t} \end{bmatrix}$	$c \in CEDU$ $t \in T$	continuation rate by cycle
(79)	$MDGVAL_{mdg2,t} = \prod_{\substack{b \in B, t' \in T11 \\ \mid mcyc(c\text{-}edup1,b,t',t)}} SHREDU_{b,c\text{-}edup1,t'}$ $\begin{bmatrix} \textit{first cycle primary school} \\ \textit{net completion rate} \end{bmatrix} = \begin{bmatrix} \textit{product of student shares (g1entry} \\ \textit{and pass) for first cycle primary} \end{bmatrix}$	$t \in T$	MDG 2
(80)	$LABPARTRAT_t = \frac{\sum_{\substack{i \in INS, f \in FLAB \\ \mid shif^0_{i,f,t}}} QFINS_{i,f,t}}{poplab_t - \sum_{c \in CELA} QENR_{c,t}}$ $\begin{bmatrix} \textit{labor force} \\ \textit{participation rate} \end{bmatrix} = \frac{[\textit{labor force}]}{[\textit{population in labor force age - enrollment in secondary and terciary}]}$	$t \in T$ $t > 1$ $flab \notin$ $FEXOG$	Labour Force Participation Rate
(81)	$QFINS_{i,f,t} = shif^0_{i,f,t}$ $\begin{bmatrix} \textit{endowment of labor type} \\ \textit{f for institution i in t} \end{bmatrix} = \begin{bmatrix} \textit{share of i in} \\ \textit{labor type f} \end{bmatrix}$ $\cdot \left\{ \left(1 - depr_{f,t-1}\right) \cdot \sum_{i' \in INS} QFINS_{i',f,t-1} \right.$ $\cdot \{ \ [\textit{non - retired labor from previous year}]$ $+ \sum_{\substack{c,c' \in C \\ \mid (f,c) \in MFC \\ \cap c \notin CEDUT}} shrdemot01_{c,c'} \cdot shrlabent_{c,t} \cdot SHREDU_{grdexit,c',t-1} \cdot SHREDU_{grdcyc,c,t-1} \cdot QENR_{c,t-1}$ $+ \begin{bmatrix} \textit{enrolled in non-tertiary cycle in t-1, who graduate,} \\ \textit{exit the school system, and enter labor force in t} \end{bmatrix}$ $+ \sum_{\substack{c \in C \\ \mid (f,c) \in MFC \\ \cap c \in CEDUT}} \left(shrlabent_{c,t} \cdot SHREDU_{grdcyc,c,t-1} \cdot QENR_{c,t-1} \right)$ $+ [\textit{enrolled in tertiary cycle in t - 1, who graduate and enter the labor force in t}]$ $+ \sum_{\substack{c \in C \\ \mid (f,c) \in MFC \\ \cap c \in CEDUT}} \left(shrlabent_{c,t} \cdot SHREDU_{\text{"}grdcyc\text{"},c,t-1} \cdot QENR_{c,t-1} \right)$ $+ [\textit{enrolled in school in t-1, who dropout + enter labor force in t at next lower level c}]$ $\left. + shrlabent2_{f,t} \cdot \left(poplabent_t - \sum_{c \in CELA} QENRNEW_{c,t} \right) \right\}$ $+ [\textit{entrants from outside educational system who are of labor-force-age}] \}$	$i \in INS$ $f \in FLAB$ $t \in T$ $t > 1$	Labour supply

(82)	$$MDGVAL_{mdg,t} = extmdg_{mdg} + \frac{\alpha mdg_{mdg}}{1 + EXP\left(\gamma mdg_{mdg} + \beta mdg_{mdg} \cdot ZMDG_{mdg,t}\right)}$$ $$\begin{bmatrix} MDG \\ value \end{bmatrix} = \begin{bmatrix} logistic\ function\ of\ intermediate \\ MDG\ value\left(ZMDG_{mdg,t}\right) \end{bmatrix}$$	$mdg \in MDGSTD$ $t \in T$	MDGs 4, 5, 7a, and 7b (logistic function)
(83)	$$ZMDG_{mdg,t} = \alpha mce_{mdg} \cdot \left(\prod_{cmdg \in CMDG} \left(\sum_{\substack{c \in C \\ \mid (cmdg,c) \in MCM}} \frac{QQ_{c,t}}{poptot_t} \right)^{\varphi m_{mdg,cmdg}} \right) \cdot \prod_{f \in FCAPGOVINF} \left(\sum_{i \in INS} QFINS_{i,f,t} \right)^{\varphi m_{mdg,f}} \cdot \left(\prod_{mdg' \in MDGSTD} MDGVAL_{mdg',t}^{\varphi m_{mdg,mdg'}} \right) \cdot QHPC_t^{\varphi m_{mdg,hhdconspc}}$$ $$\begin{bmatrix} intermediate\ variable \\ for\ MDGs\ 4\ and\ 5 \end{bmatrix} = \begin{bmatrix} exogenous \\ parameter \end{bmatrix} \cdot \begin{bmatrix} influence\ of: real\ value\ for\ services\ per\ capita; \\ level\ of\ infrastructure;\ water\ and\ sanitation\ MDGs; \\ household\ consumption\ per\ capita \end{bmatrix}$$	$mdg \in MDGSTD$ $t \in T$	MDGs 4, 5, 7a, and 7b (CE function defining intermediate variable)